

**Reading** Stewart §1.1-2.

1. Prove that every subfield  $F \subseteq \mathbb{C}$  contains  $\mathbb{Q}$ .
2. Let  $\phi : \mathbb{C} \rightarrow \mathbb{C}$  be a field automorphism (i.e. an isomorphism from the field to itself), and assume that  $\phi(a) = a$  for all  $a \in \mathbb{R}$  (such  $\phi$  is called a  $\mathbb{R}$ -automorphism of  $\mathbb{C}$ ). Prove that  $\phi(i)^2 + 1 = 0$ , and therefore that  $\phi(i) = \pm i$ . Conclude that the only automorphisms of  $\mathbb{C}$  are the identity and complex conjugation.
3. Generalizing the previous problem: let  $F \subseteq \mathbb{C}$  be a subfield, and  $\alpha \in \mathbb{C}$  a complex number such that  $\alpha \notin F$  but  $\alpha^2 \in F$ . Prove that there are exactly two automorphisms  $\phi : F(\alpha) \rightarrow F(\alpha)$  such that  $\phi(a) = a$  for all  $a \in F$ .
4. Prove that  $\{1, \sqrt{2}\}$  is a linearly independent set over  $\mathbb{Q}$ .
5. Denote by  $\zeta_n$  the complex number  $\zeta_n = e^{2\pi i/n}$ . Prove that  $\mathbb{Q}[\zeta_3] = \mathbb{Q}[\sqrt{-3}]$ .
6. Prove that  $\mathbb{Q}(\sqrt{2}, \sqrt{3}) = \mathbb{Q}(\sqrt{2} + \sqrt{3})$ .