Lecture 5: Degree distributions and power laws

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Sociology 204: Social Networks, Fall 2021
Princeton University

Monday, September 20, 2021

TODO for next time: include screenshot of Jones and Handcock paper so that you can talk about what that is important derivation of linear on log-log is confusing because we have $\propto$ and α in picture

Feedback on the feedback:

- thank you

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- if you still have questions post on Ed and/or come to office hours

Review:

- simple model (ring lattice + rewiring) predicts that many networks will be "small-world" networks

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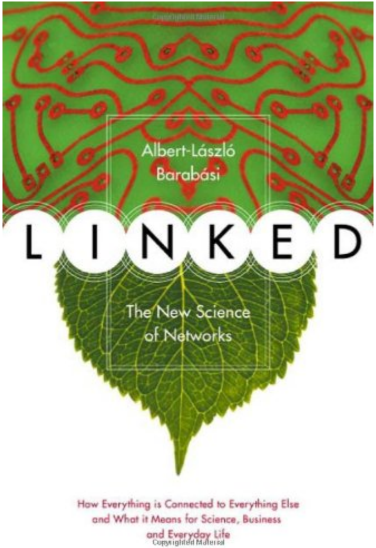
- three real networks (movie actors, power grid, and worm brain) have high clustering coefficient (relative to Erdos-Renyi random graph) and similar characteristic path length to Erdos-Renyi random graph
- abstract model helps us understand many types of networks
- these network structural properties are important for dynamics happening on the network (e.g., disease spread)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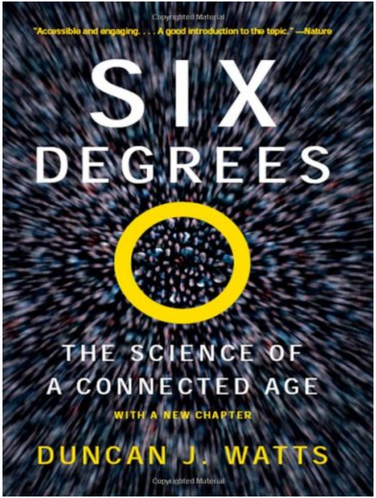
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THE SCIENCE OF
A CONNECTED AGE
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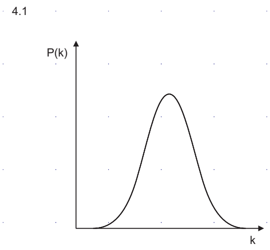
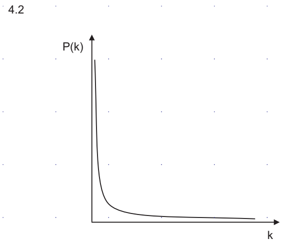
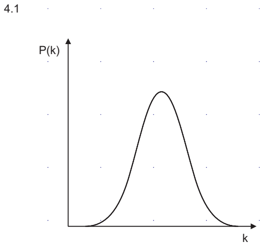
- degree: number of connections that a node has to other nodes (not related to degrees of separation)
- degree distribution: distribution of degrees



(a) Normal



(b) Power law



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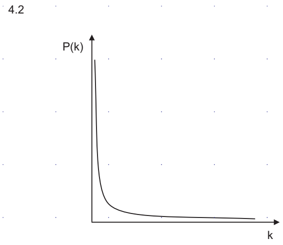
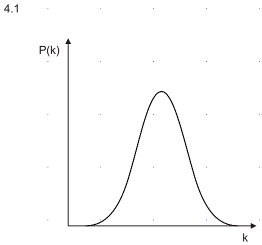
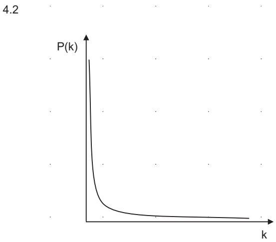


(b) Power law

Is the distribution of heights more similar to normal or scale-free?

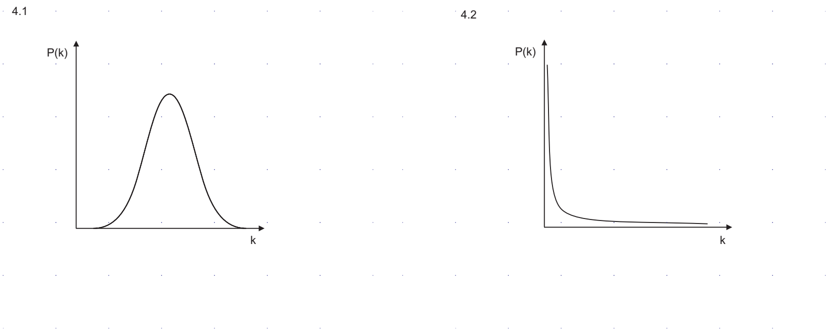


(a) Normal



(b) Power law

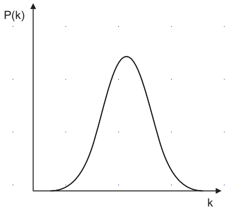
Is the distribution of heights more similar to normal or scale-free? normal



(a) Normal (b) Power law

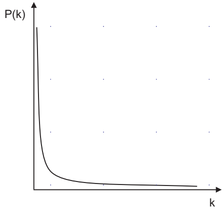
Is the distribution of wealth more similar to normal or scale-free?

4.1



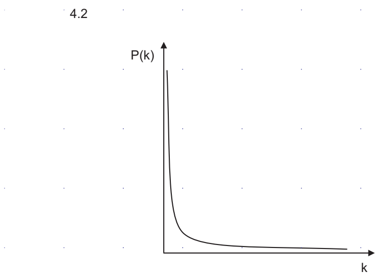
(a) Normal

4.2



(b) Power law

Is the distribution of wealth more similar to normal or scale-free? scale-free



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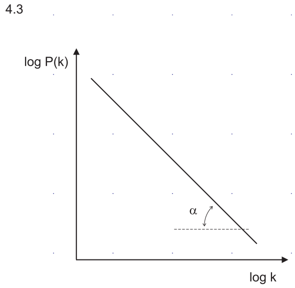
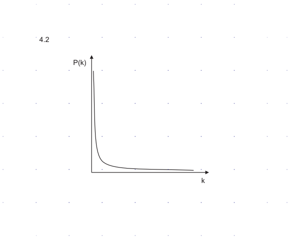


(b) log-log Power law



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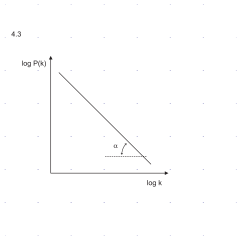

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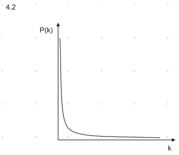

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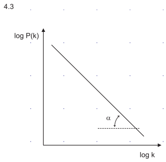
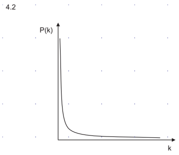


(b) log-log Power law

$p(k) \propto \frac{1}{k^n}$



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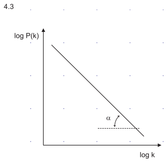
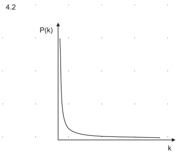


(b) log-log Power law

$p(k) \propto \frac{1}{k^n}$
$logp(k) \propto log(\frac{1}{k^n})$



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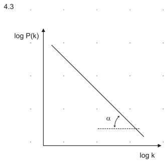
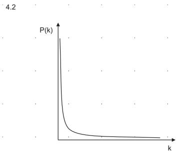


(b) log-log Power law

$p(k) \propto \frac{1}{k^n}$
$log p(k) \propto log(\frac{1}{k^n})$
$log p(k) \propto log(1) - log(k^n)$



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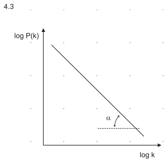


(b) log-log Power law

$$p(k) \propto \frac{1}{k^n}$$
$$log p(k) \propto log(\frac{1}{k^n})$$
$$log p(k) \propto log(1) - log(k^n)$$
$$log p(k) \propto -n log(k)$$

It turns out that many degree distributions follow a power law distribution (which Barabasi calls “scale-free”)

$p(k) \sim \frac{1}{k^\gamma}$

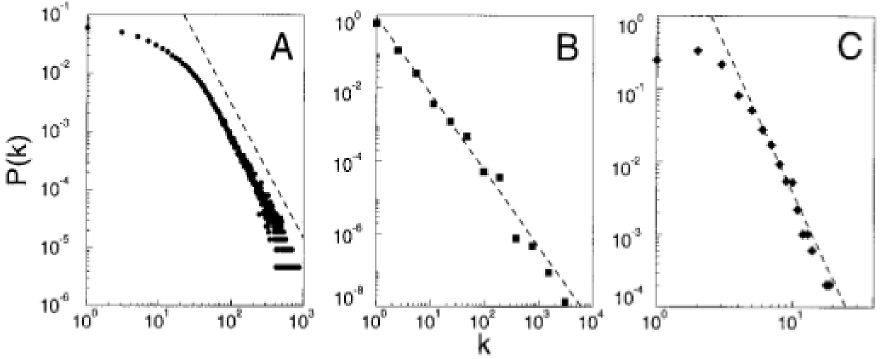
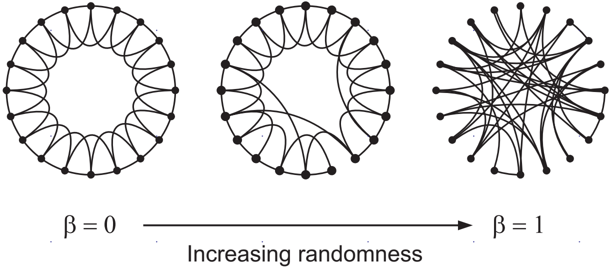


Fig. 1. The distribution function of connectivities for various large networks. **(A)** Actor collaboration graph with $N = 212{,}250$ vertices and average connectivity $\langle k \rangle = 28.78$. **(B)** WWW, $N = 325{,}729$, $\langle k \rangle = 5.46$ (6). **(C)** Power grid data, $N = 4941$, $\langle k \rangle = 2.67$. The dashed lines have slopes (A) $\gamma_{actor} = 2.3$, (B) $\gamma_{www} = 2.1$ and (C) $\gamma_{power} = 4$.

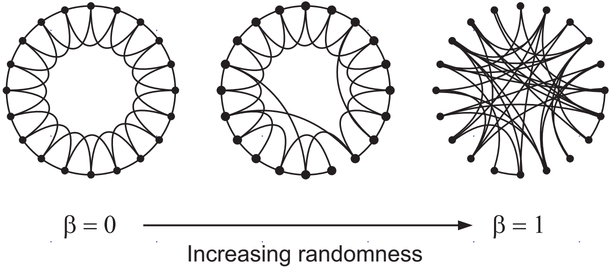
Does β model produce power law degree distribution?

3.6



Does β model produce power law degree distribution? No

3.6



Barabasi and Albert propose a very simple model that generates networks with power law degree distributions

- growth (new nodes enter the system)
- preferential attachment (more likely to connect to high degree nodes)

Demo

http://netlogoweb.org/launch#http://netlogoweb.org/assets/modelslib/Sample%20Models/Networks/Preferential%20Attachment.nlogo

Follow up work:

- Implications
- Empirical
- Modeling

Implication

Epidemic Spreading in Scale-Free Networks

Romualdo Pastor-Satorras[1] and Alessandro Vespignani[2]

[1]*Departament de Física i Enginyeria Nuclear, Universitat Politècnica de Catalunya, Campus Nord, Mòdul B4, 08034 Barcelona, Spain*

[2]*The Abdus Salam International Centre for Theoretical Physics (ICTP), P.O. Box 586, 34100 Trieste, Italy*

(Received 20 October 2000)

Implication

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- Diseases are harder to stop when spreading in scale-free networks

`http://dx.doi.org/10.1103/PhysRevLett.86.3200`

Implication

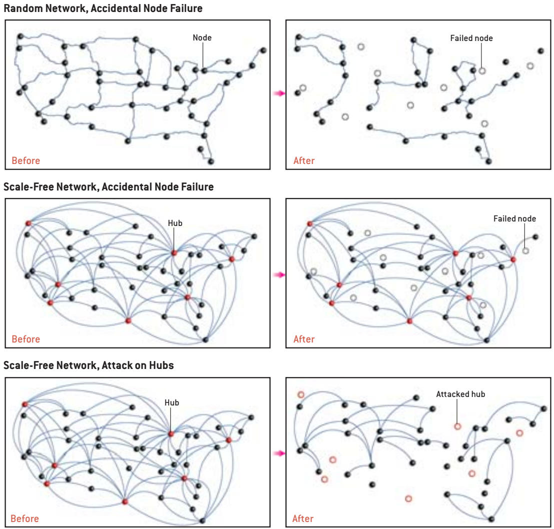
Error and attack tolerance of complex networks

Réka Albert, Hawoong Jeong & Albert-László Barabási

Department of Physics, 225 Nieuwland Science Hall, University of Notre Dame, Notre Dame, Indiana 46556, USA

http://dx.doi.org/10.1038/35019019

Implication



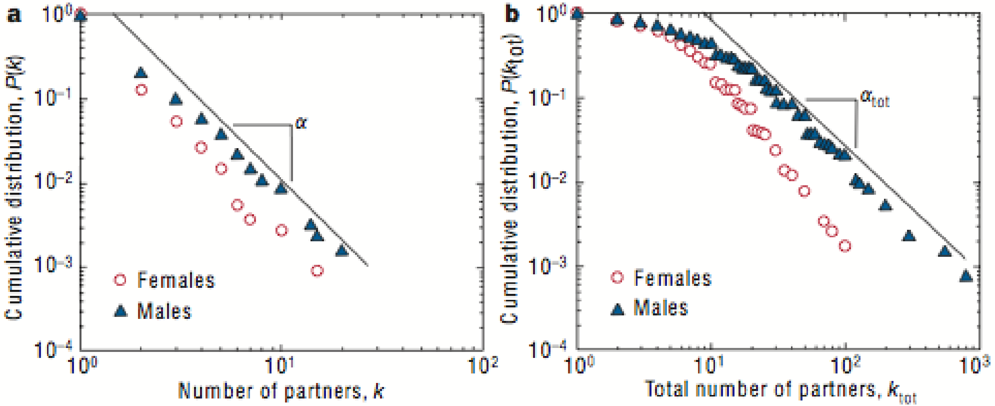
Empirical

The web of human sexual contacts

Promiscuous individuals are the vulnerable nodes to target in safe-sex campaigns.

https://doi.org/10.1038/35082140

Empirical



a
b
Cumulative distribution, P(k)
Number of partners, k
Females
Males
α
Cumulative distribution, P(k_tot)
Total number of partners, k_tot
α_tot

Empirical

ARTICLE

https://doi.org/10.1038/s41467-019-08746-5 OPEN

Scale-free networks are rare

Anna D. Broido[1] & Aaron Clauset[2,3,4]

- Formal definitions of scale-free networks: Super-weak, weakest, weak, strong, strongest

Empirical

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Empirical

COMMENT

https://doi.org/10.1038/s41467-019-09038-8 OPEN

Rare and everywhere: Perspectives on scale-free networks

Petter Holme [1]

https://doi.org/10.1038/s41467-019-09038-8

Modeling

Organization of growing random networks

P. L. Krapivsky and S. Redner
Center for BioDynamics, Center for Polymer Studies, and Department of Physics, Boston University, Boston, Massachusetts 02215
(Received 7 November 2000; published 24 May 2001)

- Generalizes preferential attachment process

`https://doi.org/10.1103/PhysRevE.63.066123`

Modeling

Scale-Free Networks from Varying Vertex Intrinsic Fitness

G. Caldarelli,[1] A. Capocci,[2] P. De Los Rios,[3,4] and M. A. Muñoz[5]
[1]*INFM UdR ROMA1 Dipartimento Fisica, Università di Roma "La Sapienza," Piazzale Aldo Moro 2 00185, Roma, Italy*
[2]*Département de Physique, Université de Fribourg-Pérolles, CH-1700 Fribourg, Switzerland*
[3]*Institut de Physique Théorique, Université de Lausanne, CH-1004 Lausanne, Switzerland*
[4]*INFM UdR Politecnico di Torino, Corso Duca degli Abruzzi 24, 10129 Torino, Italy*
[5]*Instituto de Física Teórica y Computacional Carlos I, Universidad de Granada, Facultad de Ciencias, 18071-Granada, Spain*
(Received 15 July 2002; published 3 December 2002)

- power laws can from from "good-get-richer" in addition to "rich-get-richer"

`https://doi.org/10.1103/PhysRevLett.89.258702`

Question from previous year:
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A note on terminology:

- power law
- scale-free
- hubs

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- some (but not all) real networks have a power law degree distribution
- diseases spread more easily on networks with power law degree distribution than on other types of networks
- networks with power law degree distribution are robust to random failure but fragile to targeted attack

Feedback: `http://bit.ly/soc204-2021`

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- Gladwell, M. (1999). Six degrees of Lois Weisberg. *The New Yorker*.
- Watts, Chapter 4, 114-129.
- Feld, S.L. (1981) The focused organization of social ties. *American Journal of Sociology*.

$\rightarrow$