積分ノート

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1　部分積分

1.1

$$\begin{aligned}\int x\sin x dx &= -x\cos x + \int \cos x dx \\ &= -x\cos x + \sin x + C\end{aligned}$$

1.2

$$\begin{aligned}\int xe^{2x}dx &= \frac{1}{2}xe^{2x} - \frac{1}{2}\int e^{2x}dx \\ &= \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C \\ &= \frac{1}{4}e^{2x}(2x-1) + C\end{aligned}$$

2　置換積分

2.1

$$\begin{aligned}\int \frac{1}{x(4-(\log x)^2)} &= \int \frac{1}{4-t^2}dt \\ &= \int \frac{1}{(2-t)(2+t)}dt \\ &= \frac{1}{4}\int \Big(\frac{1}{2-t} + \frac{1}{2+t}\Big)dt \\ &= \frac{1}{4}\Big(-\log|2-t| + \log|2+t|\Big) + C \\ &= \frac{1}{4}\Big(-\log|2-\log x| + \log|2+\log x|\Big)\end{aligned}$$

一行目で $t = \log x$ と置換した.

3　関数の微分形を含む積分

3.1

$$\int \frac{1}{x(\log x)^2}dx = \int \frac{(\log x)'}{(\log x)^2}dx$$

$$= -\frac{1}{\log x} + C$$

3.2

$$\begin{aligned}
\int \frac{\log(\log x)}{x\log x}dx &= \int \frac{\log t}{t}dt \\
&= \int \log t(\log t)' dt \\
&= \frac{1}{2}(\log t)^2 + C \\
&= \frac{1}{2}(\log\log x)^2 + C
\end{aligned}$$

3.3

$$\begin{aligned}
\int_0^1 (x+2x^3)\sqrt{1+2x^2}dx &= \int_0^1 x(1+2x^2)\sqrt{1+2x^2}dx \\
&= \int_0^1 x(1+2x^2)^{\frac{3}{2}}dx \\
&= \frac{1}{4}\int_0^1 (1+2x^2)'(1+2x^2)^{\frac{3}{2}}dx \\
&= \frac{1}{4}\Big[\frac{2}{5}(1+2x^2)^{\frac{5}{2}}\Big]_0^1 \\
&= \frac{1}{10}(3^{\frac{5}{2}}-1)
\end{aligned}$$

4 点対称性の利用

4.1

$$\begin{aligned}
\int_0^\pi \frac{x\sin x}{8+\sin^2 x}dx &= \frac{\pi}{2}\int_0^\pi \frac{\sin x}{8+\sin^2 x}dx \\
&= \frac{\pi}{2}\int_0^\pi \frac{\sin x}{9-\cos^2 x}dx \\
&= \frac{\pi}{2}\int_{-1}^1 \frac{1}{9-t^2}dt \\
&= \frac{\pi}{2}\int_{-1}^1 \frac{1}{(3-t)(3+t)}dt \\
&= \frac{\pi}{2}\int_{-1}^1 \frac{1}{6}\Big(\frac{1}{3-t}+\frac{1}{3+t}\Big)dt \\
&= \frac{\pi}{12}\Big[-\log|3-t|+\log|3+t|\Big]_{-1}^1 \\
&= \frac{\pi}{12}\Big[-\log 2+2\log 2+2\log 2-\log 2\Big] \\
&= \frac{\pi}{6}\log 2
\end{aligned}$$

最初の変形で $\int_0^\pi xf(\sin x)dx = \frac{\pi}{2}\int_0^\pi f(\sin x)dx$ を使った.

5 同形出現

5.1

$$\begin{aligned}
\int e^x \sin x dx &= e^x \sin x - \int e^x \cos x dx \\
&= e^x \sin x - \left(e^x \cos x + \int e^x \sin x dx\right) \\
&= e^x \sin x - e^x \cos x - \int e^x \sin x dx \\
2\int e^x \sin x dx &= e^x(\sin x - \cos x) \\
\int e^x \sin x dx &= \frac{1}{2}e^x(\sin x - \cos x)
\end{aligned}$$

5.2

$$\begin{aligned}
\int e^x \cos x dx &= e^x \cos x + \int e^x \sin x dx \\
&= e^x \cos x + e^x \sin x - \int e^x \cos x dx \\
2\int e^x \cos x dx &= e^x(\sin x + \cos x) \\
\int e^x \cos x dx &= \frac{1}{2}e^x(\sin x + \cos x)
\end{aligned}$$