CSE 211: Discrete Mathematics **(Due: 17/01/21)**

Homework #4

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Course Policy: Read all the instructions below carefully before you start working on the assignment, and before you make a submission.

- It is not a group homework. Do not share your answers to anyone in any circumstance. Any cheating means at least -100 for both sides.
- Do not take any information from Internet.
- No late homework will be accepted.
- For any questions about the homework, send an email to gizemsungu@gtu.edu.tr
- The homeworks (both latex and pdf files in a zip file) will be submitted into the course page of Moodle.
- The latex, pdf and zip files of the homeworks should be saved as "Name_Surname_StudentId".{tex, pdf, zip}.
- If the answers of the homeworks have only calculations without any formula or any explanation -when needed- will get zero.
- Writing the homeworks on Latex is strongly suggested. However, hand-written paper is still accepted **IFF** hand writing of the student is clear and understandable to read, and the paper is well-organized. Otherwise, the assistant cannot grade the student's homework.

Problem 1 (15+15=30 points)

Consider the nonhomogeneous linear recurrence relation $a_n = 3a_{n-1} + 2^n$.

(a) Show that whether $a_n = -2^{n+1}$ is a solution of the given recurrence relation or not. Show your work step by step.
(Solution)
Step -1) If $a_n = -2^{n+1}$ than we could say $a_{n-1} = -2^n$
Step -2) We have a_n and a_{n-1}. We put those in to recurrence relation
Step -3) $-2^{n+1} = 3*(-2^n) + -2^n$
Step -4) $-2^{n+1} = 2*-2^n$ from then $2^{n+1} = 2*2^n$
Step -5) Equation simply 2 = 2, from that. We can say it is a recurrence relation.

(b) Find the solution with $a_0 = 1$.
(Solution)
Step 1) $a_n = a^p + a^h$
Step 2) First of all for a^h a_n - $-2^{n+1} = 0$
Step 3) r - 3 = 0 so r = 3
Step 4) $a^h = a_n^3$
Step 5) Secondly, for a^p, a_n - 3a_n-1 = 2n
Step 6) $A2^n$ - $3A^{2n1} = 2^n$ from that -¿ A - 3/2A = 1 so A = -2
Step 7) $a_n^p = 2^{2n-1}$
Step 8) a_n = a3^n - 2^{n+1} for ($n >= 1$)
Step 9) For $a_0 = 1$ in question so a_1 = 3*1+2 = 5
Step 10) For $a_1 = 5$ from that so a_1 = a3^1 - 2^2 so a = 3
Step 11) Solution for $a_0 = 1$ is $a_n = 3^n + 1$ - $2^n + 1$ (n ¿= 1)

Problem 2 (35 points)

Solve the recurrence relation f(n) = 4f(n-1) - 4f(n-2) + n^2 for f(0) = 2 and f(1) = 5.
(Solution)
Step 1) f(n) - 4f(n-1) + 4f(n-2) = n^2
Step 2) f(n) = f^h(n) + f^p(n)
Step 3) Charactaristic equation -¿ $r2$ - 4r + 4 = 0
Step 4) So r=2 -¿ a_n^h = 2a^n + $B2^n$
Step 5) Kn^2 + Ln + M form
Step 6) Kn^2 + Ln + M - 4[K(n-1)^2 + L(n-1) + M -K(n-2)^2 - L(n-2) - M]
Step 7) n^2 = K(n^2 - 4(n-1)^2 + (n-2)^2) + L(n-4n+4+n-2) + 3M
Step 8) After solving equiation we get K = 1, L = 8, M = 20
Step 9) After replacing values f(n) = n^2 + 8n + 20 + 2^n(6n-18)
Step 10) f(0) = 2 -¿ 20 + 2 = 2 from that a = 18 -¿ f(1) = 5 so $B = 6$
Step 11) Finally f(n) = n^2 + 8n + 20 +2^n(6n-18)

Problem 3 (20+15 = 35 points)

Consider the linear homogeneous recurrence relation $a_n = 2a_{n-1}$ - $2a_{n-2}$.
(a) Find the characteristic roots of the recurrence relation.
(Solution)
Step 1) a_n - $2a_{n-1}$ + $2a_{n-2}$ = 0
Step 2) $2a_{n-2}$ as a constant as 2 for solving
Step 3) transforms to -¿ r^2 -2r + 2 = 0
Step 4) and transforms to (r-1)^2+2= 0
Step 5) (r-1) = i or -(r-1) = i cause i = $\sqrt{-1}$
Step 6) Finally r = 1- i V r = 1 + i

(b) Find the solution of the recurrence relation with $a_0 = 1$ and $a_1 = 2$.
(Solution)
Step 1) a(1-i)^n + B(1+i)^n = a_n
Step 2) a_0 = a + B = 1 and a_1 = a + B + i(B -a) = 2
Step 3) a + B = 1 then, B - a would be = -i
Step 4) $B = (1-i)/2$ and a = 1 - $(1-i)/2 = (1+i)/2$
Step 5) Finally $a_n = (i+1)/2$(1-i)^n + $(1-i)/2$(1+n)^n