Suggested reading for this week (from the textbook):

- Chapter 1.

Study items:

- Vector spaces are the most important objects in linear algebra. (This is why we do them right at the beginning so you have plenty of time to get used to them.) You should know by heart the eight properties that make up the definition of a vector space.
- Understand the phrases "for all" and "there exists" in the definition of a vector space. These phrases are extremely important in all of mathematics. What do these phrases tell you about the task of proving (or disproving!) that a given set and operations form a vector space?
- Notation: $\mathbb{R}^n$, $F(\mathbb{R})$, $P_n(\mathbb{R})$, $\mathbf{x}$ or $\vec{x}$ for vectors.
- Notation: $\mathbf{0}$ or $\vec{0}$.
- Be able to prove facts that are true in any vector space.
- Know, and be able to use, the definition of a subspace (of a vector space).
- How do you prove that a given subset is, or is not, a subspace?

Problems:

1. *(Damiano–Little 1.1.1)* **Explicit linear combinations in $\mathbb{R}^3$, parts (a)-(c).
2. *(Damiano–Little 1.1.2)* **Explicit linear combinations in $F(\mathbb{R})$, parts (a)-(c).
3. *(Damiano–Little 1.1.3, properties 4,6 only)* Complete the proof that $\mathbb{R}^n$, with the operations given in Example (1.1.2), is a vector space. ***reword to just include properties 4,6.
4. *(Damiano–Little 1.1.4, properties 3.8 only)* Complete the proof that $\mathbb{R}^n$, with the operations given in Example (1.1.2), is a vector space. ***reword to just include properties 3,8.
5. *(Damiano–Little 1.1.6(a,c))* ***Decide if $\mathbb{R}^2$, with a given bizarro operation, is a vector space. If not, state which axiom fails to hold.
6. *(Damiano–Little 1.1.7(c))* ** Similar, in $F(\mathbb{R})$.
7. *(Damiano–Little 1.1.8)* Show that in any vector space V,
   (a) If $\vec{x}, \vec{y}, \vec{z} \in V$, then $\vec{x} + \vec{y} = \vec{x} + \vec{z}$ implies $\vec{y} = \vec{z}$.
   (b) If $\vec{x}, \vec{y} \in V$ and $a, b \in \mathbb{R}$, then $(a + b)(\vec{x} + \vec{y}) = a\vec{x} + b\vec{x} + a\vec{y} + b\vec{y}$.
8. *(Damiano–Little 1.2.1)* Show that for any vector space V, the set $\{\vec{0}\}$ is a subspace of V. (It is sometimes called the *trivial subspace*)

Note. The solution in the book is incomplete, because it does not explain how we know that $c\vec{0} = \vec{0}$ for any $c \in \mathbb{R}$. You should prove this as part of your answer.

9. *(Damiano–Little 1.2.2(a,b))* ** Even functions as a subspace of $F(\mathbb{R})$
10. *(Damiano–Little 1.2.3(c,d))* ** Determine whether $(a_1 + a_2 + a_3)^2 = 0$ and $a_3 \geq 0$ define subspaces.

11. Show that the set of solutions to $f''(x) = -f(x)$ forms a vector space. **reword, choose notation.

Extra practice (not to hand in)

- *(Damiano–Little 1.1.3, all properties)*
- *(Damiano–Little 1.1.4, all properties)*
- *(Damiano–Little 1.1.6(b))*
- *(Damiano–Little 1.1.7(a,b))*
- *(Damiano–Little 1.1.10)*
- *(Damiano–Little 1.1.11)*
- *(Damiano–Little 1.2.2)*
- *(Damiano–Little 1.2.3)*
- *(Damiano–Little 1.2.8)*
- *(Damiano–Little 1.2.9)*