CSE 211: Discrete Mathematics **(Due: 17/11/20)**

Homework #1

Instructor: Dr. Zafeirakis Zafeirakopoulos
Assistant: Gizem Süngü

Course Policy: Read all the instructions below carefully before you start working on the assignment, and before you make a submission.

- It is not a group homework. Do not share your answers to anyone in any circumstance. Any cheating means at least -100 for both sides.
- Do not take any information from Internet.
- No late homework will be accepted.
- For any questions about the homework, send an email to gizemsungu@gtu.edu.tr
- The homeworks (both latex and pdf files in a zip file) will be submitted into the course page of Moodle.
- The latex, pdf and zip files of the homeworks should be saved as "Name_Surname_StudentId".{tex, pdf, zip}.
- If the answers of the homeworks have only calculations without any formula or any explanation -when needed- will get zero.
- Writing the homeworks on Latex is strongly suggested. However, hand-written paper is still accepted **IFF** hand writing of the student is clear and understandable to read, and the paper is well-organized. Otherwise, the assistant cannot grade the student's homework.

Problem 1: Conditional Statements (5+5+5=15 points)

State the converse, contrapositive, and inverse of each of these conditional statements.

(a) If it snows tonight, then I will stay at home.
(Solution)

Converse: If I stay at home, it snows tonight.

Contrapositive: If I wont stay at home, It wont snows tonight

Inverse: If it wont snows tonight, than I wont stay at home

(b) I go to the beach whenever it is a sunny summer day.
(Solution)

Converse: Whenever it is a sunny summer day, I go to the beach

Contrapositive: Whenever it is not sunny a summer day, I do not go to the beach

Inverse: I do not go to the beach, whenever it is not a sunny summer day

(c) If I stay up late, then I sleep until noon.
(Solution)

Converse: If I sleep until noon,, then I stay up late

Contrapositive: If I do not sleep until noon, then I do not stay up late

Inverse: If I do not stay up late, then I do not sleep until noon.

Problem 2: Truth Tables For Logic Operators (5+5+5=15 points)

Construct a truth table for each of the following compound propositions.

(a) (p $\oplus$ ¬ q)
(Solution)
Table 1: (p $\oplus$ ¬ q)

p	q	¬ q	p $\oplus$ ¬ q
1	1	0	1
1	0	1	0
0	1	0	0
0	0	1	1

(b) (p $\iff$ q) $\oplus$ (¬ p $\iff$ ¬ r)
(Solution)
Table 2: (p $\iff$ q) $\oplus$ (¬ p $\iff$ ¬ r)

p	q	r	¬ p	¬ r	p $\iff$ q	¬ p $\iff$ ¬ r	(p $\iff$ q) $\oplus$ (¬ p $\iff$ ¬ r)
1	1	1	0	0	1	1	0
1	1	0	0	1	1	0	1
1	0	1	0	0	0	1	1
1	0	0	0	1	0	0	0
0	1	1	1	0	0	0	0
0	1	0	1	1	0	1	1
0	0	1	1	0	1	0	1
0	0	0	1	1	1	1	0

(c) (p $\oplus$ q) $\Rightarrow$ (p $\oplus$ ¬ q)
(Solution)
Table 3: (p $\oplus$ q) $\Rightarrow$ (p $\oplus$ ¬ q)

p	q	¬ q	p $\oplus$ q	p $\oplus$ ¬ q	(p $\oplus$ q) $\Rightarrow$ (p $\oplus$ ¬ q)
1	1	0	0	1	1
1	0	1	1	0	0
0	1	0	1	0	0
0	0	1	0	1	1

Problem 3: Predicates and Quantifiers (21 points)

There are three predicate logic statements which represent English sentences as follows.

- P(x): ”x can speak English.”
- Q(x): ”x knows Python.”
- H(x): ”x is happy.”

Express each of the following sentences in terms of P(x), Q(x), H(x), quantifiers, and logical connectives or vice versa. The domain for quantifiers consists of all students at the university.

(a) There is a student at the university who can speak English and who knows Python.
(Solution) $\exists$x(P(x)$\wedge$Q(x))

(b) There is a student at the university who can speak English but who doesn't know Python.
(Solution) $\exists$x(P(x)$\wedge$ $\neg$Q(x))

(c) Every student at the university either can speak English or knows Python.
(Solution) $\forall$x(P(x)$\vee$Q(x))

(d) No student at the university can speak English or knows Python.
(Solution) $\neg\forall$x(P(x)$\vee$Q(x))

(e) If there is a student at the university who can speak English and know Python, then she/he is happy.
(Solution) (P(x)$\wedge$Q(x)) $\Rightarrow$ H(x)

(f) At least two students are happy.
(Solution) $\exists$xH(x) $\wedge$ $\exists$yH(y)

(g) $\neg\forall x(Q(x) \wedge P(x))$
(Solution) There are no student in university who speaks English and knows python

Problem 4: Mathematical Induction (21 points)

Prove that $3 + 3 \ . \ 5 + 3 \ . \ 5^2 + \ . \ . \ . + 3 \ . \ 5^n = \frac{3(5^{n+1}-1)}{4}$ whenever n is a non-negative integer.
(Solution)

Basic Steps:
for n = 0;
$3*5^0 = \frac{3(5^{0+1}-1)}{4}$
3 = 3
It means n = 0 is true

Let assume that n = k is true;
$3 + 3*5 + 3*5^2 + ... + 3*5^k = \frac{3(5^{k+1}-1)}{4}$
Apply n = k+1 is true and prove that based on n=k is true;

$3 + 3{*}5 + 3{*}5^2 + ... + 3{*}5^k + 3{*}5^{k+1} = \frac{3(5^{k+1+1}-1)}{4}$
from n = k we can say that
$\frac{3(5^{k+1}-1)}{4} + 3.5^{k+1} = \frac{3(5^{k+2}-1)}{4}$
$3.5^{k+1} = \frac{3(5^{k+2}-1)}{4}$ - $\frac{3(5^{k+1}-1)}{4}$
Some mathematical operations applied and than
$3.5^{k+1} = 3.5^{k+1}$ ✓
Conclusion: This equation is true

Problem 5: Mathematical Induction (20 points)

Prove that n^2 - 1 is divisible by 8 whenever n is an odd positive integer.
(Solution)
Assume that this exploration is true for smallest odd number n = 1
$1^2 - 1 = 0$ it is divisible to 8
if n is odd also n+2 is odd from that
$(n + 2)^2$ - 1 = n^2 +4n + 4-1
= n^2 -1 + 4(n+1)
we assume that left side is true and divisible to 8, what about right side
for all odd n numbers right side will be 4*(an even number) = 4*2.(..)
That means it will be divisible to 8 for all odd n numbers too.
Conclusion: This equation is true

Problem 6: Sets (8 points)

Which of the following sets are equal? Show your work step by step.

(a) {t : t is a root of $x^2 - 6$x + 8 = 0}

(b) {y : y is a real number in the closed interval [2, 3]}

(c) {4, 2, 5, 4}

(d) {4, 5, 7, 2} - {5, 7}

(e) {q: q is either the number of sides of a rectangle or the number of digits in any integer between 11 and 99}

(Solution)
from a) roots of this equation is 2 and 4 so t = {2,4}
from b) Y is real numbers in this interval so it could have infinite elements in this set
from c) this set could write like {2,4,5}, it is not important and not have to specify an element more than once
from d) {4, 5, 7, 2}-{5, 7} = {2,4} from e) rectangle has 4 sides and all numbers between 11 and 99 has 2 digits so this set is {2,4}
Conclusion: (a) (d) and (e) are equal sets

Problem Bonus: Logic in Algorithms (20 points)

Let p and q be the statements as follows.

- **p:** It is sunny.
- **q:** The flowers are blooming.

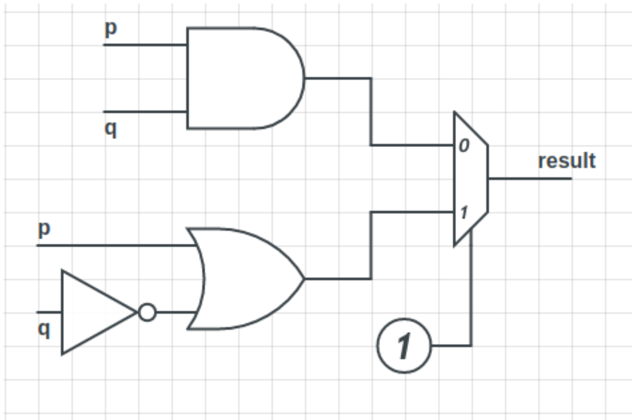

Figure 1: Combinational Circuit

In Figure 1, the two statements are used as input. The circuit has 3 gates as AND, OR and NOT operators. It has also a 2x1 multiplexer[1] which provides to select one of the two options.

(a) Write the sentence that "result" output has.

(Solution)

This logic gate is equal to

(p $\wedge$q) 2x1MUX (p$\vee$ $\neg$q)

2x1MUX selected as 1 so result is = (p$\vee$ $\neg$q)

This sentence is: It is sunny or the flowers are not blooming.

(b) Convert Figure 1 to an algorithm which you can write in any programming language that you prefer (including pseudocode).

(Solution) A C++ code example

```
#include <iostream>
using namespace std;
bool logicGate(bool p, bool q);

int main(void)
{
    bool p,q;
    cin >> p >> q;
    cout << logicGate(p,q) << endl;
}

bool logicGate(bool p, bool q)
{
    if (p == 1 || !q == 1)
        return 1;
    else
        return 0;
}
```

[1] https://www.geeksforgeeks.org/multiplexers-in-digital-logic/