Seminar 6
ML estimation
DEDP

1. We receive constant signal with unknown amplitude A, $r(t) = \underbrace{A}_{s_\Theta(t)} + noise$, where the noise is gaussian with $\mathcal{N}(\mu = 0, \sigma^2 = 2)$. The signal is sampled at moments $t_i = [0, 1.5, 3, 4]$ and the samples are $r_i = [4.6, 5.2, 5.35, 4.8]$.

   a. Estimate A using Maximum Likelihood (ML) estimation
   b. Repeat a) if the noise is uniform $U[-2, 2]$. Is it possible to find a precise value?

2. A received signal $r(t) = a \cdot t^2 + noise$ is sampled at time moments $t_i = [1, 2, 3, 4, 5]$, and the values are $r_i = [1.2, 3.7, 8.5, 18, 25.8]$. The noise distribution is $\mathcal{N}(0, \sigma^2 = 1)$. Estimate the parameter a.

   a. use Maximum Likelihood (ML) estimation

3. Fit a linear function $y = ax$ (i.e. estimate a) through the following data points $(x_i, y_i) = (1, 1.8), (2, 4.1), (2.5, 5.1), (4, 7.9), (4.3, 8.5)$, assuming the noise is $\mathcal{N}(0, \sigma^2 = 1)$

   a. use Maximum Likelihood (ML) estimation

4. A robot travels a linear road with a constant but unknown speed v cm/s, starting from position $x_0 = 0$ at time 0.

   Every second the robot measures its position using an imprecise sensor, which provides values affected by Gaussian noise $\mathcal{N}(0, \sigma^2 = 0.1)$.

   The measured values at time moments $t_i = [1, 2, 3, 4, 5]$ are $r_i = [4.9, 9.8, 14.3, 21.2, 25.7]$.

   a. Estimate the speed v using ML estimation.

      Hint: If the speed is constant, the travelled distance is $x = v \cdot t$.

   b. Predict the robot position at time 6.

c. Assuming the starting position x_0 is unknown, $x_0 \neq 0$, estimate the pair of parameters [v, x_0] using ML estimation. Predict the robot's position at time 6.

d. Assuming the movement law is $x(t) = a \cdot t^2 + v_0 \cdot t + x_0$, write the equation system for finding the unknown parameters [a, v_0, x_0]. (constant acceleration a, initial speed v_0, initial position x_0).

5. A robot travels a linear road with a constant speed $V = 10$ cm/s.

   The robot measures its position every second with with a sensor affected by gaussian noise $\mathcal{N}(0, \sigma^2 = 0.5)$.

   At time $t_0 = 0$, the robot's position is around $x_0 = 20$, being a random variable with the distribution:

$$w(x_0) = \mathcal{N}(\mu = 20, \sigma = 0.5)$$

   a. Find the distribution $w(x_1)$ of the robot's position at time $t_1 = 1$, and predict the actual value with the MAP and MMSE estimators.

   b. Suppose at this time $t_1 = 1$ we have a new measurement of the position, with value $r = 29.5$.

      Take this into account with Bayesian estimation and find the new position using MAP and MMSE estimators, considering the predicted distribution in a) as the prior distribution.

   c. What happens if the speed is not known precisely. Suppose V is a random variable $\mathcal{N}(\mu = 10 \text{ cm/s}, \sigma^2 = 0.3)$?