

# Cross-validation and performance measures

How to avoid prediction blunders

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# Overfitting



# What is overfitting?

You may fit a deep learning model to your data and then measure the “accuracy” of predictions on the same data:  
**would this be correct?**



# What is overfitting?

You may fit a deep learning model to your data and then measure the “accuracy” of predictions on the same data:  
**would this be correct?**

- short answer: **NO!**
- main reason: **overfitting**



# What is overfitting?

Overfitting:

Fitting too well the data:  $R^2$  too large ( $\approx 1$ )



# What is overfitting?

## Overfitting:

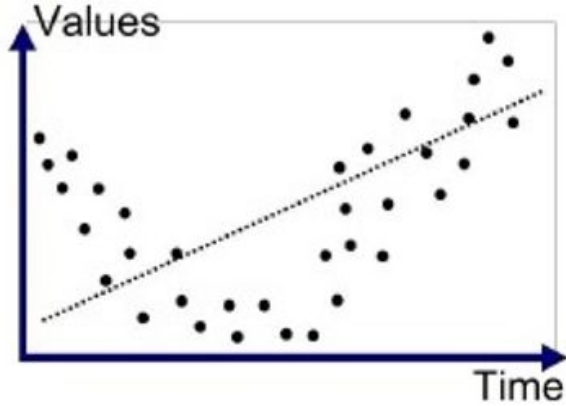
Fitting too well the data:  $R^2$  too large ( $\approx 1$ )

overfitting happens with:

- using the same data to fit the model and to make predictions
- overparameterization of the model (e.g. too many effects)
- flexible methods (e.g. polynomial functions, splines, ... and **deep learning!**)

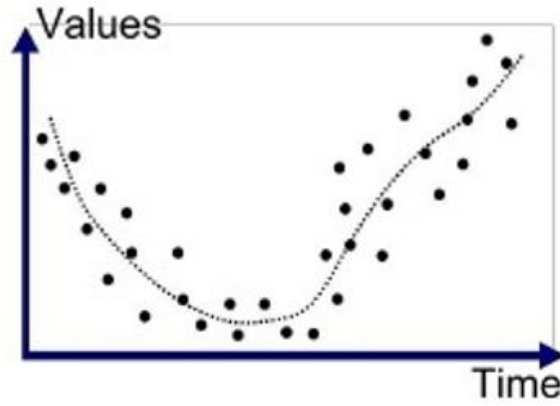


# What is overfitting?



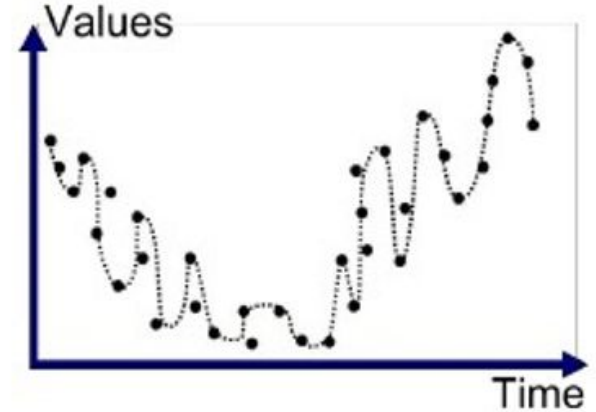
Underfitted

Linear regression



Good Fit/Robust

Polynomial (?)

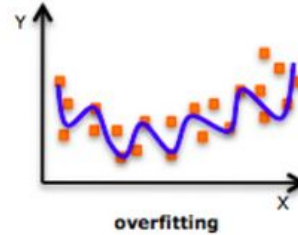
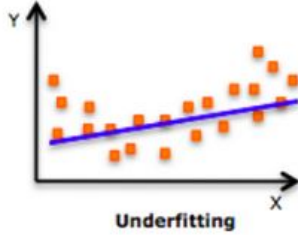


Overfitted

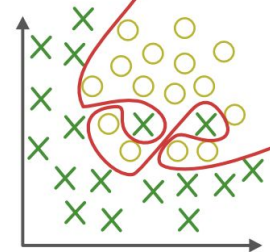
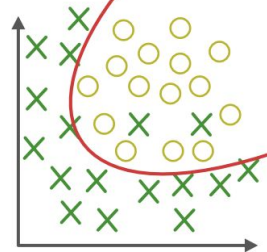
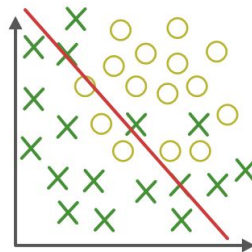
Highly non-linear  
model

# What is overfitting?

regression



classification



GG

Linear regression

Polynomial (?)

Highly non-linear model





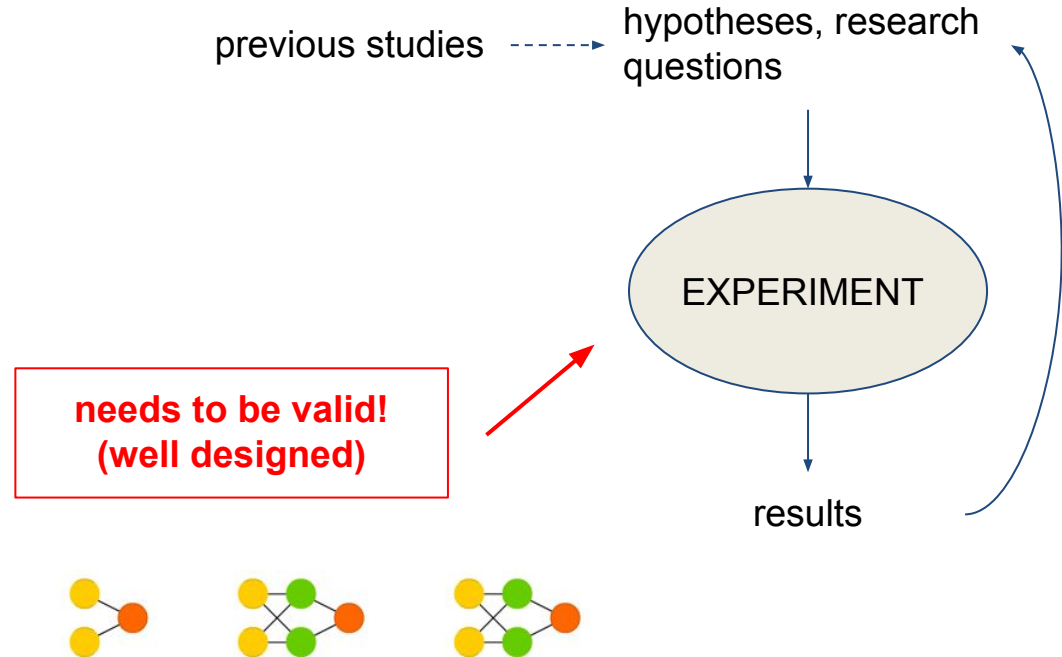
# Training and validation sets

A tool to control overfitting

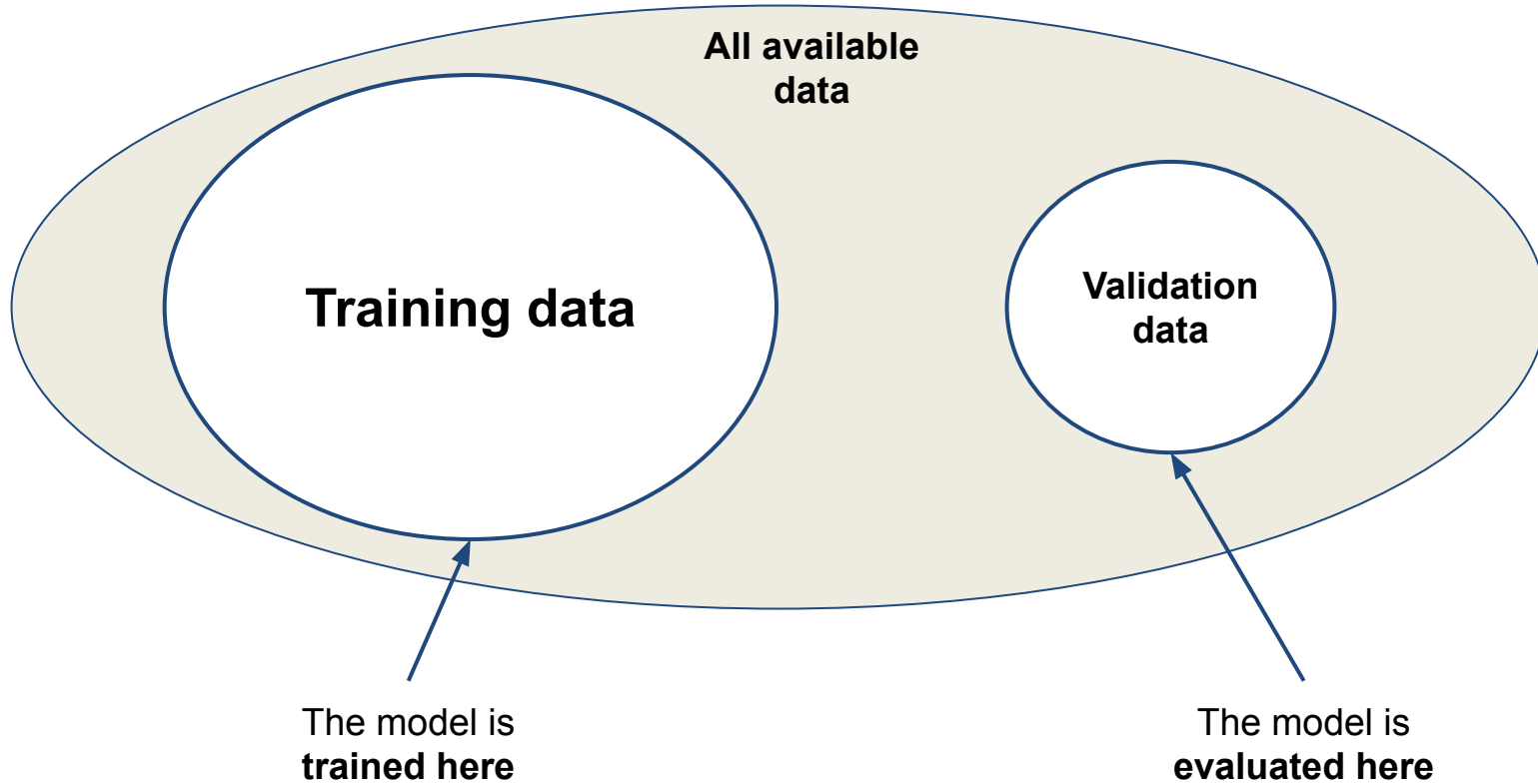


# Training and validation sets

*“The test of all knowledge is experiment”*



# Training and validation sets

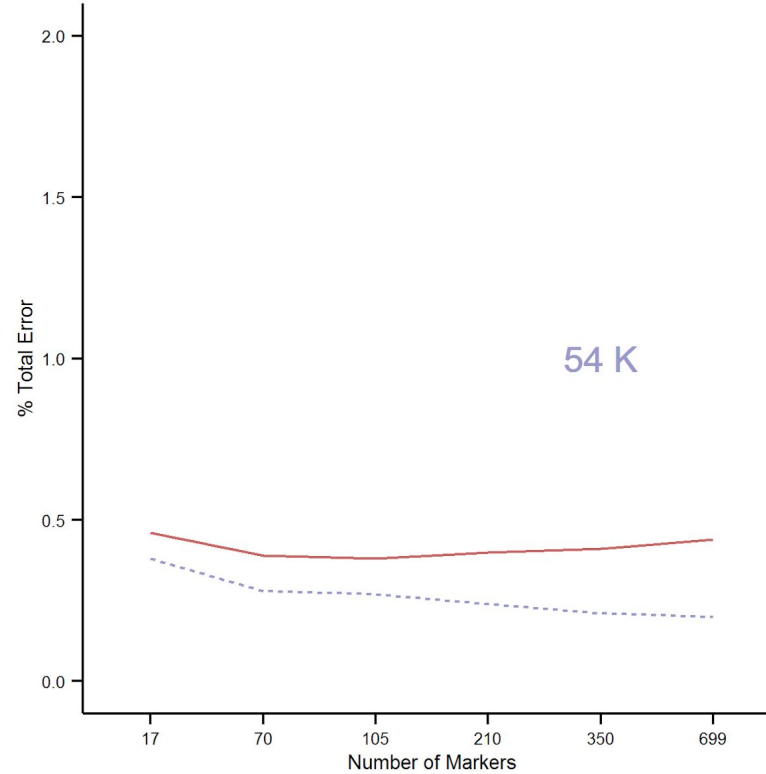
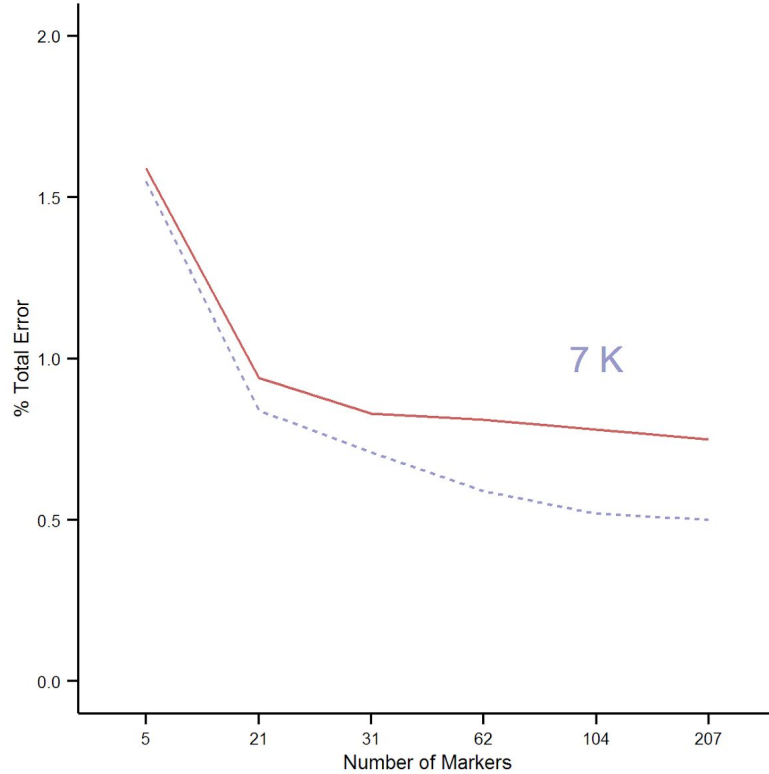


# Training and validation sets

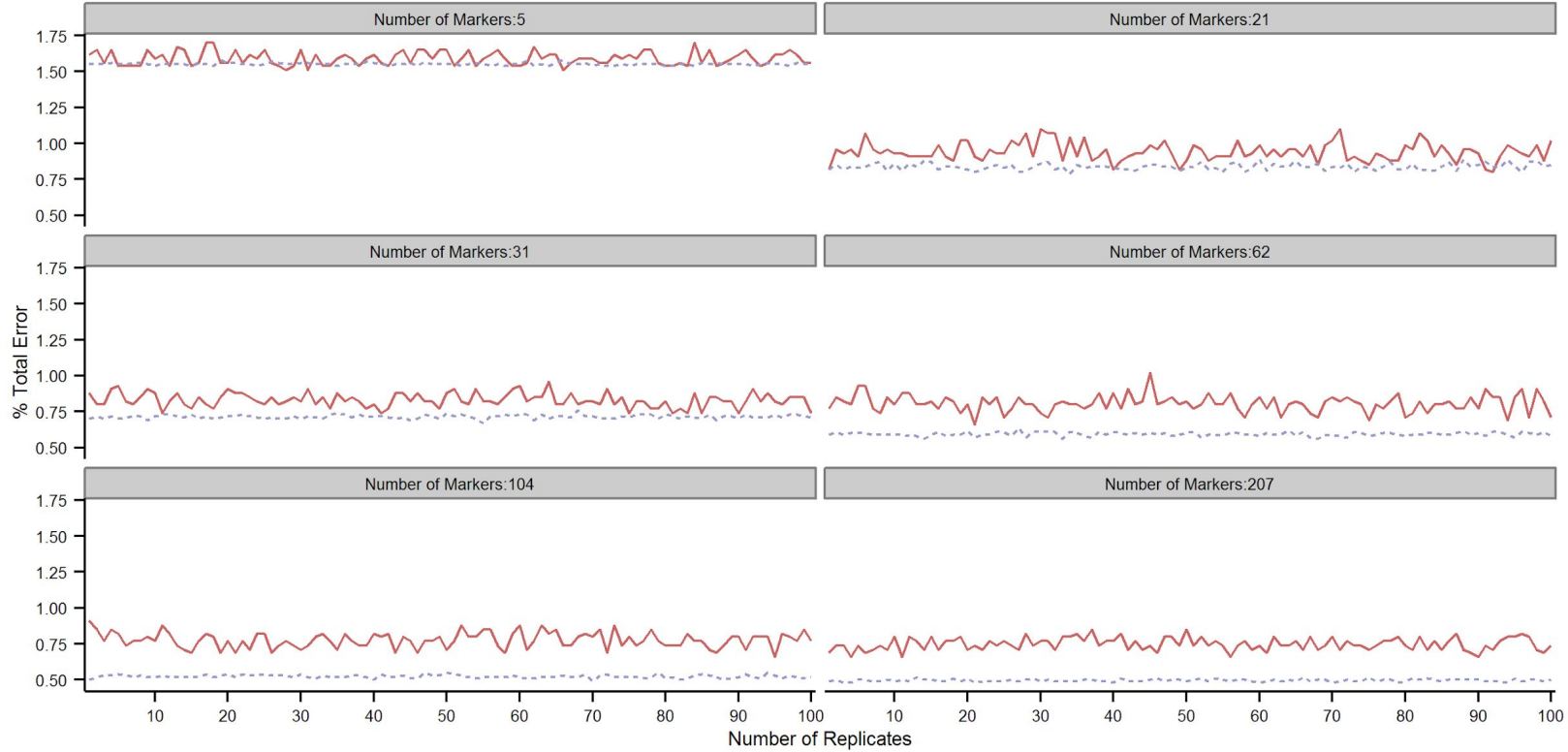
- accuracy (model performance) on the training set is “optimistic” (biased upward ← *overfitting*)
- a better estimate of model performance can be obtained from independent data
- usually we are interested in the predictive performance on new data
- accuracy in the validation set is usually lower than in the training set



# An example from genomics



# An example from genomics



# And the test set?

- “Test” and “Validation” are often considered and used as synonyms
  - But they are not! (strictly speaking)
- A test set would be a third set
  - Completely new data
  - Used only once when you finished everything else
  - An estimate of performances in real world
- We (try to) stick to Keras convention, and never talk about test sets :)



# Prediction error





# Prediction error

$$E \left( y - \hat{f}(x) \right) = Var \left( \hat{f}(x) \right) + \left[ \text{Bias} \left( \hat{f}(x) \right) \right]^2 + Var(\epsilon)$$

variance

bias<sup>2</sup>



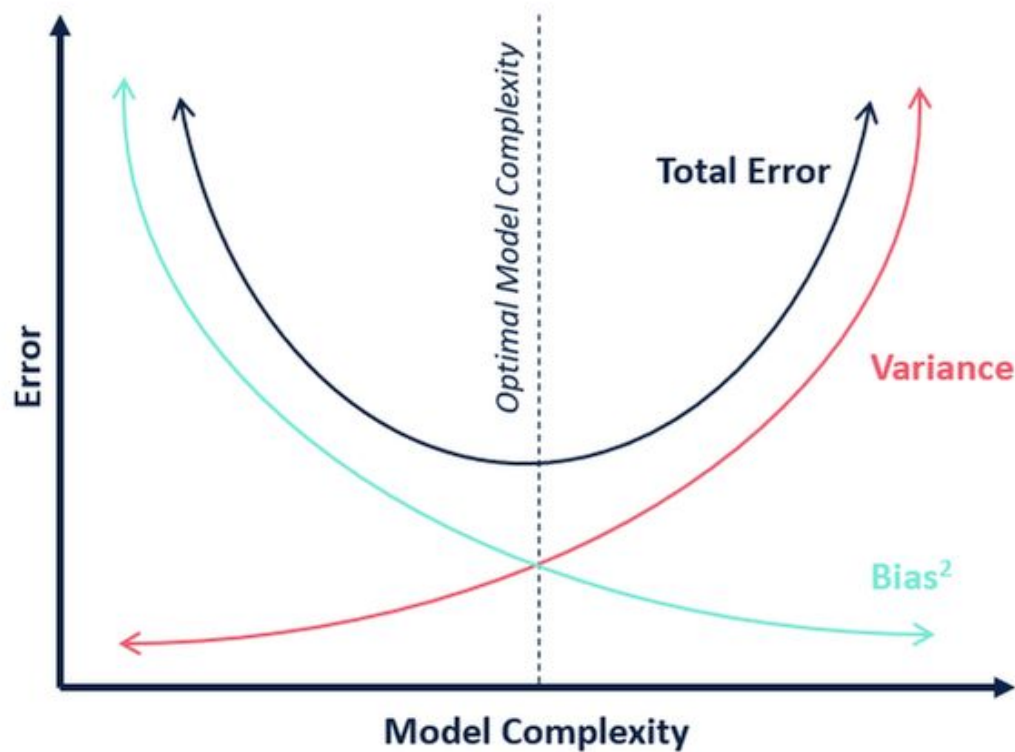
# Prediction error

$$E \left( y - \hat{f}(x) \right) = Var \left( \hat{f}(x) \right) + \left[ \text{Bias} \left( \hat{f}(x) \right) \right]^2 + Var(\epsilon)$$

- **variance** refers to the change of the predictor if estimated using different training data
- **bias** refers to the approximation of a real problem by a simpler model



# Bias-variance trade-off



- models with low bias and high variance (e.g. KNN with  $k=1$ )
- models with high bias and low variance (e.g. horizontal line crossing the data)
- → find models/methods with both low variance and low bias

Source: <https://ai-pool.com/a/s/bias-variance-tradeoff-in-machine-learning>



# Bias-variance trade-off

## Methods

### low variance and high bias

- Linear regression
- Logistic regression
- Penalised regression
- SVM (linear kernel)
- Naive Bayes
- etc.

### high variance and low bias

- Random Forest
- Boosting
- Polynomial regression
- Regression splines
- GAM
- **Deep learning**
- KNN
- SVM (Rbf)
- Loess/Lowess (local regression)
- etc.



# Bias-variance trade-off

## Related trade-offs

1. Prediction accuracy vs model interpretability:
  - e.g. linear regression is easy to interpret, splines are not
2. Parsimony vs “black-box”:
  - e.g. variable selection, all-variables models (e.g. RF), Occam’s razor



# Bias-variance trade-off

Important for:

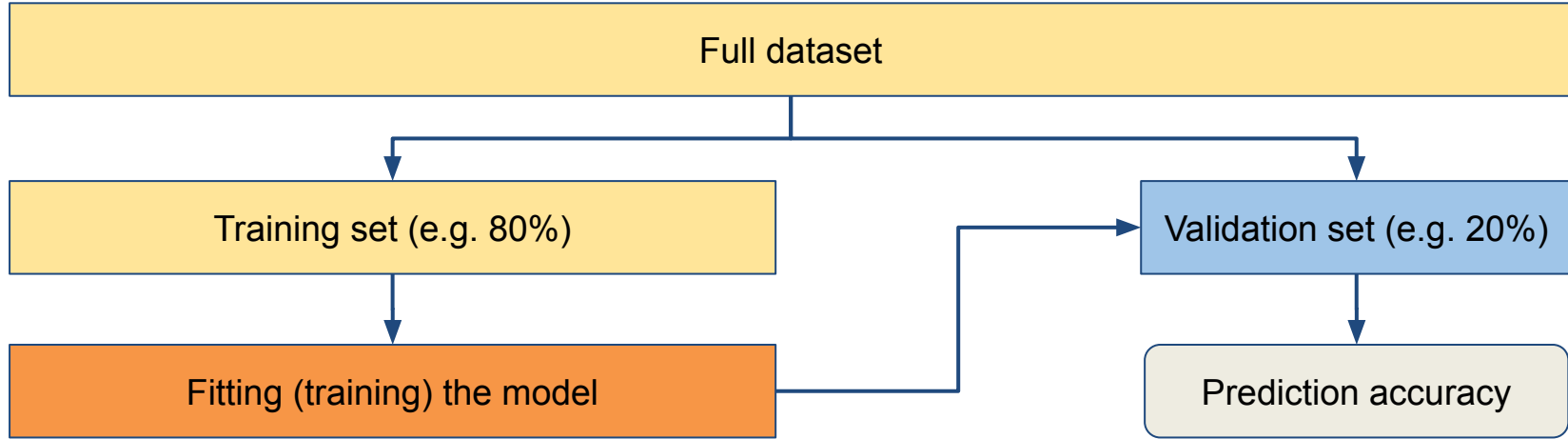
1. **Correctly estimating the performance of a predictive machine**
2. Correctly estimating model parameters
3. Selecting between models



# Resampling methods



# Sampling the training and the validation sets



- To correctly assess the performance of a predictive model we measure it on independent data → validation data
- However we can sample many different training and test sets!





# Resampling the data

- Resampling involves **repeatedly sampling** the training and validation datasets: each time, the model is **refitted** in the training set and **evaluated** in the validation set
- You can e.g. estimate the **variability** of a predictive model or the effect of modifying the model or method:
  - **Model assessment**
  - **Model selection**



# Resampling the data

- Several resampling methods exist
- We will examine two such methods:
  1. **validation set approach**
  2. **cross-validation**



# The validation set approach

training set

validation set

- We split the data in **two random subsets**: training and validation (test)
- 10%/90%, 20%/80%, 30%/70% etc.
- This is what we already did!
- Repeat this *n times* and you get **robust estimates** of the model performance



# The validation set approach

training set

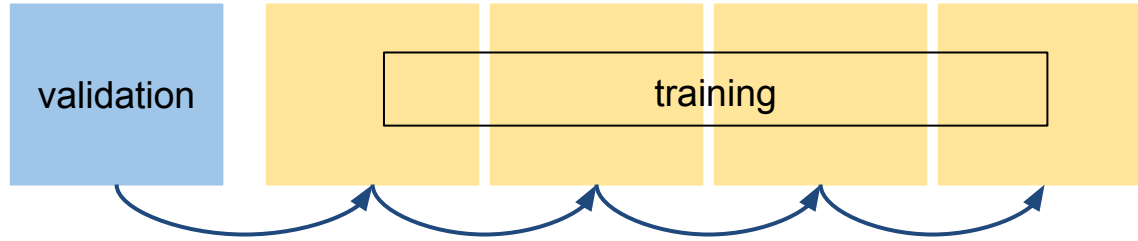
validation set

Drawbacks:

- **highly variable** (depending on the random partition of the data)
- only a subset of the data is used to train (fit) the model → **potentially underestimate model performance**



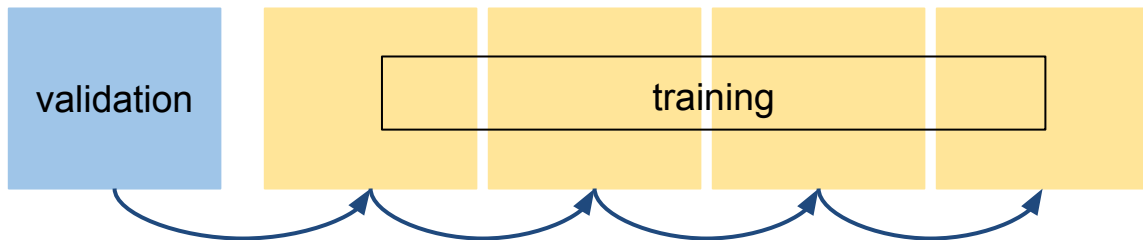
# k-fold cross-validation



- $k$  random partitions of equal size
- each partition in turn is used for validation, the rest for training
- $k$  estimates of model performance



# k-fold cross-validation



- $k$  random partitions of equal size
- each partition in turn is used for validation, the rest for training

- **$k$  estimates** of model performance  $\longrightarrow CV_{(k)} = \frac{1}{k} \sum_{i=1}^k MSE_i$



# k-fold cross-validation

- Lower variability than the validation set approach
- cross-validation works well in **finding the minimum point** in the estimated test MSE curve → model selection
- In cross-validation each observation/record is used both to train the model and to test it → more data are used here than in the validation set approach → lower bias
- cross-validation is therefore expected to have **both lower variance** and **lower bias** than the validation set approach → more accurate estimate of model performance
- typical values for  $k$  are  **$k=5$**  and  **$k=10$**  ( $k = n \rightarrow$  LOOCV)



# Cross-validation: right and wrong

- Consider a **regression problem**: 100 samples, 50,000 features (variables, e.g. 'omics data):
  - Step 1: Find the 100 features with the **strongest correlation** with the response variable
  - Step 2: Apply a **predictor** (e.g. multiple linear regression) with only these 100 **selected features**

Estimate the **prediction error**: can we apply cross-validation in step 2?





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# Cross-validation: right and wrong

Estimate the **prediction error**: can we apply cross-validation in step 2? → **NO!**

- in Step 1, the **model has already used the response** of the training data
- Features have been “**cherry picked**” based on the data: this is already **training**, and the correlation with the response may be a result of the specific configuration of this dataset (a “quirk” in the data)



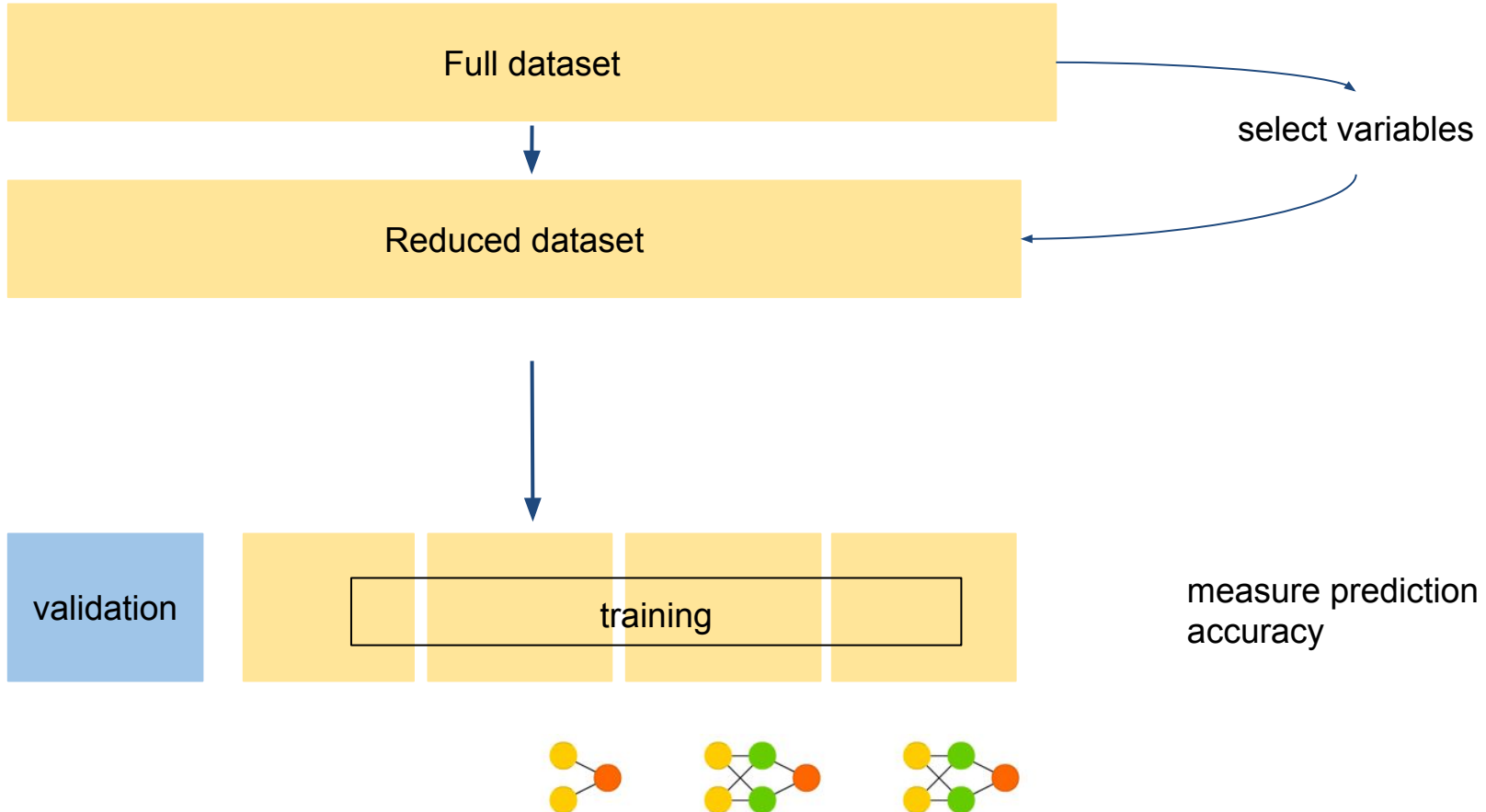
# Cross-validation: right and wrong

Estimate the **prediction error**: can we apply cross-validation in step 2? → **NO!**

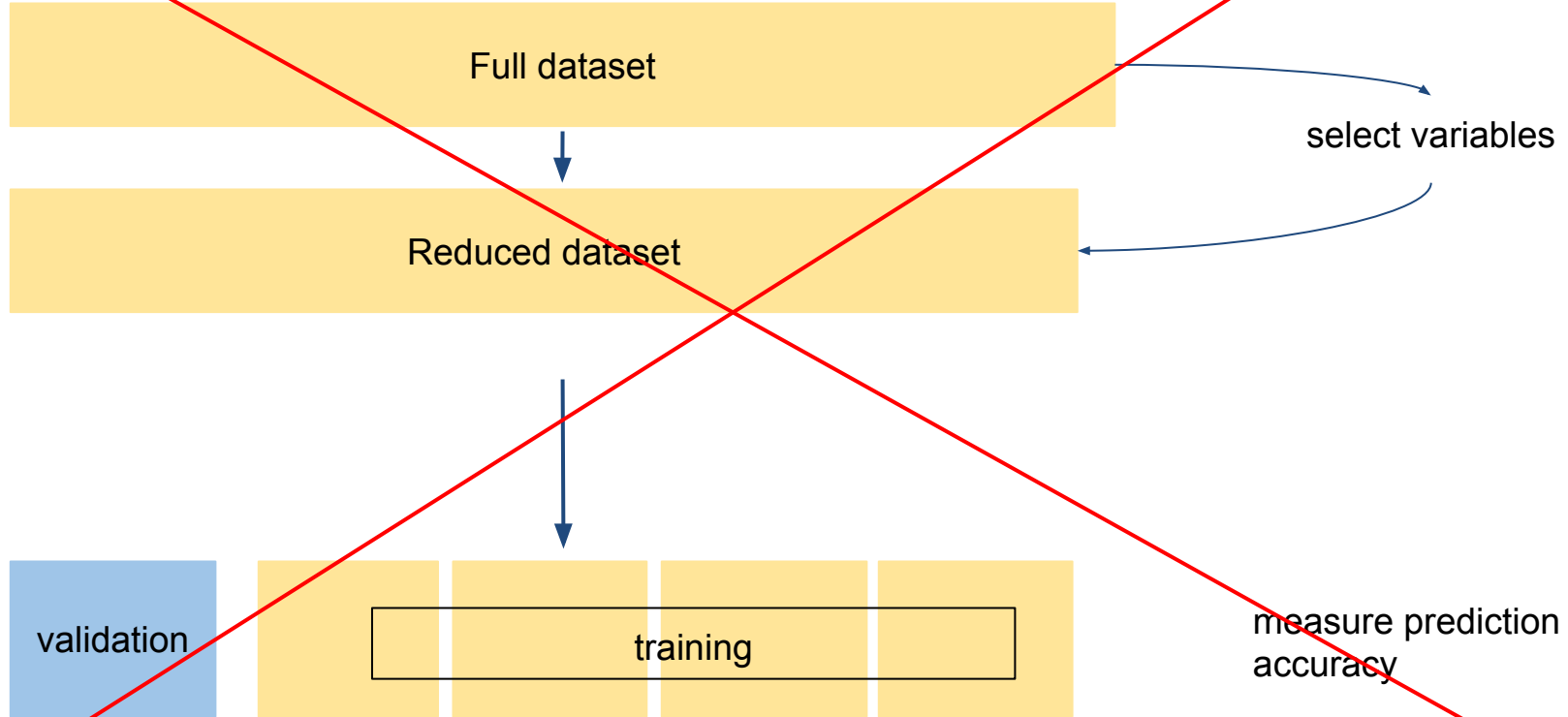
- **Wrong!** → select variables on the whole dataset, then apply cross-validation
- **Right!** → first split the data in training and test sets, then select variables (part of training)



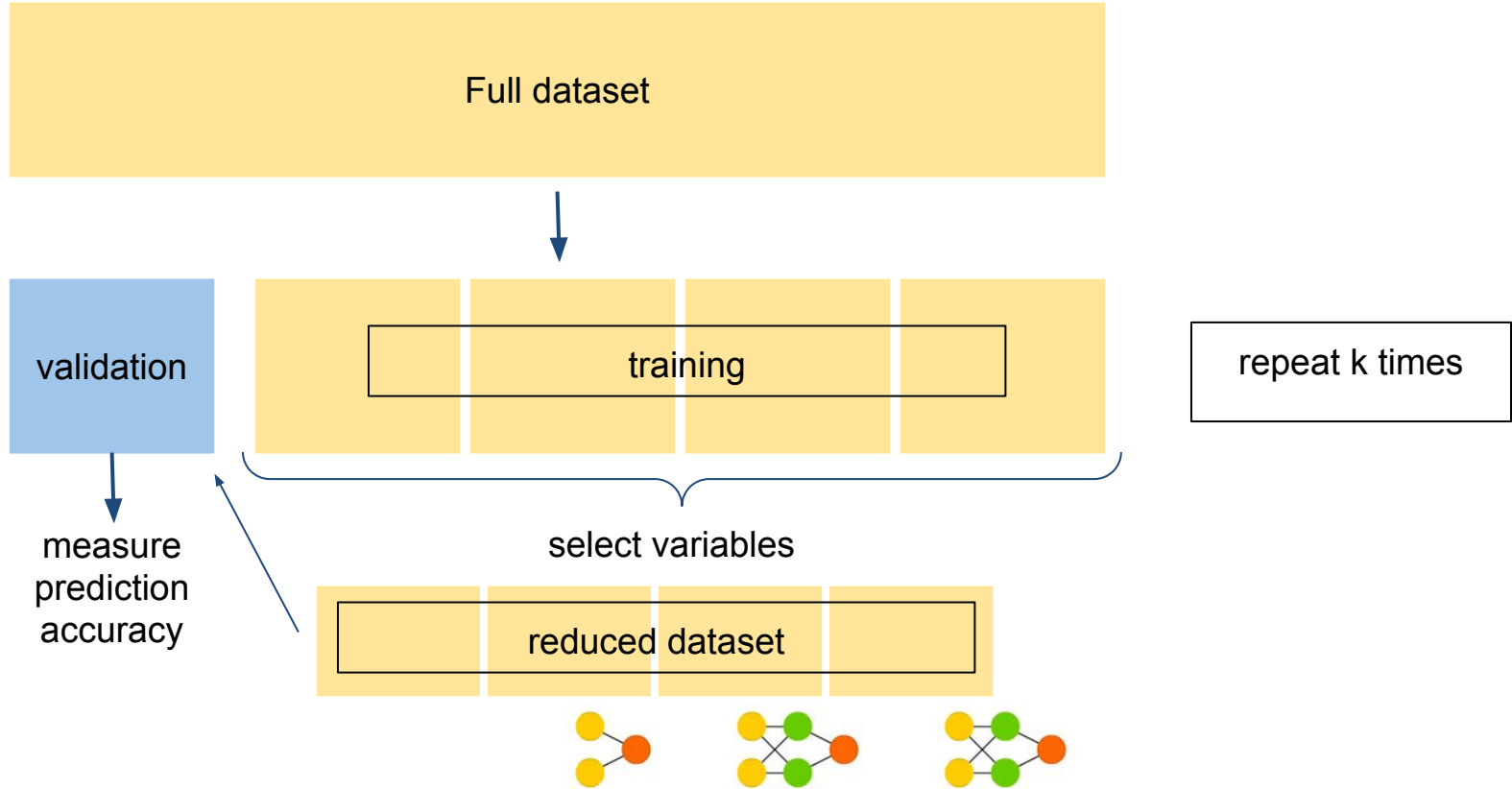
# Cross-validation: **wrong way**



# Cross-validation: **wrong way**



# Cross-validation: right way



# Cross-validation: right way

