Compact binary prefix trees

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Abstract

Our prefix-tree, digital-tree, or trie is an ordered set or map with key strings. We build a dynamic index of two-bytes *per* entry, only storing differences in a compact binary radix tree. To maximize locality of reference while descending the trie and minimizing update data, these are grouped together in a forest of fix-sized trees.

1 INTRODUCTION

A trie is a tree that stores partitioned sets of strings[1, 2, 3, 4] so that, "instead of basing a search method on comparisons between keys, we can make use of their representation as a sequence of digits or alphabetic characters [directly].[5]" It is necessarily ordered, and allows prefix range queries.

Often, only parts of the key string are important; a radix trie (compact prefix tree) skips past the parts that are not important, as [6]. If a candidate key match is found, a full match can be made with one index from the trie.

For most applications, a 256-ary trie is space-intensive; the index contains many spaces for keys that are unused. Various compression schemes are available, such as re-using a pool of memory[1], reducing our encoding alphabet, or take smaller than 8-bit chunks[2].

We use a combination binary radix trie, described in [7] as the PATRICIA automaton. Rather than being sparse, a Patricia-trie is a packed index. Recursively, it encodes which is next distinguishing bit and how many keys are on the zero path and the one path.

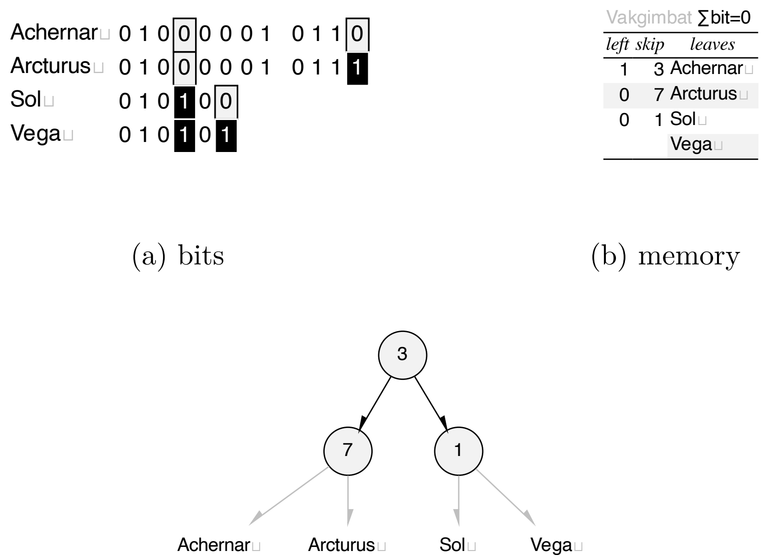
2 IMPLEMENTATION

2.1 Encoding

In practice, we talk about a string always terminated by a sentinel; this is an easy way to allow a string and it's prefix in the same trie[7]. In C, a NUL-terminated string automatically has this property, and is ordered correctly. Keys are sorted in lexicographic order by numerical value; `strcmp`-order, not by any collation algorithm.

Figure 1a is a visual example of a Patricia trie, that is, a binary radix tree and skip values when bits offer no difference. Note that, in ASCII and UTF-8, `A` is represented by an octet with the value of 65, binary 01000001; `c` 99, 01100011; `r` 114, 01110010; `S` 83, 01010011; `V` 86, 01010110.

We encode the branches in pre-order fashion, as in Figure 1b. Each branch has a `left` and a `skip`, corresponding to how many branches are descendants on the left, and how many bits we should skip before the decision bit. With the initial range set



(a) bits (b) memory

(c) tree

Figure 1: A trie with three different views of the data.

to the total number of branches, it becomes a matter of accumulating leaf values for the right branches of a key, accessing the index skip-sequentially, until the range is zero. The right values are implicit in the range. The leaves, on the other hand, are alphabetized, in-order. There will always be one less branch than leaf; that is, this is a full (strict) binary tree with $order - 1$ branches, for *order* keys as leaves.

Figure 1c shows the conventional full binary tree view of the same data as Figure 1a and 1b. The branches indicate a `do not care` for all the skipped bits. If a query might have a difference in the skipped values, one can also check the final leaf for agreement with the found value.

2.2 Range and locality

Only when the algorithm arrives at a leaf will it go outside the `left, skip`. This suggests that these be placed in a contiguous index. This index should be compact as possible to fit the maximum into cache.

However, in establishing a maximum `skip` value, one limits the contiguous bits that can be skipped; this has an effect on both on insertion and deletion. More noticeably, the maximum `left` plus one is the maximum number of leaves in the worst-case of all-left. It is also inefficient to modify the trie with more and more keys; this requires more branches to be changed and an array insertion of the leaf.

To combat these two contradictory requirements, we have broken up the trie in much the same manner as [8]. Except in tries, contrary to B-trees, the data can not be rotated at will; instead, it relaxes the rules and instead uses a bitmap of which leaves are links to other structures, called trees. Thus a trie is a forest of non-empty full binary trees. A tree corresponds to a B-tree node[5], that is, a contiguous area in memory. This would conflict with the terminology of a key as a leaf and individual branches, which are longer implicit.

Thus, on adding to a tree in a trie that has the maximum number of keys, we must split it into two trees. We use the fact that a binary tree of $n \geq 2$ nodes can be split into two trees not exceeding $\left\lceil \frac{2n-1}{3} \right\rceil$ nodes by starting `daughter` tree at the root and choosing the subtree that is larger until the bound is achieved. The `mother` will have an extra linking leaf.

We use

3 MACHINE CONSIDERATIONS

A more complex example is given in Figure 2. This has 3 fixed trees of order 7 maximum leaves and 6 maximum branches with 14 keys in total.

3.1 Linked sample keys

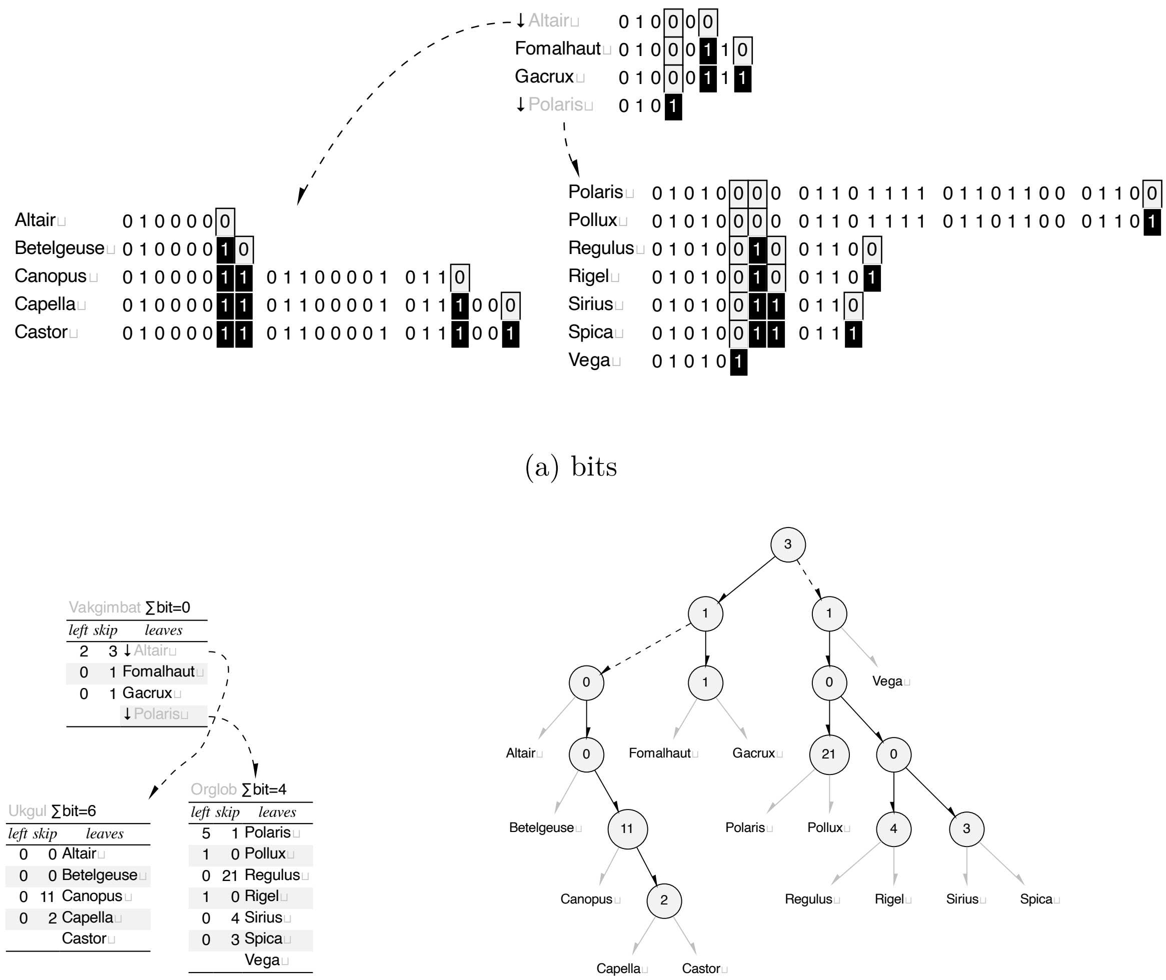
The grey `Altair` and `Polaris` in the root tree, `Vakgimbat`, in Figure 2, are samples of the the trees that are links. We could get any sample from the sub-tree, because all the bits up to bit 6 and 4, respectively, are the same in the sub-tree. Faced with this ambiguity, we arbitrarily and conveniently select the very left. This is important information to have when adding keys, because one has to test each bit of the key against something.

Picking a sample is also important when calculating asymptotic run time of adding keys. The worse case would be an engineered trie, (not with randomly distributed keys,) which with many left-links at once. On addition of a short key to the right, the algorithm must go though all the left-links to arrive at a key for comparison. This leads to worst-case performance $\mathcal{O}(|\text{trie}|)$, something we don't see in practice. We argue that the length of a key should be amortized for future samples in insertions; while there can be multiple insertions with the same trie structure, it becomes less important as the trie grows. Amortized $\mathcal{O}(|\text{key}|)$ is more what we see in practice, which is related to amortized $\mathcal{O}(\log |\text{trie}|)$[10].

3.2 Hysteresis

The non-empty criteria of the trees avoids the pathological case where empty trees from deletion pop-up. Further, we can always join a single leaf with its parent except the in the root.

With smaller, dynamic tries, it is more important to not free resources which could be used in the future. Anything less than greedy merging on deletion will have hysteresis. We also should have a zero-key-state with resources.



(a) bits

(b) memory

(c) tree

Figure 2: A trie as a forest of 3 trees of order-7.

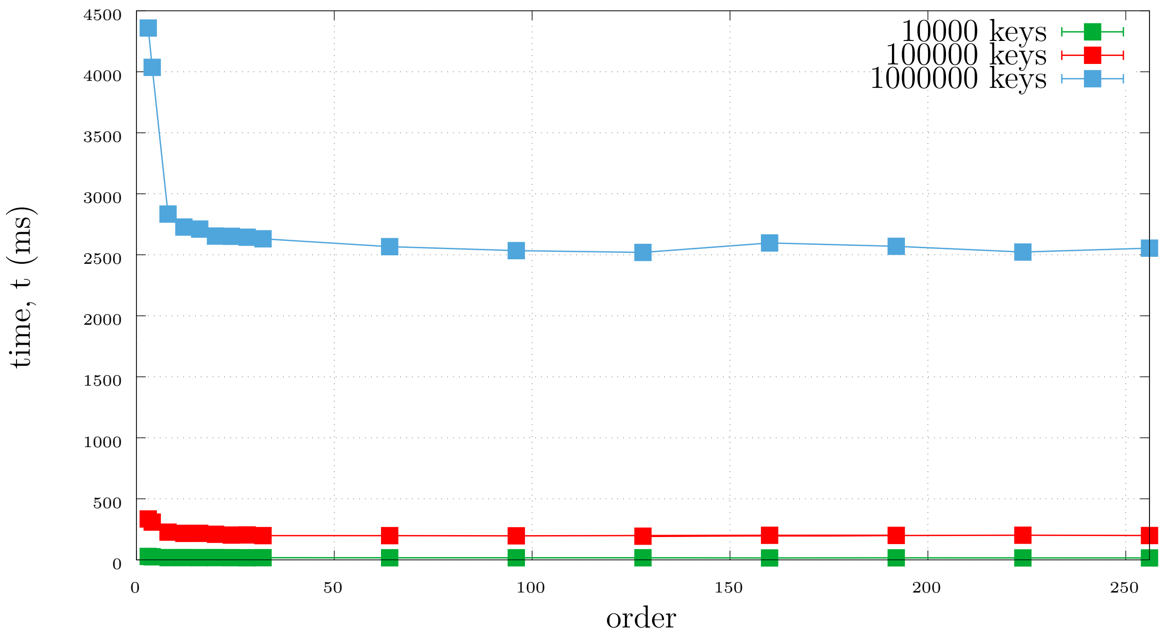


Figure 3: The effects of order on run-time.

3.3 Data size and order

An octet is presumed to be a byte, if not, this will introduce some inefficiencies. An individual `left`, in multiples of octets, is either a maximum of 255, 65 535, or more. 255 is a reasonable size for a contiguous tree. However, we reserve one flag, 254. This corresponds to the maximum branches of 255 and the maximum branching factor, or order, of 256.

Because of alignment, `skip` must also be one or three octets. We have made it one octet. One might also play around with the balance of `left` and `skip` ranges, but without a specific use case, this has been deemed acceptable. The size of the tree fits nicely in the front, giving a maximum index size of 512 octets.

The data has been engineered for maximum effectiveness of the cache in reading and traversing. That is, the tree structure and the string decisions have been reduced to each a byte and placed at the the top of the data structure of the tree. Always forward in memory. The size of this sub-structure should be a multiple of the cache line size, while also maximizing the dynamic range of *left*; a trie (also a B-tree) of order 256 is an obvious choice.[9]

The links do not affect the asymptotic run-time, but in reality, the number of links is closely related to how long it takes. Not only is this an extra node, it has to fetch it, and then jump to a new tree. Once in the cache, it is much faster.

3.4 Limits

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