

# Supervised learning

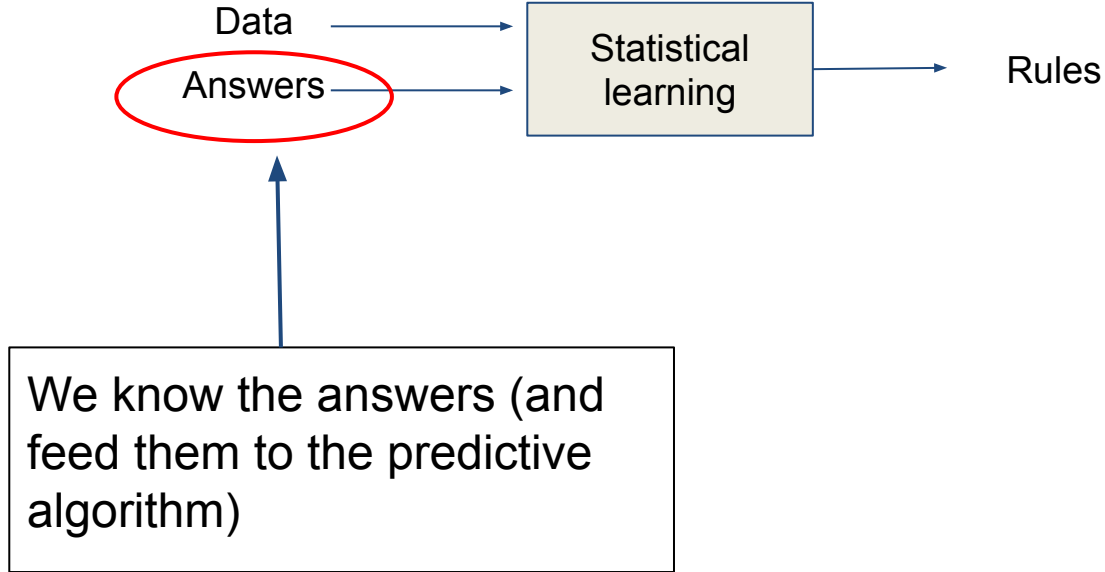
Train the learners

Filippo Biscarini  
Senior Scientist  
CNR, Milan (Italy)

Nelson Nazzicari  
Senior Scientist  
CREA, Lodi (Italy)



# Supervised learning problems



# Why supervised?

## Training examples

measured variables / features  
on  $n$  examples

$$\begin{bmatrix} 1 \\ 0 \\ \vdots \\ 1 \end{bmatrix} = \begin{bmatrix} 0.12 & 1.5 & \dots & 0.9 \\ 2.05 & 0.95 & \dots & 1.1 \\ \vdots & \vdots & \ddots & \vdots \\ 3.5 & 0.88 & \dots & 1.75 \end{bmatrix}$$

labels / target  
variables (e.g.  
phenotypes) on  $n$   
examples



# Unsupervised learning

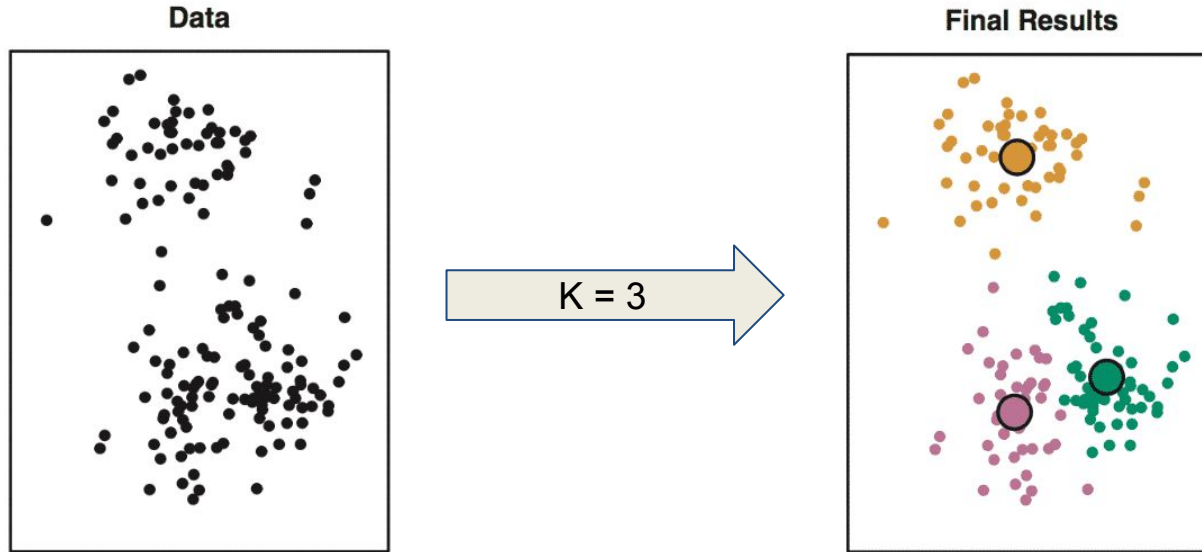
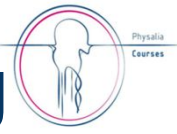
## Training examples

measured variables / features  
on  $n$  examples

$$\cancel{\begin{bmatrix} 1 \\ 0 \\ \vdots \\ 1 \end{bmatrix}} = \begin{bmatrix} 0.12 & 1.5 & \dots & 0.9 \\ 2.05 & 0.95 & \dots & 1.1 \\ \vdots & \vdots & \ddots & \vdots \\ 3.5 & 0.88 & \dots & 1.75 \end{bmatrix}$$



# Unsupervised example: K-means clustering



source:  
<https://www.iotforall.com/machine-learning-crash-course-unsupervised-learning>

# Supervised learning: Regression and classification

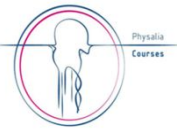


# Supervised learning problems

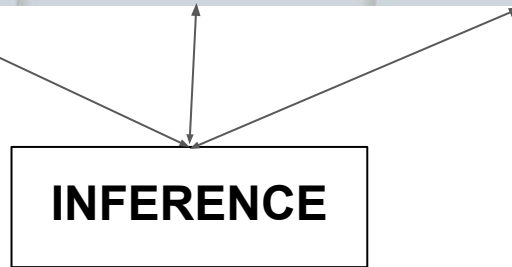
- Regression (**predictive**) problems
- Classification (**predictive**) problems



# Supervised learning (predictive) problems

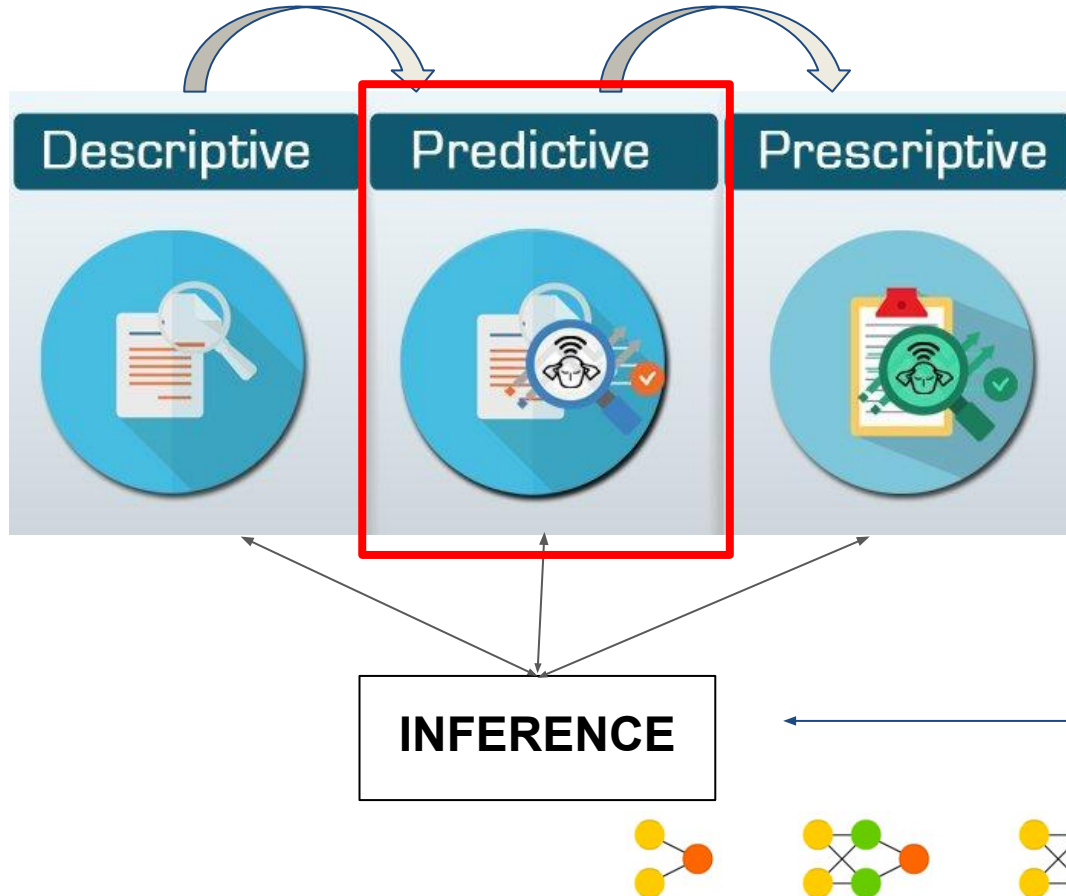
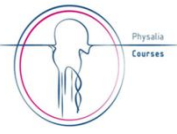


- Know the past
- Predict the future
- Act consequently





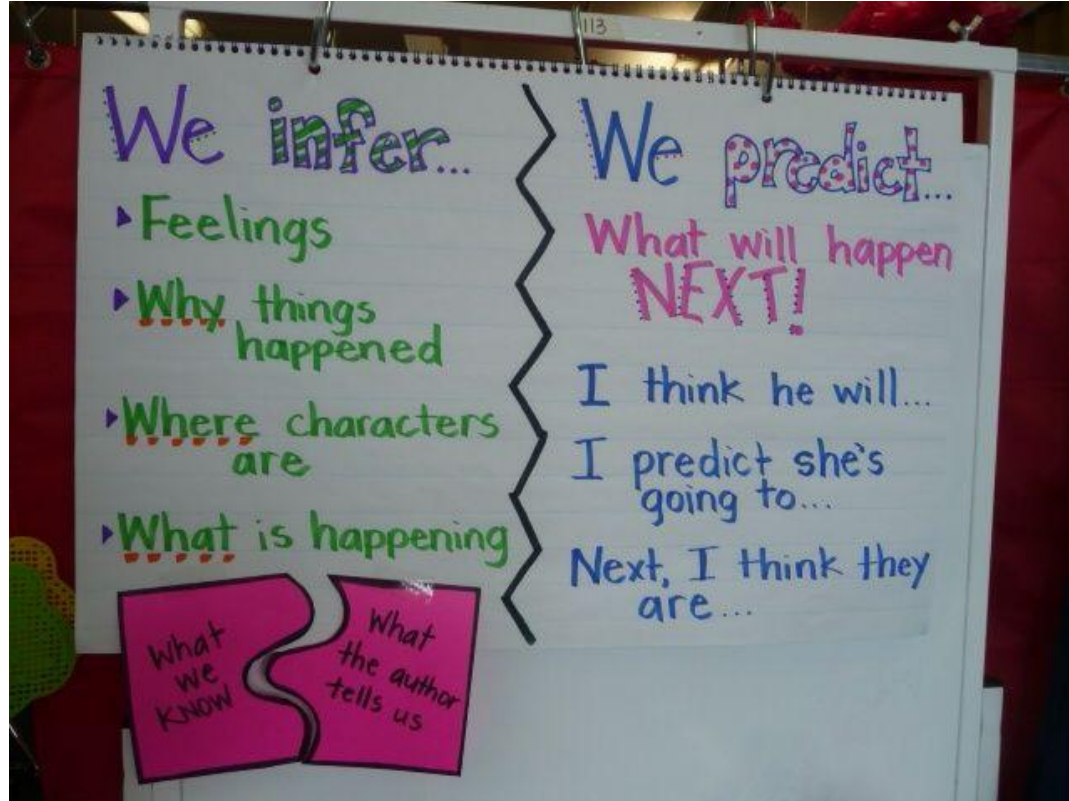
# Supervised learning (predictive) problems



- Know the past
- Predict the future
- Act consequently

- A catch-all term
- Can be confusing

# Inference vs Prediction



- different statistical problems
- different objectives, different rules ... different ballparks
- inference is in general more difficult than prediction



# Supervised learning (predictive) problems



- Regression (**predictive**) problems
- Classification (**predictive**) problems

## Predictive machines!

- Classifiers
- Regressors



source:

<https://blog.bigml.com/2013/03/12/machine-learning-from-streaming-data-two-problems-two-solutions-two-concerns-and-two-lessons/>

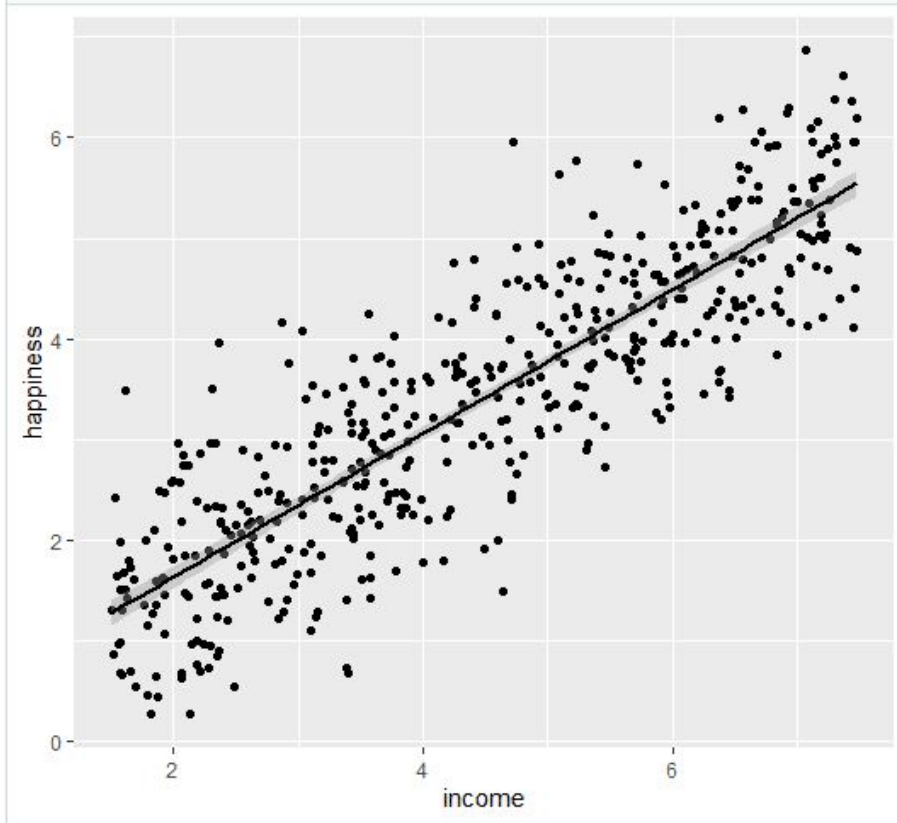
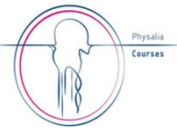


# Regression problems

- the response variable **y** is **quantitative**
- e.g.: *height, weight, yield (milk, crops), blood sugar concentration*
- **y** = **target** (dependent) variable (a.k.a. response, objective variable)
- **X** = matrix of **features** (continuous, categorical)
- **regressor**:  $y = f(x) = \mathbf{P}(\mathbf{X}) \leftarrow$  [predictive machine]



# Regression problems - simple regression



$$\text{happiness} = (\text{intercept}) + \text{beta} * \text{income}$$

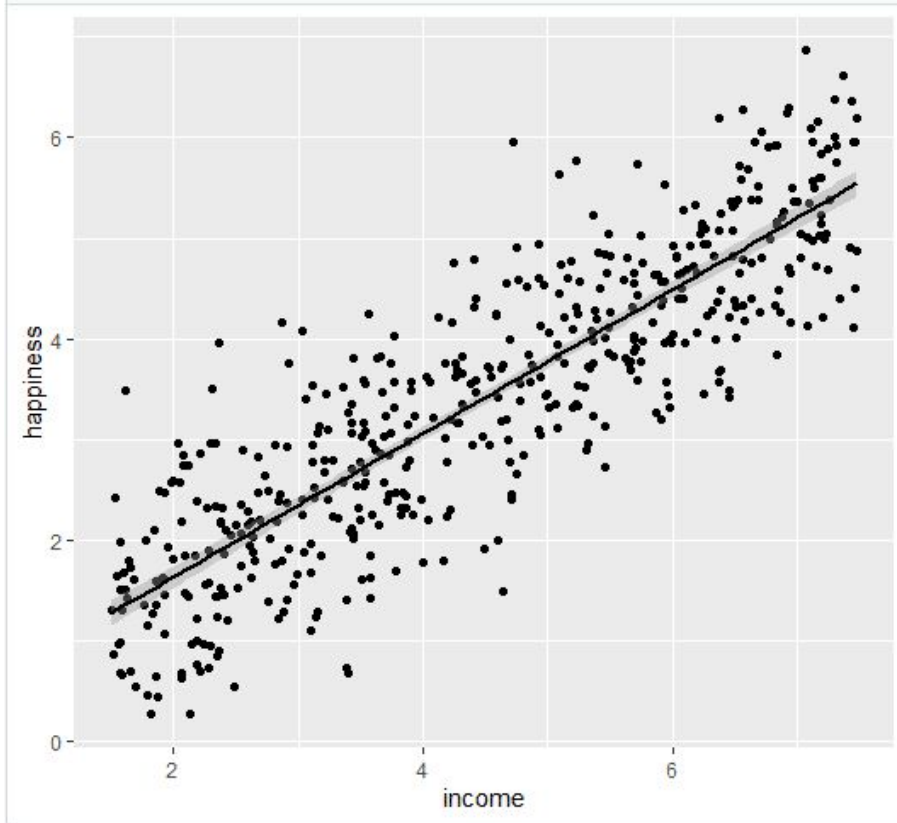
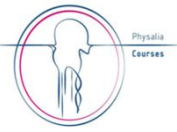
or

$$\text{income} = (\text{intercept}) + \text{beta} * \text{happiness}$$

Source: <https://www.scribbr.com/statistics/linear-regression-in-r/>



# Regression problems - simple regression



$$\text{happiness} = (\text{intercept}) + \text{beta} * \text{income}$$

or

$$\text{income} = (\text{intercept}) + \text{beta} * \text{happiness}$$

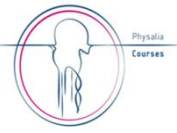
**cause → effect?**

1. Can we predict backwards?
2. Can we make predictions if there is no cause-effect relationship?

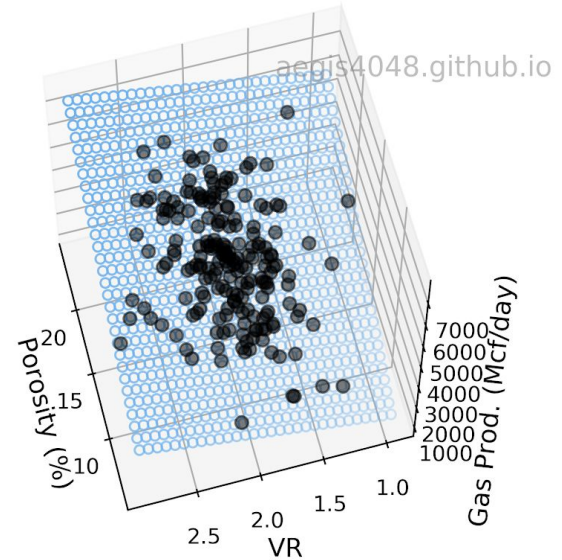
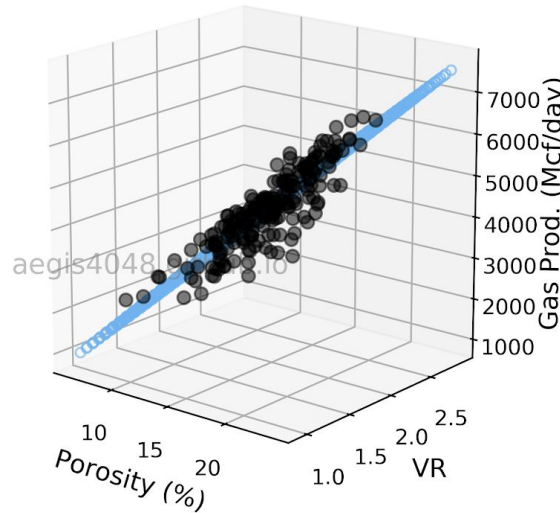
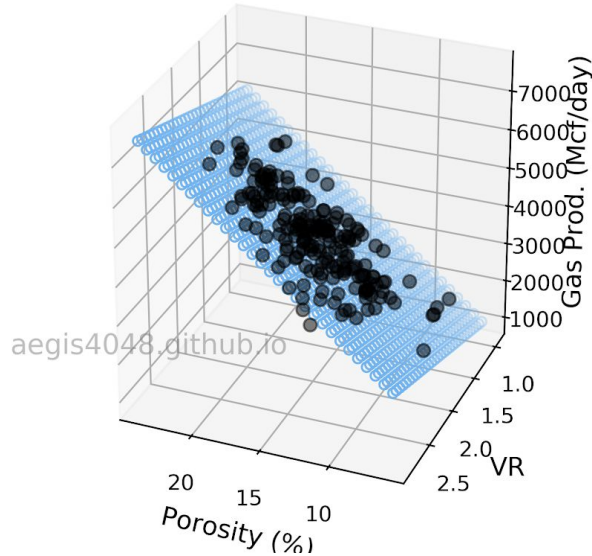
Source: <https://www.scribbr.com/statistics/linear-regression-in-r/>



# Regression problems - multiple regression



$$R^2 = 0.79$$



- Gas production
- Well porosity %
- Vitrinite reflectance %

Source: [https://aegis4048.github.io/mutiple\\_linear\\_regression\\_and\\_visualization\\_in\\_python](https://aegis4048.github.io/mutiple_linear_regression_and_visualization_in_python)



# Multiple linear regression

$$y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_p X_p + \epsilon$$

- $y$ : target variable
- $\beta$ 's: model coefficients
- $X$ 's: features (predictors, independent variables, factors)





# Multiple linear regression

$$\mathbf{y} = \beta \mathbf{X} + \mathbf{e}$$

- matrix (compact) notation
- vectors of observations ( $\mathbf{y}$ ), coefficients ( $\beta$ ) and residuals ( $\mathbf{e}$ )
- matrix of features ( $\mathbf{X}$ )



# Multiple linear regression

$$\mathbf{y} = \beta \mathbf{X} + \mathbf{e}$$

estimation of  
coefficients  
(learning!)

$$\hat{\mathbf{y}} = \hat{\beta} \mathbf{X}$$

→ predictions!

- matrix (compact) notation
- vectors of observations ( $\mathbf{y}$ ), coefficients ( $\beta$ ) and residuals ( $\mathbf{e}$ )
- matrix of features ( $\mathbf{X}$ )



# Predictions

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2 + \dots + \hat{\beta}_p x_p$$

with the estimated coefficients  $\hat{\beta}$  and the feature values  $\mathbf{X}$  we obtain the predicted values  $\hat{y}$



# Predictions

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2 + \dots + \hat{\beta}_p x_p$$

with the estimated coefficients  $\hat{\beta}$  and the feature values  $\mathbf{X}$  we obtain the predicted values  $\hat{y}$

→ **how do we obtain the model coefficients  $\beta$ ?**



# Supervised learning recap

- **deep learning** is one of many methods that can be used to solve supervised learning problems
- deep learning is mainly used in **predictive problems**
- (can be used though also for inferential problems and for unsupervised learning)
- we'll see later how to use deep learning to solve linear regression

