ECE 348: Digital Signal Processing Lab, Spring 2023

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<u>General Instructions</u>: Please submit a written report in the pdf format within 2 weeks of your lab session. The report must describe the purposes of the experiments, the methods used, including all graphs and Matlab code. The reports must be uploaded to Canvas within the allotted time frame.

LAB – 2

CONVOLUTION:

Problem 1. Consider an integrator-like filter defined by the I/O equation:

$$y(n) = \frac{1}{15}[x(n) + x(n-1) + x(n-2) + ... + x(n-14)].$$

In finance, this filter is referred to as 15-day moving averages (n represents days). The impulse response of this filter is

$$h(n) = \begin{cases} 1/15, & 0 \leq n \leq 14, \\ 0, & otherwise. \end{cases}$$

Consider a square wave input signal $x(n)$ of length $L = 200$ and period of $K = 50$ samples. Such a signal may be generated by the simple vectorized MATLAB code:

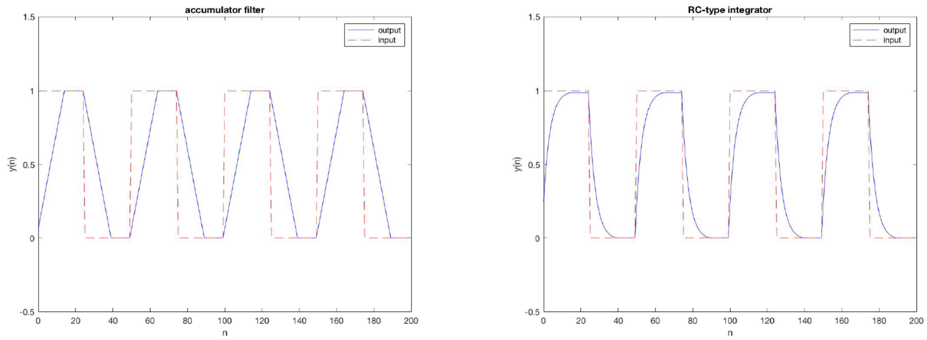
```
n = 0:L-1;
x = double(rem(n,K) < K/2);        % REM is the remainder function
```

Using the function **conv**, compute the output signal $y(n)$ and plot it versus n on the same graph with $x(n)$. As the square wave periodically goes on and off, you can observe the on-transient, steady-state, and off-transient behavior of the filter (see Fig. 1(a)).

Problem 2. Repeat Problem 1.1 for the filter:

$$h(n) = \begin{cases} 0.25(0.75)^n, & 0 \leq n \leq 14, \\ 0, & otherwise. \end{cases}$$

This filter acts more like an RC-type integrator than an accumulator (see Fig. 1(b)).



(a) (b)

Figure 1: Plots for (a) Problem 1 and (b) Problem 2.

Problem 3. To demonstrate the concepts of impulse response, linearity, and time-invariance, consider a filter with finite impulse response $h(n) = (0.95)^n$, for $0 \le n \le 24$. The input signal,

$$x(n) = \delta(n) + 2\delta(n-40) + 2\delta(n-70) + \delta(n-80), \qquad n = 0, 1, \ldots, 120,$$

consists of four impulses of the indicated strengths occurring at the indicated time instances. Note that the first two impulses are separated by more than the duration of the filter, whereas the last two are separated by less. The input vector x can be constructed with the help of the following anonymous function implementing the discrete-time delta function $\delta(n)$:

```
d = @(n) double(n==0);          % n can be a vector of indices
```

Using the function **conv**, compute the filter output $y(n)$ for $0 \le n \le 120$ and plot it on the same graph with $x(n)$. Comment on the resulting output with regard to linearity and time invariance (see Fig. 2).

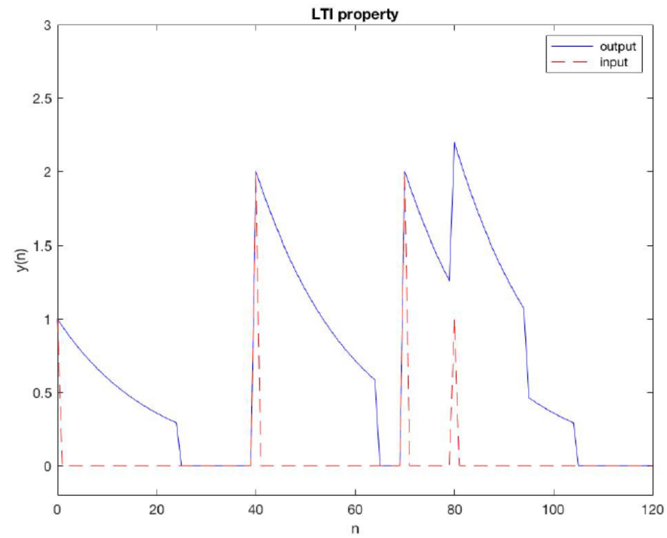


Figure 2: Plot for Problem 3.

DTFT COMPUTATION:

Problem 4.

Consider the signal

$$p(n) = u(n) - u(n-L) = \begin{cases} 1, & 0 \leq n \leq L-1, \\ 0, & otherwise. \end{cases}$$

where $u(n)$ is the unit-step function. This can be generated for any vector of n's by the MATLAB function (assuming L was already defined):

```
p = @(n) double(n>=0 & n<=L-1);
```

Now, choose $L = 20$, and construct $p(n)$ and plot it versus n over the time interval $-15 \leq n \leq 35$ using a **stem** plot (see Fig. 3(a)).

Next, show that the DTFT of $p(n)$ is given by the analytical expression:

$$P(\omega) = e^{-j\omega(L-1)/2} \frac{sin(\omega L/2)}{sin(\omega/2)} \tag{1}$$

For the purpose of programming (1) in MATLAB, re-write it in terms of MATLAB's **sinc** function, which is vectorized and avoids a computational issue at $\omega = 0$:

$$P(\omega) = Le^{-j\omega(L-1)/2}\frac{sinc(\omega L/2\pi)}{sinc(\omega/2\pi)} \tag{2}$$

where

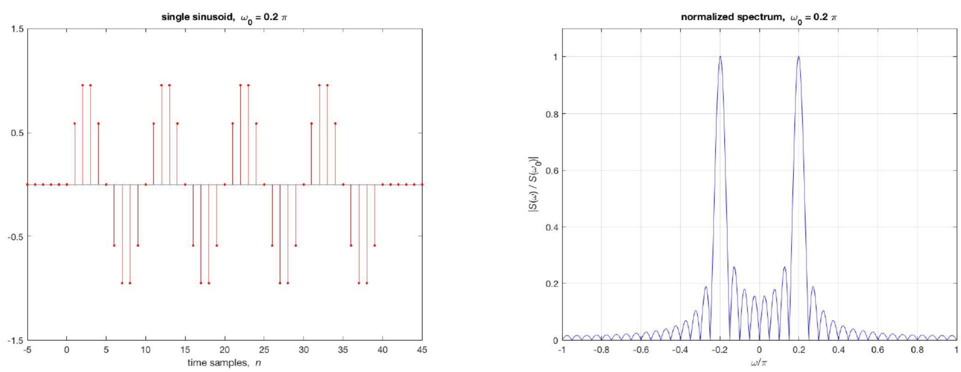
$$sinc(x) = \frac{sin(\pi x)}{\pi x}$$

Given a value for L, write an anonymous MATLAB function $P(\omega)$ implementing (2). It should be vectorized with respect to the variable ω.

Next, using your anonymous function P, calculate the corresponding DTFT of $p(n)$ for $L = 20$ at $N = 1001$ equally-spaced frequency points over the interval $-\pi \le \omega \le \pi$ and plot the following normalized quantity versus ω,

$$F(\omega) = |\frac{F(\omega)}{F(\omega_0)}|$$

where $\omega = 0$. Moreover, verify that the DTFT computed using the analytical expression (2) agrees with the numerical computation using the **freqz** function over the same set of N frequencies.



(a)

(b)

Figure 3:

Problem 5.

Consider a length-L finite portion of a sinusoid of frequency ω_0, defined as the sinusoidally modulated pulse signal $p(n)$ of Problem

$$s(n) = sin(w_0 n)p(n) = \begin{cases} sin(w_0 n), & 0 \le n \le L-1, \\ 0, & otherwise. \end{cases}$$

Using the modulation property of the DTFT, show that the DTFT of $s(n)$ is given by the following analytical expression in terms of the DTFT $P(\omega)$:

$$S(\omega) = \frac{1}{2j}[P(\omega - \omega_0) - P(\omega + \omega_0)] \tag{3}$$

Choose $L = 40$ and $\omega_0 = 0.2\pi$ radians/sample. Using your anonymous MATLAB function $P(\omega)$ of Problem 4 evaluate (3) over the same set of N frequencies as in Problem 4 and plot the following normalized quantity versus ω:

$$F(\omega) = |\frac{S(\omega)}{S(\omega_0)}|$$

for the present value of ω_0. Plot also the sinusoidal signal s(n) versus n using a **stem** plot over the time range $-5 \leq n \leq 45$. See example graphs in Fig. 4.

Moreover, re-calculate the DTFT $S(\omega)$ using **freqz** and verify that you get the same results as (3) to within the double-precision floating-point accuracy of MATLAB.

PROBLEM 6:

Next, consider a length-L signal consisting of the sum of two sinusoids of frequencies

$\omega_1 = 0.2\pi$ and $\omega_2 = 0.4\pi$, with relative amplitudes of 1 and 0.8, as defined below:

$$s(n) = [sin(w_1 n) + 0.8sin(w_2 n)]p(n) = \begin{cases} sin(w_1 n) + 0.8sin(w_2 n), & 0 \leq n \leq L-1, \\ 0, & otherwise. \end{cases}$$

Show that the DTFT of $s(n)$ is given analytically as the linear combination:

$$S(\omega) = S_1(\omega) + 0.8S_2(\omega) \tag{4}$$

where $S_1(\omega)$ and $S_2(\omega)$ are given by (3), with ω_0 replaced by ω_1 and ω_2, respectively. For convenience, you may wish to define an anonymous Matlab function for (4).

For $L = 40$ and using (4), plot the DTFT $S(\omega)$ versus ω, normalized with respect to its value at $\omega = \omega_1$, that is, evaluate and plot the quantity:

$$F(\omega) = |\frac{S(\omega)}{S(\omega_1)}|$$

Also, plot $s(n)$ versus n using a **stem** plot over the range $-5 \leq n \leq 45$.

And again, re-calculate the DTFT using **freqz** and verify that you get essentially the same results as with the analytical formula.

PROBLEM 7:

If you zoom into the spectral peaks at ω_1 and ω_2, you will notice that the peaks do not occur exactly at ω_1 and ω_2 as they are in the case of infinitely-long sinusoids. This phenomenon is due to the finite-duration of the sinusoids, which causes the spectral peaks to broaden and interact with each other, shifting them slightly relative to one another.

For the particular choices of L, ω_1, ω_2 of Problem 6 determine the actual location of the spectral peaks using the built-in MATLAB function, **fminbnd**. To do so, define a function handle for the negative magnitude of (4),

$$f(w) = -|S(\omega)|$$

and pass it into **fminbnd**, searching first near the true frequency ω_1, and then near ω_2. You should obtain the following peak frequencies, to be compared with $\omega_1 = 0.2\pi$, $\omega_2 = 0.4\pi$,

$$\omega_{1,peak} = 0.1950\pi, \qquad \omega_{2,peak} = 0.4030\pi.$$

The same biasing effect occurs also in the single-sinusoid case and gets more pronounced when ω_0 is near 0 or near π. This is so because the peaks at $\pm\omega_0$ are now closer and interact more with each other. Using the parameters from Problem 5 and **fminbnd**, verify that the actual peak maximum of the finite-length signal is at,

$$\omega_{0,peak} = 0.1983\pi.$$

as compared to $\omega_0 = 0.2\pi$.

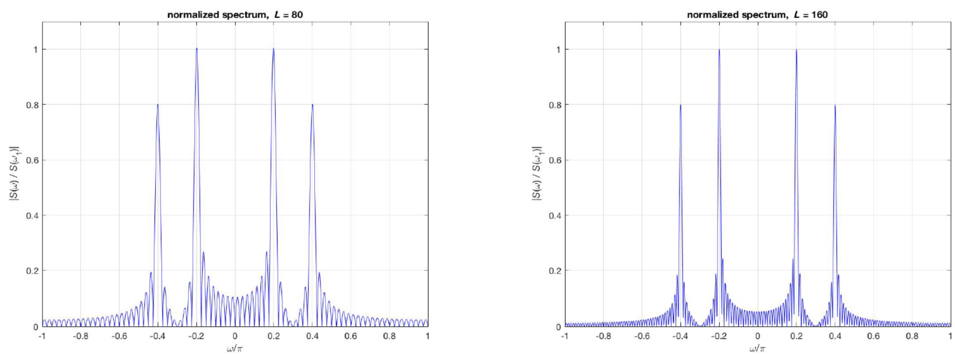
PROBLEM 8:

Graphically verify that the above effect tends to disappear as the signal length L increases, resulting in narrower peaks that interact less with each other. Listed below are the peak frequencies for the double and single sinusoidal cases for lengths $L = 80$ and $L = 160$, and shown also are the corresponding spectra in Fig. 6:

$$L = 80, \omega_{1,peak} = 0.1987\pi, \qquad \omega_{2,peak} = 0.4008\pi, \qquad \omega_{0,peak} = 0.1996\pi$$

$$L = 160, \omega_{1,peak} = 0.1997\pi, \qquad \omega_{2,peak} = 0.4002\pi, \qquad \omega_{0,peak} = 0.1999\pi$$

Some other observations are that the peak widths get narrower with increasing L, but the sidelobes immediately to the left and right of each mainlobe appear to not diminish with increasing L. These and other spectral analysis issues, such as frequency leakage and resolution, and the effect of using non-rectangular windows, will be explored in a future lab as well in future classes.



(a) (b)

Figure 4: