Mathematical Grammar

Math 111 is a writing course. This statement may surprise you, but it is true. The writing you do is mathematical writing, which is different from the writing you do in other courses but still has a grammar that you need to follow carefully.

There are two types of objects you encounter when doing math:

- *Expressions*: These are things like $x^2 + 1$ or $\lim_{x\to 1} \sqrt{1+x^3}$. Expressions are the mathematical analog of nouns.
- *Statements*: These are the mathematical analogs of sentences. An example of a statement is "$\lim_{x\to 1} \sqrt{1+x^3} = \sqrt{2}$". This has two nouns ($\lim_{x\to 1} \sqrt{1+x^3}$ and $\sqrt{2}$) connected by a verb (the equal sign). The key fact is that statements can be true or false.

I see often on student papers work that looks like

$$\lim_{x\to 2} \frac{x^2-4}{x^2-6x+8} \to \frac{(x-2)(x+2)}{(x-2)(x-4)} \to \frac{x+2}{x-4} \to \frac{2+2}{2-4} = \frac{4}{-2} = -2.$$

This has multiple problems. First, what does "$\to$" mean? For many of you, it means "and next I did". Unfortunately, this is not part of the grammar of mathematics. So you need something else.

If we convert the "$\to$" to "$=$", we get

$$\lim_{x\to 2} \frac{x^2-4}{x^2-6x+8} = \frac{(x-2)(x+2)}{(x-2)(x-4)} = \frac{x+2}{x-4} = \frac{2+2}{2-4} = \frac{4}{-2} = -2.$$

This is better, because it is now a statement. Unfortunately, it is a false statement: the first and third equal signs are false. For example, in the first equal sign, the expression on the left is the limit $\lim_{x\to 2} \frac{x^2-4}{x^2-6x+8}$, which here is just a *number*. But the expression on the right of the first equal sign is the *function* $\frac{(x-2)(x+2)}{(x-2)(x-4)}$. These can't be equal. The third equal sign is false for similar reasons.

Here are two ways to turn the above into literate mathematics:

1. First, you can write a series of equalities that links the initial problem to the final answer:

$$\lim_{x\to 2} \frac{x^2-4}{x^2-6x+8} = \lim_{x\to 2} \frac{(x-2)(x+2)}{(x-2)(x-4)} = \lim_{x\to 2} \frac{x+2}{x-4} \overset{\text{DSP}}{=} \frac{2+2}{2-4} = \frac{4}{-2_{\neq 0}} = -2.$$

2. Second, you can do the algebra on a clearly labeled separate line and then evaluate the limit at the end:

$$\begin{aligned} &\lim_{x\to 2} \frac{x^2-4}{x^2-6x+8} \\ &\text{fn inside the limit} = \frac{(x-2)(x+2)}{(x-2)(x-4)} = \frac{x+2}{x-4} \\ &\text{hence } \lim_{x\to 2} \frac{x^2-4}{x^2-6x+8} = \lim_{x\to 2} \frac{x+2}{x-4} \overset{\text{DSP}}{=} \frac{2+2}{2-4} = \frac{4}{-2_{\neq 0}} = -2. \end{aligned}$$

Note how I added "$\neq 0$" in both solutions to indicate correct use of DSP.

Finally, let me comment on $\Rightarrow$, which means "implies." This is used to connect *statements*, such as

$$x^2 = 1 \;\Rightarrow\; x = \pm 1.$$

This says that if the first statement is true, then so is the second. You *cannot* put $\Rightarrow$ between expressions. It is meaningless to write

$$\frac{(x-2)(x+2)}{(x-2)(x-4)} \;\Rightarrow\; \frac{x+2}{x-4}.$$

Be sure you understand why.