Seminar 6
ML estimation
DEDP

1. We receive constant signal with unknown amplitude A, $r(t) = \underbrace{A}_{s_\Theta(t)} + noise$, where the noise is gaussian with $\mathcal{N}(\mu = 0, \sigma^2 = 2)$. The signal is sampled at moments $t_i = [0, 1.5, 3, 4]$ and the samples are $r_i = [4.6, 5.2, 5.35, 4.8]$.

   a. Estimate A using Maximum Likelihood (ML) estimation
   b. Repeat a) if the noise is uniform $U[-2, 2]$. Is it possible to find a precise value?

2. A received signal $r(t) = a \cdot t^2 + noise$ is sampled at time moments $t_i = [1, 2, 3, 4, 5]$, and the values are $r_i = [1.2, 3.7, 8.5, 18, 25.8]$. The noise distribution is $\mathcal{N}(0, \sigma^2 = 1)$. Estimate the parameter a.

   a. use Maximum Likelihood (ML) estimation

3. Fit a linear function $y = ax$ (i.e. estimate a) through the following data points $(x_i, y_i) = (1, 1.8), (2, 4.1), (2.5, 5.1), (4, 7.9), (4.3, 8.5)$, assuming the noise is $\mathcal{N}(0, \sigma^2 = 1)$

   a. use Maximum Likelihood (ML) estimation

4. A robot travels a linear road with a constant but unknown speed v cm/s, starting from position x_0 at time 0.

   Every second the robot measures its position using an imprecise sensor, which provides values affected by Gaussian noise $\mathcal{N}(0, \sigma^2 = 0.1)$.

   The measured values at time moments $t_i = [1, 2, 3, 4, 5]$ are $r_i = [4.9, 9.8, 14.3, 21.2, 25.7]$.

   a. Estimate the speed v using ML estimation.

      Hint: If the speed is constant, the travelled distance is $x = v \cdot t$.

   b. Predict the robot position at time 6.

c. Assuming the starting position x_0 is unknown, $x_0 \neq 0$, estimate the pair of parameters [v, x_0] using ML estimation. Predict the robot's position at time 6.

d. Assuming the movement law is $x(t) = a \cdot t^2 + v_0 \cdot t + x_0$, write the equation system for finding the unknown parameters [a, v_0, x_0]. (constant acceleration a, initial speed v_0, initial position x_0).

5. Repeat point a) for the previous exercise, assuming we have some prior knowledge if the speed, as a prior distribution $w(v) = \mathcal{N}(\mu = 4.5, \sigma^2 = 1)$.