Goal estimate functions and definite integrals by Taylor approximations and the ASET. Also review interval and radius of convergence.
Reference: §11.10, 11.11

Note For these problems, you should use a handy theorem that we'll mention in class on Friday. Here it is, in case you are starting early (see also p. 775 of the textbook):

Alternating Series Estimation Theorem (ASET): If $\sum_{n=0}^{\infty}(-1)^n a_n$ is an *alternating* series that passes the three conditions of the alternating series test (a_n positive, a_n decreasing, $a_n \to 0$), then the error of partial sum $\sum_{n=1}^{m}(-1)^n a_n$ is less than a_{n+1}.

The "error of the partial sum" means the difference between it and the sums. Less formally: the error of any partial sum is less than the absolute value of the next term in the series.

1. Use Series to Estimate $\frac{1}{e}$ with error less than $\frac{1}{20}$. Justify.

2. Use Series to Estimate $\frac{1}{e}$ with error less than $\frac{1}{100}$. Justify. (Can reuse work from 1)

3. Use Series to Estimate $\frac{1}{e}$ with error less than $\frac{1}{500}$. Justify. (Can reuse work from 1)

4. Use Series to Estimate $\sin(1)$ with error less than $\frac{1}{1000}$. Justify.

5. Use Series to Estimate $e^{-\frac{1}{3}}$ with error less than $\frac{1}{100}$. Justify.

6. Use Series to Estimate $\arctan\left(\frac{1}{2}\right)$ with error less than $\frac{1}{100}$. Justify.

7. Use Series to Estimate $\int_0^1 x\ln(1+x^3)\,dx$ with error less than $\frac{1}{20}$. Justify.

8. Use Series to Estimate $\int_0^1 x\sin(x^2)\,dx$ with error less than $\frac{1}{1000}$. Justify.

Review: Find the Interval and Radius of Convergence for each of the following.

9. $\sum_{n=1}^{\infty}(n!)^2(3x-7)^n$

10. $\sum_{n=1}^{\infty}\frac{(-1)^n\,(5x-2)^n}{n^3\,8^n}$

11. $\sum_{n=1}^{\infty}\frac{(x-7)^n}{n!\,\sqrt{n}}$