Seminar 4
Decision criteria
DEDP

1. A vehicle airbag system detects a crash by evaluating a sensor which provides two values: $s_0(t) = 0$ (no crash) or $s_1(t) = A$ (crashing), where $A = 5$.

   The signal is affected by gaussian noise $\mathcal{N}$ $(\mu = 0, \sigma^2 = 2)$.

   The costs of the scenarios are: $C_{00} = 0$, $C_{01} = 100$, $C_{10} = 10$, $C_{11} = 0$.

   The probabilities of the two hypotheses are $P(H_0) = 2/3$, $P(H_1) = 1/3$.

   The receiver takes a single sample r.

   a. Find the decision regions R_0 and R_1, for all the criteria below:

      - ML
      - MPE
      - MR
      - Neyman-Pearson with false alarm (conditioned) probability $P_{fa} \leq 0.01$
      - A custom threshold value $T = 3$

   b. Find the probability of miss, for all the criteria above

   c. Find the decision taken based on a sample $r = 3.1$, with each criterion above

   d. Considering the MR criterion, what is the minimum value of A such that the miss probability (non-conditioned) is at most $P_m \leq 10^{-6}$?

   e. Repeat the whole exercise, but consider the noise is uniform $U[-3, 3]$.

2. A signal can have two values, $s_0(t) = 0$ (hypothesis H_0) or $s_1(t) = 6$ (hypothesis H_1).

   The signal is affected by AWGN $\mathcal{N}(0, \sigma^2 = 1)$.

   The receiver takes 5 samples with values $\{1.1, 4.4, 3.7, 4.1, 3.8\}$.

   a. What is decision according to Maximum Likelihood criterion?

b. What is decision according to Minimum Probability of Error criterion, assuming $P(H_0) = 2/3$ and $P(H_1) = 1/3$?
c. What is the decision according to Minimum Risk Criterion, assuming $P(H_0) = 2/3$ and $P(H_1) = 1/3$, and $C_{00} = 0$, $C_{10} = 10$, $C_{01} = 20$, $C_{11} = 5$?
d. What are the values of $P(H_0)$ such that the decision according to MPE criterion to be D_0?