

DDP Seminar 5

$$f_1 = 1$$

$$(1) H_1: \Delta_1(t) = 3 \sin(2\pi \cdot t)$$

$$H_0: \Delta_0(t) = 0$$

$$r = [1.1 \quad 4.4]$$

$$t = [0.125 \quad 0.625]$$

$$D_{NL} = ?$$

$$s_0 = [0 \quad 0]$$

$$s_1 = [2.12 \quad -2.12]$$

$$r = [1.1 \quad 4.4]$$

$$\Delta_1(t_1) = 3 \cdot \sin(2\pi \cdot \overset{1/8}{0.125}) = 3 \cdot \sin(\pi/4) = 3 \cdot \frac{\sqrt{2}}{2} = 2.12$$

$$\Delta_1(t_2) = 3 \cdot \sin(2\pi \cdot \overset{5/8}{0.625}) = 3 \cdot \sin(5\pi/4) = -3 \cdot \frac{\sqrt{2}}{2} = -2.12$$

$$d(r, \Delta_0)^2 = 1.1^2 + 4.4^2 = 20.57$$

$$d(r, \Delta_1)^2 = (1.1 - 2.12)^2 + (4.4 + 2.12)^2 = 43.508$$

$$\Rightarrow D_0 \text{ (smallest wins)}$$

$$d(r, \Delta_0)^2 \geq d(r, \Delta_1)^2 + 2\sigma^2 \underbrace{p_m(K)}_{\substack{K=1 \\ \text{ML:} \\ K=1}}$$

2

a) $S_0 = \begin{bmatrix} 2 & 2 & -2 \end{bmatrix}$

$S_1 = \begin{bmatrix} -2 & -2 & 2 \end{bmatrix}$

$R = \begin{bmatrix} -1 & -1 & 1 \end{bmatrix}$

$d(R, S_0)^2 = 9 + 9 + 9 = 27$

$d(R, S_1)^2 = 1 + 1 + 1 = 3$

$\Rightarrow D_1$

b)

a

b

$d(a, b) = \sqrt{\sum_i (a_i - b_i)^2}$

Geometric distance:
for vectors

$a(t)$

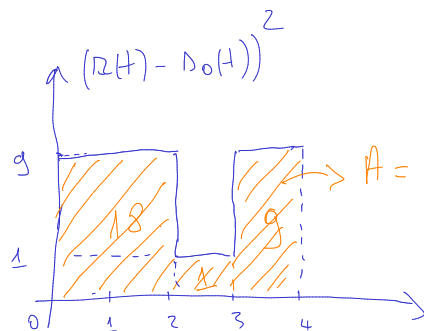
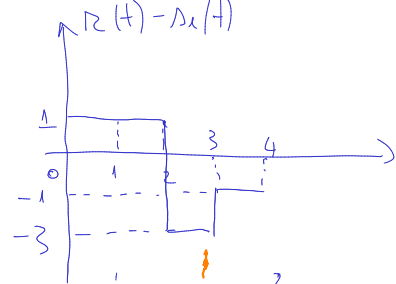
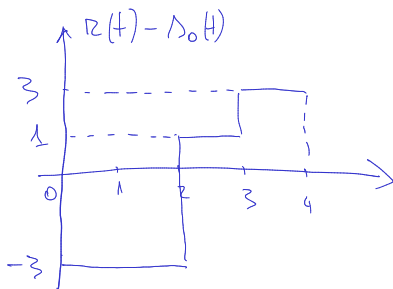
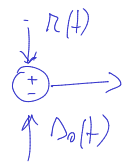
$b(t)$

$d(a, b) = \sqrt{\int (a(t) - b(t))^2 dt}$

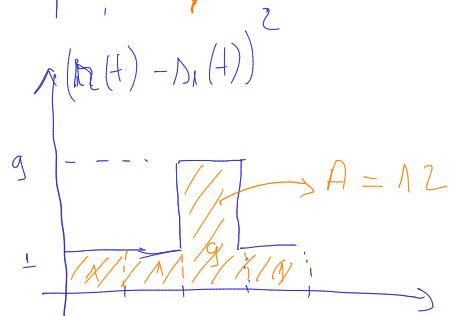
for cont. signals

$d(R, S_0)^2 = \int (R(t) - S_0(t))^2 dt = 28$

$d(R, S_1)^2 = \int (R(t) - S_1(t))^2 dt = 12$



$A = 28 = \int_0^4 (R(t) - S_0(t))^2 dt$



$\left. \begin{aligned} d(R, S_0)^2 &= 28 \\ d(R, S_1)^2 &= 12 \end{aligned} \right\} \Rightarrow D_1$

③

$$d(x, v_1) =$$

$$d(x, v_2) =$$

⋮

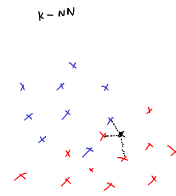
$$d(x, v_{10}) =$$

• Class A:

$$\vec{v}_1 = \begin{bmatrix} 2 \\ -4 \end{bmatrix} \quad \vec{v}_2 = \begin{bmatrix} 1 \\ -5 \end{bmatrix} \quad \vec{v}_3 = \begin{bmatrix} -2 \\ 6 \end{bmatrix} \quad \vec{v}_4 = \begin{bmatrix} -3 \\ 4 \end{bmatrix} \quad \vec{v}_5 = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$$

• Class B:

$$\vec{v}_6 = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \quad \vec{v}_7 = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \quad \vec{v}_8 = \begin{bmatrix} -4 \\ -3 \end{bmatrix} \quad \vec{v}_9 = \begin{bmatrix} -3 \\ 0 \end{bmatrix} \quad \vec{v}_{10} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$



$$x = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$$

$$d(x, v_1) = \sqrt{16 + 81} = \sqrt{97}$$

$$d(x, v_2) = \sqrt{9 + 100} = \sqrt{109}$$

$$d(x, v_3) = \sqrt{1}$$

$$d(x, v_4) = \sqrt{2}$$

$$d(x, v_5) = \sqrt{116}$$

$$d(x, v_6) = \sqrt{41}$$

$$d(x, v_7) = \sqrt{17}$$

$$d(x, v_8) = \sqrt{68}$$

$$d(x, v_9) = \sqrt{26}$$

$$d(x, v_{10}) = \sqrt{4}$$

increasing distance
→

v_3	v_4	v_{10}	v_7	v_9	v_6	v_8	v_1	v_2	v_5
A	A	B	B	B	B	B	A	A	A

$$k = 1$$

⇒ A

$$k = 3$$

⇒ A

$$k = 5$$

⇒ B

$$k = 7$$

⇒ B

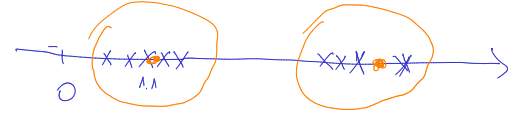
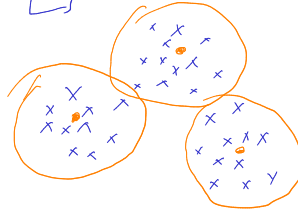
$$k = 9$$

⇒ B

4

$$\vec{v} = \{v_i\} = [1.1, 0.9, 5.5, 0.6, 5, 6, 1.3, 4.8, 6, 0.8]$$

K-Means
clusterization



$$C_1 = 0.95$$

$$C_2 = 0.96$$

1) a) $C_1 : 0.9 \quad 0.6 \quad 0.8$

$C_2 : 1.1 \quad 5.5 \quad 5 \quad 6 \quad 1.3 \quad 4.8 \quad 6$

b) $C_1 = \frac{0.9 + 0.6 + 0.8}{3} = 0.76$ (average)

$C_2 = 4.24$

2) a) $C_1 : 1.1 \quad 0.9 \quad 0.6 \quad 1.3 \quad 0.8$

$C_2 : 5.5 \quad 5 \quad 6 \quad 4.8 \quad 6$

b) $C_1 = 0.94$

$C_2 = 5.46$

$$\vec{v} = \{v_i\} = [1.1, 0.9, 5.5, 0.6, 5, 6, 1.3, 4.8, 6, 0.8]$$

3) a) $C_1 : 1.1 \quad 0.9 \quad 0.6 \quad 1.3 \quad 0.8$

$C_2 : 5.5 \quad 5 \quad 6 \quad 4.8 \quad 6$

b) $C_1 = 0.94$

$C_2 = 5.46$

Algorithm converged