HW4

July 2019

1 Q1 Information matrix

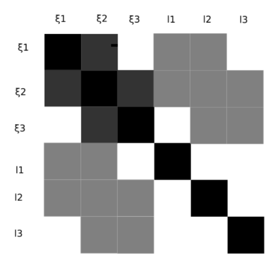

Figure 1: Information matrix before marginalization

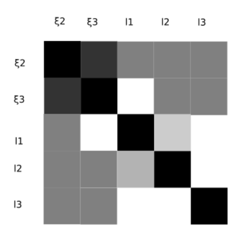

Figure 2: Information Matrix after marginalization

2 Q2 Bundle Adjustment

The reprojection error can be represented as:

$$e = z - h(\xi, p) \tag{1}$$

where z is observation data, h is the observation function, ξ is the camera pose in the form of lie algebra, and p is the 3-dimensional point in the world coordinate.

The the cost function can be written as

$$\frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{n}||e_{ij}||^2 \tag{2}$$

where i represents the index of the camera and j represents the index of the landmark.

For the problem of bundle adjustment, the variable x is defined as

$$x = [\xi_1, ..., \xi_m, p_1, ..., p_m]$$

Then the cost function can be represented using Tylor Expansion:

$$\frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{n}||e_{ij} + F_{ij}\delta\xi_i + E_{ij}\delta p_j||^2 \tag{3}$$

where F_{ij} represents the Jacobian with respect to the camera pose, and E_{ij} represents the Jacobian with respect to the landmarks.

The Jacobian for the re-projection error with respect to the point in the current camera coordinate:

$$\frac{\partial e}{\partial P_c} = \begin{bmatrix} \frac{f_x}{Z} & 0 & -\frac{f_x X}{Z^2} \\ 0 & f_y & -\frac{f_y Y}{Z^2} \end{bmatrix} \tag{4}$$

Then, the Jacobian for the re-projection error with respect to the point in the world coordinate:

$$\frac{\partial e}{\partial P_w} = \begin{bmatrix} \frac{f_x}{Z} & 0 & -\frac{f_x X}{Z^2} \\ 0 & f_y & -\frac{f_y Y}{Z^2} \end{bmatrix} R \tag{5}$$

where R is the rotation matrix for the transformation from the world coordinate to the camera coordinate.

The Jacobian for the re-projection error with respect to the camera pose (in the form of Lie algebra):

$$\frac{\partial e}{\partial \delta\xi} = \begin{bmatrix} -\frac{f_x XY}{Z^2} & f_x + \frac{f_x X^2}{Z^2} & -\frac{f_x Y}{Z} & \frac{f_x}{Z} & 0 & -\frac{f_x X}{Z^2} \\ -f_y - \frac{f_y X^2}{Z^2} & \frac{f_y XY}{Z^2} & \frac{f_y Y}{Z} & 0 & \frac{f_y}{Z} & -\frac{f_y Y}{Z^2} \end{bmatrix} \tag{6}$$

where $\delta\xi$ represents the perturbation.

The the Jacobian matrix can be written as:

$$J = \begin{bmatrix} F & E \end{bmatrix} \tag{7}$$

Then the Hessian matrix is

$$H = J^T J = \begin{bmatrix} F^T F & F^T E \\ E^T F & E^T E \end{bmatrix} \tag{8}$$

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 0.00608493
 0.00547299
  0.0053236
 0.00520788
 0.00502341
  0.0048434
 0.00451083
  0.0042627
 0.00386223
 0.00351651
 0.00302963
 0.00253459
 0.00230246
 0.00172459
0.000422374
3.21708e-17
2.06732e-17
1.43188e-17
7.66992e-18
6.08423e-18
6.05715e-18
3.94363e-18
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Figure 3: Result of Q2