

Topics trig. subs. involving secant and tangent; completing the square; partial fractions
Reference: §7.3, §7.4

Examples to study first

Example Evaluate $\int \frac{6}{x^2 + 2x + 8} dx$.

Solution

$$\begin{aligned}
 \int \frac{6}{x^2 + 2x + 8} dx &\stackrel{\text{complete square}}{=} \int \frac{6}{(x+1)^2 + 7} dx & \begin{array}{l} u = x+1 \\ du = dx \end{array} \\
 &= 6 \int \frac{1}{u^2 + 7} du \\
 &= 6 \left(\frac{1}{\sqrt{7}} \right) \arctan \left(\frac{u}{\sqrt{7}} \right) + C \\
 &\quad (\text{"a-rule", or by substituting } v = u/\sqrt{7}) \\
 &= \boxed{\frac{6}{\sqrt{7}} \arctan \left(\frac{x+1}{\sqrt{7}} \right) + C}
 \end{aligned}$$

Example Evaluate $\int_{-3}^1 \frac{6}{x^2 + 2x - 8} dx$.

Solution

$$\begin{aligned}
 \int_{-3}^1 \frac{6}{x^2 + 2x - 8} dx &\stackrel{\text{factor}}{=} \int_{-3}^1 \frac{6}{(x-2)(x+4)} dx \\
 &\stackrel{\text{PFD}}{=} \int_{-3}^1 \frac{1}{x-2} - \frac{1}{x+4} dx & (\text{see algebra below}) \\
 &= \ln|x-2| - \ln|x+4| \Big|_{-3}^1 \\
 &= \ln|-1| - \ln 5 - (\ln|-5| - \ln 1) \\
 &= 0 - \ln 5 - \ln 5 + 0 \\
 &= \boxed{-2 \ln 5}
 \end{aligned}$$

Partial Fractions Decomposition:

$$\frac{6}{(x-2)(x+4)} = \frac{A}{x-2} + \frac{B}{x+4}$$

Clearing the denominator yields:

$$6 = A(x+4) + B(x-2) = Ax + 4A + Bx - 2B = (A+B)x + (4A-2B)$$

so that $A+B=0$ and $4A-2B=6$.

Solve to obtain $A=1$ and $B=-1$.

Problems to hand in

Compute each of the following Integrals. Simplify when possible.

1. $\int \frac{1}{\sqrt{4-4x-x^2}} dx$

2. $\int_{-1}^1 \frac{1}{x^2+4x+7} dx$

3. $\int \sqrt{3-2x-x^2} dx$

4. $\int \frac{x+4}{x^2+2x+5} dx$

5. $\int_3^5 \frac{6}{x^2-4x+7} dx$

6. $\int_0^3 \frac{6}{x^2-4x-5} dx$

7. $\int_0^1 \frac{x-4}{x^2-5x+6} dx$

8. $\int \frac{\arctan x}{x^2} dx = \int \arctan x \cdot (x^{-2}) dx$

9. $\int_{\ln 2}^{\ln 5} \frac{2e^x}{e^{2x}-1} dx$

10. $\int \frac{10}{(x-1)(x^2+9)} dx$

↑ hint:
$$\begin{aligned} \frac{2}{u^2-1} &= \frac{2}{(u-1)(u+1)} \\ &= \frac{A}{u-1} + \frac{B}{u+1} \end{aligned}$$

↑ use PFD
$$\frac{10}{(x-1)(x^2+9)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+9}$$