Bu-Ali Sina University
Department of Computer Engineering

Synthesis of Digital Systems (SDS2022)

Homework Solution #2

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Homework and Project Solutions
https://github.com/nedaraad/MSc-Synthesis.git

Q.1 States Reduction of FSM

Given is Machine M

a) Find the minimal machine (in the number of states) that is equivalent to machine M
b) Draw the triangular table of machine M
c) Solve the triangular table
d) Find the maximal compatible groups of states.
e) Solve graphically the covering/closure problem.
f) Formulate algebraically the binate covering problem.
g) Realize the machine using JK flipflops and combinational gates.

a	d/-	f/0
b	c/1	h/0
c	d/0	h/0
d	c/0	a/0
e	-/-	f/0
f	e/0	a/0
g	c/1	-/-
h	b/1	a/1

Machine M

Q1. Solution

Implication Table
(Triangular Table)

b	cd / fh					
c	fh	✗				
d	cd / fh	✗	ah			
e	✓	fh	fh	af		
f	de	✗	de / ah	cd	af	
g	cd	✓	✗	✗	✓	✗
h	✗	✗	✗	✗	✗	✗	bc

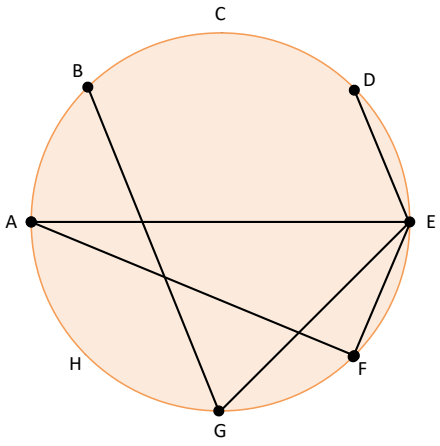
Compatibility Classes

g	—
f	• —
e	(ef) (eg)
d	(de) (ef) (eg)
c	(de) (ef) (eg)
b	(bg) (de) (ef) (eg)
a	(ae) (af) (bg) (de) (ef) (rg)
a	(aef) (bg) (de) (eg)

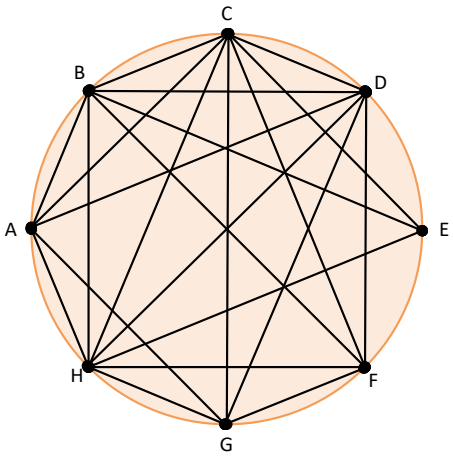
Incompatibility Classes

g	(gh)
f	(fg) (fh) (gh)
e	(ef) (fgh)
⋮	⋮
d	(dfgh) (eh)
⋮	⋮
c	(cdfgh) (ceh)
⋮	⋮
b	(bcdfh) (bceh) (cdfgh)
⋮	⋮
a	(acdgh) (abcdh) (bcdfh) (bceh)

Maximal Compatibility



Maximal Incompatibility



Closure Table and Equations

		0	1
A ←	aef	de/0	af/0
B ←	bg	c/1	h/0
C ←	de	c/0	af/0
	eg	c/1	f/0
D ←	c	d/0	h/0
E ←	h	b/1	a/1

$$U = \min\{NSMC, NSOC\}$$
$$U = \min\{6, 8\} = 6$$
$$L = \max\{NSMI_1, NSMI_2, \ldots., NSMI_i\}$$
$$L = \max\{5, 5, 5, 4, 5\} = 5$$
$$L \le K \le U \rightarrow 5 \le K \le 6 \rightarrow K = 5 \checkmark$$
$$K = 5$$

Reduced State Table

	0	1
A	C/0	A/0
B	D/1	E/0
C	D/0	A/0
D	C/0	E/0
E	B/1	A/1

Most Transferred Rules

Rule 1: $A\ adj\ D, B\ adj\ C, E\ \rightarrow (AD, BC, E)$

Rule 2: $C\ adj\ A, D\ adj\ E, D\ adj\ A, C\ adj\ E, B\ adj\ A$

Rule 3: (AE, BCD)

Rule 4 $most\ transferred\ to: A\ \rightarrow A \equiv 000$

$A \equiv 000$

States Code Assignment

The combination of rules 1 and 3 is initially selected for state code assessment. Then rule 2 is considered.

$A \equiv 000$
$E \equiv 010$
$C \equiv 110$
$B \equiv 111$
$D \equiv 101$

Transition Table

Q	Q^+	J	K	D
0	0	0	—	0
0	1	1	—	1
1	0	—	1	0
1	1	—	0	1

		X	
	$Q_1Q_2Q_3$	0	1
A ←	000	110/0	000/0
	001	—	—
	011	—	—
E ←	010	111/1	000/1
C ←	110	101/0	000/0
B ←	111	101/1	010/0
D ←	101	110/0	010/0
	100	—	—

Q_1	Q_2	Q_3	Q_1^+	Q_2^+	Q_3^+	X	J_1	K_1	J_2	K_2	J_3	K_3	D_1	D_2	D_3	Z
0	0	0	1	1	0	0	1	—	1	—	0	—	1	1	30	0
0	0	1	—	—	—	0	—	—	—	—	—	—	—	—	—	—
0	1	0	—	—	—	0	—	—	—	—	—	—	—	—	—	—
0	1	1	1	1	1	—	1	—	—	0	1	—	1	1	1	1
1	0	0	1	0	1	0	—	0	—	1	1	—	1	—	1	0
1	0	1	1	0	1	0	—	0	—	1	—	0	1	0	1	1
1	1	0	1	1	0	0	—	0	1	—	—	1	1	1	0	0
1	1	1	—	—	—	0	—	—	—	—	—	—	—	—	—	—
0	0	0	0	0	0	1	0	—	0	—	0	—	0	0	0	0
0	0	1	-	-	-	1	-	-	-	-	-	-	-	-	-	-
0	1	0	-	-	-	1	-	-	-	-	-	-	-	-	-	-
0	1	1	0	0	0	1	0	-	0	-	0	-	0	0	0	1
1	0	0	0	0	0	1	-	1	-	1	0	-	0	0	0	0
1	0	1	0	1	0	1	-	1	-	1	-	1	0	1	0	0
1	1	0	0	1	0	1	-	1	-	1	-	1	0	1	0	0
1	1	1	-	-	-	1	-	-	-	-	-	-	-	-	-	-

Q_3X \ Q_1Q_2	00	01	11	10
00	1	1		
01	0	0		
11				
10				

$J_1 = \bar{X}$

Q_3X \ Q_1Q_2	00	01	11	10
00	1			
01	0			
11				1
10				1

$J_2 = \bar{X} + Q_3$

Q_3X \ Q_1Q_2	00	01	11	10
00	0	1	1	
01	0	0	0	
11				
10				

$J_3 = \bar{X} \cdot Q_2$

Q_3X \ Q_1Q_2	00	01	11	10
00			0	
01			1	
11			1	1
10				

$K_1 = X$

Q_3X \ Q_1Q_2	00	01	11	10
00	0		1	
01		1	1	
11			0	
10			1	

$K_2 = X \cdot \overline{Q_3} + \bar{X} \cdot Q_1$

Q_3X \ Q_1Q_2	00	01	11	10
00				
01				
11			1	1
10			0	1

$K_3 = Q_2 + X$

Q_3X \ Q_1Q_2	00	01	11	10
00	1	1	1	
01	0	0	0	
11			0	0
10			1	1

$D_1 = \bar{X}$

Q_3X \ Q_1Q_2	00	01	11	10
00	1	1	0	
01	0	0	0	
11			1	1
10			0	1

$D_2 = \bar{X} \cdot (\overline{Q_1 . Q_2}) + X \cdot Q_3$

Q_3X \ Q_1Q_2	00	01	11	10
00	0	1	1	
01	0	0	0	
11			0	0
10			1	0

$D_3 = \bar{X} \cdot Q_2$

Q_3X \ Q_1Q_2	00	01	11	10
00	1	1	1	
01	0	0	0	
11			0	0
10			1	1

$Z = \overline{Q_1} \cdot Q_2 + \bar{X} \cdot Q_2 \cdot Q_3$

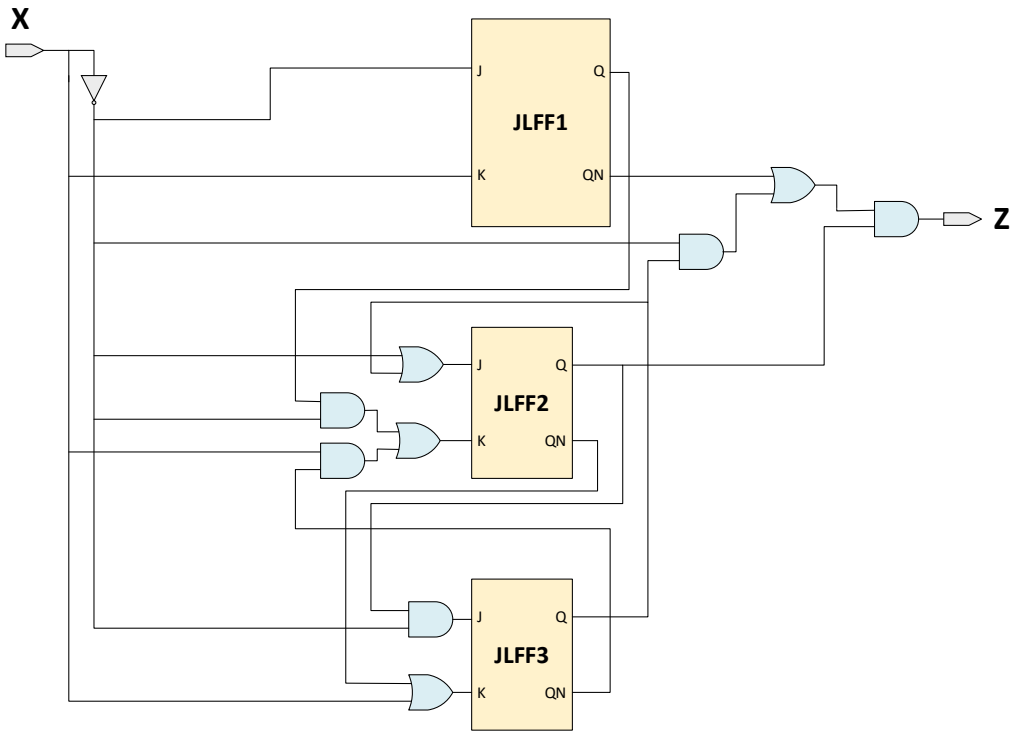
$Z = Q_2 \cdot (\overline{Q_1} + \bar{X} \cdot Q_3)$

$D_2 = \bar{X} \cdot \overline{Q_1} + X \cdot Q_3 + \bar{X} \cdot \overline{Q_2}$

$D_2 = \bar{X} \cdot (\overline{Q_1} + \overline{Q_2}) + X \cdot Q_3$

$D_2 = \bar{X} \cdot (\overline{Q_1 . Q_2}) + X \cdot Q_3$

Realize the machine using JKFF and Logic Gates



Realize the machine using DFF and Logic Gates