

# Seminar 6

## ML estimation

### DEDP

1. We receive constant signal with unknown amplitude  $A$ ,  $r(t) = \underbrace{A}_{s_{\Theta}(t)} + noise$ , where the noise is gaussian with  $\mathcal{N}(\mu = 0, \sigma^2 = 2)$ . The signal is sampled at moments  $t_i = [0, 1.5, 3, 4]$  and the samples are  $r_i = [4.6, 5.2, 5.35, 4.8]$ .
  - a. Estimate  $A$  using Maximum Likelihood (ML) estimation
  - b. Repeat a) if the noise is uniform  $U[-2, 2]$ . Is it possible to find a precise value?
2. A received signal  $r(t) = a \cdot t^2 + noise$  is sampled at time moments  $t_i = [1, 2, 3, 4, 5]$ , and the values are  $r_i = [1.2, 3.7, 8.5, 18, 25.8]$ . The noise distribution is  $\mathcal{N}(0, \sigma^2 = 1)$ . Estimate the parameter  $a$ .
  - a. use Maximum Likelihood (ML) estimation
3. Fit a linear function  $y = ax$  (i.e. estimate  $a$ ) through the following data points  $(x_i, y_i) = (1, 1.8), (2, 4.1), (2.5, 5.1), (4, 7.9), (4.3, 8.5)$ , assuming the noise is  $\mathcal{N}(0, \sigma^2 = 1)$ 
  - a. use Maximum Likelihood (ML) estimation
4. A robot travels a linear road with a constant but unknown speed  $v$  cm/s, starting from position  $x_0 = 0$  at time 0.

Every second the robot measures its position using an imprecise sensor, which provides values affected by Gaussian noise  $\mathcal{N}(0, \sigma^2 = 0.1)$ .

The measured values at time moments  $t_i = [1, 2, 3, 4, 5]$  are  $r_i = [4.9, 9.8, 14.3, 21.2, 25.7]$ .

  - a. Estimate the speed  $v$  using ML estimation.

*Hint:* If the speed is constant, the travelled distance is  $x = v \cdot t$ .
  - b. Predict the robot position at time 6.

- c. Assuming the starting position  $x_0$  is unknown,  $x_0 \neq 0$ , estimate the pair of parameters  $[v, x_0]$  using ML estimation. Predict the robot's position at time 6.
  - d. Assuming the movement law is  $x(t) = a \cdot t^2 + v_0 \cdot t + x_0$ , write the equation system for finding the unknown parameters  $[a, v_0, x_0]$ . (constant acceleration  $a$ , initial speed  $v_0$ , initial position  $x_0$ ).
5. A robot travels a linear road with a constant speed  $V = 10$  cm/s.

The robot measures its position every second with a sensor affected by gaussian noise  $\mathcal{N}(0, \sigma^2 = 0.5)$ .

At time  $t_0 = 0$ , the robot's position is around  $x_0 = 20$ , being a random variable with the distribution:

$$w(x_0) = \mathcal{N}(\mu = 20, \sigma = 0.5)$$

- a. Find the distribution  $w(x_1)$  of the robot's position at time  $t_1 = 1$ , and predict the actual value with the MAP and MMSE estimators.
- b. Suppose at this time  $t_1 = 1$  we have a new measurement of the position, with value  $r = 29.5$ .

Take this into account with Bayesian estimation and find the new position using MAP and MMSE estimators, considering the predicted distribution in a) as the prior distribution.

- c. What happens if the speed is not known precisely. Suppose  $V$  is a random variable  $\mathcal{N}(\mu = 10 \text{ cm/s}, \sigma^2 = 0.3)$ ?