UNIVERSITY PARTNER

UNIVERSITY OF WOLVERHAMPTON

HERALD COLLEGE KATHMANDU

4MM013 - Computational Mathematics

Final Examination

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Date of Submission	: August 26, 2022

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ASSIGNMENT 2

1. $2x_1 + x_2 - x_3 = 0$

$x_1 + x_3 = 4$

$x_1 + x_2 + x_3 = 0$

Now, writing the following set of equations in matrix form,

$$\begin{bmatrix} 2 & 1 & -1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix}$$

Now,

$$\Delta = \begin{vmatrix} 2 & 1 & -1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= 2 \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} - 1 \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} - 1 \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}$$

$= -2 - 0 - 1$

$= -3$

$\because$ ~~Hence~~, $-3 \neq 0$

$$\therefore \Delta x_1 = \begin{vmatrix} 0 & 1 & -1 \\ 4 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix}$$

$$= 0 \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} - 1 \begin{vmatrix} 4 & 1 \\ 0 & 1 \end{vmatrix} - 1 \begin{vmatrix} 4 & 0 \\ 0 & 1 \end{vmatrix}$$

$= 0 - 4 - 4$

$= -8$

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$$\Delta x_2 = \begin{vmatrix} 2 & 0 & -1 \\ 1 & 4 & 1 \\ 1 & 0 & 1 \end{vmatrix}$$

$$= 2\begin{vmatrix} 4 & 1 \\ 0 & 1 \end{vmatrix} - 0\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & 4 \\ 1 & 0 \end{vmatrix}$$

$$= 8 - 0 + 4$$

$$= 12$$

$$\Delta x_3 = \begin{vmatrix} 2 & 1 & 0 \\ 1 & 0 & 4 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= 2\begin{vmatrix} 0 & 4 \\ 1 & 0 \end{vmatrix} - 1\begin{vmatrix} 1 & 4 \\ 1 & 0 \end{vmatrix} + 0\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}$$

$$= -8 + 4 + 0$$

$$= -4$$

Finally,

$$x_1 = \frac{\Delta x_1}{\Delta} \Rightarrow \frac{-8}{-3} \Rightarrow \frac{8}{3}$$

$$x_2 = \frac{\Delta x_2}{\Delta} \Rightarrow \frac{12}{-3} \Rightarrow -4$$

$$x_3 = \frac{\Delta x_3}{\Delta} \Rightarrow \frac{-4}{-3} \Rightarrow \frac{4}{3}$$

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2. a) $x_1 + x_2 + x_3 = 2$
$2x_1 + 3x_2 + 4x_3 = 3$
$x_1 - 2x_2 - x_3 = 1$

Now, writing the following sets of equations in matrix form,

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 2 & 3 & 4 & 3 \\ 1 & -2 & -1 & 1 \end{array}\right]$$

$R_2 \rightarrow R_2 - 2R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 2 & -1 \\ 1 & -2 & -1 & 1 \end{array}\right]$$

$R_3 \rightarrow R_3 - R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 2 & -1 \\ 0 & -3 & -2 & -1 \end{array}\right]$$

$R_3 \rightarrow R_3 + 3R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 4 & -4 \end{array}\right]$$

Hence, we can write,

$x_1 + x_2 + x_3 = 2 \rightarrow$ ①
$x_2 + 2x_3 = -1 \rightarrow$ ②
$4x_3 = -4 \rightarrow$ ③
$\therefore x_3 = -1$

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Putting the value of x_3 in eq^n (2), we get

$$x_2 + 2x_3 = -1$$
$$x_2 + 2(-1) = -1$$
$$x_2 = 1$$

Putting the value of x_3 & x_2 in eq^n (1), we get

$$x_1 + x_2 + x_3 = 2$$
$$x_1 + 1 - 1 = 2$$
$$x_1 = 2$$

Finally,

$$x_1 = 2$$
$$x_2 = 1$$
$$x_3 = -1$$

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b) Now to find the inverse of the matrix,

Writing in matrix form,

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 2 & 3 & 4 & 0 & 1 & 0 \\ 1 & -2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$R_2 \rightarrow R_2 - 2R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -2 & 1 & 0 \\ 1 & -2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$R_3 \rightarrow R_3 - R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -2 & 1 & 0 \\ 0 & -3 & -2 & -1 & 0 & 1 \end{array}\right]$$

$R_3 \rightarrow R_3 + 3R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -2 & 1 & 0 \\ 0 & 0 & 4 & -7 & 3 & 1 \end{array}\right]$$

$R_2 \rightarrow R_2 - 1/2\, R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 3/2 & -1/2 & -1/2 \\ 0 & 0 & 4 & -7 & 3 & 1 \end{array}\right]$$

$R_1 \rightarrow R_1 - 1/4\, R_3$

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$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 11/4 & -3/4 & -1/4 \\ 0 & 1 & 0 & 3/2 & -1/2 & -1/2 \\ 0 & 0 & 4 & -7 & 3 & 1 \end{array}\right]$$

$R_1 \rightarrow R_1 - R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 5/4 & -1/4 & 1/4 \\ 0 & 1 & 0 & 3/2 & -1/2 & -1/2 \\ 0 & 0 & 4 & -7 & 3 & 1 \end{array}\right]$$

$R_3 \rightarrow R_3/4$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 5/4 & -1/4 & 1/4 \\ 0 & 1 & 0 & 3/2 & -1/2 & -1/2 \\ 0 & 0 & 1 & -7/4 & 3/4 & 1/4 \end{array}\right]$$

$\therefore$ The inverse of the matrix is

$$A^{-1} = \begin{bmatrix} 5/4 & -1/4 & 1/4 \\ 3/2 & -1/2 & -1/2 \\ -7/4 & 3/4 & 1/4 \end{bmatrix}$$

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3. $t_n = (-1)^{n+1} \dfrac{n+1}{n^2+3}$

$a_n = \dfrac{n+1}{n^2+3}$

$\lim_{n \to \infty} a_n = \lim_{n \to \infty} \dfrac{n+1}{n^2+3}$

$\lim_{n \to \infty} a_n = \lim_{n \to \infty} \dfrac{\cancel{n}(1+1/n)}{\cancel{n}(n+3/n)}$

$= \lim_{n \to \infty} \dfrac{(1+1/n)}{(n+3/n)}$

$= \dfrac{1+1/\infty}{\infty+3/\infty}$

$= \dfrac{1}{\infty}$

$= 0$

Hence, the series converges.

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4. Maclaurin series expansion of Sin$x$,

$f(x) = \sin x$

For Maclaurin series expansion,

$$f(x) = f(0) + f'(0)x + f''(0)\frac{x^2}{2!} + f'''(0)\frac{x^3}{3!} + f^{iv}(0)\frac{x^4}{4!} + f^{v}(0)\frac{x^5}{5!} + \ldots$$

$f(x) = \sin x$, $f(0) = 0$
$f'(x) = \cos x$, $f'(0) = 1$
$f''(x) = -\sin x$, $f''(0) = 0$
$f'''(x) = -\cos x$, $f'''(0) = -1$
$f^{iv}(x) = \sin x$, $f^{iv}(0) = 0$
$f^{v}(x) = \cos x$, $f^{v}(0) = 1$

Now,

$$f(x) = f(0) + f'(0)x + f''(0)\frac{x^2}{2!} + f'''(0)\frac{x^3}{3!} + f^{iv}(0)\frac{x^4}{4!} + f^{v}(0)\frac{x^5}{5!}$$

$$= 0 + x + 0 - \frac{x^3}{3!} + 0 + \frac{x^5}{5!}$$

$$= \sum_{n=0}^{\infty} (-1)^{n} \frac{x^{2n+1}}{(2n+1)!}$$

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Now, calculating the radius of convergence using the ratio test,

$$\Rightarrow \lim_{n \to \infty} \left| \frac{a_n + 1}{a_n} \right|$$

$$\Rightarrow \lim_{n \to \infty} \left| \frac{\dfrac{x^{2(n+1)}}{\{2(n+1)+1\}!}}{\dfrac{x^{2n}}{(2n+1)!}} \right|$$

$$\Rightarrow \lim_{n \to \infty} \left| \frac{\dfrac{x^{2n+2}}{(2n+3)!}}{\dfrac{x^{2n}}{(2n+1)!}} \right|$$

$$\Rightarrow \lim_{n \to \infty} \frac{x^{2n+2}}{(2n+3)!} \times \frac{(2n+1)!}{x^{2n}}$$

$$\Rightarrow \lim_{n \to \infty} \frac{x^2}{(2n+3)!(2n+2)!}$$

$$\Rightarrow 0$$