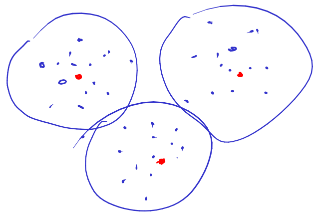
DEDP Seminar 6

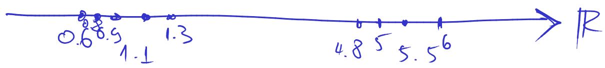


Ex 5 from Sem. 5:

5. Consider the following 10 values:

$$\vec{v} = \{v_i\} = [1.1, 0.9, 5.5, 0.6, 5, 6, 1.3, 4.8, 6, 0.8]$$

Perform 5 iterations of the k-Means algorithm in order to find two centroids $\vec{c}_1$ și $\vec{c}_2$, starting from two random values $\vec{c}_1 = 0.95$ și $\vec{c}_2 = 0.96$.



$c_1 = \boxed{0.95}$

$c_2 = \boxed{0.96}$

Iteration 1 : 1). Classif.

C_1 : 0.9 0.6 0.8

C_2 : 1.1 5.5 5 6 1.3 4.8 6

2). Update :

$$c_1 = \frac{0.9 + 0.6 + 0.8}{3} = \boxed{0.76}$$

$$c_2 = \frac{1.1 + 5.5 + 5 + 6 + 1.3 + 4.8 + 6}{7} = \boxed{4.24}$$

Iteration 2 : $\vec{v} = \{v_i\} = [1.1, 0.9, 5.5, 0.6, 5, 6, 1.3, 4.8, 6, 0.8]$

1). C_1 : 1.1 0.9 0.6 1.3 0.8

C_2 : 5.5 5 6 4.8 6

2).

$$c_1 = \frac{1.1 + \dots + 0.8}{5} = \boxed{0.94}$$

$$c_2 = \frac{5.5 + \dots + 6}{5} = \boxed{5.46}$$

Iteration 3 :

1). C_1 : 1.1 0.9 0.6 1.3 0.8

C_2 : 5.5 5 6 4.8 6

Nothing changed.
Converged.

2). $c_1 = 0.94$

$c_2 = 5.46$

Result :

$c_1 = 0.94$ (1.1, 0.9, 0.6, 1.3, 0.8)

$c_2 = 5.46$ (5.5 5 6 4.8 6)

Centroids Clusters

1

1. We receive a constant signal with unknown amplitude A, $r(t) = \underbrace{A}_{s_\Theta(t)} + noise$, where the noise is gaussian with $\mathcal{N}(\mu = 0, \sigma^2 = 2)$. The signal is sampled at moments $t_i = [0, 1.5, 3, 4]$ and the samples are $r_i = [4.6, 5.2, 5.35, 4.8]$.

   a. Estimate A using Maximum Likelihood (ML) estimation

   ~~b. Repeat a) if the noise is uniform $U[-2, 2]$. Is it possible to find a precise value?~~

a). $\boxed{\hat{A}_{ML} = \underset{A}{\operatorname{argmin}}\ d(r, s_\Theta)^2}$ (M.L. estimation with gaussian noise)

$$\left.\begin{array}{l} r = [4.6 \quad 5.2 \quad 5.35 \quad 4.8] \\ s_\Theta = [A \quad A \quad A \quad A] \end{array}\right\} \Rightarrow d(r, s_\Theta)^2 = (A-4.6)^2 + (A-5.2)^2 + (A-5.35)^2 + (A-4.8)^2$$

$$\frac{d}{dA}\, d(r, s_\Theta)^2 = 0$$

$$\cancel{2}\cdot(A-4.6)\cdot 1 + \cancel{2}(A-5.2) + \cancel{2}(A-5.35) + \cancel{2}(A-4.8) = 0 \Leftrightarrow$$

$$\Leftrightarrow 4A = 4.6 + 5.2 + 5.35 + 4.8 \Rightarrow \hat{A}_{ML} = \frac{4.6 + 5.2 + 5.35 + 4.8}{4} = \dots$$

b). Later

(2)

2. A received signal $r(t) = \overset{s_\theta(t)}{a \cdot t^2} + noise$ is sampled at time moments $t_i = [1, 2, 3, 4, 5]$, and the values are $r_i = [1.2, 3.7, 8.5, 18, 25.8]$. The noise distribution is $\mathcal{N}(0, \sigma^2 = 1)$. Gaussian ✓
Estimate the parameter a.

a. use Maximum Likelihood (ML) estimation

$$t = [1 \quad 2 \quad 3 \quad 4 \quad 5]$$

$$r = [1.2 \quad 3.7 \quad 8.5 \quad 18 \quad 25.8]$$

$$s_\theta = [a \quad 4a \quad 9a \quad 16a \quad 25 \cdot a]$$

$$d(r, s_\theta)^2 = (a - 1.2)^2 + (4a - 3.7)^2 + (9a - 8.5)^2 + (16a - 18)^2 + (25a - 25.8)^2$$

$$\frac{d}{da} d(r, s_\theta)^2 = \cancel{2}(a - 1.2) \cdot 1 + \cancel{2}(4a - 3.7) \cdot 4 + \cancel{2}(9a - 8.5) \cdot 9 + \cancel{2}(16a - 18) \cdot 16 + \\ + \cancel{2}(25a - 25.8) \cdot 25 = 0$$

$$\hat{a}_{ML} = \frac{1.2 + 3.7 \cdot 4 + 8.5 \cdot 9 + 18 \cdot 16 + 25.8 \cdot 25}{1 + 16 + 81 + 256 + 625} = \ldots$$

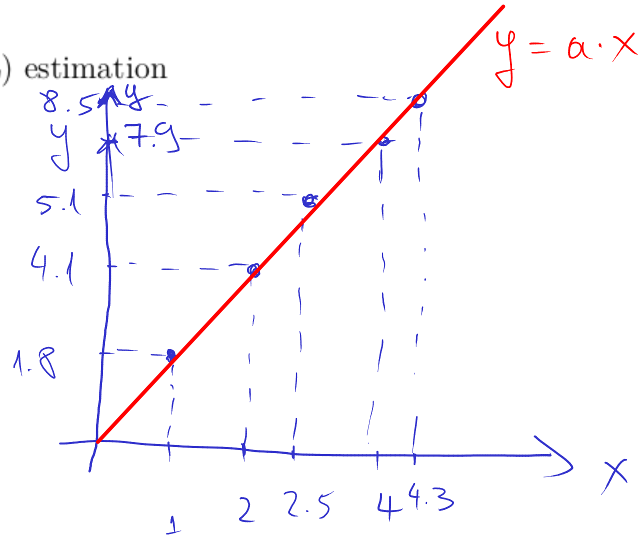
3

3. Fit a linear function $\boxed{y = ax}$ (i.e. estimate a) through the following data points $(x_i, y_i) = (1, 1.8), (2, 4.1), (2.5, 5.1), (4, 7.9), (4.3, 8.5)$, assuming the noise is $\mathcal{N}(0, \sigma^2 = 1)$

   a. use Maximum Likelihood (ML) estimation

x_i	y_i
1	1.8
2	4.1
2.5	5.1
4	7.9
4.3	8.5

t r



$$\hat{a}_{ML} = \underset{a}{\operatorname{argmin}}\ d(r, s_\theta)^2$$

$$r = [1.8 \quad 4.1 \quad 5.1 \quad 7.9 \quad 8.5]$$

$$s_\theta = [a \quad 2a \quad 2.5a \quad 4a \quad 4.3a]$$

$$d(r, s_\theta)^2 = (a-1.8)^2 + (2a-4.1)^2 + \ldots + (4.3a-8.5)^2$$

$$\frac{d}{da} d(r, s_\theta)^2 = 0 \iff 2(a-1.8) + 2(2a-4.1)\cdot 2 + \ldots + 2(4.3a-8.5)\cdot 4.3 = 0$$

$$\Rightarrow \hat{a}_{ML} = \frac{1.8 + 2\cdot 4.1 + 2.5\cdot 5.1 + 4\cdot 7.9 + 4.3\cdot 8.5}{1.8 + 2\cdot 2 + 2.5^2 + 4^2 + 4.3^2} = \ldots$$