Simple Mechanical Systems
are
Truly Simple

MECHANICS AND CONTROL WORKSHOP — IITB
18/03/2023

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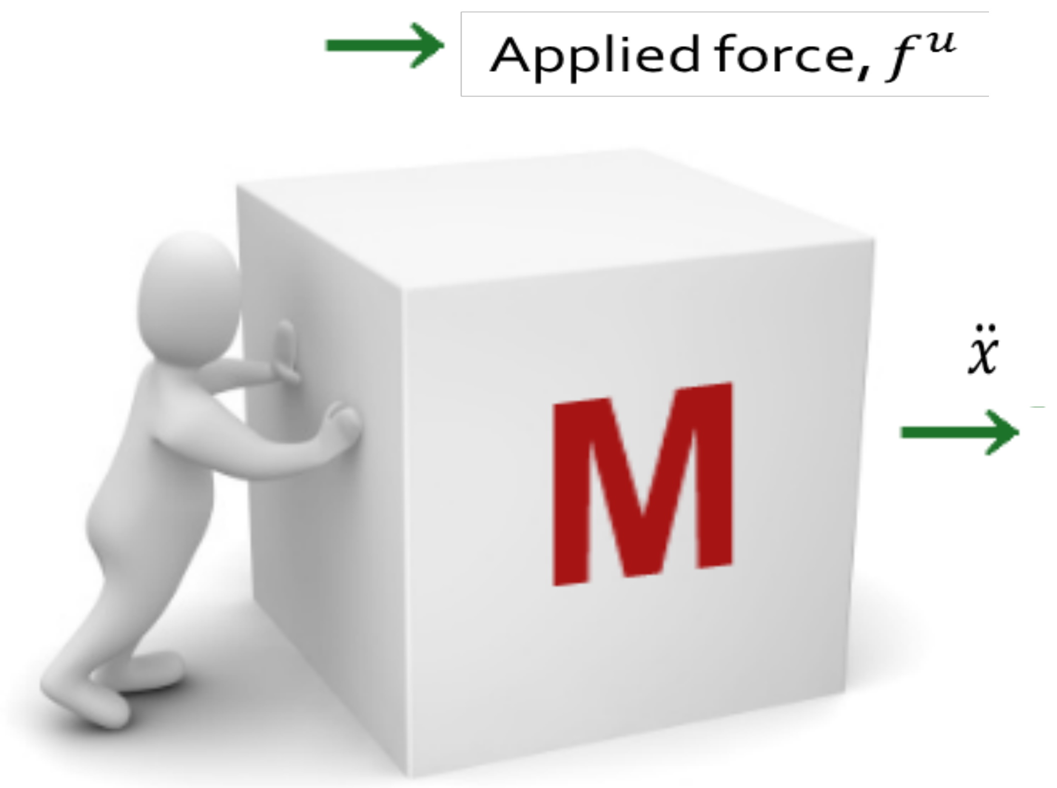
Mechanical Systems on $\mathbb{R}^n$



Configuration Space $\rightarrow$ $\mathbb{R}^n$

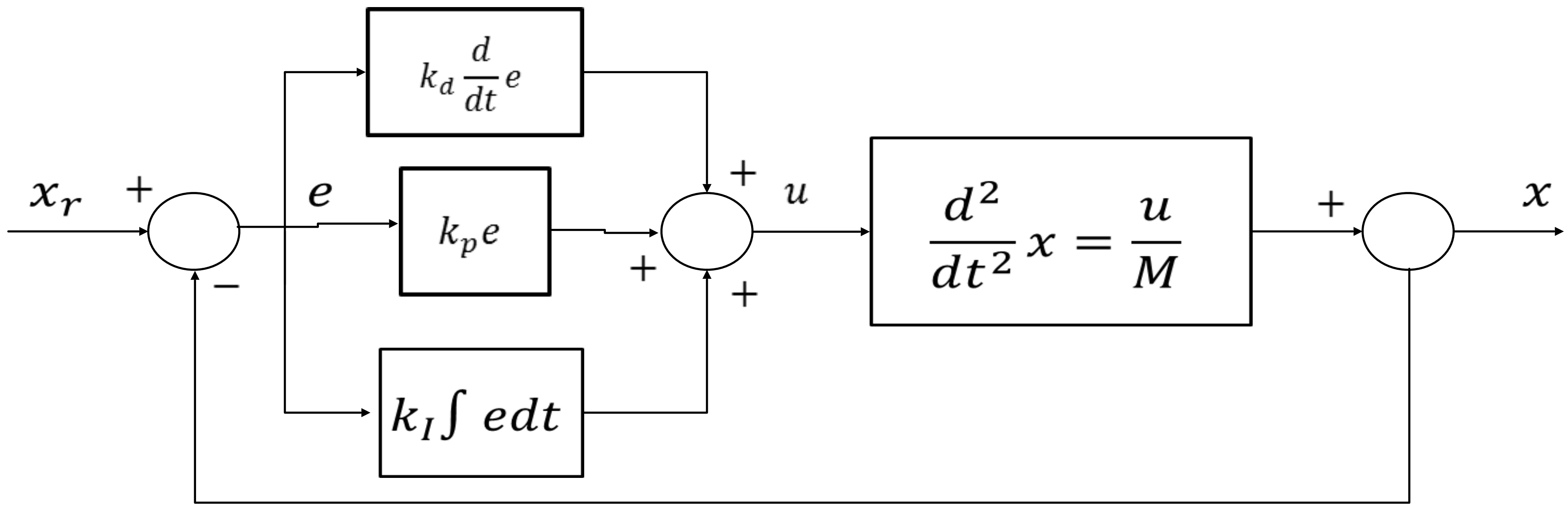
Kinetic Energy $\rightarrow$ $\frac{1}{2}\langle M\dot{x}, \dot{x}\rangle = \frac{M}{2}\dot{x}^T\dot{x}$

Newton's Equations

$$M\ddot{x} = f^d + f^u$$

$$\begin{aligned} \dot{x} &= v \\ M\dot{v} &= f^d + f^u \end{aligned}$$

PID Control on $\mathbb{R}^n$



PID Without Feedforward Control

$$e \triangleq (x - x_r)$$

$$\dot{e}_I = e$$

$$u_{PID} = -M(k_p e + k_d \dot{e} + k_I e_I) + M \ddot{x}_r$$

$$\boxed{\lim_{t\to\infty} x(t) = x_r(t)}$$

Globally Exponentially

- Global stability
- Robust
- Can track constant acceleration references
- Can reject constant input disturbances

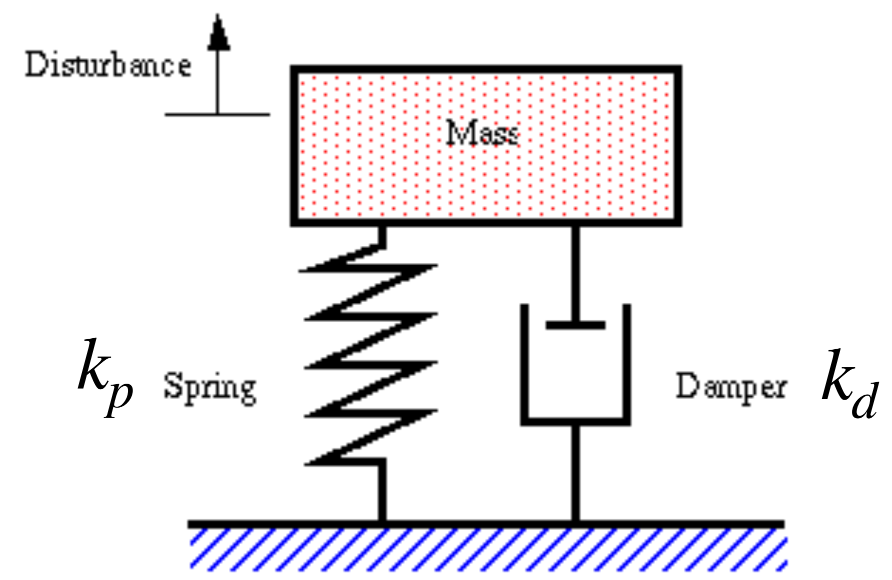
PD Control on $\mathbb{R}^n$

$$e \triangleq (x - x_r)$$

$$f^u = -M(k_p e + k_d \dot{e}) + M\ddot{x}_r$$

$$\ddot{e} + k_p e + k_d \dot{e} = 0$$

Error Dynamics



Momentum error $\rightarrow$ $p_e \triangleq M\dot{e}$

$$W(e, \dot{e}) = \frac{k_p}{2}\langle Me\,, e\rangle + \frac{1}{2}\langle p_e\,, \dot{e}\rangle \geq 0$$

$$f^u = -M(k_p e + k_d \dot{e}) + M\ddot{x}_r \;\Rightarrow\; \frac{dW}{dt} = k_p\langle Me\,, \dot{e}\rangle + \langle M\ddot{e}\,, \dot{e}\rangle = -k_d\langle M\dot{e}\,, \dot{e}\rangle \leq 0$$

LaSalle's Invariance Principle $\Rightarrow$ $\lim_{t\to\infty} e(t) = 0$

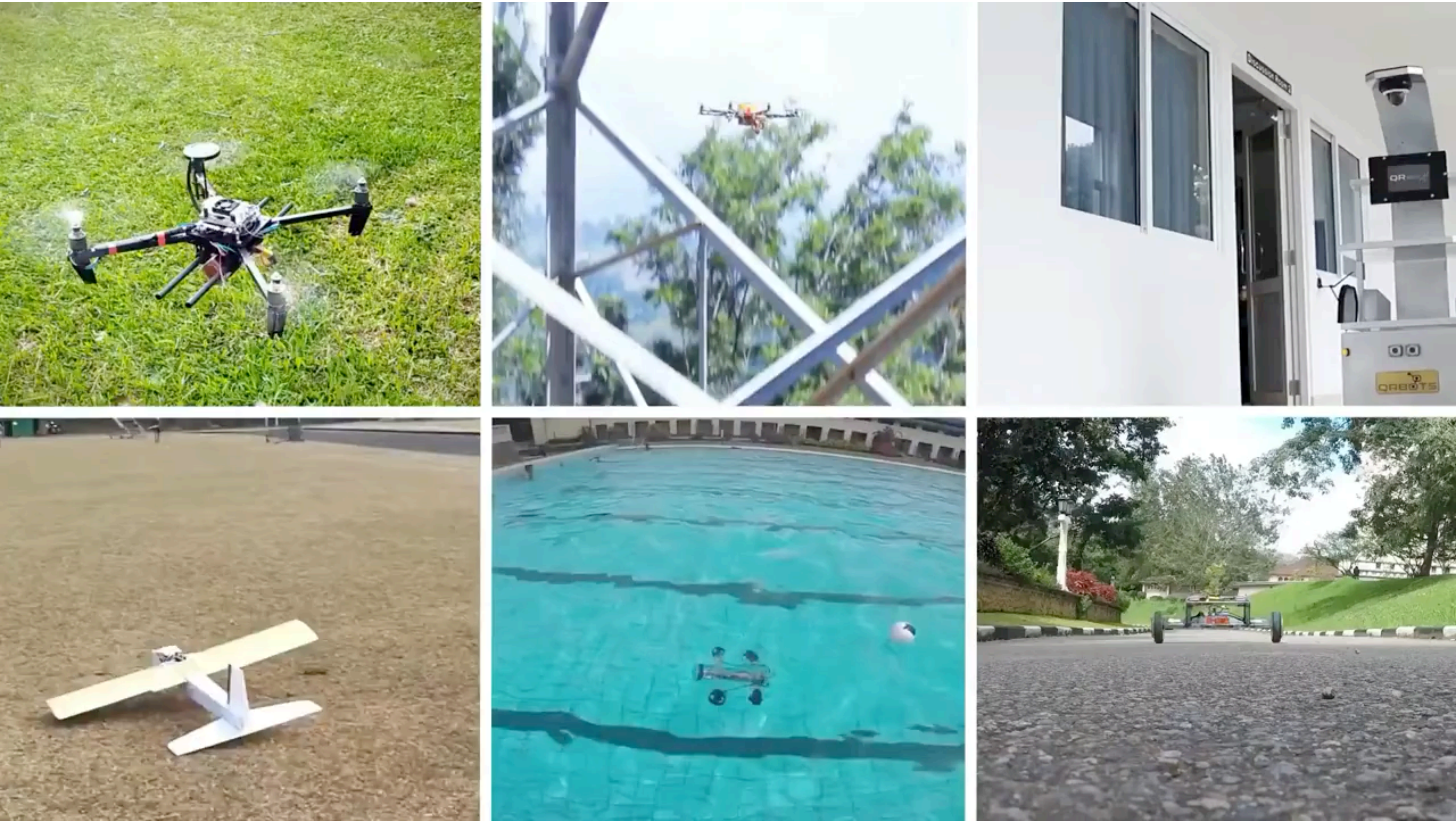
Integral Control $u_I = -k_I e_I$ **Adds Robustness**

Can We Always Use PID ?

Nonlinear Systems are Tricky

https://youtu.be/ZNpi-IHnAh0

What We Have Developed



https://youtu.be/6E9WDQNVSYA

Almost Globally Stable PID

https://youtu.be/zq05N8m_9SA

Fully Autonomous Control and Navigation

https://youtu.be/SQLYFBChrAk

Fully Onboard Autonomous Obstacle Avoidance

CODEGEN
www.codegen.co.uk

Development of an Autonomous Robot Control and Navigation Platform

Faculty of Engineering University of Peradeniya

&

CodeGen International (Pvt) Ltd

https://youtu.be/9dfmOx7TPgs

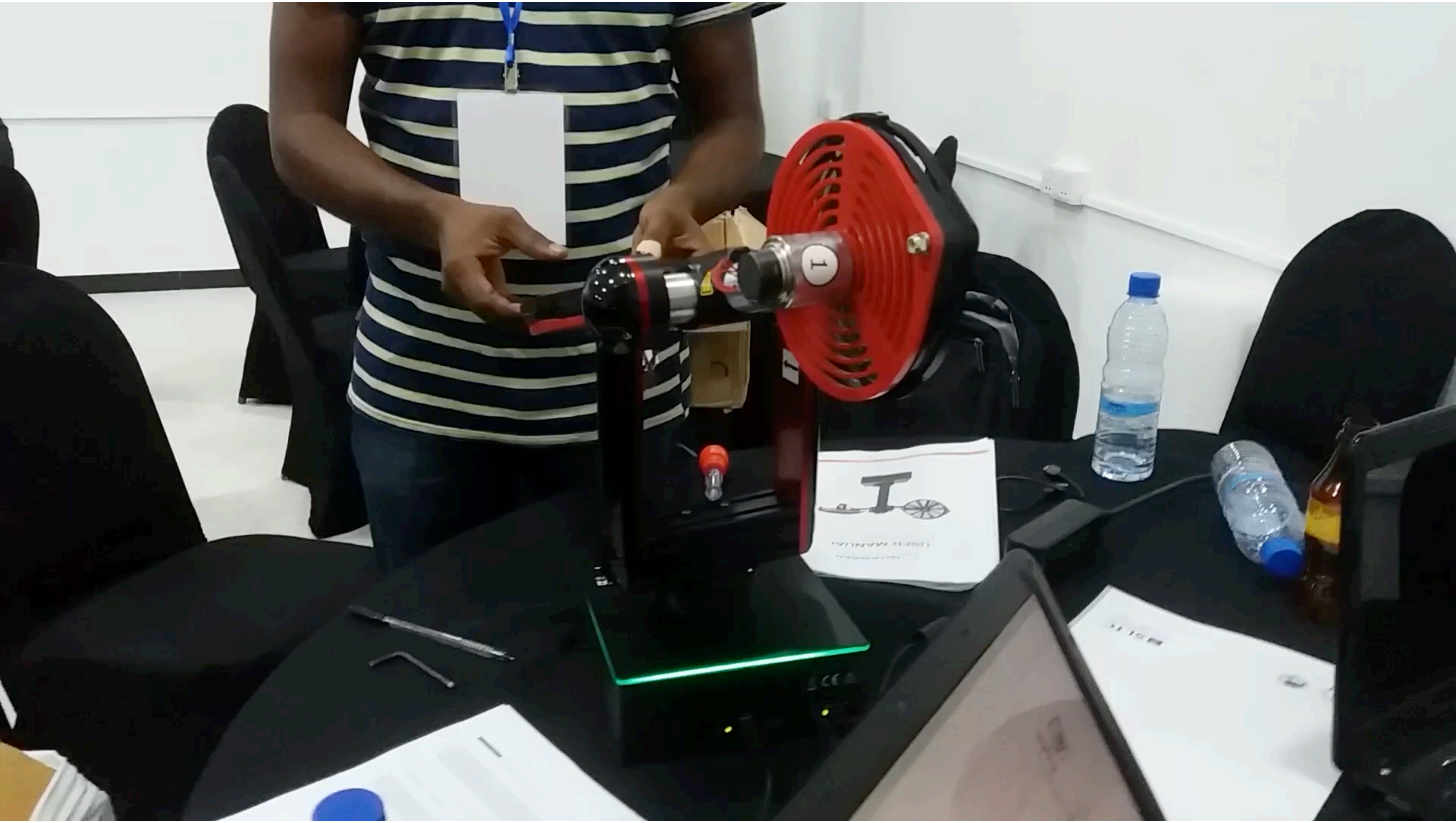
Underwater Vehicle Control and Navigation

https://youtu.be/AZrR0rp7-p0

Balancing Vehicle Control and Navigation

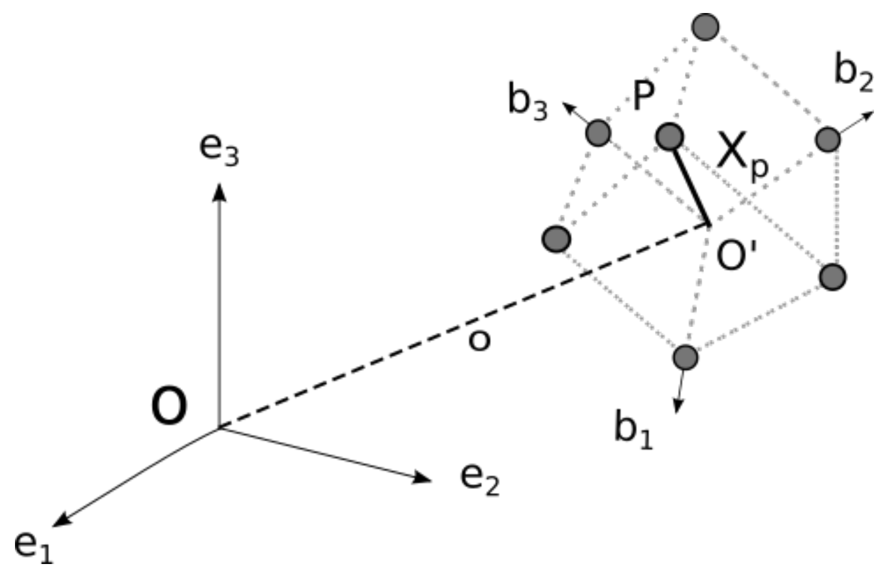
https://youtu.be/uUKxXImRMOA

Control of the QUANSER AEORO



https://youtu.be/J5MMp6Be3tU

Rigid Body Control



$$\pi = \mathbb{I}_R \omega$$
$$\dot{R} = \widehat{\omega} R$$
$$\dot{\pi} = \tau^u$$

OR

$$\dot{R} = R\,\widehat{\Omega}$$
$$\mathbb{I}\dot{\Omega} = \mathbb{I}\Omega \times \Omega + T^u$$

$$\omega = R\Omega$$
$$\tau^u = RT^u$$

Fully actuated

$$\left(R_r(t), \dot{R}_r = R_r\,\widehat{\Omega_r(t)}\right)$$

Reference Trajectory

Control Objective

find T^u such that

$$\lim_{t\to\infty} R(t) = R_r(t)$$

Almost Globally and Locally Exponentially

Intrinsic PD on SO(3)

On $\mathbb{R}^n$

$$\boxed{\begin{gathered} e \triangleq (x - x_r) \\ f^u = -M(k_p e + k_d \dot{e}) + M\ddot{x}_r \\ \ddot{e} + k_p e + k_d \dot{e} = 0 \\ \textbf{Error Dynamics} \end{gathered}}$$

$$V : \mathbb{R}^n \to \mathbb{R} \quad \to \quad V(e) = \frac{k_p}{2}\langle Me\,,e\rangle$$

$$W(e,\dot{e}) = V(e) + \frac{1}{2}\langle M\dot{e}\,,\dot{e}\rangle \geq 0$$

$$f^u = -M(k_p e + k_d \dot{e}) + M\ddot{x}_r$$

$$\frac{dW}{dt} = -k_d\langle M\dot{e}\,,\dot{e}\rangle \leq 0$$

On $SO(3)$

$$\boxed{\begin{gathered} \left(R_r^T \dot{R}_r = R_r \widehat{\Omega_r(t)}\right) \\ \textbf{Reference Trajectory} \end{gathered}} \qquad \boxed{\begin{gathered} E = R_r^T R \\ \textbf{Tracking Error} \end{gathered}}$$

Velocity Error $\to$ $\dot{E} = E \cdot \underbrace{\left(\widehat{\Omega - E^T\Omega_r}\right)}_{\widehat{\Omega}_E}$

$$V : SO(3) \to \mathbb{R} \quad \to \quad V(E) = \frac{k_p}{2}\text{trace}\left(K(I - E)\right)$$

$$K = \text{diag}\{\mathbb{I}_2 + \mathbb{I}_3 - \mathbb{I}_1, \mathbb{I}_3 + \mathbb{I}_1 - \mathbb{I}_2, \mathbb{I}_1 + \mathbb{I}_2 - \mathbb{I}_3\}$$

$$E = I + \sin\theta_e\,\hat{n}_e + (1 - \cos\theta_e)\,\hat{n}_e^2$$

$$V(E) = \alpha_n \frac{k_p}{2}\left(1 - \cos\theta_e\right) \geq 0$$

$$\alpha_n \triangleq \mathbb{I}_1 n_{e_1}^2 + \mathbb{I}_2 n_{e_2}^2 + \mathbb{I}_3 n_{e_3}^2$$

Intrinsic PD on SO(3)

On $\mathbb{R}^n$

$$e \triangleq (x - x_r)$$
$$f^u = -M(k_p e + k_d \dot e) + M\ddot x_r$$
$$\ddot e + k_p e + k_d \dot e = 0$$

Error Dynamics

$$V : \mathbb{R}^n \to \mathbb{R} \quad \to \quad V(e) = \frac{k_p}{2}\langle Me\,, e\rangle$$

$$W(e, \dot e) = V(e) + \frac{1}{2}\langle M\dot e\,, \dot e\rangle \geq 0$$

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On $SO(3)$

$$\left(R_r(t), \dot R_r = R_r \widehat{\Omega_r(t)}\right)$$

Reference Trajectory

$$E = R_r^T R$$

Tracking Error

Velocity Error $\quad \to \quad \dot E = E \cdot \underbrace{\left(\widehat{\Omega - E^T\Omega_r}\right)}_{\widehat{\Omega}_E}$

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$$V(E) = \alpha_n \frac{k_p}{2}\left(1 - \cos\theta_e\right) \geq 0$$

Unique minimum when $\quad E = I_{3\times3}$

Three maxima when $\quad E = \mathrm{diag}\{1, -1, -1\}$

$$E = \mathrm{diag}\{-1, 1, -1\} \quad E = \mathrm{diag}\{-1, -1, 1\}$$

$$\alpha_n \triangleq \mathbb{I}_1 n_{e_1}^2 + \mathbb{I}_2 n_{e_2}^2 + \mathbb{I}_3 n_{e_3}^2$$

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$$\omega_E \triangleq E\Omega_E \quad \pi_E \triangleq E\mathbb{I}\Omega_E \quad \to \quad \langle \pi_E\,, \omega_E,\rangle = \langle \mathbb{I}\Omega_E\,, \Omega_E,\rangle$$

$$W(E, \Omega_E) = V(E) + \frac{1}{2}\langle \mathbb{I}\Omega_E\,, \Omega_E,\rangle \geq 0$$

$$\frac{d}{dt}W(E, \Omega_E) = \frac{d}{dt}V(E) + \frac{1}{2}\frac{d}{dt}\langle \mathbb{I}\Omega_E\,, \Omega_E,\rangle$$

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$$V : SO(3) \to \mathbb{R} \quad \to \quad V(E) = \frac{k_p}{2}\text{trace}\left(K\left(I - E\right)\right)$$

$$\frac{d}{dt}V(E) = -\frac{k_p}{2}\text{trace}(K\dot{E}) = -\frac{k_p}{2}\text{trace}(KE\widehat{\Omega}_E)$$

$$= -\frac{k_p}{2}\text{trace}\left(\underbrace{\frac{(E - E^T)}{2}}_{\widehat{\zeta}_E} K\widehat{\Omega}_E\right) = k_p\,\langle \mathbb{I}\zeta_E\,, \Omega_E\rangle$$

$$K = \text{diag}\{\mathbb{I}_2 + \mathbb{I}_3 - \mathbb{I}_1, \mathbb{I}_3 + \mathbb{I}_1 - \mathbb{I}_2, \mathbb{I}_1 + \mathbb{I}_2 - \mathbb{I}_3\}$$

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$$\frac{d}{dt}V(E) = -\frac{k_p}{2}\text{trace}\left(\underbrace{\frac{(E - E^T)}{2}}_{\widehat{\zeta}_E} K\widehat{\Omega}_E\right)$$

grad of V $\rightarrow$ $\nabla V \triangleq \zeta_E$

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$$\omega_E \triangleq E\Omega_E \qquad \pi_E \triangleq E\mathbb{I}\Omega_E$$

$$\langle \pi_E\,, \omega_E,\rangle = \langle \mathbb{I}\Omega_E\,, \Omega_E,\rangle$$

$$\frac{d}{dt}\langle \pi_E\,, \omega_E,\rangle = \frac{d}{dt}\langle \mathbb{I}\Omega_E\,, \Omega_E\rangle = 2\,\langle \mathbb{I}\dot{\Omega}_E\,, \Omega_E\rangle$$

Intrinsic PD on SO(3)

On $\mathbb{R}^n$

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On $SO(3)$

$$\boxed{\begin{gathered} \left(R_r(t), \dot{R}_r = R_r \widehat{\Omega_r(t)}\right) \\ \textbf{Reference Trajectory} \end{gathered}} \qquad \boxed{\begin{gathered} E = R_r^T R \\ \textbf{Tracking Error} \end{gathered}}$$

Velocity Error $\quad \to \quad \dot{E} = E \cdot \underbrace{\left(\widehat{\Omega - E^T\Omega_r}\right)}_{\widehat{\Omega}_E}$

$$\begin{aligned} \mathbb{I}\dot{\Omega}_E &= \mathbb{I}\dot{\Omega} - \mathbb{I}E^T\dot{\Omega}_r + \mathbb{I}\Omega_E \times (E^T\Omega_r) \\ &= \mathbb{I}\Omega \times \Omega + T^u - \mathbb{I}E^T\dot{\Omega}_r + \mathbb{I}\Omega_E \times (E^T\Omega_r) \end{aligned}$$

$$\begin{aligned} \mathbb{I}\Omega \times \Omega &= \mathbb{I}(\Omega_E + E^T\Omega_r) \times (\Omega_E + E^T\Omega_r) \\ &= \mathbb{I}\Omega_E \times \Omega_E + \mathbb{I}\Omega_E \times E^T\Omega_r + \mathbb{I}E^T\Omega_r \times \Omega_E + \mathbb{I}E^T\Omega_r \times E^T\Omega_r \end{aligned}$$

$$\mathbb{I}\dot{\Omega}_E = \mathbb{I}\Omega_E \times \Omega_E + T^u - F_r(E, \Omega_E, \Omega_r)$$

$$F_r = \mathbb{I}E^T\dot{\Omega}_r - 2\mathbb{I}\Omega_E \times (E^T\Omega_r) - \mathbb{I}E^T\Omega_r \times \Omega_E - \mathbb{I}E^T\Omega_r \times E^T\Omega_r$$

Intrinsic PD on SO(3)

On $\mathbb{R}^n$

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$$f^u = -M(k_p e + k_d \dot{e}) + M\ddot{x}_r$$
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Error Dynamics

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On $SO(3)$

$$\left(R_r(t), \dot{R}_r = R_r\,\widehat{\Omega_r(t)}\right)$$

Reference Trajectory

$$E = R_r^T R$$

Tracking Error

Velocity Error $\to$ $\dot{E} = E \cdot \underbrace{\left(\widehat{\Omega - E^T\Omega_r}\right)}_{\widehat{\Omega}_E}$

grad of V $\to$ $\nabla V \triangleq \zeta_E$

$$\mathbb{I}\dot{\Omega}_E = \mathbb{I}\Omega_E \times \Omega_E + T^u - F_r(E, \Omega_E, \Omega_r)$$

$$\frac{1}{2}\frac{d}{dt}\langle \pi_E\,, \omega_E\rangle = \langle \mathbb{I}\dot{\Omega}_E\,, \Omega_E\rangle = \langle (T^u - F_r)\,, \Omega_E\rangle$$

$$\frac{d}{dt}V(E) = k_p\,\langle \mathbb{I}\zeta_E\,, \Omega_E\rangle$$

$$W(E, \Omega_E) = V(E) + \frac{1}{2}\langle \mathbb{I}\Omega_E\,, \Omega_E,\rangle \geq 0$$

$$\frac{d}{dt}W(E, \Omega_E) = \Big\langle\ (k_p\mathbb{I}\zeta_E + T^u - F_r)\ , \Omega_E \Big\rangle$$

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On $SO(3)$

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grad of V $\rightarrow$ $\nabla V \triangleq \zeta_E$

$$W(E,\Omega_E) = V(E) + \frac{1}{2}\langle \mathbb{I}\Omega_E\,,\Omega_E\rangle$$

$$\frac{d}{dt}W(E,\Omega_E) = \left\langle\ (k_p\mathbb{I}\zeta_E + T^u - F_r)\,,\Omega_E\right\rangle$$

Intrinsic PD on SO(3)

On $\mathbb{R}^n$

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On $SO(3)$

$$\left(R_r(t), \dot{R}_r = R_r\,\widehat{\Omega_r(t)}\right)$$

Reference Trajectory

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Tracking Error

Velocity Error $\quad \to \quad \dot{E} = E \cdot \underbrace{\left(\widehat{\Omega - E^T\Omega_r}\right)}_{\widehat{\Omega}_E}$

grad of V $\quad \to \quad \nabla V \triangleq \zeta_E$

$$\frac{d}{dt}W(E, \Omega_E) = \Big\langle\, (k_p \mathbb{I}\zeta_E + T^u - F_r)\,,\Omega_E \Big\rangle$$

$$T^u = -\mathbb{I}(k_p\zeta_E + k_d\Omega_E) + F_r$$

$$\frac{d}{dt}W(E, \Omega_E) = -k_d\langle \mathbb{I}\Omega_E\,,\Omega_E\rangle \leq 0$$

Intrinsic PD on SO(3)

$$\left(R_r(t), \dot{R}_r = R_r \widehat{\Omega_r(t)}\right)$$

Reference Trajectory

$$E = R_r^T R$$

Tracking Error

Velocity Error $\rightarrow$ $\dot{E} = E \cdot \underbrace{\left(\Omega \widehat{- E^T} \Omega_r\right)}_{\widehat{\Omega}_E}$

grad of V $\rightarrow$ $\nabla V \triangleq \zeta_E$

$$V : SO(3) \rightarrow \mathbb{R} \quad \rightarrow \quad V(E) = \frac{k_p}{2} \text{trace}\left(K\left(I - E\right)\right)$$

$$W(E, \Omega_E) = V(E) + \frac{1}{2} \langle \mathbb{I} \Omega_E, \Omega_E \rangle$$

$$T^u = -\mathbb{I}(k_p \zeta_E + k_d \Omega_E) + F_r$$

Intrinsic PD control

$$\frac{d}{dt} W(E, \Omega_E) = -k_d \langle \mathbb{I} \Omega_E, \Omega_E \rangle \leq 0$$

LaSalle's Invariance Principle

$$\lim_{t \to \infty} R(t) = R_r(t)$$

For all ICs except three !

$$\mathbb{I} \dot{\Omega}_E = \mathbb{I} \Omega_E \times \Omega_E - (k_p \zeta_E + k_d \Omega_E)$$

$$\mathbb{I} \nabla_{\Omega_E} \Omega_E \triangleq \mathbb{I} \dot{\Omega}_E - \mathbb{I} \Omega_E \times \Omega_E$$

$$\mathbb{I} \left(\nabla_{\Omega_E} \Omega_E + k_p \zeta_E + k_d \Omega_E \right) = 0$$

Error Dynamics on $SO(3)$

Intrinsic PID

Velocity Error $\rightarrow$ $\dot{E} = E \cdot \underbrace{\left(\widehat{\Omega - E^T\Omega_r}\right)}_{\widehat{\Omega}_E}$

grad of V $\rightarrow$ $\nabla V \triangleq \zeta_E$

$$
\begin{aligned}
\mathbb{I}\,\nabla_{\zeta_E}\Omega_I &= \mathbb{I}\zeta_E, \\
T^u &= -\,\mathbb{I}(k_p\zeta_E + k_d\Omega_E + k_I\Omega_I)
\end{aligned}
$$

$$V : SO(3) \rightarrow \mathbb{R} \quad \rightarrow \quad V(E) = \frac{k_p}{2}\mathrm{trace}\left(K\,(I - E)\right)$$

$$W(E, \Omega_E, \Omega_I) = V(E) + \frac{1}{2}\langle \mathbb{I}\Omega_E\,, \Omega_E\rangle + \frac{\gamma}{2}\langle \mathbb{I}\Omega_I\,, \Omega_I\rangle + \text{cross terms}$$

Works even for problems with singular Inertia Tensors !

D.H.S. Maithripala and J.M. Berg, "**An intrinsic robust PID on Lie groups**", IEEE 53rd Annual Conference on CDC, USA, 2014

D.H.S. Maithripala and J.M. Berg, "**An intrinsic PID controller for mechanical systems on Lie groups**", Automatica, vol. 54, no. 0, pp. 89-200, 2015

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Thank You