

Seminar 7

ML and Bayesian estimation, multiple unknowns

DEDP

1. A robot travels a linear road with a constant but unknown speed v cm/s, starting from position $x_0 = 0$ at time 0.

Every second the robot measures its position using an imprecise sensor, which provides values affected by Gaussian noise $\mathcal{N}(0, \sigma^2 = 0.1)$.

The measured values at time moments $t_i = [1, 2, 3, 4, 5]$ are $r_i = [4.9, 9.8, 14.3, 21.2, 25.7]$.

- a. Estimate the speed v using ML estimation.

Hint: If the speed is constant, the travelled distance is $x = v \cdot t$.

- b. Predict the robot position at time 6.
 - c. Assuming the starting position x_0 is unknown, $x_0 \neq 0$, estimate the pair of parameters $[v, x_0]$ using ML estimation. Predict the robot's position at time 6.
 - d. Assuming the movement law is $x(t) = a \cdot t^2 + v_0 \cdot t + x_0$, write the equation system for finding the unknown parameters $[a, v_0, x_0]$. (constant acceleration a , initial speed v_0 , initial position x_0).
2. A robot travels a linear road with a constant speed $V = 10$ cm/s.

The robot measures its position every second with a sensor affected by gaussian noise $\mathcal{N}(0, \sigma^2 = 0.5)$.

At time $t_0 = 0$, the robot's position is around $x_0 = 20$, being a random variable with the distribution:

$$w(x_0) = \mathcal{N}(\mu = 20, \sigma = 0.5)$$

- a. Find the distribution $w(x_1)$ of the robot's position at time $t_1 = 1$.
- b. Suppose at this time $t_1 = 1$ we have a new measurement of the position, with value $r = 29.5$.

Take this into account with Bayesian estimation and find the new position using MAP and MMSE estimators, considering the predicted distribution in a) as the prior distribution.

- c. What happens if the speed is not known precisely. Suppose V is a random variable $\mathcal{N}(\mu = 10 \text{ cm/s}, \sigma^2 = 0.3)$?