Amherst College
Department of Mathematics and Statistics

MATH 272 | MIDTERM 2 | 18 APRIL 2025

NAME:____

Read This First!

- The exam uses **both sides of the page**.
- Keep cell phones off and out of sight.
- Do not talk during the exam.
- You are allowed one page of notes, front and back. No other books, notes, calculators, cell phones, communication devices of any sort, webpages, or other aids are permitted.
- Show **ALL** work clearly in the space provided or on the blank pages.
- In order to receive full credit on a problem, solution methods must be complete, logical and understandable.
- You may cite any theorems proved in class or on the homework in your proofs, except in cases where the statement to be proved is essentially the same as a theorem proved earlier. In that case you should write out the full proof. Please ask me if you are uncertain about whether you should prove a theorem or if it is enough to cite it.

Grading - For Instructor Use Only

Question:	1	2	3	4	5	$\sum$
Points:	12	12	12	12	12	60
Score:						

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1. [12 points] Suppose that $B = \{\vec{u}, \vec{v}, \vec{w}\}$ is a basis for a vector space V. Prove that $B' = \{\vec{u}, \vec{v} + \vec{w}, \vec{v} - \vec{w}\}$ is also a basis for V.

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2. [12 points] Consider the following set of vectors in $\mathbb{R}^5$.

$$S = \left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 7 \\ 2 \\ 5 \end{bmatrix}, \begin{bmatrix} 4 \\ 9 \\ 1 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \right\}.$$

Find a basis for the span of S and determine $\dim \operatorname{span}(S)$.

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3. [12 points] Consider the following 3×4 matrix.

$$A = \begin{bmatrix} 1 & -2 & 1 & 5 \\ 0 & 1 & -2 & -5 \\ 1 & 0 & -3 & -5 \end{bmatrix}$$

Let $W \subseteq \mathbb{R}^4$ denote the set

$$W = \left\{ \vec{x} \in \mathbb{R}^4 : \; A\vec{x} = \vec{0} \right\}.$$

(a) Show that W is a subspace of $\mathbb{R}^4$.

(b) Find a basis for W and determine $\dim W$.

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4. [12 points] Define three vectors as follows.

$$\vec{x} = \begin{bmatrix} -1 \\ 0 \\ 2 \\ 3 \end{bmatrix}, \quad \vec{1} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad \vec{y} = \begin{bmatrix} 0 \\ 0 \\ 2 \\ 2 \end{bmatrix}.$$

For the values $a, b \in \mathbb{R}$ minimizing $\|a\vec{x} + b\vec{1} - \vec{y}\|^2$. In other words, for which a, b is $a\vec{x} + b\vec{1}$ as close as possible to $\vec{y}$?

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5. [12 points] The following two sets are both bases of $\mathbb{R}^3$ (you do not need to prove this).

$$B = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}, \quad B' = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \right\}$$

Find the change of basis matrix from B to B', i.e. the matrix $[I]_B^{B'}$.

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