Practical in-memory B-tree design

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Abstract

Several design decisions for a minimal in-memory B-tree are discussed, covering bulk-loading, insertion, and deletion. Experimentally, a default order of 65 was reached based on insertions.

1 DESIGN

A tree is used as an ordered set or map. For memory locality, this is implemented B-tree[1]. In an implementation for C, we would expect memory usage to be low, performance to be high, and simplicity over complexity. Practically, this means that B$^+$-trees and B*-trees are less attractive, along with an added layer of order statistic tree. The nodes are linked one-way and iteration is very simple. This precludes multi-maps. The use-case has no concurrency and places importance on modification with operations being as lazy as is reasonably possible. Any allocations are fail-fast before modifying the tree so that it remains in a valid state. Concurrency not considered.

1.1 Branching factor

The branching factor, or order as [2], is a fixed value between $[3, \text{UINT\_MAX} + 1]$. The implementation has no buffering or middle-memory management. Thus, a high-order means greater memory allocation granularity, leading to asymptotically desirable caching. This conflicts with small values, which produce much more compact trees. The default order is 65, a value arrived at experimentally in Section 3.3.

In general, it is left-leaning where convenient. This is because keys on the right side are faster to move because of the array configuration of the nodes.

1.2 Minimum and maximum keys

We use fixed-length nodes. In [3], these are (a, b)-trees as $(\text{minimum} + 1, \text{maxumum} + 1)$-trees. That is, the maximum is the node's key capacity. Since the keys can be thought of as an implicit complete binary tree, necessarily maximum+1 is the order. Unlike [2], we differentiate by branch and leaf; a leaf node has no need for null-children, so we don't include them.

Performance will be $\mathcal{O}(\log \text{size})$. Equation 1 gives the standard B-tree key minimum in terms of the key maximum. (Equivalent to the specifying the minimum children as a function of the order.)

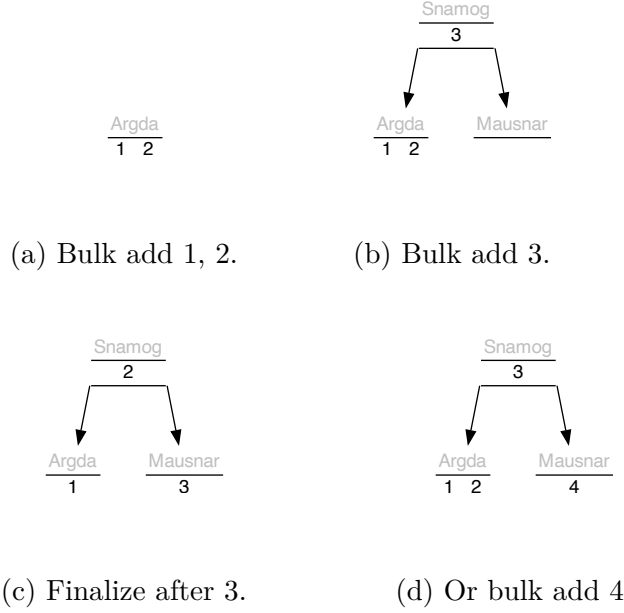$$\text{minimum} = \left\lfloor \frac{\text{maximum}}{2} \right\rfloor \tag{1}$$

Snamog
3

Argda
1 2

Argda
1 2   Mausnar

(a) Bulk add 1, 2.   (b) Bulk add 3.

Snamog
2

Argda   Mausnar
1   3

Snamog
3

Argda   Mausnar
1 2   4

(c) Finalize after 3.   (d) Or bulk add 4.

Figure 1: Order-3, maximum keys 2, bulk-addition, with labels for nodes. 1a. Keys 1, 2: full tree. 1b. Adding 3 increases the height, with new nodes (branch) `Snamog` and (leaf) `Mausnar` . Minimum invariant is violated for `Mausnar` . 1c. Finalize would balance all the right nodes below the root. 1d. Or continue adding 4 to 1b.

Equation 1 represents the maximum minimum without borrowing from the siblings. However, freeing at empty gives good results in [4]. We compromise with Equation 2. Designed to be less-eager and provides some hysteresis while balancing asymptotic performance.

$$\text{minimum} = \max\left(1, \left\lfloor \frac{\text{maximum}}{3} \right\rfloor\right) \tag{2}$$

In a sense, it is the opposite of a B*-tree[2, 5], where $\frac{1}{3}$ instead of $\frac{2}{3}$ of the capacity is full. It doesn't affect the shape of the tree unless deletions are performed, where instead of being stricter, it is lazier.

2 OPERATIONS

2.1 Bulk loading

Bulk loading buffers, as described in [6], are too complex for our purposes. However, bulk loading is useful when the initial keys are in order. A key undergoing bulk-add is packed to the lowest height on the right side, ignoring the rules for splitting, and recruiting zero-key nodes below as needed. When the tree is full, the height increases.
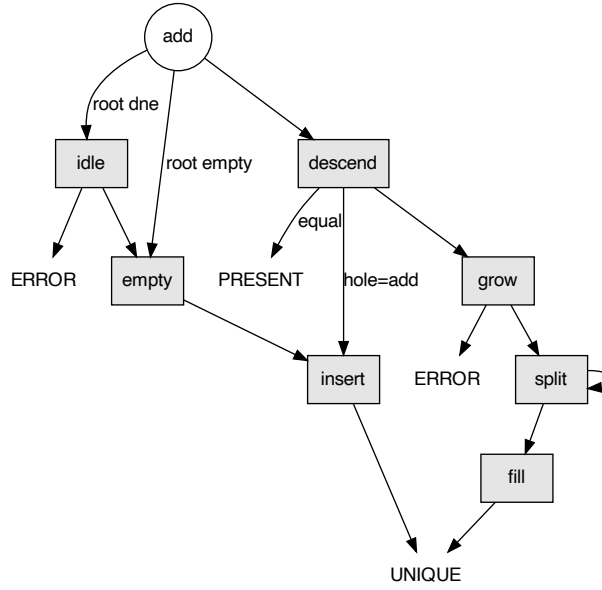
Figure 2: The requirements for `add` are $\mathcal{O}(1)$ space and $\mathcal{O}(\log \text{size})$ time, with no parent pointers. State diagram of adding a key, traversing a maximum twice.

Because the B-tree rule for the minimum number of keys may be violated on bulk-add, it's important to finalize the tree. This balances and restores the B-tree invariants on the right side of the tree, under the root.

Initially, this will efficiently produce a more compact tree. For example, Figure 1 shows bulk addition of natural numbers in order. Compare adding them normally: after a split, there's not any more keys to be added on the low side; this asymptotically results in one-half occupancy.

2.2 Adding a key

In [1, 2, 5], insertion is a $\mathcal{O}(\log \text{size})$ operation. In theory, the key is inserted in a leaf node. If this operation causes the leaf to be overfull, the leaf is split into two, with the median element promoted. This repeats until all the nodes are within the specified key maximum. If implemented directly, this requires an extra temporary element to be added to each node and double-linking to access the parent node.

It is advantageous, then, to flip the insertion and keep track of the hole, getting rid of the temporary size overload and moving the intermediate keys only once. A potential state machine is shown schematically in Figure 2. `idle` is no dynamic memory; `empty` contains an unused node. `ERROR` from `idle` and `grow` is a memory error; the state of the tree remains unchanged.

The `descend` path is taken by any non-empty tree, and descends to find the space
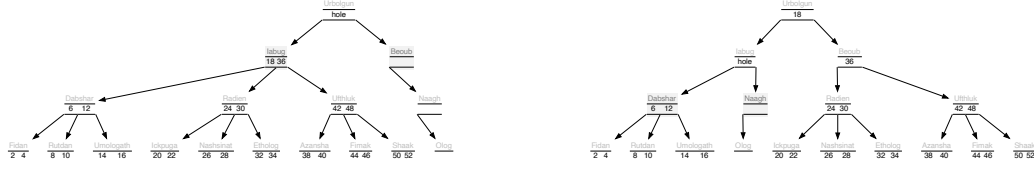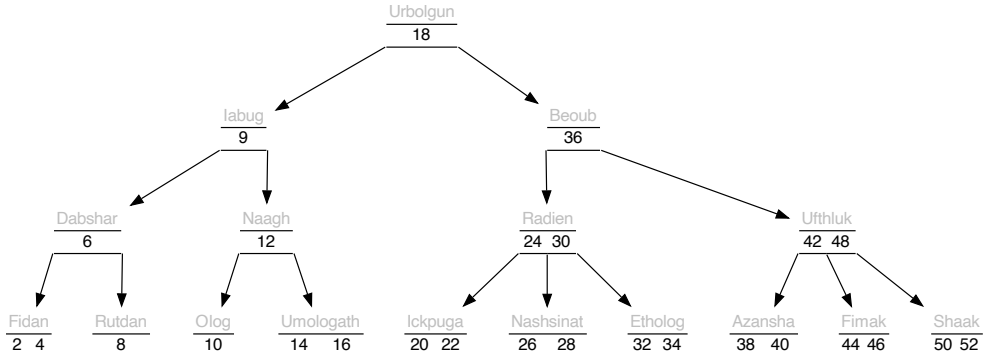
(a) Add 9; four new nodes, the first split.

(b) After one split.
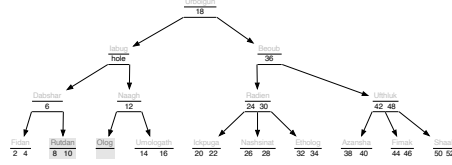
(c) Further split.

(d) After fill.

Figure 3: Full order-3 tree: addition of any number will cause the tree height to increase. 3a – 3c: working down the tree in `split`. 3d after `fill`.

in the leaf that it will go. It stops with `PRESENT` if it finds an already present match. When it completes finding a new key in a leaf node, the `hole` will be the lowest-height node that has free space in the path from the root to that leaf.

A non-empty leaf node results in the `insert` path, and a single time down the tree. Specifying a high-order makes taking this path more likely than having to repeat the `grow` path.

An add must `grow` if it has the maximum keys in the leaf node. The height of the `hole` (zero-based) is the number of nodes extra that need to be reserved. A null `hole` means all of the path is full of keys; this requires increased tree height: tree height + 2 new nodes. These new empty nodes are then strung together and added to the tree.

`split` introduces `cursor`, along the path to the leaf, and `sibling`, the new node ancestry. The `cursor` is initially the `hole`; it descends a second time, balancing with
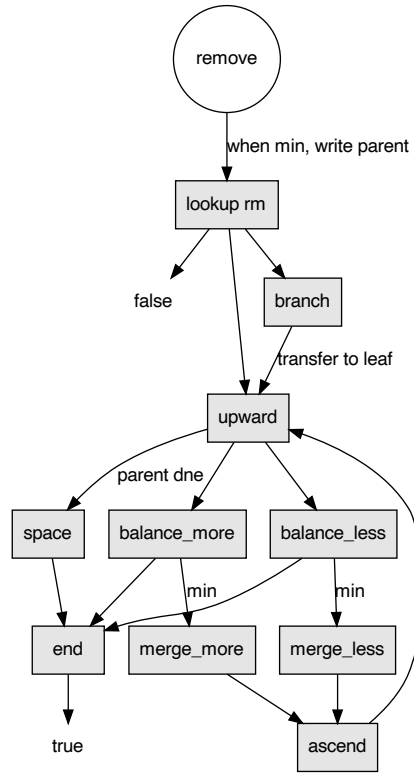
4

Figure 4: `remove` is the opposite of `add` in Figure 2. `rm` starts at the bottom; works up to find any node or siblings that have excess.

`sibling`, thus splits it with the new node. `hole` can either go to the left, right, or exactly in between. `cursor` descends on the path, and `sibling` descends on the new nodes. In `fill`, the added key then goes in `hole`.

In Figure 3, the path from before 3a to 3d to insert 9 on Figure 2 is `descend`, `grow`, `split` four times (the maximum for this size), `fill`, and returns `UNIQUE`. The new nodes are (branches) `Urbolgun`, `Beoub`, `Naagh`, and (leaf) `Olog`. This is the most complicated path, when the height increases.

2.3   Deleting a key

Figure 4 shows a schematic diagram of removal of one key. This has been done directly, working up the tree. Since nodes necessarily have the minimum amount of keys by `ascend`, we can write the a temporary pointer to the parent node of those that have the minimum number of keys (or less) in `lookup rm` and `branch` in the unused space for child nodes.

We require a leaf node for removal. In the event that the key is in a branch, we
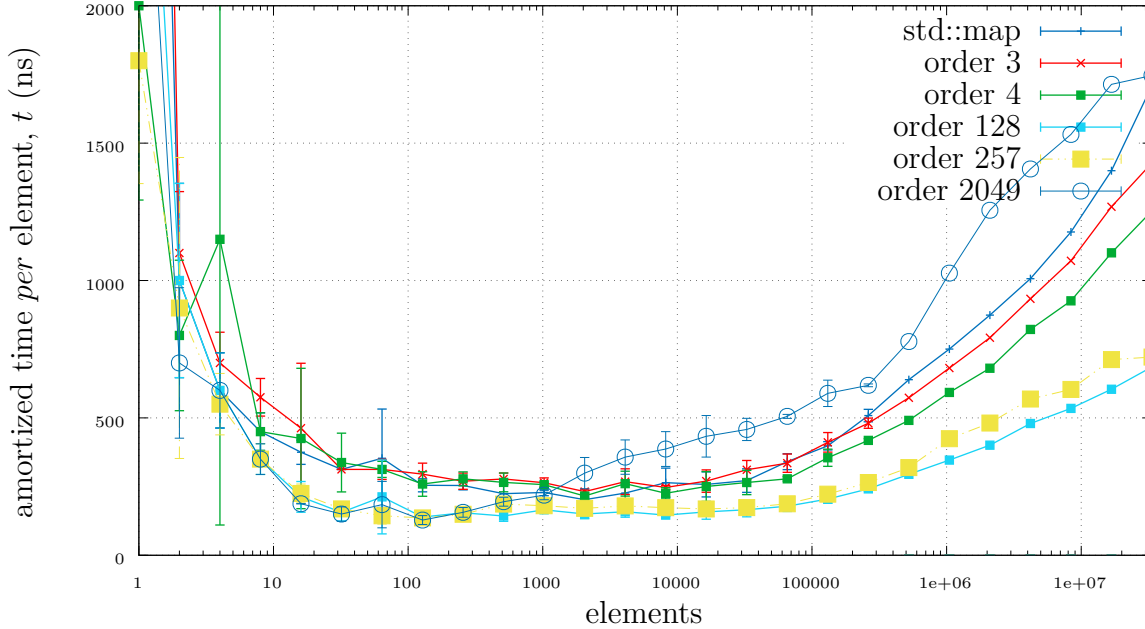
Figure 5: A comparison different orders and `std::map` in C^{++} benchmark, with a log-x scale. Lower t numbers are better performing.

move the predecessor or successor, which is in a leaf, to the current spot and delete the leaf key. We must be careful to replace the provisional upwards-bubbling key with the predecessor or successor.

This is relying on being the only process which modifies the tree during deletion. A potentially more robust way would be from the top-down, like insertion. More work is needed to determine the optimal way of removing keys.

3 PERFORMANCE

3.1 Map

When the tree set has an extra value *per* key that is not associated with the tree operations, it becomes a map. To facilitate cache performance, these values are not interspersed with the keys. That is, only packed keys are ever accessed on lookup. The disadvantage of this approach is the mapped key and value are not in contiguous memory; on rotation of the tree, we must move two parts in tandem.

We expect some kind of map from an integral type to some kind of pointer to be the most common use-case. This covers polymorphic types. Figure 5 and Figure 6 are using a map from `unsigned int` to pointer.
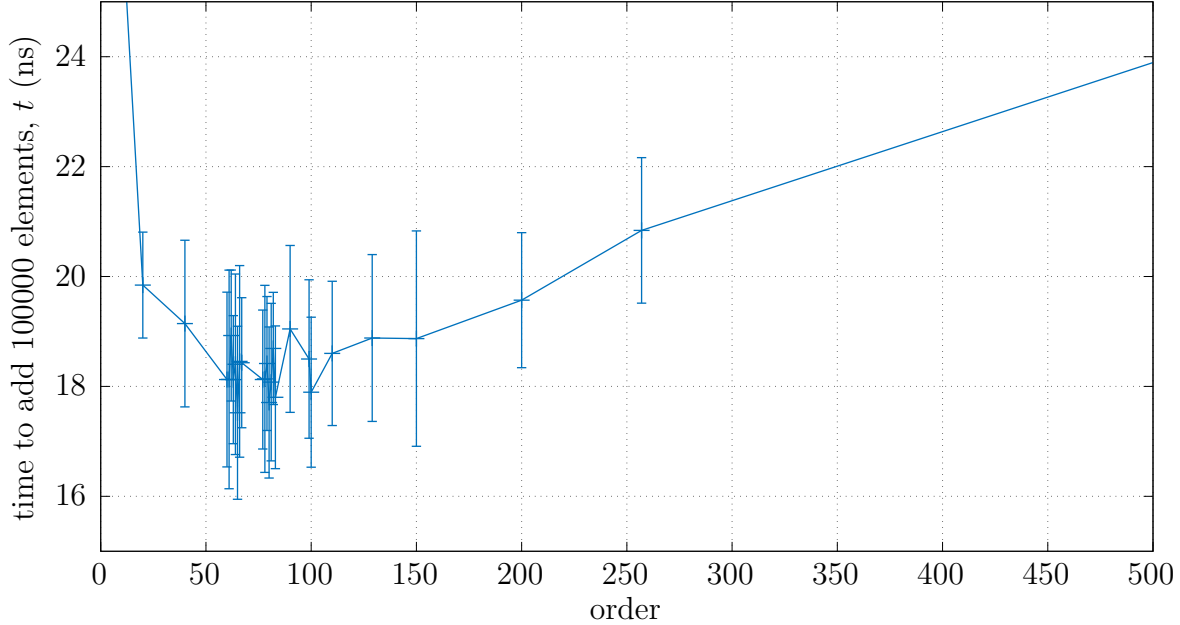
Figure 6: Different orders benchmark for an addition of 100 000 elements. Note that this is zoomed in on the region of interest. Especially, t does not extend to zero; thus, the differences are more apparent.

3.2   Compared with $std :: map$

Figure 5 shows the common case where one has an ordered map of `unsigned int` to pointer. It is of straight insertion of random elements, (not in any order.) The keys are drawn from `rand` in $[0, 2\,147\,483\,647]$. (This causes some probability of collision.)

`std::map` is a red-black tree in `C++`. Though order 4 produces an isomorphism with left-leaning red-black trees[7], it is very close in performance to an order-3 tree. This is probably because of the importance of caching: locality-of-reference in order-3 trees, which have up to 2 elements grouped together is very similar to having single, stable, elements.

Higher orders are generally more cache-friendly, but it gets to the point where all the data is in one node. This node is essentially an ordered array. Order 2049 in Figure 5 is an example; with more data to be transferred every insertion, the lower the performance. The more the size in bytes of the individual keys (and values, if applicable,) the more large the nodes become, and this negative effect is exacerbated.

3.3   Default order

From Figure 5, we see that it resembles a level curve after the variance *per* element settles. This is what we would expect. When choosing a default fixed order, it is therefore appropriate to pick a fixed number of elements. Figure 6 is a cut at 100 000 elements. This suggests that the default order be from 20 to 200.

We expect that this is on the low-end of size in use-cases. We would also expect an entry of twice the size to behave performance-wise under rotations as if it has twice the order. This means that it is best to choose on the low-side. Thus, experimentally and heuristically, a reasonable default is 65. This means the leaves are allocated to hold 64 entries, and the branches an extra 65 pointers for the maximum branching factor.

4 CONCLUSION

A B-tree can be superior to a single node-based approach used in red-black or AVL trees, used by many languages to implement ordered sets or maps. It is not without cost, since this is now not a stable data-structure.

REFERENCES

[1] R. Bayer and E. McCreight, "Organization and maintainence of large ordered indices," *Acta Informatica*, vol. 1, no. 3, p. 1, 1972.

[2] D. E. Knuth, *The Art of Computer Programming, Sorting and Searching*, vol. 3. Addison-Wesley, Reading, Massachusetts, 2 ed., 1998.

[3] M. T. Goodrich, R. Tamassia, and D. M. Mount, *Data structures and algorithms in C++*. John Wiley & Sons, 2011.

[4] T. Johnson and D. Shasha, "B-trees with inserts and deletes: Why free-at-empty is better than merge-at-half," *Journal of Computer and System Sciences*, vol. 47, no. 1, pp. 45–76, 1993.

[5] D. Comer, "Ubiquitous b-tree," *ACM Computing Surveys (CSUR)*, vol. 11, no. 2, pp. 121–137, 1979.

[6] G. Graefe *et al.*, "Modern b-tree techniques," *Foundations and Trends® in Databases*, vol. 3, no. 4, 2011.

[7] R. Sedgewick, "Left-leaning red-black trees," in *Dagstuhl Workshop on Data Structures*, vol. 17, 2008.