DEDP Seminar 5

28. 11. 2023

Ex 2 from Seminar 4
Left-over last week:

2). $s_0(t) = 0$

$s_1(t) = 6$

$r = [1.1, 4.4, 3.7, 4.1, 3.8]$

Gaussian noise:

$$d(r, s_0)^2 \underset{H_0}{\overset{H_1}{\gtrless}} d(r, s_1)^2 + 2\sigma^2 \ln(K)$$

$$K = \begin{cases} 1, & M.L. \\ \frac{P(H_0)}{P(H_1)}, & MPE = 2 \\ \frac{(C_{10} - C_{00}) \cdot P(H_0)}{(C_{01} - C_{11}) \cdot P(H_1)}, & M.R. = \frac{10}{15} \cdot 2 \end{cases}$$

$$\langle r, s_0 \rangle - \frac{1}{2} E_0 \gtrless \langle r, s_1 \rangle - \frac{1}{2} E_1 + \sigma^2 \ln(K)$$

$s_0 = [0 \quad 0 \quad 0 \quad 0 \quad 0]$

$s_1 = [6 \quad 6 \quad 6 \quad 6 \quad 6]$

$r = [1.1, 4.4, 3.7, 4.1, 3.8]$

$$d(a, b) = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + \ldots + (a_N - b_N)^2}$$

$$d(r, s_0)^2 = (1.1-0)^2 + (4.4-0)^2 + (3.7-0)^2 + (4.1-0)^2 + (3.8-0)^2 = 65.51$$

$$d(r, s_1)^2 = (1.1-6)^2 + (4.4-6)^2 + (3.7-6)^2 + (4.1-6)^2 + (3.8-6)^2 = 40.31$$

M.L.: $65.51 \underset{H_0}{\overset{H_1}{\gtrless}} 40.31 \Rightarrow D_1$

MPE: $65.51 \underset{H_0}{\overset{H_1}{\gtrless}} 40.31 + \underbrace{2 \cdot 1 \cdot \ln(2)}_{\text{small} = 1.3} \Rightarrow D_1$

M.R.: $65.51 \underset{H_0}{\overset{H_1}{\gtrless}} 40.31 + \underbrace{2 \cdot 1 \cdot \ln\left(\frac{10}{15} \cdot 2\right)}_{\text{small} = 0.5} \Rightarrow D_1$

d). $P(H_0)$? such that M.P.E. $\Rightarrow D_0$

$$65.51 \underset{H_0}{\overset{H_1}{\gtrless}} 40.31 + 2 \cdot 1 \cdot \ln(K) \Rightarrow K$$

For D_0: $40.31 + 2 \ln(K) > 65.51 \Leftrightarrow$

$$\Leftrightarrow \ln(K) > \frac{25.20}{2} = 12.6$$

$$\Leftrightarrow \ln(K) > 12.6 \quad \Big| \; e^x$$

$$\Leftrightarrow K > e^{12.6}$$

MPE: $K = \frac{P(H_0)}{P(H_1)} = \frac{P(H_0)}{1 - P(H_0)} > e^{12.6} \Leftrightarrow \quad \Big| \; (1 - P(H_0))$

$$P(H_0) > e^{12.6} - P(H_0) \cdot e^{12.6}$$

$$P(H_0)(1 + e^{12.6}) > e^{12.6}$$

$$P(H_0) > \frac{e^{12.6}}{1 + e^{12.6}} = 0.9999966$$

(1) (from Seminar 5)

$s_1(t) = 3 \cdot \sin(2\pi f_1 t)$

$s_0(t) = 0$

$\mathcal{N}(\mu = 0, \sigma^2 = 1)$ ✓

$r = \{1.1,\ 4.4\}$

$t_1 = \frac{0.125}{f_1}$

$t_2 = \frac{0.625}{f_1}$

Decision (ML) = ?

$d(r, s_0)^2 \gtrless d(r, s_1)^2 + 2\sigma^2 \ln(K)$

$K = 1$ (ML)

$r = [1.1 \quad 4.4]$

$s_0 = [0 \quad 0]$

$s_1 = [2.12 \quad -2.12]$

$d(r, s_0)^2 < d(r, s_1)^2 \Rightarrow s_0$

3/Seminar 5:

a) $r = [-1 \quad -1 \quad 1]$

$s_0 = [2 \quad 2 \quad -2]$

$s_1 = [-2 \quad -2 \quad 2]$

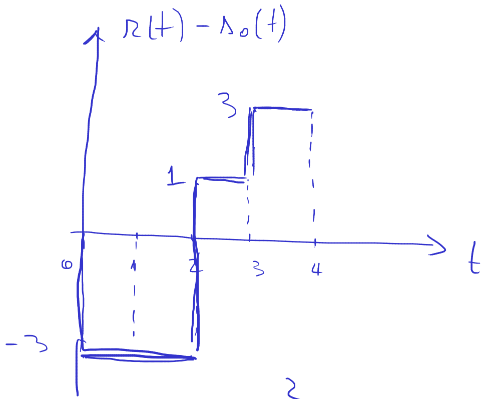
$d(r, s_0)^2 = 3^2 + 3^2 + 3^2 = 27$

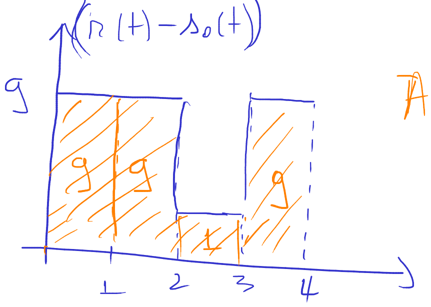
$d(r, s_1)^2 = 1^2 + 1^2 + 1^2 = 3$

$d(r, s_1) < d(r, s_0) \Rightarrow s_1$

b) Use cont. signals:

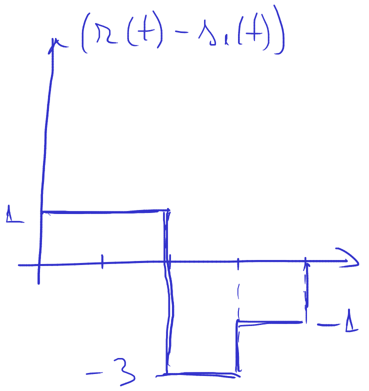
$d(r, s_0)^2 = \int (r(t) - s_0(t))^2 dt = 28$

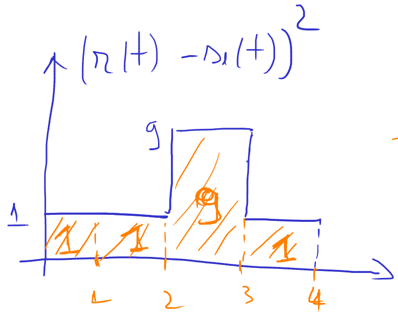




Area $= \int (r(t) - s_0(t))^2 dt = 28$

$$d(r, s_1) = \int (r(t) - s_1(t))^2 \, dt$$





$$\text{Area} = \int (r(t) - s_1(t))^2 \, dt = 12$$

$$\underbrace{d(r, s_0)^2}_{28} \underset{H_0}{\overset{H_1}{\gtrless}} \underbrace{d(r, s_1)}_{12} + \underbrace{2\sigma^2 \ln(k)}_{0 \ (M.L)}$$

$$\Rightarrow \underline{\underline{s_1}}$$

4 K-NN

4. Consider the k-NN algorithm with the following training set, composed of 5 vectors of class A and another 5 vectors from class B:

- Class A:

$$\vec{v}_1 = \begin{bmatrix} 2 \\ -4 \end{bmatrix} \; \vec{v}_2 = \begin{bmatrix} 1 \\ -5 \end{bmatrix} \; \vec{v}_3 = \begin{bmatrix} -2 \\ 6 \end{bmatrix} \; \vec{v}_4 = \begin{bmatrix} -3 \\ 4 \end{bmatrix} \; \vec{v}_5 = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$$

- Class B:

$$\vec{v}_6 = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \; \vec{v}_7 = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \; \vec{v}_8 = \begin{bmatrix} -4 \\ -3 \end{bmatrix} \; \vec{v}_9 = \begin{bmatrix} -3 \\ 0 \end{bmatrix} \; \vec{v}_{10} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

Compute the class of the vector $\vec{x} = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$ using the k-NN algorithm, with $k = 1$, $k = 3$, $k = 5$, $k = 7$ and $k = 9$

$d(x, v_1) = \sqrt{4^2 + 9^2} = \sqrt{97}$

$d(x, v_2) = \sqrt{9 + 100} = \sqrt{109}$

$d(x, v_3) = \sqrt{1}$

$d(x, v_4) = \sqrt{2}$

$d(x, v_5) = \sqrt{116}$

$d(x, v_6) = \sqrt{41}$

$d(x, v_7) = \sqrt{17}$

$d(x, v_8) = \sqrt{68}$

$d(x, v_9) = \sqrt{26}$

$d(x, v_{10}) = \sqrt{4}$

SORT:

V_3	V_4	V_{10}	V_7	V_9	V_6	V_8	V_1	V_2	V_5
A	A	B	B	B	B	B	A	A	A

$k=1$ ⇒ A

$k=3$ ⇒ A

$k=5$ ⇒ B

$k=7$ ⇒ B

$k=9$ ⇒ B

✓