HW6

July 2019

1 Q1

SVD-decomposition of $D^T D$

$$D^T D = \sum_{i=1}^{4} \sigma_i^2 u_i u_j^T = \sigma_{max}^2 u_0 u_0^T + ... + \sigma_{min}^2 u_4 u_4^T \tag{1}$$

where u_i, u_j are orthogonal.

Assume y' has basis of $[u_1, u_2, u_3, u_4]$. Then y' can be represented as

$$y' = \sum_{i=1}^{4} k_i u_i = u_4 + v \tag{2}$$

where k_i is a constant coefficient, and v is orthogonal to u_4. v can be represented as

$$v = \sum_{i=1}^{3} k_i u_i \tag{3}$$

Then,

$$||Dy'||_2^2 = y'^T D^T D y' = \sigma_{max}^2 k_0^2 u_0^T u_0 u_0^T u_0 + ... + \sigma_{min}^2 k_4^2 u_4^T u_4 u_4^T u_4 = \sigma_{max}^2 k_0^2 + ... + \sigma_{min}^2 k_4^2 \tag{4}$$

As we know, $||y|| = 1$ and $\sum_i^4 k_i^2 = 1$, then when $k_4 = 1$ the Equation 4 observes the minimum. $(\sigma_4 << \sigma_3)$

If $y = u_4$ is the optimum, then

$$||Dy||_2^2 = y^T D^T D y = \sigma_{min}^2 k_4^2 u_4^T u_4 u_4^T u_4 = \sigma_{min}^2 k_4^2 \tag{5}$$

That is,

$$y'^T D^T D y' \geq y^T D^T D y$$

2 Q2

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21:17:45: Starting /home/meng7/HW6/build/estimate_depth...
ground truth:
  -2.9477 -0.330799   8.43792
your result:
  -2.9477 -0.330799   8.43792
```

Figure 1: Result of Q2

As we have the transformation matrix in the k_{th} frame $P_{cw,k} \in R^{3X4}$,

$$\forall k, z_k x_k = P_k y \tag{6}$$

where z_k is the depth information of the observation, x_k is the observation in the $2D$ normalized plane in the k_{th} frame, and y is the unknown $3D$ point in the world coordinate.

The depth information can be represented as

$$z_k = P_{k,3}^T y \tag{7}$$

where $P_{k,i}$ represents the i_{th} row in the transformation matrix of the k_{th} frame.

Then, we have,

$$Dy = \begin{bmatrix} u_1 P_{1,3}^T - P_{1,1}^T \\ v_1 P_{1,3}^T - P_{1,2}^T \\ \vdots \\ u_n P_{n,3}^T - P_{n,1}^T \\ v_n P_{n,3}^T - P_{n,2}^T \end{bmatrix} y = 0 \tag{8}$$

Then, use the method mentioned in $Q1$ to solve Equation 8.