From logistic regression to neural networks

Binary classification problems

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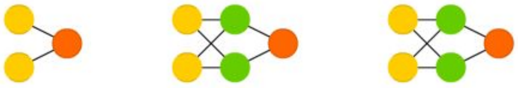
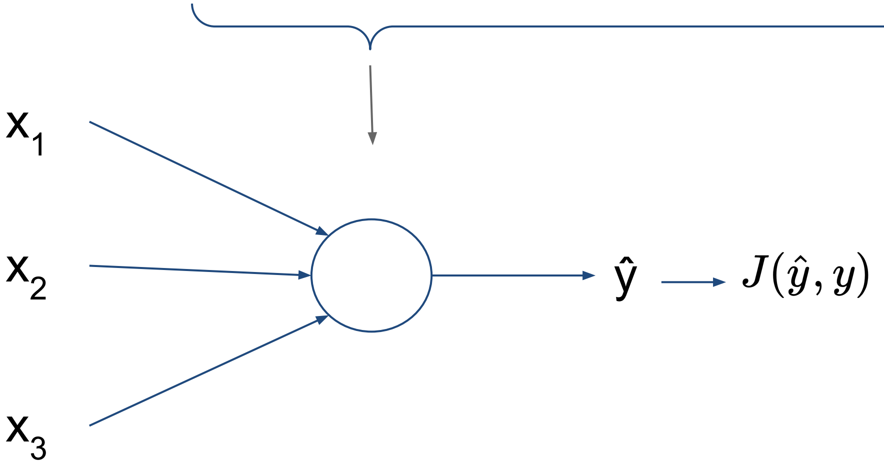
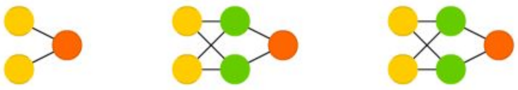


Logistic regression as a neural network

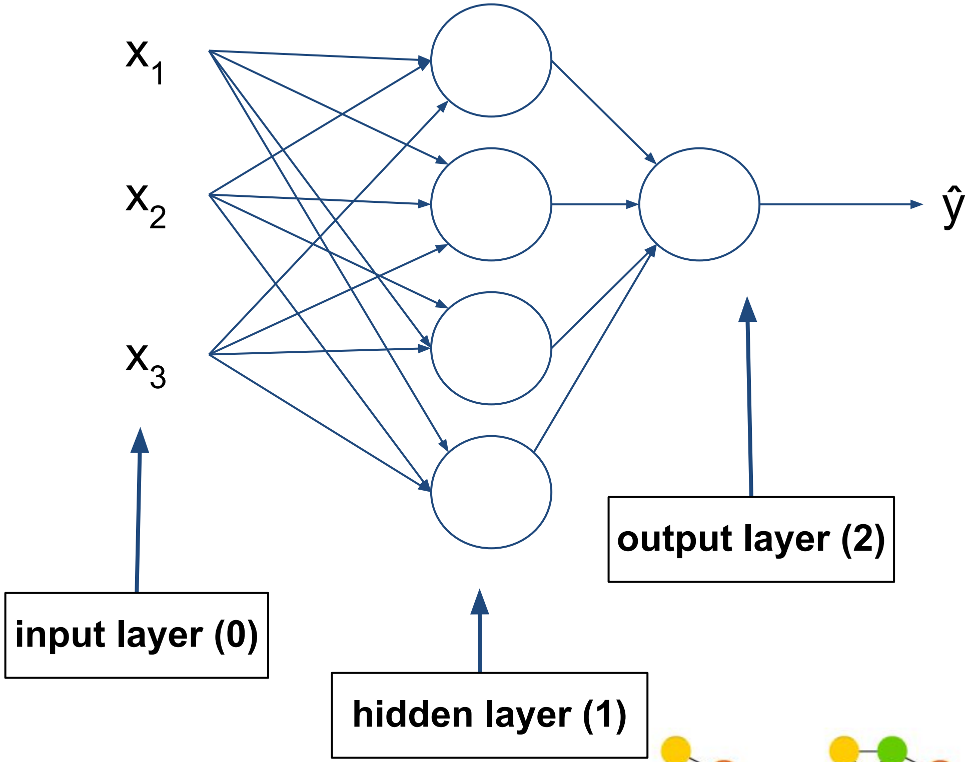
$$z = w \cdot x + b \longrightarrow \hat{y} = \sigma(z)$$



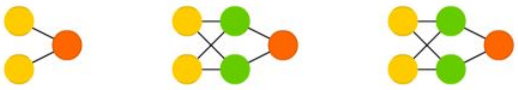
- input variables (features): x1, x2, x3
- calculations in the unit/neuron
- forward and back propagation
- this is just one single neuron!



Logistic regression as a neural network

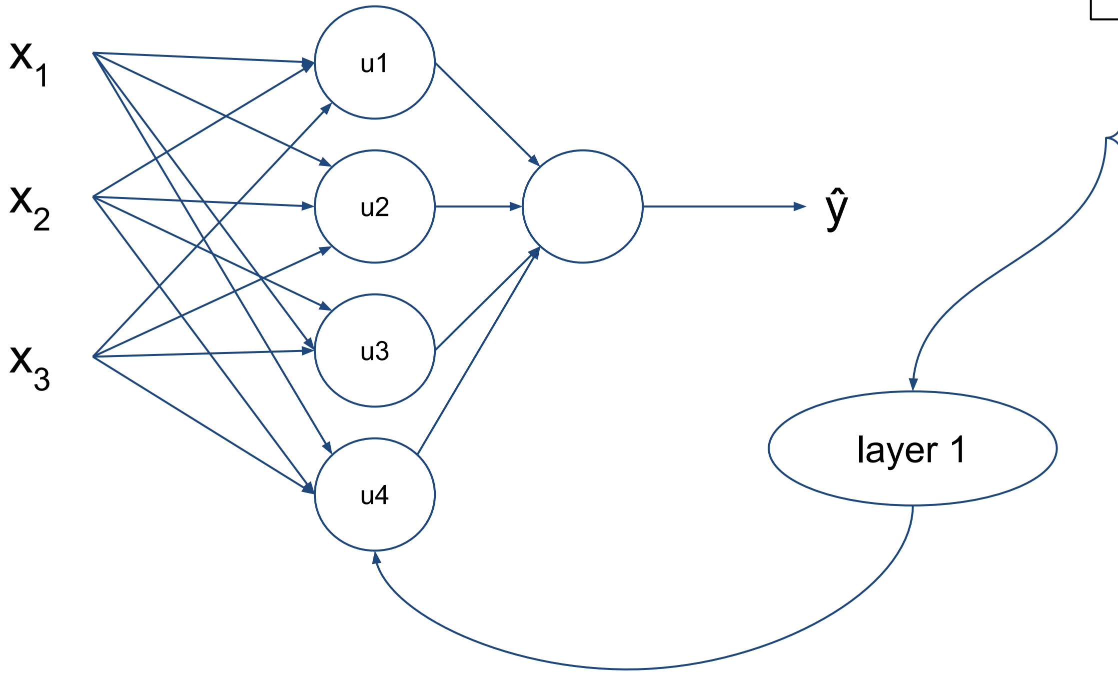


- <u>two layers</u>:
  - **1 hidden layer, 4 nodes**
  - **1 output layer, 1 node**
- logistic regression is performed in each node
- each node in the hidden layer receives all input variables
- the node in the output layer receives all outputs (**activations**) from the hidden layer nodes



Logistic regression as a neural network

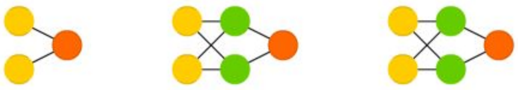
n observations, **m** features, **u** units



$$\begin{cases} \mathbf{Z}^{[1]}_{(n,u)} = \mathbf{X}_{(n,m)} \cdot \mathbf{W}'^{[1]}_{(m,u)} + \mathbf{b}^{[1]}_{(1,u)} \\ \mathbf{A}^{[1]}_{(n,u)} = \sigma(\mathbf{Z}^{[1]}) \end{cases}$$

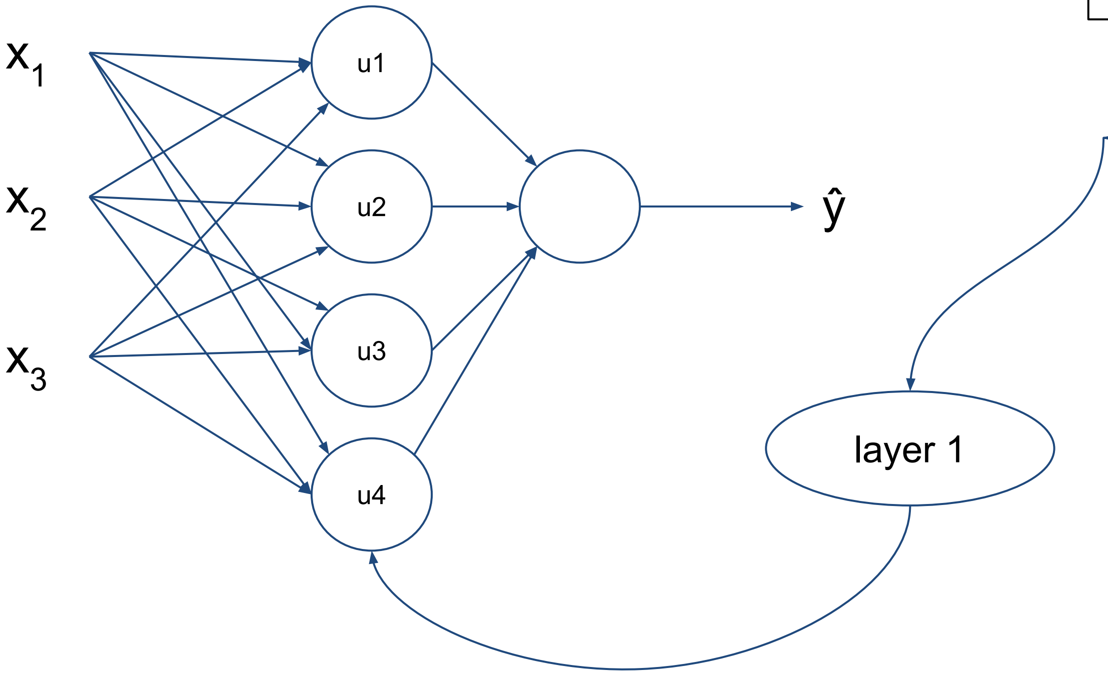
original input data
matrix **X**

A for activation
(output of the
hidden layer)



Logistic regression as a neural network

n observations, **m** features, **u** units



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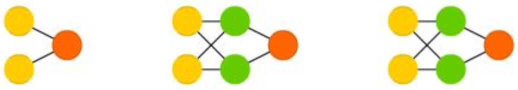
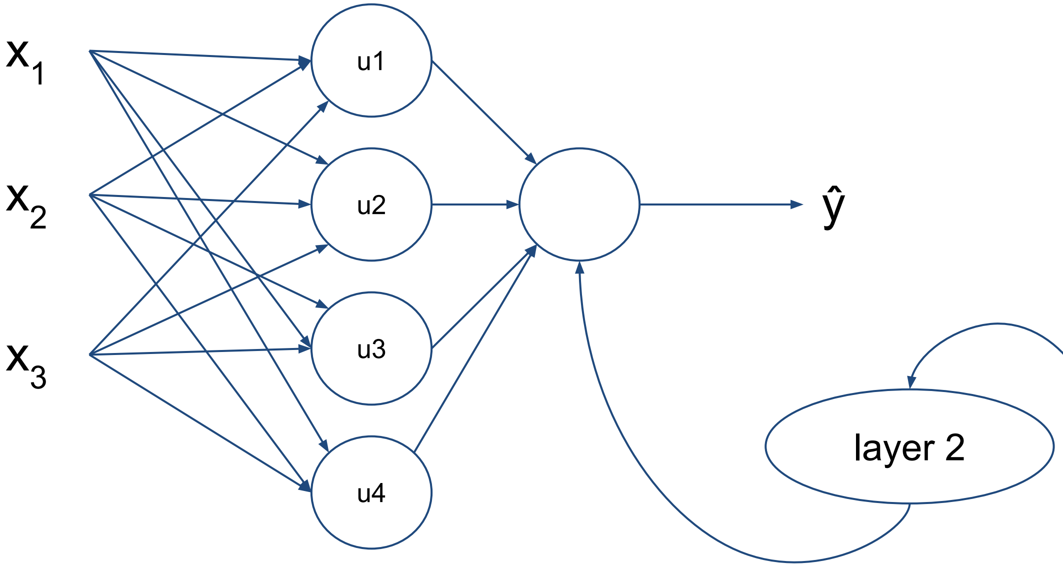
original input data matrix **X**

A for activation (output of the hidden layer)

What do we have in matrix A?



Logistic regression as a neural network



n observations, **m** features, **u** units

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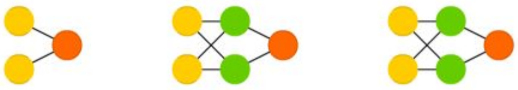
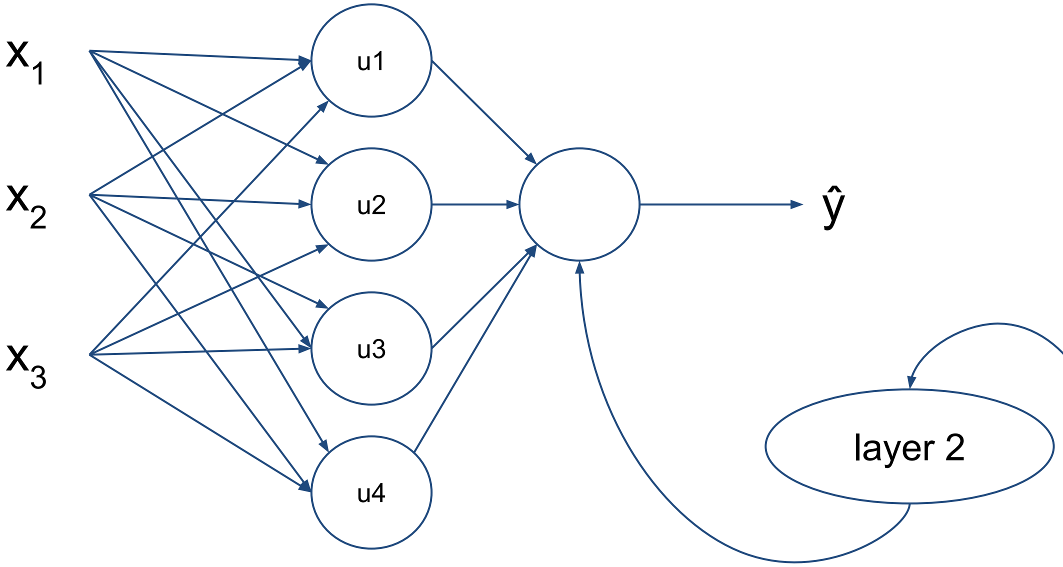
$$\begin{cases} \mathbf{Z}^{[2]}_{(n,1)} = \mathbf{A}^{[1]}_{(n,u)} \cdot \mathbf{W}'^{[2]}_{(u,1)} + \mathbf{b}^{[2]}_{(1,1)} \\ \hat{\mathbf{y}}_{(n,1)} = \sigma(\mathbf{Z}^{[2]}) \end{cases}$$



Logistic regression as a neural network



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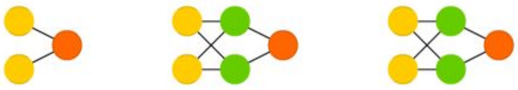
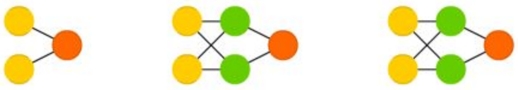
What do we have in vector ŷ?



Take away messages

- You can build a neural network (NN) for binary classification
- NNs are logistic regressions repeated several times! → n. of nodes/units, n. of layers
- probably a bit of an overkill to use NNs in place of a simple logistic regression model → used for illustration
- however, when you have many observations and many features (big data), NN will do the job



Let’s go deep

From NNs to deep learning

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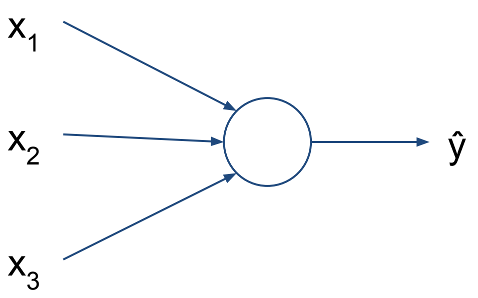
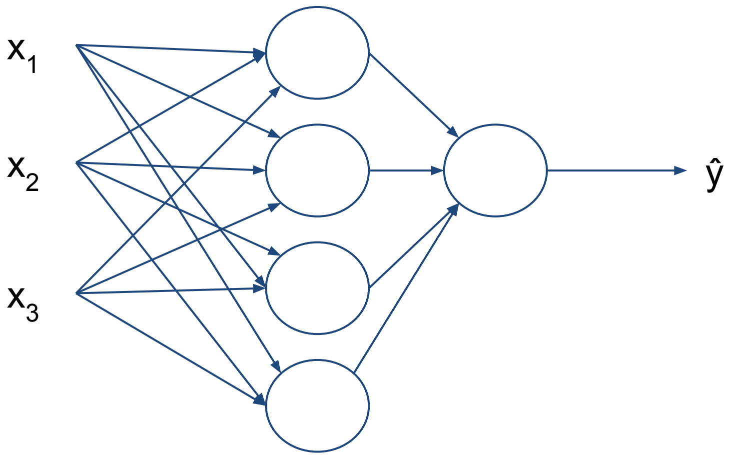
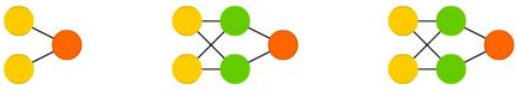
It’s a matter of layers



logistic regression
(1 layer)

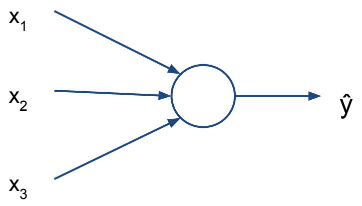


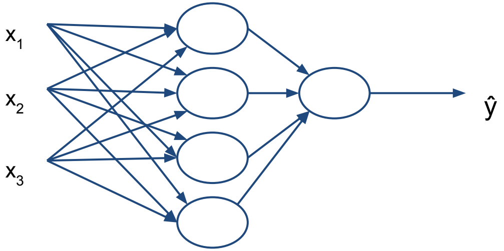
shallow NN
(2 layers)

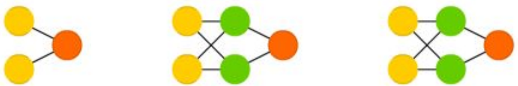


It’s a matter of layers

- we had a (cursory) look at the calculations involved in neural networks models
- Lots of details, but it’s **not a black box**!

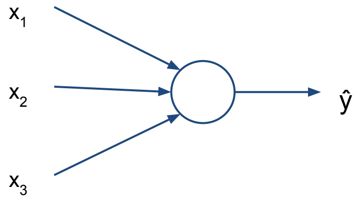


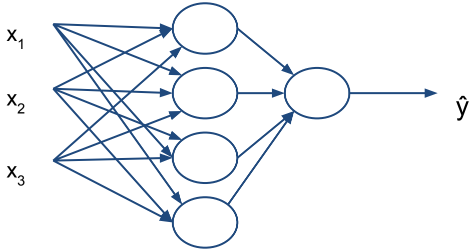




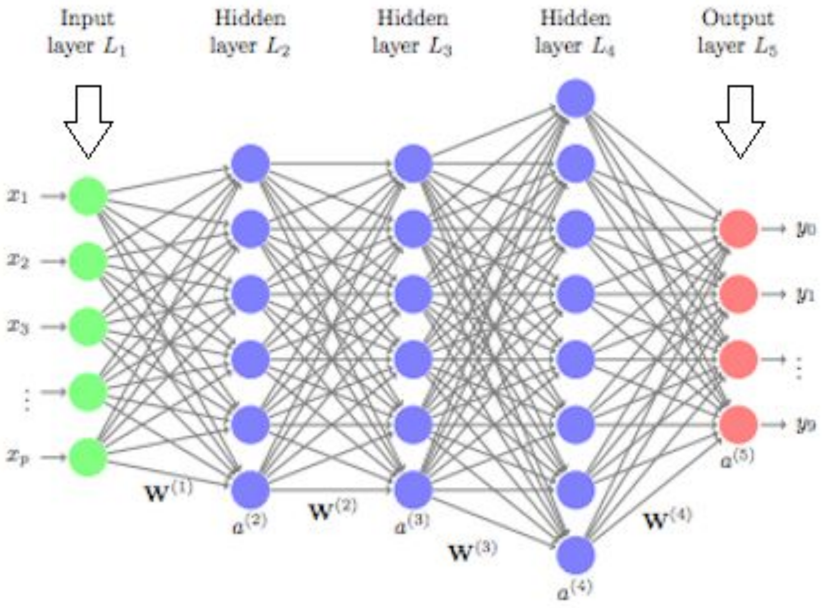
It’s a matter of layers

- we had a (cursory) look at the calculations involved in neural networks models
- Lots of details, but it’s **not a black box**!
- however, when you go deep (more layers), the magic gets back in the play!



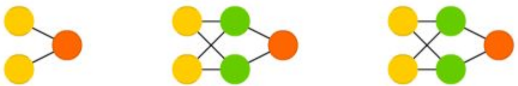


It’s a matter of layers

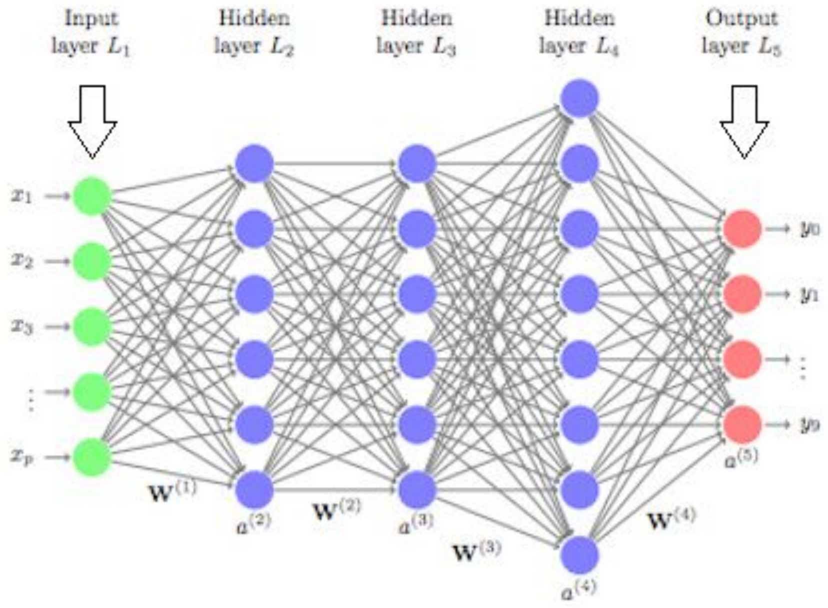


Source: University of Cincinnati

- 4 layers (“deep” NN)
- n. of layers in a deep learning model: hyperparameter to tune (one of many)

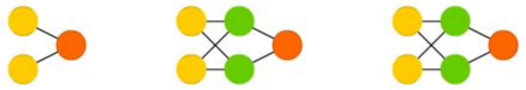


Layers matter!

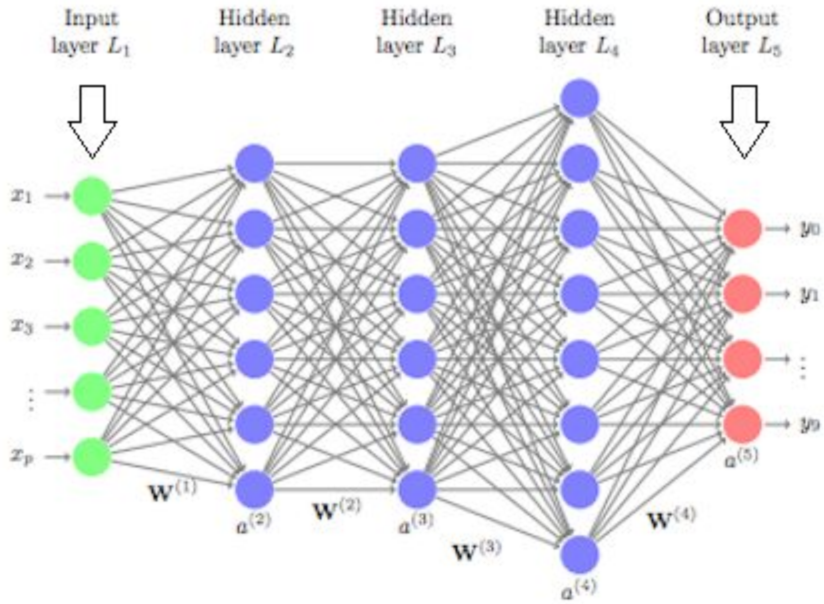


Source: University of Cincinnati

- 4 layers (“deep” NN)
- n. of layers in a deep learning model: hyperparameter to tune (one of many)
- research in the AI/machine learning communities has shown that there are functions that deep NN can learn which can not be learnt by shallower models



Forward propagation

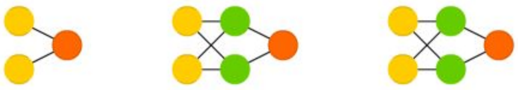


Source: University of Cincinnati

- L: n. of layers (l in 1 to L)
- X: matrix of input features → $A^{[0]}$

$$\begin{cases} \mathbf{Z}^{[l]} = \mathbf{A}^{[l-1]} \cdot \mathbf{W}'^{[l]} + \mathbf{b}^{[l]} \\ \mathbf{A}^{[l]} = g^{[l]}(\mathbf{Z}^{[l]}) \end{cases}$$

- for each layer
- iterate over n. of layers (unavoidable for loop)
  - N. nodes → memory
  - N. layers → CPU

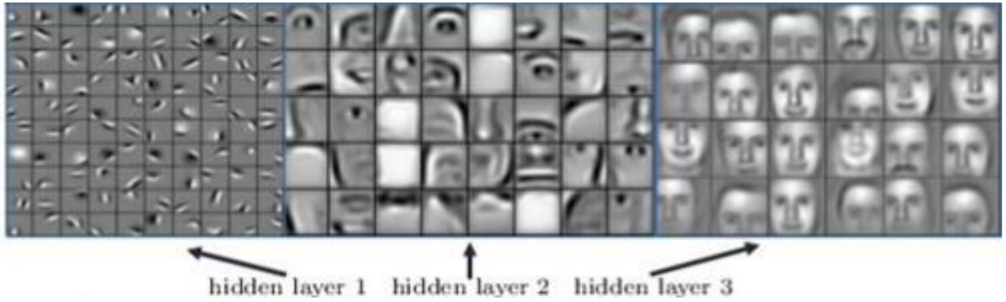


Compositional representation

- why do deep NNs work better?
- **each layer** focuses on **one representation** of the data
- representations are then **combined** to get the final result → **compositional representation**

<u>Simplified example from face recognition</u>:

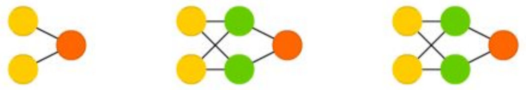
- Layer 1 → edges
- Layer 2 → pieces of faces
- Layer 3 → contours



Source: https://medium.com/@fenjiro/face-id-deep-learning-for-face-recognition-324b50d916d1

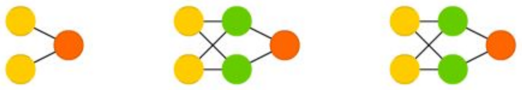
Compositional representation

- works also with other types of data
- e.g. speech recognition: i) high/low soundwaves; ii) combinations of soundwaves into phonemes; iii) combination of phonemes into words; iv) from words to sentences
- relatively simple functions of the input data in the first layers → **progressively more complex functions** of the data in the later layers



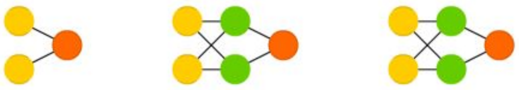
Depth vs width

- deep learning works by stacking together multiple (many) hidden layers with relatively few nodes
- alternatively, one could use a shallow but very wide (many nodes) neural network
- **depth is more efficient than width**: shallow NN require exponentially more hidden units (nodes) compared to deep NN:
    - deep NN → $O(\log(n))$
    - wide NN → $O(2^n)$



Hyperparameters

- we saw that there are many ingredients that make up a deep learning model (and many still yet to come) → deep learning has **many hyperparameters**:
    - learning rate α
    - n. of hidden layers
    - n. of nodes (total, each layer)
    - activation function
    - and many more (mini-batch size, NN architecture, regularization etc.)
- to be fine-tuned (→ cross-validation)



Neural networks models

- lab 5

→ day2_code02 keras shallow neural networks.ipynb

Neural networks models: recap

- students' (collaborative) exercise n. 1 (write your own code, in groups or independently)

→ day2_code03_neural_networks_[EXERCISE].ipynb