Lecture 5: Degree distributions and power laws

Matthew J. Salganik

Sociology 204: Social Networks, Fall 2021
Princeton University

Monday, September 20, 2021

Feedback on the feedback:

- thank you

Feedback on the feedback:

- thank you
- slides are posted before class: `https://github.com/msalganik/soc204_f2021/tree/main/slides`

Feedback on the feedback:

- thank you
- slides are posted before class: `https://github.com/msalganik/soc204_f2021/tree/main/slides`
- trying new lighting for this class

Feedback on the feedback:

- thank you
- slides are posted before class: `https://github.com/msalganik/soc204_f2021/tree/main/slides`
- trying new lighting for this class
- you should not always expect to come out of each lecture understanding everything

Feedback on the feedback:

- thank you
- slides are posted before class: `https://github.com/msalganik/soc204_f2021/tree/main/slides`
- trying new lighting for this class
- you should not always expect to come out of each lecture understanding everything
- if you still have questions post on Ed and/or come to office hours

Review:

- simple model (ring lattice $+$ rewiring) predicts that many networks will be "small-world" networks

Review:

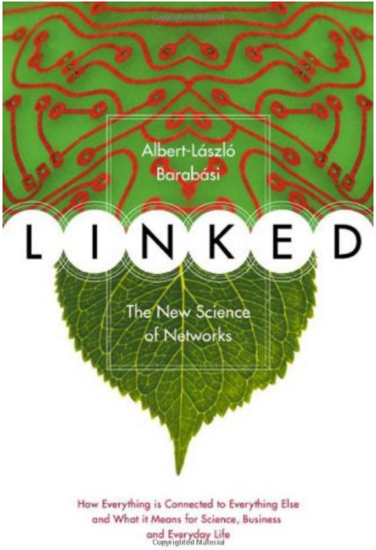
- simple model (ring lattice + rewiring) predicts that many networks will be "small-world" networks
- three real networks (movie actors, power grid, and worm brain) have high clustering coefficient (relative to Erdos-Renyi random graph) and similar characteristic path length to Erdos-Renyi random graph

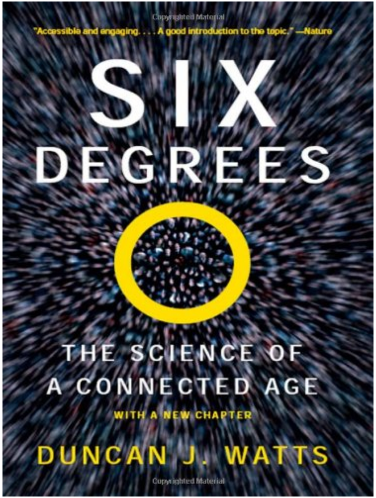
Review:

- simple model (ring lattice + rewiring) predicts that many networks will be "small-world" networks
- three real networks (movie actors, power grid, and worm brain) have high clustering coefficient (relative to Erdos-Renyi random graph) and similar characteristic path length to Erdos-Renyi random graph
- abstract model helps us understand many types of networks

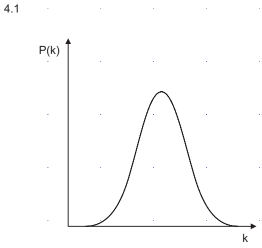
Review:

- simple model (ring lattice + rewiring) predicts that many networks will be "small-world" networks
- three real networks (movie actors, power grid, and worm brain) have high clustering coefficient (relative to Erdos-Renyi random graph) and similar characteristic path length to Erdos-Renyi random graph
- abstract model helps us understand many types of networks
- these network structural properties are important for dynamics happening on the network (e.g., disease spread)

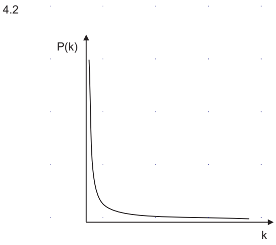

Copyrighted Material
Albert-László Barabási
LINKED
The New Science of Networks
How Everything is Connected to Everything Else and What it Means for Science, Business and Everyday Life
Copyrighted Material


Copyrighted Material
"Accessible and engaging. . . . A good introduction to the topic." —Nature
SIX DEGREES
THE SCIENCE OF A CONNECTED AGE
WITH A NEW CHAPTER
DUNCAN J. WATTS
Copyrighted Material

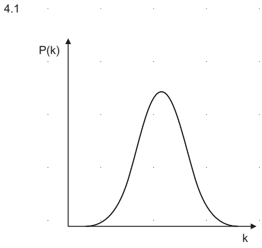
- degree: number of connections that a node has to other nodes (not related to degrees of separation)
- degree distribution: distribution of degrees



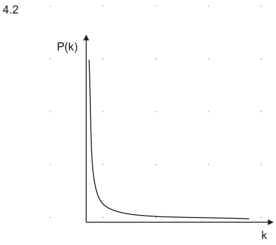
(a) Normal



(b) Power law



(a) Normal

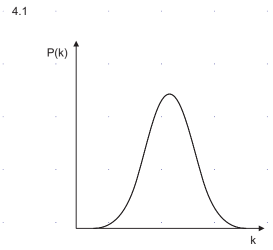


(b) Power law

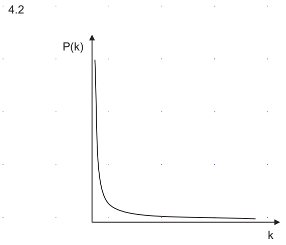
Is the distribution of heights more similar to normal or scale-free?

(a) Normal (b) Power law

Is the distribution of heights more similar to normal or scale-free? normal

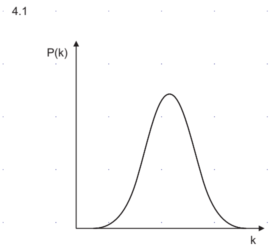


(a) Normal

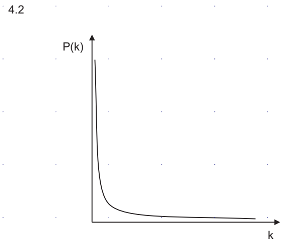


(b) Power law

Is the distribution of wealth more similar to normal or scale-free?

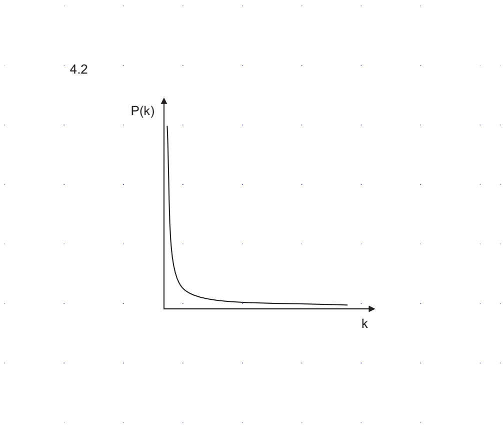


(a) Normal

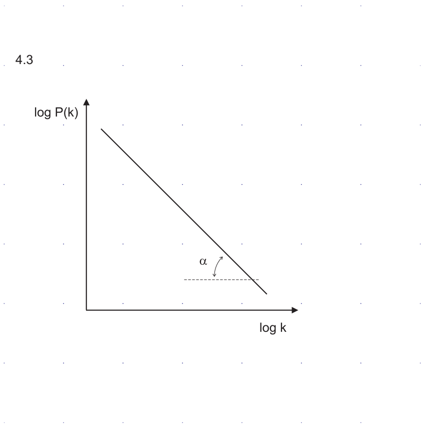


(b) Power law

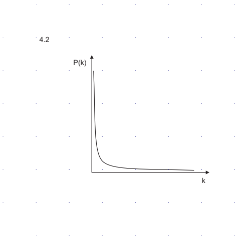
Is the distribution of wealth more similar to normal or scale-free? scale-free



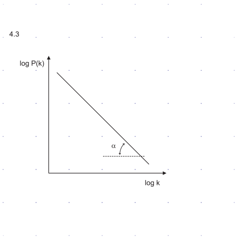
(a) Power law



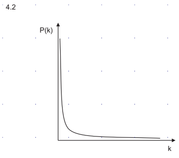
(b) log-log Power law



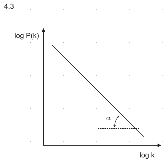
(a) Power law



(b) log-log Power law

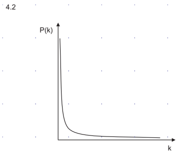


(a) Power law

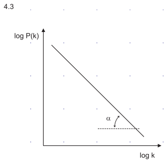


(b) log-log Power law

$$p(k) \propto \frac{1}{k^n}$$

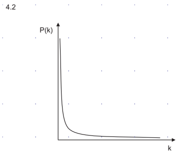


(a) Power law

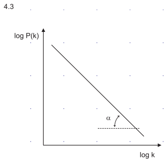


(b) log-log Power law

$p(k) \propto \frac{1}{k^n}$
$log p(k) \propto log(\frac{1}{k^n})$

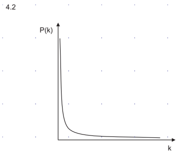


(a) Power law

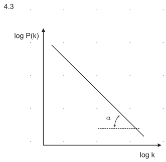


(b) log-log Power law

$$p(k) \propto \frac{1}{k^n}$$
$$logp(k) \propto log(\frac{1}{k^n})$$
$$logp(k) \propto log(1) - log(k^n)$$



(a) Power law



(b) log-log Power law

$$p(k) \propto \frac{1}{k^n}$$
$$logp(k) \propto log(\frac{1}{k^n})$$
$$logp(k) \propto log(1) - log(k^n)$$
$$logp(k) \propto -nlog(k)$$

It turns out that many degree distributions follow a power law distribution (which Barabasi calls "scale-free")

$p(k) \sim \frac{1}{k^\gamma}$

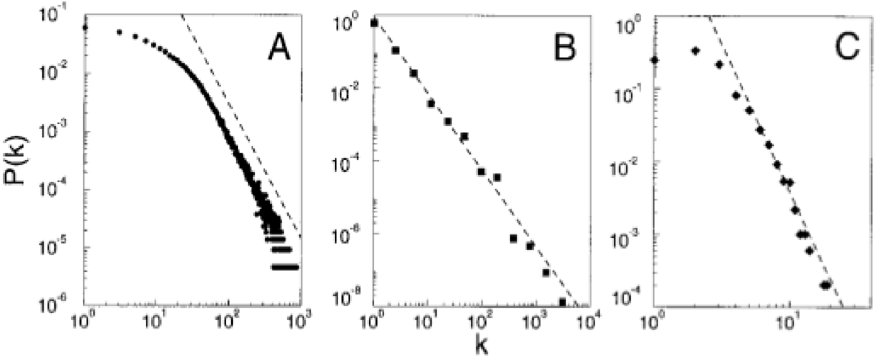
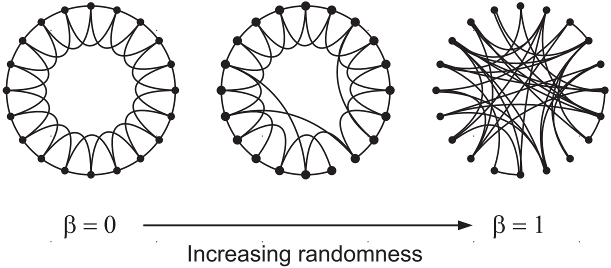


Fig. 1. The distribution function of connectivities for various large networks. **(A)** Actor collaboration graph with $N = 212{,}250$ vertices and average connectivity $\langle k \rangle = 28.78$. **(B)** WWW, $N = 325{,}729$, $\langle k \rangle = 5.46$ (6). **(C)** Power grid data, $N = 4941$, $\langle k \rangle = 2.67$. The dashed lines have slopes (A) $\gamma_{actor} = 2.3$, (B) $\gamma_{www} = 2.1$ and (C) $\gamma_{power} = 4$.

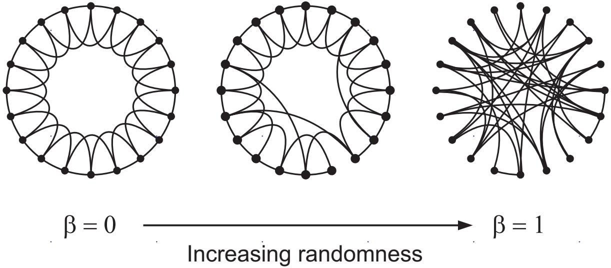
Does β model produce power law degree distribution?

3.6



Does β model produce power law degree distribution? No

3.6



Barabasi and Albert propose a very simple model that generates networks with power law degree distributions

- growth (new nodes enter the system)
- preferential attachment (more likely to connect to high degree nodes)

Demo

`http://netlogoweb.org/launch#http://netlogoweb.org/assets/modelslib/Sample%20Models/Networks/Preferential%20Attachment.nlogo`

Follow up work:

- Implications
- Empirical
- Modeling

Implication

Epidemic Spreading in Scale-Free Networks

Romualdo Pastor-Satorras[1] and Alessandro Vespignani[2]

[1] *Departament de Física i Enginyeria Nuclear, Universitat Politècnica de Catalunya, Campus Nord, Mòdul B4, 08034 Barcelona, Spain*

[2] *The Abdus Salam International Centre for Theoretical Physics (ICTP), P.O. Box 586, 34100 Trieste, Italy*

(Received 20 October 2000)

Implication

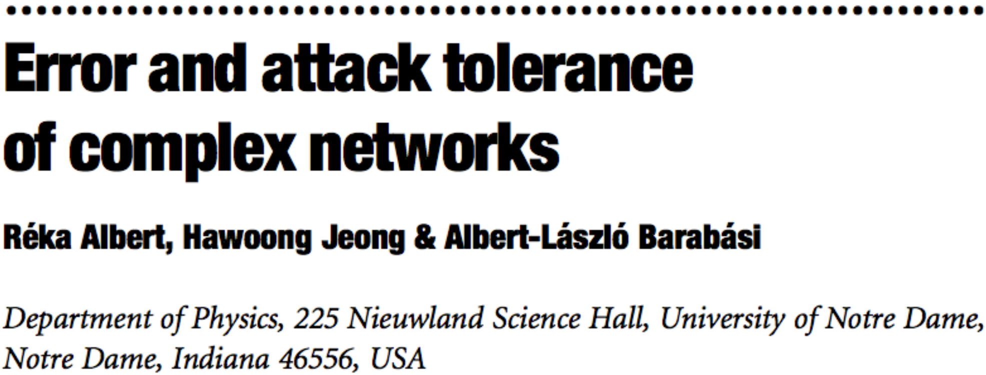
Epidemic Spreading in Scale-Free Networks

Romualdo Pastor-Satorras[1] and Alessandro Vespignani[2]
[1]*Departament de Física i Enginyeria Nuclear, Universitat Politècnica de Catalunya, Campus Nord, Mòdul B4, 08034 Barcelona, Spain*
[2]*The Abdus Salam International Centre for Theoretical Physics (ICTP), P.O. Box 586, 34100 Trieste, Italy*
(Received 20 October 2000)

- Diseases are harder to stop when spreading in scale-free networks

`http://dx.doi.org/10.1103/PhysRevLett.86.3200`

Implication

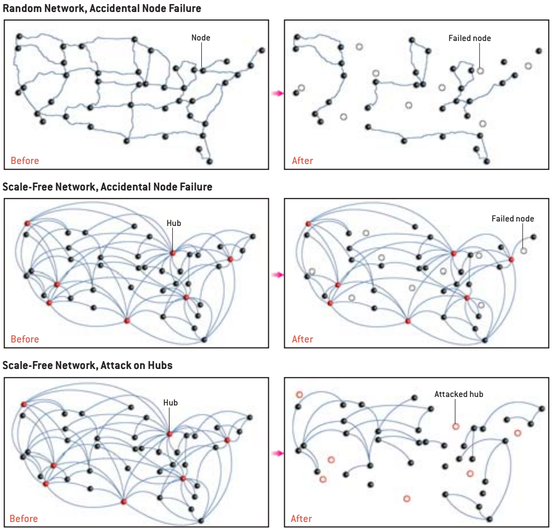

Error and attack tolerance of complex networks

Réka Albert, Hawoong Jeong & Albert-László Barabási

Department of Physics, 225 Nieuwland Science Hall, University of Notre Dame, Notre Dame, Indiana 46556, USA

http://dx.doi.org/10.1038/35019019

Implication



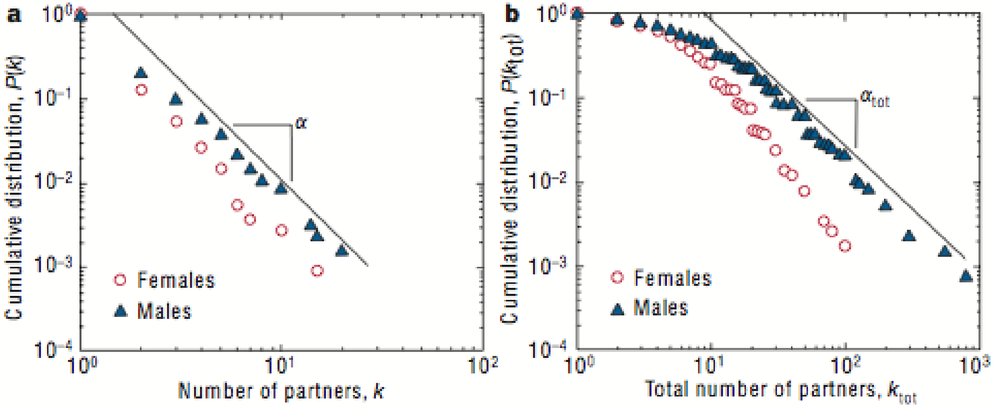
Emprical

The web of human sexual contacts

Promiscuous individuals are the vulnerable nodes to target in safe-sex campaigns.

https://doi.org/10.1038/35082140

Emprical


a
b
Cumulative distribution, P(k)
Number of partners, k
Cumulative distribution, P(k_tot)
Total number of partners, k_tot
α
α_tot
Females
Males
10^0
10^-1
10^-2
10^-3
10^-4
10^1
10^2
10^3

Empirical

ARTICLE

https://doi.org/10.1038/s41467-019-08746-5 OPEN

Scale-free networks are rare

Anna D. Broido[1] & Aaron Clauset[2,3,4]

- Formal definitions of scale-free networks: Super-weak, weakest, weak, strong, strongest

Empirical

ARTICLE

https://doi.org/10.1038/s41467-019-08746-5 OPEN

Scale-free networks are rare

Anna D. Broido[1] & Aaron Clauset[2,3,4]

- Formal definitions of scale-free networks: Super-weak, weakest, weak, strong, strongest
- Analyzed nearly 1,000 social, biological, technological, transportation, and information networks

Empirical

ARTICLE

https://doi.org/10.1038/s41467-019-08746-5 OPEN

Scale-free networks are rare

Anna D. Broido[1] & Aaron Clauset[2,3,4]

- Formal definitions of scale-free networks: Super-weak, weakest, weak, strong, strongest
- Analyzed nearly 1,000 social, biological, technological, transportation, and information networks
- Strongest form of scale-free structure is very rare

Empirical

ARTICLE

https://doi.org/10.1038/s41467-019-08746-5

OPEN

Scale-free networks are rare

Anna D. Broido[1] & Aaron Clauset[2,3,4]

- Formal definitions of scale-free networks: Super-weak, weakest, weak, strong, strongest
- Analyzed nearly 1,000 social, biological, technological, transportation, and information networks
- Strongest form of scale-free structure is very rare
- Social networks seem least scale-free, whereas technical and biological seem more scale-free

Empirical

ARTICLE

https://doi.org/10.1038/s41467-019-08746-5 OPEN

Scale-free networks are rare

Anna D. Broido[1] & Aaron Clauset[2,3,4]

- Formal definitions of scale-free networks: Super-weak, weakest, weak, strong, strongest
- Analyzed nearly 1,000 social, biological, technological, transportation, and information networks
- Strongest form of scale-free structure is very rare
- Social networks seem least scale-free, whereas technical and biological seem more scale-free

`https://doi.org/10.1038/s41467-019-08746-5`

Empirical

COMMENT

https://doi.org/10.1038/s41467-019-09038-8

OPEN

Rare and everywhere: Perspectives on scale-free networks

Petter Holme [1]

https://doi.org/10.1038/s41467-019-09038-8

Modeling

Organization of growing random networks

P. L. Krapivsky and S. Redner
Center for BioDynamics, Center for Polymer Studies, and Department of Physics, Boston University, Boston, Massachusetts 02215
(Received 7 November 2000; published 24 May 2001)

- Generalizes preferential attachment process

`https://doi.org/10.1103/PhysRevE.63.066123`

Modeling

Scale-Free Networks from Varying Vertex Intrinsic Fitness

G. Caldarelli,[1] A. Capocci,[2] P. De Los Rios,[3,4] and M. A. Muñoz[5]

[1]*INFM UdR ROMA1 Dipartimento Fisica, Università di Roma "La Sapienza," Piazzale Aldo Moro 2 00185, Roma, Italy*
[2]*Département de Physique, Université de Fribourg-Pérolles, CH-1700 Fribourg, Switzerland*
[3]*Institut de Physique Théorique, Université de Lausanne, CH-1004 Lausanne, Switzerland*
[4]*INFM UdR Politecnico di Torino, Corso Duca degli Abruzzi 24, 10129 Torino, Italy*
[5]*Instituto de Física Teórica y Computacional Carlos I, Universidad de Granada, Facultad de Ciencias, 18071-Granada, Spain*
(Received 15 July 2002; published 3 December 2002)

- power laws can from from "good-get-richer" in addition to "rich-get-richer"

`https://doi.org/10.1103/PhysRevLett.89.258702`

Question from previous year:
"Is it possible for hubs to exist even where a network doesn't follow a power law distribution? Meaning, the fact that some nodes will be more connected than other nodes, but without the entire network being scale-free?"

Question from previous year:
"Is it possible for hubs to exist even where a network doesn't follow a power law distribution? Meaning, the fact that some nodes will be more connected than other nodes, but without the entire network being scale-free?"
A note on terminology:

- power law
- scale-free
- hubs

- growth $+$ preferential attachment $\rightarrow$ power law degree distribution

- growth $+$ preferential attachment $\rightarrow$ power law degree distribution
- some (but not all) real networks have a power law degree distribution

- growth $+$ preferential attachment $\rightarrow$ power law degree distribution
- some (but not all) real networks have a power law degree distribution
- diseases spread more easily on networks with power law degree distribution than on other types of networks

- growth + preferential attachment $\rightarrow$ power law degree distribution
- some (but not all) real networks have a power law degree distribution
- diseases spread more easily on networks with power law degree distribution than on other types of networks
- networks with power law degree distribution are robust to random failure but fragile to targeted attack

Feedback: `http://bit.ly/soc204-2021`

Feedback: `http://bit.ly/soc204-2021`

- Gladwell, M. (1999). Six degrees of Lois Weisberg. *The New Yorker*.
- Watts, Chapter 4, 114-129.
- Feld, S.L. (1981) The focused organization of social ties. *American Journal of Sociology*.

$\rightarrow$