Lecture 4: Understanding the small world phenomena

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Social Networks (Soc 204)
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Review:

- empirical vs modeling approaches

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- empirical approach runs into difficulties

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Today we will see two different small world models and then an empirical assessment

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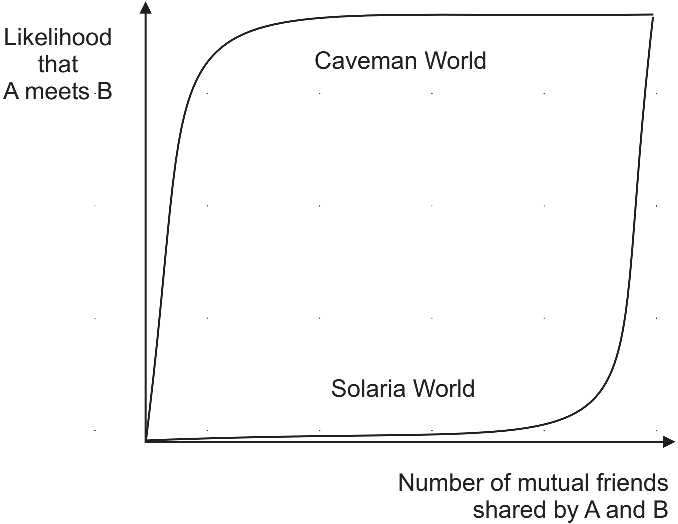
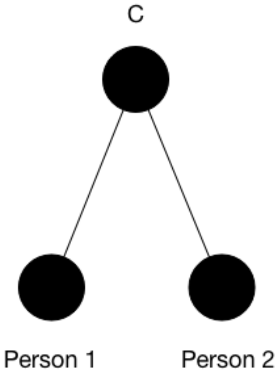
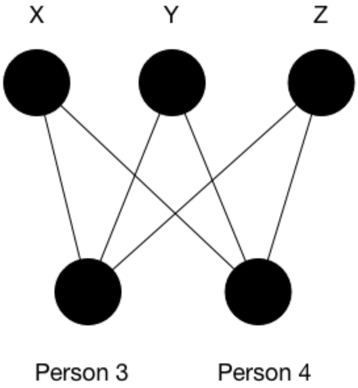
- small overlapping groups that are linked by people who belong to multiple groups
- social network evolve
- not all relationships are equally likely
- occasionally we do things that are not determined by existing network structure

3.1


Likelihood that A meets B
Caveman World
Solaria World
Number of mutual friends shared by A and B

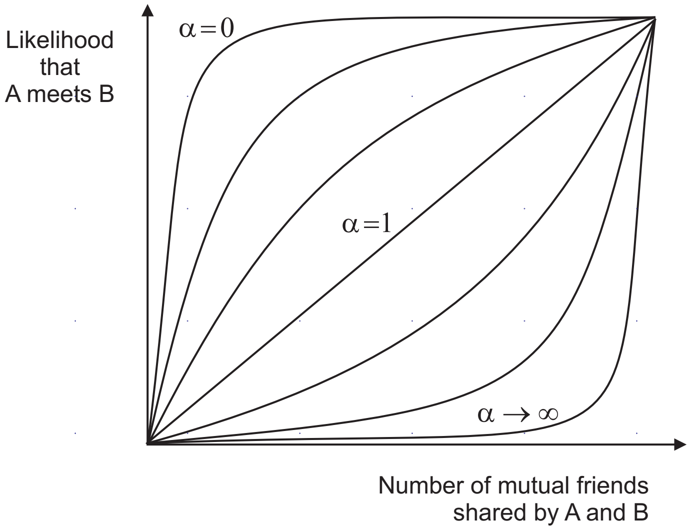


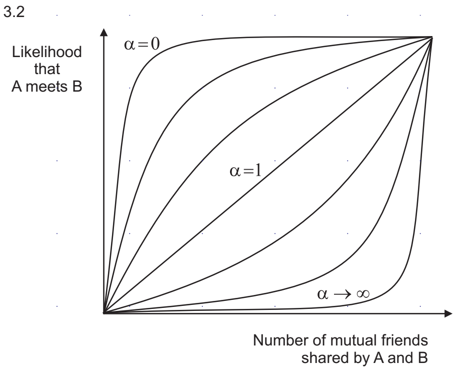
1 mutual friend



3 mutual friend

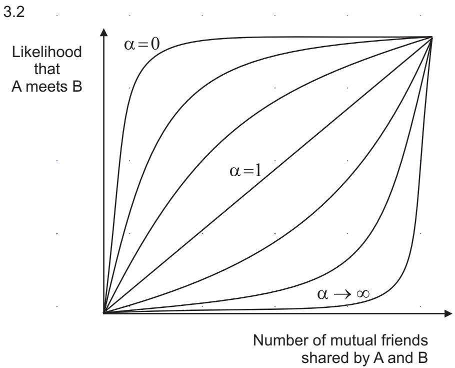
3.2


Likelihood
that
A meets B
$\alpha = 0$
$\alpha = 1$
$\alpha \to \infty$
Number of mutual friends
shared by A and B



As technology changes do you think we are moving more toward:

1. caveman world ($\alpha = 0$)
2. solaria world ($\alpha \to \infty$)



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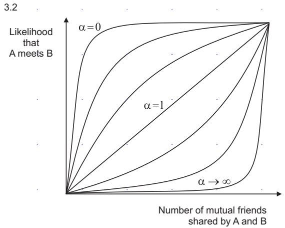
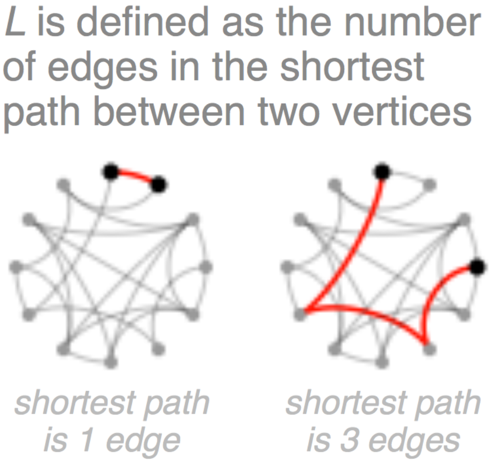


$$R_{i,j} = \begin{cases} 1 & m_{i,j} \geq k \\ \left[\dfrac{m_{i,j}}{k}\right]^{\alpha}(1-p)+p & k > m_{i,j} > 0, \\ p & m_{i,j} = 0 \end{cases} \tag{5}$$

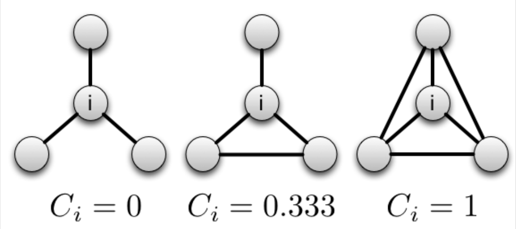
Note on this process of modeling: the graph came before the equation.

First metric:

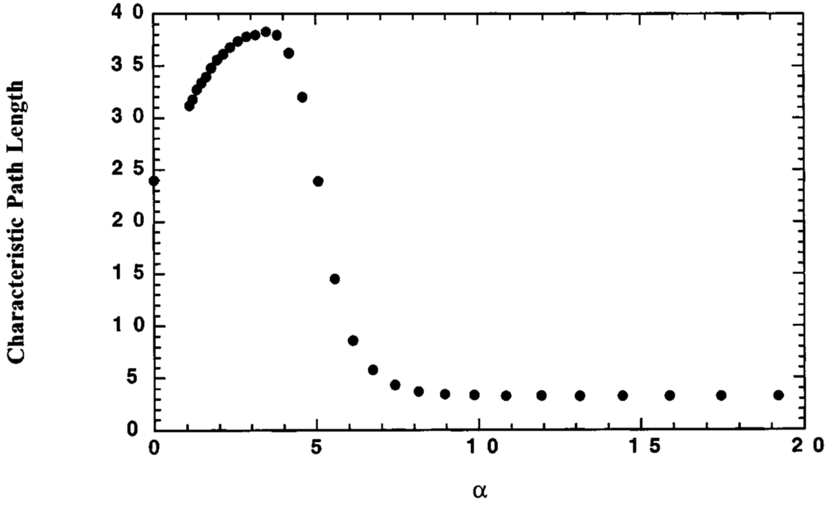
Characteristics path length L: number of edges in shortest path, averaged over all paths

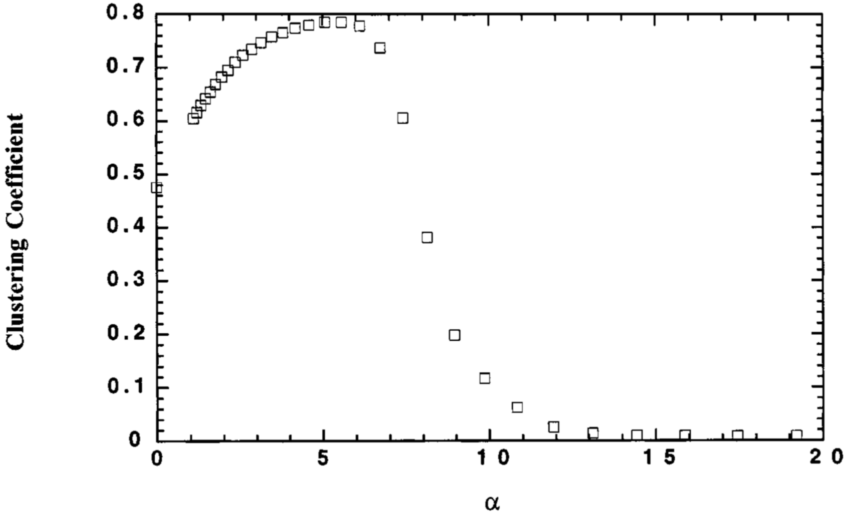


Second metric:
Clustering coefficient C: probability that a two friends of a randomly chosen person are friends

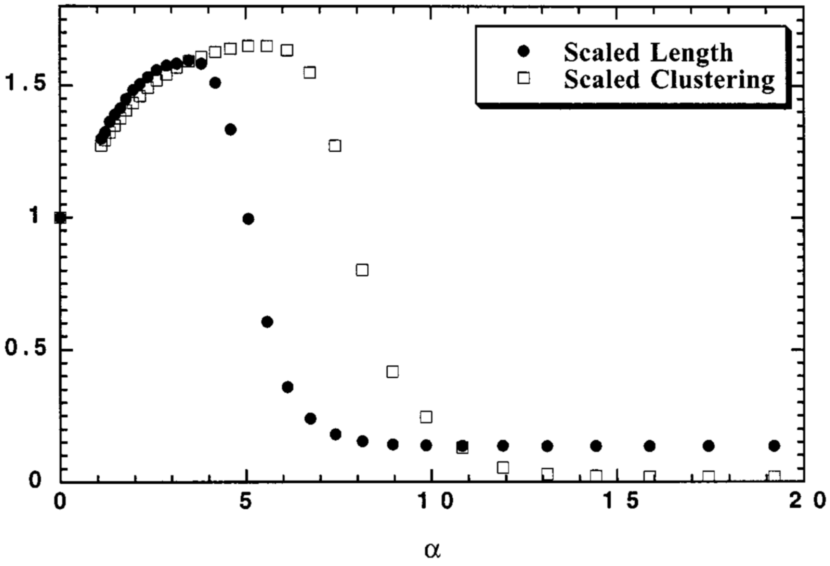


If we simulate lots of people following these rules, what kinds of networks get created?

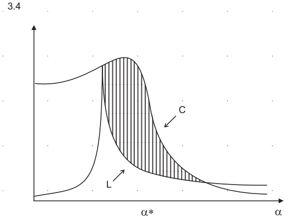

Characteristic Path Length
4 0
3 5
3 0
2 5
2 0
1 5
1 0
5
0
0
5
1 0
1 5
2 0
α

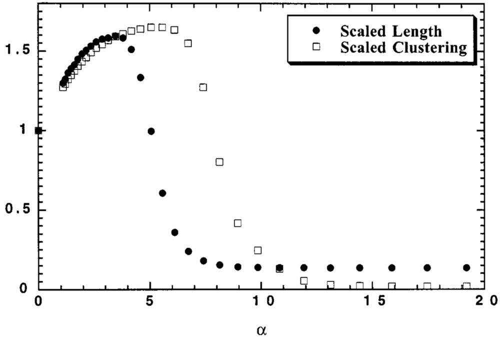


Clustering Coefficient
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
5
10
15
20
α

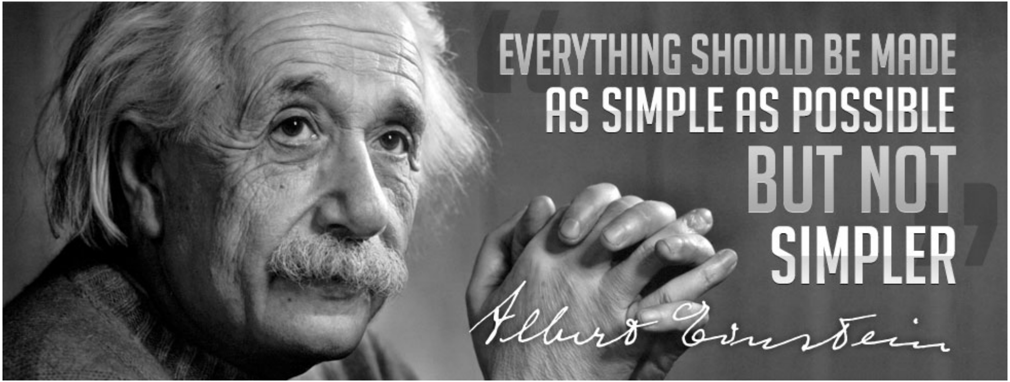


Scaled Length
Scaled Clustering
1.5
1
0.5
0
0
5
10
15
20
α



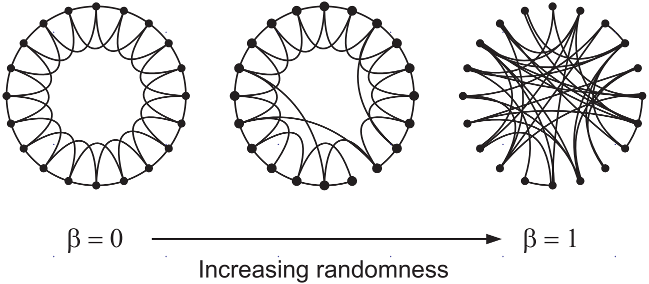


Paper and book look different because paper does not include the disconnected region

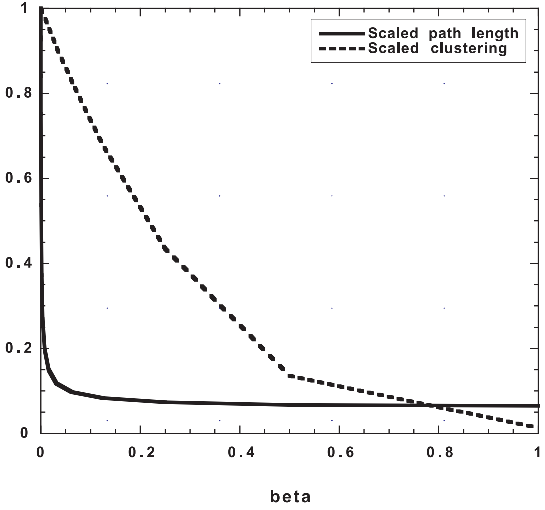


http://vireomd.net/blog/dhc/einstein-kiss.html

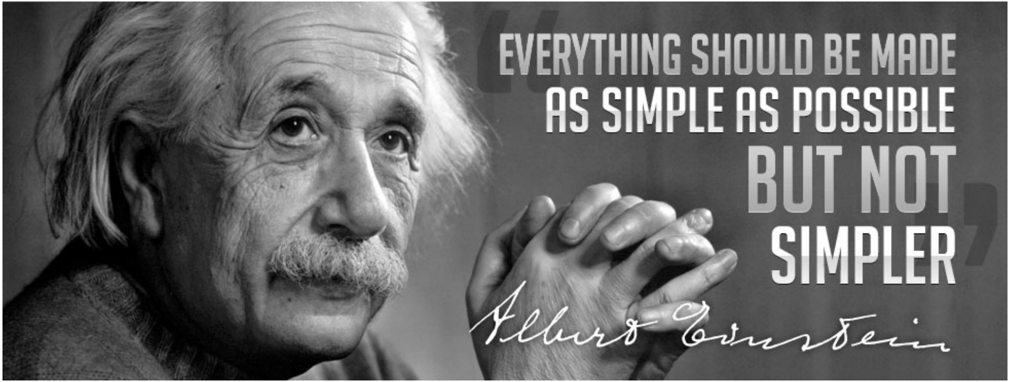
3.6



3.7


Scaled path length
Scaled clustering
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1
beta

Demo: `http://mathinsight.org/small_world_network`



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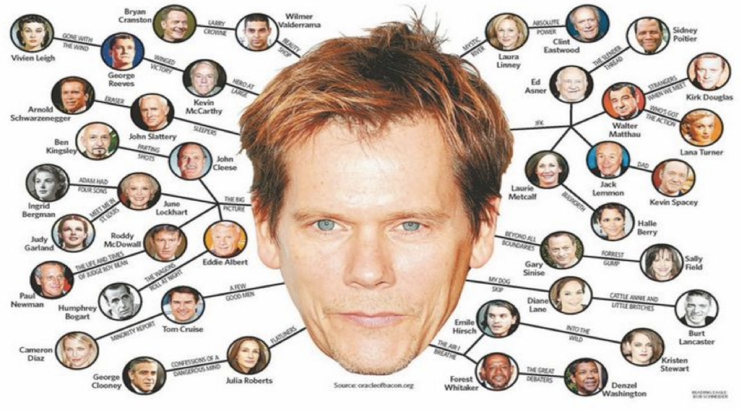
Are real networks small world networks?



Small world means:

- $L_{actual} \approx L_{random}$
- $C_{actual} >> C_{random}$

	L_{actual}	L_{random}	C_{actual}	C_{random}

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Movie actors				

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Movie actors	3.65	2.99	0.79	0.00027

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	L_{actual}	L_{random}	C_{actual}	C_{random}
Movie actors	3.65	2.99	0.79	0.00027
Power Grid				

Small world means:

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	L_{actual}	L_{random}	C_{actual}	C_{random}
Movie actors	3.65	2.99	0.79	0.00027
Power Grid	18.7	12.4	0.080	0.005

Small world means:

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	L_{actual}	L_{random}	C_{actual}	C_{random}
Movie actors	3.65	2.99	0.79	0.00027
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C. Elegans				

Small world means:

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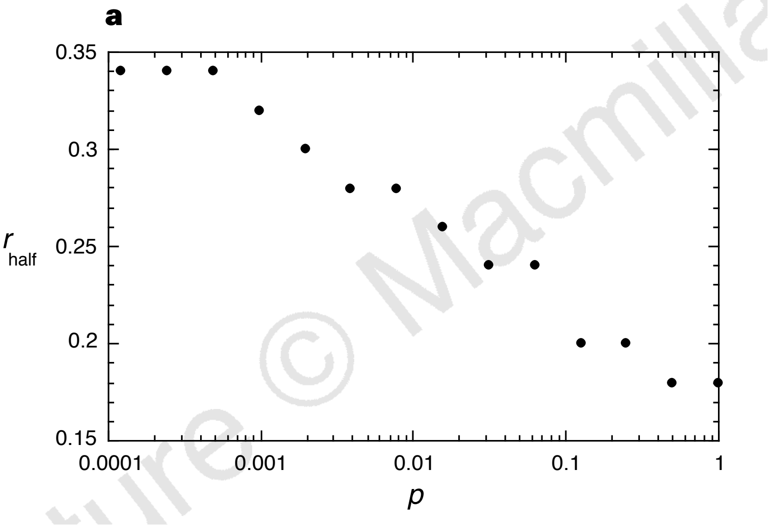
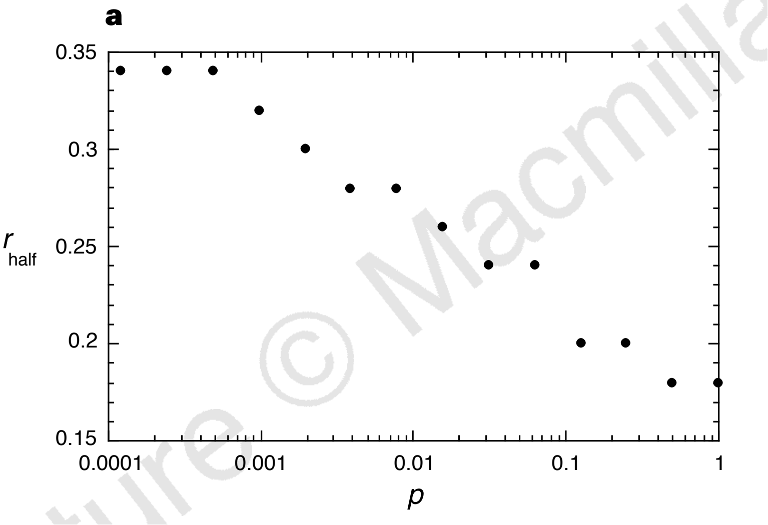
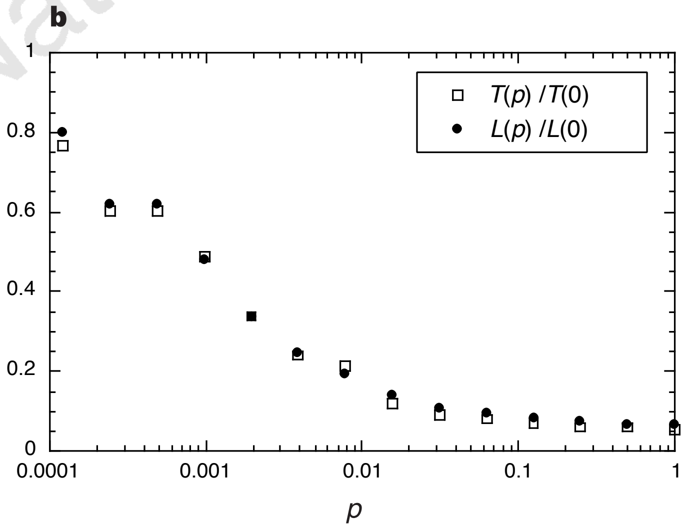
	L_{actual}	L_{random}	C_{actual}	C_{random}
Movie actors	3.65	2.99	0.79	0.00027
Power Grid	18.7	12.4	0.080	0.005
C. Elegans	2.65	2.25	0.28	0.05

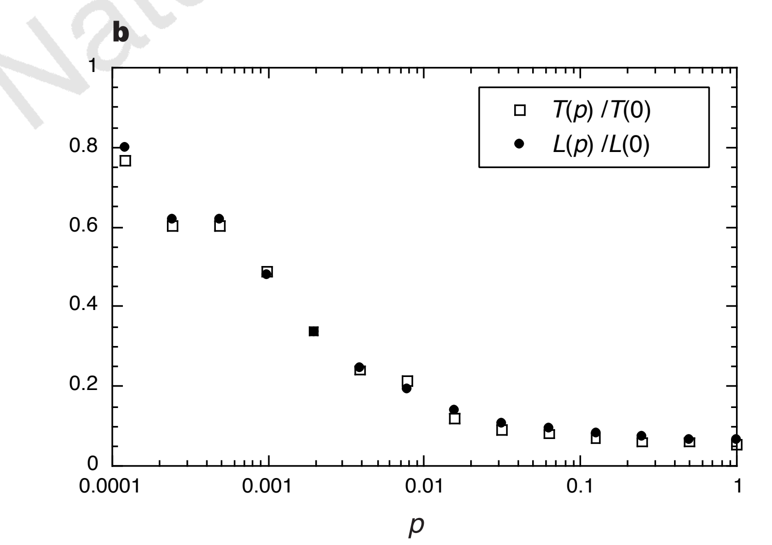
Who cares?


a
0.35
0.3
0.25
0.2
0.15
r half
0.0001
0.001
0.01
0.1
1
p
Nature © Macmillan



The more shortcuts the less infectious (r) a disease needs to be to spread


b
T(p) / T(0)
L(p) / L(0)
1
0.8
0.6
0.4
0.2
0
0.0001
0.001
0.01
0.1
1
p



The more shortcuts the faster a disease spreads

Making length contractions concrete . . . .

- abstract modeling leads to deep and non-obvious insights

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- shortcuts are key the small world property (characteristic path length changes fast, clustering changes slow)
- small local changes can have global impacts
- similarity across networks of different types
- network structure impacts dynamics

Feedback: `http://bit.ly/soc204-2021`

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Next class. Power law networks

- Watts, Chapter 4, 101-114.
- Barabasi, A.L. and Bonabeau, E. (2003) Scale-free networks. *Scientific American*, 50-59.
- Barabasi, A.L. and Albert, R. (1999) The emergence of scaling in random networks. *Science*, 286:509-512.
- Liljeros, F. et al. (2001). The web of human sexual contacts. *Nature*, 411:907-908 with comment and rejoinder.