

Note Like last week, I will add a couple more problems after Friday's class.

1. Write out a complete (not abbreviated, this time) deduction in our proposition calculus of formula $\alpha \rightarrow \neg\neg\alpha$. Note that for this problem, you should write a deduction in full, i.e. each line is a premise, an axiom, or follows from previous lines by modus ponense. **In all other problems on this problem set, it is enough to write an “abbreviated deduction” as we have defined in class.**
2. Recall that, in our formulation of first-order logic, the \wedge symbol is an *abbreviation*: $\alpha \wedge \beta$ is always understood to be *literally the same formula* as $\neg(\neg\alpha \vee \neg\beta)$. Give an abbreviated deduction (in the sense stated in class) showing each of the following. Together these give the main tools for interacting with the \wedge symbol.
 - (a) $\vdash \alpha \wedge \beta \rightarrow \alpha$
 - (b) $\vdash \alpha \wedge \beta \rightarrow \beta$
 - (c) $\vdash \alpha \rightarrow (\beta \rightarrow \alpha \wedge \beta)$
3. Give an abbreviated deduction for each of the following *associativity* formulas for \vee . Note: if you wish, you may prove one and then use it as a premise in your deduction of the other.
 - (a) $\vdash (\alpha \vee \beta) \vee \gamma \rightarrow \alpha \vee (\beta \vee \gamma)$
 - (b) $\vdash \alpha \vee (\beta \vee \gamma) \rightarrow (\alpha \vee \beta) \vee \gamma$
4. We proved one version of “contraposition” in class, namely

$$\alpha \rightarrow \neg\beta \vdash \beta \rightarrow \neg\alpha.$$

Prove the following other version of contraposition:

$$\alpha \rightarrow \beta \vdash \neg\beta \rightarrow \neg\alpha.$$

You may assume in your argument that we have already demonstrated the two “double negation” rules (DN1) and (DN2) from class.

5+ One or two problems will be added to this set after Friday's class.