Suggested reading for this week (from the textbook):
Study items:

- Definitions: *linear combination*, *span*.
- Proofs: basic facts about subspaces, linear combinations, and the span of a set of vectors.
- Understand what is means to say that a subspace is generated by a set of vectors
- Definitions: *linearly dependent*, *linearly independent*.
- How do you determine whether a set of vectors is linearly dependent or linearly independent?
- Proofs: basic facts about linearly dependent and independent sets of vectors.

Problems:

1. *(Damiano–Little 1.2.3(h,i))* ** subspace? polys with $p(\sqrt{2}) = 0$; polys with $p(1) = 1$ and $p(2) = 0$.
2. *(Damiano–Little 1.2.5)* Let W be a subspace of a vector space V, let $\vec{y} \in V$, and define the set $\vec{y} + W = \{\vec{x} \in V : \vec{x} = \vec{y} + \vec{w} \text{ for some } \vec{w} \in W\}$. Show that $\vec{y} + W$ is a subspace of V if and only if $y \in W$.
3. *(Damiano–Little 1.2.8)* ** Continuous functions $C([a, b])$ is a subspace of the vector space $F([a, b])$ introduced in Exercise 1.1.10.
4. ** Define an *affine subspace* of a vector space. Prove that defn is equivalent to translation of a subspace, or to set of differences being a subspace. Example: $x + y + z = 1$. Also $f''(x) = -f(x)$ and $f(0) = 1$. In 1.2.3 above, which of the times you answered "no" is this thing an affine subspace instead?
5. *(Damiano–Little 1.3.1(a,d))* ** Examples of spans in $\mathbb{R}^3$ and $P_4(\mathbb{R})$; give another name for both sets.

Note. In part (d), ignore the word "geometrically;" just give another name for the set $\mathrm{Span}(S)$.

6. *(Damiano–Little 1.3.3)* In $V = P_2(\mathbb{R})$, let $S = \{1, 1 + x, 1 + x + x^2\}$. Show that $\mathrm{Span}(S) = P_2(\mathbb{R})$.
7. *(Damiano–Little 1.3.6(a))* ** Show that $W_1 \cap W_2 \supset \mathrm{Span}(S_1 \cap S_2)$.
8. *(Damiano–Little 1.3.7)* Show that if S is a subset of a vector space V and W is a subspace of V that contains S, then $\mathrm{Span}(S) \subset W$.
9. *(Damiano–Little 1.4.1(a,b,c,e))* ** Determine whether LI or LD... in $\mathbb{R}^2, \mathbb{R}^3, \mathbb{R}^3, P_2(\mathbb{R})$.
10. *(Damiano–Little 1.4.5)* Let $\vec{v}, \vec{w} \in V$ Show that $\{v, w\}$ is linearly independent if and only if $\{\vec{v} + \vec{w}, \vec{v} - \vec{w}\}$ is linearly independent.
11. *(Damiano–Little 1.4.8)* Let W_1, W_2 be subspaces of a vector space, satisfying $W_1 \cap W_2 = \{\vec{0}\}$. Show that if $S_1 \subset W_1$ and $S_2 \subset W_2$ are linearly independent sets, then their union $S_1 \cup S_2$ is linearly independent.
12. *(Damiano–Little 1.4.9(a))* ** If S is LI and spans V, then adding any vector results in an LD set.

Extra practice (not to hand in)

- *(Damiano–Little 1.2.6)*
- *(Damiano–Little 1.2.10)*
- *(Damiano–Little 1.3.5)*
- *(Damiano–Little 1.3.6)*
- *(Damiano–Little 1.3.8)*
- *(Damiano–Little 1.4.2)*
- *(Damiano–Little 1.4.9(b))*