How to reduce the example from class to RREF:
(not RREF yet)

$$\begin{bmatrix} 0 & 1 & -1 & 3 & 4 & | & 33 \\ ②& 1 & 7 & 3 & -1 & | & 22 \\ -2 & 0 & -8 & 0 & -1 & | & -31 \\ 4 & -1 & 17 & -3 & 7 & | & 92 \end{bmatrix}$$

swap, $R_1 \leftrightarrow R_2$

To get that 2 as the first pivot.

$-2\times \quad +1\times$

$$\begin{bmatrix} ② & 1 & 7 & 3 & -1 & | & 22 \\ 0 & 1 & -1 & 3 & 4 & | & 33 \\ -2 & 0 & -8 & 0 & -1 & | & -31 \\ 4 & -1 & 17 & -3 & 7 & | & 92 \end{bmatrix}$$

use the pivot "2" to eliminate all other first-column entries.

$R_3 + R_1 \longrightarrow R_3$
$R_4 - 2R_1 \longrightarrow R_4$

$+3\times \quad -1\times$

$$\begin{bmatrix} 2 & 1 & 7 & 3 & -1 & | & 22 \\ 0 & ① & -1 & 3 & 4 & | & 33 \\ 0 & 1 & -1 & 3 & -2 & | & -9 \\ 0 & -3 & 3 & -9 & 9 & | & 48 \end{bmatrix}$$

use the pivot "1" in row 2 to eliminate all nonzero entries below it.

$R_3 - R_2 \longrightarrow R_3$
$R_4 + 3 \cdot R_2 \longrightarrow R_4$

$+\frac{7}{2}\times$

$$\begin{bmatrix} 2 & 1 & 7 & 3 & -1 & | & 22 \\ 0 & 1 & -1 & 3 & 4 & | & 33 \\ 0 & 0 & 0 & 0 & ⑦-6 & | & -42 \\ 0 & 0 & 0 & 0 & 21 & | & 147 \end{bmatrix}$$

use the pivot "-6" in row 3 to cancel the "21" below it.

$R_4 + \frac{7}{2} R_3 \longrightarrow R_4$

$$\begin{bmatrix} 2 & 1 & 7 & 3 & -1 & | & 22 \\ 0 & 1 & -1 & 3 & 4 & | & 33 \\ 0 & 0 & 0 & 0 & -6 & | & -42 \\ 0 & 0 & 0 & 0 & 0 & | & 0 \end{bmatrix}$$

and we obtain the REF matrix from the handout in class.

How to reduce the example from class from REF
to reduced REF (RREF):

scale by $\frac{1}{2}$
$$\left[\begin{array}{ccccc|c} \boxed{2} & 1 & 7 & 3 & -1 & 22 \\ 0 & 1 & -1 & 3 & 4 & 33 \\ 0 & 0 & 0 & 0 & \boxed{-6} & -42 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$
scale by $\frac{-1}{6}$

Turn the non-1 pivots
to 1 by scaling rows.
$$\frac{1}{2}R_1 \longrightarrow R_1$$
$$-\frac{1}{6}R_3 \longrightarrow R_3$$

$-\frac{1}{2}\times$
$$\left[\begin{array}{ccccc|c} 1 & \frac{1}{2} & \frac{7}{2} & \frac{3}{2} & -\frac{1}{2} & 11 \\ 0 & \boxed{1} & -1 & 3 & 4 & 33 \\ 0 & 0 & 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

eliminate entries above the
row-2 pivot.
$$R_1 - \tfrac{1}{2}R_2 \longrightarrow R_1$$

$+\frac{5}{2}\times$
$-4\times$
$$\left[\begin{array}{ccccc|c} 1 & 0 & 4 & 0 & -\frac{5}{2} & -\frac{11}{2} \\ 0 & 1 & -1 & 3 & 4 & 33 \\ 0 & 0 & 0 & 0 & \boxed{1} & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

eliminate entries above the
row-3 pivot.
$$R_1 + \tfrac{5}{2}R_3 \longrightarrow R_1$$
$$R_2 - 4R_3 \longrightarrow R_2$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & 4 & 0 & 0 & 12 \\ 0 & 1 & -1 & 3 & 0 & 5 \\ 0 & 0 & 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

and we obtain the
RREF matrix
from the handout.