1. Prove that if L/K is a finite field extension, and $\phi : K \to \Omega$ is a nonzero field homomorphism from K to an algebraically closed field Ω, then there exists an extension of K to L. That is, there exists a field homomorphism $\psi : L \to \Omega$ such that $\psi|_K = \phi$.

   Hint First consider the case where L/K is a simple extension. Then use induction to obtain the general result.

2. Prove the following extension of Problem 7 on PSet 4: if L/K is a finite extension in $\mathbb{C}$, $\alpha \in L$, and α' is a K-conjugate of α, then there exists a homomorphism (not necessarily unique) $\phi : L \to \mathbb{C}$ such that $\phi(\alpha) = \alpha'$.

3. Recall that we called a field extension L/K *Galois* if, for every $\alpha \in L$, all K-conjugates $\alpha' \in \mathbb{C}$ of α are also in L. This definition can be unwieldy since there are infinitely many choices of α. Show that it suffices to check only α in a generating set. That is: if $L = K(\alpha_1, \cdots, \alpha_n)$, and L contains all K-conjugates of $\alpha_1, \cdots, \alpha_n$, then L is Galois.

4. Prove that a degree 2 field extension in $\mathbb{C}$ is always Galois.

5. We showed in class that $\mathbb{Q}(\sqrt[3]{2})/\mathbb{Q}$ is not Galois. Show that $\mathbb{Q}(\sqrt[3]{2}, \zeta_3)/\mathbb{Q}$, however, is Galois. What is the degree of that extension?