

# Agent Based Model on Collision and Bursting of Bubbles

ES 491 : Modeling and Simulation of Complex Systems  
IIT Gandhinagar

# Motivation and Relevance

Bubbles are fascinating entities in nature. We observe them in our daily lives.

They indicate numerous properties about a fluid and are relevant to be studied.

They are fragile and short-lasting in real life. This makes observing and analyzing them challenging.

A decorative graphic on the left side of the slide consisting of two blue squares. The top square is a lighter shade of blue and is positioned to the right of a dark blue vertical bar that extends from the bottom of the slide.

# Agent Based Model

Studying bubbles in real life through experiments can be resource-consuming

A convenient method to study the complex system of bubbles, which is hard to model using equation-based modeling.

The model can answer questions such as how the number of bubbles and their dimensions vary with time. The model is simple, yet it shows an emergent pattern.

# Agent Based Model

## Agents

The agents in this model are bubbles (turtles) that collide with each other and with the container walls where fluid is placed.

## State Variables

Current radius (size / 2)

XY-coordinates of the center

Heading direction.

## Environment

The patches represent the fluid, i.e., the environment where the bubbles exist.

The boundaries of the world represent the boundaries of the container.

The world is made up of 33 by 33 patches with no wrapping at its edges.

# Scheduling

01

Processing  
Collision  
with Other  
Bubbles

02

Bounce  
Back from  
Wall

03

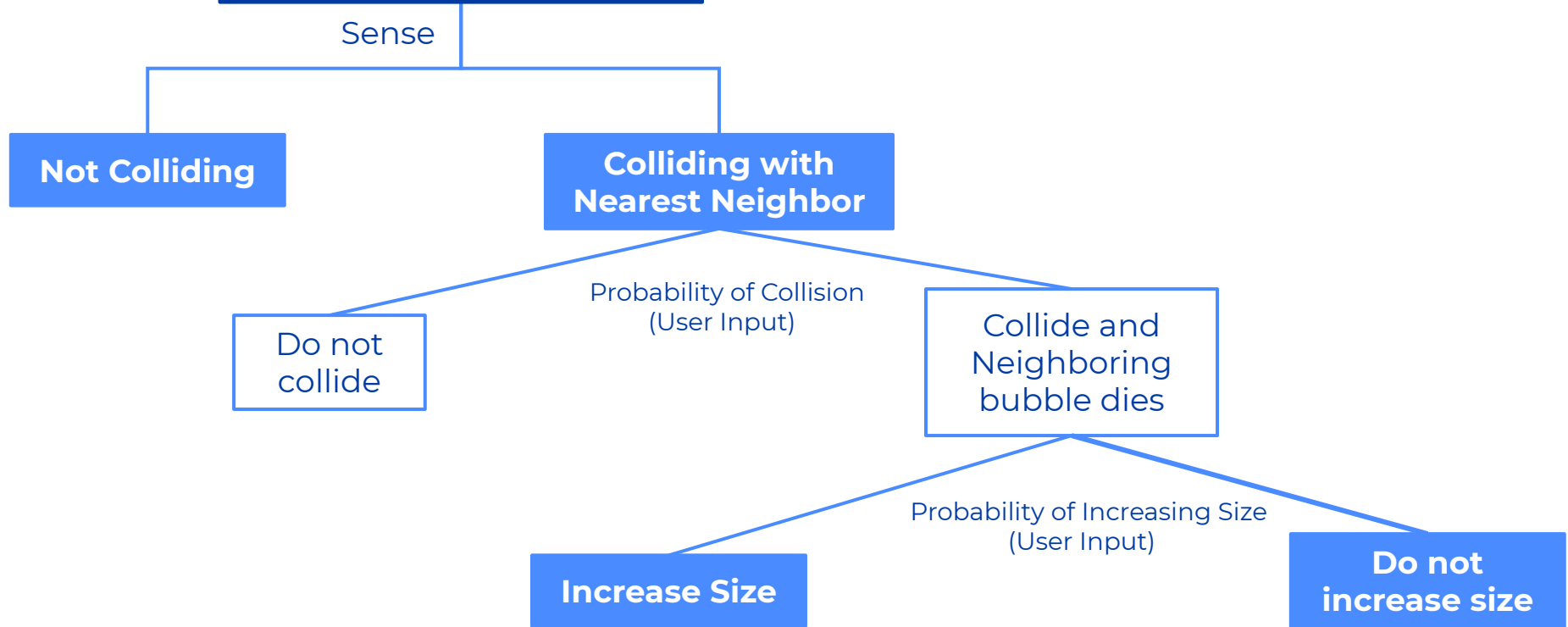
Turn angle  
randomly  
and move  
ahead

04

Add new  
bubbles  
from  
bottom of  
the  
container

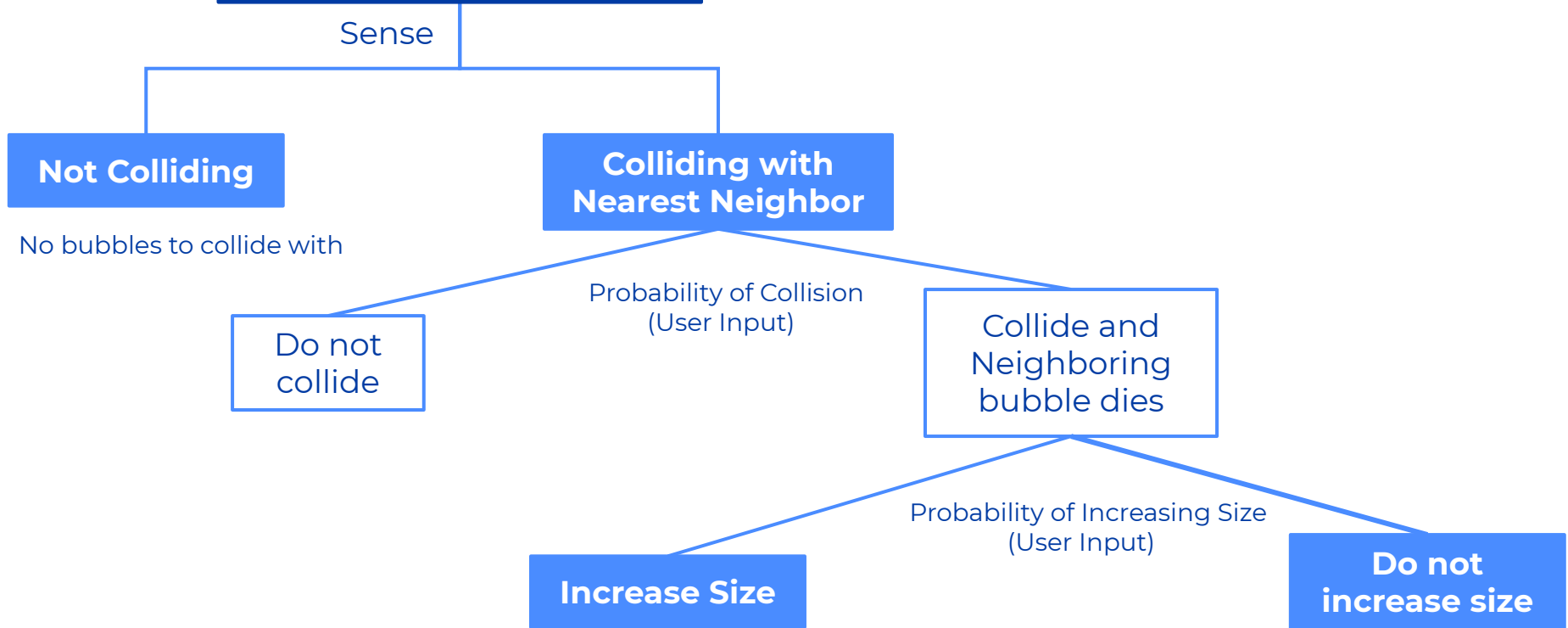
# Processing Collision

## Bubble in Context



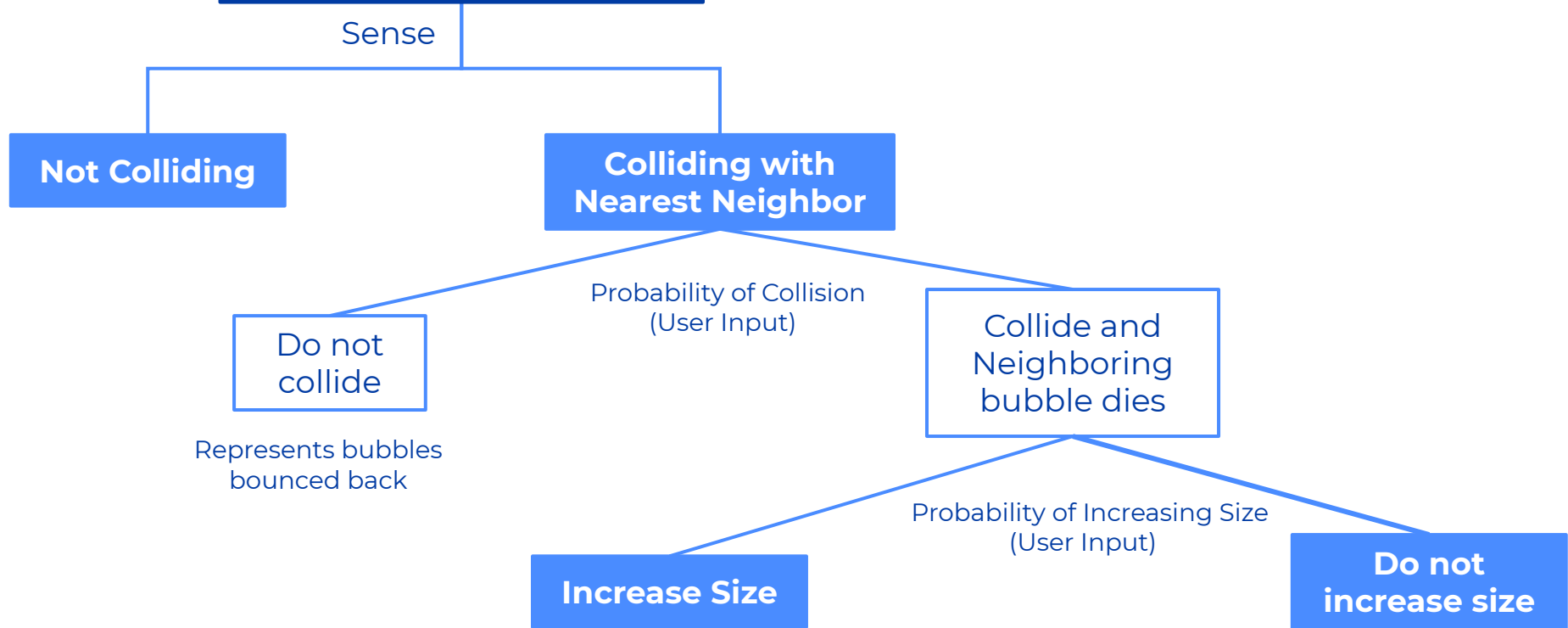
# Processing Collision

## Bubble in Context



# Processing Collision

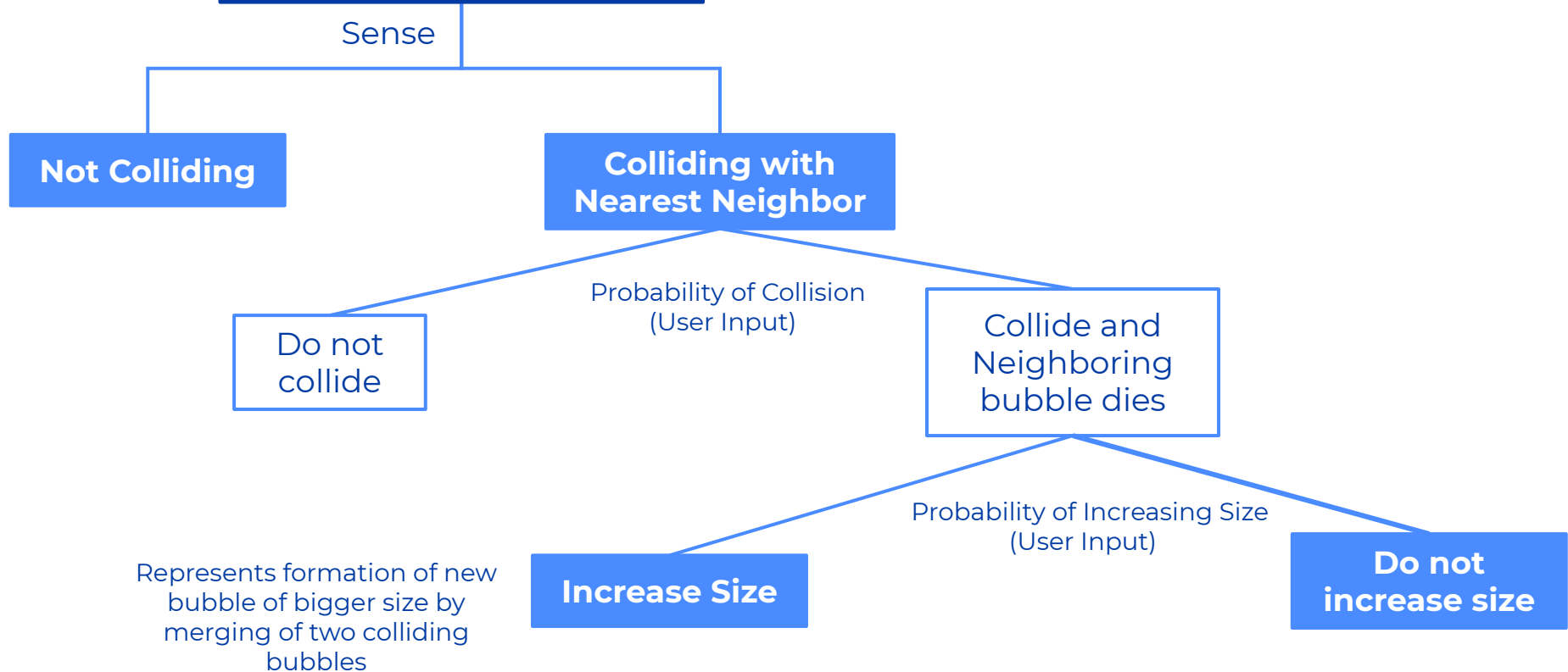
## Bubble in Context





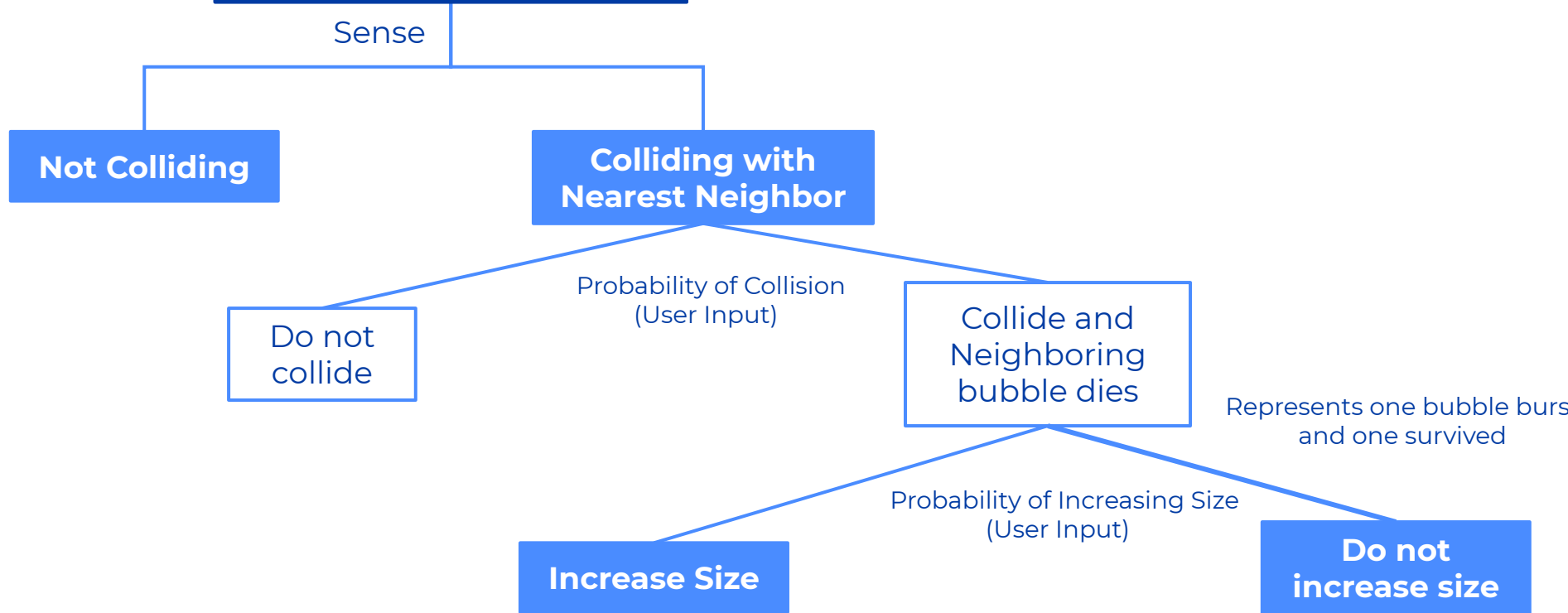
# Processing Collision

## Bubble in Context



# Processing Collision

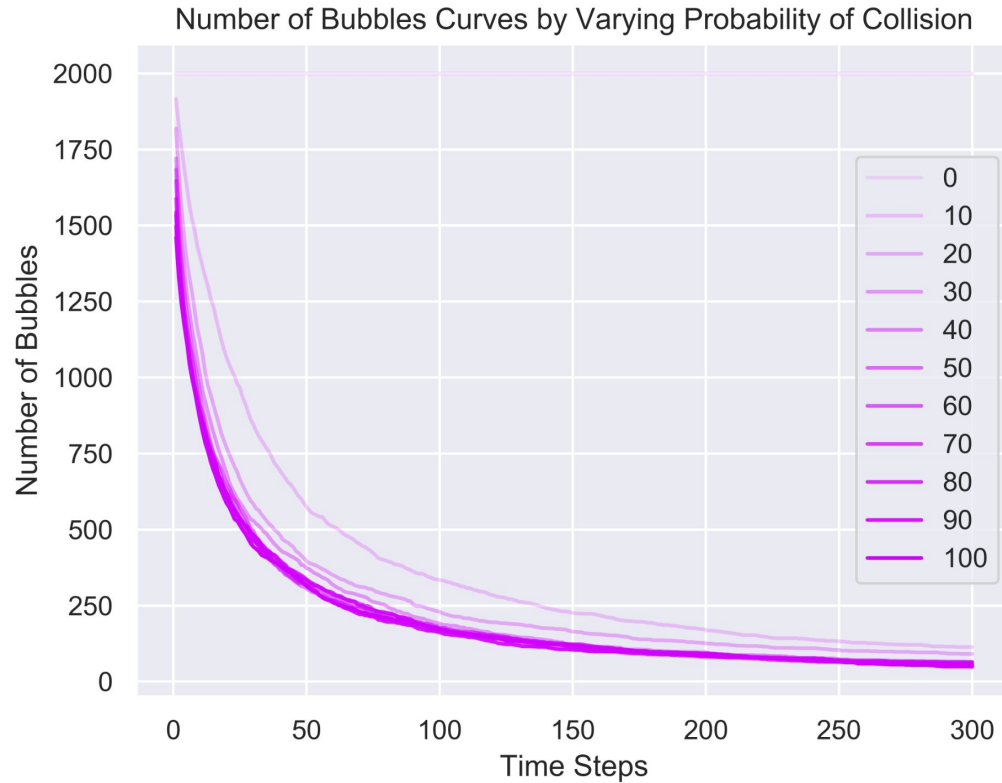
## Bubble in Context



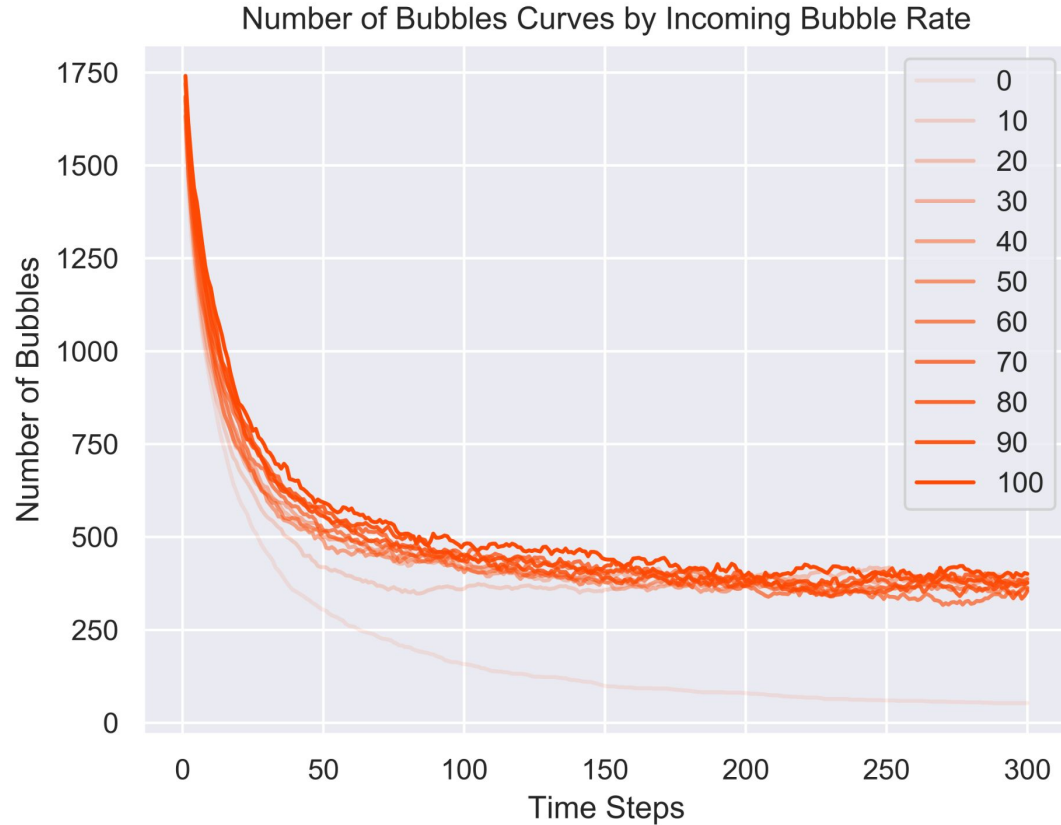
# DEMO



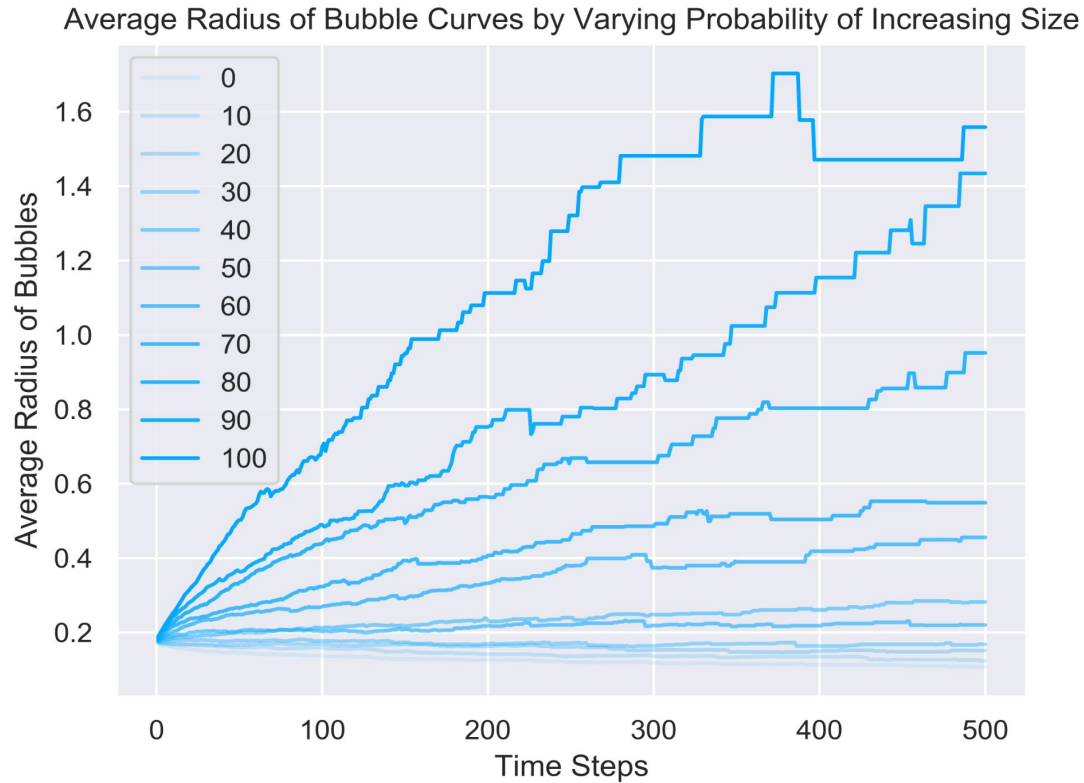
# Analyzing Outputs (Behavior Space)



# Analyzing Outputs (Behavior Space)



## Analyzing Outputs (Behavior Space)



# Literature

Calibrating Model Against  
Hypotheses Researched

# Demonstration of the exponential decay law using beer froth

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## Abstract

The volume of beer froth decays exponentially with time. This property is used to demonstrate the exponential decay law in the classroom. The decay constant depends on the type of beer and can be used to differentiate between different beers. The analysis shows in a transparent way the techniques of data analysis commonly used in science—consistency checks of theoretical models with the data, parameter estimation and determination of confidence intervals.

Exponential laws are common to many physical phenomena. Examples are the amplitude of an oscillator subject to linear friction, the discharge of a capacitor, cooling processes or radioactive decays. The demonstration described here has the advantages that it is cheap, clear and motivating because it investigates an everyday phenomenon. It can easily be repeated by the students elsewhere.

The decay of beer froth is mentioned as a very short notice in [1]. It is described in several German textbooks of mathematics. Recently, it also attracted the attention of Bavarian pupils [2].

The data analysis proposed in this paper has much in common with real science—see, for example, the determination of the Higgs mass by the LEP collaborations [3]. The techniques involved are of great practical importance but are often poorly understood by students [4].

Exponential decay can be demonstrated using beer froth, the volume of which reduces exponentially with time [1]. The exponential law can readily be derived from the assumption that the volume of froth  $dV$  disappearing in the time between  $t$  and  $t + dt$  is proportional to the volume  $V$  present at the time  $t$ ,  $dV = -(V/\tau) dt$ . In a cylindrical beer mug with an area  $A$ , the volume is proportional to the height,  $dV = A dh$ . The phenomenological theory of exponential decay predicts the height as a function of time

$$h^{th}(t) = h(0) \exp\left(-\frac{t}{\tau}\right). \quad (1)$$

The constant  $\tau$  is a free parameter of the theory. It defines how fast the froth decays; during the time  $\tau$  the amount  $1 - 1/e \approx 63\%$  of the froth disappears. Different kinds of beer have, in general, different parameters  $\tau$ .

$$dV = -(V/\tau) dt$$

$$dV/V = -(1/\tau)dt$$

$$\ln V = -(t/\tau)$$

$$V = \exp(-t/\tau)$$

## Reference -

A Leike. *Demonstration of the exponential decay law using beer froth.*

Ludwig-Maximilians-Universität, Sektion Physik, Theresienstr. 37, D-80333 München, Germany.



# THE APOLLONIAN DECAY OF BEER FOAM BUBBLE SIZE DISTRIBUTION AND THE LATTICES OF YOUNG DIAGRAMS AND THEIR CORRELATED MIXING FUNCTIONS

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## 3. Results

**3.1. Temporal behaviour of the foam volume.** We investigated the volume dependence  $V(t)$  of beer foam decay measuring the height of the foam and the level of liquid beer as a function of time. In every case we averaged arithmetically five independent measurements under the same conditions. The data obtained were approximated by various curve fitting calculations. Out of the tested fitting functions we always found the following fitting formula  $V(t)$  with highest priority:

$$\ln V(t) = a - bt - ct^{2.5} \quad (3.1)$$

fulfilling the constraints

$$\lim_{t \rightarrow \infty} V(t) = 0, \quad V(t) \geq 0 \quad \forall t > t_0. \quad (3.2)$$

Using (3.1), we estimated the coefficients for many kinds of beer and different temperatures in different glasses.

Taking *Beck's Pils* from Bremen, for instance, we obtained in case of the (20/100/24° C) experiment equation (3.3) (see Figure 3.1):

$$\ln V(t) = 3.64 - 4.34 \cdot 10^{-3}t - 6.66 \cdot 10^{-7}t^{2.5}, \quad R^2 = 0.998. \quad (3.3)$$

$$V(t) = \exp(a - bt - ct^{2.5})$$

$$\begin{aligned} a &= 3.64 \\ b &= 4.34 \times 10^{-3} \\ c &= 6.66 \times 10^{-7} \end{aligned}$$

In a 20mL beer in 100mL glass at 24 deg C

## Reference -

S. Sauerbrei, E. C. Haß, P. J. Plath. *The apollonian decay of beer foam bubble size distribution and the lattices of young diagrams and their correlate mixing functions.*

# Our Model

We are measuring number of bubbles instead of volume

But, volume is proportional to the number of bubbles

Approximating,

$$V(t) = C \times N(t)$$

where,  $N(t)$  denotes number of bubbles at time  $t$

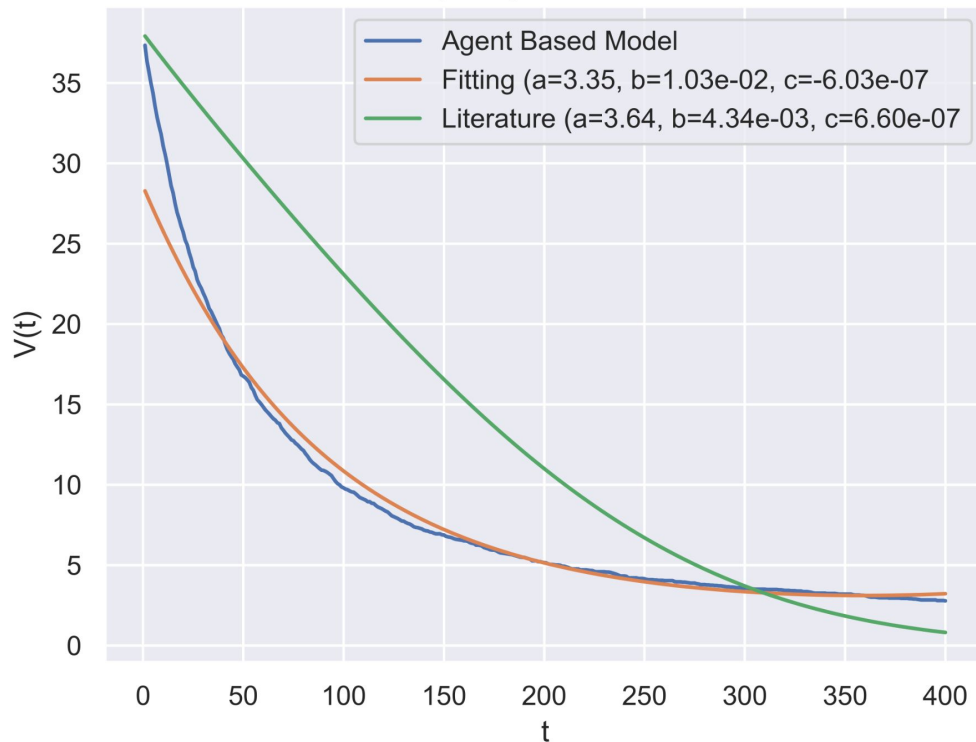
$$\text{But, } V(0) = 38.092$$

Suppose we start our model with 2000 bubbles, taking  $N(0) = 2000$ .

$$\Rightarrow C = 0.019046$$

# Calibrating Parameters

Comparing with Literature



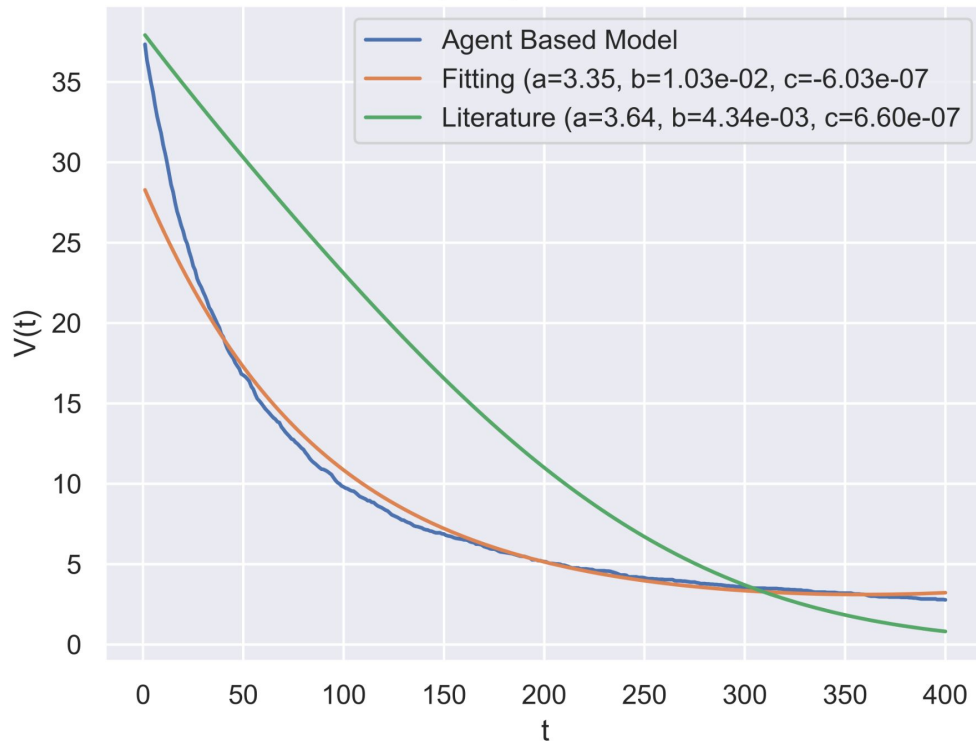
Ground truth by experiments  
 $V(t)$

Calculated  $V(t)$  by using  $V(t) = C \times N(t)$ , where  $N(t)$  is obtained from the agent based model

Fitting  $C \times N(t)$  using curve of the form  $\exp(a - bt - ct^{2.5})$

# Calibrating Parameters

Comparing with Literature



## Observations -

- Similar decay
- $V(t) = f(\dots, N(t), \dots)$  in reality which varies the results
- Currently, time scale from the ground truth (1 sec) is being equated with 1 time step of Netlogo, this could vary.

$$V(t) = C \times N(t')$$

- Value of probability of collision could be varied. Currently, 5%.

# Future Improvements

- Currently, the fluid is stationary. We can add external forces, viscosity, and momentum to the fluid. The interactions of bubbles can then be observed and compared against the current model.
- Two or more bubbles can stick and move together instead of bursting.
- Laws of Physics can be utilized to know the direction of the bubble after colliding or bouncing back.
- The model can be used to answer questions about various properties of the fluid and predict its behavior under certain conditions by studying the interaction of bubbles formed in the fluid. This could help us test the fluids without actually experimenting in a laboratory.

# Thanks

Presented By -  
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