

# Math 271, Linear Algebra, Fall 2022

## Midterm 1 Practice Test 1

(This is a modified version of Harris Daniels's Midterm 1 practice test from Fall 2016)

### Instructions:

- You may not use notes, books, calculators, cell phones or any other aids.
- You must show all your work to get full credit.
- You have 50 minutes to complete the exam.

### Answer the following questions:

1. Let  $V = \mathbb{R}^+$  be the vector space whose objects are the positive real numbers with addition and scalar multiplication operations defined by

$$x \oplus y = xy, \quad c \odot x = x^c.$$

- (a) Prove that 1 is the additive identity for the vector space  $V$ .
  - (b) Prove one of the two distributive laws for the vector space  $V$ .
  - (c) Prove  $V = \text{span}(\{e\})$  where  $e$  is the base of the natural logarithm function  $\ln(x)$ .
  - (d) What is the dimension of  $V$ ? Justify your answer.
2. Is the vector  $(4, 0, 6, 9)$  in the span of the set  $\{(2, 1, 0, 0), (0, 1, 0, 0), (0, 1, -2, -3)\}$ ? Justify your answer.
3. Put the following linear system into echelon form and use your answer to write down an expression for the solution set in terms of free variables

$$\begin{array}{rrrrr} x_2 & + & 2x_3 & - & x_4 & + & x_5 & = & 1 \\ x_1 & & & + & 4x_3 & & & + & x_5 & = & 2 \\ x_1 & - & 2x_2 & & & & & & & = & 0 \end{array}$$

4. Let  $V$  be a vector space and suppose  $\{\mathbf{u}, \mathbf{v}\}$  is a basis for  $V$ . Prove that  $\{2\mathbf{u} - \mathbf{v}, \mathbf{u} + \mathbf{v}\}$  is also a basis for  $V$ .
5. Find a basis for the subspace of  $P_2(\mathbb{R})$  given by  $W = \{f \in P_2(\mathbb{R}) \mid f'(2) = 0\}$ . Find a set that spans  $W$ . (Note:  $f'$  here refers to the derivative of  $f$ .)