Note Like last week, I will add a couple more problems after Friday's class.

1. Write out a complete (not abbreviated, this time) deduction in our proposition calculus of formula $\alpha \to \neg\neg\alpha$. Note that for this problem, you should write a deduction in full, i.e. each line is a premise, an axiom, or follows from previous lines by modus ponense. **In all other problems on this problem set, it is enough to write an "abbreviated deduction" as we have defined in class.**

2. Recall that, in our formulation of first-order logic, the $\wedge$ symbols is an *abbreviation*: $\alpha \wedge \beta$ is always understood to be *literally the same formula* as $\neg(\neg\alpha \vee \neg\beta)$. Give an abbreviated deduction (in the sense stated in class) showing each of the following. Together these give the main tools for interacting with the $\wedge$ symbol.

   (a) $\vdash \alpha \wedge \beta \to \alpha$

   (b) $\vdash \alpha \wedge \beta \to \beta$

   (c) $\vdash \alpha \to (\beta \to \alpha \wedge \beta)$

3. Give an abbreviated deduction for each of the following *associativity* formulas for $\vee$. Note: if you wish, you may prove one and then use it as a premise in your deduction of the other.

   (a) $\vdash (\alpha \vee \beta) \vee \gamma \to \alpha \vee (\beta \vee \gamma)$

   (b) $\vdash \alpha \vee (\beta \vee \gamma) \to (\alpha \vee \beta) \vee \gamma$

4. We proved one version of "contraposition" in class, namely

$$\alpha \to \neg\beta \vdash \beta \to \neg\alpha.$$

   Prove the following other version of contraposition:

$$\alpha \to \beta \vdash \neg\beta \to \neg\alpha.$$

   You may assume in your argument that we have already demonstrated the two "double negation" rules (DN1) and (DN2) from class.