Lecture 5: Degree distributions and power laws

Matthew J. Salganik

Sociology 204: Social Networks, Fall 2021
Princeton University

Monday, September 20, 2021

Feedback on the feedback:

- thank you

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- slides are posted before class: `https://github.com/msalganik/soc204_f2021/tree/main/slides`

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- if you still have questions post on Ed and/or come to office hours

Review:

- simple model (ring lattice + rewiring) predicts that many networks will be "small-world" networks

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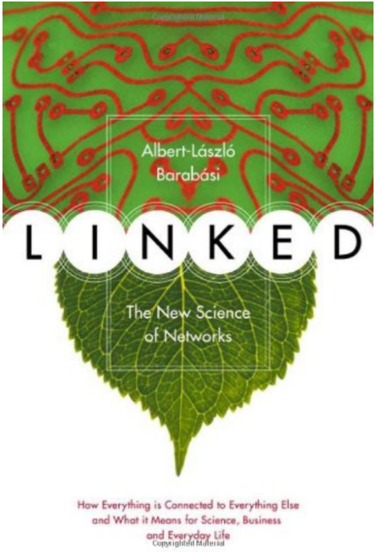
- abstract model helps us understand many types of networks
- these network structural properties are important for dynamics happening on the network (e.g., disease spread)


Albert-László Barabási
LINKED
The New Science of Networks
How Everything is Connected to Everything Else and What it Means for Science, Business and Everyday Life
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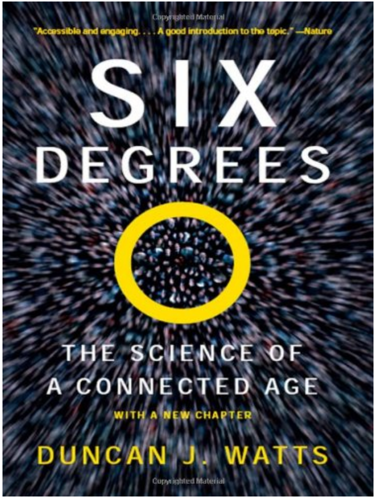
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THE SCIENCE OF A CONNECTED AGE
WITH A NEW CHAPTER
DUNCAN J. WATTS
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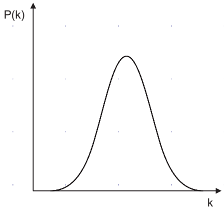
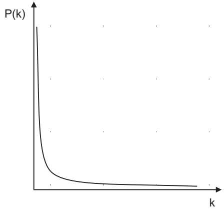
- degree: number of connections that a node has to other nodes (not related to degrees of separation)
- degree distribution: distribution of degrees

4.1



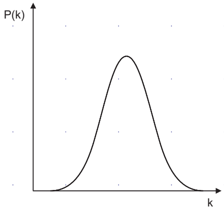
(a) Normal

4.2



(b) Power law

4.1



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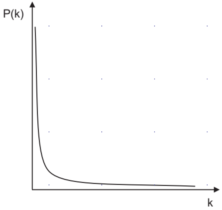
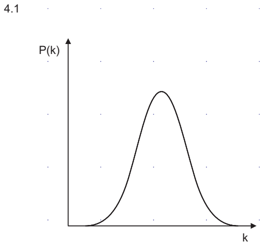
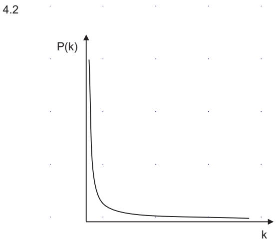


(b) Power law

Is the distribution of heights more similar to normal or scale-free?

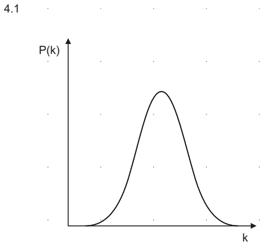


(a) Normal

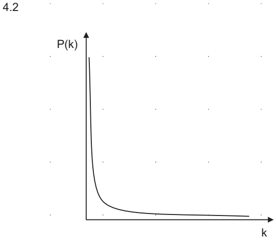


(b) Power law

Is the distribution of heights more similar to normal or scale-free? normal



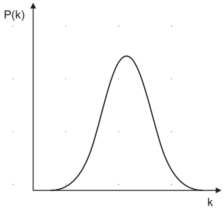
(a) Normal



(b) Power law

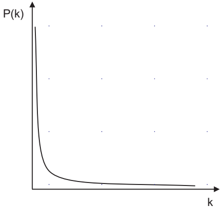
Is the distribution of wealth more similar to normal or scale-free?

4.1



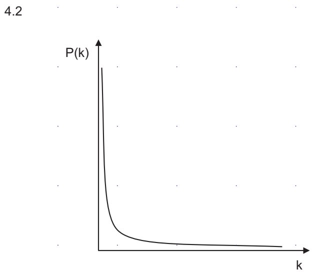
(a) Normal

4.2

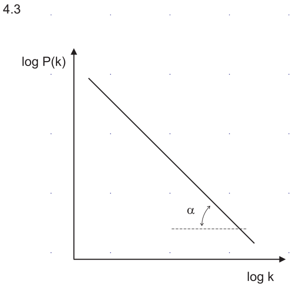


(b) Power law

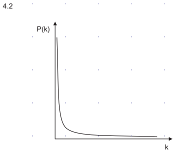
Is the distribution of wealth more similar to normal or scale-free? scale-free



(a) Power law



(b) log-log Power law



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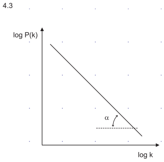

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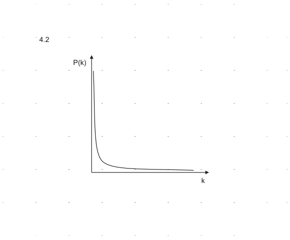

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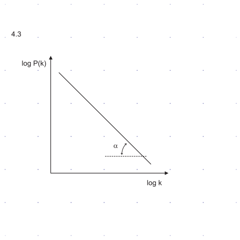
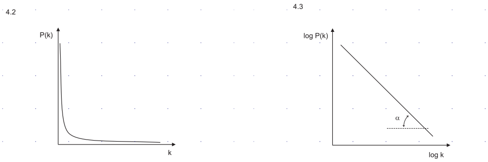


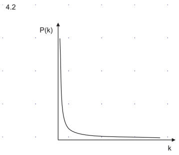
(b) log-log Power law

$p(k) \propto \frac{1}{k^n}$

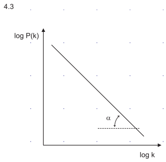


(a) Power law (b) log-log Power law

$$p(k) \propto \frac{1}{k^n}$$
$$logp(k) \propto log(\frac{1}{k^n})$$



(a) Power law



(b) log-log Power law

$$p(k) \propto \frac{1}{k^n}$$
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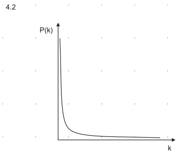
$$logp(k) \propto log(1) - log(k^n)$$



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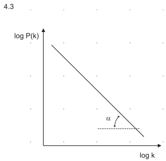

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$$logp(k) \propto log(1) - log(k^n)$$
$$logp(k) \propto -nlog(k)$$

It turns out that many degree distributions follow a power law distribution (which Barabasi calls “scale-free”)

$p(k) \sim \frac{1}{k^\gamma}$

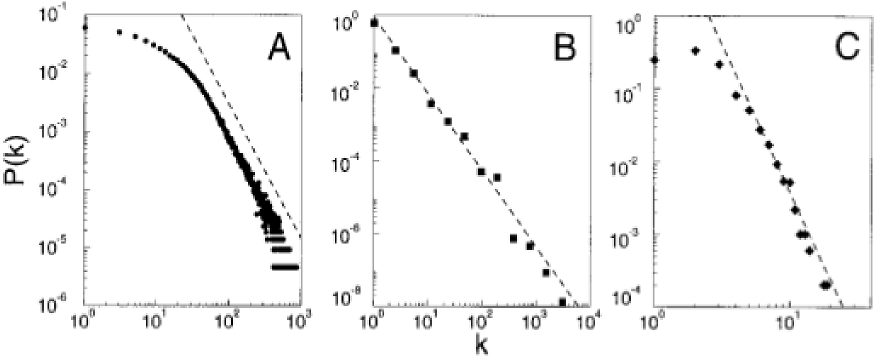
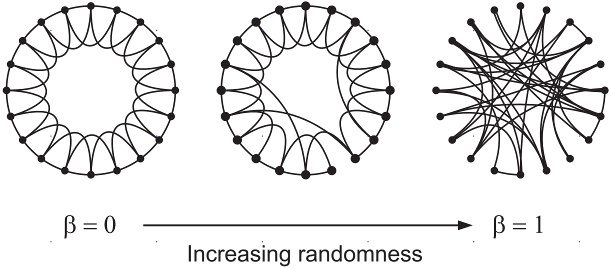


Fig. 1. The distribution function of connectivities for various large networks. **(A)** Actor collaboration graph with N = 212,250 vertices and average connectivity $\langle k \rangle$ = 28.78. **(B)** WWW, N = 325,729, $\langle k \rangle$ = 5.46 (6). **(C)** Power grid data, N = 4941, $\langle k \rangle$ = 2.67. The dashed lines have slopes (A) $\gamma_{\text{actor}} = 2.3$, (B) $\gamma_{\text{www}} = 2.1$ and (C) $\gamma_{\text{power}} = 4$.

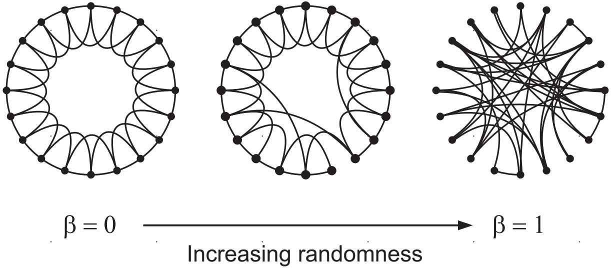
Does β model produce power law degree distribution?

3.6



Does β model produce power law degree distribution? No

3.6



Barabasi and Albert propose a very simple model that generates networks with power law degree distributions

- growth (new nodes enter the system)
- preferential attachment (more likely to connect to high degree nodes)

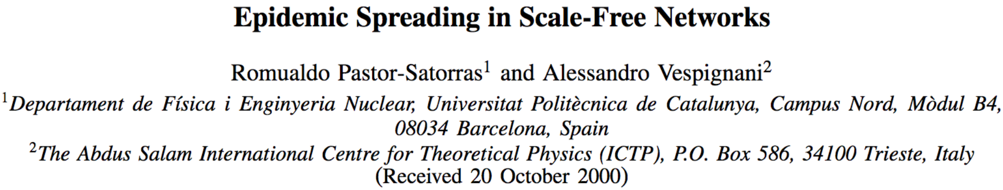
Demo

`http://netlogoweb.org/launch#http://netlogoweb.org/assets/modelslib/Sample%20Models/Networks/Preferential%20Attachment.nlogo`

Follow up work:

- Implications
- Empirical
- Modeling

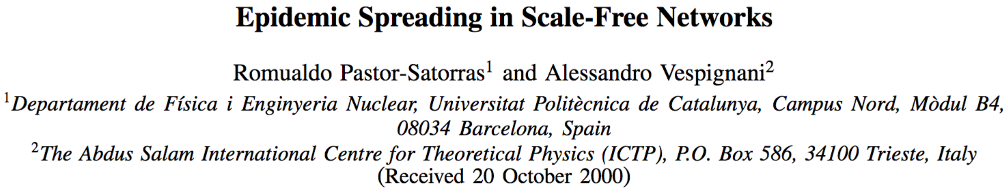
Implication



Epidemic Spreading in Scale-Free Networks

Romualdo Pastor-Satorras[1] and Alessandro Vespignani[2]
[1] *Departament de Física i Enginyeria Nuclear, Universitat Politècnica de Catalunya, Campus Nord, Mòdul B4, 08034 Barcelona, Spain*
[2] *The Abdus Salam International Centre for Theoretical Physics (ICTP), P.O. Box 586, 34100 Trieste, Italy*
(Received 20 October 2000)

Implication



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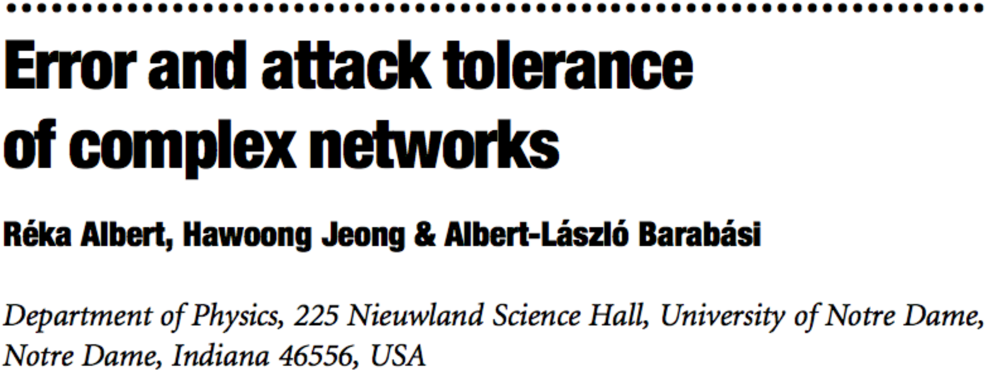
(Received 20 October 2000)

- Diseases are harder to stop when spreading in scale-free networks

`http://dx.doi.org/10.1103/PhysRevLett.86.3200`

Implication



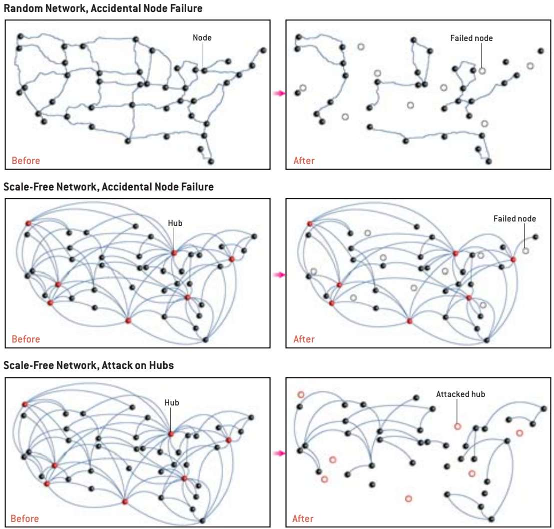
Error and attack tolerance of complex networks

Réka Albert, Hawoong Jeong & Albert-László Barabási

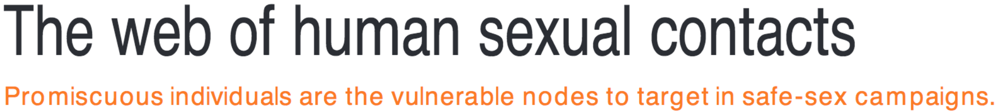
Department of Physics, 225 Nieuwland Science Hall, University of Notre Dame,
Notre Dame, Indiana 46556, USA

`http://dx.doi.org/10.1038/35019019`

Implication



Empirical

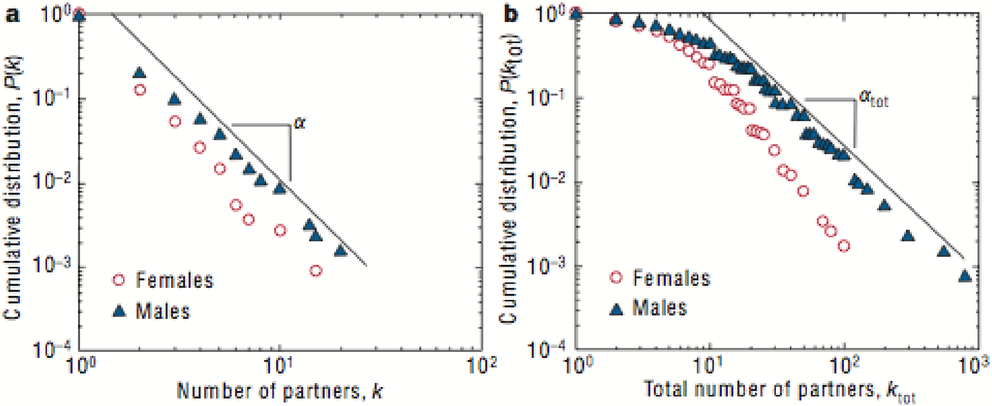


The web of human sexual contacts

Promiscuous individuals are the vulnerable nodes to target in safe-sex campaigns.

https://doi.org/10.1038/35082140

Empirical


a
b
Cumulative distribution, $P(k)$
Number of partners, k
Cumulative distribution, $P(k_{tot})$
Total number of partners, k_{tot}
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^1
10^2
10^3
α
α_{tot}
○ Females
▲ Males

Empirical

ARTICLE

https://doi.org/10.1038/s41467-019-08746-5 OPEN

Scale-free networks are rare

Anna D. Broido[1] & Aaron Clauset[2,3,4]

- Formal definitions of scale-free networks: Super-weak, weakest, weak, strong, strongest

Empirical

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Empirical

ARTICLE

https://doi.org/10.1038/s41467-019-08746-5 **OPEN**

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`https://doi.org/10.1038/s41467-019-08746-5`

Empirical

COMMENT

https://doi.org/10.1038/s41467-019-09038-8

OPEN

Rare and everywhere: Perspectives on scale-free networks

Petter Holme[1]

https://doi.org/10.1038/s41467-019-09038-8

Modeling

Organization of growing random networks

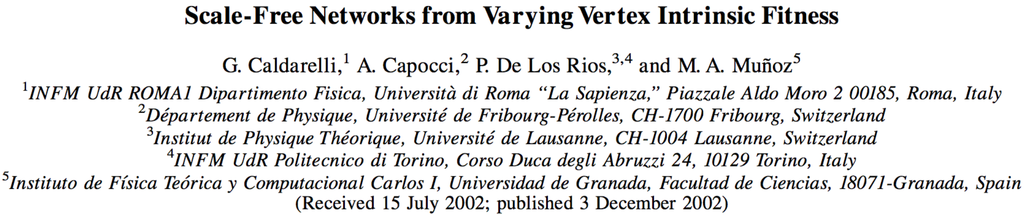
P. L. Krapivsky and S. Redner
Center for BioDynamics, Center for Polymer Studies, and Department of Physics, Boston University, Boston, Massachusetts 02215

(Received 7 November 2000; published 24 May 2001)

- Generalizes preferential attachment process

`https://doi.org/10.1103/PhysRevE.63.066123`

Modeling



Scale-Free Networks from Varying Vertex Intrinsic Fitness

G. Caldarelli,[1] A. Capocci,[2] P. De Los Rios,[3,4] and M. A. Muñoz[5]
[1] *INFM UdR ROMA1 Dipartimento Fisica, Università di Roma “La Sapienza,” Piazzale Aldo Moro 2 00185, Roma, Italy*
[2] *Département de Physique, Université de Fribourg-Pérolles, CH-1700 Fribourg, Switzerland*
[3] *Institut de Physique Théorique, Université de Lausanne, CH-1004 Lausanne, Switzerland*
[4] *INFM UdR Politecnico di Torino, Corso Duca degli Abruzzi 24, 10129 Torino, Italy*
[5] *Instituto de Física Teórica y Computacional Carlos I, Universidad de Granada, Facultad de Ciencias, 18071-Granada, Spain*

(Received 15 July 2002; published 3 December 2002)

- power laws can from from “good-get-richer” in addition to “rich-get-richer”

`https://doi.org/10.1103/PhysRevLett.89.258702`

Question from previous year:
“Is it possible for hubs to exist even where a network doesn’t follow a power law distribution? Meaning, the fact that some nodes will be more connected than other nodes, but without the entire network being scale-free?”

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A note on terminology:

- power law
- scale-free
- hubs

- growth + preferential attachment → power law degree distribution

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- some (but not all) real networks have a power law degree distribution

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- growth $+$ preferential attachment $\rightarrow$ power law degree distribution
- some (but not all) real networks have a power law degree distribution
- diseases spread more easily on networks with power law degree distribution than on other types of networks
- networks with power law degree distribution are robust to random failure but fragile to targeted attack

Feedback: `http://bit.ly/soc204-2021`

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- Gladwell, M. (1999). Six degrees of Lois Weisberg. *The New Yorker*.
- Watts, Chapter 4, 114-129.
- Feld, S.L. (1981) The focused organization of social ties. *American Journal of Sociology*.

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