Smoothness
Multivariable Calculus

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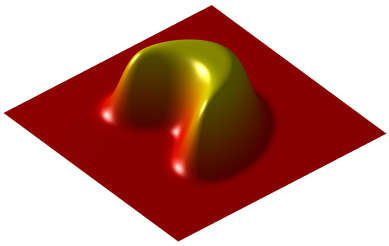
1 Intuitive Sense



For a curve to be considered smooth, it needs to be *smooth*. There cannot be corners.

2 Mathematical Sense

To express that some function $f(x)$ is smooth, we express it with the differentiability class

$$f(x) \in C^{(n)} | n \in \mathbb{N}$$

In other words, all derivative functions of $f(x)$, up to the n^{th} derivative of the function, are continuous.

If a functions is in itself continuous, then

$$f(x) \in C^{(0)}$$

This is also the minimum requirement for a "smooth" curve.

A curve can also be differentiable for derivative of all orders in its domain, where it is called "infinity differentiable"

$$f(x) \in C^{(\infty)}$$

3 Examples

1. $f(x) = 0$ is considered smooth, and also infinitely differentiable, since derivatives of any order is 0.

2. e^x is also smooth and infinitely differentiable, since it stays the same.
3. $\sin(x)$ is also a function that is differentiable for all orders.