A 405 - Problem Set 4

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1.

Here I'm assuming the numerator is actually n_w

$a_w = \frac{\frac{(\frac{4}{3}\pi r^3\rho - m)}{M_w}}{[\frac{(\frac{4}{3}\pi r^3\rho - m)}{M_w}] + \frac{im}{M_s}}$, Assume $m << \frac{4}{3}\pi r^3\rho$ and and use first order taylor series approximation

$$\frac{\frac{(\frac{4}{3}\pi r^3\rho)}{M_w}}{[\frac{(\frac{4}{3}\pi r^3\rho)}{M_w}] + \frac{im}{M_s}} = \frac{1}{1+\frac{im}{M_s}(\frac{M_w}{\frac{4\pi}{3}ir^3\rho})} = 1 - \frac{b}{r^3}$$

2.

First order taylor series expansion of $exp(\frac{2\sigma}{\rho R_v Tr})$ gives $1 + \frac{2\sigma}{\rho R_v Tr}$

Multiplying $(1 - \frac{b}{r^3})$ by $(1 + \frac{2\sigma}{\rho R_v Tr}) = (1 + \frac{a}{r})$ and dropping the product of the two small terms yields $1 - \frac{b}{r^3} + \frac{a}{r}$

Differentiating this, and setting equal to zero: $-\frac{a}{r^2} + s\frac{b}{r^4} = 0, \quad r^* = \sqrt{\frac{3b}{a}}$

and $S^* = 1 + \frac{a}{\sqrt{\frac{3b}{a}}} - \frac{b}{\sqrt{\frac{3b}{a}}^3} = 1 + a\sqrt{\frac{a}{3b}} - \frac{a}{3}\sqrt{\frac{a}{3b}} = 1 + \sqrt{\frac{4a^3}{27b}}$

3.

(a)

$$S^* - 1 = (\frac{4(\frac{2\sigma}{\rho R_v T})^3}{27(\frac{imM_w}{\frac{4}{3}M_s\pi\rho})})^{.5} = (\frac{4(\frac{2\times .076}{1000\times 461.51\times T})^3}{27\frac{3\times m_{aero}\times 18}{\frac{4}{3}\times 132.14\times\pi\times 1000}})^{.5} = (\frac{5.4968\times 10^{-24}}{T^3 m_{aero}})^{.5}$$

(b)

$S^* - 1 \approx 1.54 \times 10^{-12} m_{aer}^{-.5} = 1.54 \times 10^{-12}(\frac{4}{24}\pi\rho_{aer}D^3)^{-.5}$

So $N = 10^9(S^* - 1)^{0.5} = 1.2409 \times 10^3(\frac{\pi}{6})^{-.25}(D^3)^{-.25} = 1.4588 \times 10^3 \times \rho_{aero}^{-.25} D^{-\frac{3}{4}}$

Differentiating and multiplying by -1 yields $n(D) = 1.0941 \times 10^3 \rho^{-.25} \times (D^{-\frac{7}{4}})$

(c)

$M(D) = \frac{4}{3}\pi\rho_{aer}\int_D^\infty (\frac{D}{2})^3 n(D')dD' = 5.728 \times 10^2 \rho_{aero}^{.75} \int_D^{D_{limit}} D^{\frac{5}{4}}$

$= 5.728 \times 10^2 \rho_{aero}^{.75}[\frac{4}{9}D^{\frac{9}{4}}]_D^{D_{limit}} = 2.546 \times 10^2 \times \rho^{.75}[D^{\frac{9}{4}}]_D^{D_{limit}} = 6.962 \times 10^3 \times \rho^{.75}[D^{\frac{9}{4}}]_D^{D_{limit}}$

(d)

See python script

4.

See python script

5.

$\frac{\partial^2 E}{\partial r^2} \approx -4\pi R_v T \rho_l (2a - \frac{3b}{r^2}) + 8\pi\sigma$

Substituting in $r^* = (\frac{3b}{a})^{\frac{1}{2}}$ yields $-4\pi R_v T \rho_l a + 8\pi\sigma = 0$

If $r > r^*$, $\frac{\partial^2 E}{\partial r^2} < 0$ and the curvature is concave implying a stable equilibrium

When $r < r^*$ the curvature is convex.