Seminar 5
Decision with multiple samples
DEDP

1. Consider detecting a signal $s_1(t) = 3\sin(2\pi f_1 t)$ that can be present (hypothesis H_1) or not ($s_0(t) = 0$, hypothesis H_0). The signal is affected by AWGN $\mathcal{N}(0, \sigma^2 = 1)$. The receiver takes 2 samples.

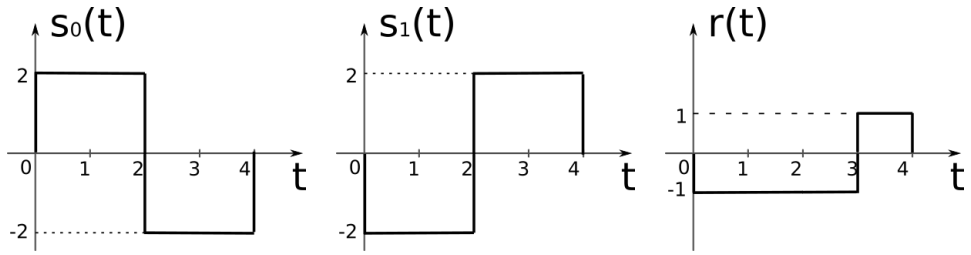
    a. What are the best sample times t_1 and t_2 to maximize detection performance (depending on f_1)?
    b. The receiver takes 2 samples with values $\{1.1, 4.4\}$, at sample times $t_1 = \frac{0.125}{f_1}$ and $t_2 = \frac{0.625}{f_1}$. What is decision according to Maximum Likelihood criterion?
    c. What if we take the decision with Minimum Probability of Error criterion, assuming $P(H_0) = 2/3$ and $P(H_1) = 1/3$?
    d. What is the decision according to Minimum Risk Criterion, assuming $P(H_0) = 2/3$ and $P(H_1) = 1/3$, and $C_{00} = 0$, $C_{10} = 10$, $C_{01} = 20$, $C_{11} = 5$?
    e. What if the receiver takes an extra third sample at time $t_3 = \frac{0.5}{f_1}$. Will the detection be improved?

2. A signal can have two values, 0 (hypothesis H_0) or 6 (hypothesis H_1). The signal is affected by AWGN $\mathcal{N}(0, \sigma^2 = 1)$. The receiver takes 5 samples with values $\{1.1, 4.4, 3.7, 4.1, 3.8\}$.

    a. What is decision according to Maximum Likelihood criterion?
    b. What is decision according to Minimum Probability of Error criterion, assuming $P(H_0) = 2/3$ and $P(H_1) = 1/3$?
    c. What is the decision according to Minimum Risk Criterion, assuming $P(H_0) = 2/3$ and $P(H_1) = 1/3$, and $C_{00} = 0$, $C_{10} = 10$, $C_{01} = 20$, $C_{11} = 5$?
    d. How large should $P(H_0)$ be, in order for the decision according to MPE criterion to be D_0?

3. A transmitted signal can be one of the following $s_0(t)$ or $s_1(t)$ (depicted below). The received signal is $r(t)$. The signal is affected by AWGN $\mathcal{N}(0, \sigma^2 = 2)$. Find the receiverÈ›s decision based on the Maximum Likelihood criterion, in two ways:

a. based on 3 samples taken at moments $t_1 = 0.5$, $t_2 = 1.5$ and $t_2 = 3.5$
b. with continuous observation



4. Consider the k-NN algorithm with the following training set, composed of 5 vectors of class A and another 5 vectors from class B:

   - Class A:

   $$\vec{v}_1 = \begin{bmatrix} 2 \\ -4 \end{bmatrix} \quad \vec{v}_2 = \begin{bmatrix} 1 \\ -5 \end{bmatrix} \quad \vec{v}_3 = \begin{bmatrix} -2 \\ 6 \end{bmatrix} \quad \vec{v}_4 = \begin{bmatrix} -3 \\ 4 \end{bmatrix} \quad \vec{v}_5 = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$$

   - Class B:

   $$\vec{v}_6 = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \quad \vec{v}_7 = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \quad \vec{v}_8 = \begin{bmatrix} -4 \\ -3 \end{bmatrix} \quad \vec{v}_9 = \begin{bmatrix} -3 \\ 0 \end{bmatrix} \quad \vec{v}_{10} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

   Compute the class of the vector $\vec{x} = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$ using the k-NN algorithm, with $k = 1$, $k = 3$, $k = 5$, $k = 7$ and $k = 9$