

From logistic regression to neural networks

Binary classification problems

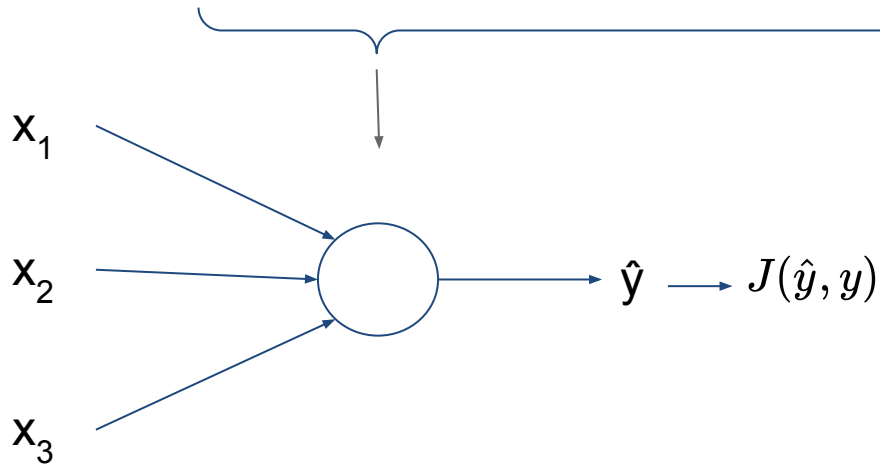
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Logistic regression as a neural network

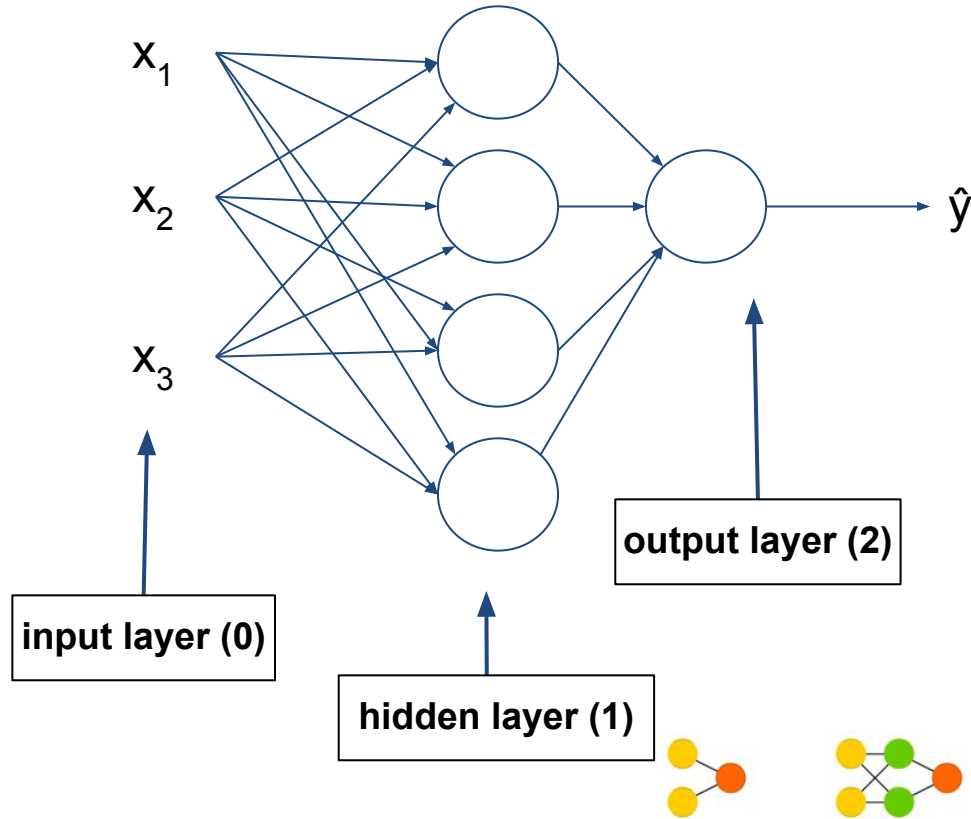
$$z = w \cdot x + b \longrightarrow \hat{y} = \sigma(z)$$



- input variables (features): x_1 , x_2 , x_3
- calculations in the unit/neuron
- forward and back propagation
- **this is just one single neuron!**



Logistic regression as a neural network

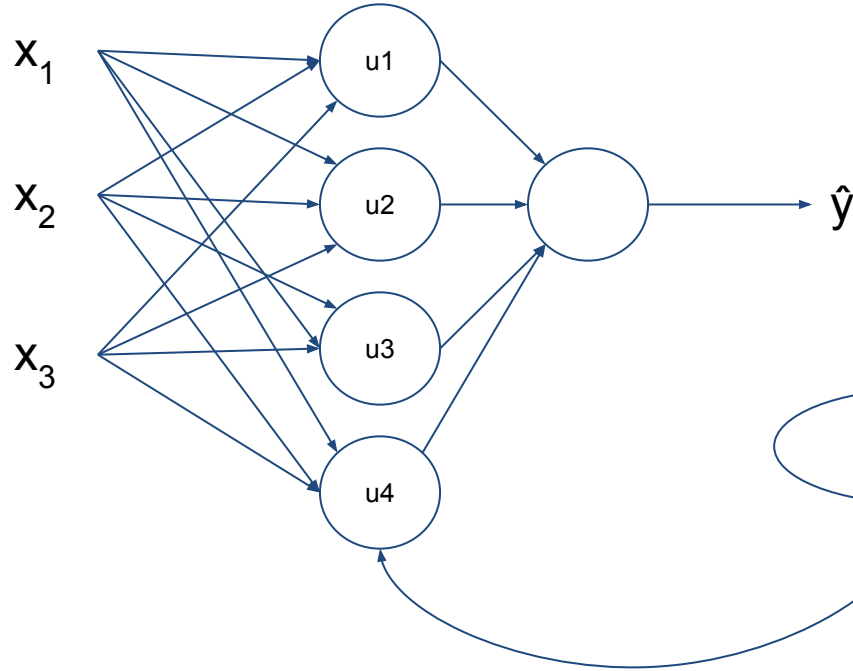


- two layers:
 - 1 hidden layer, 4 nodes
 - 1 output layer, 1 node
- logistic regression is performed in each node
- each node in the hidden layer receives all input variables
- the node in the output layer receives all outputs (**activations**) from the hidden layer nodes

Q: what differs between one logistic regression model and the other?

Logistic regression as a neural network

n observations, m features, u units



$$\begin{cases} \mathbf{Z}_{(n,u)}^{[1]} = \mathbf{X}_{(n,m)} \cdot \mathbf{W}_{(m,u)}'^{[1]} + \mathbf{b}_{(1,u)}^{[1]} \\ \mathbf{A}_{(n,u)}^{[1]} = \sigma(\mathbf{Z}^{[1]}) \end{cases}$$

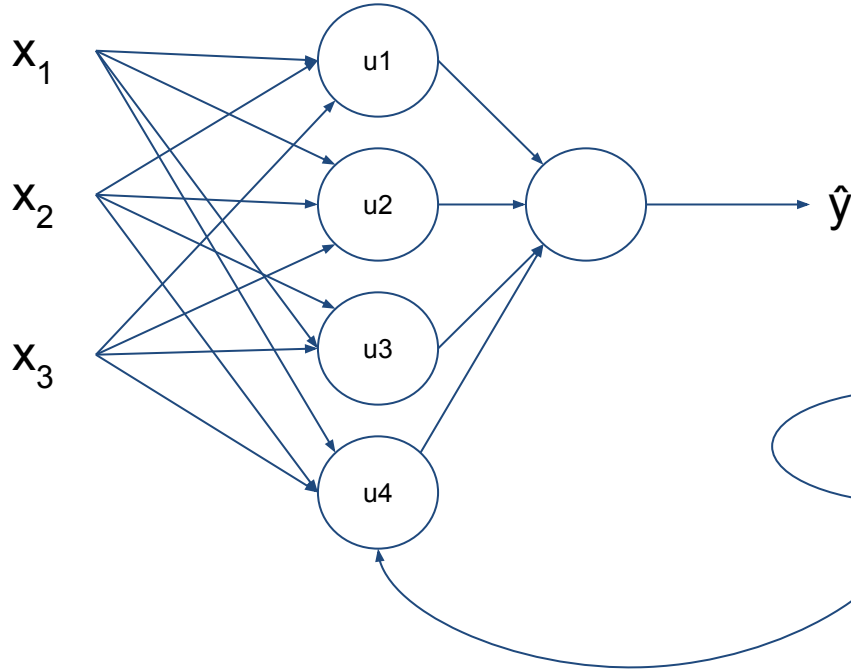
original input data matrix \mathbf{X}

\mathbf{A} for activation
(output of the hidden layer)



Logistic regression as a neural network

n observations, **m** features, **u** units



$$\begin{cases} \mathbf{Z}_{(n,u)}^{[1]} = \mathbf{X}_{(n,m)} \cdot \mathbf{W}_{(m,u)}'^{[1]} + \mathbf{b}_{(1,u)}^{[1]} \\ \mathbf{A}_{(n,u)}^{[1]} = \sigma(\mathbf{Z}_{(n,u)}^{[1]}) \end{cases}$$

original input data matrix **X**

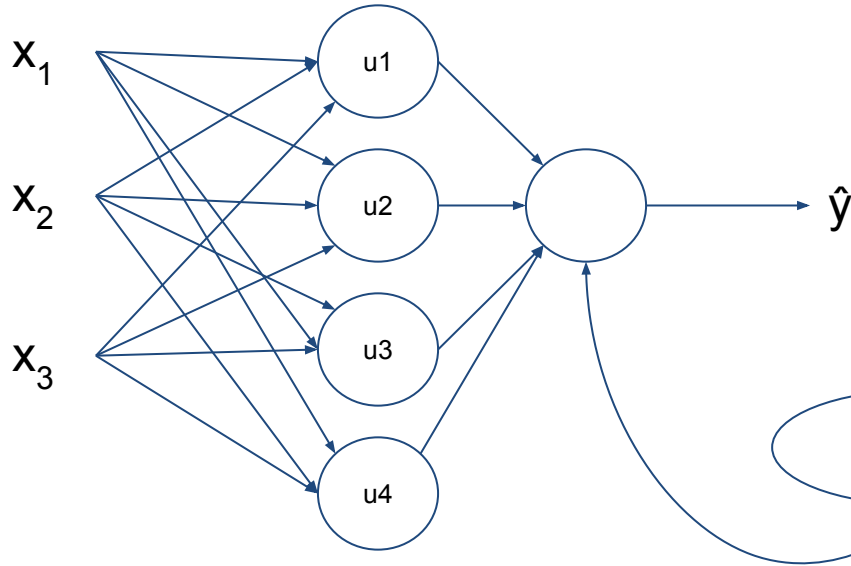
A for activation
(output of the hidden layer)

What do we have in matrix **A**?



Logistic regression as a neural network

n observations, **m** features, **u** units



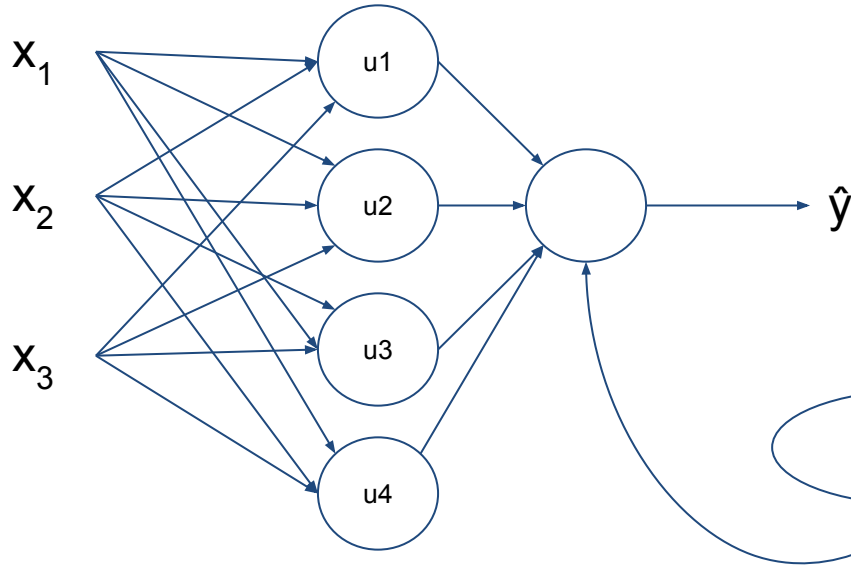
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$$\begin{cases} \mathbf{Z}_{(n,1)}^{[2]} = \mathbf{A}_{(n,u)}^{[1]} \cdot \mathbf{W}_{(u,1)}'^{[2]} + \mathbf{b}_{(1,1)}^{[2]} \\ \hat{\mathbf{y}}_{(n,1)} = \sigma(\mathbf{Z}^{[2]}) \end{cases}$$



Logistic regression as a neural network

n observations, **m** features, **u** units



$$\begin{cases} \mathbf{Z}_{(n,u)}^{[1]} = \mathbf{X}_{(n,m)} \cdot \mathbf{W}_{(m,u)}'^{[1]} + \mathbf{b}_{(1,u)}^{[1]} \\ \mathbf{A}_{(n,u)}^{[1]} = \sigma(\mathbf{Z}^{[1]}) \end{cases}$$

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What do we have in vector $\hat{\mathbf{y}}$?



Take away messages

- You can build a neural network (NN) for binary classification
- NNs are logistic regressions repeated several times! → n. of nodes/units, n. of layers
- probably a bit of an overkill to use NNs in place of a simple logistic regression model → used for illustration
- however, when you have many observations and many features (big data), NN will do the job



Let's go deep

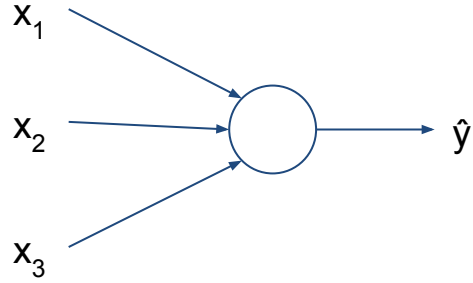
From NNs to deep learning

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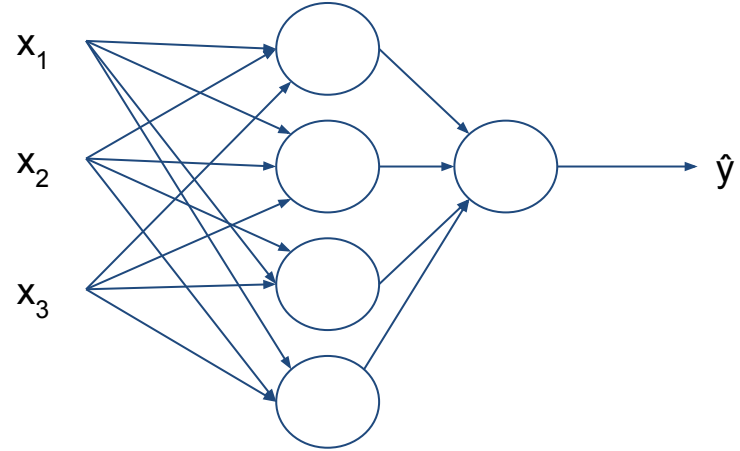
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It's a matter of layers



logistic regression
(1 layer)

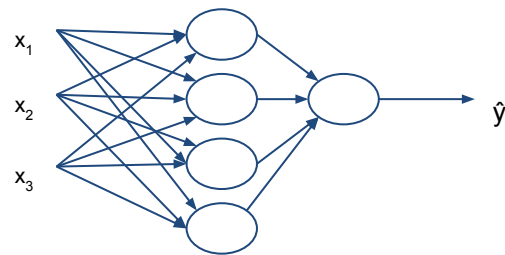
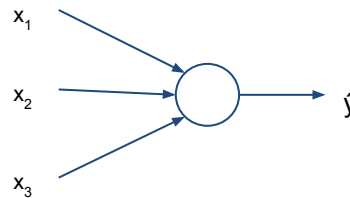


shallow NN
(2 layers)



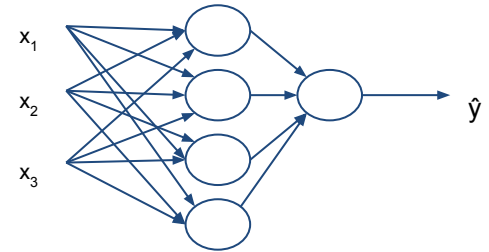
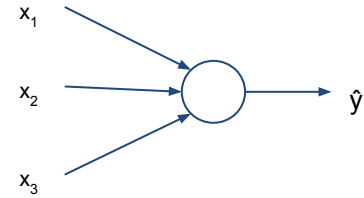
It's a matter of layers

- we had a (cursory) look at the calculations involved in neural networks models
- Lots of details, but it's **not a black box!**

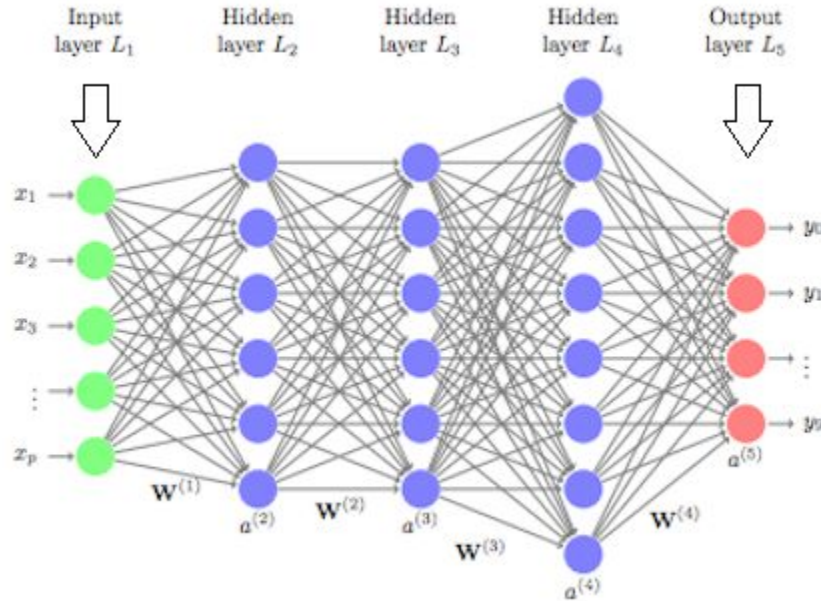


It's a matter of layers

- we had a (cursory) look at the calculations involved in neural networks models
- Lots of details, but it's **not a black box**!
- however, when you go deep (more layers), the “magic” gets back in the play!



It's a matter of layers

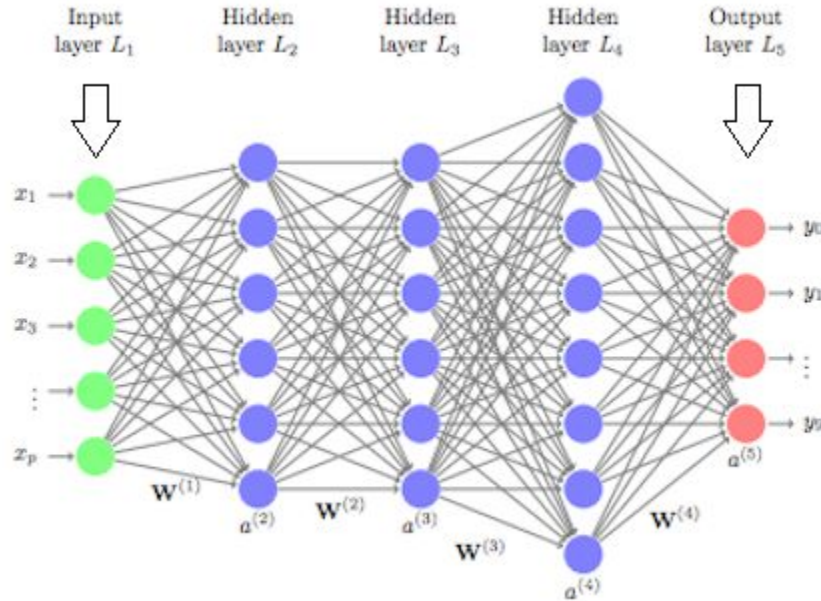


- 4 layers (“deep” NN)
- n. of layers in a deep learning model: hyperparameter to tune (one of many)

Source: University of Cincinnati



Layers matter!

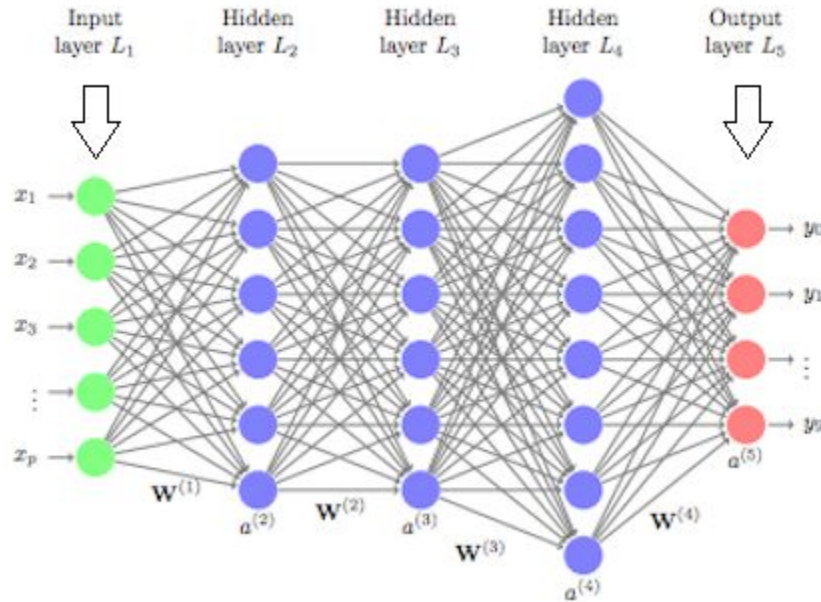


Source: University of Cincinnati

- 4 layers (“deep” NN)
- n. of layers in a deep learning model: hyperparameter to tune (one of many)
- research in the AI/machine learning communities has shown that there are functions that deep NN can learn which can not be learnt by shallower models



Forward propagation



Source: University of Cincinnati

- L : n. of layers (1 in 1 to L)
- X : matrix of input features $\rightarrow A^{[0]}$

$$\begin{cases} \mathbf{Z}^{[l]} = \mathbf{A}^{[l-1]} \cdot \mathbf{W}'^{[l]} + \mathbf{b}^{[l]} \\ \mathbf{A}^{[l]} = g^{[l]}(\mathbf{Z}^{[l]}) \end{cases}$$

- for each layer
- iterate over n. of layers (unavoidable for loop)
 - N. nodes \rightarrow memory
 - N. layers \rightarrow CPU



Compositional representation

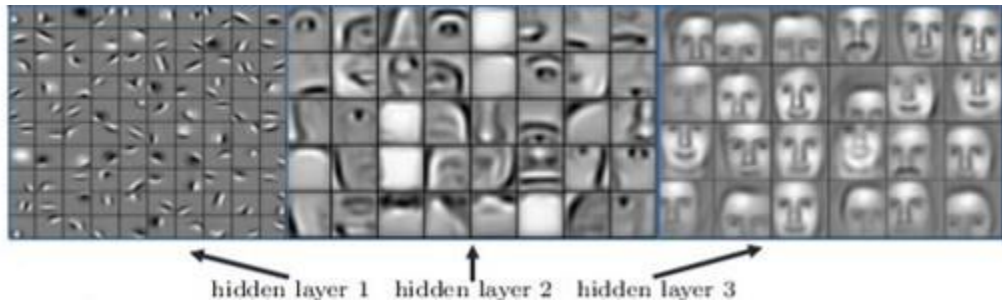
- why do deep NNs work better?
- **each layer** focuses on **one representation** of the data
- representations are then **combined** to get the final result → **compositional representation**

Simplified example from face recognition:

- Layer 1 → edges
- Layer 2 → pieces of faces
- Layer 3 → contours



[mind you: this is just intuition]



Source: <https://medium.com/@fenjiro/face-id-deep-learning-for-face-recognition-324b50d916d1>



Compositional representation

- works also with other types of data
- e.g. speech recognition: i) high/low soundwaves; ii) combinations of soundwaves into phonemes; iii) combination of phonemes into words; iv) from words to sentences
- relatively simple functions of the input data in the first layers → **progressively more complex functions** of the data in the later layers



Depth vs width

- deep learning works by stacking together multiple (many) hidden layers with relatively few nodes
- alternatively, one could use a shallow but very wide (many nodes) neural network
- **depth is more efficient than width:** shallow NN require exponentially more hidden units (nodes) compared to deep NN:
 - deep NN $\rightarrow O(\log(n))$
 - wide NN $\rightarrow O(2^n)$
- deep NN can learn functions that can not be learnt by shallow networks



Hyperparameters

- we saw that there are many ingredients that make up a deep learning model (and many still yet to come) → deep learning has **many hyperparameters**:
 - learning rate α
 - n. of hidden layers
 - n. of nodes (total, each layer)
 - activation function
 - and many more (mini-batch size, NN architecture, regularization etc.)
- to be fine-tuned (→ cross-validation)



Neural networks models

- lab 5

→ `day2_code02 keras shallow neural networks.ipynb`



Neural networks models: recap

- students' (collaborative) exercise n. 1 (write your own code, in groups or independently)

→ `day2_code03_neural_networks_[EXERCISE].ipynb`

