

積分ノート

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1 置換積分

1.1

$$\begin{aligned}\int_0^\pi \frac{x \sin x}{8 + \sin^2 x} dx &= \frac{\pi}{2} \int_0^\pi \frac{\sin x}{8 + \sin^2 x} dx \\&= \frac{\pi}{2} \int_0^\pi \frac{\sin x}{9 - \cos^2 x} dx \\&= \frac{\pi}{2} \int_{-1}^1 \frac{1}{9 - t^2} dt \\&= \frac{\pi}{2} \int_{-1}^1 \frac{1}{(3 - t)(3 + t)} dt \\&= \frac{\pi}{2} \int_{-1}^1 \frac{1}{6} \left(\frac{1}{3 - t} + \frac{1}{3 + t} \right) dt \\&= \frac{\pi}{12} \left[-\log |3 - t| + \log |3 + t| \right]_{-1}^1 \\&= \frac{\pi}{12} \left[-\log 2 + 2 \log 2 + 2 \log 2 - \log 2 \right] \\&= \frac{\pi}{6} \log 2\end{aligned}$$

最初の変形で $\int_0^\pi x f(\sin x) dx = \frac{\pi}{2} \int_0^\pi f(\sin x) dx$ を使った.