

EIOPA RISK-FREE CURVE MARCH-23 RECALCULATION

The risk-free curve is one of the principal inputs into an economic scenario generator. This notebook recalculates the risk-free curve using the parameters that are claimed to be used. The European Insurance and Occupational Pensions Authority (EIOPA) publishes their own yield curve prediction. To do this they use the Smith & Wilson algorithm.

Summary

The goal of this test is to replicate the EIOPA yield curve. This test will use the methodology that EIOPA claims it is using and the calibration vector that they publish. If the test is passed, the user can be more confident, that EIOPA risk free rate (RFR) curve was generated using the described methodology/calibration and that the process was implemented correctly.

Table of Contents

1. [Note on Smith & Wilson algorithm](#)
2. [Data requirements](#)
3. [Success criteria](#)
4. [External dependencies](#)
5. [Calibration parameters and calibration vector provided by EIOPA](#)
6. [Smith & Wilson calculation functions](#)
7. [Generation of the risk-free curve](#)
8. [Test 1; Comparison test](#)
9. [Test 1; Success criteria](#)
10. [Test 1; Comparison test](#)
11. [Conclusion](#)

Note on Smith & Wilson algorithm

To replicate the calculations, this example uses a modified Smith&Wilson implementation (The original implementation is available on [GitHub](#):

- [Python](#)
- [Matlab](#)
- [JavaScript](#)

Limitations of the implementation

Current implementation only looks at a single currency and with/without Volatility Adjustment (VA). The day count convention assumes that each year has the same number of days.

Data requirements

This script contains the EIOPA risk-free rate publication for March 2023. The publication can be found on the [EIOPA RFR website](#).

The observed maturities `M_Obs` and the calibrated vector `Qb` can be found in the Excel sheet `EIOPA_RFR_20230331_Qb_SW.xlsx`.

For this example, the curve without the volatility adjustment (VA) is used. It can be found in the sheet `SW_Qb_no_VA`. This example is focused on the EUR curve, but this example can be easily modified for any other curve.

The target maturities (`T_Obs`), the additional parameters (`UFR` and `alpha`), and the given curve can be found in the Excel `EIOPA_RFR_20230331_Term_Structures.xlsx`, sheet `RFR_spot_no_VA` if the test looks at the curve without the Volatility Adjustment and the sheet `RFR_spot_with_VA` if the test looks at the curve with the Volatility Adjustment.

[Back to the top](#)

Success criteria

The following success criteria is defined:

- Maximum difference between the calculated curve and the one provided by EIOPA is less than 0.1 bps
- Average difference between the calculated curve and the one provided by EIOPA is less than 0.05 bps

In [156...]

```
test_statistics_max_diff_in_bps = 0.1
test_statistics_average_diff_in_bps = 0.05
```

The success function is called at the end of the test to confirm if the success criteria have been met.

In [157...]

```
def SuccessTest(TestStatistics, threshold_max, threshold_mean):
    out1 = False
    out2 = False
    if max(TestStatistics) < threshold_max:
        print("Test passed")
        out1 = True
    else:
        print("Test failed")

    if np.mean(TestStatistics) < threshold_mean:
        print("Test passed")
        out2 = True
    else:
        print("Test failed")
    return [out1, out2]
```

[Back to the top](#)

External dependencies

This implementation uses three well established Python packages widely used in the financial industry. Pandas (<https://pandas.pydata.org/docs/>), Numpy (<https://numpy.org/doc/>), and Matplotlib (<https://matplotlib.org/stable/index.html>)

In [158...]

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import matplotlib.ticker as mtick
%matplotlib notebook
pd.options.display.max_rows = 150
```

Importing data

In [159...]

```
selected_param_file = 'Param_VA.csv'
selected_curves_file = 'Curves_VA.csv'

#selected_param_file = 'Param_no_VA.csv'
#selected_curves_file = 'Curves_no_VA.csv'
```

In [160...]

```
param_raw = pd.read_csv(selected_param_file, sep=',', index_col=0)
```

Parameter input

Parameters sheet

In [161...]

```
param_raw.head()
```

Out[161...]

	Euro_Maturities	Euro_Values	Austria_Maturities	Austria_Values	Belgium_Maturities	Be
Country						
Coupon_freq	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
LLP	20.000000	20.000000	20.000000	20.000000	20.000000	20.000000
Convergence	40.000000	40.000000	40.000000	40.000000	40.000000	40.000000
UFR	3.450000	3.450000	3.450000	3.450000	3.450000	3.450000
alpha	0.113689	0.113689	0.113689	0.113689	0.113689	0.113689

5 rows × 106 columns

The country selected is:

In [162...]

```
country = "Italy"
```

In [163...]

```
maturities_country_raw = param_raw.loc[:,country+"_Maturities"].iloc[6:]
param_country_raw = param_raw.loc[:,country + "_Values"].iloc[6:]
extra_param = param_raw.loc[:,country + "_Values"].iloc[:6]
```

Extra parameters

Smith-Wilson calibration parameters

In [164...]

```
extra_param
```

Out[164...]

```
Country
Coupon_freq      1.000000
LLP              20.000000
Convergence      40.000000
UFR              3.450000
alpha             0.113689
CRA               10.000000
Name: Italy_Values, dtype: float64
```

In [165...]

```
relevant_positions = pd.notna(maturities_country_raw.values)
```

In [166...]

```
maturities_country = maturities_country_raw.iloc[relevant_positions]
```

Maturity vector

Vector of maturities used in the calibration

In [167...]

```
maturities_country.head(15)
```

Out[167...]

```
Country
1      1.0
2      2.0
3      3.0
4      4.0
5      5.0
6      6.0
7      7.0
8      8.0
9      9.0
10     10.0
11     11.0
12     12.0
13     13.0
14     14.0
15     15.0
Name: Italy_Maturities, dtype: float64
```

In [168...]

```
Qb = param_country_raw.iloc[relevant_positions]
```

Calibration vector

Vector Qb provided as input

In [169...]

```
Qb
```

Out[169...]

```
Country
1      -1.994869
2      -2.183116
3       0.583192
4       0.748252
5       0.759139
6      -0.750781
7       0.677648
8      -1.655131
9       6.983692
10     -16.586837
11      20.229207
12     -9.376203
13      0.015868
14      0.003562
15     -0.625900
16      0.021149
```

```
17      0.052772
18     -0.073404
19      0.391200
20      0.377356
Name: Italy Values, dtype: float64
```

```
In [170...]: curve_raw = pd.read_csv(selected_curves_file, sep=',', index_col=0)
```

```
In [171...]: curve_country = curve_raw.loc[:, country]
```

[Back to the top](#)

Calibration parameters and calibration vector provided by EIOPA

```
In [172...]: # Maturity of observations:
M_Obs = np.transpose(np.array(maturities_country.values))

# Ultimate forward rate ufr represents the rate to which the rate curve will converge
ufr = extra_param.iloc[3]/100

# Convergence speed parameter alpha controls the speed at which the curve converges
alpha = extra_param.iloc[4]

# For which maturities do we want the SW algorithm to calculate the rates. In this case, we want rates for all maturities
M_Target = np.transpose(np.arange(1, 151))

# Qb calibration vector published by EIOPA for the curve calibration:
Qb = np.transpose(np.array(Qb.values))
```

[Back to the top](#)

Smith & Wilson calculation functions

In this step, the independent version of the Smith&Wilson algorithm is implemented. To do this, two functions are taken from the publicly available repository and modified to accept the product of Q^*b instead of the calibration vector b .

In [173...]

```

def SWExtrapolate(M_Target, M_Obs, Qb, ufr, alpha):
    # SWEXTRAPOLATE Interpolate or/and extrapolate rates for targeted maturities using a
    # out = SWExtrapolate(M_Target, M_Obs, Qb, ufr, alpha) calculates the rates for maturities
    #
    # Arguments:
    #     M_Target = k x 1 ndarray. Each element represents a bond maturity of interest.
    #     M_Obs = n x 1 ndarray. Observed bond maturities used for calibrating the curve.
    #     Qb = n x 1 ndarray. Calibration vector calculated on observed bonds.
    #     ufr = 1 x 1 floating number. Representing the ultimate forward rate.
    #         Ex. ufr = 0.042
    #     alpha = 1 x 1 floating number. Representing the convergence speed parameter of the
    #
    #
    # Returns:
    #     k x 1 ndarray. Represents the targeted rates for a zero-coupon bond. Each rate is
    #
    # For more information see https://www.eiopa.europa.eu/sites/default/files/risk_free_curve.ipynb

def SWHeart(u, v, alpha):
    # SWHEART Calculate the heart of the Wilson function.
    # H = SWHeart(u, v, alpha) calculates the matrix H (Heart of the Wilson
    # function) for maturities specified by vectors u and v. The formula is
    # taken from the EIOPA technical specifications paragraph 132.
    #
    # Arguments:
    #     u = n_1 x 1 vector of maturities. Ex. u = [1; 3]
    #     v = n_2 x 1 vector of maturities. Ex. v = [1; 2; 3; 5]
    #     alpha = 1 x 1 floating number representing the convergence speed parameter of the
    #
    # Returns:
    #     n_1 x n_2 matrix representing the Heart of the Wilson function for selected
    #
    # For more information see https://www.eiopa.europa.eu/sites/default/files/risk_free_curve.ipynb

    u_Mat = np.tile(u, [v.size, 1]).transpose()
    v_Mat = np.tile(v, [u.size, 1])
    return 0.5 * (alpha * (u_Mat + v_Mat) + np.exp(-alpha * (u_Mat + v_Mat)) - alpha)

H = SWHeart(M_Target, M_Obs, alpha) # Heart of the Wilson function from paragraph 132
p = np.exp(-np.log(1+ufr)* M_Target) + np.diag(np.exp(-np.log(1+ufr) * M_Target))
return p ** (-1/ M_Target) -1 # Convert obtained prices to rates and return price

```

[Back to the top](#)

Generation of the risk-free curve

The observed maturities, target maturities, and the model parameters provided by EIOPA are used to generate the target curve.

In [174...]

```

r_Target = SWExtrapolate(M_Target,M_Obs, Qb, ufr, alpha)
r_Target = pd.DataFrame(r_Target,columns=['Recalculated rates'])

```

Yield curve calculated

Yield curve calculated using the calibration vector Q_b

In [175...]

```
r_Target.head(15)
```

Out[175...]

Recalculated rates

0	0.036720
1	0.035154
2	0.033400
3	0.032091
4	0.031299
5	0.030860
6	0.030598
7	0.030473
8	0.030454
9	0.030501
10	0.030373
11	0.030698
12	0.030894
13	0.030842
14	0.030604

[Back to the top](#)

Test 1; Comparison test

Comparison of the calculated yield curve with the yield curve provided by EIOPA. The test is passed if the success criteria is reached.

The provided yield curve can be found in file *EIOPA_RFR_20230331_Term_Structures.xlsx*, sheet *RFR_spot_no_VA* if the test looks at the curve without the Volatility Adjustment and the sheet *RFR_spot_with_VA* if the test looks at the curve with the Volatility Adjustment.

In [176...]

```
target_curve = np.transpose(np.array(curve_country.values))
```

This implementation looks at two kinds of test statistics. The average deviation and the maximum deviation.

The average deviation is defined as:

$$S_{AVERAGE} = \frac{1}{T} \sum_{t=0}^T |r_{EIOPA}(t) - r_{EST}(t)|$$

The maximum deviation is defined as:

$$S_{MAX} = \max_t |r_{EIOPA}(t) - r_{EST}(t)|$$

Where T is the maximum maturity available.

The average difference test is successful if:

$$S_{AVERAGE} < 0.05bps$$

The maximum difference test is successful if:

$$S_{MAX} < 0.1bps$$

In [177...]

```
target_curve = pd.DataFrame(target_curve,columns=['Given rates'])
```

EIOPA curve provided

Yield curve provided by EIOPA

In [178...]

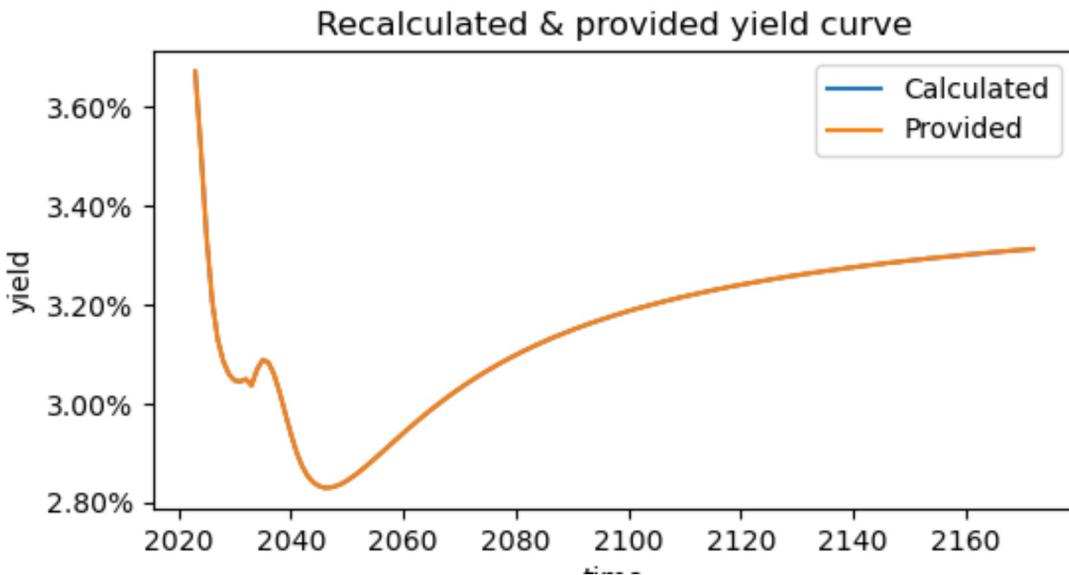
```
target_curve.head()
```

Out[178...]

	Given rates
0	0.03672
1	0.03515
2	0.03340
3	0.03209
4	0.03130

```
In [179...]:  
x_data_label = range(2023, 2023+r_Target.shape[0], 1)
```

```
In [180...]:  
fig, ax1 = plt.subplots(1,1)  
ax1.plot(x_data_label, r_Target.values*100, color='tab:blue', label="Calculated")  
ax1.plot(x_data_label, target_curve.values*100, color='tab:orange', label="Provided")  
  
ax1.set_ylabel("yield")  
ax1.set_title('Recalculated & provided yield curve')  
ax1.set_xlabel("time")  
ax1.legend()  
ax1.yaxis.set_major_formatter(mtick.PercentFormatter())  
fig.set_figwidth(6)  
fig.set_figheight(3)  
plt.show()
```



```
In [181...]:  
test_statistics_bdp = pd.DataFrame(abs(r_Target.values-target_curve.values)*10000, co
```

EIOPA curve comparison

Absolute difference in bps

```
In [182...]:  
test_statistics_bdp.head()
```

```
Out[182...]:  
Abs diff in bps
```

0	6.255937e-07
1	4.490431e-02
2	2.995177e-03
3	1.095525e-02

Abs diff in bps

[Back to the top](#)

Test 1; Success criteria

The successful application of the success criteria marks the completion of the test.

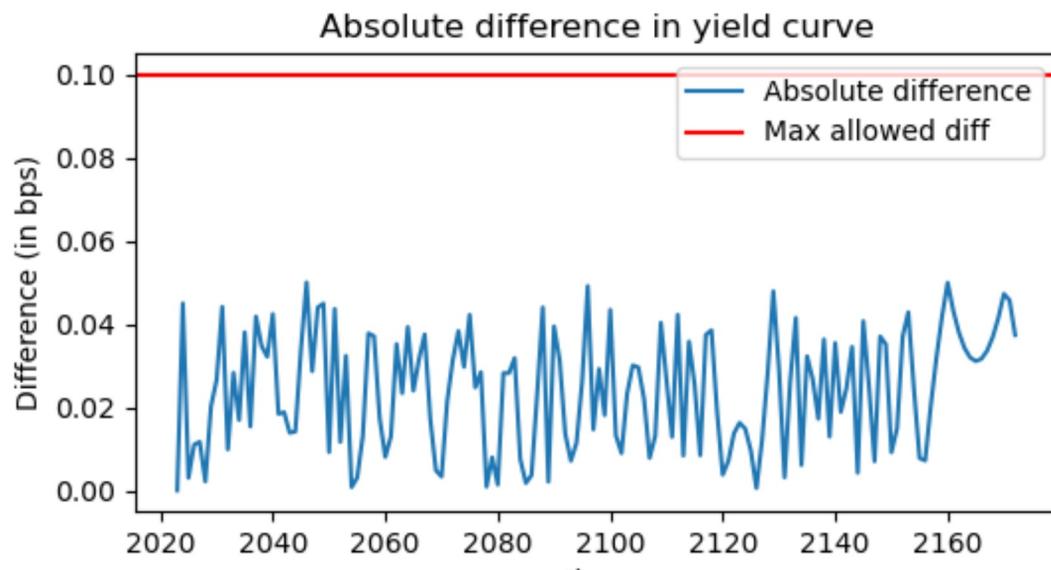
In [183...]

```
result1 = SuccessTest(test_statistics_bdp.values, test_statistics_max_diff_in_bps, tolerance)
```

```
Test passed  
Test passed
```

In [184...]

```
x_data_label = range(2023,2023+r_Target.shape[0],1)  
fig, ax1 = plt.subplots(1,1)  
ax1.plot(x_data_label, test_statistics_bdp, label= "Absolute difference")  
ax1.axhline(y = test_statistics_max_diff_in_bps, color = 'r', linestyle = '--',label="Max allowed diff")  
  
ax1.set_xlabel("time")  
ax1.set_ylabel("Difference (in bps)")  
ax1.set_title('Absolute difference in yield curve')  
ax1.legend()  
fig.set_figwidth(6)  
fig.set_figheight(3)  
  
plt.show()
```



[Back to the top](#)

Conclusion

This test checks the success criteria on the EIOPA curve generated for March 2023. If the tests are passed, it is likely that the curve was generated using the Smith & Wilson algorithm with the

In [185...]

```
pd.DataFrame(data = [result1], columns = ["Mean test", "Max test"], \
              index= ["Provided vs calculated"])
```

Out[185...]

	Mean test	Max test
Provided vs calculated	True	True

Final yield curve

Full yield curve provided by EIOPA in %

In [186...]

```
(curve_country*100).head(150)
```

Out[186...]

Country	
1	3.672
2	3.515
3	3.340
4	3.209
5	3.130
6	3.086
7	3.060
8	3.047
9	3.045
10	3.050
11	3.037
12	3.070
13	3.089
14	3.084
15	3.060
16	3.024
17	2.981
18	2.940
19	2.903
20	2.874
21	2.854
22	2.841
23	2.834
24	2.830
25	2.831
26	2.834
27	2.838
28	2.845
29	2.853
30	2.861
31	2.870
32	2.880
33	2.890

34	2.900
35	2.910
36	2.921
37	2.931
38	2.941
39	2.951
40	2.961
41	2.970
42	2.980
43	2.989
44	2.998
45	3.006
46	3.015
47	3.023
48	3.031
49	3.039
50	3.046
51	3.054
52	3.061
53	3.068
54	3.074
55	3.081
56	3.087
57	3.093
58	3.099
59	3.105
60	3.110
61	3.116
62	3.121
63	3.126
64	3.131
65	3.136
66	3.140
67	3.145
68	3.149
69	3.154
70	3.158
71	3.162
72	3.166
73	3.170
74	3.173
75	3.177
76	3.181
77	3.184
78	3.188
79	3.191
80	3.194
81	3.197
82	3.200
83	3.203
84	3.206
85	3.209
86	3.212
87	3.215
88	3.217
89	3.220
90	3.222
91	3.225
92	3.227

93	3.230
94	3.232
95	3.234
96	3.237
97	3.239
98	3.241
99	3.243
100	3.245
101	3.247
102	3.249
103	3.251
104	3.253
105	3.255
106	3.257
107	3.259
108	3.260
109	3.262
110	3.264
111	3.265
112	3.267
113	3.269
114	3.270
115	3.272
116	3.273
117	3.275
118	3.276
119	3.278
120	3.279
121	3.281
122	3.282
123	3.283
124	3.285
125	3.286
126	3.287
127	3.289
128	3.290
129	3.291
130	3.292
131	3.294
132	3.295
133	3.296
134	3.297
135	3.298
136	3.299
137	3.300
138	3.302
139	3.303
140	3.304
141	3.305
142	3.306
143	3.307
144	3.308
145	3.309
146	3.310
147	3.311
148	3.312
149	3.312
150	3.313

Name: Italv. dtype: float64

