

Reading Stewart §1.1-2.

1. Prove that every subfield $F \subseteq \mathbb{C}$ contains \mathbb{Q} .
2. Let $\phi : \mathbb{C} \rightarrow \mathbb{C}$ be any field automorphism (i.e. an isomorphism from the field to itself).
 - (a) Prove that $\phi(a) = a$ for every $a \in \mathbb{Q}$.
 - (b) Prove that $\phi(i)^2 + 1 = 0$, and therefore that $\phi(i) = \pm i$.
 - (c) Conclude that the only automorphisms of \mathbb{C} are the identity and complex conjugation.
3. Generalizing the previous problem: let $F \subseteq \mathbb{C}$ be a subfield, and $\alpha \in \mathbb{C}$ a complex number such that $\alpha \notin F$ but $\alpha^2 \in F$. Prove that there are exactly two automorphisms $\phi : F(\alpha) \rightarrow F(\alpha)$ such that $\phi(a) = a$ for all $a \in F$.
4. Prove that $\{1, \sqrt{2}\}$ is a linearly independent set over \mathbb{Q} .
5. Denote by ζ_n the complex number $\zeta_n = e^{2\pi i/n}$. Prove that $\mathbb{Q}[\zeta_3] = \mathbb{Q}[\sqrt{-3}]$.
6. Prove that $\mathbb{Q}(\sqrt{2}, \sqrt{3}) = \mathbb{Q}(\sqrt{2} + \sqrt{3})$.