ODE Derivations for nchap _ ode _ littlerock.py

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1 ODEs

The function to be integrated F calls $calcVars$ to calculate the rates of change with respect to time dW, dT, dSS, dr. The reaction scheme is summarized as follows:

$\frac{dW}{dt} = g_0 \frac{(T_v - T_v^*)}{T_v}$

$\frac{dr}{dt} = \frac{D_v \rho_{vs}}{\rho_l} SS$, where $SS = \frac{e}{e_s} - 1$
if unsaturated

$\frac{dT}{dt} = \frac{-R_d T}{c_{pd} P} g_0 \rho w$

if saturated

$= \left(\frac{\frac{R_d T}{c_{pd} P_d} - \frac{l_v}{c_{pd}} \frac{\partial W_s}{\partial P}}{1 - \frac{l_v w_s}{c_{pd} T} + \frac{l_v}{c_{pd}} \frac{\partial w_s}{\partial T}} \right) w(-\rho g_0)$

$\frac{dSS}{dt} = \frac{P}{\epsilon e_s}(\frac{dw_v}{dt} - (1 + SS)(\frac{-\epsilon e_s}{P^2} \frac{-g_0 P w}{R_d T} + \frac{\epsilon}{P} \frac{de_s}{dT} \frac{dT}{dt}))$

2 Derivation of ODEs

Rate of Change of Temperature wrt Time - $\frac{dT}{dt}$

$\theta_e = T \times (\frac{P_0}{P_d})^{\frac{R_d}{C_p}} \times exp(\frac{L_v w_{sat}}{C_p T})$

Conserved with respect to time, so $\frac{d\theta_e}{dt} =$

$\frac{dT}{dt}(\frac{P_0}{P_d})^{\frac{R_d}{C_p}} - \frac{R_d T}{C_p}(\frac{P_0}{P_d})^{\frac{R_d}{C_p}} \frac{1}{P_d} \frac{dP_d}{dt} + \frac{d}{dt}(\frac{L_v w_s}{C_p T}) \times T(\frac{P_0}{P_d})^{\frac{R_d}{C_p}} = 0$

$\frac{dT}{dt} - \frac{R_d T}{C_p} \frac{1}{P_d} \frac{dP_d}{dt} - \frac{L_v w_s}{C_p T} \frac{dT}{dt} + \frac{L_v}{C_p} \frac{dT}{dt} \frac{dw_s}{dT} + \frac{L_v}{C_p} \frac{dw_s}{dP_d} \frac{dP_d}{dt} = 0$

$\frac{dT}{dt}(1 - \frac{L_v w_s}{C_p T} + \frac{L_v}{C_p}\frac{dw_s}{dT}) = (\frac{R_d T}{C_p P_d} - \frac{L_v}{C_p}\frac{dw_s}{dP})\frac{dP_d}{dt} = 0$

$\frac{dT}{dt} = (\frac{R_d T}{C_p P_d} - \frac{L_v}{C_p}\frac{dw_s}{dP})(1 - \frac{L_v w_s}{C_p T} + \frac{L_v}{C_p}\frac{dw_s}{dT})^{-1} \times w \times (-\rho g)$

$\frac{dw_s}{dT} = (w_s + w_s^2)\frac{1}{e_s}\frac{de_s}{dT}, \frac{de_s}{dT} = \frac{L_v e_s}{R_v T^2}, \frac{dw_s}{dP} = -\frac{\epsilon \times e_s}{(P - e_s)^2}$

Analogously for the unsaturated case, using θ

Another Way

$h_m = C_p T + L_v w_s + gz$

$\frac{dh_m}{dz} = C_p \frac{dT}{dz} + L_v \frac{dw_s}{dT}\frac{dT}{dz} + L_v \frac{dw_s}{dP}\frac{dP}{dz} + g$

$\frac{dT}{dz}w = \frac{dT}{dt} = -\frac{(g + L_v \frac{dw_s}{dP}\frac{dP}{dz})w}{C_p + L_v \frac{dw_s}{dT}}$

Note: it's now taken into consideration that w_s is dependent on pressure.

What would the dry/unsaturated version be?

Droplet Radius – $\frac{dr}{dt}$

$\frac{dr}{dt} = \frac{D_v \rho_v(parcel)}{r\rho_{l\ in\ droplet} e(parcel)} \times [e(parcel) - e(r)]$

if we assume saturation around droplet is equal to that over a bulk water surface

$= \frac{D_v \rho_v(parcel)}{r\rho_{l\ in\ droplet} e(parcel)} e_s \times [SS]$

Assume density of droplet is that of water

$= \frac{D_v \rho_{vs}(parcel)}{r\rho_l} \times SS$

Supersaturation – $\frac{dSS}{dt}$

$w_v = \frac{\epsilon e}{p - e} \approx (1 + SS)\frac{\epsilon e_s}{p}$

$\frac{dw_v}{dt} = (1 + SS)\frac{\epsilon}{p}(\frac{de_s}{dT}\frac{dT}{dt} - \frac{e_s}{p}(-g_0 \rho w)) + \frac{\epsilon e_s}{p}\frac{dSS}{dt}$

So

$\frac{dSS}{dt} = [\frac{dw_v}{dt} - (1 + SS)\frac{\epsilon}{p}(\frac{de_s}{dt}\frac{dT}{dt} - \frac{e_s}{p}(-g_0 \rho w))]\frac{p}{\epsilon e_s}$
$\frac{dw_v}{dt} = -\frac{dw_l}{dt} = -\frac{d}{dt}(N_{aero}\frac{4\pi}{3}r^3) = N_{aero}\frac{4}{\pi}r^2$