

**Note.** This “problem set” need not be handed in, since you have an exam this week. You should still work through the problems, but rather than requiring you to submit solutions, I will post sample solutions in the middle of next week for you to check over your work.

**Textbook reading** for this week:

- §4.3 (geometry in  $\mathbb{R}^n$ )
- (Not required for the last exam) the last several classes will discuss material up to the *spectral theorem for symmetric matrices*, which provides a theoretical foundation for principal components analysis (as described in the guest lecture earlier). In our book, this material occupies sections §4.4-4.6, so you may be interested in skimming those sections to learn more.

**Study items:**

- Decide if a given matrix, or linear transformation, is diagonalizable.
- For a diagonalizable matrix  $A$ , find an invertible matrix  $Q$  and diagonal matrix  $D$  such that  $A = QDQ^{-1}$ .
- Calculate the geometric and algebraic multiplicity of an eigenvalue of a matrix or linear transformation.
- Know the definition of the *standard inner product* or *dot product* on  $\mathbb{R}^n$ , and the two most common notations for it:  $\langle \vec{v}, \vec{w} \rangle$  and  $\vec{v} \cdot \vec{w}$ .
- How do you use the dot product to measure lengths and angles in  $\mathbb{R}^n$ ?
- What does it mean for a set of vectors to be *orthogonal* or *orthonormal*? Why does this imply that they are linearly independent?

**Problems:**

1. (*Damiano–Little 4.2.1(a)*) (determine whether  $2 \times 2$  matrix is diagonalizable, and find basis of eigenvalues if so)
2. (*Damiano–Little 4.2.3*) (necessary and sufficient conditions for a  $3 \times 3$  upper-triangular matrix to be diagonalizable)
3. (*Damiano–Little 4.2.6(a)*) (if  $A$  is diagonalizable, then so is  $A^k$ )
4. (*Damiano–Little 4.2.14(a)*) (eigenvectors and commuting matrices)
5. Calculate the matrix  $A^{10}$  where  $A$  is the matrix  $\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 2 \end{bmatrix}$ . (Hint: use the fact that  $A$  is diagonalizable to write  $A$  as  $QDQ^{-1}$ . Also:  $2^{10} = 1024$ .)
6. (*Damiano–Little 4.3.1(a,b,c)*) (inner products and norms in  $\mathbb{R}^3$ )
7. (*Damiano–Little 4.3.7*) (determine whether a set of vectors is orthogonal)
8. (*Damiano–Little 4.3.8*) (an orthogonal set of vectors is linearly independent)
9. (*Damiano–Little 4.4.4*) (if vectors are orthogonal, then a Pythagorean theorem holds)
10. (*Damiano–Little Ch 4 supplemental 4(a,c)*) (diagonalizable? If so, give diagonal matrix and change of basis;  $2 \times 2$  and  $3 \times 3$ )