The Effect of Image Interpolation on the Generalization of Generative and Discriminative Models

William Watson
The Johns Hopkins University
billwatson@jhu.edu

Abstract

In statistical classification, there are two main approaches to learning: *generative* and *discriminative*. In computer vision, *image interpolation* is a method to rescale images analogous to dimensionality reduction. We explore the generalization ability of *generative-discriminative pairs* when interpolating images to smaller sizes.

1 Introduction

Probabilistic graphical models have many applications in computer science, ranging from image segmentation to named entity recognition. However, these models can become unwieldily when our feature space $\mathbf{x}$ grows. In the case of images, as we increase the number of pixels per row and column, the total number of features grows exponentially. For large images, if we treat the input space as a rasterized image, then the total number of features is $3 \times size^2$.

This leads to issues of tractability for both parameter estimation and inference. This paper concerns itself with using image interpolation as a dimensionality reduction technique to make learning tractable. We vary the dimensionality to see both the generalization ability of our models and the efficiency of parameter estimation. We describe our data in Section 4, our experimental procedure in Section 7, and image interpolation (Section 3).

We experiment with this concept through two approaches: *generative* and *discriminative* learning. Both approaches are outlined in Section 2. Our models are formally described in Section 5 and Section 6. Our results are presented for both the parameter estimation (Section 8) and inference (Section 9) tasks.

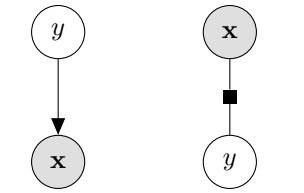
2 Statistical Classification

Classification is the task of assigning a label y to a set of observed features $\mathbf{x}$. Yet the way we approach modeling these relationships can be broken down into either the *generative* or the *discriminative* approach.

The generative approach models the joint distribution $p(y, \mathbf{x})$, and can assign labels through Bayes rule [7].

$$\hat{y} \;=\; \underset{y}{\operatorname{argmax}}\; p(y, \mathbf{x}) \;=\; \underset{y}{\operatorname{argmax}}\; p(y) \cdot p(\mathbf{x}|y) \tag{1}$$

Generative models are attractive in that distributions are learned over the feature space for each individual class. However, an important limitation mentioned by McCallum et. al [6] is that modeling a distribution *per feature* quickly becomes intractable as the dimensionality of our observed variables



$\mathbf{x}$ grows. While simple models can mitigate these issues by assuming independence among the features, allowing complex dependencies between inputs offers the ability to increase performance.

An alternative approach is to model the conditional probabilities directly, ignoring the feature distributions. This is the discriminative approach, and is sometimes referred to as a *distribution-free classifier*. By ignoring the feature distributions, parameters are learned only on the conditional likelihood $p(y|\mathbf{x})$, resulting in a compact model that can handle large feature spaces, as dependencies are ignored [6].

To compare generative and discriminative modeling, one can experiment with pairs of classifiers that can be considered analogous to each other. More formally, a *generative-discriminative pair* is a parametric family of probabilistic models that can either be fit to maximize the joint probability $p(y, \mathbf{x})$ or the conditional likelihood $p(y|\mathbf{x})$ [7]. The simplest pairing is the Naive Bayes classifier (generative) and Logistic Regression (discriminative). This paradigm can be extended to sequential and general models to form more pairs. We discuss our experimental pairs in Sections 5 and 6.

3 Image Interpolation

The most common problem encountered in machine learning is the *curse of dimensionality*. Essentially, as our feature space $\mathbf{x}$ grows in size, we require a larger number of parameters to estimate. This can quickly become intractable in complex models, and reasonably difficult in simple ones. We can see the growth in features as we increase the image size in Figure 2. One method to reduce the number of features for a data set is Principle Component Analysis, which projects data to a smaller dimension while maximizing the variance. However, interpretability is lost, and the structure of the painting is unintelligible.

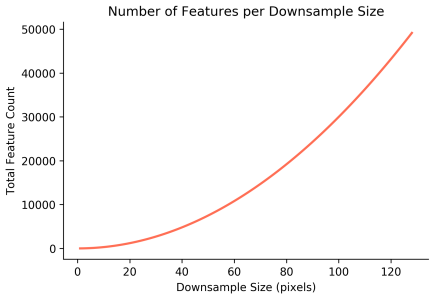


Figure 2: Exponential growth in the number features quickly makes modeling intractable.

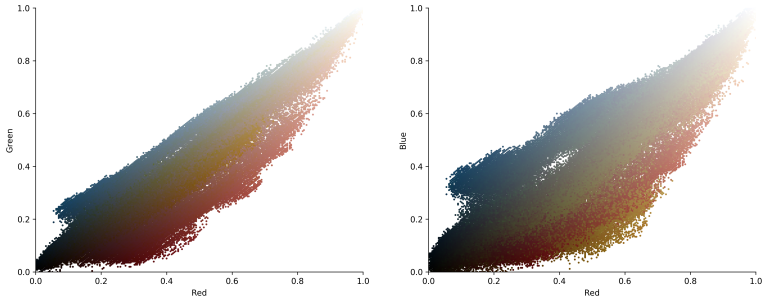
Image interpolation offers a convenient way to downsample images according to their local neighborhood, aggregating otherwise noisy estimates into a single value. This may be beneficial for classification tasks, as we combine a method analogous to dimensionality reduction while preserving the fundamental structure of the original content. We use OpenCV's implementation of interpolation [1].

Formally, image interpolation is the task of rescaling an image of one size to another. This can be used to both decimate and expand the image. Several methods exists for this task, including linear, bicubic, nearest neighbor, and *area relation*.

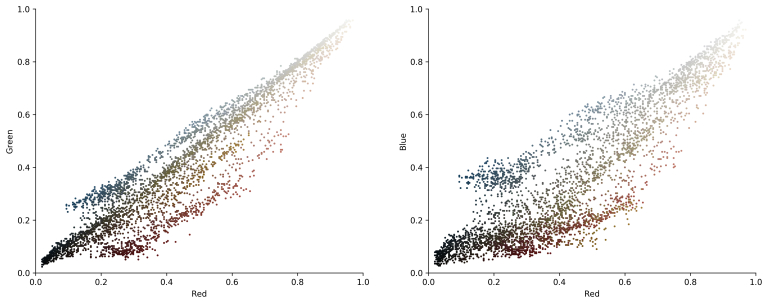
Figure 3: Results of Interpolating a 249,000 pixel image to a 4,096 pixel image.

(a) Original Image (498×500)

(b) Decimated Image (64×64)



(c) Pixel Plot for the Original Image



(d) Pixel Plot for the Decimated Image

The preferred method for image decimation, or downsampling, is to resampling using pixel area relation as it gives moire'-free results. We can see in Figure 3 that the general space of the pixels in relation to their values is replicated, albeit with information loss.

4 Data

Our data is drawn from a subset of the Rijkmuseum Dataset [3]. In order to have enough samples for a challenging classification task, we elected to perform type categorization. We used the top 4 art types: paintings, photos, drawings, and prints. Each category is limited to the size of the smallest set, which is $2,269$ photos. Hence we have a dataset of $9,076$ images split evenly between the 4 classes. For training purposes, we use 10% of the data as our test set ($8,168$ training, 908 test).

From Figure 4, we can see that paintings are vastly different from the other classes, while photos and prints may be difficult to distinguish between. Photos and prints share a similar hue and tone, and drawings do not always have the vibrant colors found in paintings. Hence, our 4-way classification task should be difficult to learn, allow ample room to experiment with complex models.

Figure 4: Class Examples

(a) Sample Painting

(b) Sample Photo

(c) Sample Drawing

(d) Sample Print

5 Baseline Models

5.1 Naive Bayes

Naive Bayes is a simple generative classifier based on applying Bayes' theorem to extract the conditional likelihood $p(y \mid \mathbf{x})$. Its' fundamental assumption for the features is "naive" in that it assumes that every pair of features is conditionally independent given then class label y for K features.

We can formulate our model from the joint distribution:

$$p(y, \mathbf{x}) \;=\; p(y) \cdot \prod_{k=1}^{K} p(x_k \mid y) \tag{2}$$

Our task is to find the class label y that maximizes the posterior conditional likelihood:

$$\begin{aligned} \hat{y} \;&=\; \underset{y}{\operatorname{argmax}}\; p(y|\mathbf{x}) \\ &=\; \underset{y}{\operatorname{argmax}}\; \frac{p(y) \cdot \prod_{k=1}^{K} p(x_k \mid y)}{p(\mathbf{x})} \\ &=\; \underset{y}{\operatorname{argmax}}\; p(y) \cdot \prod_{k=1}^{K} p(x_k \mid y) \end{aligned} \tag{3}$$

We make use of the conditional independence assumption that all features are solely dependent on y alone, and that $p(\mathbf{x})$ is constant.

Here, $p(y)$ is the frequency of class y is the training set. For our purposes we assume the features are normally distributed per class. Hence, our conditional feature likelihood is:

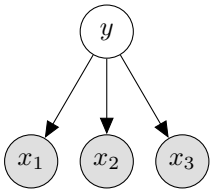
$$p(x_i \mid y) = \frac{1}{\sqrt{2\pi\sigma_y^2}} \exp\left(-\frac{(x_i - \mu_y)^2}{2\sigma_y^2}\right) \tag{4}$$

To learn a Gaussian Naive Bayes model, we use Maximum Likelihood Estimates for the parameters μ_{kc} and σ_{kc} per feature k and class c, and class weights θ_c. Hence the MLE estimates for N samples are:

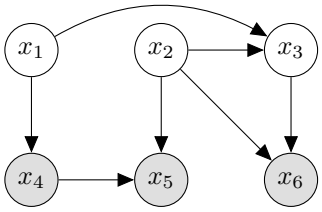
$$\begin{aligned} \hat{\theta}_c \;&=\; \frac{1}{N} \sum_{i=1}^{N} \mathbb{1}\left\{y_i = y_c\right\} = \frac{N_c}{N} \\ \hat{\mu}_{kc} \;&=\; \frac{1}{N_c} \sum_{i=1}^{N} \mathbb{1}\left\{y_i = y_c\right\} \cdot x_{ik} \\ \hat{\sigma}_{kc} \;&=\; \frac{1}{N_c} \sum_{i=1}^{N} \mathbb{1}\left\{y_i = y_c\right\} \cdot (x_{ik} - \mu_{kc})^2 \end{aligned} \tag{5}$$

where $N_c = \sum_{i=1}^{N} \mathbb{1}\left\{y_i = y_c\right\}$, or the number of data points belonging to class c [4].

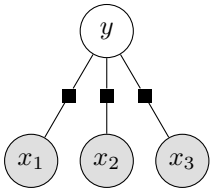
Generative: Naive Bayes



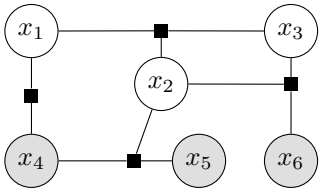
Generative: General Bayesian Networks



Discriminative: Logistic Regression



Discriminative: Conditional Random Fields



5.2 Logistic Regression

Logistic Regression is the discriminative analog to the Naive Bayes classifier, sometimes referred to as the *maxmimum-entropy classifer* or a normalized *log-linear model*. As a discriminative model, we can optimize directly for the conditional likelihood $p(y \mid \mathbf{x})$, with $Z(\mathbf{x})$ as the normalizing partition function.

$$p(y \mid \mathbf{x}) = \frac{1}{Z(\mathbf{x})} \exp\left\{ \sum_{j=1}^{K} \theta_k f_k\left(y, \mathbf{x}\right) \right\} \tag{6}$$

McCallum et. al formulates the logistic regression model, parameterized by weights $\theta \in \mathbb{R}^K$, in which a single set of weights is shared across all the classes [6]. This is achieved through a set of feature functions that are nonzero for a single class and is defined as follows:

$$\begin{aligned} f_{y',j}\left(y, \mathbf{x}\right) &= \mathbb{1}\{y' = y\} \cdot x_j \\ f_{y'}\left(y, \mathbf{x}\right) &= \mathbb{1}\{y' = y\} \end{aligned} \tag{7}$$

This notation, while not standard, mirrors our later formulation of Conditional Random Fields in Section 6.2.

For parameter estimation, we opted to use scikit-learn's implementation of the L-BFGS algorithm, which approximates the Broyden-Fletcher-Goldfarb-Shanno algorithm [5]. This is a quasi-Newton optimization method that approximates the Hessian matrix to reduce memory usage. Scikit-learn optimizes the L2 penalized multinomial loss [5] [4].

$$\min_{\theta} \; \frac{1}{2} \cdot \theta^T \theta - \sum_{i=1}^{N} \left[\sum_{c=1}^{C} \left[\mathbb{1}\{y_i = y_c\} \cdot \sum_{k=1}^{K} \theta_k f_k\left(y_i, \mathbf{x}_i\right) \right] - \log Z(\mathbf{x}_i) \right] \tag{8}$$

where y_i is the true label and y_c could be any label.

6 General Models

6.1 General Discrete Bayesian Networks (GBNs)

General Discrete Bayesian Networks (GBNs) are directed graphical models that are comprised of a set of nodes and edges. The nodes are our random variables, and they interrelate through directed edges. This allows us to encode complex relations between our features $\mathbf{x}$ that are not captured by the Naive Bayes classifier. We can exploit these relationships to decompose the joint distribution of the network as follows:

$$p(\mathbf{x}) \;=\; \prod_{k=1}^{K} p(x_k \mid pa(x_k)) \;=\; \prod_{k=1}^{K} \theta_{x_k | \mathbf{x}_{pa(k)}} \tag{9}$$

Hence the joint distribution factorizes into a distribution for a node conditioned on the parents. Given N observations $\mathbf{x}_1, \ldots, \mathbf{x}_n$, we can express the likelihood function as:

$$\mathcal{L} \;=\; \prod_{i=1}^{N} \prod_{k=1}^{K} \theta_{x_k^i | \mathbf{x}_{pa(k)}^i} \tag{10}$$

GBNs have a closed form solution to the maximum likelihood estimates, which can be expressed as the counts per node states over the total number of samples with equivalent parent states. However, caution must be exercised in the construction of the network, where as the number of parents increase we encounter *data fragmentation*. Limiting the number of parents can mitigate this issue by introducing *sparsity*.

$$\hat{\theta}_{x_k | \mathbf{x}_{pa(k)}} = \frac{\sum_{i=1}^{N} \mathbb{1}\{x_k^i = x_k\} \cdot \mathbb{1}\{\mathbf{x}_{pa(k)}^i = \mathbf{x}_{pa(k)}\}}{\sum_{i=1}^{N} \mathbb{1}\{\mathbf{x}_{pa(k)}^i = \mathbf{x}_{pa(k)}\}} \tag{11}$$

6.2 Conditional Random Fields (CRFs)

Conditional Random Fields (CRFs) are discriminative graphical models that can model output variables $\mathbf{y}$ conditioned on features $\mathbf{x}$ compactly, in contrast to generative models. CRFs are natural extensions to logistic regression for general graphs. We describe the model formulation, parameter estimation, and inference tasks as outlined by McCallum et. al [6], and introduced by Lafferty et. al [2].

CRFs model a conditional likelihood $p(\mathbf{y} \mid \mathbf{x})$ on a general graph $\mathcal{G}$, composed of a set of factors $F = \{\psi_a\}_{a=1}^{A}$. We reiterate McCallum's description of a CRF with parameter tying, where a set of factors can share a set of feature functions $\{f_{pk}(\mathbf{x}_c, \mathbf{y}_c)\}_{k=1}^{K(p)}$ and parameters $\theta_p \in \mathbb{R}^{K(p)}$ for a given *clique template* C_p. These clique templates come from partitioning the factors of graph $\mathcal{G}$ into $\mathcal{C} = \{C_1, C_2, \ldots, C_p\}$. Hence, our model is composed of the following:

$$\begin{aligned} p(\mathbf{y} \mid \mathbf{x}) \;&=\; \frac{1}{Z(\mathbf{x})} \prod_{C_p \in \mathcal{C}} \prod_{\psi_c \in C_p} \psi_c(\mathbf{x}_c, \mathbf{y}_c; \theta_p) \\ \psi_c(\mathbf{x}_c, \mathbf{y}_c; \theta_p) \;&=\; \exp\left\{ \sum_{k=1}^{K(p)} \theta_{pk} f_{pk}(\mathbf{x}_c, \mathbf{y}_c) \right\} \\ Z(\mathbf{x}) \;&=\; \sum_{\mathbf{y}} \prod_{C_p \in \mathcal{C}} \prod_{\psi_c \in C_p} \psi_c(\mathbf{x}_c, \mathbf{y}_c; \theta_p) \end{aligned} \tag{12}$$

Where p is a factor index, f_{pk} is a feature function, and θ_{pk} is the weight associated with it. K is the number of features and $K(p)$ are the features found in factor p. Because our labels are discrete, we can write our feature functions f_pk in terms of observation functions q_{pk} to make *label-observation features*.

$$f_{pk}(\mathbf{y}_c, \mathbf{x}_c) = \mathbb{1}\{\mathbf{y}_c = \widetilde{\mathbf{y}}_c\} \cdot q_{pk}(\mathbf{x}_c) \tag{13}$$

Parameter estimation for CRFs can be done through maximum likelihood estimation, where we can minimize the conditional negative log likelihood:

$$\ell(\theta) \;=\; -\sum_{C_p \in \mathcal{C}} \sum_{\psi_c \in C_p} \sum_{k=1}^{K(p)} \theta_{pk} f_{pk}(\mathbf{x}_c, \mathbf{y}_c) + \log Z(\mathbf{x}) \tag{14}$$

Hence, the partial derivative with respect to our paramter θ_{pk} associated with the clique template C_p is:

$$\frac{\partial \ell}{\partial \theta_{pk}} \;=\; -\sum_{\psi_c \in C_p} f_{pk}(\mathbf{x}_c, \mathbf{y}_c) + \sum_{\psi_c \in C_p} \sum_{\mathbf{y'}_c} f_{pk}(\mathbf{x}_c, \mathbf{y'}_c) \cdot p(\mathbf{y'}_c \mid \mathbf{x}) \tag{15}$$

We can then minimize our cost function $\ell(\theta)$ using L-BFGS optimization algorithm as described in Section 5.2.

7 Experimental Task

For a single downsample size, we performed a 3-fold cross validation on the full $9,076$ image set, using 908 images randomly as test. This gives us three points of accuracy, in which we can average and extract confidence intervals around. Variables measured for the cross-validation include: training accuracy, testing accuracy, fit time, and score time. These measures provide two fronts of analysis: generalization and tractability.

To effectively study these objective measures, we conduct this experiment per model per downsample size. Our choices of sizes included: 1, 2, 4, 8, 16, 32, 64, and 128. We did not test higher pixel sizes as 256×256 pixel images quickly became impossible due to memory constraints.

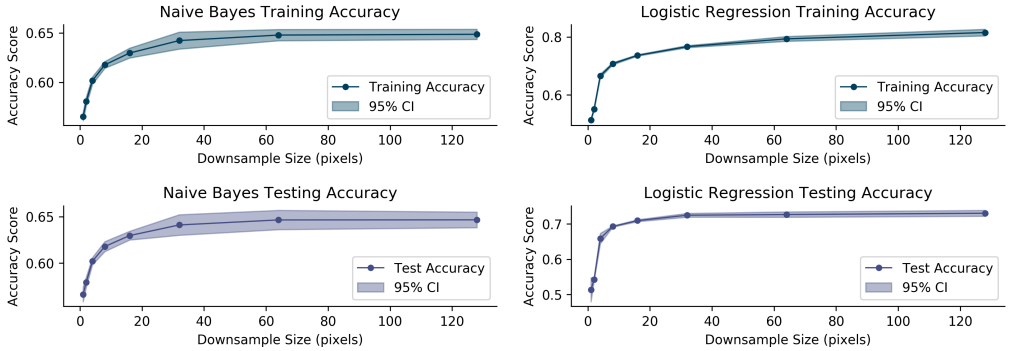


Figure 6: Training/Testing Accuracy Results for Baseline Models.

8 Results: Parameter Estimation

8.1 Baseline

8.2 General

9 Results: Inference

9.1 Baseline

9.2 General

10 Conclusion and Future Work

References

[1] G. Bradski. The OpenCV Library. *Dr. Dobb's Journal of Software Tools*, 2000.

[2] J. Lafferty. Conditional random fields: Probabilistic models for segmenting and labeling sequence data. pages 282–289. Morgan Kaufmann, 2001.

[3] T. E. J. Mensink and J. C. van Gemert. The rijksmuseum challenge: Museum-centered visual recognition. In *ACM International Conference on Multimedia Retrieval*, 2014.

[4] K. P. Murphy. *Machine Learning: A Probabilistic Perspective*. The MIT Press, 2012.

[5] F. Pedregosa, G. Varoquaux, A. Gramfort, V. Michel, B. Thirion, O. Grisel, M. Blondel, P. Prettenhofer, R. Weiss, V. Dubourg, J. Vanderplas, A. Passos, D. Cournapeau, M. Brucher, M. Perrot, and E. Duchesnay. Scikit-learn: Machine learning in Python. *Journal of Machine Learning Research*, 12:2825–2830, 2011.

[6] C. Sutton and A. McCallum. An Introduction to Conditional Random Fields. *arXiv e-prints*, page arXiv:1011.4088, Nov 2010.

[7] A. Y. Ng and M. Jordan. On discriminative vs. generative classifiers: A comparison of logistic regression and naive bayes. *Adv. Neural Inf. Process. Sys*, 2, 04 2002.