The first block are the "official" axioms and inference rule for our propositional calculus. The remaining lines are all facts that can be proved from these axioms and inference rules, and which we are free to use as inference rules in "abbreviated deductions."

Most of these were given names in class, listed in the "shorthand" column. Some of them were stated in class or homework but not officially given a shorthand. For easier reference, I have given each one a shorthand below; those marked with an asterisk are those that were not given a shorthand in class.

Shorthand	Hypotheses		Conclusion	Rule name	proved
$(P1)$	$\emptyset$	$\vdash_P$	$\alpha \vee \alpha \to \alpha$		
$(P2)$	$\emptyset$	$\vdash_P$	$\alpha \to \alpha \vee \beta$		
$(P3)$	$\emptyset$	$\vdash_P$	$\alpha \vee \beta \to \beta \vee \alpha$		
$(P4)$	$\emptyset$	$\vdash_P$	$(\beta \to \gamma) \to (\alpha \vee \beta \to \alpha \vee \gamma)$		
(MP)	$\alpha,\ \alpha \to \beta$	$\vdash_P$	β	modus ponens	
$(COMP)$	$\alpha \to \beta,\ \beta \to \gamma$	$\vdash_P$	$\alpha \to \gamma$	composition	2/26
(CW)	$\alpha \to \gamma,\ \beta \to \gamma$	$\vdash_P$	$\alpha \vee \beta \to \gamma$	casework	2/28
$(CONTRA1)$	$\alpha \to \neg\beta$	$\vdash_P$	$\beta \to \neg\alpha$	contrapositive	2/28
$(CONTRA2)^*$	$\alpha \to \beta$	$\vdash_P$	$\neg\beta \to \neg\alpha$	"	PS5 4
$(EM1)$	$\emptyset$	$\vdash_P$	$\neg\alpha \vee \alpha$	excluded middle	2/28
$(EM2)$	$\emptyset$	$\vdash_P$	$\alpha \vee \neg\alpha$	"	2/28
$(DN1)$	$\emptyset$	$\vdash_P$	$\alpha \to \neg\neg\alpha$	double negation	2/28
$(DN2)$	$\emptyset$	$\vdash_P$	$\neg\neg\alpha \to \alpha$	"	3/1
(RAA)	$\alpha \to \beta,\ \alpha \to \neg\beta$	$\vdash_P$	$\neg\alpha$	reductio ad absurdum	3/1
$(RAA-2)^*$	$\neg\alpha \to \beta,\ \alpha \to \neg\beta$	$\vdash_P$	α	"	3/1
(IH)	$\emptyset$	$\vdash_P$	$\beta \to (\alpha \to \beta)$	intro. of hypothesis	3/4
(MPH)	$\alpha \to \gamma,\ \alpha \to (\gamma \to \delta)$	$\vdash_P$	$\alpha \to \delta$	MP w/ hypothesis	3/4
$(P2.5)$	$\emptyset$	$\vdash_P$	$\alpha \to \beta \vee \alpha$		PS4 5(b)
$(P4.5)$	$\emptyset$	$\vdash_P$	$(\beta \to \gamma) \to (\beta \vee \alpha \to \gamma \vee \alpha)$		(omitted)
$(\wedge 1)^*$	$\emptyset$	$\vdash_P$	$\alpha \wedge \beta \to \alpha$		PS5 2(a)
$(\wedge 2)^*$	$\emptyset$	$\vdash_P$	$\alpha \wedge \beta \to \beta$		PS5 2(b)
$(\wedge 3)^*$	$\emptyset$	$\vdash_P$	$\alpha \to (\beta \to \alpha \wedge \beta)$		PS5 2(c)
$(ASSOC1)^*$	$\emptyset$	$\vdash_P$	$(\alpha \vee \beta) \vee \gamma \to \alpha \vee (\beta \vee \gamma)$		PS5 3(a)
$(ASSOC2)^*$	$\emptyset$	$\vdash_P$	$\alpha \vee (\beta \vee \gamma) \to (\alpha \vee \beta) \vee \gamma$		PS5 3(b)
$(CH)^*$	$\alpha \to (\beta \to \gamma)$	$\vdash_P$	$\alpha \wedge \beta \to \gamma$	comb. of hypotheses	PS5 6(a)
$(SH)^*$	$\alpha \wedge \beta \to \gamma$	$\vdash_P$	$\alpha \to (\beta \to \gamma)$	sep. of hypotheses	PS5 6(b)
$(RH)^*$	$\alpha \to (\beta \to \gamma)$	$\vdash_P$	$\beta \to (\alpha \to \gamma)$	reorder hypotheses	3/27

In addition to all of these inference rules, we also have an important meta-theorem, the "Propositional Deduction Theorem" (PDT), proved in class on 3/4, which says:

$$(PDT) \quad \Gamma, \alpha \vdash_P \beta \quad \Leftrightarrow \quad \Gamma \vdash_P \alpha \to \beta.$$