# DEDP Seminar 5

28.11.2023

Ex 2 from Seminar 4
Left-over last week:

2). $s_0(t) = 0$
$s_1(t) = 6$
$r = [1.1, 4.4, 3.7, 4.1, 3.8]$

Gaussian noise:

$$d(r, s_0)^2 \underset{H_0}{\overset{H_1}{\gtrless}} d(r, s_1)^2 + 2\sigma^2 \ln(K)$$

$$K = \begin{cases} 1, & \text{M.L.} \\ \frac{P(H_0)}{P(H_1)}, & \text{MPE} = 2 \\ \frac{(C_{10} - C_{00}) \cdot P(H_0)}{(C_{01} - C_{11}) \cdot P(H_1)}, & \text{MR} = \frac{10}{15} \cdot 2 \end{cases}$$

$$\langle r, s_0 \rangle - \tfrac{1}{2} E_0 \gtrless \langle r, s_1 \rangle - \tfrac{1}{2} E_1 + \sigma^2 \ln(K)$$

$s_0 = [0 \;\; 0 \;\; 0 \;\; 0 \;\; 0]$

$s_1 = [6 \;\; 6 \;\; 6 \;\; 6 \;\; 6]$

$r = [1.1, 4.4, 3.7, 4.1, 3.8]$

$$d(a,b) = \sqrt{(a_1-b_1)^2 + (a_2-b_2)^2 + \dots + (a_N-b_N)^2}$$

$$d(r, s_0)^2 = (1.1-0)^2 + (4.4-0)^2 + (3.7-0)^2 + (4.1-0)^2 + (3.8-0)^2 = 65.51$$

$$d(r, s_1)^2 = (1.1-6)^2 + (4.4-6)^2 + (3.7-6)^2 + (4.1-6)^2 + (3.8-6)^2 = 40.31$$

M.L.: $65.51 \underset{H_0}{\overset{H_1}{\gtrless}} 40.31 \Rightarrow D_1$

MPE: $65.51 \underset{H_0}{\overset{H_1}{\gtrless}} 40.31 + \underbrace{2 \cdot 1 \cdot \ln(2)}_{\text{small} = 1.3} \Rightarrow D_1$

MR: $65.51 \underset{H_0}{\overset{H_1}{\gtrless}} 40.31 + \underbrace{2 \cdot 1 \cdot \ln\left(\frac{10}{15} \cdot 2\right)}_{\text{small} = 0.5} \Rightarrow D_1$

d). $P(H_0)$? such that M.P.E. $\Rightarrow D_0$

$$65.51 \underset{H_0}{\overset{H_1}{\gtrless}} 40.31 + 2 \cdot 1 \cdot \ln(K) \Rightarrow K$$

For $D_0$:

$$40.31 + 2 \cdot \ln(K) > 65.51 \iff$$

$$\iff \ln(K) > \frac{25.20}{2} = 12.6$$

$$\iff \ln(K) > 12.6 \quad | \; e^x$$

$$\iff K > e^{12.6}$$

MPE: $K = \frac{P(H_0)}{P(H_1)} = \frac{P(H_0)}{1 - P(H_0)} > e^{12.6} \iff | \cdot (1 - P(H_0))$

$$P(H_0) > e^{12.6} - P(H_0) \cdot e^{12.6}$$

$$P(H_0)\left(1 + e^{12.6}\right) > e^{12.6}$$

$$P(H_0) > \frac{e^{12.6}}{1 + e^{12.6}} = 0.9999966$$

(1) (from Seminar 5)

$s_1(t) = 3 \cdot \sin(2\pi f_1 t)$

$s_0(t) = 0$

$N(\mu = 0, \sigma^2 = 1)$

$r = \{1.1, 4.4\}$

$t_1 = \frac{0.125}{f_1}$

$t_2 = \frac{0.625}{f_1}$

Decision (ML) = ?

$d(r, s_0)^2 \gtrless d(r, s_1)^2 + 2\sigma^2 \ln(k)$

$k = 1$ (ML)

$r = [1.1 \quad 4.4]$

$s_0 = [0 \quad 0]$

$s_1 = [2.12 \quad -2.12]$

$\sin\left(\frac{3\pi}{8}\right)$

$\sin\left(\frac{5\pi}{8}\right)$

$s_1(t_1) = 3 \cdot \sin\left(2\pi f_1 \cdot \frac{0.125}{f_1}\right) = 3\sin(0.375\,\pi)$

$= 2.12$ ?

$s_1(t_2) = 3 \cdot \sin\left(2\pi f_1 \frac{0.625}{f_1}\right) = -2.12$

$d(r, s_0)^2 = 1.1^2 + 4.4^2 = 20.57$

$d(r, s_1)^2 = (1.1 - 2.12)^2 + (4.4 + 2.12)^2 = 43.55$

M.L.: $k = 1 \Rightarrow$ smal