

# 積分ノート

Masanari Kimura

## 1 部分積分

### 1.1

$$\begin{aligned}\int x \sin x dx &= -x \cos x + \int \cos x dx \\ &= -x \cos x + \sin x + C\end{aligned}$$

## 2 関数の微分形を含む積分

### 2.1

$$\begin{aligned}\int \frac{1}{x(\log x)^2} dx &= \int \frac{(\log x)'}{(\log x)^2} dx \\ &= -\frac{1}{\log x} + C\end{aligned}$$

### 2.2

$$\begin{aligned}\int \frac{\log(\log x)}{x \log x} dx &= \int \frac{\log t}{t} dt \\ &= \int \log t (\log t)' dt \\ &= \frac{1}{2}(\log t)^2 + C \\ &= \frac{1}{2}(\log \log x)^2 + C\end{aligned}$$

### 2.3

$$\begin{aligned}\int_0^1 (x + 2x^3) \sqrt{1 + 2x^2} dx &= \int_0^1 x(1 + 2x^2) \sqrt{1 + 2x^2} dx \\ &= \int_0^1 x(1 + 2x^2)^{\frac{3}{2}} dx \\ &= \frac{1}{4} \int_0^1 (1 + 2x^2)' (1 + 2x^2)^{\frac{3}{2}} dx \\ &= \frac{1}{4} \left[ \frac{2}{5} (1 + 2x^2)^{\frac{5}{2}} \right]_0^1\end{aligned}$$

$$= \frac{1}{10}(3^{\frac{5}{2}} - 1)$$

### 3 点対称性の利用

#### 3.1

$$\begin{aligned} \int_0^\pi \frac{x \sin x}{8 + \sin^2 x} dx &= \frac{\pi}{2} \int_0^\pi \frac{\sin x}{8 + \sin^2 x} dx \\ &= \frac{\pi}{2} \int_0^\pi \frac{\sin x}{9 - \cos^2 x} dx \\ &= \frac{\pi}{2} \int_{-1}^1 \frac{1}{9 - t^2} dt \\ &= \frac{\pi}{2} \int_{-1}^1 \frac{1}{(3 - t)(3 + t)} dt \\ &= \frac{\pi}{2} \int_{-1}^1 \frac{1}{6} \left( \frac{1}{3 - t} + \frac{1}{3 + t} \right) dt \\ &= \frac{\pi}{12} \left[ -\log |3 - t| + \log |3 + t| \right]_{-1}^1 \\ &= \frac{\pi}{12} \left[ -\log 2 + 2 \log 2 + 2 \log 2 - \log 2 \right] \\ &= \frac{\pi}{6} \log 2 \end{aligned}$$

最初の変形で  $\int_0^\pi x f(\sin x) dx = \frac{\pi}{2} \int_0^\pi f(\sin x) dx$  を使った.

### 4 同形出現

#### 4.1

$$\begin{aligned} \int e^x \sin x dx &= e^x \sin x - \int e^x \cos x dx \\ &= e^x \sin x - \left( e^x \cos x + \int e^x \sin x dx \right) \\ &= e^x \sin x - e^x \cos x - \int e^x \sin x dx \\ 2 \int e^x \sin x dx &= e^x (\sin x - \cos x) \\ \int e^x \sin x dx &= \frac{1}{2} e^x (\sin x - \cos x) \end{aligned}$$

#### 4.2

$$\begin{aligned} \int e^x \cos x dx &= e^x \cos x + \int e^x \sin x dx \\ &= e^x \cos x + e^x \sin x - \int e^x \cos x dx \end{aligned}$$

$$\begin{aligned}
2 \int e^x \cos x dx &= e^x (\sin x + \cos x) \\
\int e^x \cos x dx &= \frac{1}{2} e^x (\sin x + \cos x)
\end{aligned}$$