Seminar 3

(1) $s_0(t) = -1 \quad (H_0)$

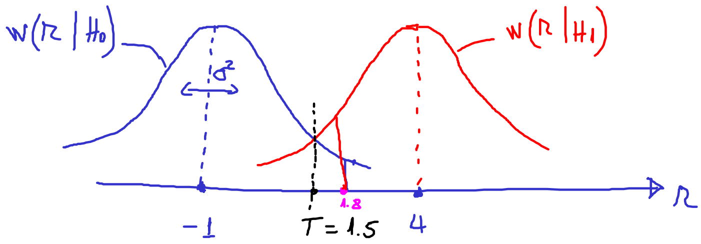
$s_1(t) = 4 \quad (H_1)$

$\mathcal{N}(\mu = 0, \sigma^2 = 4)$

$t_0 = 0.75$

$r = 1.8$

a)



$$w(r|H_0) = \frac{1}{2\sqrt{2\pi}} \cdot e^{-\frac{(r+1)^2}{8}}$$

$$w(r|H_1) = \frac{1}{2\sqrt{2\pi}} \cdot e^{-\frac{(r-4)^2}{2\sigma^2}}$$

b). $T = \frac{-1+4}{2} = 1.5$

$r > T \Rightarrow D_1$

OR

ML, gaussian noise

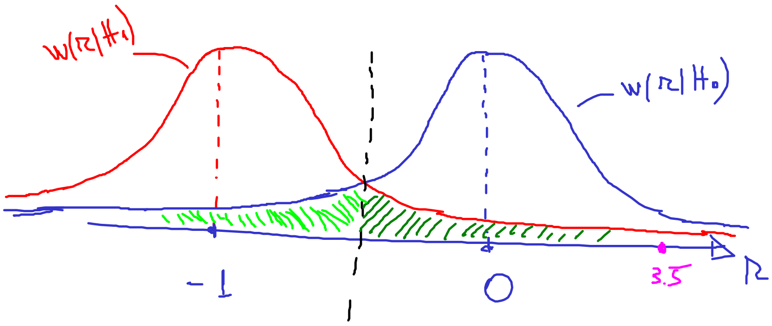
1.8 closer to 4 than to $-1 \Rightarrow D_1$

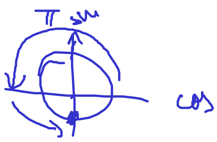
(2) $s_0(t) = \cos(2\pi t) \quad (H_0)$ ($t = 2\pi f t$)

$s_1(t) = \sin(2\pi t) \quad (H_1)$

$\mathcal{N}(\mu = 0, \sigma^2 = 4)$

$t_0 = 0.75, \quad r = 3.5$





$s_0(t_0) = \cos(2\pi\, 0.75) = 0$

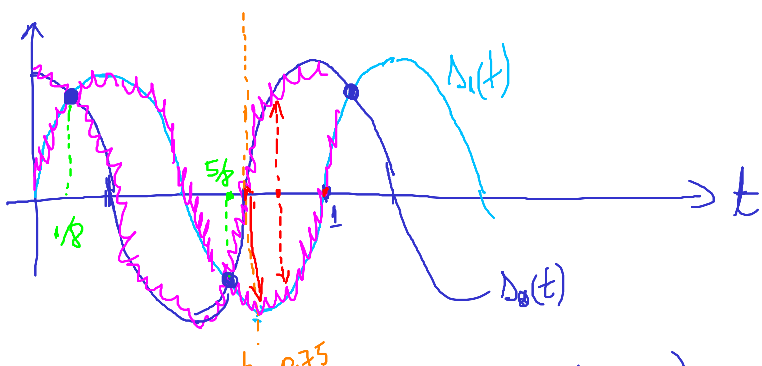
$s_1(t_0) = \sin(2\pi\, 0.75) = -1$

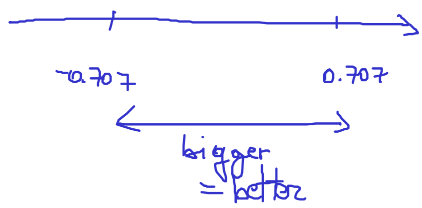
$$w(r|H_0) = \frac{1}{2\sqrt{2\pi}} e^{-\frac{(r-0)^2}{8}}$$

$$w(r|H_1) = \frac{1}{2\sqrt{2\pi}} e^{-\frac{(r+1)^2}{8}}$$

b). For $r = 3.5$, $\quad w(r|H_0) > w(r|H_1) \Rightarrow \boxed{D_0}$

c).





$s_0(0.875) = \cos(2\pi\, 0.875) = 0.707 = \frac{\sqrt{2}}{2}$

$t_1 = 0.75 + \frac{0.25}{2} = 0.875$

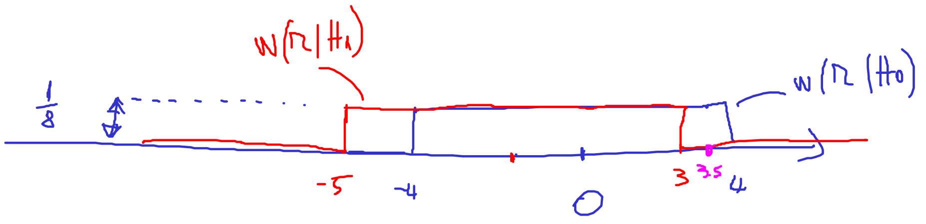
$s_1(0.875) = \sin(2\pi\, 0.875) = -0.707 = -\frac{\sqrt{2}}{2}$

d). Worst moment : when $s_0(t) = s_1(t)$

$\cos(2\pi t) = \sin(2\pi t)$

$\Leftrightarrow$ when $2\pi t = \pi/4 \Rightarrow t = \frac{1}{8}$

e) $\mathcal{U}\{-4, 4)$

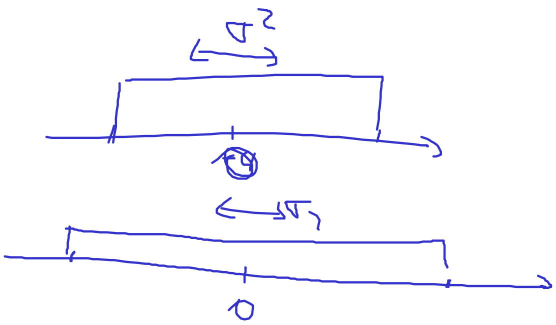


$r = 3.5 \Rightarrow w(r \mid H_0) = \frac{1}{8}$

$w(r \mid H_1) = 0$ $\Rightarrow \boxed{D_0}$

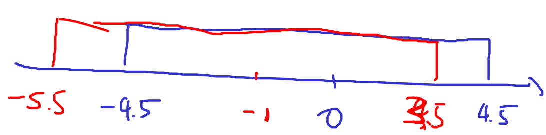
$r = 1.2 \Rightarrow$ Indecision because $w(r \mid H_0) = w(r \mid H_1)$

f). $\mathcal{U}[-4, 4]$



Max noise is $\mathcal{U}\{-4.5, 4.5]$

-4.4999 4.4999

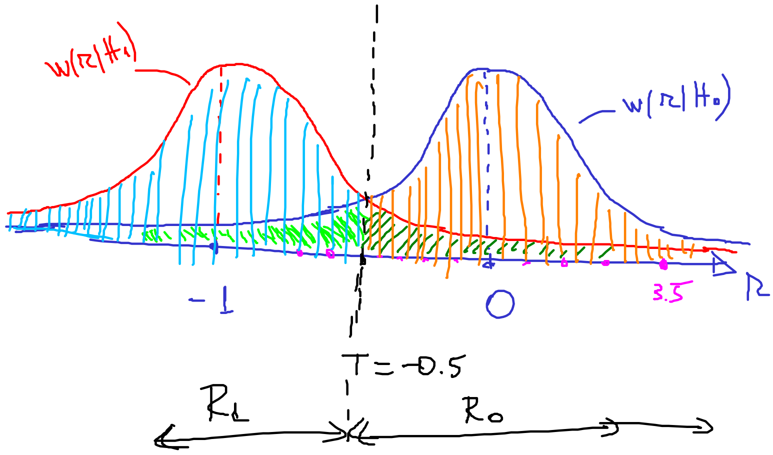


Max. variance is for noise $\mathcal{U}\{-4.5, 4.5]$

$$\sigma^2_{max} = \overline{X^2} - \underbrace{\mu^2}_{=0} = \overline{x^2} = \int_{-4.5}^{4.5} x^2 \cdot \frac{1}{9}\, dx = \frac{1}{9} \cdot \frac{x^3}{3} \Big|_{-4.5}^{4.5} = \ldots$$

g).

$\cdot\ \underset{\text{correct rej.}}{P}(D_0 \mid H_0) = \int_{-0.5}^{\infty} w(r|H_0) =$



$= \underbrace{F(\infty)}_{1} - \underbrace{F(-0.5)}_{\frac{1}{2}\left(1+\operatorname{erf}\left(\frac{-0.5-0}{2\sqrt{2}}\right)\right) = 0.4} = 0.6$

$$\boxed{F(x) = \frac{1}{2}\left(1+\operatorname{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right)}$$

$\cdot\ P(D_1 \mid H_0) = \int_{-\infty}^{-0.5} w(r|H_0) = \underbrace{F(-0.5)}_{0.4} - \underbrace{F(-\infty)}_{0} = 0.4$

false alarm

$\cdot\ P(D_0 \mid H_1) = \int_{-0.5}^{\infty} w(r|H_1) = \underbrace{F(\infty)}_{1} - \underbrace{F(-0.5)}_{\frac{1}{2}\left(1+\operatorname{erf}\left(\frac{-0.5+1}{2\sqrt{2}}\right)\right) = 0.6} = 0.4$

miss

$\cdot\ P(D_1 \mid H_1) = \int_{-\infty}^{-0.5} w(r|H_1) = \underbrace{F(-0.5)}_{0.6} - \underbrace{F(-\infty)}_{0} = 0.6$

correct detection