Company induced systemic risk in the Austrian economy

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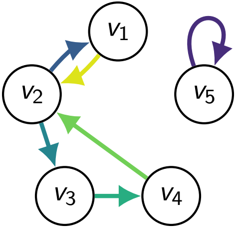
Graphs

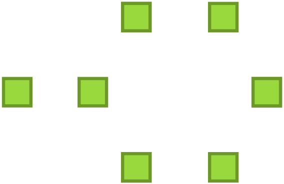


$$A_{5\times 5} = \begin{array}{c} \\ \\ \\ \\ \\ \\ \end{array}\begin{array}{cc} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} & \\ \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} & \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \end{array}$$

A graph and its adjacency matrix representation.

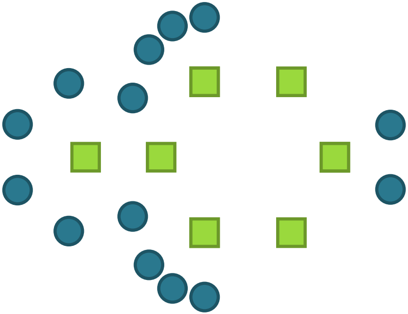
Financial networks

- Network of entities: banks



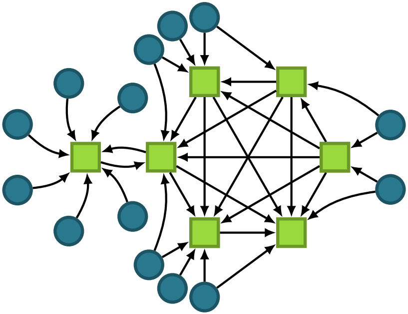
Financial networks

- Network of entities: banks, companies, . . .



Financial networks

- Network of entities: banks , companies , ...
- Connected by financial dependencies: loans



Financial networks

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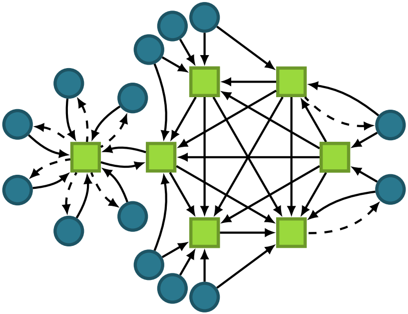

Systemic Risk

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  - Spreads in network along liability–edges $\Rightarrow$ Contagion Risk
  - Nodes that lend to risky nodes inherit some of the risk

Systemic risk

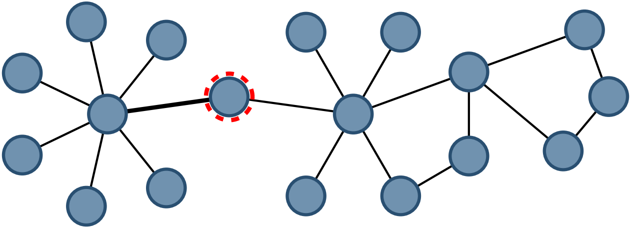
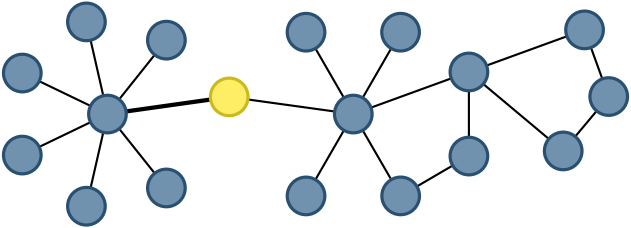


Figure: Propagation from market shock on marked node

Systemic risk



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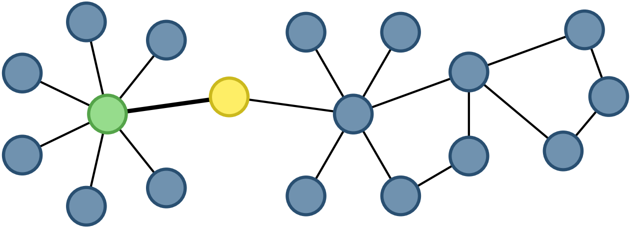

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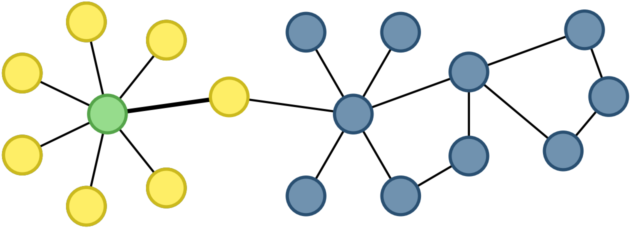


Figure: Propagation from market shock on marked node

Diffusion equation

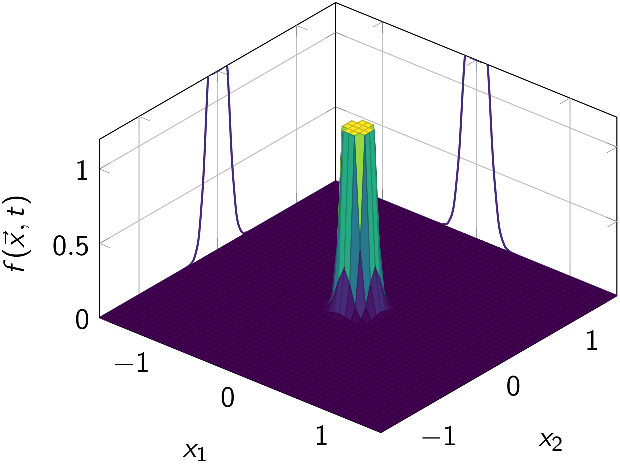
- Particles in a solution start with initial distribution $f(x,t)|_{t=0} = f(x,0) = \delta(x - x_0)$
- Frequent statistically independent collisions with solvent molecules cause Brownian motion
- Particle density $f(x,t)$ obeys diffusion equation derived by [Einstein, 1905]

$$\frac{\partial f}{\partial t} = D\frac{\partial^2 f}{\partial x^2}$$

- Analytical solution

$$f(x,t) = \frac{n}{\sqrt{4\pi Dt}} \exp\left(-\frac{(x - x_0)^2}{4Dt}\right)$$

Diffusion in 2 dimensions



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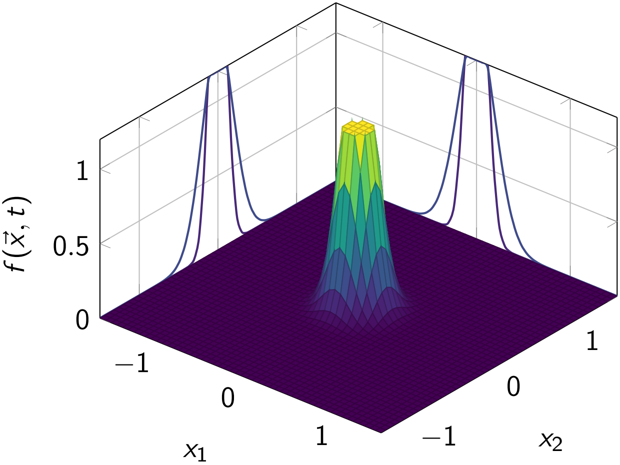

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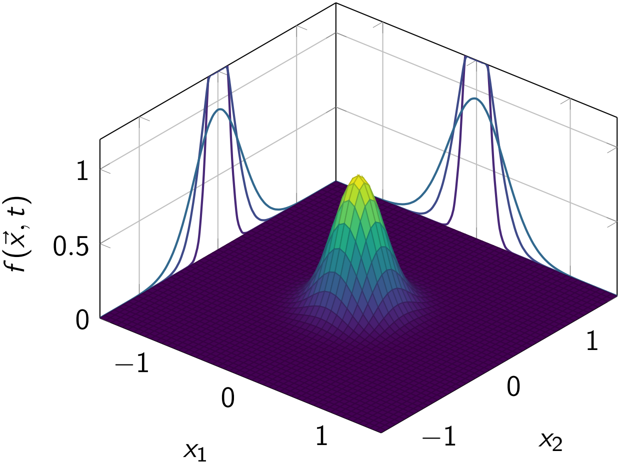

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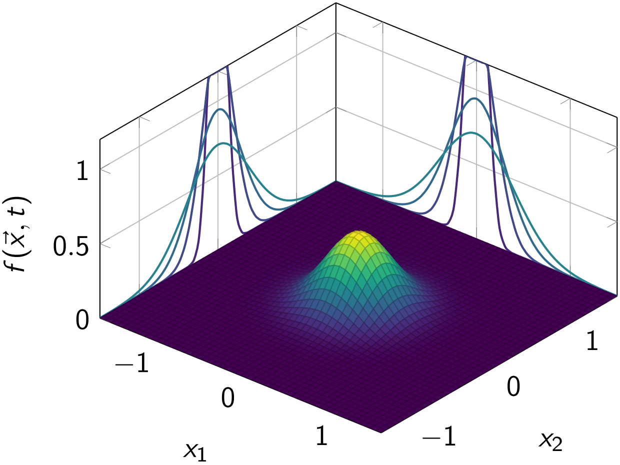

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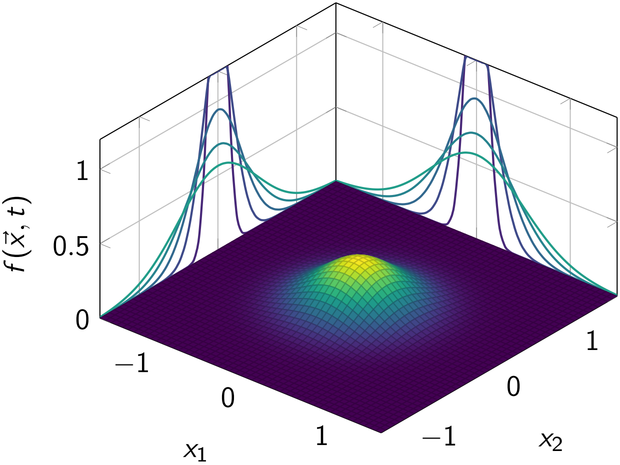

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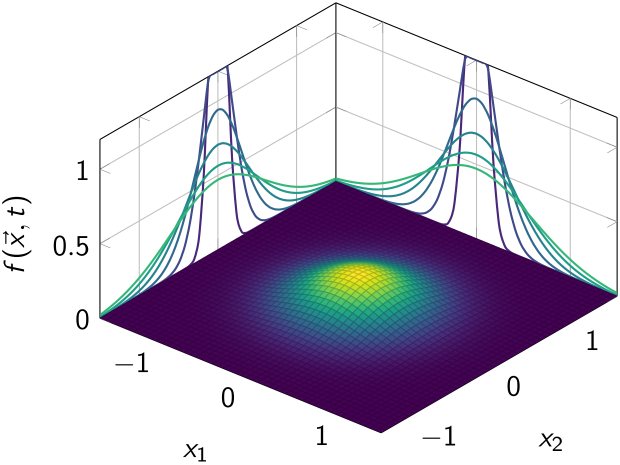

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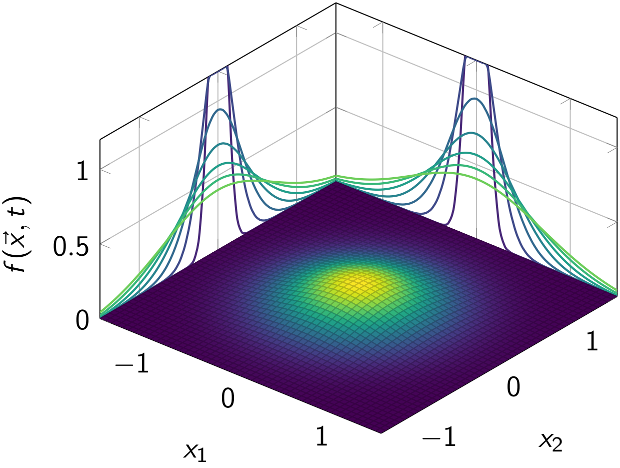

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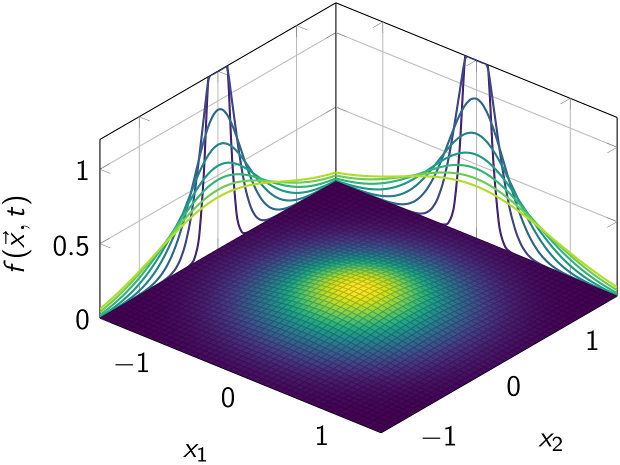
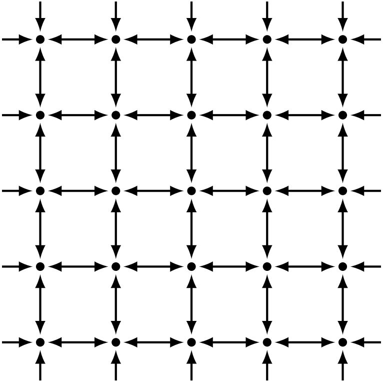
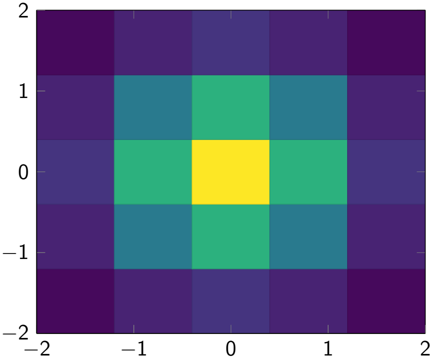


Discrete lattice





Degree centrality

Centrality measures try to measure importance of a node according to some criterion.

Source: [Jackson and Watts, 2002]

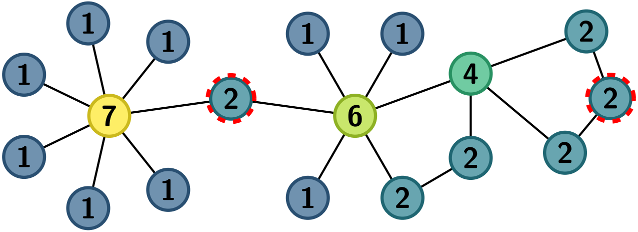


Figure: Difference of marked nodes not captured by degree centrality

Markov process & stationary distribution

- Markovian process with n states specified by $n \times n$ matrix W with entries W_{nm} specifying the flow from state n to state m (Master equation)

$$\frac{\mathrm{d}p(t)}{\mathrm{d}t} = Wp(t)$$

$$\frac{\mathrm{d}p_i(t)}{\mathrm{d}t} = \sum_j w_{ji} \cdot p_j(t) - w_{ij} \cdot p_i(t)$$

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- $p(t)$ is the probability distribution at time t, a vector with n entries.
- p^S is a stationary distribution if p^S is a right EV of W with eigenvalue $\lambda = 0$

$$\frac{\mathrm{d}p^S}{\mathrm{d}t} = 0 = Wp^S$$

Eigenvector centrality

$$\lambda x = Ax$$

- x is the eigenvector corresponding to the largest eigenvalue λ
- Take the i-th component of the eigenvector to get EV-centrality of i-th node

$$C_E(v_i) = x_i = \frac{1}{\lambda} \sum_j A_{ij} x_j = \frac{1}{\lambda} \sum_{v_j \in N(v_i)} x_j$$

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- Nodes that are connected to nodes with high centrality have higher centrality themselves.

Eigenvector centrality

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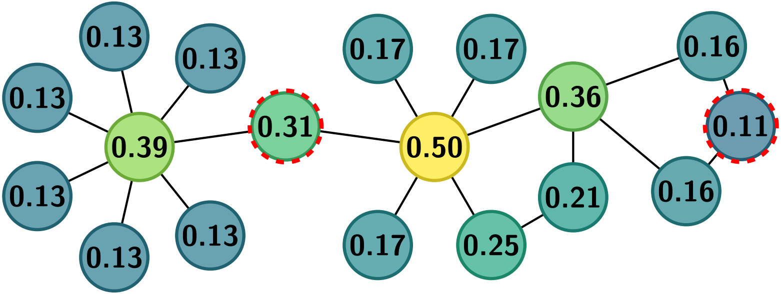

Figure: Eigenvector centrality of nodes increases by being connected to other nodes with high EV-centrality

Related measures

Various similar measures exist:

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- Katz centrality
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- PageRank
  - Divide centrality contribution of node by the number of outgoing links.
  - Measures distribution of random walkers on network choosing edges without any bias

Applicable for Systemic Risk?

Methods present so far cannot be used because spread of risk has additional constraints

- Liability network is directed and weighted
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- Market shock only spreads when capitalization of a node is insufficient to compensate for loss $\Rightarrow$ Take equity into account

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3. Result: DebtRank $R(v_x) \in [0, 1]$ measures the fraction of economic value in the network that is impacted by the default of node v_x

Aim of my work

- Systemic risk stems from loans that can't be payed back if node is in distress
- Companies usually aren't in the business of lending money (some qualifications may apply)
- Research mainly focussed on interbank market ([Battiston et al., 2012, Thurner and Poledna, 2013, Martinez-Jaramillo et al., 2014, Poledna et al., 2015, Poledna and Thurner, 2016]. . .)

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$\Rightarrow$ expand upon existing research by also taking companies into account

Empirical data

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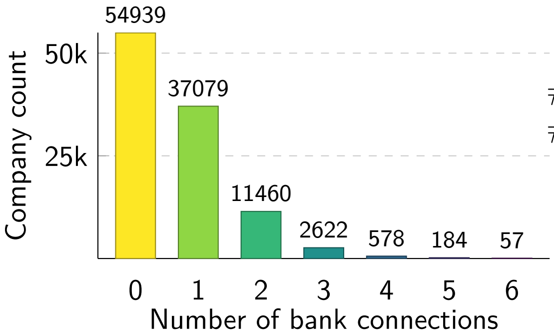

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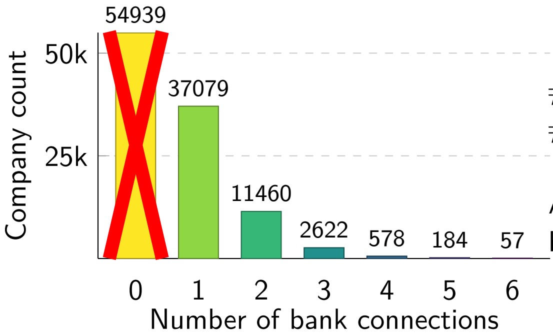

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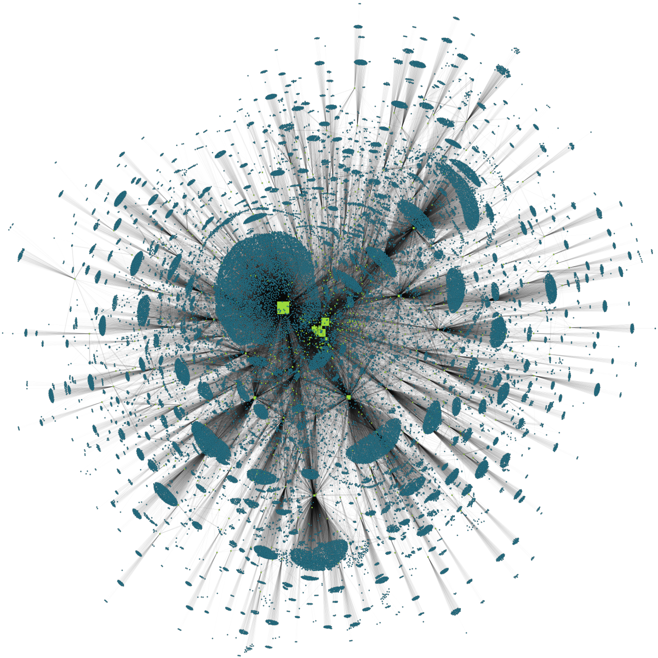
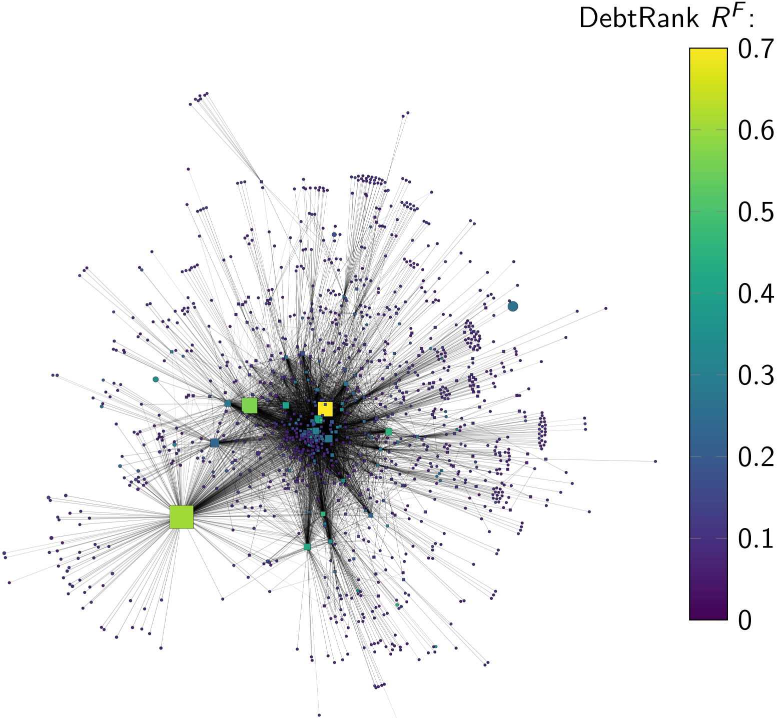
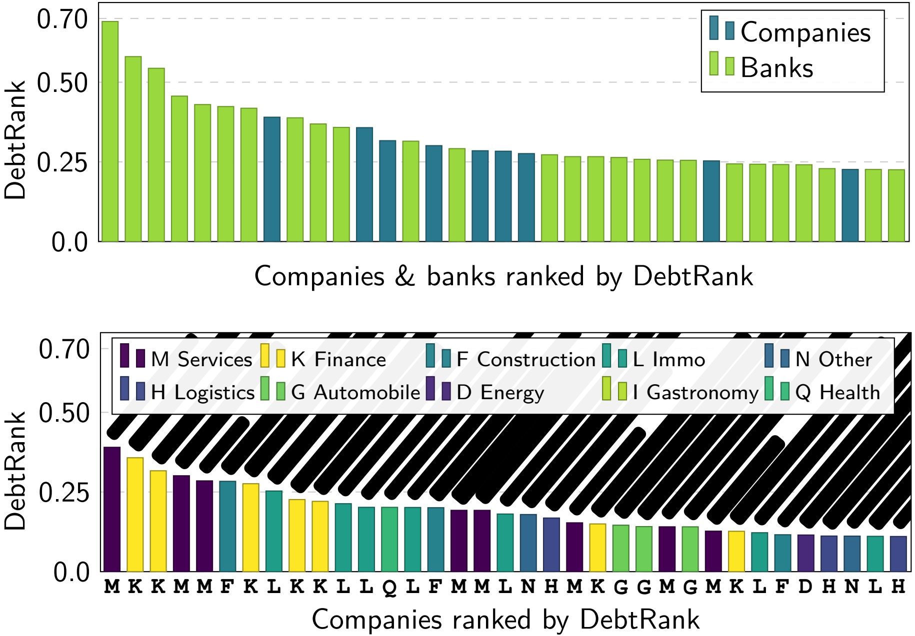
Affects 51% of companies but only 24% of liabilities




DebtRank R^F:
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0

0.70
0.50
0.25
0.0
DebtRank
Companies
Banks
Companies & banks ranked by DebtRank



DebtRank
0.0
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Companies
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Companies & banks ranked by DebtRank
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0.0
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0.70
M Services
K Finance
F Construction
L Immo
N Other
H Logistics
G Automobile
D Energy
I Gastronomy
Q Health
M K K M M F K L K K L L Q L F M M L N H M K G G M G M K L F D H N L H
Companies ranked by DebtRank

Summary

- Used methods derived from stochastic processes to analyze systemic risk of national economy
- Systemic risk of interbank network underestimates systemic risk (only 29% of absolute value)
- In the full network, companies contribute 55% of the systemic risk

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Centrality Measures

Details from centrality measures mentioned previously

- Katz centrality: Not only sum of centralities of nearest neighbors but also the centrality of nodes at arbitrary distance k taken into account (with a factor α^k where $\alpha < 1$):

$$C_K(v_i) = \sum_{k=1}^{\infty} \sum_{v_j \in V} \alpha^k (A^k)_{ij} \qquad \vec{K} = ((I - \alpha A)^{-1} - I) \cdot \vec{I}$$

- PageRank: Divide centrality contribution of node by the number of outgoing links (with $D_{ij} = \delta_{ij} \max(1, k_j^{\text{out}})$)

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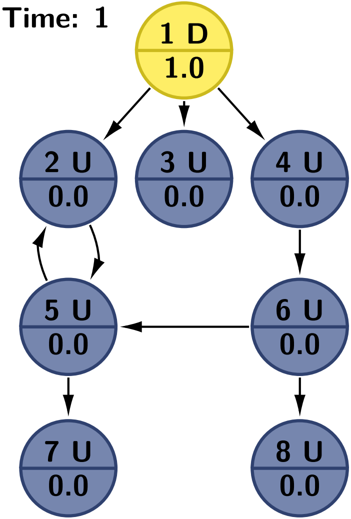
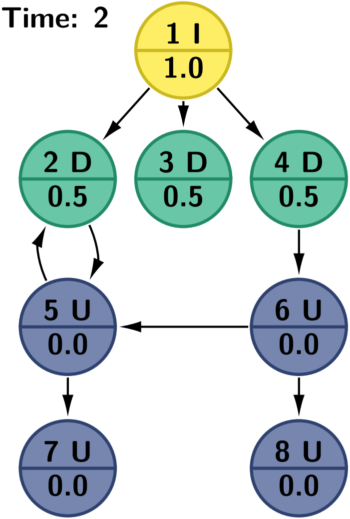
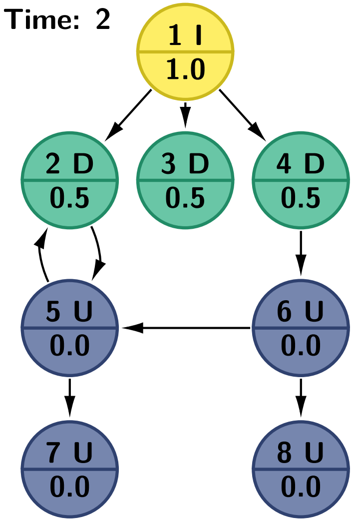
- If a node has been in distress in the previous timestep it will be inactive from then on ($\Rightarrow$ no loops)

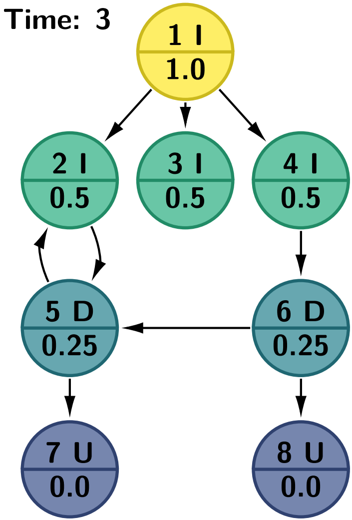
DebtRank: Example ($W_{ij} = 0.5$)



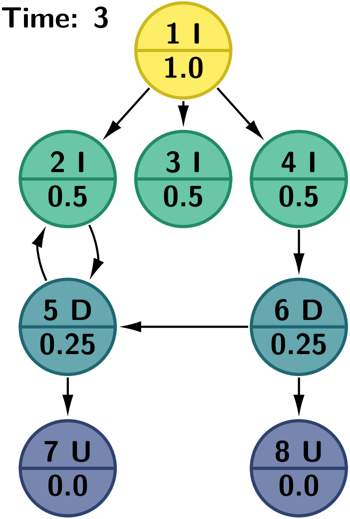


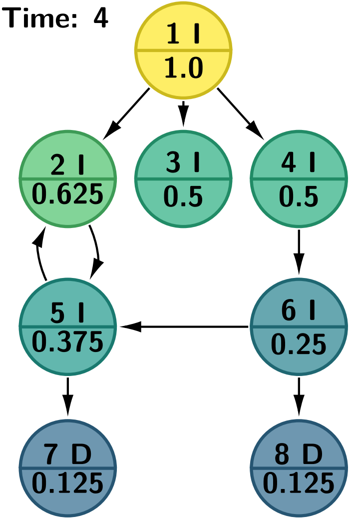
DebtRank: Example ($W_{ij} = 0.5$)



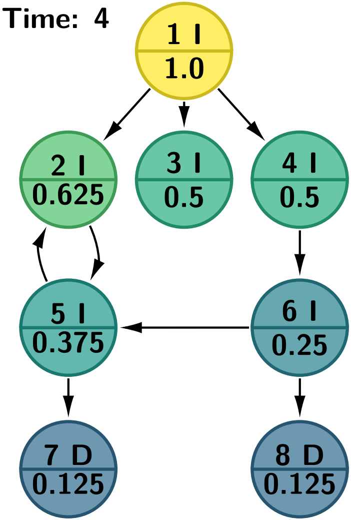


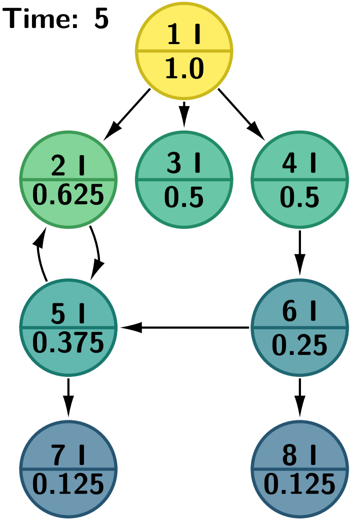
DebtRank: Example ($W_{ij} = 0.5$)

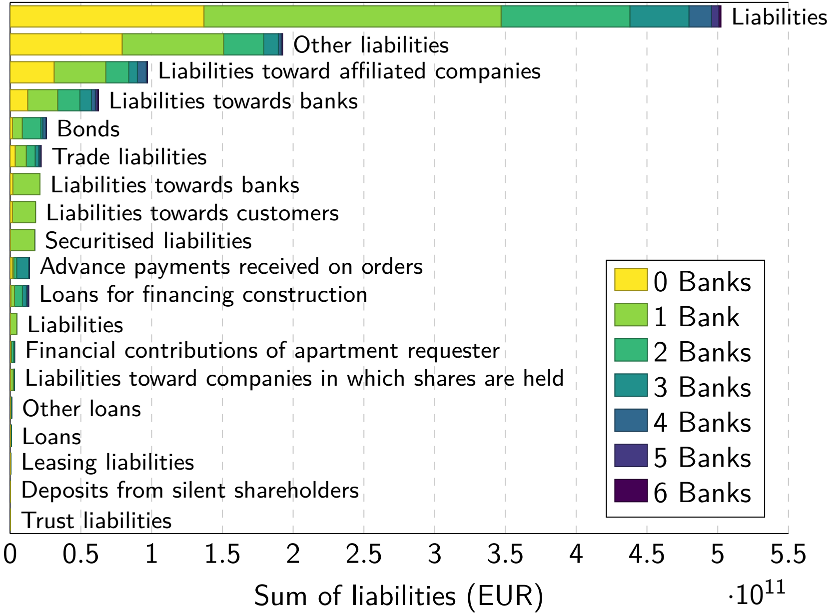




DebtRank: Example ($W_{ij} = 0.5$)






Liabilities
Other liabilities
Liabilities toward affiliated companies
Liabilities towards banks
Bonds
Trade liabilities
Liabilities towards banks
Liabilities towards customers
Securitised liabilities
Advance payments received on orders
Loans for financing construction
Liabilities
Financial contributions of apartment requester
Liabilities toward companies in which shares are held
Other loans
Loans
Leasing liabilities
Deposits from silent shareholders
Trust liabilities
0 Banks
1 Bank
2 Banks
3 Banks
4 Banks
5 Banks
6 Banks
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5
Sum of liabilities (EUR)
$\cdot 10^{11}$