EIOPA RISK-FREE CURVE JULY-23 RECALCULATION

The risk-free curve is one of the principal inputs into an economic scenario generator. This notebook recalculates the risk-free curve using the parameters that are claimed to be used. The European Insurance and Occupational Pensions Authority (EIOPA) publishes their own yield curve prediction. To do this they use the Smith & Wilson algorithm.

Summary

The goal of this test is to replicate the EIOPA yield curve. This test will use the methodology that EIOPA claims it is using and the calibration vector that they publish. If the test is passed, the user can be more confident, that EIOPA risk free rate (RFR) curve was generated using the described methodology/calibration and that the process was implemented correctly.

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Note on Smith & Wilson algorithm

To replicate the calculations, this example uses a modified Smith&Wilson implementation (The original implementation is availible on GitHub):

- Python
- Matlab
- JavaScript

Limitations of the implementation

Current implementation only looks at a single currency and with/without Volatility Adjustment (VA). The day count convention assumes that each year has the same number of days.

Data requirements

This script contains the EIOPA risk-free rate publication for July 2023. The publication can be found on the EIOPA RFR website.

The observed maturities `M_Obs` and the calibrated vector `Qb` can be found in the Excel sheet *EIOPA_RFR_20230731_Qb_SW.xlsx*.

The target maturities (`T_Obs`), the additional parameters (`UFR` and `alpha`), and the given curve can be found in the Excel *EIOPA_RFR_20230731_Term_Structures.xlsx*, sheet *RFR_spot_no_VA* if the test looks at the curve without the Volatility Adjustment and the sheet *RFR_spot_with_VA* if the test looks at the curve with the Volatility Adjustment.

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Success criteria

The following success criteria is defined:

- Maximum difference between the calculated curve and the one provided by EIOPA is less than 0.1 bps
- Average difference between the calculated curve and the one provided by EIOPA is less than 0.05 bps

In [63]:
```
test_statistics_max_diff_in_bps = 0.1
test_statistics_average_diff_in_bps = 0.05
```

The success function is called at the end of the test to confirm if the success criteria have been met.

In [64]:
```
def SuccessTest(TestStatistics, threshold_max, threshold_mean):
    out1 = False
    out2 = False
    if max(TestStatistics)<threshold_max:
        print("Test passed")
        out1 = True
    else:
        print("Test failed")

    if np.mean(TestStatistics)<threshold_mean:
        print("Test passed")
        out2 = True
    else:
        print("Test failed")
    return [out1, out2]
```

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External dependencies

This implementation uses three well established Python packages widely used in the financial industry. Pandas (https://pandas.pydata.org/docs/), Numpy (https://numpy.org/doc/), and Matplotlib (https://matplotlib.org/stable/index.html)

In [65]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import matplotlib.ticker as mtick
%matplotlib notebook
pd.options.display.max_rows = 150
```

Importing data

In [66]:
```
#selected_param_file = 'Param_VA.csv'
#selected_curves_file = 'Curves_VA.csv'

selected_param_file = 'Param_no_VA.csv'
selected_curves_file = 'Curves_no_VA.csv'
```

In [67]:
```
param_raw = pd.read_csv(selected_param_file, sep=',',index_col=0)
```

Parameter input

Parameters sheet

In [68]:
```
param_raw.head()
```

Out[68]:

Country	Euro_Maturities	Euro_Values	Austria_Maturities	Austria_Values	Belgium_Maturities	Be
Coupon_freq	1.000000	1.000000	1.000000	1.000000	1.000000	
LLP	20.000000	20.000000	20.000000	20.000000	20.000000	
Convergence	40.000000	40.000000	40.000000	40.000000	40.000000	
UFR	3.450000	3.450000	3.450000	3.450000	3.450000	
alpha	0.112203	0.112203	0.112203	0.112203	0.112203	

5 rows × 106 columns

The country selected is:

In [69]:
```
country = "United Kingdom"
```

In [70]:
```
maturities_country_raw = param_raw.loc[:,country+"_Maturities"].iloc[6:]
param_country_raw = param_raw.loc[:,country + "_Values"].iloc[6:]
extra_param = param_raw.loc[:,country + "_Values"].iloc[:6]
```

Extra parameters

Smith-Wilson calibration parameters

In [71]:
```
extra_param
```

Out[71]:
```
Country
Coupon_freq     1.000000
LLP            50.000000
Convergence    40.000000
UFR             3.450000
alpha           0.095952
CRA             0.000000
Name: United Kingdom_Values, dtype: float64
```

In [72]:
```
relevant_positions = pd.notna(maturities_country_raw.values)
```

In [73]:
```
maturities_country = maturities_country_raw.iloc[relevant_positions]
```

Maturity vector

Vector of maturities used in the calibration

In [74]:
```
maturities_country.head(15)
```

Out[74]:
```
Country
1      1.0
2      2.0
3      3.0
4      4.0
5      5.0
6      6.0
7      7.0
8      8.0
9      9.0
10    10.0
11    11.0
12    12.0
13    13.0
14    14.0
15    15.0
Name: United Kingdom_Maturities, dtype: float64
```

In [75]:
```
Qb = param_country_raw.iloc[relevant_positions]
```

Calibration vector

Vector **Qb** provided as input

In [76]:
```
Qb
```

Out[76]:
```
Country
1      1.003151
2    -13.713247
3      8.535140
4     -2.583714
5      1.159440
6     -1.122329
7      1.538413
8      0.005807
9      0.125714
10     0.325141
11    -0.012990
12    -0.157560
13    -0.006266
14    -0.006057
15    -0.140148
16    -0.000316
```

```
17    -0.000305
18    -0.000295
19    -0.000285
20     0.089859
21    -0.003775
22    -0.003649
23    -0.003527
24    -0.003410
25    -0.003296
26    -0.003186
27    -0.003080
28    -0.002977
29    -0.002878
30    -0.262258
31     0.007033
32     0.006798
33     0.006572
34     0.006352
35     0.006141
36     0.005936
37     0.005738
38     0.005546
39     0.005361
40     0.005183
41     0.005010
42     0.004843
43     0.004681
44     0.004525
45     0.004374
46     0.004228
47     0.004087
48     0.003951
49     0.003819
50     0.105816
Name: United Kingdom Values, dtype: float64
```

In [77]:
```
curve_raw = pd.read_csv(selected_curves_file, sep=',',index_col=0)
```

In [78]:
```
curve_country = curve_raw.loc[:,country]
```

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Calibration parameters and calibration vector provided by EIOPA

In [79]:

```
# Maturity of observations:
M_Obs = np.transpose(np.array(maturities_country.values))

# Ultimate froward rate ufr represents the rate to which the rate curve will converge
ufr = extra_param.iloc[3]/100

# Convergence speed parameter alpha controls the speed at which the curve converges t
alpha = extra_param.iloc[4]

# For which maturities do we want the SW algorithm to calculate the rates. In this ca
M_Target = np.transpose(np.arange(1,151))

# Qb calibration vector published by EIOPA for the curve calibration:
Qb = np.transpose(np.array(Qb.values))
```

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Smith & Wilson calculation functions

In this step, the independent version of the Smith&Wilson algorithm is implemented. To do this, two functions are taken from the publicly available repository and modified to accept the product of `Q*b` instead of the calibration vector `b`.

In [80]:

```
def SWExtrapolate(M_Target, M_Obs, Qb, ufr, alpha):
# SWEXTRAPOLATE Interpolate or/and extrapolate rates for targeted maturities using a
# out = SWExtrapolate(M_Target, M_Obs, Qb, ufr, alpha) calculates the rates for matu
#
# Arguments:
#    M_Target = k x 1 ndarray. Each element represents a bond maturity of interest. E
#    M_Obs =    n x 1 ndarray. Observed bond maturities used for calibrating the cal
#    Qb =       n x 1 ndarray. Calibration vector calculated on observed bonds.
#    ufr =      1 x 1 floating number. Representing the ultimate forward rate.
#       Ex. ufr = 0.042
#    alpha =    1 x 1 floating number. Representing the convergence speed parameter o
#
#
# Returns:
#    k x 1 ndarray. Represents the targeted rates for a zero-coupon bond. Each rate E
#
# For more information see https://www.eiopa.europa.eu/sites/default/files/risk_free_

    def SWHeart(u, v, alpha):
    # SWHEART Calculate the heart of the Wilson function.
    # H = SWHeart(u, v, alpha) calculates the matrix H (Heart of the Wilson
    # function) for maturities specified by vectors u and v. The formula is
    # taken from the EIOPA technical specifications paragraph 132.
    #
    # Arguments:
    #    u =     n_1 x 1 vector of maturities. Ex. u = [1; 3]
    #    v =     n_2 x 1 vector of maturities. Ex. v = [1; 2; 3; 5]
    #    alpha = 1 x 1 floating number representing the convergence speed parameter o
    #
    # Returns:
    #    n_1 x n_2 matrix representing the Heart of the Wilson function for selected
    #
    # For more information see https://www.eiopa.europa.eu/sites/default/files/risk_f

        u_Mat = np.tile(u, [v.size, 1]).transpose()
        v_Mat = np.tile(v, [u.size, 1])
        return 0.5 * (alpha * (u_Mat + v_Mat) + np.exp(-alpha * (u_Mat + v_Mat)) - al

    H = SWHeart(M_Target, M_Obs, alpha) # Heart of the Wilson function from paragraph
    p = np.exp(-np.log(1+ufr)* M_Target) + np.diag(np.exp(-np.log(1+ufr) * M_Target))
    return p ** (-1/ M_Target) -1 # Convert obtained prices to rates and return price
```

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Generation of the risk-free curve

The observed maturities, target maturities, and the model parameters provided by EIOPA are used to generate the target curve.

In [81]:

```
r_Target = SWExtrapolate(M_Target,M_Obs, Qb, ufr, alpha)
r_Target = pd.DataFrame(r_Target,columns=['Recalculated rates'])
```

Yield curve calculated

Yield curve calculated using the calibration vector **Qb**

In [82]:
```
r_Target.head(15)
```

Out[82]:

	Recalculated rates
0	0.058146
1	0.055991
2	0.052640
3	0.049933
4	0.047675
5	0.045821
6	0.044331
7	0.043221
8	0.042407
9	0.041812
10	0.041377
11	0.041041
12	0.040763
13	0.040518
14	0.040294

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Test 1; Comparison test

Comparison of the calculated yield curve with the yield curve provided by EIOPA. The test is passed if the success criteria is reached.

The provided yield curve can be found in file *EIOPA_RFR_20230731_Term_Structures.xlsx*, sheet *RFR_spot_no_VA* if the test looks at the curve without the Volatility Adjustment and the sheet *RFR_spot_with_VA* if the test looks at the curve with the Volatility Adjustment.

In [83]:
```
target_curve = np.transpose(np.array(curve_country.values))
```

This implementation looks at two kinds of test statistics. The average deviation and the maximum deviation.

The average deviation is defined as:

$$S_{AVERAGE} = \frac{1}{T}\sum_{t=0}^{T}|r_{EIOPA}(t) - r_{EST}(t)|$$

The maximum deviation is defined as:

$$S_{MAX} = \max_{t}|r_{EIOPA}(t) - r_{EST}(t)|$$

Where `T` is the maximum maturity available.

The average difference test is successful if:

$$S_{AVERAGE} < 0.05bps$$

The maximum difference test is successful if:

$$S_{MAX} < 0.1bps$$

In [84]:
```
target_curve = pd.DataFrame(target_curve,columns=['Given rates'])
```

EIOPA curve provided

Yield curve provided by EIOPA

In [85]:
```
target_curve.head()
```

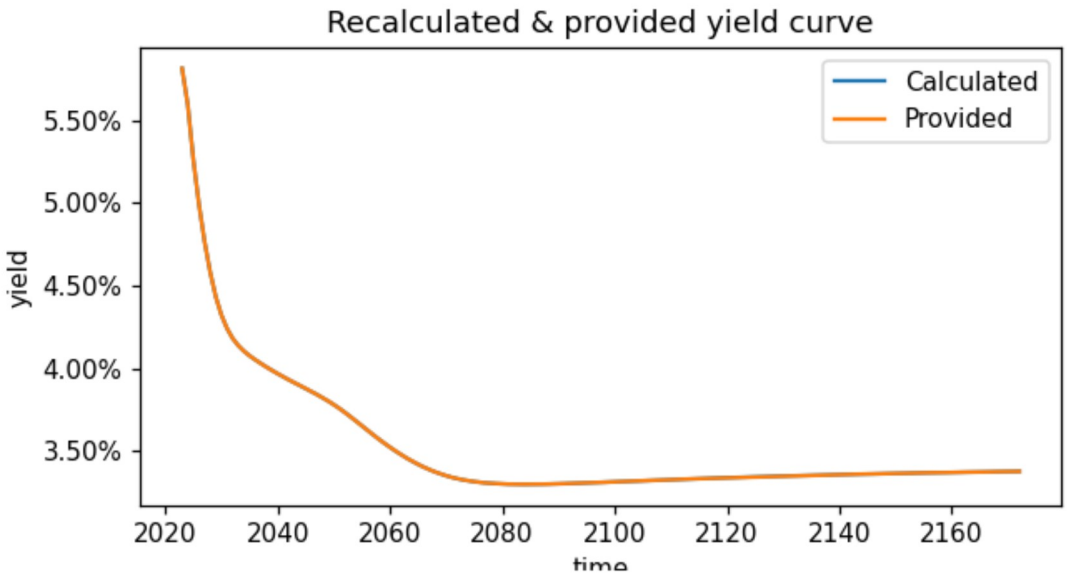
Out[85]:

	Given rates
0	0.05815
1	0.05599
2	0.05264
3	0.04993
4	0.04767

In [86]:
```
x_data_label = range(2023,2023+r_Target.shape[0],1)
```

In [87]:
```
fig, ax1 = plt.subplots(1,1)
ax1.plot(x_data_label, r_Target.values*100, color='tab:blue',label="Calculated")
ax1.plot(x_data_label, target_curve.values*100, color='tab:orange',label="Provided")

ax1.set_ylabel("yield")
ax1.set_title('Recalculated & provided yield curve')
ax1.set_xlabel("time")
ax1.legend()
ax1.yaxis.set_major_formatter(mtick.PercentFormatter())
fig.set_figwidth(6)
fig.set_figheight(3)
plt.show()
```



In [88]:
```
test_statistics_bdp = pd.DataFrame(abs(r_Target.values-target_curve.values)*10000, c
```

EIOPA curve comparison

Absolute difference in bps

In [89]:
```
test_statistics_bdp.head()
```

Out[89]:

	Abs diff in bps
0	0.039999
1	0.013809
2	0.000588
3	0.027765

Abs diff in bps

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Test 1; Success criteria

The successful application of the success criteria marks the completion of the test.

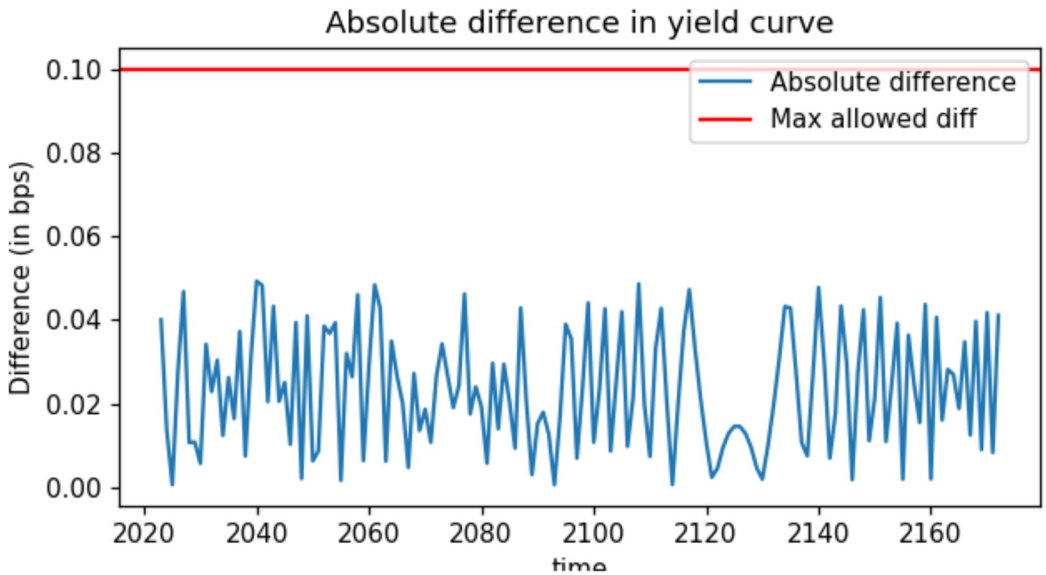
In [90]:
```
result1 = SuccessTest(test_statistics_bdp.values, test_statistics_max_diff_in_bps, te
```

```
Test passed
Test passed
```

In [91]:
```
x_data_label = range(2023,2023+r_Target.shape[0],1)
fig, ax1 = plt.subplots(1,1)
ax1.plot(x_data_label, test_statistics_bdp, label= "Absolute difference")
ax1.axhline(y = test_statistics_max_diff_in_bps, color = 'r', linestyle = '-',label='

ax1.set_xlabel("time")
ax1.set_ylabel("Difference (in bps)")
ax1.set_title('Absolute difference in yield curve')
ax1.legend()
fig.set_figwidth(6)
fig.set_figheight(3)

plt.show()
```



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Conclusion

This test checks the success criteria on the EIOPA curve generated for July 2023. If the tests are passed, it is likely that the curve was generated using the Smith & Wilson algorithm with the calibration vector that was provided in the file *EIOPA_RFR_20230731_Qb_SW.xlsx* and the parameters displayed in the file *EIOPA_RFR_20230731_Term_Structures.xlsx*.

In [92]:
```
pd.DataFrame(data = [result1], columns = ["Mean test","Max test"],  \
             index= ["Provided vs calculated"])
```

Out[92]:

	Mean test	Max test
Provided vs calculated	True	True

Final yield curve

Full yield curve provided by EIOPA in %

In [93]:
```
(curve_country*100).head(150)
```

Out[93]:
```
Country
1      5.815
2      5.599
3      5.264
4      4.993
5      4.767
6      4.582
7      4.433
8      4.322
9      4.241
10     4.181
11     4.138
12     4.104
13     4.076
14     4.052
15     4.029
16     4.008
17     3.987
18     3.968
19     3.948
20     3.930
21     3.913
22     3.895
23     3.878
24     3.860
25     3.841
26     3.822
27     3.802
28     3.780
```

29	3.757
30	3.732
31	3.706
32	3.679
33	3.652
34	3.625
35	3.598
36	3.571
37	3.546
38	3.522
39	3.499
40	3.477
41	3.456
42	3.437
43	3.419
44	3.402
45	3.387
46	3.373
47	3.361
48	3.350
49	3.340
50	3.332
51	3.325
52	3.319
53	3.314
54	3.310
55	3.306
56	3.304
57	3.302
58	3.300
59	3.299
60	3.298
61	3.298
62	3.298
63	3.298
64	3.298
65	3.299
66	3.299
67	3.300
68	3.301
69	3.302
70	3.303
71	3.304
72	3.305
73	3.306
74	3.308
75	3.309
76	3.310
77	3.312
78	3.313
79	3.314
80	3.316
81	3.317
82	3.318
83	3.320
84	3.321
85	3.322
86	3.324
87	3.325

88	3.326
89	3.327
90	3.329
91	3.330
92	3.331
93	3.332
94	3.333
95	3.335
96	3.336
97	3.337
98	3.338
99	3.339
100	3.340
101	3.341
102	3.342
103	3.343
104	3.344
105	3.345
106	3.346
107	3.347
108	3.348
109	3.349
110	3.350
111	3.351
112	3.352
113	3.352
114	3.353
115	3.354
116	3.355
117	3.356
118	3.357
119	3.357
120	3.358
121	3.359
122	3.360
123	3.360
124	3.361
125	3.362
126	3.362
127	3.363
128	3.364
129	3.364
130	3.365
131	3.366
132	3.366
133	3.367
134	3.368
135	3.368
136	3.369
137	3.369
138	3.370
139	3.371
140	3.371
141	3.372
142	3.372
143	3.373
144	3.373
145	3.374
146	3.374

```
147    3.375
148    3.375
149    3.376
150    3.376
Name: United Kingdom, dtype: float64
```