

1. Use the definition of continuity and the properties of limits to explain (briefly) why $g(x) = \frac{3x^2 - 7x}{x - 7}$ is continuous at $x = 4$.

Note As always, actually follow the instructions! In particular, to get full credit, you'll need to write out the formal definition of what it means to say that g is continuous at $x = 4$, and then use properties of limits to verify that the mathematical statement you just wrote out is true. Note that you'll be providing more detail that will be needed in future assignments, where it will be fine to simply say that this function is continuous on its domain.

2. For each of the following functions, do three things: sketch the graph, compute the limits from the left and right at $x = 2$, and classify the discontinuity (if any) at $x = 2$ as removable, jump, or infinite.

$$\text{a) } f(x) = \begin{cases} \frac{1}{x-2} & \text{if } x \neq 2, \\ 4 & \text{if } x = 2. \end{cases} \quad \text{b) } g(x) = \begin{cases} \cos(x-2) & \text{if } x < 2, \\ 0 & \text{if } x = 2, \\ x^2 - 3 & \text{if } x > 2. \end{cases}$$

3. Find a number a so that the function

$$h(x) = \begin{cases} \frac{x^2 - 4x + 3}{x^2 - 9} & \text{if } x \neq 3, \\ a & \text{if } x = 3 \end{cases}$$

is continuous at $x = 3$. Don't forget to (briefly) justify why your choice of a makes h continuous at $x = 3$.

4. Find the domain of the function $f(x) = \frac{\sin x}{(x+4)^2}$ and explain, using the theorems of Sections 1.6 and 1.8, why f is continuous on its domain.

5. Use properties of continuity to compute $\lim_{x \rightarrow \pi} \cos(x + \sin x)$

6. Show that the function $g(x) = \begin{cases} \cos x & \text{if } x \leq \pi/4 \\ \sin x & \text{if } x > \pi/4 \end{cases}$ is continuous on the whole real line $(-\infty, \infty)$.

7. Classify the discontinuities each of the following functions. That is, list each value of x where the function is discontinuous, and identify it as removable, jump, or infinite, with justification.

a) $\frac{x^2 - 10x + 24}{x^2 - 6x + 8}$

b) $\frac{x^2 - 9}{|x + 2|}$

c) $\frac{\sqrt{x+1} - 2}{|x - 3|}$

d) $\frac{1}{x} - \frac{1}{|x|}$