(This is a modified version of Harris Daniels's Midterm 1 practice test from Fall 2018)

1. True or False: (No justification necessary.)

   (a) [3 points] There exists a set of three vectors $\{\vec{u}, \vec{v}, \vec{w}\} \subseteq \mathbb{R}^2$ that is linearly independent. **T** **F**

   (b) [3 points] Every system of linear equations has at least one solution. **T** **F**

   (c) [3 points] The set of solutions to a system of linear equations in n unknowns is a subspace of $\mathbb{R}^n$. **T** **F**

   (d) [3 points] The set $\{f \in C^1(\mathbb{R}) \mid f + f' = 0\}$ is a subspace of $C^1(\mathbb{R})$. **T** **F**

2. Let V be a vector space, and let $S \subseteq V$. Define the following terms and phrases. You may use other standard terms without defining them.

   (a) [5 points] The set S spans V.

   (b) [5 points] The set S is linearly independent.

   (c) [5 points] The set S is a basis of V.

3. [15 points] Find a set of vectors spanning the set of solutions to the following system of equations.

$$\begin{cases} x_1 + 2x_2 \phantom{{}+4x_3} - x_4 = 0 \\ -2x_1 - 3x_2 + 4x_3 - 5x_4 = 0 \\ 2x_1 + 4x_2 \phantom{{}+4x_3} - 2x_4 = 0 \end{cases}$$

4. (a) [5 points] Let $S_1 = \{(1,1,1),(1,1,0),(1,0,0)\} \subseteq \mathbb{R}^3$. Is S_1 linearly independent? Why or why not?

(b) [5 points] Let $S_2 = \{1 + x + x^2, 2 - x, 3 - 2x + x^2, x - 2x^2\} \subseteq P_2(\mathbb{R})$. Is S_2 linearly independent? Why or why not?

5. [15 points] Let $S = \{1+2x+3x^2, 1+x^2+x^3, x^2+x^3\} \subseteq P_3(\mathbb{R})$. Is $3+2x+4x^2+x^3 \in \mathrm{Span}(S)$? Justify your answer.

6. [15 points] Let V be a vector space and let S and T be subsets of V. Show that if $\mathrm{Span}(S) = V$ and $S \subseteq \mathrm{Span}(T)$, then $\mathrm{Span}(T) = V$.