Statistical Inference: Project Phase I

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Question 0

Student's Performance includes various information about a sample of students studying in two different *schools*.
A sense of responsibility towards one's education and academic future is a notable information which can be mined from each individual's *study time* and their rate of *going out* which has an effect on their *failures* and their *grades*.
This dataset also contains some semi-relevent factors like each student's parent's job as well as their love life.

X	school	sex	age	Fjob	Mjob	goout	internet	romantic	studytime	failures	health	absences	G1	G2	G3
0	GP	F	18	teacher	at_home	4	no	no	2	0	3	6	5.000000	7.529856	9.289229
1	GP	F	17	other	at_home	3	yes	no	2	0	3	4	5.000000	7.192039	9.424835
2	GP	F	15	other	at_home	2	yes	no	2	3	3	10	3.807703	8.000000	7.354029
3	GP	F	15	services	health	2	yes	yes	3	0	5	2	15.000000	16.373208	17.796916
4	GP	F	16	other	other	2	no	no	2	0	5	4	6.000000	12.138542	12.800024
5	GP	M	16	other	services	2	yes	no	2	0	5	10	15.000000	16.804680	18.347259
6	GP	M	16	other	other	4	yes	no	2	0	3	0	12.000000	13.691091	14.187810
7	GP	F	17	teacher	other	4	no	no	2	0	1	6	6.000000	6.794185	9.012740
8	GP	M	15	other	services	2	yes	no	2	0	1	0	16.000000	19.852952	20.000000
9	GP	M	15	other	other	1	yes	no	2	0	5	0	14.000000	17.180466	18.073614
10	GP	F	15	health	teacher	3	yes	no	2	0	2	0	10.000000	9.609179	11.950918

Figure 1: Head of the dataset

Question 1

Chosen Categorical Variables : *sex* and *Mjob*

a.

In this part we indent to compare the proportion of mothers who are *teachers* between *Male* and *Female* students.

Conditions for inference for comparing two independent proportions :

- Independence :
  - random sample/assignment
  - if sampling without replacement, $n < 10\%$ of population
- Sample size / skew : samples should meet the success-failure condition (at least 10 *successes* and 10 *failures*) :
  - $n_1\hat{p_1} \geq 10 \rightarrow n_1\hat{p_1} = 200 \times 0.053 = 10.6 \geq 10$
  - $n_1(1-\hat{p_1}) \geq 10 \rightarrow n_1(1-\hat{p_1}) = 200 \times 0.947 = 189.4 \geq 10$
  - $n_2\hat{p_2} \geq 10 \rightarrow n_2\hat{p_2} = 200 \times 0.126 = 25.2 \geq 10$
  - $n_2(1-\hat{p_2}) \geq 10 \rightarrow n_2(1-\hat{p_2}) = 200 \times 0.874 = 174.8 \geq 10$

All is met.

$$\text{Confidence Interval} : \textit{point estimate } \pm \textit{margin of error} \longrightarrow \hat{p_1} - \hat{p_2} \pm z^{\star} SE_{\hat{p_1}-\hat{p_2}}$$

$$SE_{\hat{p_1}-\hat{p_2}} = \sqrt{\frac{\hat{p_1}(1-\hat{p_1})}{n_1} + \frac{\hat{p_2}(1-\hat{p_2})}{n_2}} = 0.047$$

$$\text{Confidence Interval} : (0.0655, 0.0811)$$

If we take repeated samples from this population, and make a confidence interval using each sample, we expect about 95% of the resulting confidence intervals to contain $\hat{p_1} - \hat{p_2}$.

We are 95% confident that the difference of population proportion of g*Male* and *Female* students whose mother's job is *teachers* is between 0.0655 and 0.0811.

Other confidence intervals can be computed accordingly : mothers who are *at-home* between *Male* and *Female* students.

$$\text{Confidence Interval} : (-0.0632, -0.0567)$$

We are 95% confident that the difference of population proportion of g*Male* and *Female* students whose mother's job is *being home* is between -0.0632 and -0.0567.

mothers who are *health* between *Male* and *Female* students.

$$\text{Confidence Interval} : (-0.0239, -0.016)$$

We are 95% confident that the difference of population proportion of g*Male* and *Female* students whose mother's job is *health* is between -0.0239 and -0.016.

Question 1 continued on next page. . .

mothers who are *services* between *Male* and *Female* students.

$$\text{Confidence Interval}: (-0.040, -0.019)$$

We are 95% confident that the difference of population proportion of g*Male* and *Female* students whose mother's job is *services* is between −0.040 and −0.019.

b.

To test the independence, I tested my hypothesis using 2 different methods :
H_0 : *Mother's job is independent from sex of the student. (Mother's job does not vary with the sex of the child)*
H_A : *Mother's job is dependent to sex of the student. (Mother's job varies with the sex of the child)*

Method 1 : Pooling :

$$\hat{p_{pool}} = \frac{\#\ \text{success}}{\#\ \text{total}} = 0.085$$
$$SE_{\hat{p_1}-\hat{p_2}} = \sqrt{\frac{\hat{p_{pool}}(1-\hat{p_{pool}})}{n_1} + \frac{\hat{p_{pool}}(1-\hat{p_{pool}})}{n_2}} = 0.039$$

Conditions for inference for comparing two independent proportions (pooling) :

- Independence :
  - random sample/assignment
  - if sampling without replacement, $n < 10\%$ of population
- Sample size / skew : samples should meet the success-failure condition (at least 10 *successes* and 10 *failures*) :
  - $n_1\hat{p_{pool}} \geq 10 \rightarrow n_1\hat{p_{pool}} = 200 \times 0.085 = 17 \geq 10$
  - $n_1(1-\hat{p_{pool}}) \geq 10 \rightarrow n_1(1-\hat{p_{pool}}) = 200 \times 0.947 = 183 \geq 10$
  - $n_1\hat{p_{pool}} \geq 10 \rightarrow n_1\hat{p_{pool}} = 200 \times 0.085 = 17 \geq 10$
  - $n_1(1-\hat{p_{pool}}) \geq 10 \rightarrow n_1(1-\hat{p_{pool}}) = 200 \times 0.947 = 183 \geq 10$

All is met.

Due to the fact that p-value (0.105) is larger than 0.05 ,we fail to reject the null hypothesis. $\longrightarrow$ There are evidence that *Mother's job (teaching specifically) does not vary with the sex of the child .*

Method 2 : χ^2 test :

$$Expected\ count = \frac{row\ total \times column\ total}{table\ total}$$

$$text\ statistic : \frac{point\ estimate\ \text{-}\ null\ value}{SE\ of\ point\ estimate}$$

$$\chi^2 = \sum_{i=1}^{k} \frac{(O-E)^2}{E} \qquad and \qquad df = (R-1)(C-1)$$

Conditions for χ^2 test :

- Independence :
  - random sample/assignment
  - if sampling without replacement, $n < 10\%$ of population
  - each case only contributes to one cell in the table other (non-paired)
- Sample size : Each particular scenario (i.e.cell) must have atleast 5 expected cases. $\rightarrow \times$

```
   Fjob
sex at_home health other services teacher
  F       5      6    61       33       6
  M       4      5    55       16       9
```

Figure 2: dataset table

Which will give out the following warning :

```
Chi-squared approximation may be incorrect
        Pearson's Chi-squared test

data:  sp.sampled.table
X-squared = 4.6465, df = 4, p-value = 0.3255
```

Figure 3: χ^2 test

For our last condition to meet, we have to merge two columns, *at-home* and *health* :

```
      other services teacher
F 11     61       33       6
M  9     55       16       9
```

Figure 4: dataset table

So our χ^2 test won't give any warnings :

```
        Pearson's Chi-squared test

data:  sp.sampled.table.bind
X-squared = 4.6445, df = 3, p-value = 0.1998
```

Figure 5: χ^2 test

Either way, due to the fact that p-value is larger than 0.05 , we fail reject the null hypothesis. $\longrightarrow$ There are evidence that *Mother's job does not vary with the sex of the child .*

Note : It's important to mention that in hypothesis testing in categorical variables, CI approach and p-value approach might not always give out the same result.

Question 2

Chosen Categorical Variable : *romantic*

$$H_0 : p = 0.5$$

$$H_A : p < 0.5$$

Conditions for inference for comparing two independent proportions :

- Independence :
  - random sample/assignment
  - if sampling without replacement, $n < 10\%$ of population
- Sample size / skew : samples should meet the success-failure condition (at least 10 *successes* and 10 *failures*) :
  - $n\hat{p} \geq 10 \rightarrow n\hat{p} = 15 \times 0.53 = 5.3 \ngeq 10$
  - $n(1-\hat{p}) \geq 10 \rightarrow n(1-\hat{p}) = 400 \times 0.47 = 4.7 \ngeq 10$

Due to the fact that our conditions did not meet, we will use simulation.

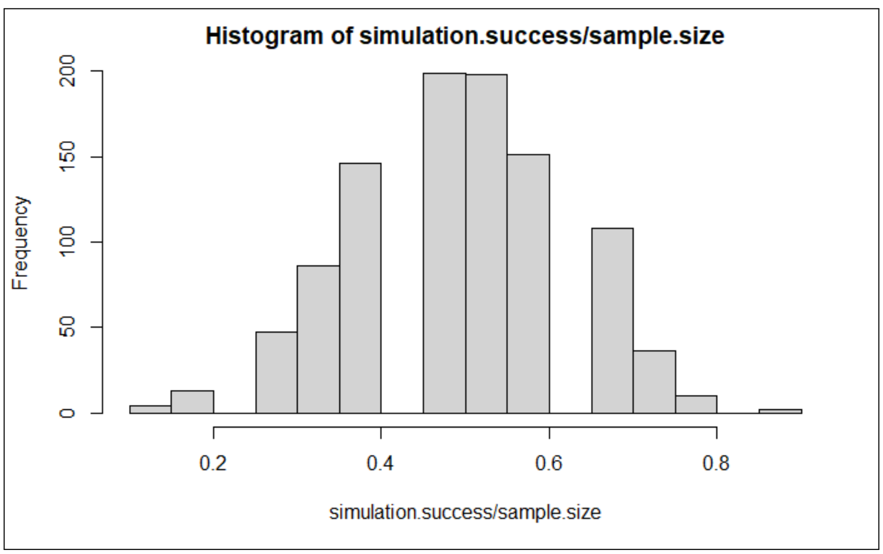


Figure 6: Histogram

Since, the p-value (0.505) is larger than 0.05, we fail to reject the null hypothesis and declare that there is not convincing evidence to accept the alternative hypothesis.
This means that each person is 50% likely to be in a romantic relationship.

Question 3

Chosen Categorical Variable : *Mjob*

```
sample.original
 at_home    health     other services   teacher
      59        34       141      103        58
```

Figure 7: Mjob

```
sample.original
 at_home    health     other services   teacher
  0.1494    0.0861    0.3570   0.2608    0.1468
```

Figure 8: Mjob - probability distribution

a.

Conditions for χ^2 test :

- Independence :
  - random sample/assignment
  - if sampling without replacement, $n < 10\%$ of population
  - each case only contributes to one cell in the table other (non-paired)
- Sample size : Each particular scenario (i.e.cell) must have atleast 5 expected cases.

$$H_0 : \textit{Samples are randomly chosen and there is nothing going on}$$

$$H_0 : \textit{Samples are not randomly chosen and there is something going on}$$

Randomly selected sample :

```
sample.unbiased
 at_home    health     other services   teacher
      16         9        39       18        18
```

Figure 9: 100 samples - randomly

χ^2 test :

```
        Chi-squared test for given probabilities

data:  unbiased.table
X-squared = 3.6496, df = 4, p-value = 0.4555
```

Figure 10: χ^2 test - randomly

Due to the fact that p-value (0.455) is larger than 0.05, we fail to reject the null hypothesis. There is convincing evidence to accept the null hypothesis.

Randomly selected sample with 0.6 bias through teachers :

```
sample.biased
 at_home    health     other services   teacher
      10        12        33        21        24
```

Figure 11: 100 samples - biased

χ^2 test :

```
        Chi-squared test for given probabilities

data:  biased.table
X-squared = 10.071, df = 4, p-value = 0.03924
```

Figure 12: χ^2 test - biased

Due to the fact that p-value (0.0392) is smaller than 0.05, we fail to reject the null hypothesis, there is convincing evidence that the samples are randomly chosen. (!)

b.

Chosen Categorical Variable : *Fjob*

$$H_0 : \textit{Mother's job and father's job are 2 independent variables}$$

$$H_0 : \textit{Mother's job and father's job are dependent variables}$$

$$\textit{Expected count} = \frac{\textit{row total} \times \textit{column total}}{\textit{table total}}$$

$$\textit{text statistic :} \ \frac{\textit{point estimate - null value}}{\textit{SE of point estimate}}$$

$$\chi^2 = \sum_{i=1}^{k} \frac{(O-E)^2}{E} \qquad \textit{and} \qquad df = (R-1)(C-1)$$

Conditions for χ^2 test :

- Independence :
  - random sample/assignment
  - if sampling without replacement, $n <$ 10% of population
  - each case only contributes to one cell in the table other (non-paired)
- Sample size : Each particular scenario (i.e.cell) must have atleast 5 expected cases. $\rightarrow \times$

```
          Fjob
Mjob       at_home health other services teacher
  at_home        2      2    21        7       0
  health         0      4     8        3       0
  other          4      0    51        8       4
  services       2      2    24       18       5
  teacher        1      3    12       13       6
```

Figure 13: table

Our last condition is not met, so we get a warning :

```
Chi-squared approximation may be incorrect
        Pearson's Chi-squared test

data:  Mjob.Fjob
X-squared = 44.517, df = 16, p-value = 0.0001645
```

Figure 14: χ^2 test

We combine *at-home* , *health, services* and *teacher* of *Mjob* and compare it to *other* :

```
         [,1] [,2]
at_home    11   21
health      7    8
other      16   51
services   27   24
teacher    23   12
```

Figure 15: combined table

```
        Pearson's Chi-squared test

data:  Mjob.Fjob.bind
X-squared = 20.514, df = 4, p-value = 0.0003952
```

Figure 16: χ^2 test

Both p-values indicate that we should reject the null hypothesis meaning that parent's job are dependent to each other.

Question 4

Chosen Variables : *G1 - failure* and *studytime*

a.

In phase 1, we used *pearson correlation* and *Correlogram* for our predictions.
(In order not to confuse the report of this phase with the previous phase, the question related to phase 1 of the project is also placed in the zip file of this project, although its abstract is also described here.)

Quoting phase 1 : *'Judging by Figure 34, G1 and G2 and G3 have positive linear associations with each other and with studytime as expected. Failure and goout both have a negative linear associations with G1, G2 and G3.'*

From all the variables mentioned,I chose one of the grades (G_1) as my response variable and from failures, goout and studytime i chose 2 of them that had the most correlation with G1 (absolute value of them are aimed).
(Note : Although G2 and G3 had a very high correlation with G1, i didn't pick them, because all three of these variables are scores in different classes and it is better to use other variables to better understand each person and do not estimate their G1 score only based on their other scores. (Each one of them can be a great response variable) Although in the end I built the model based on these two variables, because I do not know exactly was the exact aim of this question, to choose only based on scores or not, simply because I myself though a better model should be based on a student's other characteristics, I explained more about this.)

$$cor(G_1, failure) = -0.463$$

$$cor(G_1, studytime) = 0.176$$

$$cor(G_1, goout) = -0.161$$

Using those codes here, judging by the results, we can say *failures* is the more significant predictor :

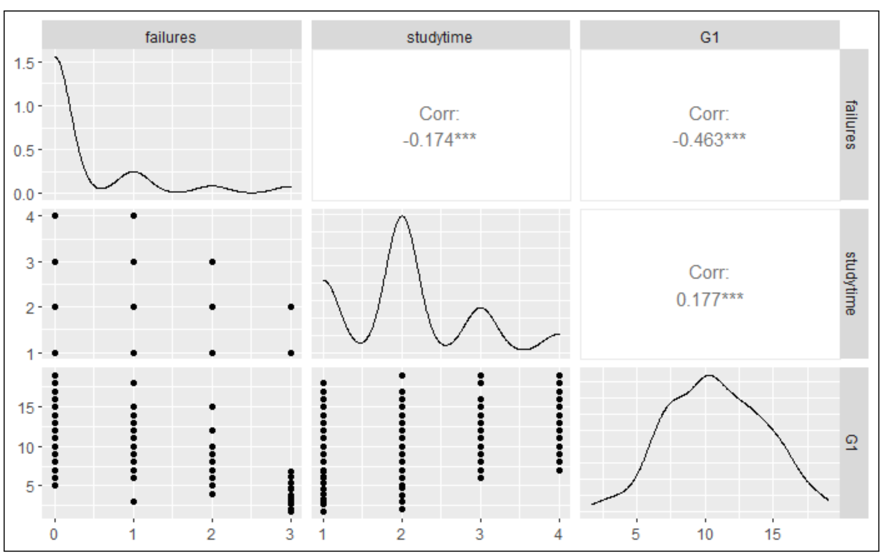


Figure 17: Correlogram

b.

Conditions for linear regression :

- Residuals vs Fitted : Used to check the linear relationship assumptions. A horizontal line, without distinct patterns is an indication for a linear relationship, what is good.
- Normal Q-Q : Used to examine whether the residuals are normally distributed. It's good if residuals points follow the straight dashed line.
- Scale-Location :(or Spread-Location). Used to check the homogeneity of variance of the residuals (homoscedasticity). Horizontal line with equally spread points is a good indication of homoscedasticity.
- Residuals vs Leverage : Used to identify influential cases, that is extreme values that might influence the regression results when included or excluded from the analysis.

failures :

```
Call:
lm(formula = G1 ~ failures, data = StudentsPerformance)

Residuals:
    Min      1Q  Median      3Q     Max
-6.5154 -2.5154 -0.5154  2.4846  8.6768

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  11.5154     0.1724   66.79   <2e-16 ***
failures     -2.1922     0.2117  -10.36   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3.125 on 393 degrees of freedom
Multiple R-squared:  0.2144,	Adjusted R-squared:  0.2124
F-statistic: 107.2 on 1 and 393 DF,  p-value: < 2.2e-16
```

Figure 18: LM model

$$R^2 = 0.214$$

$$p-value < 2.2e-16$$

According to R^2, 0.214 of the variability of the model is explained by failures.

According to the $p-value$, by modeling $G_1 \sim failures$, we can reject the null hypothesis that suggests there is no relationship between these two variables (slope is zero.)

```
Call:
lm(formula = G1 ~ failures, data = StudentsPerformance)

Coefficients:
(Intercept)     failures
     11.515       -2.192
```

Figure 19: LM model

$$G_1 = 11.515 - 2.192 \times failures$$

Intercept: When $failures = 0$, G_1 is expected to equal the intercept (11.515). (Maybe meaningless in context of the data, and only serve to adjust the height of the line.)

In our case when the student has not failed at all , their G_1 score is nearly 11 .

Slope: For each unit increase in $failures$, G_1 is expected to be 2.192 lower on average.

We also need to check whether conditions for using linear regression are met :

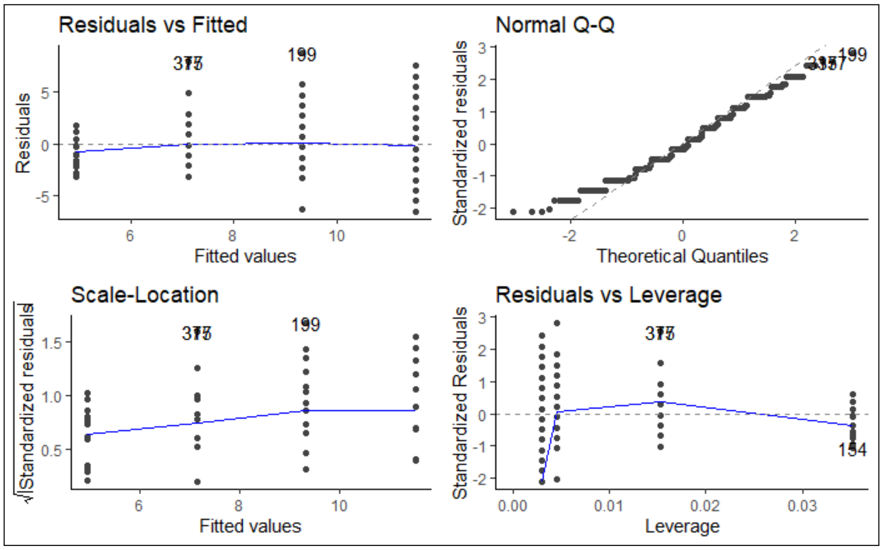


Figure 20: LM conditions - all met

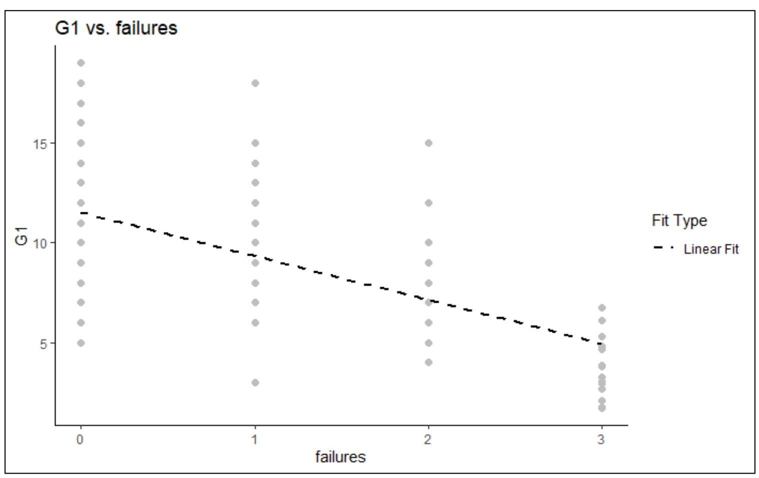


Figure 21: Scatter plot

studytime :

```
Call:
lm(formula = G1 ~ studytime, data = StudentsPerformance)

Residuals:
    Min      1Q  Median      3Q     Max
-8.6592 -2.7566 -0.0149  2.3726  8.2434

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   9.2732     0.4585  20.224  < 2e-16 ***
studytime     0.7417     0.2083   3.561 0.000415 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 3.47 on 393 degrees of freedom
Multiple R-squared:  0.03125,	Adjusted R-squared:  0.02879
F-statistic: 12.68 on 1 and 393 DF,  p-value: 0.0004154
```

Figure 22: LM model

$$R^2 = 0.031$$

$$p - value = 0.00041$$

According to R^2, only 0.03 of the variability of the model is explained by studytime (which is alot smaller than failures).

According to the $p - value$, by modeling $G_1 \sim studytime$, we can reject the null hypothesis that suggests there is no relationship between these two variables (slope is zero.)

```
Call:
lm(formula = G1 ~ studytime, data = StudentsPerformance)

Coefficients:
(Intercept)    studytime
     9.2732       0.7417
```

Figure 23: LM model

$$G_1 = 9.2732 + 0.7417 \times studytime$$

intercept: When $studytime = 0$, G_1 is expected to equal the intercept (9.2732). Maybe meaningless in context of the data, and only serve to adjust the height of the line. In our case when the studentdoes not study at all , their G_1 score is nearly 9 .

slope: For each unit increase in $studytime$, G_1 is expected to be 0.7417 higher on average.

We also need to check whether conditions for using linear regression are met :

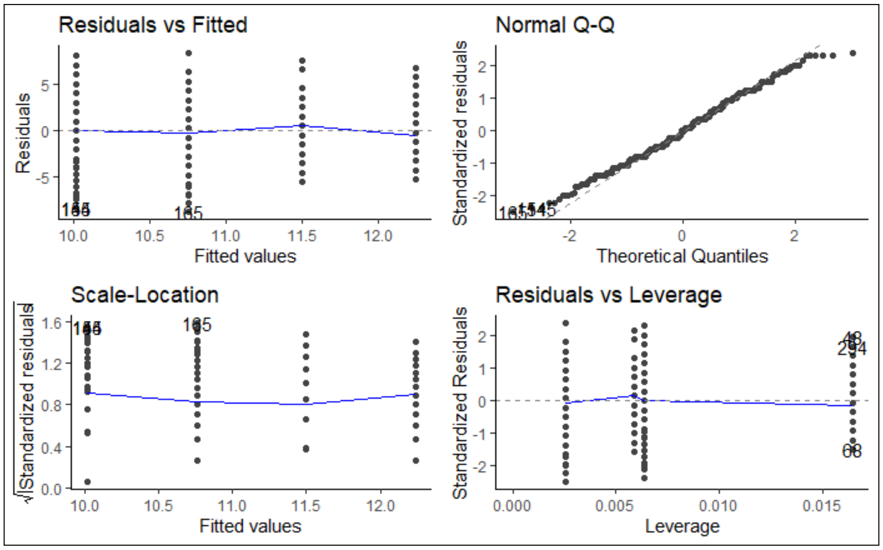


Figure 24: LM conditions - all met

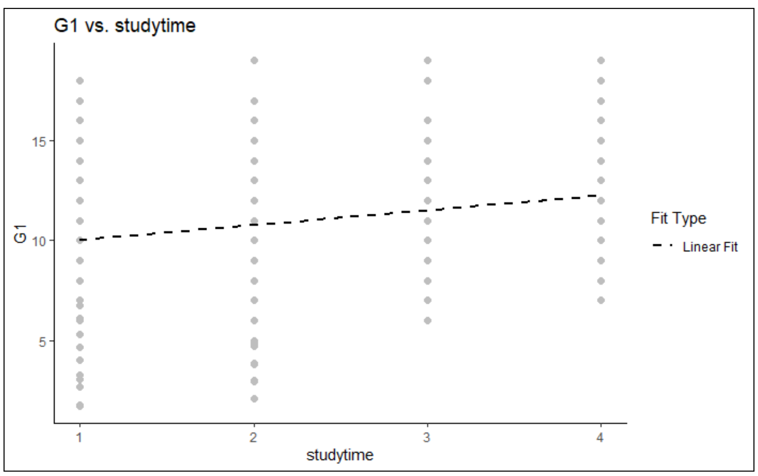


Figure 25: Scatter plot

c.

Judging by above figures, in order to pick the the more significant predictor we can use both R^2_{adj} and $p - value$:

	Adj. R-squared	p-value
failures	0.2124	2.2e-16
studytime	0.02879	0.00041

The more significant predictor is the one with the lowest $p - value$ and highest R^2_{adj}. Both of these point to failures being the best one.

Chosen Variables : *G1* - *G2* and *G3*

$$cor(G_1, G2) = 0.85$$

$$cor(G_1, G3) = 0.80$$

Using those codes here, judging by the results, we can say *G2* is the more significant predictor :

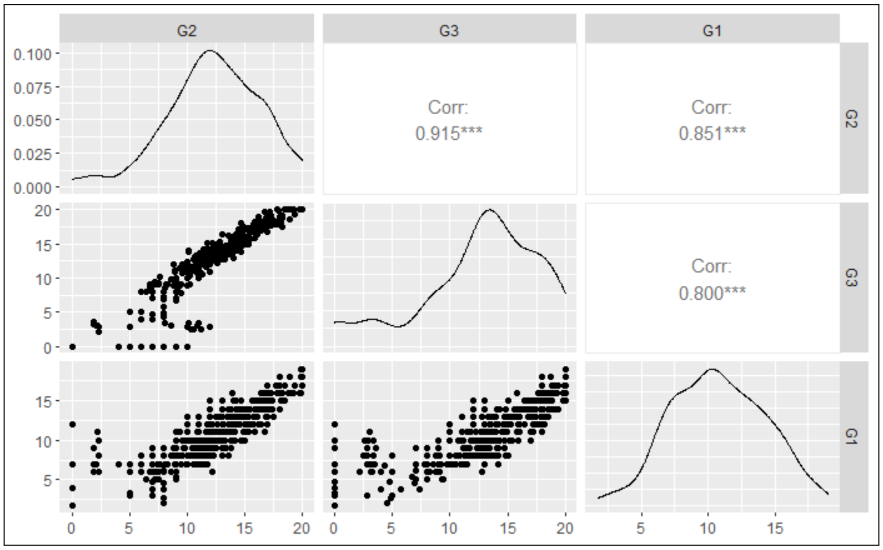


Figure 26: Correlogram

b.
Conditions for linear regression :

- Residuals vs Fitted : Used to check the linear relationship assumptions. A horizontal line, without distinct patterns is an indication for a linear relationship, what is good.
- Normal Q-Q : Used to examine whether the residuals are normally distributed. It's good if residuals points follow the straight dashed line.
- Scale-Location :(or Spread-Location). Used to check the homogeneity of variance of the residuals (homoscedasticity). Horizontal line with equally spread points is a good indication of homoscedasticity.
- Residuals vs Leverage : Used to identify influential cases, that is extreme values that might influence the regression results when included or excluded from the analysis.

G2 :

```
Call:
lm(formula = G1 ~ G2, data = StudentsPerformance)

Residuals:
    Min      1Q  Median      3Q     Max
-5.5525 -1.1545 -0.0471  1.0380 10.2153

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.78475    0.29527   6.045 3.49e-09 ***
G2           0.73313    0.02283  32.115  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.852 on 393 degrees of freedom
Multiple R-squared:  0.7241,    Adjusted R-squared:  0.7234
F-statistic:  1031 on 1 and 393 DF,  p-value: < 2.2e-16
```

Figure 27: LM model

$$R^2 = 0.724$$

$$p - value < 2.2e - 16$$

According to R^2, 0.724 of the variability of the model is explained by failures (which is pretty good).

According to the $p - value$, by modeling $G1 \sim G2$, we can reject the null hypothesis that suggests there is no relationship between these two variables (slope is zero.)

$$G1 = 1.7847 + 0.7331 \times G2$$

Intercept: When $G2 = 0$, $G1$ is expected to equal the intercept (1.7847). (Maybe meaningless in context of the data, and only serve to adjust the height of the line.)
In our case when the student has not failed at all , their G_1 score is nearly 1.78 .

Slope: For each unit increase in $G2$, $G1$ is expected to be 0.7331 higher on average.

We also need to check whether conditions for using linear regression are met :

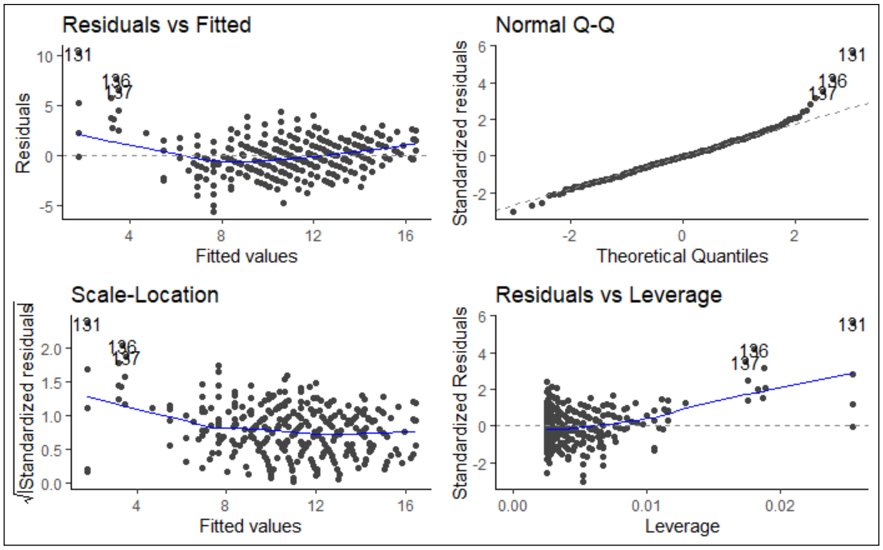


Figure 28: LM conditions - all are hardly met

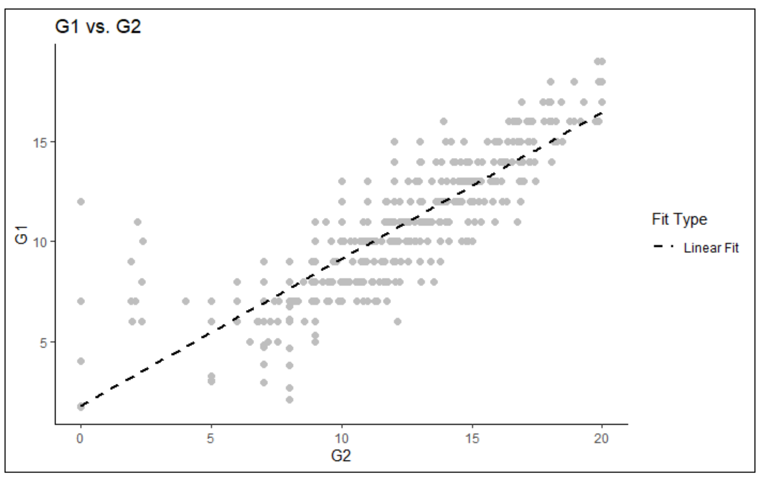


Figure 29: Scatter plot

G3 :

```
Call:
lm(formula = G1 ~ G3, data = StudentsPerformance)

Residuals:
   Min     1Q Median     3Q    Max 
-4.870 -1.623 -0.080  1.338  8.140 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)   3.8602     0.2825   13.66   <2e-16 ***
G3            0.5476     0.0207   26.45   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 2.114 on 393 degrees of freedom
Multiple R-squared:  0.6403,	Adjusted R-squared:  0.6394 
F-statistic: 699.6 on 1 and 393 DF,  p-value: < 2.2e-16
```

Figure 30: LM model

$$R^2 = 0.64$$

$$p - value < 2.2e - 16$$

According to R^2, 0.64 of the variability of the model is explained by failures (which is pretty good).

According to the $p - value$, by modeling $G1 \sim G3$, we can reject the null hypothesis that suggests there is no relationship between these two variables (slope is zero.)

$$G1 = 3.860 + 0.5476 \times G3$$

Intercept: When $G3 = 0$, $G1$ is expected to equal the intercept (3.860). (Maybe meaningless in context of the data, and only serve to adjust the height of the line.)
In our case when the student has not failed at all , their $G3$ score is nearly 1.78 .

Slope: For each unit increase in $G3$, $G1$ is expected to be 0.5476 higher on average.

We also need to check whether conditions for using linear regression are met :

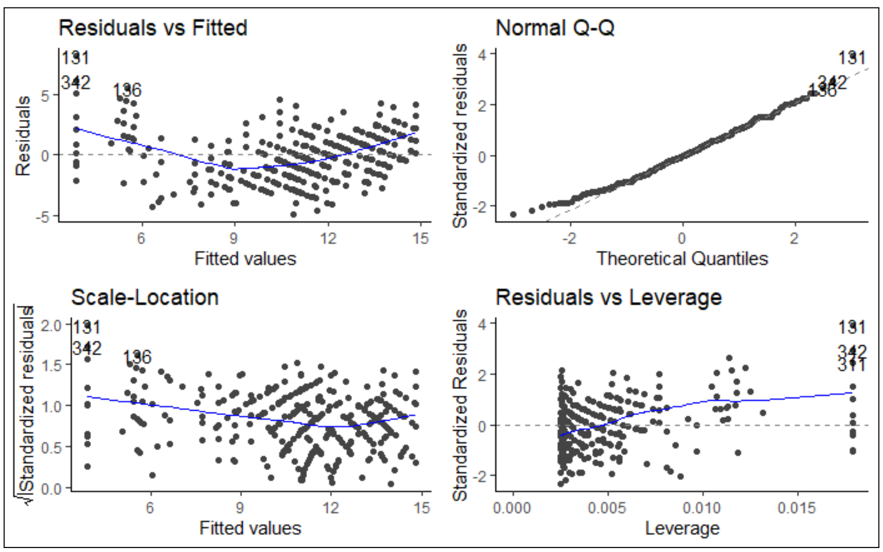


Figure 31: LM conditions - all are hardly met

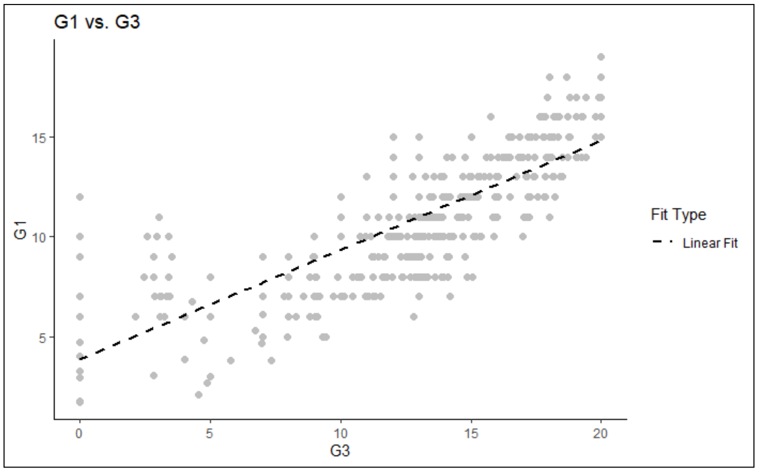


Figure 32: Scatter plot

	Adj. R-squared	p-value
G2	0.724	2.2e-16
G3	0.64	2.2e-16

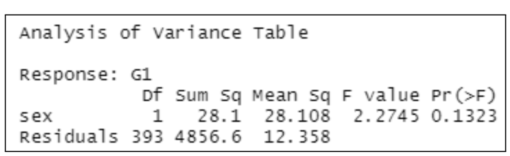
The more significant predictor is the one with the lowest $p - value$ and highest R^2_{adj}. Although there is no difference in $p - value$, according to Adj. R-squared, G2 is the best one.
(Note : Between G2 and failures, G2 has a better R^2_{adj}, but it did not meet the conditions very well. ButR^2_{adj} is more important so if we have to choose one variable, we choose G2)
d.
From this part forward , i will compare both of the models i made till now :

Adj. R-squared :
As was also mentioned in part c., Comparing failure vs. studytime using R^2_{adj} will result in $G_1 \sim failures$ to be the better model.
As was also mentioned in part c., Comparing G2 vs. G3 using R^2_{adj} will result in $G_1 \sim G2$ to be the better model.
As was also mentioned in part c., Comparing failure vs. G2 using R^2_{adj} will result in $G_1 \sim G2$ to be the better model.

ANOVA table :

In order to compare my models, we first consider a base model, for example :

$$G1 \sim sex$$



```
Analysis of Variance Table

Response: G1
           Df Sum Sq Mean Sq F value Pr(>F)
sex         1   28.1  28.108  2.2745 0.1323
Residuals 393 4856.6  12.358
```

Figure 33: anove

failure vs. studytime :

```
Analysis of Variance Table

Response: G1
           Df Sum Sq Mean Sq  F value  Pr(>F)
sex         1   28.1   28.11   2.9056 0.08906 .
failures    1 1064.5 1064.54 110.0451 < 2e-16 ***
Residuals 392 3792.1    9.67
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Figure 34: anove

```
Analysis of Variance Table

Response: G1
           Df Sum Sq Mean Sq F value    Pr(>F)
sex         1   28.1  28.108  2.3741    0.1242
studytime   1  215.6 215.641 18.2140 2.479e-05 ***
Residuals 392 4641.0  11.839
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Figure 35: anove

$$R^2 = \frac{SS_{reg}}{SS_{total}}$$

	Base	+ failures	+ studytime
R2	0.01	0.22	0.05

Figure 36: computed R2 base on anova

Comparing failure vs. studytime using R^2 will result in $G_1 \sim failures$ to be the better model.
G2 vs. G3 :

```
Analysis of Variance Table

Response: G1
           Df Sum Sq Mean Sq   F value    Pr(>F)
sex         1   28.1    28.1    8.1791  0.004464 **
G2          1 3509.5  3509.5 1021.2354 < 2.2e-16 ***
Residuals 392 1347.1     3.4
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Figure 37: anove

```
Analysis of Variance Table

Response: G1
           Df  Sum Sq Mean Sq  F value  Pr(>F)
sex         1   28.11   28.11   6.2773 0.01263 *
G3          1 3101.39 3101.39 692.6334 < 2e-16 ***
Residuals 392 1755.25    4.48
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Figure 38: anove

$$R^2 = \frac{SS_{reg}}{SS_{total}}$$

```
Analysis of Variance Table

Response: G1
           Df  Sum Sq Mean Sq  F value  Pr(>F)
sex         1   28.11   28.11   6.2773 0.01263 *
G3          1 3101.39 3101.39 692.6334 < 2e-16 ***
Residuals 392 1755.25    4.48
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Figure 39: computed R2 base on anova

Comparing G2 vs. G3 using R^2 will result in $G_1 \sim G2$ to be the better model.
Due to the fact that $n-1$ and $n-k-1$ are approximately the same, R^2 and Adjusted R^2 doesn't have a noticeable difference.

e.

When there are many possible predictors, we need some strategy for selecting the best predictors to use in a regression model.

- Adjusted R^2 : Under this criterion, the best model is the one with the highest value of Adjusted R^2 .
- Cross-validation (explained in the next question) :Under this criterion, the best model is the one with the smallest value of MSE.
- Corrected Akaike's Information Criterion :Under this criterion, the best model is the one with the smallest value of AIC.
- Schwarz's Bayesian Information Criterion :Under this criterion, the best model is the one with the smallest value of BIC.

While R^2 is widely used, and has been around longer than the other measures, its tendency to select too many predictor variables makes it less suitable for forecasting.
Many statisticians like to use the BIC because it has the feature that if there is a true underlying model, the BIC will select that model given enough data. However, in reality, there is rarely, if ever, a true underlying model, and even if there was a true underlying model, selecting that model will not necessarily give the best forecasts (because the parameter estimates may not be accurate).

f.

H_0 : *The explanatory variable is not a significant predictor of the response variable, i.e. no relationship* $\rightarrow \beta = 0$

H_A : *The explanatory variable is a significant predictor of the response variable, i.e. relationship* $\rightarrow \beta \neq 0$

(a)

	p-value	significant
studytime	0.385	×
failure	2.2e-16	✓
G2	2.2e-16	✓
G3	2.2e-16	✓

Comparing failure vs. studytime : failure is significant predictor of the response variable.

Comparing G2 vs. G3 : Both are significant predictor of the response variable.

Comparing failure vs. G2 : Both are is significant predictor of the response variable.

(b)

$$failure\ CI\ : (-2.448, -1.769)$$

We are 95% confident that for each additional point on failure, G1 is expected on average to be lower by 1.769 to 2.448 points.

$$studytime\ CI\ : (0.019, 0.845)$$

We are 95% confident that for each additional point on studytime, G1 is expected on average to be higher by 0.019 to 0.845 points.

$$G2\ CI\ : (0.613, 0.698)$$

We are 95% confident that for each additional point on G2, G1 is expected on average to be higher by 0.613 to 0.846985 points.

$$G3\ CI\ : (0.530, 0.565)$$

We are 95% confident that for each additional point on G3, G1 is expected on average to be higher by 0.530 to 0.565 points.

(c)

	Actual	Predicted studytime	Predicted failues	Predicted G2	Predicted G3
209	9	10.2	11.5	10.2	10.9
244	13	10.2	11.5	11.9	12.2
6	15	10.6	11.5	13.8	13.9
358	12	10.6	11.5	12.0	11.6
178	6	10.6	11.5	7.3	8.7
44	8	10.2	11.5	9.4	11.5
201	16	10.6	11.5	14.6	14.3
82	11	11.0	11.5	10.6	11.4
295	14	11.0	11.5	12.4	13.2
30	10	10.6	11.5	11.8	11.5

Figure 40: Predicted

(d)

	Actual	Predicted studytime	Predicted failues	Predicted G2	Predicted G3
209	0	1.2	2.5	1.2	1.9
244	0	2.8	1.5	1.1	0.8
6	0	4.4	3.5	1.2	1.1
358	0	1.4	0.5	0.0	0.4
178	0	4.6	5.5	1.3	2.7
44	0	2.2	3.5	1.4	3.5
201	0	5.4	4.5	1.4	1.7
82	0	0.0	0.5	0.4	0.4
295	0	3.0	2.5	1.6	0.8
30	0	0.6	1.5	1.8	1.5

Figure 41: Prediction error

In order to compute success rate, I accept an 0.1 error which is 2 (data range $\times$ accepted error = 20 $\times$ 0.1 = 2).

If the predicted result is ± 2 of my actual value, I accept it.

	Predicted studytime	Predicted failues	Predicted G2	Predicted G3
1	40 %	40 %	100 %	80 %

Figure 42: Success rate

Comparing failure vs. studytime : no difference !

Comparing G2 vs. G3 : G2 is the best predictor of the response variable.

Comparing failure vs. G2 : G2 is the best predictor of the response variable.

G2 is by far the best predictor.

Using *min-max calculation* :

$$MinMaxAccuracy = mean\left(\frac{min(actual, predicted)}{max(actual, predicted)}\right)$$

	Predicted studytime	Predicted failues	Predicted G2	Predicted G3
1	79.99 %	79.76 %	90 %	86.93 %

Figure 43: Success rate

Comparing failure vs. studytime :studytime is the best predictor of the response variable.

Comparing G2 vs. G3 : G2 is the best predictor of the response variable.

Comparing failure vs. G2 : G2 is the best predictor of the response variable.

G2 is by far the best predictor.

Using *MAPE calculation* :

$$MeanAbsolutePercentageError = mean\left(\frac{abs(actual - predicted)}{actual}\right)$$

	Predicted studytime	Predicted failues	Predicted G2	Predicted G3	
1	79.99 %	79.76 %	90 %	86.93 %	86.93 %

Figure 44: Success rate

Comparing failure vs. studytime :studytime is the best predictor of the response variable.

Comparing G2 vs. G3 : G2 is the best predictor of the response variable.

Comparing failure vs. G2 : G2 is the best predictor of the response variable.

Question 5

Chosen Categorical Variables : *G1 - G2*, *goout* , *sex* ,*failures* ,*age* and *studytime*

a.

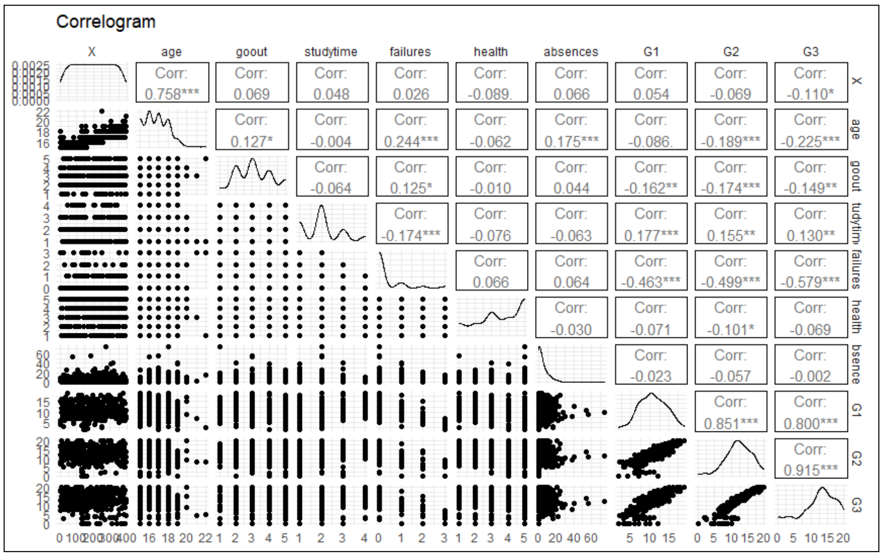


Figure 45: Correlogram

Considering all the analysis and inferences made in the previous question, for this question, from all the choices we have, we will definitely put G2 and failure among our options. Adding both G2 and G3 won't add anything new to the table since they are collinear.
We will add studytime, goout, age, and sex too, but we have to be careful not to use too many variables; we should pay attention to *occam's razor*; prefer the simplest best model!
The correlation between variables was also explained in the last question, but to summarize, as it was expected, failures and G2 have the highest correlation. Surprisingly, age has a higher correlation with G2 than studytime (considering their absolute value).
Age and sex are not correlated, just as sex and G2. sex has nothing to do with score or age, so it was probably expected.
More explanations can be found on phase 1, so to avoid lengthening the report, we move on to the next part.
Mjob and Fjob seemed hardly important to G1, so I didn't use them.

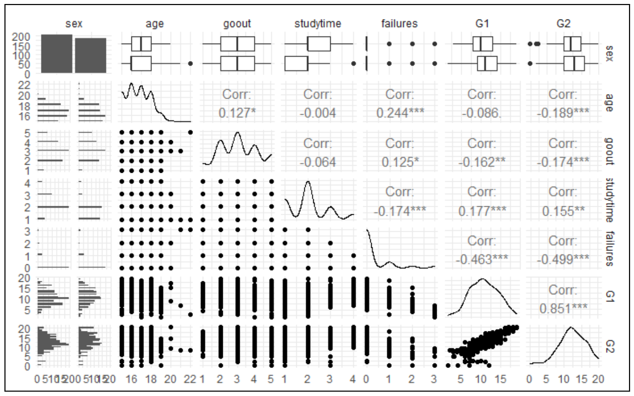


Figure 46: Correlogram

We don't want any collinearity between the variables that we chose and the below figure shows that we did a good job :

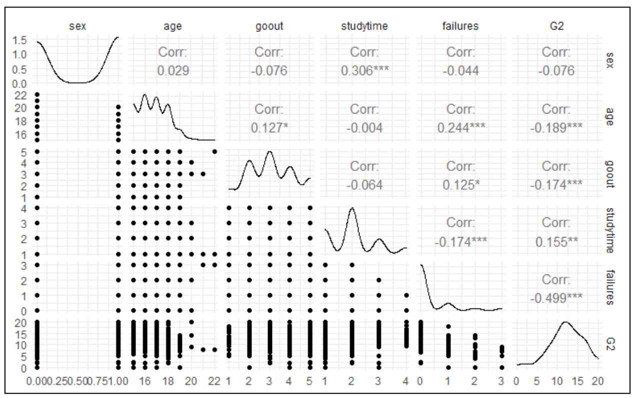


Figure 47: Correlogram - response variable omitted

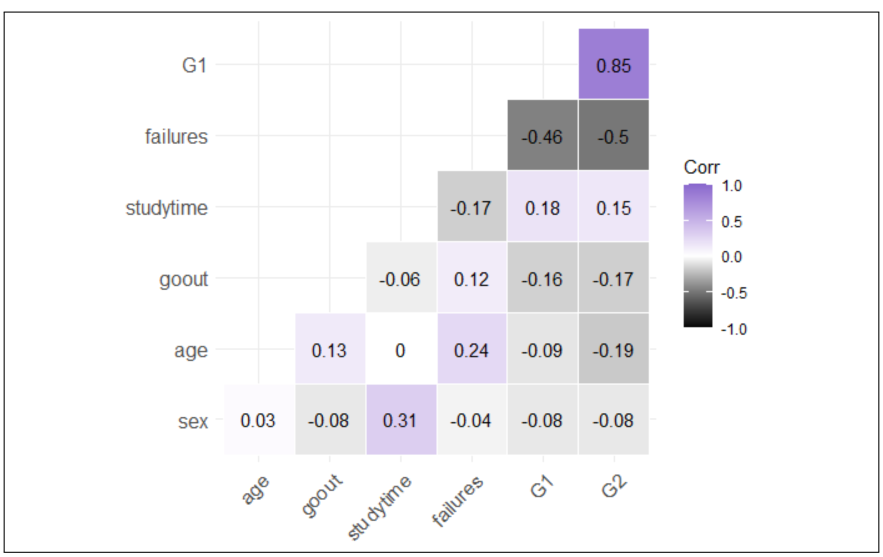


Figure 48: Correlogram

As was said multiple times in the previous question, $G2$ plays a more significant role in prediction.

b.

```
Call:
lm(formula = G1 ~ G2 + goout + failures + studytime + sex + age,
    data = StudentsPerformance)

Residuals:
    Min      1Q  Median      3Q     Max
-5.3720 -1.1898 -0.1367  1.0890 10.6786

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -1.97651    1.33371  -1.482   0.1392
G2           0.70774    0.02661  26.599   <2e-16 ***
goout       -0.06969    0.08454  -0.824   0.4102
failures    -0.30731    0.14614  -2.103   0.0361 *
studytime    0.20212    0.11755   1.719   0.0863 .
sex         -0.24841    0.19572  -1.269   0.2051
age          0.24627    0.07487   3.289   0.0011 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.823 on 388 degrees of freedom
Multiple R-squared:  0.7361,    Adjusted R-squared:  0.732
F-statistic: 180.4 on 6 and 388 DF,  p-value: < 2.2e-16
```

Figure 49: Correlogram

$$G1 = -1.97 + 0.7 \times G2 - 0.06 \times goout + -0.3 \times failures + 0.2 \times studytime - 0.24 \times sex : M + 0.24 \times age$$

c.

R^2 shows what percent of variability in the response variable is explained by the model. In our case nearly 74% of the variablity was explained using 6 variables out of the 15 variables available which is pretty good.

d.

Higher R^2 doesn't necessarily guarantee that the model fits the data well, we might face over-fitting if we are not careful.

Adjusted R^2 can be a good indicator of when the model fits the data well, it compares the explanatory power of regression models that contain different numbers of predictors. Adjusted R^2 is around 73% in our fitted model.

The fact that R^2 and Adjusted R^2 are this close is very good which means we don't have overfitting in our model.

Other techniques can help us know whether we have a good fit or not, for example Residuals : A good way to test the quality of the fit of the model is to look at the residuals The idea in here is that the sum of the residuals is approximately zero or as low as possible.

Although all our analyzes so far have promised a good model, the figure below shows that the model is not the best possible model.

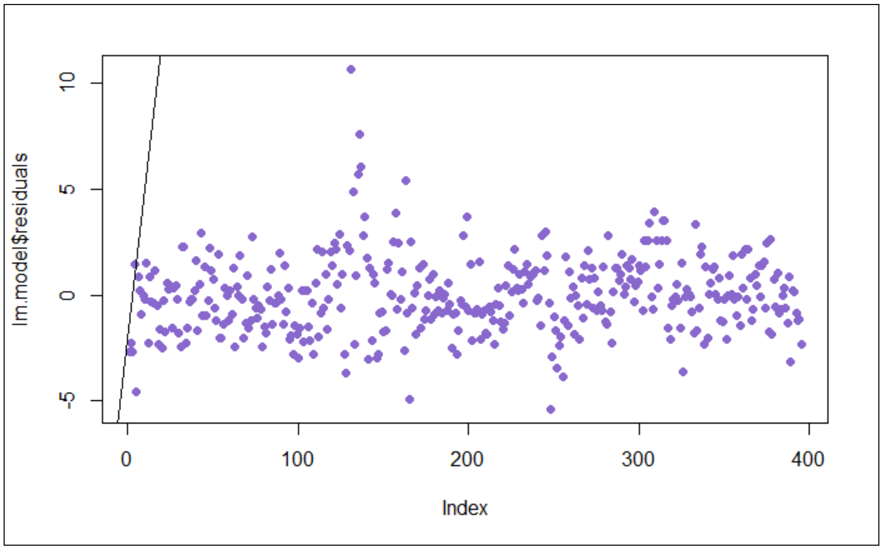


Figure 50: residuals

e.

To develop the best possible model, there are 4 different approaches :

Forward selection : start with an empty model and add one predictor at a time until the parsimonious model is reached.

(a) *p-value* :

Start with single predictor regressions of response vs. each explanatory variable (G2, G3 and failure all had pvalues smaller than 2.2 e-16 , doesn't matter which one we will choose)

Pick the variable with the lowest significant p-value

Add the remaining variables one at a time to the existing model, and pick the variable with the lowest significant p-value

Repeat until any of the remaining variables do not have a significant p-value

For this part i used $ols_step_forward_p$, detailes can also be found in my project's file.

```
Variables Entered:

+ G2
+ age
+ failures
+ studytime
+ sex


Final Model Output
------------------

                        Model Summary
--------------------------------------------------------------
R                       0.858       RMSE                1.822
R-Squared               0.736       Coef. Var          16.896
Adj. R-Squared          0.732       MSE                 3.319
Pred R-Squared          0.722       MAE                 1.361
--------------------------------------------------------------
 RMSE: Root Mean Square Error
 MSE: Mean Square Error
 MAE: Mean Absolute Error

                                ANOVA
----------------------------------------------------------------------
                Sum of
               Squares        DF    Mean Square       F         Sig.
----------------------------------------------------------------------
Regression    3593.549         5        718.710    216.526    0.0000
Residual      1291.200       389          3.319
Total         4884.749       394
----------------------------------------------------------------------

                                  Parameter Estimates
----------------------------------------------------------------------------------------
      model      Beta    Std. Error    Std. Beta      t        Sig      lower     upper
----------------------------------------------------------------------------------------
(Intercept)    -2.139         1.319                 -1.622    0.106    -4.731     0.453
         G2     0.711         0.026        0.825    26.946    0.000     0.659     0.762
        age     0.240         0.075        0.087     3.227    0.001     0.094     0.387
   failures    -0.309         0.146       -0.065    -2.119    0.035    -0.597    -0.022
  studytime     0.203         0.117        0.048     1.728    0.085    -0.028     0.434
        sex    -0.235         0.195       -0.033    -1.206    0.229    -0.618     0.148
----------------------------------------------------------------------------------------
```

Figure 51: Forward - p-value

Final model :

$$G1 \sim G2 + age + failures + sex + studytime$$

(b) *Adjusted* R^2 :

Start with single predictor regressions of response vs. each explanatory variable Pick the model with the highest adjusted R^2
Add the remaining variables one at a time to the existing model, and pick the model with the highest adjusted R^2
Repeat until the addition of any of the remaining variables does not result in a higher adjusted R^2

	best.pred	all.adj.r.squared
1	G2	0.7507277
2	G2 + age	0.7630615
3	G2 + age + failures	0.7639030
4	G2 + age + failures + studytime	0.7635379

Figure 52: Forward - Adjusted R^2

As we can see, adding the fourth variable, studytime, didn't help us with gaining a better fit for our model, so we wrapped this approach up after obtaining this model :

$$G1 \sim G2 + age + failures$$

with Adjusted $R^2 \approx 73.4$ %

Question 5 continued on next page. . .

Backward elimination : start with a full model (containing all predictors), drop one predictor at a time until the parsimonious model is reached.

(a) *p-value* :

Start with the full model
Drop the variable with the highest p-value and refit a smaller model
Repeat until all variables left in the model are significant

For this part i used $ols_step_backward_p$, detailes can also be found in my project's file.

```
Variables Removed:

x goout
x sex
x studytime

Final Model Output
------------------

                          Model Summary
---------------------------------------------------------------
R                        0.856       RMSE                1.825
R-Squared                0.733       Coef. Var          16.928
Adj. R-Squared           0.731       MSE                 3.332
Pred R-Squared           0.723       MAE                 1.363
---------------------------------------------------------------
 RMSE: Root Mean Square Error
 MSE: Mean Square Error
 MAE: Mean Absolute Error

                                  ANOVA
-----------------------------------------------------------------------
                Sum of
               Squares         DF    Mean Square       F          Sig.
-----------------------------------------------------------------------
Regression    3582.019          3       1194.006    358.368     0.0000
Residual      1302.730        391          3.332
Total         4884.749        394
-----------------------------------------------------------------------

                                    Parameter Estimates
-----------------------------------------------------------------------------------------
      model        Beta    Std. Error    Std. Beta      t        Sig      lower     upper
-----------------------------------------------------------------------------------------
(Intercept)      -1.994         1.311                -1.521    0.129    -4.573     0.584
         G2       0.718         0.026        0.834   27.563    0.000     0.667     0.769
   failures      -0.323         0.145       -0.068   -2.225    0.027    -0.608    -0.038
        age       0.244         0.075        0.088    3.270    0.001     0.097     0.390
-----------------------------------------------------------------------------------------
```

Figure 53: Backward - p-value

Final model :

$$G1 \sim G2 + age + failures$$

Forward and backward p-value approach did not gain the same result.

(b) *Adjusted* R^2 :

Start with the full model
Drop one variable at a time and record adjusted R^2 of each smaller model
Pick the model with the highest increase in adjusted R^2
Repeat until none of the models yield an increase in adjusted R^2

	best.pred	all.adj.r.squared
1	G2 + failures + studytime + sex + age	0.7630107
2	G2 + failures + studytime + age	0.7635379
3	G2 + failures + age	0.7639030

Figure 54: Backward - Adjusted R^2

As we can see, omiting either G2, failure nor age, didn't help us with gaining a better fit for our model and increasing our Adjusted R^2 , so we wrapped this approach up after obtaining this model :

$$G1 \sim G2 + age + failures$$

with Adjusted $R^2 \approx 73.4$ %

Surprisingly, both forward and backward Adjusted R^2 gained the same result.

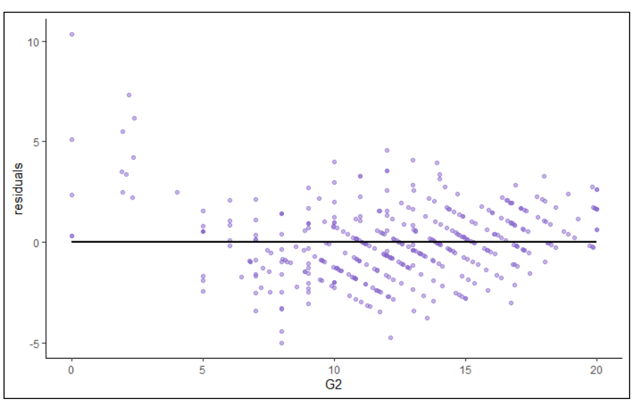
After completing these 4 methods, we see that both Adjusted R^2 approaches and the backward elimination pvalue approach all came to the same result, which is different from the result reached by forward selection pvalue.
According to the criterias mentioned in part d, $G1 \sim G2 + age + failures$ model is better than $G1 \sim G2 + age + failures + studytime + sex$, so we will use the same model in the following parts.
f.

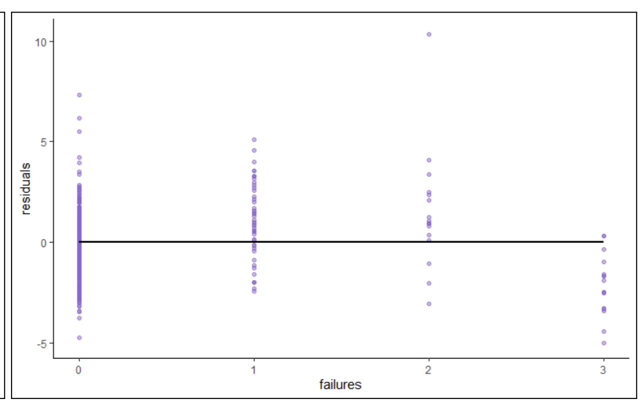
Conditions for linear regression :

- Linear relationships between x and y :
  Each (numerical) explanatory variable linearly related to the response variable
  Check using residuals plots (e vs. x)
  Looking for a random scatter around 0
  Instead of scatterplot of y vs. x : allows for considering the other variables that are also in the model, and not just the bivariate relationship between a given x and y

- Nearly normal residuals :
  Some residuals will be positive and some negative
  On a residuals plot we look for random scatter of residuals around 0
  This translates to a nearly normal distribution of residuals centered at 0
  Check using histogram or normal probability plot

- Constant variability of residuals :
  Residuals should be equally variable for low and high values of the predicted response variable
  Check using residuals plots of residuals vs. predicted (e vs. y)
  Residuals vs. predicted instead of residuals vs. x because it allows for considering the entire model (with all explanatory variables) at once
  Residuals randomly scattered in a band with a constant width around 0 (no fan shape)
  Also worthwhile to view absolute value of residuals vs. predicted to identify unusual observations easily





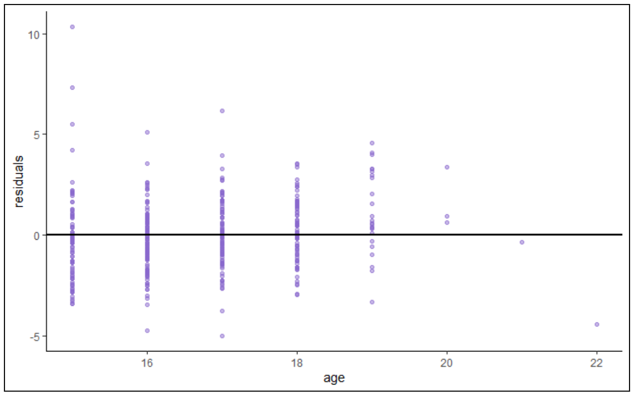


Figure 55: Linear relationship

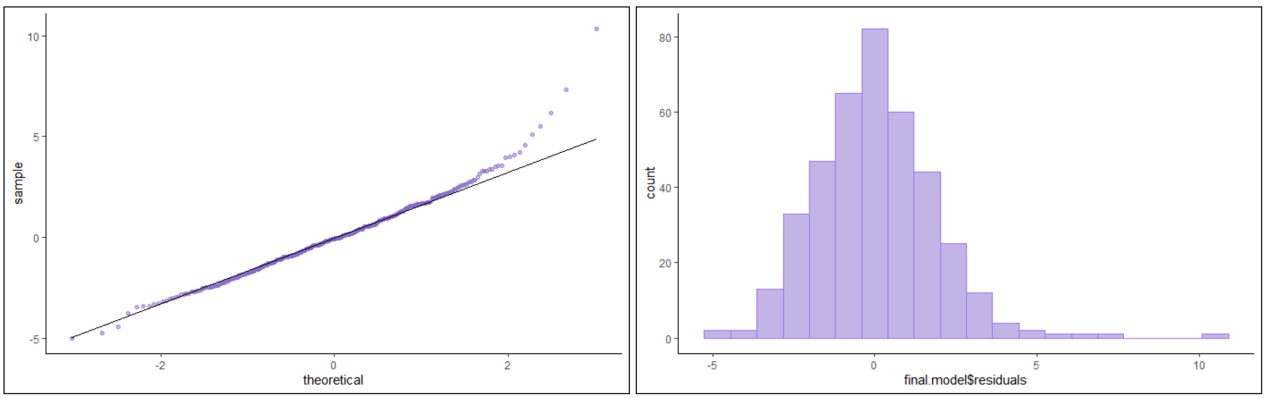


Figure 56: Nearly normal residuals

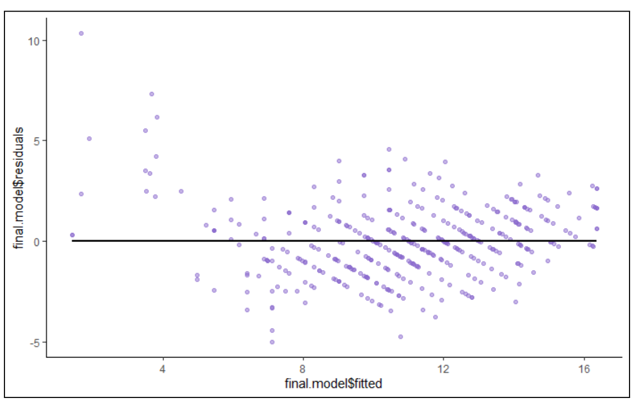


Figure 57: Constant var.

All conditions are met, not fully and perfectly, but they are met.
There are outliers that are effecting our model, if we eliminate them, we will meet them perfectly.
More on ourliers and detecting them in conditions and plots below :
(Note : this conditions were not mentioned in the slides, i checked them too just to be sure, some of them might overlap with the prev. conditions) Conditions for linear regression :

- Residuals vs Fitted : Used to check the linear relationship assumptions. A horizontal line, without distinct patterns is an indication for a linear relationship, what is good.
- Normal Q-Q : Used to examine whether the residuals are normally distributed. It's good if residuals points follow the straight dashed line.
- Scale-Location :(or Spread-Location). Used to check the homogeneity of variance of the residuals (homoscedasticity). Horizontal line with equally spread points is a good indication of homoscedasticity.
- Residuals vs Leverage : Used to identify influential cases, that is extreme values that might influence the regression results when included or excluded from the analysis.

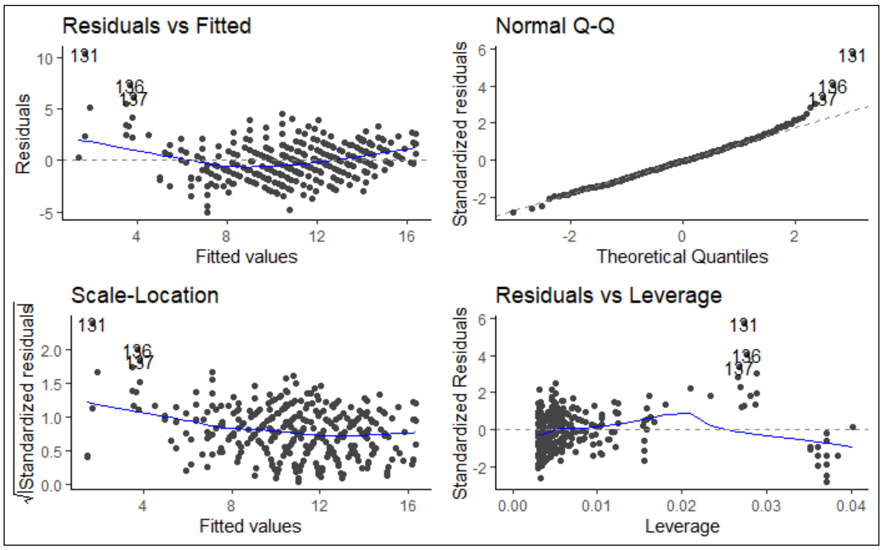


Figure 58: Conditions

All in all, we have a reliable model !

g.

The basic idea, behind cross-validation techniques, consists of dividing the data into two sets:
The training set, used to train (i.e. build) the model; and the testing set (or validation set), used to test (i.e. validate) the model by estimating the prediction error. Cross-validation is also known as a resampling method because it involves fitting the same statistical method multiple times using different subsets of the data.
The k-fold cross-validation method evaluates the model performance on different subset of the training data and then calculate the average prediction error rate. The algorithm is as follow:

- Randomly split the data set into k-subsets (or k-fold) (for example 5 subsets)
- Reserve one subset and train the model on all other subsets
- Test the model on the reserved subset and record the prediction error
- Repeat this process until each of the k subsets has served as the test set.
- Compute the average of the k recorded errors. This is called the cross-validation error serving as the performance metric for the model.

Root Mean Squared Error, which measures the model prediction error. It corresponds to the average difference between the observed known values of the outcome and the predicted value by the model. The lower the RMSE, the better the model.

```
Linear Regression

395 samples
  6 predictor

No pre-processing
Resampling: Cross-Validated (5 fold)
Summary of sample sizes: 317, 315, 316, 316, 316
Resampling results:

  RMSE     Rsquared   MAE
  1.85564  0.7268193  1.389432

Tuning parameter 'intercept' was held constant at a value of TRUE
```

Figure 59: Full model

```
Linear Regression

395 samples
  3 predictor

No pre-processing
Resampling: Cross-Validated (5 fold)
Summary of sample sizes: 316, 316, 315, 317, 316
Resampling results:

  RMSE      Rsquared   MAE
  1.836361  0.7357864  1.379812

Tuning parameter 'intercept' was held constant at a value of TRUE
```

Figure 60: Best model

Due to the fact that RMSE is lower in the so called 'Best model', we trust what we have done till now.

	RMSE	Rsquared	MAE	Resample
1	1.637994	0.7763565	1.284930	Fold1
2	2.069837	0.6892742	1.364075	Fold2
3	1.712149	0.7383289	1.322173	Fold3
4	1.848170	0.7410741	1.559756	Fold4
5	1.912499	0.7349491	1.378221	Fold5

Figure 61: Different metrics of all 5-fold , best model

Question 6

Chosen Variables : *catG3* - *failures*, *studytime*, *G2* and *sex*

Due to the fact that my dataset lacked a good binary categorical variable, i made G_3 into a binary categorical variable $\longrightarrow$ if $G_3 < 10 : Fail(0)$ else $Pass(1)$

a.

```
Call:
glm(formula = catG3 ~ failures + studytime + G2 + sex, family = binomial(link = "logit"),
    data = StudentsPerformance)

Deviance Residuals:
     Min        1Q    Median        3Q       Max
-2.81360   0.00018   0.01265   0.13763   2.19874

Coefficients:
            Estimate Std. Error z value Pr(>|z|)
(Intercept) -14.5388     2.4952  -5.827 5.65e-09 ***
failures     -0.7129     0.3903  -1.827   0.0678 .
studytime    -0.2745     0.3209  -0.856   0.3922
G2            1.6466     0.2553   6.449 1.13e-10 ***
sex          -0.4254     0.5741  -0.741   0.4587
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 433.50  on 394  degrees of freedom
Residual deviance: 102.11  on 390  degrees of freedom
AIC: 112.11

Number of Fisher Scoring iterations: 8
```

Figure 62: GLM model

$$log(\frac{p}{1-p}) = -14.538 - 0.712 \times failures - 0.2745 \times studytime + 1.646 \times G2 + -0.42 \times sex : M$$

intercept : keeping all other predictors zero, the log odds ratio / odds radio of catG3 is -14.538 / exp(-14.538) = 4.85e-7

failures : keeping all other predictors constant for a unit increase in failures, the log odds ratio / odds radio of catG3 will decrease -0.712 / exp(-0.712) = 0.49

studytime : keeping all other predictors constant for a unit increase in studytime, the log odds ratio / odds radio of catG3 will decrease -0.2745 / exp(-0.2745) = 0.763

G2 : keeping all other predictors constant for a unit increase in G2 , the log odds ratio / odds radio of catG3 will increase 1.6462 / exp(1.646) = 5.15

sex : keeping all other predictors constant, the log odds ratio / odds radio of catG3 for reference point (M) is - 0.42 / exp(-0.712) = 0.657 less than F

b.

Odds ratio (OR) is a statistic that quantifies the strength of the association between two events, A and B. The odds ratio is defined as the ratio of the odds of A in the presence of B and vise versa, which, due to symmetry, is equal to the ratio of the odds of B in the presence of A and the odds of B in the absence of A.

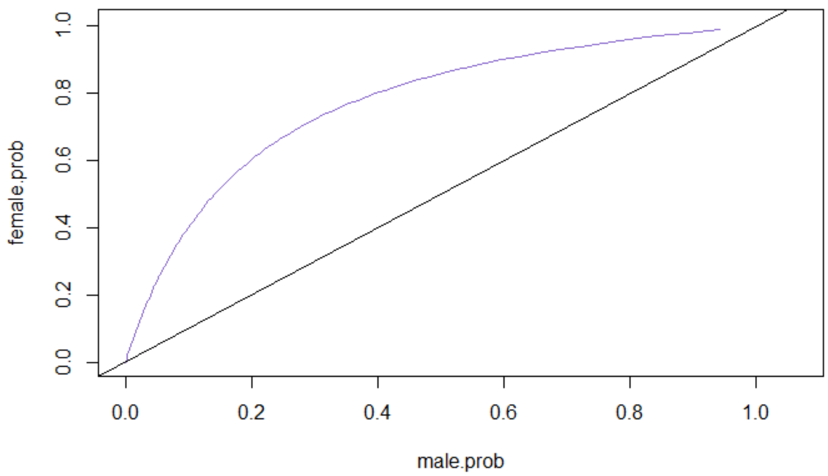


Figure 63: Odds ratio curve for sex - ref : M

This curve indicates the probability of passing G3 (catG3 = 1), for male refrence point :

$$x : P(catG3|Male) \sim y : P(catG3|Female)$$

c.

ROC stands for Receiver Operating Characteristics, and it is used to evaluate the prediction accuracy of a classifier model. ROC curve is a metric describing the trade-off between the sensitivity (true positive rate, TPR) and specificity (false positive rate, FPR) of a prediction in all probability cutoffs (thresholds).
It can be used for binary and multi-class classification accuracy checking.
To evaluate the ROC in multi-class prediction, we create binary classes by mapping each class against the other classes.

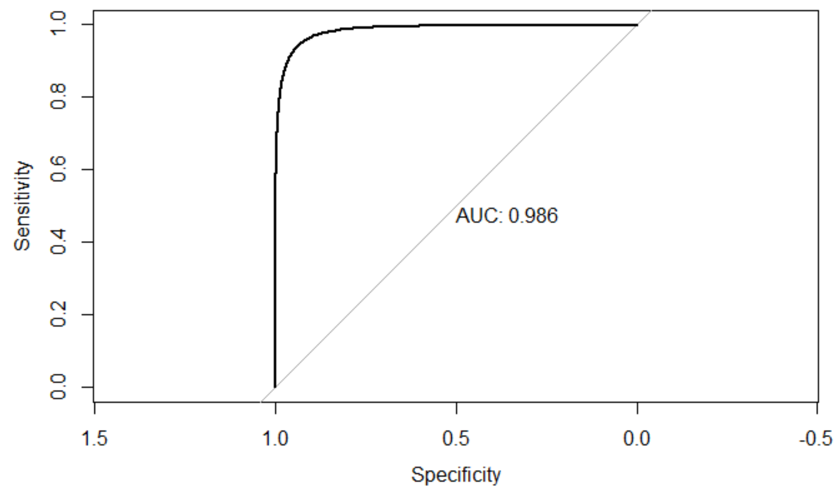


Figure 64: ROC curve - test

The AUC represents the area under the ROC curve. We can evaluate the model the performance by the value of AUC. Higher than 0.5 shows a better model performance. If the curve changes to rectangle it is perfect classifier with AUC value 1.

In our case, AUC is nearly 0.98 which is really good considering all that was mentioned.

d.
The explanatory variable with the lowest p-value in the model, plays the most significant role in the prediction.
e.
According to the summary of our model, *G2* and *failures* are the explanatory variables with the most significant contribution to the model.

```
Call:
glm(formula = catG3 ~ failures + G2, family = binomial(link = "logit"),
    data = StudentsPerformance)

Deviance Residuals:
     Min        1Q    Median        3Q       Max
-2.91186   0.00028   0.01588   0.14760   2.39928

Coefficients:
            Estimate Std. Error z value Pr(>|z|)
(Intercept) -14.9789     2.4607  -6.087 1.15e-09 ***
failures     -0.6456     0.3882  -1.663   0.0963 .
G2            1.6030     0.2453   6.536 6.31e-11 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 433.50  on 394  degrees of freedom
Residual deviance: 104.12  on 392  degrees of freedom
AIC: 110.12

Number of Fisher Scoring iterations: 8
```

Figure 65: GLM model

$$log(\frac{p}{1-p}) = -14.978 - 0.645 \times failures + 1.603 \times G2$$

intercept : keeping all other predictors zero, the log odds ratio / odds radio of catG3 is -14.978 / exp(-14.978) = 4.85e-7

failures : keeping all other predictors constant for a unit increase in failures, the log odds ratio / odds radio of catG3 will decrease -0.645 / exp(-0.645) = 0.52

G2 : keeping all other predictors constant for a unit increase in G2 , the log odds ratio / odds radio of catG3 will increase 1.603 / exp(1.603) = 4.96

Produces a table of fit statistics for multiple glm models: AIC, AICc, BIC, p-value, pseudo R-squared (McFadden, Cox and Snell, Nagelkerke).
Smaller values for AIC, AICc, and BIC indicate a better balance of goodness-of-fit of the model and the complexity of the model. The goal is to find a model that adequately explains the data without having too many terms.
BIC tends to choose models with fewer parameters relative to AIC.

Rank <dbl>	Df.res <dbl>	AIC <dbl>	AICc <dbl>	BIC <dbl>	McFadden <dbl>	Cox.and.Snell <dbl>	Nagelkerke <dbl>	p.value <dbl>
5	390	114.1	114.3	138	0.7645	0.5678	0.8523	9.060e-71
3	392	112.1	112.2	128	0.7598	0.5656	0.8489	1.498e-72

Figure 66: GLM model comparison

Model analysis :

```
Confusion Matrix and Statistics

          Reference
Prediction  0  1
         0 21  2
         1  3 73

               Accuracy : 0.9495
                 95% CI : (0.8861, 0.9834)
    No Information Rate : 0.7576
    P-Value [Acc > NIR] : 3.298e-07

                  Kappa : 0.8605

 Mcnemar's Test P-Value : 1

            Sensitivity : 0.8750
            Specificity : 0.9733
         Pos Pred Value : 0.9130
         Neg Pred Value : 0.9605
             Prevalence : 0.2424
         Detection Rate : 0.2121
   Detection Prevalence : 0.2323
      Balanced Accuracy : 0.9242

       'Positive' Class : 0
```

Figure 67: Confusion matrix and accuracy

f.

catG3 is a binary numerical variable indicating whether you pass the test or not.
A perfect regression model needs to have a low false-positive rate and a low false-negative rate.
In minimizing these factors, we face a dilemma, and we have to decide in which case it is more harmful for us to make mistakes.
It will be costly to have a large false-positive. False-positive might ruin your study plans; failing a course might have some harmful effects on your future. But having false-negative, although still bad, is not as costly as false-positive. False-negative will make you study more, although it might cause depression. :))

Outcome	Utility
True Positive	1
True Negetive	1
False positive	-80
False Negetive	-10

$$U(p) = TP(p) + TN(p) - 80 \times FP(p) - 10 \times FN(p)$$

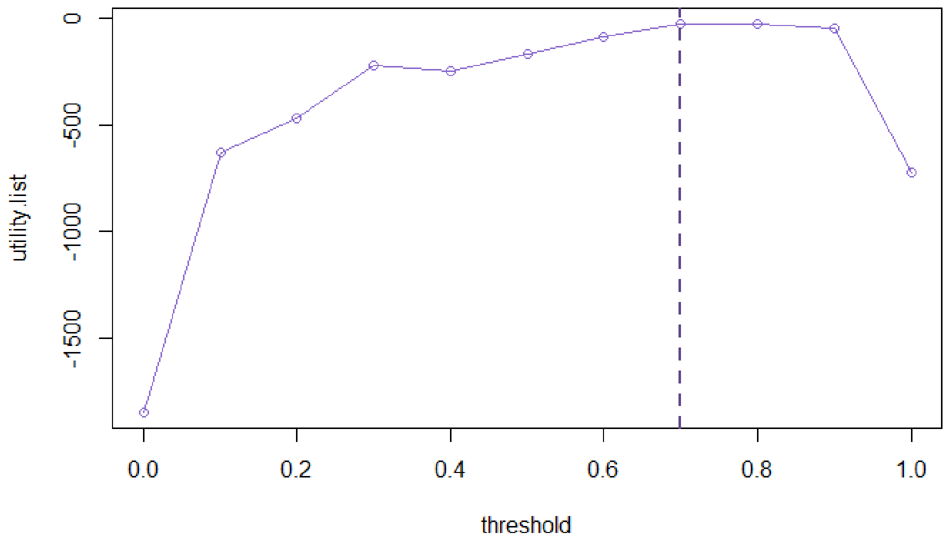


Figure 68: Utility curve

Best threshold : 0.7

Question 7

After converting the sums of Gs to a numeric binary variable :

```
Call:
glm(formula = Gsum ~ school + age + Fjob + Mjob + internet +
    romantic + health + failures + goout + studytime + absences +
    sex, family = binomial, data = train)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-1.7955  -0.5313  -0.3384  -0.1397   2.9123

Coefficients:
             Estimate Std. Error z value Pr(>|z|)
(Intercept)  -6.44436    3.08877  -2.086  0.03694 *
schoolMS     -0.15419    0.55473  -0.278  0.78105
age           0.28826    0.17245   1.672  0.09461 .
Fjobhealth   -0.33615    1.49316  -0.225  0.82188
Fjobother     0.54473    0.79098   0.689  0.49103
Fjobservices -0.45337    0.82518  -0.549  0.58272
Fjobteacher   0.79923    1.00316   0.797  0.42561
Mjobhealth   -1.90779    1.02396  -1.863  0.06244 .
Mjobother    -0.58612    0.53176  -1.102  0.27036
Mjobservices -0.45011    0.56543  -0.796  0.42600
Mjobteacher  -0.54123    0.75417  -0.718  0.47297
internetyes   0.25312    0.51771   0.489  0.62490
romanticyes   0.38277    0.39121   0.978  0.32788
health        0.05246    0.13736   0.382  0.70252
failures      1.80570    0.31693   5.697 1.22e-08 ***
goout         0.45450    0.17554   2.589  0.00962 **
studytime    -0.74356    0.28326  -2.625  0.00867 **
absences     -0.10105    0.03580  -2.822  0.00477 **
sexM         -0.78753    0.42097  -1.871  0.06138 .
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 301.62  on 296  degrees of freedom
Residual deviance: 201.84  on 278  degrees of freedom
AIC: 239.84

Number of Fisher Scoring iterations: 6
```

Figure 69: GLM model of all variables

Significant predictors are the ones with the $p-value$ smaller than 0.05 :

Question 7 continued on next page. . .

	p_value
(Intercept)	1
schoolMS	0
age	0
Fjobhealth	0
Fjobother	0
Fjobservices	0
Fjobteacher	0
Mjobhealth	0
Mjobother	0
Mjobservices	0
Mjobteacher	0
internetyes	0
romanticyes	0
health	0
failures	1
goout	1
studytime	1
absences	1
sexM	0

Figure 70: GLM model of all variables

According to figure 63, the variables that have significant p-value will be selected

$$Gsum \sim failures + goout + studytime + absence$$

Accuracy is 0.86, which is good enough for this model.
86% of the time, we can correctly predict whether a student will be on academic probation or not.
There are several statistics that can help us determine which predictor variables are most important in regression models. These statistics might not agree because the manner in which each one defines "most important" is a bit different :

- P-value : Look for the predictor variable with the lowerst p-value
- Standardized regression coefficients : Look for the predictor variable with the largest absolute value for the standardized coefficient.
- Change in R-squared when the variable is added to the model last : Look for the predictor variable that is associated with the greatest increase in R-squared. (explained comprehensively in next question)

The variable with the most effect on academic probation is the variable with the least p-value, which is failures which makes sense.

```
Confusion Matrix and Statistics

          Reference
Prediction  0  1
         0 10  1
         1 12 75

               Accuracy : 0.8673
                 95% CI : (0.7838, 0.9274)
    No Information Rate : 0.7755
    P-Value [Acc > NIR] : 0.015801

                  Kappa : 0.5367

 Mcnemar's Test P-Value : 0.005546

            Sensitivity : 0.4545
            Specificity : 0.9868
         Pos Pred Value : 0.9091
         Neg Pred Value : 0.8621
             Prevalence : 0.2245
         Detection Rate : 0.1020
   Detection Prevalence : 0.1122
      Balanced Accuracy : 0.7207

       'Positive' Class : 0
```

Figure 71: Prediction accuracy

R Codes

````
---
title: "Statistical Inference"
output:
  pdf_document: default
  #html_notebook: default
---
<h1> Phase 2 </h1>
<h2> Dataset : Students Performance </h2>
<h4> Narjes Noorzad - 810196626 </h4>
### Question 0
####Refreshing the memory:

```{r}
set.seed(NULL)
StudentsPerformance <- read.csv("StudentsPerformance.csv")
head(StudentsPerformance)
summary(StudentsPerformance)

```
### Question 1
#### Chosen varibales : *sex* and *Mjob*

##### a.



First we have to compute the proportions :
```{r warning=FALSE}
sample.size <- 200
sp.sample <- StudentsPerformance[sample(nrow(StudentsPerformance), sample.size), ]
sp.sampled.table <- table(sp.sample[,c("sex", "Fjob")])
sp.sampled.table

F.phat <- sp.sampled.table["F", "teacher"]/sum(sp.sampled.table["F",])
F.phat

M.phat <- sp.sampled.table["M", "teacher"]/sum(sp.sampled.table["M",])
M.phat
```

```{r}
SE <- sqrt( M.phat*(1-M.phat)/sum(sp.sampled.table["F",]) + M.phat*(1-M.phat)/sum(sp.sampled.table["M",]) )
SE
```

```{r}
(M.phat - F.phat) + c(-1, 1)*pnorm(0.975, lower.tail = F)*SE
```


##### b.
```{r}

p.pool <- (sp.sampled.table["F", "teacher"] + sp.sampled.table["M", "teacher"]) / (sum(sp.sampled.table["F",]) + sum(sp.sampled.table["M",]))
p.pool

SE.pool <- sqrt( p.pool * (1-p.pool)*( 1/ (sum(sp.sampled.table["F",])) + 1/ (sum(sp.sampled.table["M",]))))
````
````

```
SE.pool

p_value <- pnorm((M.phat - F.phat) / SE.pool, lower.tail = FALSE)

hypothesis.test <-function(pvalue, alpha = 0.05){
  if (pvalue < alpha){cat("Due to the fact that p-value (", round(pvalue, 3) , ") is smaller
       than", alpha, ", we reject the null hypothesis.")}
  else {cat("Due to the fact that p-value (", round(pvalue, 3) , ") is larger than " ,alpha
      , ",we fail to reject the null hypothesis.")}
  }

hypothesis.test(p_value)

```

```{r}

sp.sampled.table
sp.sampled.table.bind <- cbind(sp.sampled.table[,1] + sp.sampled.table[, 2], sp.sampled.
    table[, 3:5] )
sp.sampled.table.bind

chisq.test(sp.sampled.table , rescale.p = T)
chisq.test(sp.sampled.table.bind , rescale.p = T)

```

Question 2
Chosen varibales : *romantic*

```{r}
sample.size <- 15
romantic.sample <- StudentsPerformance[sample(nrow(StudentsPerformance), sample.size), ]$
    romantic
p.hat <- length(which(romantic.sample == 'yes' ))/sample.size
p.hat


simulation <-  data.frame(t(replicate(n = 1000, sample(levels(as.factor(StudentsPerformance$
    romantic)), size = sample.size, replace = TRUE))))

simulation.success <- apply(simulation, 1, function(x) length(which(x == 'yes')))
p_value <- length(which(simulation.success >= 8))/1000
hypothesis.test(p_value)

hist(simulation.success/sample.size)
```


Question 3
Chosen varibales : *Mjob*

a.
```{r}

sample.original <- StudentsPerformance$Mjob
round(table(sample.original) / length(StudentsPerformance$Mjob), 4)

sample.size <- 100

```

```
sample.unbiased <- sample(StudentsPerformance$Mjob, sample.size, replace = FALSE)
unbiased.table <- table(sample.unbiased)
unbiased.table


biased.prob <- ifelse(StudentsPerformance$Mjob == "teacher", 0.6, 0.4)
sample.biased <- sample(StudentsPerformance$Mjob, sample.size, prob = biased.prob)
biased.table <- table(sample.biased)
biased.table


original_prob <- c(prop.table(table(StudentsPerformance$Mjob)))
chisq.test(unbiased.table, p = original_prob)
chisq.test(biased.table, p = original_prob)
```
Chosen varibales : *Fjob*

b.
```{r}
Mjob.Fjob <- table(sp.sample[,c("Mjob","Fjob")])
Mjob.Fjob
chisq.test(Mjob.Fjob)

Mjob.Fjob.bind <- cbind(Mjob.Fjob[,1] + Mjob.Fjob[, 2] + Mjob.Fjob[, 4] + Mjob.Fjob[, 5],
    Mjob.Fjob[, 3] )
Mjob.Fjob.bind

chisq.test(Mjob.Fjob.bind, rescale.p = T)
```
Question 4
Chosen varibales : *G1* , *failure* and *studytime*

a.

```{r}
library(ggplot2)
library("ggpubr")
library(GGally)
cor(StudentsPerformance$failures, StudentsPerformance$G1)
cor(StudentsPerformance$studytime, StudentsPerformance$G1)
cor(StudentsPerformance$goout, StudentsPerformance$G1)

ggpairs(StudentsPerformance[, c(11, 10, 14)])
```

b.
a. and b.
```{r}
#just failure
lm.G1.failure <- lm(G1 ~ failures, data = StudentsPerformance)
summary(lm.G1.failure)
lm.G1.failure
```


```{r}
#condition
library(ggplot2)
library(ggfortify)
autoplot(lm.G1.failure)+ theme_classic()
```

```
```

```{r}
#just studytime
lm.G1.studytime <- lm(G1 ~ studytime, data = StudentsPerformance)
summary(lm.G1.studytime)
lm.G1.studytime
```

```{r}
#condition
library(ggplot2)
library(ggfortify)
autoplot(lm.G1.studytime)+ theme_classic()
```

c.
```{r}
G1.failures <- ggplot(StudentsPerformance, aes(x = failures)) + geom_point(aes(y = G1), size = 2, colour = "grey") + stat_smooth(aes(x = failures, y = G1, linetype = "Linear Fit"), method = "lm", formula = y ~ x, se = F, color = "black")+ scale_linetype_manual(name = "Fit Type", values = c(2, 2)) + ggtitle("G1 vs. failures")

G1.failures + theme_classic()

G1.studytime <- ggplot(StudentsPerformance, aes(x = studytime)) + geom_point(aes(y = G1), size = 2, colour = "grey") + stat_smooth(aes(x = studytime, y = G1, linetype = "Linear Fit"), method = "lm", formula = y ~ x, se = F, color = "black")+ scale_linetype_manual(name = "Fit Type", values = c(2, 2)) + ggtitle("G1 vs. studytime")

G1.studytime + theme_classic()

```


e.
```{r}

compute.R.sqr <- function(model){
  SS.reg <- (anova(model)[[2]])[1] + (anova(model)[[2]])[2]
  SS.res <- (anova(model)[[2]])[3]
  R.sqr.f <- SS.reg / (SS.res + SS.reg)
  return(R.sqr.f)
}


base.model <- lm(G1 ~ sex, data = StudentsPerformance)
anova(base.model)
SS.reg <- (anova(base.model)[[2]])[1]
SS.res <- (anova(base.model)[[2]])[2]
R.sqr <- SS.reg / (SS.res + SS.reg)


#failure vs. studytime :
model.s.f <- lm(G1 ~ sex + failures, data = StudentsPerformance)
anova(model.s.f)
R.sqr.f <- compute.R.sqr(model.s.f)

model.s.s <- lm(G1 ~ sex + studytime, data = StudentsPerformance)
```

```
anova(model.s.s)
R.sqr.s <- compute.R.sqr(model.s.s)

R.square <- c(R.sqr, R.sqr.f, R.sqr.s)
df <- data.frame(R2 = round(R.square, 2))
df <- t(df)
colnames(df) <-c ("Base" , " + failures", " + studytime")
df

#G2 vs. G3
model.s.2 <- lm(G1 ~ sex + G2, data = StudentsPerformance)
anova(model.s.2)
R.sqr.2 <- compute.R.sqr(model.s.2)

model.s.3 <- lm(G1 ~ sex + G3, data = StudentsPerformance)
anova(model.s.3)
R.sqr.3 <- compute.R.sqr(model.s.3)

R.square. <- c(R.sqr, R.sqr.2, R.sqr.3)
df <- data.frame(R2 = round(R.square., 2))
df <- t(df)
colnames(df) <-c ("Base" , " + G2", " + G3")
df

```

e.
a.
```{r}
require(caTools)
set.seed(101)

sample.size <- 100
sp.sample <- StudentsPerformance[sample(nrow(StudentsPerformance), sample.size), ]

sample <- sample.split(sp.sample$G1, SplitRatio = 9/10)
G1.train <- subset(sp.sample, sample == TRUE)
G1.test  <- subset(sp.sample, sample == FALSE)
```


```{r}
#failues
lm.G1.failures <- lm(G1 ~ failures, data = G1.train)
summary(lm.G1.failures)
p_value <- summary(lm.G1.failures)$coefficients[8]
hypothesis.test(p_value)
```


```{r}
#studytime
lm.G1.studytime <- lm(G1 ~ studytime, data = G1.train)
summary(lm.G1.studytime)
p_value <- summary(lm.G1.studytime)$coefficients[8]
hypothesis.test(p_value)
```

```{r}
#G2
lm.G1.G2 <- lm(G1 ~ G2, data = G1.train)
summary(lm.G1.G2)
```

```
p_value <- summary(lm.G1.G2)$coefficients[8]
hypothesis.test(p_value)
```


```{r}
#G3
lm.G1.G3 <- lm(G1 ~ G3, data = G1.train)
summary(lm.G1.G3)
p_value <- summary(lm.G1.G3)$coefficients[8]
hypothesis.test(p_value)
```

b.
```{r}

calculate.CI <- function(model, alpha = 0.05){

  point.est <- summary(model)$coefficients[2]
  std.error <- summary(model)$coefficients[4]

  round(point.est + c(-1, 1) * pnorm(1 - alpha/2) * std.error, 3)
}

calculate.CI(lm.G1.failures)

calculate.CI(lm.G1.studytime)

calculate.CI(lm.G1.G2)

calculate.CI(lm.G1.G3)

```

c.
```{r}
predicted.s <-  round(predict(lm.G1.studytime, G1.test, type = "response"),1)
predicted.f <-  round(predict(lm.G1.failures, G1.test, type = "response"),1)
predicted.2 <-  round(predict(lm.G1.G2, G1.test, type = "response"),1)
predicted.3 <-  round(predict(lm.G1.G3, G1.test, type = "response"),1)



pred.actual <- data.frame(G1.test$G1, predicted.s, predicted.f, predicted.2, predicted.3)
colnames(pred.actual) <- c("Actual", "Predicted studytime", "Predicted failues", "Predicted
    G2", "Predicted G3")

```
d.
```{r}
# 0.1 * data_range =  error
error <- abs(G1.test$G1 - pred.actual)
error

succes.rate.list <- c()
for (predictor in 1:length(error)) {
  error.accepted <- length(which(error[predictor] <= 2))
  succes.rate <- paste((error.accepted / length(G1.test$G1))*100, "%")
  succes.rate.list <- c(succes.rate.list, succes.rate)
}


```

```
succes.rate <- data.frame(t(succes.rate.list[2:5]))
colnames(succes.rate) <- c("Predicted studytime", "Predicted failues", "Predicted G2", "
    Predicted G3")
succes.rate
```


```{r}
# Min-Max Accuracy Calculation
predictors <- data.frame(predicted.s, predicted.f, predicted.2, predicted.3)

mm.succes.rate.list <- c()
for (p in 1:length(predictors)) {
  actuals.preds <- data.frame(cbind(actuals = G1.test$G1, predicteds = predictors[p]))
  min.max.succes.rate <- paste(round((mean(apply(actuals.preds, 1, min) / apply(actuals.
      preds, 1, max)))*100, 2), "%")
  mm.succes.rate.list <- c(mm.succes.rate.list, min.max.succes.rate)
}

succes.rate <- data.frame((t(mm.succes.rate.list)))
colnames(succes.rate) <- c("Predicted studytime", "Predicted failues", "Predicted G2", "
    Predicted G3")
succes.rate


```


```{r}
# MAPE Calculation


mape.succes.rate.list <- c()
for (p in 1:length(predictors)) {
  actuals.preds <- data.frame(cbind(actuals = G1.test$G1, predicteds = predictors[p]))
  mape.succes.rate <- paste(round((mean(abs((actuals.preds$predicteds - actuals.preds$
      actuals))/actuals.preds$actuals) )*100, 2), "%")
  mape.succes.rate.list <- c(mm.succes.rate.list, min.max.succes.rate)
}



succes.rate <- data.frame((t(mape.succes.rate.list)))
colnames(succes.rate) <- c("Predicted studytime", "Predicted failues", "Predicted G2", "
    Predicted G3")
succes.rate



```


extra part
```{r}
library(ggplot2)
library("ggpubr")
library(GGally)
cor(StudentsPerformance$G2, StudentsPerformance$G1)
cor(StudentsPerformance$G3, StudentsPerformance$G1)


ggpairs(StudentsPerformance[, c(15, 16, 14)])
```

```
```
```{r}
#G2
lm.G1.G2 <- lm(G1 ~ G2, data = StudentsPerformance)
summary(lm.G1.G2)
lm.G1.G2

library(ggplot2)
library(ggfortify)
autoplot(lm.G1.G2)+ theme_classic()

G1.G2 <- ggplot(StudentsPerformance, aes(x = G2)) + geom_point(aes(y = G1), size = 2, colour = "grey") + stat_smooth(aes(x = G2, y = G1, linetype = "Linear Fit"), method = "lm", formula = y ~ x, se = F, color = "black")+ scale_linetype_manual(name = "Fit Type", values = c(2, 2)) + ggtitle("G1 vs. G2")

G1.G2 + theme_classic()

```
```{r}
#G2
lm.G1.G3 <- lm(G1 ~ G3, data = StudentsPerformance)
summary(lm.G1.G3)
lm.G1.G3

library(ggplot2)
library(ggfortify)
autoplot(lm.G1.G3)+ theme_classic()

G1.G3 <- ggplot(StudentsPerformance, aes(x = G3)) + geom_point(aes(y = G1), size = 2, colour = "grey") + stat_smooth(aes(x = G3, y = G1, linetype = "Linear Fit"), method = "lm", formula = y ~ x, se = F, color = "black")+ scale_linetype_manual(name = "Fit Type", values = c(2, 2)) + ggtitle("G1 vs. G3")

G1.G3 + theme_classic()
```

Question 5
Chosen response varibale : *G1*
####Chosen explanatory variables : *G2*, *goout*, *failures*, *studytime*, *sex* , *age*

a.

```{r message=FALSE, warning=FALSE}

library(GGally)
p_ <- GGally::print_if_interactive
pm <- ggpairs(StudentsPerformance[, c(3, 4, 7, 10, 11, 14, 15)], progress = FALSE) + theme_minimal()
p_(pm)

pm <- ggpairs(StudentsPerformance[, c(3, 4, 7, 10, 11, 15)], progress = FALSE) + theme_minimal()
p_(pm)


StudentsPerformance$sex <- ifelse(StudentsPerformance$sex == "F", 1, 0)
library(ggcorrplot)
ggcorrplot(cor(StudentsPerformance[, c(3, 4, 7, 10, 11, 14, 15)]) , type = "lower", lab = TRUE, outline.color = "white", colors = c("black", "white", "mediumpurple3"))

```

```
```


b.
```{r}
lm.model <- lm(G1 ~ G2 + goout +  failures + studytime + sex + age , data =
    StudentsPerformance)
summary(lm.model)

```

```{r}
plot(lm.model$residuals , pch = 16, col = "mediumpurple3") + abline(lm.model)
```


e.
```{r}
library(olsrr)
#forward - p-value
forward.selection.p <- ols_step_forward_p(lm.model, details = TRUE, prem  = 0.05)


#backward - p-value
backward.elimination.p <- ols_step_backward_p(lm.model, details = TRUE, prem  = 0.05)

```

```{r}
#forward - adjusted R-sqrt
library(rms)

best.pred <- c()

adj.r.square <- function(formula, dataset, k = 1) {
  n <- length(StudentsPerformance$G1)
  r.squared <- lrm(formula = formula , data = dataset)$stat["R2"]
  adjR2 <- 1 - (((n-1)/(n-k-1)) * (1-r.squared))
}

#step 1
adj.r.squared.list1 <- c()
names <- c("G2" , "goout" ,  "failures" , "studytime" , "sex" , "age")
adj.r.squared.list1 <- c(adj.r.square(G1 ~ G2, StudentsPerformance),
                         adj.r.square(G1 ~ goout, StudentsPerformance),
                         adj.r.square(G1 ~ failures, StudentsPerformance),
                         adj.r.square(G1 ~ studytime, StudentsPerformance),
                         adj.r.square(G1 ~ sex, StudentsPerformance),
                         adj.r.square(G1 ~ age, StudentsPerformance))



max.adj.r.squared <- names[which.max(adj.r.squared.list1)]
if (max(adj.r.squared.list1, 0) > 0) { best.pred <- c(best.pred, max.adj.r.squared) }
best.pred


#step 2
names <- c("G2 + goout" ,  "G2 + failures" , "G2 + studytime" , "G2 + sex" , "G2 + age")
adj.r.squared.list2 <- c(adj.r.square(G1 ~ goout + G2, StudentsPerformance, k = 2),
                         adj.r.square(G1 ~ failures + G2, StudentsPerformance, k = 2),
```

```
                          adj.r.square(G1 ~ studytime + G2, StudentsPerformance, k = 2),
                          adj.r.square(G1 ~ sex + G2, StudentsPerformance, k = 2),
                          adj.r.square(G1 ~ age + G2, StudentsPerformance, k = 2))


max.adj.r.squared <- names[which.max(adj.r.squared.list2 - max(adj.r.squared.list1))]
if (max(adj.r.squared.list2 - max(adj.r.squared.list1)) > 0) { best.pred <- c(best.pred, max.adj.r.squared) }
best.pred

#step 3
names <- c("G2 + age + goout" , "G2 + age + failures" , "G2 + age + studytime" , "G2 + age + sex" )
adj.r.squared.list3 <- c(adj.r.square(G1 ~ goout + G2 + age , StudentsPerformance, k = 3),
                          adj.r.square(G1 ~ failures + G2 + age , StudentsPerformance, k = 3),
                          adj.r.square(G1 ~ studytime + G2 + age , StudentsPerformance, k = 3),
                          adj.r.square(G1 ~ sex + G2+ age , StudentsPerformance, k = 3))

max.adj.r.squared <- names[which.max(adj.r.squared.list3 - max(adj.r.squared.list2))]

if (max(adj.r.squared.list3 - max(adj.r.squared.list2)) > 0) { best.pred <- c(best.pred, max.adj.r.squared) }
best.pred


#step 4
names <- c("G2 + age + failures + goout" , "G2 + age + failures + studytime" , "G2 + age + failures + sex" )
adj.r.squared.list4 <- c(adj.r.square(G1 ~ goout + G2 + age + failures ,
    StudentsPerformance, k = 4),
                          adj.r.square(G1 ~ studytime + G2 + age + failures ,
                              StudentsPerformance, k = 4),
                          adj.r.square(G1 ~ sex + G2 + age + failures , StudentsPerformance,
                              k = 4))

adj.r.squared.list4
max.adj.r.squared <- names[which.max(adj.r.squared.list4 - max(adj.r.squared.list4))]
adj.r.squared.list4 - max(adj.r.squared.list3)

if (max(adj.r.squared.list3 - max(adj.r.squared.list2)) > 0) { best.pred <- c(best.pred, max.adj.r.squared) }
best.pred


all.adj.r.squared <- c(max(adj.r.squared.list1), max(adj.r.squared.list2), max(adj.r.squared.list3), max(adj.r.squared.list4))

model <- data.frame(best.pred, all.adj.r.squared)
model


```
```{r}
#backward - adjusted R-sqrt
library(rms)

fullmodel.adj.r.sqr <- adj.r.square(G1 ~ G2 + goout + failures + studytime + sex + age,
    StudentsPerformance ,k = 6)
best.pred <- c()


```

```
#step 1
adj.r.squared.list1 <- c()
names <- c("G2 + goout +  failures + studytime + sex" , "G2 + goout +  failures + studytime
    + age" ,
           "G2 + goout +  failures + sex + age" , "G2 + goout + studytime + sex + age" ,
           "G2 +  failures + studytime + sex + age" , "goout +  failures + studytime + sex +
                age")
adj.r.squared.list1 <- c(adj.r.square(G1 ~ G2 + goout +  failures + studytime + sex,
    StudentsPerformance, k = 5),
                         adj.r.square(G1 ~ G2 + goout +  failures + studytime + age,
                             StudentsPerformance, k = 5),
                         adj.r.square(G1 ~ G2 + goout +  failures + sex + age,
                             StudentsPerformance, k = 5),
                         adj.r.square(G1 ~ G2 + goout + studytime + sex + age,
                             StudentsPerformance, k = 5),
                         adj.r.square(G1 ~ G2 +  failures + studytime + sex + age,
                             StudentsPerformance, k = 5),
                         adj.r.square(G1 ~ goout +  failures + studytime + sex + age,
                             StudentsPerformance, k = 5))



max.adj.r.squared <- names[which.max(adj.r.squared.list1 - fullmodel.adj.r.sqr)]
if ( (max(adj.r.squared.list1) - fullmodel.adj.r.sqr) > 0) { best.pred <- c(best.pred, max.
    adj.r.squared) }
best.pred


#step 2
adj.r.squared.list2 <- c()
names <- c("G2 + failures + studytime + sex" , "G2 +  failures + studytime + age" ,
           "G2 + failures + sex + age" , "G2 + studytime + sex + age",
           "failures + studytime + sex + age")
adj.r.squared.list2 <- c(adj.r.square(G1 ~ G2 + failures + studytime + sex,
    StudentsPerformance, k = 4),
                         adj.r.square(G1 ~ G2 + failures + studytime + age,
                             StudentsPerformance, k = 4),
                         adj.r.square(G1 ~ G2 + failures + sex + age, StudentsPerformance, k
                            = 4),
                         adj.r.square(G1 ~ G2 + studytime + sex + age, StudentsPerformance, k
                             = 4),
                         adj.r.square(G1 ~  failures + studytime + sex + age,
                             StudentsPerformance, k = 4))



max.adj.r.squared <- names[which.max(adj.r.squared.list2 - max(adj.r.squared.list1))]
if ((max(adj.r.squared.list2) - max(adj.r.squared.list1)) > 0) { best.pred <- c(best.pred,
    max.adj.r.squared) }
best.pred


#step 3
adj.r.squared.list3 <- c()
names <- c("G2 + failures + studytime" , "G2 + failures + age" ,
           "G2 + studytime + age", "failures + studytime + age")
adj.r.squared.list3 <- c(adj.r.square(G1 ~ G2 + failures + studytime, StudentsPerformance, k
     = 3),
                         adj.r.square(G1 ~ G2 + failures + age, StudentsPerformance, k = 3),
                         adj.r.square(G1 ~ G2 + studytime + age, StudentsPerformance, k = 3),
                         adj.r.square(G1 ~  failures + studytime + age, StudentsPerformance, k
                             = 3))
```

```
max.adj.r.squared <- names[which.max(adj.r.squared.list3 - max(adj.r.squared.list2))]
if ((max(adj.r.squared.list3) - max(adj.r.squared.list2)) > 0) { best.pred <- c(best.pred,
    max.adj.r.squared) }
best.pred


#step 4
adj.r.squared.list4 <- c()
names <- c("G2 + failures" , "G2 + age", "failures + age")
adj.r.squared.list4 <- c(adj.r.square(G1 ~ G2 + failures, StudentsPerformance, k = 2),
                         adj.r.square(G1 ~ G2 + age, StudentsPerformance, k = 2),
                         adj.r.square(G1 ~ failures + age, StudentsPerformance, k = 2))



max.adj.r.squared <- names[which.max(adj.r.squared.list4 - max(adj.r.squared.list3))]
if ((max(adj.r.squared.list4) - max(adj.r.squared.list3)) > 0) { best.pred <- c(best.pred,
    max.adj.r.squared) }
best.pred


all.adj.r.squared <- c(max(adj.r.squared.list1), max(adj.r.squared.list2), max(adj.r.squared
    .list3))

model <- data.frame(best.pred, all.adj.r.squared)
model

```


```{r}
final.model <- lm(G1 ~ G2 + failures + age , data = StudentsPerformance)
summary(final.model)
```

f.
```{r}
#linearity
data <- data.frame(G2 = StudentsPerformance$G2, residuals = final.model$residuals)
ggplot(data = data,aes(G2, residuals)) + geom_point(color = "mediumpurple3", alpha = 0.5) +
    stat_smooth(method = lm, se= F, color = "black") + theme_classic()

data <- data.frame(failures = StudentsPerformance$failures, residuals = final.model$
    residuals)
ggplot(data = data,aes(failures, residuals)) + geom_point(color = "mediumpurple3", alpha =
    0.5) + stat_smooth(method = lm, se= F, color = "black") + theme_classic()

data <- data.frame(age = StudentsPerformance$age, residuals = final.model$residuals)
ggplot(data = data,aes(age, residuals)) + geom_point(color = "mediumpurple3", alpha = 0.5) +
     geom_hline( yintercept  = 0, size = 1) + theme_classic()

#nearly normal
ggplot(final.model, aes(sample = final.model$residuals)) + stat_qq(col = "mediumpurple3",
    alpha = 0.5) + stat_qq_line() + theme_classic()

ggplot(data = final.model,aes(final.model$residuals)) + geom_histogram(bins = 20, col = "
    mediumpurple2", fill="mediumpurple3", alpha = 0.5) + theme_classic()

#cons. var
```

```
ks.test(unique(final.model$residuals), "pnorm", mean=0, sd=1)
ggplot(data = final.model,aes(final.model$fitted, final.model$residuals)) + geom_point(color
    = "mediumpurple3", alpha = 0.5) + stat_smooth(method = lm, se= F, color = "black") +
    theme_classic()

```



```{r}
library(ggplot2)
library(ggfortify)
autoplot(final.model)+ theme_classic()
```

g.
```{r}
library(caret)
model <- trainControl(method = "cv", number = 5)
fullmodel.cv <- train(G1 ~ G2 + goout +  failures + studytime + sex + age, data =
    StudentsPerformance, trControl = model, method = "lm")

bestmodel.cv <- train(G1 ~ G2 + failures + age, data = StudentsPerformance, trControl =
    model, method = "lm")


fullmodel.cv
bestmodel.cv


fullmodel.cv$finalModel
bestmodel.cv$finalModel

allfolds <- bestmodel.cv$resample
```

Question 6

```{r}
StudentsPerformance$catG3 <- ifelse(StudentsPerformance$G3 < 10, 0, 1)

sample <- sample.split(StudentsPerformance$catG3, SplitRatio = 3/4)
train <- subset(StudentsPerformance, sample == TRUE)
test  <- subset(StudentsPerformance, sample == FALSE)


```


Chosen response varibale : *catG3*
####Chosen explanatory variables : *failures*, *studytime*, *G2* and *sex*

a.

```{r}
model.gml <- glm(catG3 ~ failures + studytime + G2 + sex , family = binomial(link='logit'),
     data = train)

summary(model.gml)
```

```
```

```
##### b.
```{r}

female.prob <- seq(0, 1.01, 0.01)
OR.ratio = abs(summary(model.gml)$coefficients[3])

pred.y <- function(x) {
  return ((OR.ratio*x/(1-x)) / (1 + (OR.ratio*x/(1-x))))
}
male.prob <- sapply(female.prob, pred.y)
plot(male.prob, female.prob, type = "l", col = "mediumpurple3", lwd = 1.3) + abline(a=0, b
    =1)


```

c.
```{r message=FALSE, warning=FALSE}
library(pROC)
require(ROCR)

pred <-  predict(model.gml, train , type="response")
roc(catG3 ~ pred, data = train, plot = TRUE, print.auc = TRUE, smooth = TRUE)


pred.t <-  predict(model.gml, test , type="response")
roc(catG3 ~ pred.t, data = test, plot = TRUE, print.auc = TRUE, smooth = TRUE)


```

e.
```{r}
library(rcompanion)
better.model.gml <- glm(catG3 ~ failures + G2 , family = binomial(link='logit'),  data =
    StudentsPerformance)
summary(better.model.gml)

compareGLM(model.gml, better.model.gml)

```

f.


```{r}
library(caret)


confusion.matrix <- function(threshold){
  prediction.probability <- predict(better.model.gml, newdata = test, type = "response")
  pos.neg <- ifelse(prediction.probability > threshold, "1", "0")
  p.class <- factor(pos.neg, levels = c("0", "1"))
  cm <- confusionMatrix(p.class, as.factor(test$catG3))
  return(cm)}

confusion.matrix(0.5)


threshold <- seq(0, 1, by = 0.1)
utility.list <- c()
for (i in 1:length(threshold)){
```
```

```
  cm <- confusion.matrix(threshold[i])

 TP <- cm$table[1]
 FP <- cm$table[2]
 FN <- cm$table[3]
 TN <- cm$table[4]

  utility <- TP + TN - 80*FP - 10*FN
  utility.list <- c(utility.list, utility)

}

plot(threshold, utility.list, type = "o", col = "mediumpurple3", lwd = 1.3) + abline(v =
    threshold[which.max(utility.list)], col="mediumpurple4", lwd = 2, lty=2)




```
```
### Question 7
```{r}

G.sum <- StudentsPerformance$G1 + StudentsPerformance$G2 + StudentsPerformance$G3
StudentsPerformance$Gsum <- ifelse(G.sum < 25, 1, 0)


sample <- sample.split(StudentsPerformance$Gsum, SplitRatio = 3/4)
train <- subset(StudentsPerformance, sample == TRUE)
test  <- subset(StudentsPerformance, sample == FALSE)



model.gml <- glm(Gsum ~ school + age + Fjob + Mjob + internet + romantic + health  +failures
    +goout + studytime + absences + sex , family = binomial,  data = train)

summary(model.gml)

```


```{r}

p.values <- coef(summary(model.gml))[,4]

p_value <- ifelse(p.values < 0.05, 1, 0)
significant.pvalue <- data.frame(p_value)


```


```{r}

prediction.probability <- predict(model.gml, newdata = test, type = "response")
pos.neg <- ifelse(prediction.probability > 0.5, "0", "1")
p.class <- factor(pos.neg, levels = c("0", "1"))
cm <- confusionMatrix(p.class, as.factor(test$catG3))

cm

```
```

```
‘ ‘ ‘
```

code.Rmd

Forward Selection Method

Candidate Terms:

1. G2
2. goout
3. failures
4. studytime
5. sex
6. age

We are selecting variables based on p value...

Forward Selection: Step 1

+ G2

Model Summary

R	0.851	RMSE	1.852
R-Squared	0.724	Coef. Var	17.174
Adj. R-Squared	0.723	MSE	3.429
Pred R-Squared	0.719	MAE	1.383

RMSE: Root Mean Square Error
MSE: Mean Square Error
MAE: Mean Absolute Error

ANOVA

	Sum of Squares	DF	Mean Square	F	Sig.
Regression	3537.013	1	3537.013	1031.393	0.0000
Residual	1347.737	393	3.429		
Total	4884.749	394			

Parameter Estimates

model	Beta	Std. Error	Std. Beta	t	Sig	lower	upper
(Intercept)	1.785	0.295		6.045	0.000	1.204	2.365
G2	0.733	0.023	0.851	32.115	0.000	0.688	0.778

Forward Selection: Step 2

+ age

Model Summary

R	0.854	RMSE	1.834
R-Squared	0.730	Coef. Var	17.013
Adj. R-Squared	0.729	MSE	3.365
Pred R-Squared	0.722	MAE	1.348

RMSE: Root Mean Square Error
MSE: Mean Square Error
MAE: Mean Absolute Error

ANOVA

	Sum of Squares	DF	Mean Square	F	Sig.
Regression	3565.520	2	1782.760	529.735	0.0000
Residual	1319.229	392	3.365		

```
Total         4884.749        394
----------------------------------------------------------------------

                                  Parameter Estimates
--------------------------------------------------------------------------------------
-
      model      Beta    Std. Error    Std. Beta      t        Sig      lower    upper
--------------------------------------------------------------------------------------
-
(Intercept)    -1.956         1.318                 -1.484    0.139    -4.547    0.636
         G2     0.746         0.023        0.866    32.383    0.000     0.701    0.791
        age     0.215         0.074        0.078     2.910    0.004     0.070    0.360
--------------------------------------------------------------------------------------
-


Forward Selection: Step 3

+ failures

                        Model Summary
--------------------------------------------------------------
R                       0.856       RMSE                1.825
R-Squared               0.733       Coef. Var          16.928
Adj. R-Squared          0.731       MSE                 3.332
Pred R-Squared          0.723       MAE                 1.363
--------------------------------------------------------------
 RMSE: Root Mean Square Error
 MSE: Mean Square Error
 MAE: Mean Absolute Error

                                ANOVA
----------------------------------------------------------------------
                Sum of
               Squares        DF    Mean Square       F         Sig.
----------------------------------------------------------------------
Regression    3582.019         3       1194.006    358.368    0.0000
Residual      1302.730       391          3.332
Total         4884.749       394
----------------------------------------------------------------------

                                  Parameter Estimates
--------------------------------------------------------------------------------------
--
      model      Beta    Std. Error    Std. Beta      t        Sig      lower
upper
--------------------------------------------------------------------------------------
--
(Intercept)    -1.994         1.311                 -1.521    0.129    -4.573
0.584
         G2     0.718         0.026        0.834    27.563    0.000     0.667
0.769
        age     0.244         0.075        0.088     3.270    0.001     0.097
0.390
   failures    -0.323         0.145       -0.068    -2.225    0.027    -0.608    -
0.038
--------------------------------------------------------------------------------------
--


Forward Selection: Step 4

+ studytime

                        Model Summary
--------------------------------------------------------------
R                       0.857       RMSE                1.823
R-Squared               0.735       Coef. Var          16.906
Adj. R-Squared          0.732       MSE                 3.323
Pred R-Squared          0.723       MAE                 1.361
```

```
--------------------------------------------------------------
 RMSE: Root Mean Square Error
 MSE: Mean Square Error
 MAE: Mean Absolute Error

                                ANOVA
----------------------------------------------------------------------
                Sum of
                Squares        DF    Mean Square       F         Sig.
----------------------------------------------------------------------
Regression     3588.724         4        897.181     269.98    0.0000
Residual       1296.025       390          3.323
Total          4884.749       394
----------------------------------------------------------------------

                                  Parameter Estimates
--------------------------------------------------------------------------------------
--
      model      Beta    Std. Error    Std. Beta       t        Sig      lower
upper
--------------------------------------------------------------------------------------
--
(Intercept)    -2.205         1.318                 -1.673    0.095    -4.796
0.387
         G2     0.715         0.026        0.830    27.384    0.000     0.664
0.766
        age     0.239         0.075        0.087     3.204    0.001     0.092
0.385
   failures    -0.298         0.146       -0.063    -2.043    0.042    -0.585    -
0.011
  studytime     0.159         0.112        0.038     1.420    0.156    -0.061
0.378
--------------------------------------------------------------------------------------
--


Forward Selection: Step 5

+ sex

                        Model Summary
--------------------------------------------------------------
R                       0.858       RMSE                1.822
R-Squared               0.736       Coef. Var          16.896
Adj. R-Squared          0.732       MSE                 3.319
Pred R-Squared          0.722       MAE                 1.361
--------------------------------------------------------------
 RMSE: Root Mean Square Error
 MSE: Mean Square Error
 MAE: Mean Absolute Error

                                ANOVA
----------------------------------------------------------------------
                Sum of
                Squares        DF    Mean Square       F         Sig.
----------------------------------------------------------------------
Regression     3593.549         5        718.710    216.526    0.0000
Residual       1291.200       389          3.319
Total          4884.749       394
----------------------------------------------------------------------

                                  Parameter Estimates
--------------------------------------------------------------------------------------
--
      model      Beta    Std. Error    Std. Beta       t        Sig      lower
upper
--------------------------------------------------------------------------------------
--
(Intercept)    -2.139         1.319                 -1.622    0.106    -4.731
0.453
```

```
         G2     0.711         0.026         0.825    26.946    0.000     0.659
0.762
        age     0.240         0.075         0.087     3.227    0.001     0.094
0.387
   failures    -0.309         0.146        -0.065    -2.119    0.035    -0.597    -
0.022
  studytime     0.203         0.117         0.048     1.728    0.085    -0.028
0.434
        sex    -0.235         0.195        -0.033    -1.206    0.229    -0.618
0.148
-------------------------------------------------------------------------------------
--


No more variables to be added.

Variables Entered:

+ G2
+ age
+ failures
+ studytime
+ sex


Final Model Output
------------------

                        Model Summary
--------------------------------------------------------------
R                       0.858       RMSE                1.822
R-Squared               0.736       Coef. Var          16.896
Adj. R-Squared          0.732       MSE                 3.319
Pred R-Squared          0.722       MAE                 1.361
--------------------------------------------------------------
 RMSE: Root Mean Square Error
 MSE: Mean Square Error
 MAE: Mean Absolute Error

                                ANOVA
----------------------------------------------------------------------
                Sum of
               Squares         DF    Mean Square       F          Sig.
----------------------------------------------------------------------
Regression    3593.549          5        718.710    216.526    0.0000
Residual      1291.200        389          3.319
Total         4884.749        394
----------------------------------------------------------------------

                                  Parameter Estimates
-------------------------------------------------------------------------------------
--
      model      Beta    Std. Error    Std. Beta      t        Sig      lower
upper
-------------------------------------------------------------------------------------
--
(Intercept)    -2.139         1.319                  -1.622    0.106    -4.731
0.453
         G2     0.711         0.026         0.825    26.946    0.000     0.659
0.762
        age     0.240         0.075         0.087     3.227    0.001     0.094
0.387
   failures    -0.309         0.146        -0.065    -2.119    0.035    -0.597    -
0.022
  studytime     0.203         0.117         0.048     1.728    0.085    -0.028
0.434
        sex    -0.235         0.195        -0.033    -1.206    0.229    -0.618
0.148
-------------------------------------------------------------------------------------
--
```

Backward Elimination Method

Candidate Terms:

1 . G2
2 . goout
3 . failures
4 . studytime
5 . sex
6 . age

We are eliminating variables based on p value...

x goout

Backward Elimination: Step 1

Variable goout Removed

Model Summary

R	0.858	RMSE	1.822
R-Squared	0.736	Coef. Var	16.896
Adj. R-Squared	0.732	MSE	3.319
Pred R-Squared	0.722	MAE	1.361

RMSE: Root Mean Square Error
MSE: Mean Square Error
MAE: Mean Absolute Error

ANOVA

	Sum of Squares	DF	Mean Square	F	Sig.
Regression	3593.549	5	718.710	216.526	0.0000
Residual	1291.200	389	3.319		
Total	4884.749	394			

Parameter Estimates

model	Beta	Std. Error	Std. Beta	t	Sig	lower	upper
(Intercept)	-2.139	1.319		-1.622	0.106	-4.731	0.453
G2	0.711	0.026	0.825	26.946	0.000	0.659	0.762
failures	-0.309	0.146	-0.065	-2.119	0.035	-0.597	-0.022
studytime	0.203	0.117	0.048	1.728	0.085	-0.028	0.434
sex	-0.235	0.195	-0.033	-1.206	0.229	-0.618	0.148
age	0.240	0.075	0.087	3.227	0.001	0.094	0.387

x sex

Backward Elimination: Step 2

Variable sex Removed

Model Summary

```
R                        0.857       RMSE                1.823
R-Squared                0.735       Coef. Var          16.906
Adj. R-Squared           0.732       MSE                 3.323
Pred R-Squared           0.723       MAE                 1.361
--------------------------------------------------------------
 RMSE: Root Mean Square Error
 MSE: Mean Square Error
 MAE: Mean Absolute Error

                                ANOVA
----------------------------------------------------------------------
                Sum of
               Squares         DF    Mean Square       F          Sig.
----------------------------------------------------------------------
Regression    3588.724          4        897.181     269.98    0.0000
Residual      1296.025        390          3.323
Total         4884.749        394
----------------------------------------------------------------------

                                  Parameter Estimates
--------------------------------------------------------------------------------------
--
      model      Beta    Std. Error    Std. Beta      t        Sig      lower
upper
--------------------------------------------------------------------------------------
--
(Intercept)    -2.205         1.318                 -1.673    0.095    -4.796
0.387
         G2     0.715         0.026        0.830    27.384    0.000     0.664
0.766
   failures    -0.298         0.146       -0.063    -2.043    0.042    -0.585    -
0.011
  studytime     0.159         0.112        0.038     1.420    0.156    -0.061
0.378
        age     0.239         0.075        0.087     3.204    0.001     0.092
0.385
--------------------------------------------------------------------------------------
--


x studytime

Backward Elimination: Step 3

 Variable studytime Removed

                        Model Summary
--------------------------------------------------------------
R                        0.856       RMSE                1.825
R-Squared                0.733       Coef. Var          16.928
Adj. R-Squared           0.731       MSE                 3.332
Pred R-Squared           0.723       MAE                 1.363
--------------------------------------------------------------
 RMSE: Root Mean Square Error
 MSE: Mean Square Error
 MAE: Mean Absolute Error

                                ANOVA
----------------------------------------------------------------------
                Sum of
               Squares         DF    Mean Square       F          Sig.
----------------------------------------------------------------------
Regression    3582.019          3       1194.006    358.368    0.0000
Residual      1302.730        391          3.332
Total         4884.749        394
----------------------------------------------------------------------

                                  Parameter Estimates
--------------------------------------------------------------------------------------
--
      model      Beta    Std. Error    Std. Beta      t        Sig      lower
upper
```

```
--------------------------------------------------------------------------------------
--
(Intercept)    -1.994         1.311                  -1.521    0.129    -4.573
0.584
         G2     0.718         0.026         0.834    27.563    0.000     0.667
0.769
   failures    -0.323         0.145        -0.068    -2.225    0.027    -0.608    -
0.038
        age     0.244         0.075         0.088     3.270    0.001     0.097
0.390
--------------------------------------------------------------------------------------
--


No more variables satisfy the condition of p value = 0.05


Variables Removed:

x goout
x sex
x studytime


Final Model Output
------------------

                        Model Summary
--------------------------------------------------------------
R                       0.856       RMSE               1.825
R-Squared               0.733       Coef. Var         16.928
Adj. R-Squared          0.731       MSE                3.332
Pred R-Squared          0.723       MAE                1.363
--------------------------------------------------------------
 RMSE: Root Mean Square Error
 MSE: Mean Square Error
 MAE: Mean Absolute Error

                                ANOVA
----------------------------------------------------------------------
                Sum of
               Squares         DF    Mean Square       F         Sig.
----------------------------------------------------------------------
Regression    3582.019          3       1194.006    358.368    0.0000
Residual      1302.730        391          3.332
Total         4884.749        394
----------------------------------------------------------------------

                                  Parameter Estimates
--------------------------------------------------------------------------------------
--
      model      Beta    Std. Error    Std. Beta      t        Sig      lower
upper
--------------------------------------------------------------------------------------
--
(Intercept)    -1.994         1.311                  -1.521    0.129    -4.573
0.584
         G2     0.718         0.026         0.834    27.563    0.000     0.667
0.769
   failures    -0.323         0.145        -0.068    -2.225    0.027    -0.608    -
0.038
        age     0.244         0.075         0.088     3.270    0.001     0.097
0.390
--------------------------------------------------------------------------------------
--
```