MATHEMATICAL ASSOCIATION OF AMERICA

Mathematics in Data Science

Mathematical Association of America Distinguished Lecture Series

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Waltham, MA

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What is data science?

Quick Definition of Data Science

If you're using **mathematics or statistics**
powered by **modern computing tools**
to answer a **real world question**,
you are probably doing data science.

Example Data Science Questions

Data science = math/stats + computing + application

- **Social justice:** Do the U.S. government's records of mortgage applications reveal any patterns of discrimination?
- **Sports:** What baseball players offer the best value in my fantasy league draft?
- **Business:** How can shipping companies ensure every container on a ship gets filled?
- **Public health:** What percentage of the people in my state will eventually be vaccinated?

Mathematics in Data Science

Working with Data

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Doing an Analysis

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Unsolved Problems

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Part 1

Working with Data

Example data: U.S. COVID vaccinations, 2021

Date	Location	Total vacc's	Total Distrib'd	People vacc'd	People vacc'd per hundred	···
2021-01-12	Alabama	78134.0	377025.0	70861.0	0.15	···
2021-01-13	Alabama	84040.0	378975.0	74792.0	0.19	···
2021-01-14	Alabama	92300.0	435350.0	80480.0	NaN	···
2021-01-15	Alabama	100567.0	444650.0	86956.0	0.28	···
2021-01-16	Alabama	NaN	NaN	NaN	NaN	···
2021-01-17	Alabama	NaN	NaN	NaN	NaN	···
2021-01-18	Alabama	NaN	NaN	NaN	NaN	···
2021-01-19	Alabama	130795.0	444650.0	114319.0	0.33	···
⋮	⋮	⋮	⋮	⋮	⋮	
2021-12-12	Wyoming	655151.0	840845.0	316311.0	46.42	···
2021-12-13	Wyoming	655179.0	840845.0	316355.0	46.42	···
2021-12-14	Wyoming	662705.0	845255.0	317645.0	46.72	···

Source: `https://ourworldindata.org/us-states-vaccinations`

Task #1: Lookup

How many people per hundred were vaccinated in Alabama as of January 19, 2021?

Date	Location	Total vacc's	Total Distrib'd	People vacc'd	People vacc'd per hundred	···
2021-01-12	Alabama	78134.0	377025.0	70861.0	0.15	···
2021-01-13	Alabama	84040.0	378975.0	74792.0	0.19	···
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2021-01-16	Alabama	NaN	NaN	NaN	NaN	···
2021-01-17	Alabama	NaN	NaN	NaN	NaN	···
2021-01-18	Alabama	NaN	NaN	NaN	NaN	···
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⋮	⋮	⋮	⋮	⋮	⋮	
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- Domain and range
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Doing an Analysis

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Unsolved Problems

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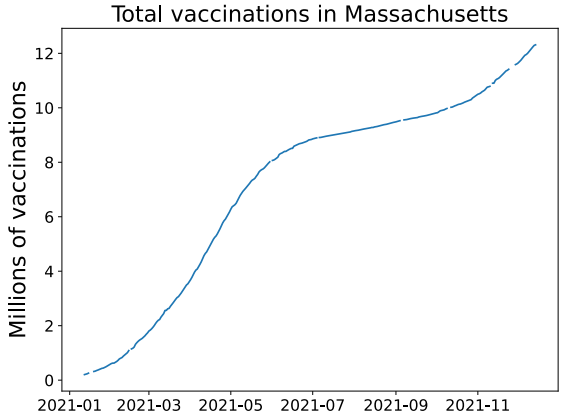
Task #2: Visualization

How has the vaccine distribution gone in my state?



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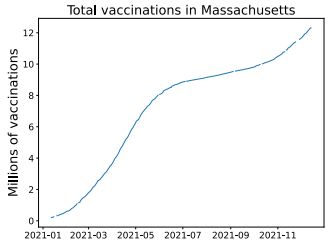

Data	Visualization
function on real numbers	line/curve plot
function from pairs to real numbers	surface plot/map
function with small (finite) domain	bar chart
subset of the plane	scatter plot
subset of the real line	histogram
matrix	heat map/graph

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- Sets, subsets, finite sets
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Doing an Analysis

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Unsolved Problems

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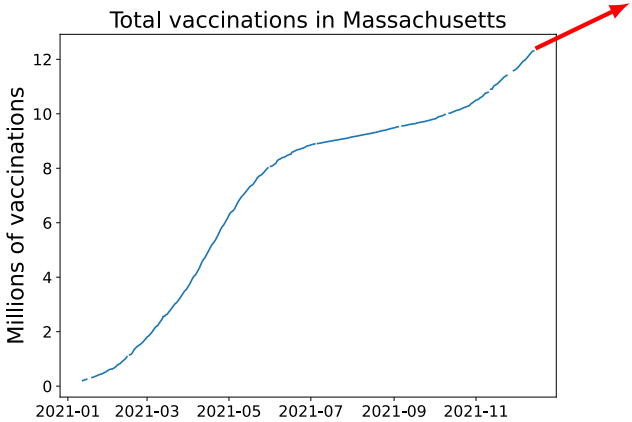
Task #3: Prediction

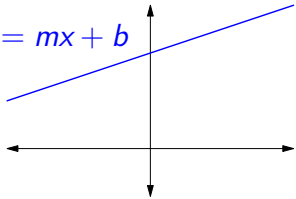
How many vaccinations will eventually be given in my state?



Famous function families

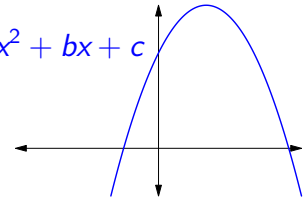
Linear

$y = mx + b$



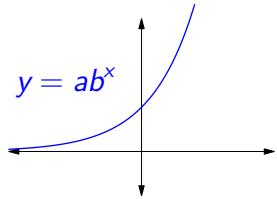
Quadratic

$y = ax^2 + bx + c$



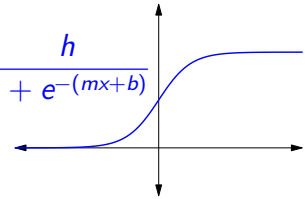
Exponential

$y = ab^x$

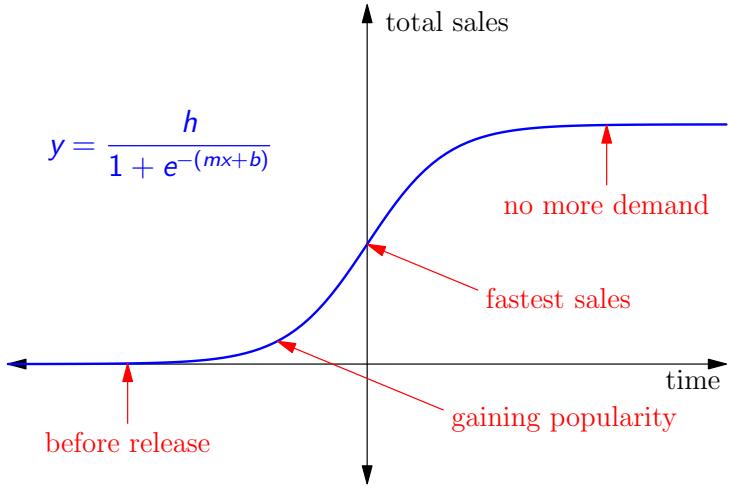


Logistic

$y = \dfrac{h}{1 + e^{-(mx+b)}}$



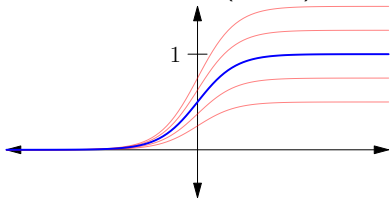
The logistic family: Rolling out a new product



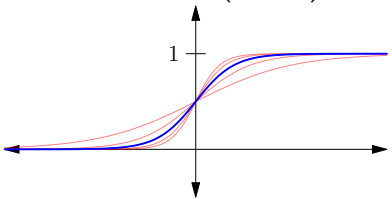
The logistic family: Three parameters

$$y = \frac{h}{1 + e^{-(mx+b)}}$$

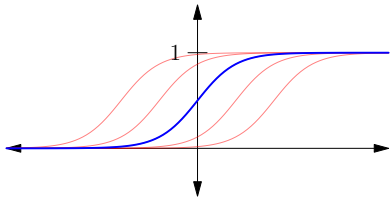
As h varies ($h > 0$)



As m varies ($m > 0$)

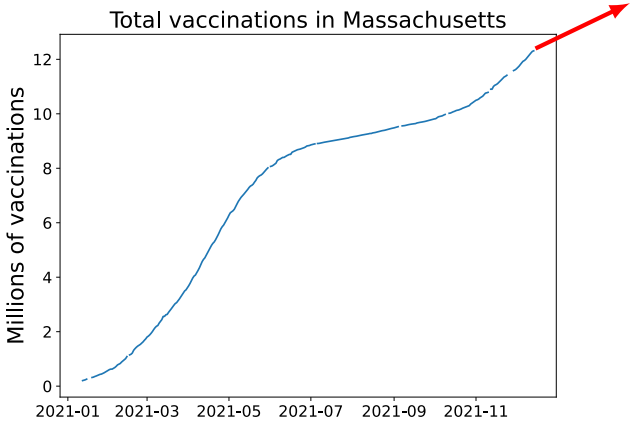


As b varies



Task #3: Prediction

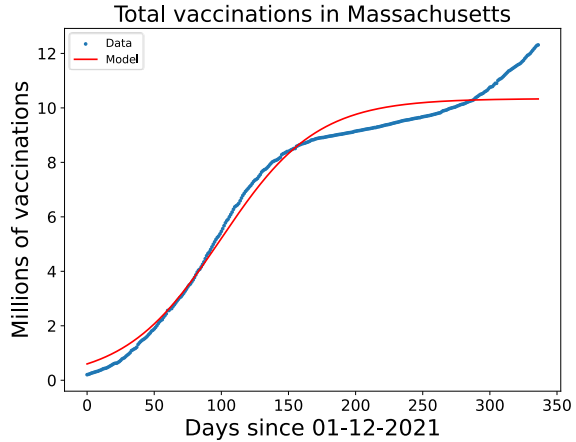
How many vaccinations will eventually be given in my state?



Task #3: Prediction

How many vaccinations will eventually be given in my state?

$$y = \frac{10.3}{1 + e^{-(0.028x - 2.79)}}$$



Task #3: Prediction

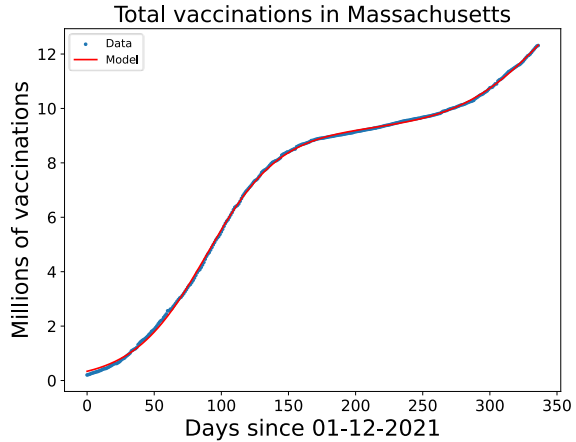
How many vaccinations will eventually be given in my state?

$$y = \frac{h_1}{1 + e^{-(m_1 x + b_1)}} + \frac{h_2}{1 + e^{-(m_2 x + b_2)}}$$

Task #3: Prediction

How many vaccinations will eventually be given in my state?

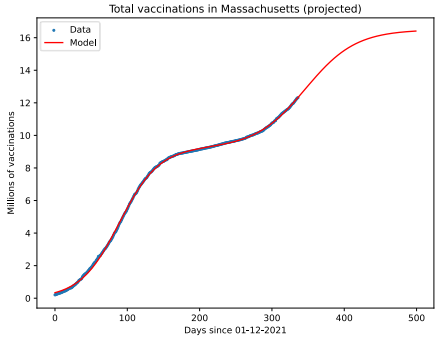
$$y = \frac{9.24}{e^{-(0.037x-3.27)}} + \frac{7.26}{e^{-(0.029x-9.97)}}$$



Task #3: Prediction

How many vaccinations will eventually be given in my state?

$$\lim_{t\to\infty}\left(\frac{h_1}{1+e^{-(m_1x+b_1)}}+\frac{h_2}{1+e^{-(m_2x+b_2)}}\right)=h_1+h_2 \quad (\approx 16.5)$$



Mathematics in Data Science

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- Functions and relations
- Domain and range
- Famous sets (reals, etc.)
- Sets, subsets, finite sets
- Asymptotes/limits
- Famous function families
- Algebraic transformations

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Unsolved Problems

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Part 2

Doing an Analysis

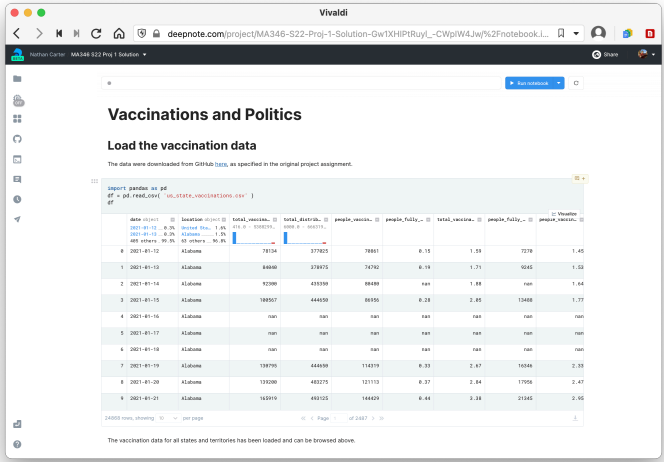
What do I mean by an “analysis?”

Example

Are vaccination rates in a state related to the state’s politics?

- Find and load vaccination and voting data for each state.
- Perform the prediction we just saw on each state separately.
- Examine correlations between the predictions and voting data.
- Formulate those correlations as a hypothesis test.
- Summarize the results in a report with visualizations.

Example computational narrative (in Jupyter)



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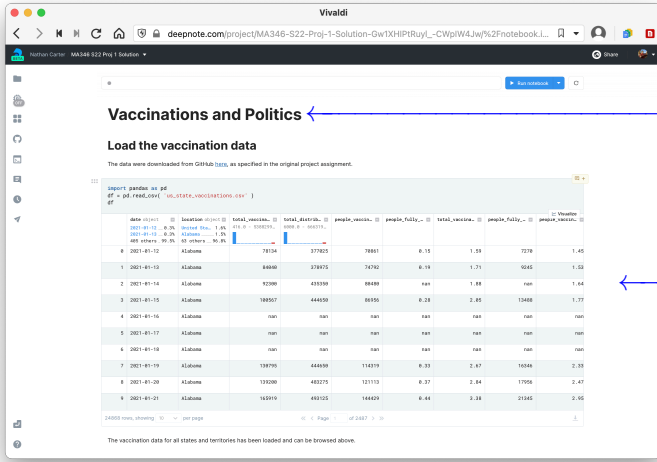
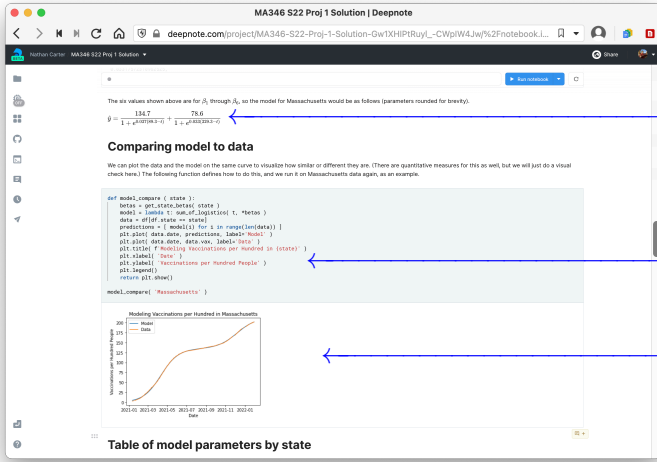


Headings

Tables

Example computational narrative (in Jupyter)



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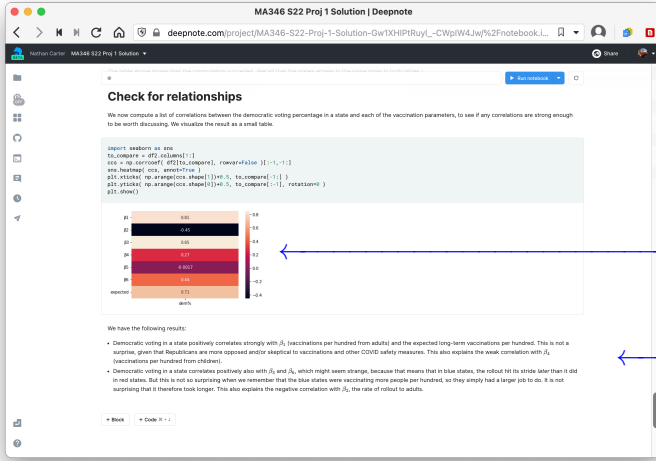

Visualizations

Text

Computational narratives are like proofs

Computational narratives are like proofs

Vaccination data timeframe

What time period does the vaccination data cover?

```
[8]
df.Date.iloc[0], df.Date.iloc[-1]
```

```
(Timestamp('2021-01-12 00:00:00'), Timestamp('2021-12-14 00:00:00'))
```

The minimum and maximum dates in the dataset are therefore:

- January 12, 2021
- December 14, 2021

Computational narratives are like proofs

Vaccination data timeframe

What time period does the vaccination data cover?

```
[7]
df.Date.min(), df.Date.max()
```

```
(Timestamp('2021-01-12 00:00:00'), Timestamp('2021-12-14 00:00:00'))
```

The minimum and maximum dates in the dataset are therefore:

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Doing an Analysis

- Questioning assumptions
- Using official definitions
-
-

Unsolved Problems

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Great Expectations, from Superconductive

great_expectations

COMMUNITY EXPECTATIONS DOCUMENTATION CASE STUDIES BLOG

Greetings! We now have a cloud offering Learn More

Case studies from Great Expectations

Stay up to date on our releases and comments on the current state of data management

Great Expectations Case Study: Vimeo

March 24, 2021

About Vimeo Vimeo is one of the world's leading service providers of HD streaming video, allowing over 200M professionals, teams, and...

Great Expectations Case Study: Heineken

November 16, 2020

Check out the video of the HEINEKEN team presenting at our first Great Expectations Community Show & Tell in October 2020! About HEINEKEN...

Great Expectations Case Study: Avanade

October 08, 2020

About Avanade Avanade is a global professional services company providing IT consulting and services focused on the Microsoft platform.

Don't actually read this proof

Theorem

Every prime $p > 3$ is adjacent to a multiple of 6.

Proof.

Let $p > 3$ be prime and therefore divisible by neither 2 nor 3. So for some k we have $p = 2k + 1$ and for some m we have either $p = 3m + 1$ or $p = 3m + 2$.

Case 1: When $p = 3m + 1$, $p - 1 = 3m$ and $p - 1$ is divisible by 3. But $p - 1 = 2k$ as well, so $p - 1$ is also divisible by 2. So $p - 1$ is divisible by 6 and adjacent to p.

Case 2: When $p = 3m + 2$, $p + 1 = 3(m + 1)$ and $p + 1$ is divisible by 3. But $p + 1 = 2(k + 1)$ as well, so $p + 1$ is also divisible by 2. So $p + 1$ is divisible by 6 and adjacent to p. □

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Case 2: When $p = 3m + 2$, $p + 1 = 3(m + 1)$ and $p + 1$ is divisible by 3. But $p + 1 = 2(k + 1)$ as well, so $p + 1$ is also divisible by 2. So $p + 1$ is divisible by 6 and adjacent to p. □

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Computational narratives are step-by-step constructions

← Computer's memory at A

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← Computer's memory at B

Don't actually read this proof

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□

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Case 1: When $p = 3m + 1$, $p - 1 = 3m$ and $p - 1$ is divisible by 3. But $p - 1 = 2k$ as well, so $p - 1$ is also divisible by 2. So $p - 1$ is divisible by 6 and adjacent to p.

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Since the theorem holds in both cases, it holds in general. □

Computational narratives are *narratives*

Vaccination data timeframe ← Reader's intuition and comfort at A

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- Asymptotes/limits
- Famous function families
- Algebraic transformations

Doing an Analysis

- Questioning assumptions
- Using official definitions
- Step-by-step constructions
- Supplying intuition

Unsolved Problems

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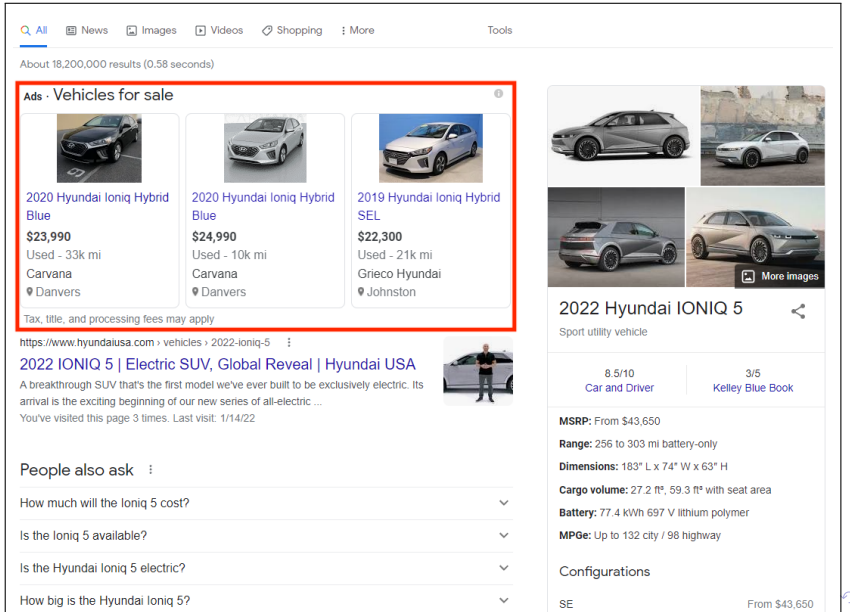
Which advertisements to show?

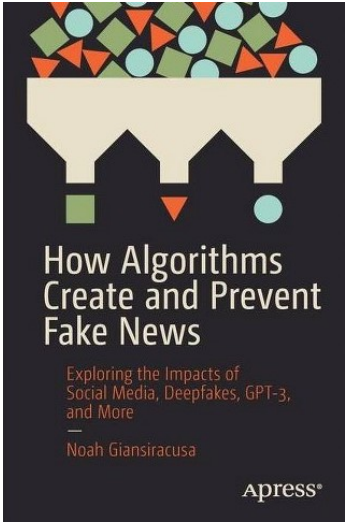


Which advertisements to show?

Google's assumption

If we let the behavior of people on our site determine which ads should show up for *everyone* that will be good, in all cases.

Which advertisements to show?




SEARCH ENGINES

acmqueue Discrimination in Online Ad Delivery

Google ads, black names and white names, racial discrimination, and click advertising

Latanya Sweeney

Do online ads suggestive of arrest records appear more often with searches of black-sounding names than white-sounding names? What is a black-sounding name or white-sounding name, anyway? How many more times would an ad have to appear adversely affecting one racial group for it to be considered discrimination? Is online activity so ubiquitous that computer scientists have to think about societal consequences such as structural racism in technology design? If so, how is this technology to be built? Let's take a scientific dive into online ad delivery to find answers.

"Have you ever been arrested?" Imagine this question appearing whenever someone enters your name in a search engine. Perhaps you are in competition for an award, a scholarship, an appointment, a promotion, or a new job, or maybe you are in a position of trust, such as a professor, a physician, a banker, a judge, a manager, or a volunteer. Perhaps you are completing a rental application, selling goods, applying for a loan, joining a social club, making new friends, dating, or engaged in any one of hundreds of circumstances for which someone wants to learn more about you online. Appearing alongside your list of accomplishments is an advertisement implying you may have a criminal record, whether you actually have one or not. Worse, the ads may not appear for your competitors.

Job applications frequently include questions such as: Have you ever been arrested? Have you ever been charged with a crime? Other than a traffic ticket, have you been convicted of a crime? Employers ask these questions to establish trustworthiness. Because others often equate a criminal record with not being reliable or honest, protections exist for those having criminal records.

If an employer disqualifies a job applicant based solely upon information indicating an arrest record, the company may face legal consequences. The U.S. EEOC (Equal Employment Opportunity Commission) is the federal agency charged with enforcing Title VII of the Civil Rights Act of 1964, a law that applies to most employers, prohibiting employment discrimination based on race, color, religion, sex, or national origin. Guidance issued in 1973 extended protections to people with criminal records.[1,2] Title VII does not prohibit employers from obtaining criminal background information. Certain uses of criminal information, however, such as a blanket policy or practice of excluding applicants or disqualifying employees based solely upon information indicating an arrest record, can result in a charge of discrimination.

To make a determination, the EEOC uses an adverse impact test that measures whether certain practices, intentional or not, have a disproportionate effect on a group of people whose defining characteristics are covered by Title VII. To decide, you calculate the percentage of people affected in each group and then divide the smaller value by the larger to get the ratio and compare the result to 80. For example, suppose a company laid off comparable black and white workers at the same rate—25 percent of blacks and 25 percent of whites—then the ratio, 25 divided by 25, would be 100 percent. If the ratio is less than 80 percent, then the EEOC considers the effect disproportionate and may hold the employer responsible for discrimination.[3]

1

Mathematics in Data Science

Working with Data

- Functions and relations
- Domain and range
- Famous sets (reals, etc.)
- Sets, subsets, finite sets
- Asymptotes/limits
- Famous function families
- Algebraic transformations

Doing an Analysis

- Questioning assumptions
- Using official definitions
- Step-by-step constructions
- Supplying intuition

Unsolved Problems

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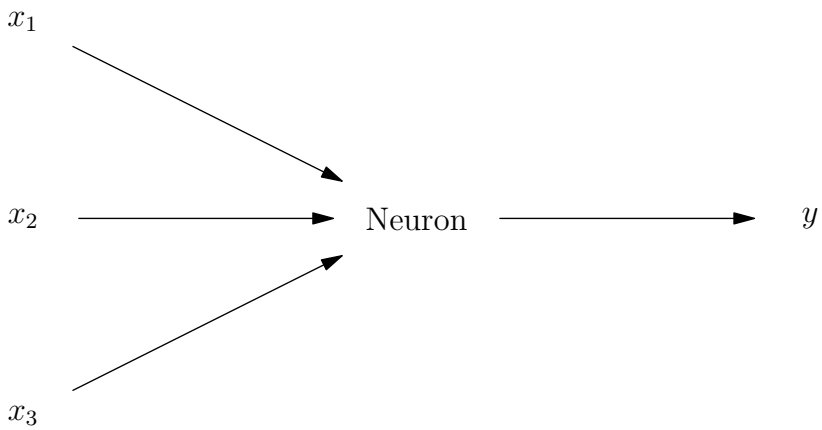
Part 3

Unsolved Problems

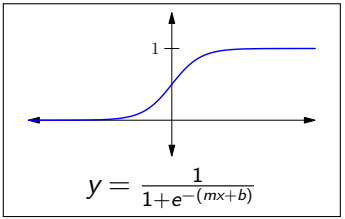
Neural Networks

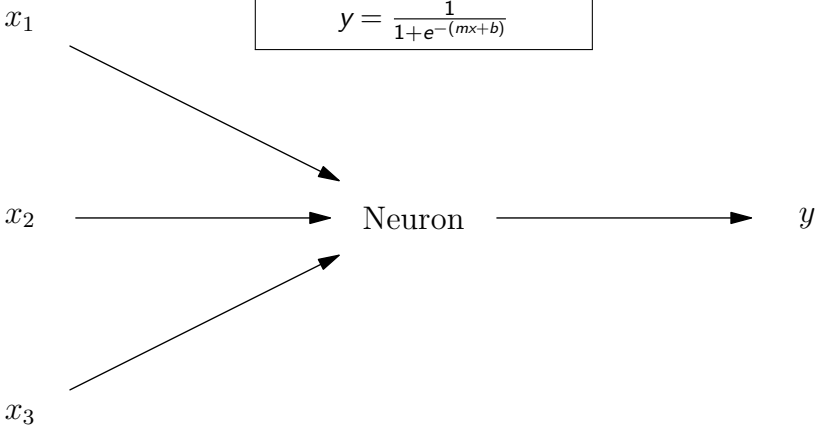
Inspiration



Neural Networks

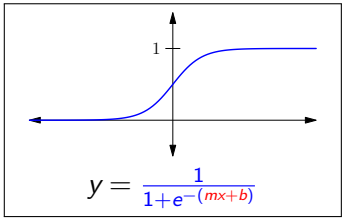
Inspiration

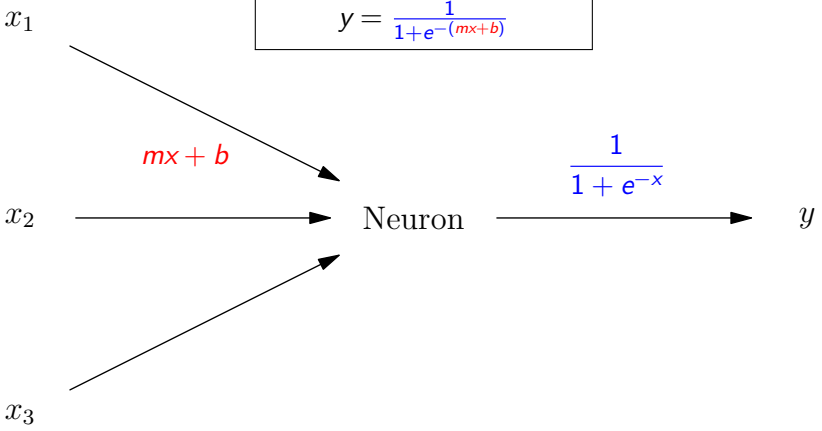




Neural Networks

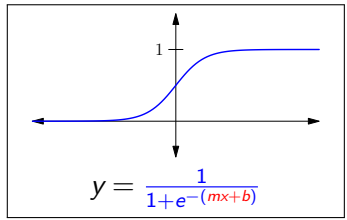
Inspiration

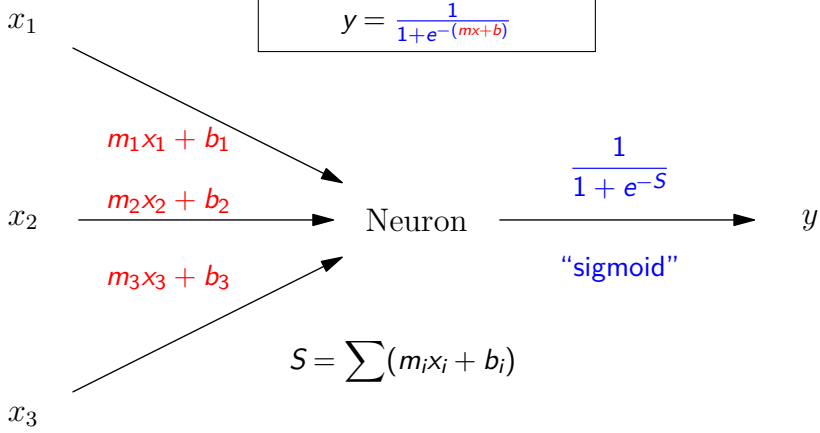




Neural Networks

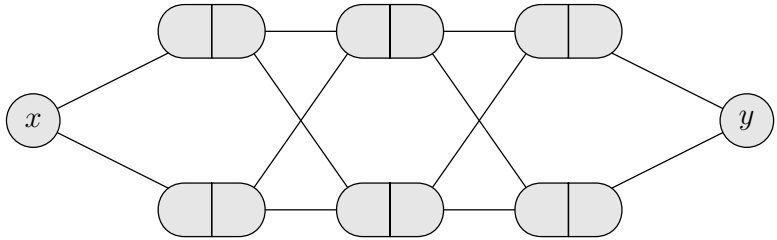
Inspiration





Neural Networks

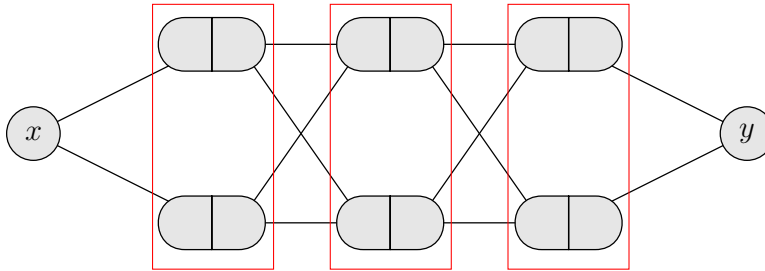
Example network



with sigmoid activation function $(\frac{1}{1+e^{-x}})$

Neural Networks

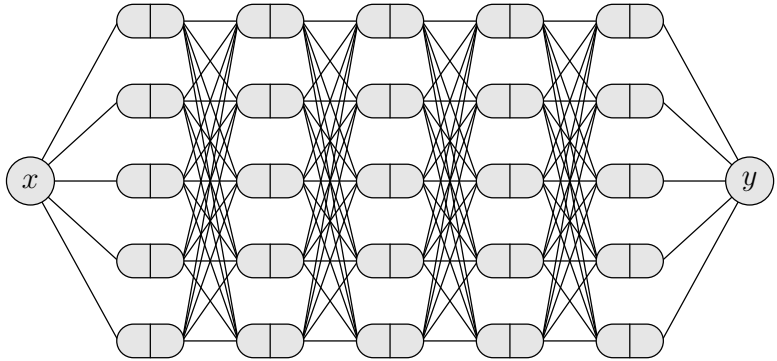
Example network



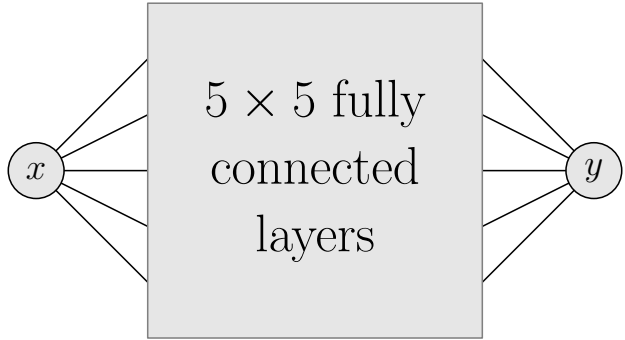
with sigmoid activation function $(\frac{1}{1+e^{-x}})$

Neural Networks

Network with 5 dense layers of width 5

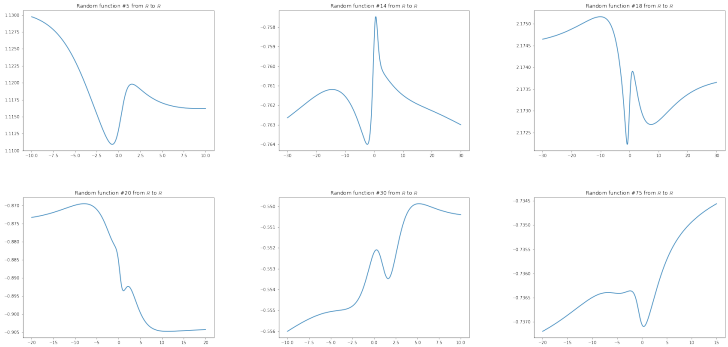


Neural Networks

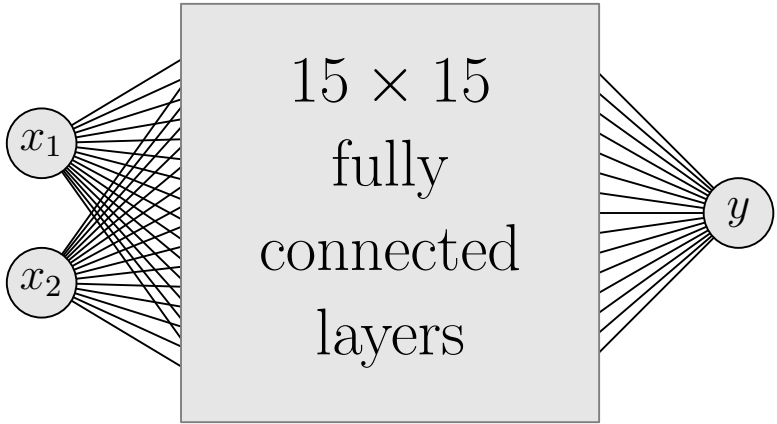


Example Neural Network functions $f : \mathbb{R} \to \mathbb{R}$

5 dense layers of width 5 with sigmoid activation function

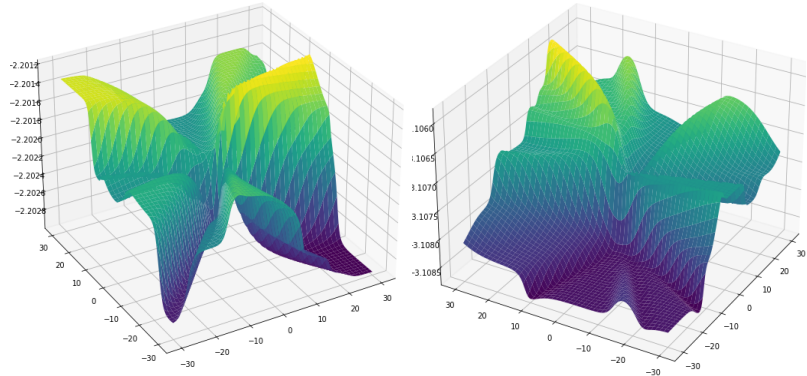


Neural Networks



Example Neural Network functions $f : \mathbb{R}^2 \to \mathbb{R}$

15 dense layers of width 15 with sigmoid activation function



Example Neural Network functions $f : \mathbb{R}^2 \to \mathbb{R}$

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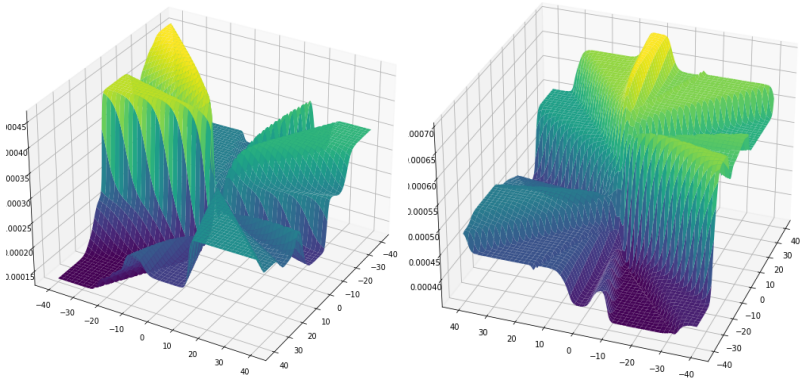
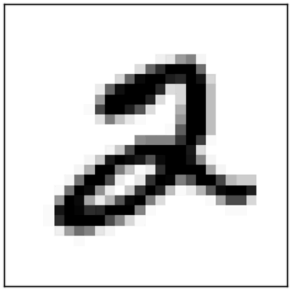


What can neural networks do?

Example: Classify handwritten digits

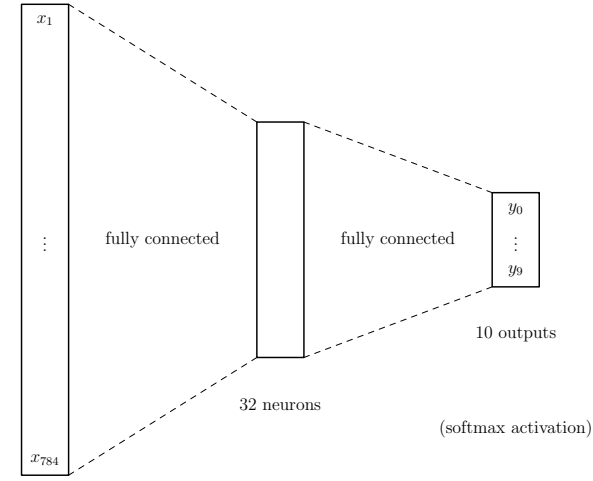


2

Grayscale image, 28×28
$x_1, x_2, \ldots, x_{784} \in [0, 1]$

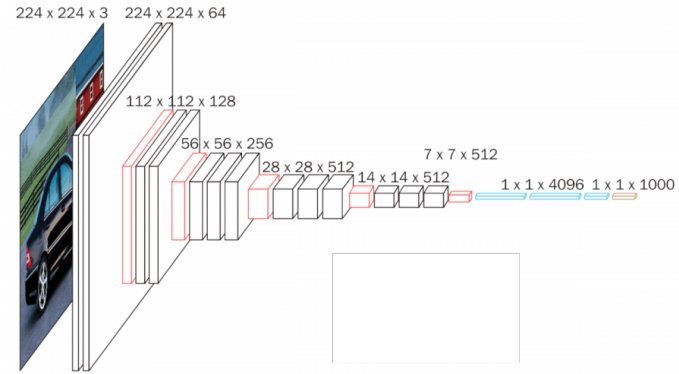
Predicted digit
$y \in \{0, \ldots, 9\}$

Architecture to classify handwritten digits



VGG16 Architecture

Authors: Karen Simonyan and Andrew Zisserman, 2014

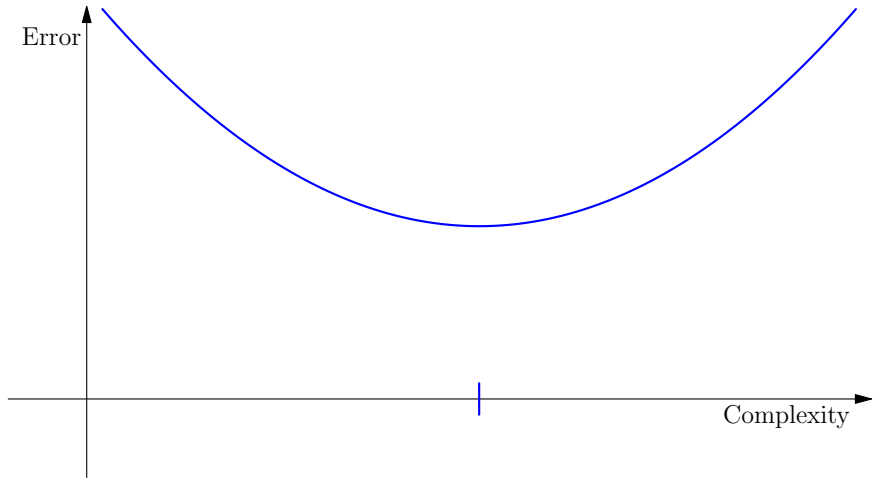


(about 14.7 million parameters)

Two Unsolved Problems in Neural Networks

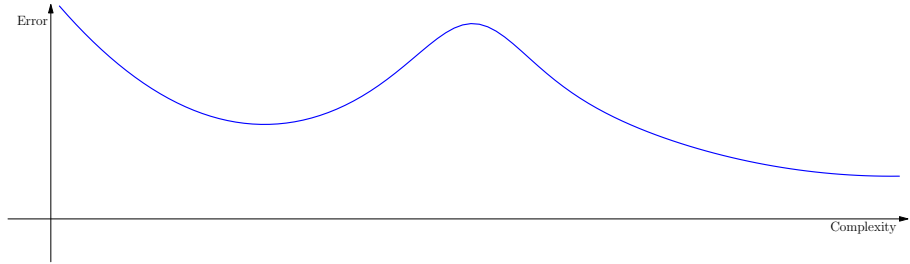
1. Are models with millions/billions of parameters *too* flexible?
2. What do all of those parameters mean (if anything)?

Traditional Wisdom in Mathematical Modeling



Double descent

(Made famous in 2018 by Belkin, Hsu, Ma, and Mandal)



What causes double descent?

Theories and partial answers:

- The way we train neural networks ensures we get "smooth" ones. (Belkin et al., 2019)
- The parameters might have much less freedom than we think they do, because they outnumber the data. (Advani et al., 2019)
- The number of parameters required to achieve smoothness is much greater than a naive estimate would suggest ($\propto n \times d$). (Bubeck and Sellke, Dec.2021)

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Unsolved Problems

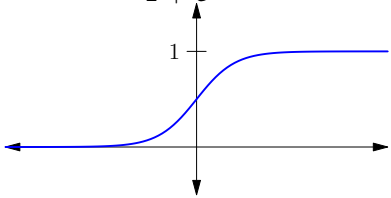
- Probability
- Real analysis
-
-

Two Unsolved Problems in Neural Networks

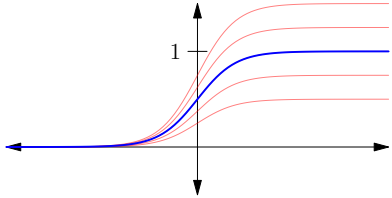
1. Are models with millions/billions of parameters *too* flexible?
2. What do all of those parameters mean (if anything)?

Interpreting Model Parameters

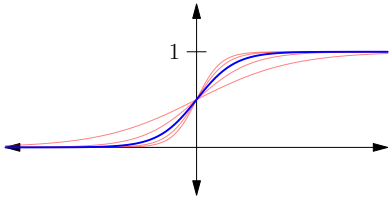
$$y = \frac{h}{1 + e^{-(mx+b)}}$$



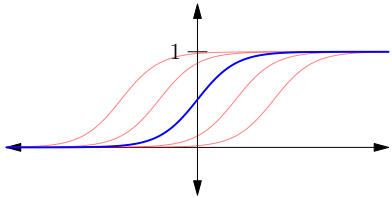
h = total vaccinations long-term



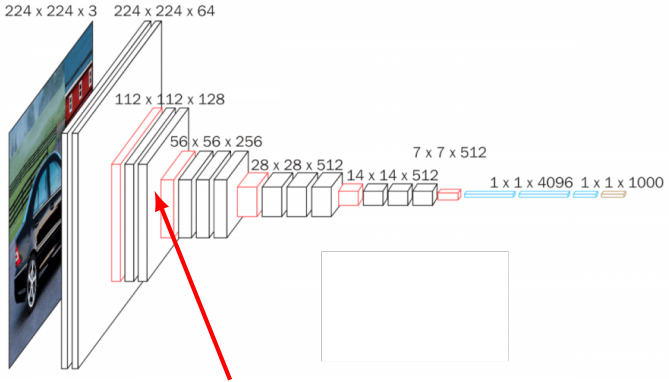
$m \approx$ speed of vaccine distribution



b = time when half complete

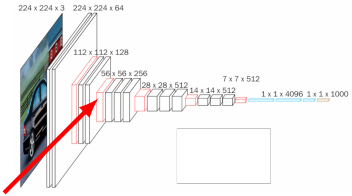


Interpretability



What does parameter $m_{5,903,646}$ mean?

Interpreting Model Parameters



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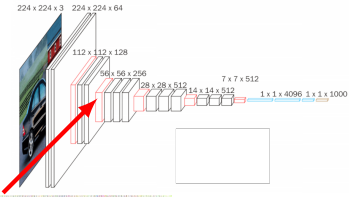

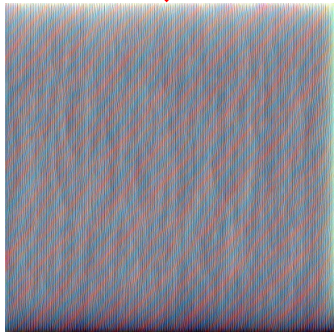

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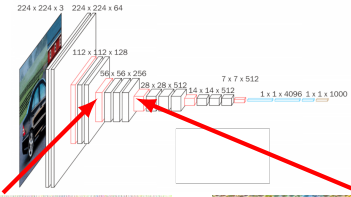

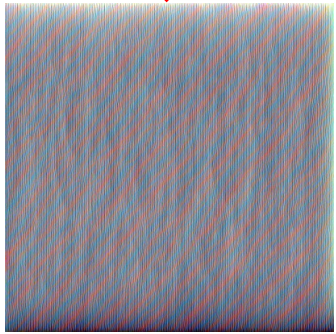

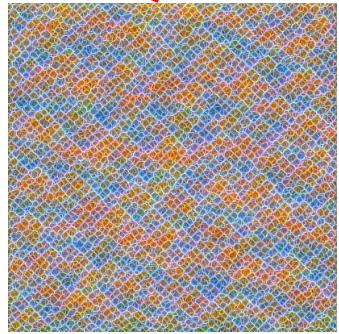

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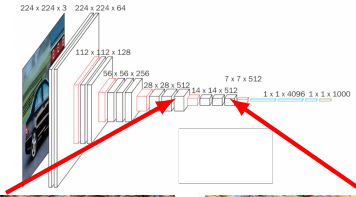

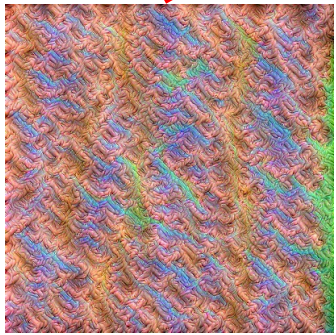

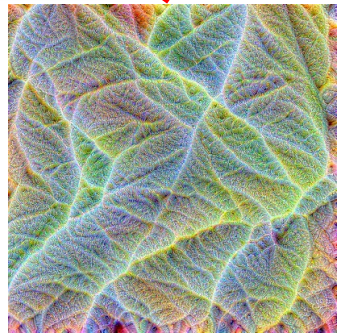

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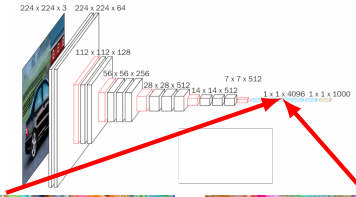

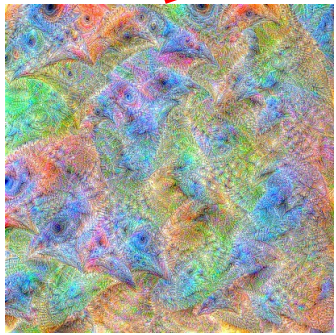

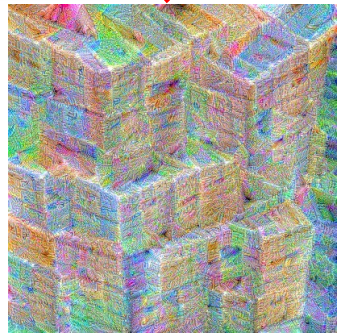

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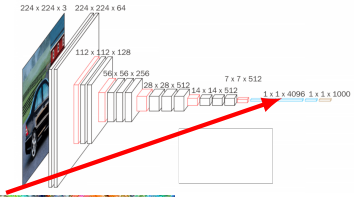

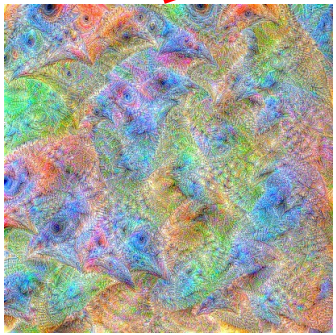

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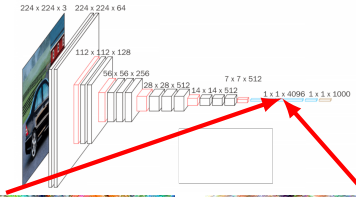
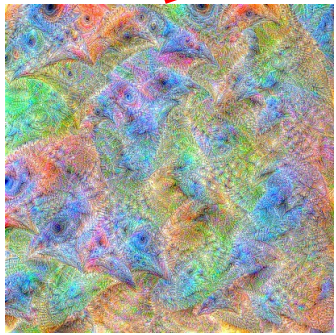
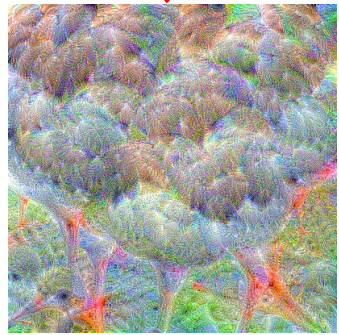
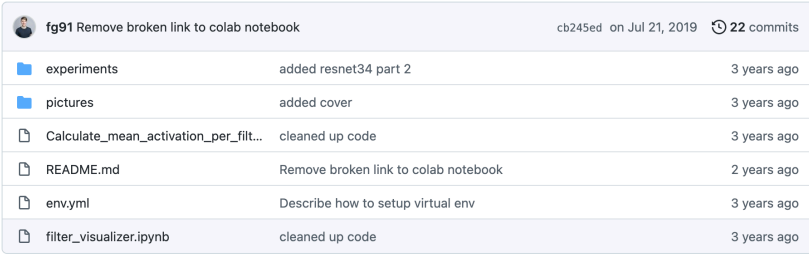






Interpreting Model Parameters

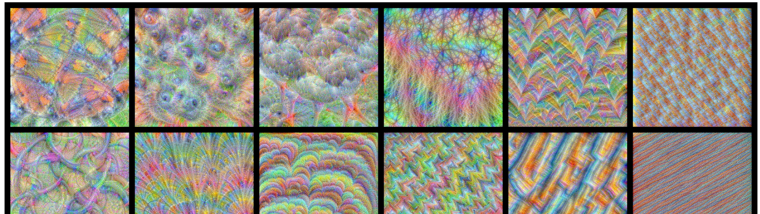
Author: Fabio Graetz, https://github.com/fg91/visualizing-cnn-feature-maps



fg91 Remove broken link to colab notebook — cb245ed on Jul 21, 2019 — **22** commits

experiments	added resnet34 part 2	3 years ago
pictures	added cover	3 years ago
Calculate_mean_activation_per_filt...	cleaned up code	3 years ago
README.md	Remove broken link to colab notebook	2 years ago
env.yml	Describe how to setup virtual env	3 years ago
filter_visualizer.ipynb	cleaned up code	3 years ago

README.md



Mathematics in Data Science

Working with Data

- Functions and relations
- Domain and range
- Famous sets (reals, etc.)
- Sets, subsets, finite sets
- Asymptotes/limits
- Famous function families
- Algebraic transformations

Doing an Analysis

- Questioning assumptions
- Using official definitions
- Step-by-step constructions
- Supplying intuition

Unsolved Problems

- Probability
- Real analysis
- Multivariate calculus
- Linear algebra

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Thank you for joining us for this MAA Distinguished Lecture Series presentation.