MODULE *LearnerGraph*

A learner graph is a record with four fields:

- learners: a set of learners
- acceptors: a set of acceptors
- *quorums*: a function mapping a learner to its minimal *quorums*
- *safeSets*: a function mapping a pair of learners $l1$ and $l2$ to their minimal safe sets.

$Reverse(p) \triangleq \langle p[2],\ p[1]\rangle$
$X \subset Y \triangleq X \neq Y \land X \subseteq Y$

$IsLearnerGraph(lg) \triangleq$
NOTE *quorums* and safe sets should be minimal
$\land\ lg.quorums \in [lg.learners \to \text{SUBSET SUBSET } lg.acceptors]$
$\land\ \forall\, l \in lg.learners : \forall\, Q1 \in lg.quorums[l] : \neg(\exists\, Q2 \in lg.quorums[l] : Q2 \subset Q1)$
$\land\ lg.safeSets \in [lg.learners \times lg.learners \to \text{SUBSET SUBSET } lg.acceptors]$
$\land\ \forall\, p \in lg.learners \times lg.learners :$
$\quad \land\ lg.safeSets[p] = lg.safeSets[Reverse(p)]$
$\quad \land\ \forall\, s1 \in lg.safeSets[p] : \neg(\exists\, s2 \in lg.safeSets[p] : s2 \subset s1)$

$IsValidLearnerGraph(lg) \triangleq$
$\land\ IsLearnerGraph(lg)$
$\land\ \forall\, l1,\ l2 \in lg.learners : l1 \neq l2 \Rightarrow$
$\quad \forall\, s \in lg.safeSets[\langle l1,\ l2\rangle] :$
$\quad\quad \forall\, q1 \in lg.quorums[l1] :$
$\quad\quad\quad \forall\, q2 \in lg.quorums[l2] :$
$\quad\quad\quad\quad s \cap q1 \cap q2 \neq \{\}$

$Condensed(lg) \triangleq \forall\, l1,\ l2,\ l3 \in lg.learners :$
$l1 \neq l2 \land l2 \neq l3 \land l1 \neq l3 \Rightarrow$
$\forall\, s1 \in lg.safeSets[\langle l1,\ l2\rangle] :$
$\forall\, s2 \in lg.safeSets[\langle l2,\ l3\rangle] :$
$\exists\, s3 \in lg.safeSets[\langle l1,\ l3\rangle] : s3 \subseteq s1 \cup s2$