1. Denote by N the list of axioms for number theory on page 68 our our textbook. Denote by $\mathcal{H}$ the *Henkin structure* of N (as defined in class). Refamiliarize yourself with the notation in that section if necessary, especially the "canonical term" $\overline{n}$. You may freely use anything proved in that section, e.g. Lemma 2.8.4, without proving it again.

   (a) Prove that for every term t in $\mathcal{L}_{NT}$, there exists a canonical term $\overline{n}$ such that $N \vdash t = \overline{n}$.

   (b) Prove that the universe of $\mathcal{H}$ may be identified with the set $\mathbb{N}$ of natural numbers.

   (c) Prove that for any sentence σ, $\mathcal{N} \vDash \sigma$ if and only if $N \vdash \sigma$. Here $\mathcal{N}$ denotes the standard structure on $\mathcal{L}_{NT}$. So the Henkin construction faithfully captures the "usual" semantics of number theory (remember that this is by no means guaranteed!).

   Hint In part (a), use induction on term structure. Part (b) will follow quickly, once you check that all canonical terms give distinct equivalence classes. In part (c), use induction on complexity of the sentence.