Agent Based Modeling of Collision and Bursting of Bubbles in a Fluid

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Abstract

Bubbles are fascinating entities in nature. We observe them in our daily lives. They indicate numerous properties about a fluid and are relevant to be studied. But, they are fragile and short-lasting in real life. This makes observing and analyzing them challenging.

This article proposes an agent-based modeling approach to model bubbles in a fluid. The primary purpose of the model is to explore the phenomenon of bubble bursting and collision. The model can answer questions such as how the number of bubbles and their dimensions vary with time. The model is simple, yet it shows an emergent pattern. The model has been taken through tough experiments such as analysing outputs from the model and their dependence on the parameters used and calibrating the model against the research outcomes obtained in the literature.

Introduction

In 2001, A Leike [1] published a paper about his observations on the decay of beer foam. He used a cylindrical mug and filled it with beer and observed that the volume of beer froth dV disappearing in a short time dt is proportional to the volume V, i.e.,

$$dV = -(V/\tau)dt \tag{1}$$

Which shows that volume decays exponentially with time.

In 2004, S. Sauerbrei, E. C. Haß, And P. J. Plath [2] published a paper where they presented different methods to characterize foam decay. They investigated the volume dependence $V(t)$ of beer foam decay by measuring the foam heights and level of liquid beer and recording images as a function of time. They averaged arithmetically five independent measurements under the same conditions and approximated them by various curve-fitting calculations. With the highest preference, they always found the following fitting function -

$$V(t) = e^{a-bt-ct^{2.5}} \tag{2}$$

Following these arguments, we can observe that volume of foam which is proportional to the number of bubbles in the foam, follows an exponential or a higher-order exponential decay. Hence, the hypothesis is that naturally, the number of bubbles inside a fluid should also follow an exponential decay.

Method

This article proposes an agent-based modeling approach to model the bursting and collision of bubbles as a complex system. The model has been implemented on Netlogo [3] software.

The bubbles in the model act as agents that move randomly inside the fluid and collide with other bubbles and container walls. New bubbles can also be generated from the bottom of the container.

State Variables of the Model

- Agents: The agents in this model are bubbles that collide with each other and with the container walls.
- Environment: The patches in Netlogo [3] represent the fluid, i.e., the environment where the bubbles exist. The boundaries of the world represent the boundaries of the container.
- State Variables: The current radius, XY-coordinates of the center of the bubble, and heading direction are state variables of each bubble.

Actions

The bubbles in the model perform the following actions in the order in one-time step -

- Bubbles sense their nearest neighbor and check if they are colliding by comparing the Euclidean distance between their centers with the sum of their radius.

  $$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} <= r_1 + r_2 \tag{3}$$

  where, x_1, y_1 and x_2, y_2 are co-ordinates of the bubble in context and neighboring bubble and r_1 and r_2 are their radius respectively.

  Now, the bubble can choose to react to collision by a probability (provided as a parameter by the user). If they decide to collide, then the neighboring bubble bursts. The size of the bubble in context increases by a probability (which is also provided as a parameter by the user). In this case, the size of the bubble in context is increased such that the total area now covered by giant bubble (the bubble in context) is equal to the sum of areas of the two bubbles, i.e.

  $$R^2 = r_1^2 + r_2^2 \tag{4}$$

  where R is the radius of a new, more giant bubble. In real-life this represents as if a giant bubble is formed by merging of two smaller bubbles.
- If the bubbles are colliding with a wall, then they change their direction to a new random direction
- The bubbles slightly change their heading angle and move forward.
- New bubbles are added from the bottom of the container as per incoming bubble rate.

These actions happen over a randomized order of the bubbles in each time step.

Parameters

The user can adjust the following parameters -

- Probability of collision of bubbles if they happen to be colliding.
- Initial number of bubbles in the container.
- Probability of increasing size of the bubble as a result of the collision of two small bubbles.
- Incoming bubble rate from the bottom of the container.

Output Measures

The following outcomes can be observed from the model -

- Number of bubbles with time.
- Distribution of radius of bubbles with time.
- Mean and maximum radius of bubbles with time.

Results

Emergent Pattern

The model's primary results are the number of bubbles and the distribution of radius that show emergent behavior. Significant secondary output are the mean and the maximum radius of bubbles over time.

Figure 1 represents outputs that emerge from how individual bubbles make their decisions, and also from the probability of collision, the probability of increasing the size if they choose to react to collision, and the incoming new bubble rate.

The number of bubbles with time shows more emergence since it has an exponential decay which can indeed not be predicted from individual decisions of agents. Also, the distribution of radius follows a power-law distribution.

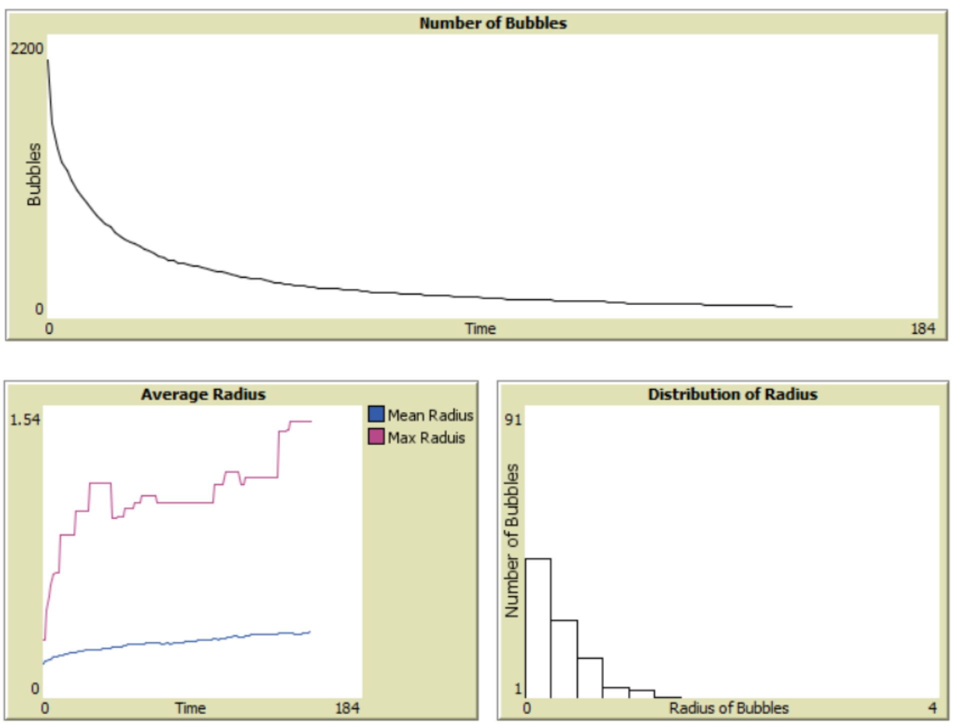

Figure 1: Emergent outputs from the model. Parameters values : probability of collision and probability of increasing size - 70%, initial number of bubbles - 2000, incoming bubble rate - 0

Analyzing Effects of Parameters

The parameters in the model were varied using Behavior Space tool of Netlogo [3] software; the data obtained were plotted using Matplotlib [5] library in Python [4].

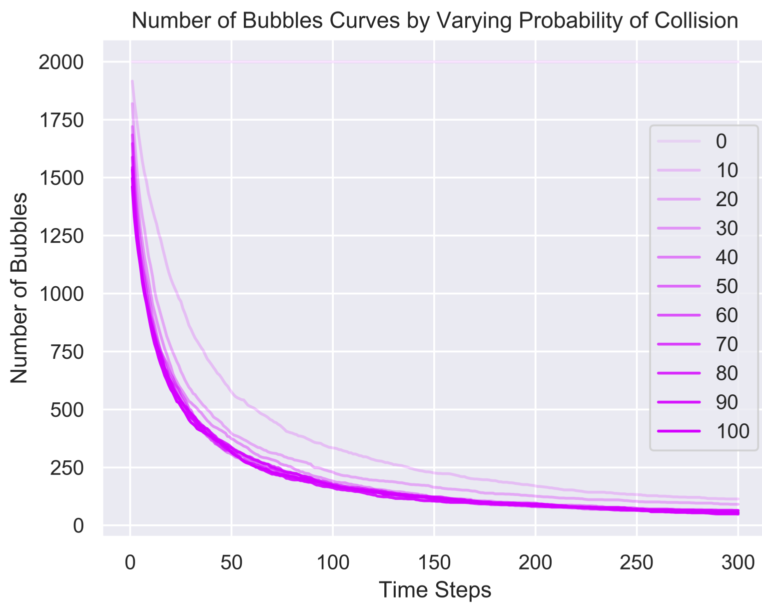

Figure 2: The labels in right represent the probability of collision. Other parameters : probability of increasing size - 50%, initial number of bubbles - 2000, incoming bubble rate - 0

Number of bubbles with time against the probability of collision

In Figure 2, the collision probability was varied while keeping the other parameters constant, and the plot was obtained. The darker curves represent a high probability of collision. We can observe from the plot that the larger the probability of collision, the faster the number of bubbles decay.

Number of bubbles with time against incoming bubble rate

In Figure 3, the incoming bubble rate was varied while keeping the other parameters constant, and the plot was obtained. The darker curves represent high incoming bubble rate. We can observe from the plot that the more significant the value of the incoming bubble rate, the number of bubbles settle to a higher value after decaying. The curves with lower incoming bubble rates have a relatively smaller number of bubbles in the end. Another interesting observation from this plot is that no matter what the incoming bubble rate with each time step, the number of bubbles does not grow; instead, it eventually converges to a constant value that depends on the incoming bubble rate (Provided that the initial number of bubbles is high enough). The new bubbles increase the system's energy with each step, but the system adjusts and stabilizes.

Average radius of the bubble with time against the probability of increasing size

In Figure 4, the probability of increasing size after the collision was varied while keeping the other parameters constant. The darker curves represent a higher probability of increasing size. It can be noticed that the higher the probability, the average radius of bubbles grows at a faster rate.

Discussion

The number of bubbles in the model with time was calibrated to the hypothesis provided by the literature. Assuming that volume of bubbles is proportional to the number of bubbles.

$$V(t) = C \times N(t) \tag{5}$$

Here, N(t) represents the number of bubbles at time t.

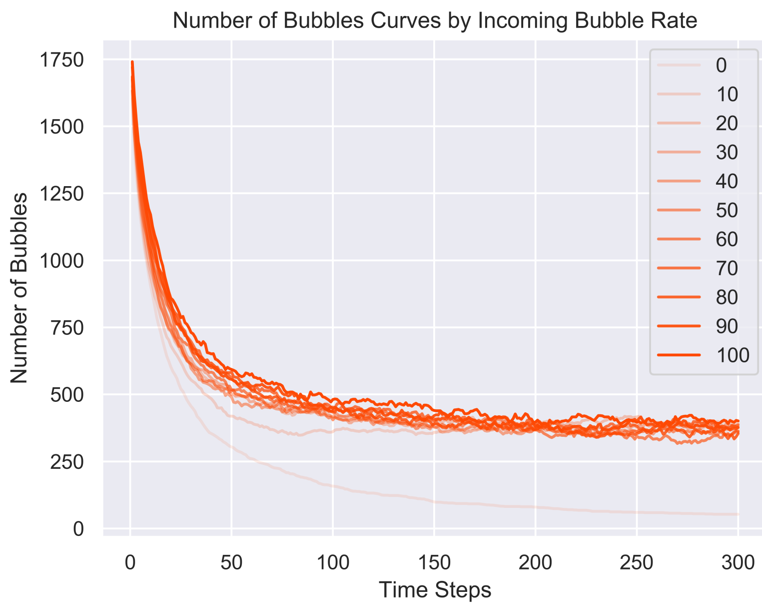

Figure 3: The labels in right represent the incoming bubble rate. Other parameters : probability of collision - 50%, probability of increasing size - 50%, initial number of bubbles - 2000

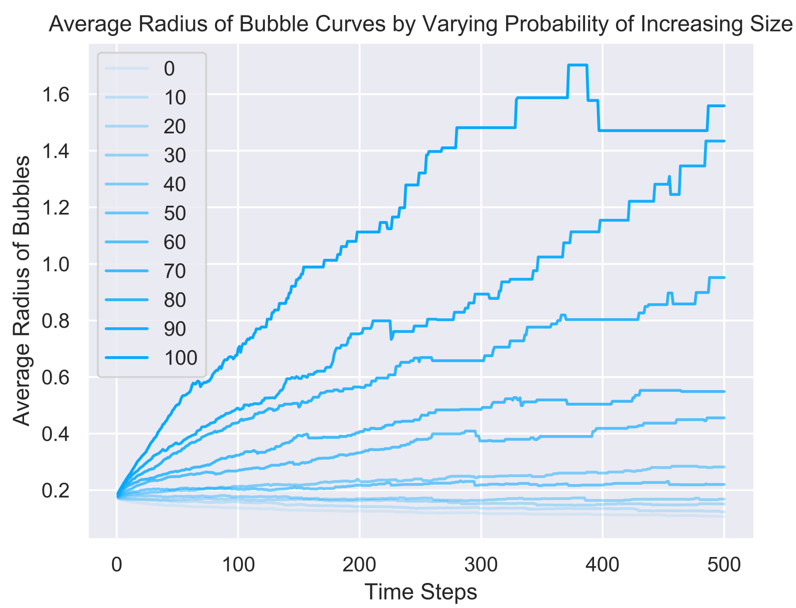

Figure 4: The labels in left represent the probability of increasing size. Other parameters : probability of collision - 50%, initial number of bubbles - 2000, incoming bubble rate - 0

The original values of constants a, b and c in the S. Sauerbrei, E. C. Haß, And P. J. Plath paper [2] are 3.64, 4.34×10^{-3} and 6.66×10^{-7} respectively in 20mL beer in a 100mL glass at temperature 24 degree celcius.

Hence, $V(0) = 38.092$. Now, assuming that the time scale are same and if we run our model with initial value of $N(0) = 2000$, then C becomes 0.019046.

The value of $N(t)$ was obtained from the model with the initial number of bubbles as 2000, probability of collision as 5%, probability of increasing size as 50%, and incoming bubble rate as 0 using Behaviour Space. These values were then converted to corresponding $V(t)$ values using the above equation.

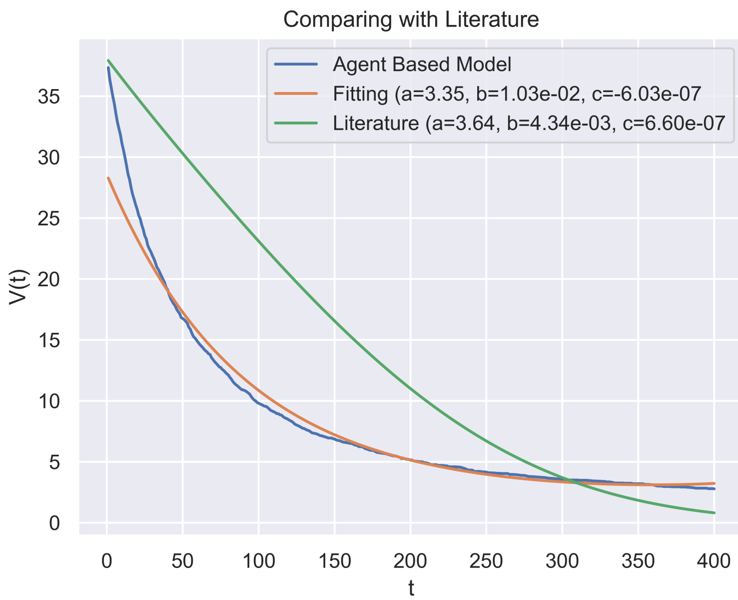

Figure 5: Calibrating Model with Literature

Figure 5 represents three curves. The green curve represents the actual values of $V(t)$ obtained in the paper, the blue curve represents the value of $V(t)$ obtained from $N(t)$ using the model. The orange curve represents the approximation fitting of the blue curve obtained using model to the curve $V(t) = e^{a-bt-ct^{2.5}}$. The approximation has been done by Scipy [6] library in Python [4] and the values of a, b and c obtained for are shown in the plot.

We have assumed that the volume of bubbles is proportional to the number of bubbles; in real life, the volume may depend upon various factors as well. Also, the time scale from the paper (1 second) has been assumed to be equal to a one-time step in Netlogo [3], which may also vary.

Hence, better plots can be obtained using similar environmental conditions in the model as in the experiment. Nevertheless, the two papers suggest an exponential decay of volume and number of bubbles play a significant role in total volume. This verifies the hypotheses mentioned in the literature.

Conclusion

Studying bubbles in real life through experiments can be resource-consuming; hence in this article, we discussed a convenient method to study the complex system of bubbles, which is hard to model using equation-based modeling.

We also showed outputs and dependence of the model on various parameters and successfully verified the hypotheses suggested by the literature.

Future Improvements

- Currently, the fluid is stationary. We can add external forces, viscosity, and momentum to the fluid. The interactions of bubbles can then be observed and compared against the current model.
- Two or more bubbles can stick and move together instead of bursting.

- Laws of Physics can be utilized to know the direction of the bubble after colliding or bouncing back.
- The model can be used to answer questions about various properties of the fluid and predict its behavior under certain conditions by studying the interaction of bubbles formed in the fluid. This could help us test the fluids without actually experimenting in a laboratory.

References

[1] A Leike. *Demonstration of the exponential decay law using beer froth*. Ludwig–Maximilians-Universitat, Sektion Physik, Theresienstr. 37, D-80333 Munchen, Germany.

[2] S. Sauerbrei, E. C. Haß, P. J. Plath. *The apollonian decay of beer foam bubble size distribution and the lattices of young diagrams and their correlate mixing functions*.

[3] Netlogo. `https://ccl.northwestern.edu/netlogo/`.

[4] Python. `https://www.python.org/`.

[5] MatplotLib Library. `https://matplotlib.org/`.

[6] Scipy Library. `https://www.scipy.org/`.