MODULE *LearnerGraph*

A learner graph is a record with four fields:

- learners: a set of learners
- acceptors: a set of acceptors
- *quorums*: a function from learners to sets of sets of acceptors
- *safeSets*: a function from pairs of learners to sets of sets of acceptors

$$Reverse(p) \triangleq \langle p[2],\ p[1] \rangle$$

$$\begin{aligned}
&IsLearnerGraph(lg) \triangleq \\
&\quad \land\ lg.quorums \in [lg.learners \to \text{SUBSET SUBSET } lg.acceptors] \\
&\quad \land\ lg.safeSets \in [lg.learners \times lg.learners \to \text{SUBSET SUBSET } lg.acceptors] \\
&\quad \land\ \forall\, p \in lg.learners \times lg.learners : lg.safeSets[p] = lg.safeSets[Reverse(p)]
\end{aligned}$$

$$\begin{aligned}
&IsValidLearnerGraph(lg) \triangleq \\
&\quad \land\ IsLearnerGraph(lg) \\
&\quad \land\ \forall\, l1,\ l2 \in lg.learners : l1 \neq l2 \Rightarrow \\
&\qquad\quad \forall\, s\ \ \in lg.safeSets[\langle l1,\ l2 \rangle] : \\
&\qquad\qquad \forall\, q1 \in lg.quorums[l1] : \\
&\qquad\qquad\quad \forall\, q2 \in lg.quorums[l2] : \\
&\qquad\qquad\qquad s \cap q1 \cap q2 \neq \{\}
\end{aligned}$$