

Seminar 6

ML estimation

DEDP

1. We receive constant signal with unknown amplitude A , $r(t) = \underbrace{A}_{s_{\Theta}(t)} + noise$, where the noise is gaussian with $\mathcal{N}(\mu = 0, \sigma^2 = 2)$. The signal is sampled at moments $t_i = [0, 1.5, 3, 4]$ and the samples are $r_i = [4.6, 5.2, 5.35, 4.8]$.
 - a. Estimate A using Maximum Likelihood (ML) estimation
 - b. Repeat a) if the noise is uniform $U[-2, 2]$. Is it possible to find a precise value?
2. A received signal $r(t) = a \cdot t^2 + noise$ is sampled at time moments $t_i = [1, 2, 3, 4, 5]$, and the values are $r_i = [1.2, 3.7, 8.5, 18, 25.8]$. The noise distribution is $\mathcal{N}(0, \sigma^2 = 1)$. Estimate the parameter a .
 - a. use Maximum Likelihood (ML) estimation
3. Fit a linear function $y = ax$ (i.e. estimate a) through the following data points $(x_i, y_i) = (1, 1.8), (2, 4.1), (2.5, 5.1), (4, 7.9), (4.3, 8.5)$, assuming the noise is $\mathcal{N}(0, \sigma^2 = 1)$
 - a. use Maximum Likelihood (ML) estimation
4. A robot travels a linear road with a constant but unknown speed v cm/s, starting from position x_0 at time 0.

Every second the robot measures its position using an imprecise sensor, which provides values affected by Gaussian noise $\mathcal{N}(0, \sigma^2 = 0.1)$.

The measured values at time moments $t_i = [1, 2, 3, 4, 5]$ are $r_i = [4.9, 9.8, 14.3, 21.2, 25.7]$.

 - a. Estimate the speed v using ML estimation.

Hint: If the speed is constant, the travelled distance is $x = v \cdot t$.
 - b. Predict the robot position at time 6.

- c. Assuming the starting position x_0 is unknown, $x_0 \neq 0$, estimate the pair of parameters $[v, x_0]$ using ML estimation. Predict the robot's position at time 6.
 - d. Assuming the movement law is $x(t) = a \cdot t^2 + v_0 \cdot t + x_0$, write the equation system for finding the unknown parameters $[a, v_0, x_0]$. (constant acceleration a , initial speed v_0 , initial position x_0).
5. A robot travels a linear road with a constant speed $V = 10$ cm/s.

The robot measures its position every second with a sensor affected by gaussian noise $\mathcal{N}(0, \sigma^2 = 0.5)$.

At time $t_0 = 0$, the robot's position is around $x_0 = 20$, being a random variable with the distribution:

$$w(x_0) = \mathcal{N}(\mu = 20, \sigma = 0.5)$$

- a. Find the distribution $w(x_1)$ of the robot's position at time $t_1 = 1$, and predict the actual value with the MAP and MMSE estimators.
- b. Suppose at this time $t_1 = 1$ we have a new measurement of the position, with value $r = 29.5$.

Take this into account with Bayesian estimation and find the new position using MAP and MMSE estimators, considering the predicted distribution in a) as the prior distribution.

- c. What happens if the speed is not known precisely. Suppose V is a random variable $\mathcal{N}(\mu = 10 \text{ cm/s}, \sigma^2 = 0.3)$?