EIOPA RISK-FREE CURVE MARCH-23 RECALCULATION

The risk-free curve is one of the principal inputs into an economic scenario generator. This notebook recalculates the risk-free curve using the parameters that are claimed to be used. The European Insurance and Occupational Pensions Authority (EIOPA) publishes their own yield curve prediction. To do this they use the Smith & Wilson algorithm.

Summary

The goal of this test is to replicate the EIOPA yield curve. This test will use the methodology that EIOPA claims it is using and the calibration vector that they publish. If the test is passed, the user can be more confident, that EIOPA risk free rate (RFR) curve was generated using the described methodology/calibration and that the process was implemented correctly.

Table of Contents

Note on Smith & Wilson algorithm

To replicate the calculations, this example uses a modified Smith&Wilson implementation (The original implementation is availible on GitHub:

- Python
- Matlab
- JavaScript

Limitations of the implementation

Current implementation only looks at a single currency and with/without Volatility Adjustment (VA). The day count convention assumes that each year has the same number of days.

Data requirements

This script contains the EIOPA risk-free rate publication for March 2023. The publication can be found on the EIOPA RFR website.

The observed maturities `M_Obs` and the calibrated vector `Qb` can be found in the Excel sheet *EIOPA_RFR_20230331_Qb_SW.xlsx*.

For this example, the curve without the volatility adjustment (VA) is used. It can be found in the sheet *SW_Qb_no_VA*. This example is focused on the EUR curve, but this example can be easily modified for any other curve.

The target maturities (`T_Obs`), the additional parameters (`UFR` and `alpha`), and the given curve can be found in the Excel *EIOPA_RFR_20230331_Term_Structures.xlsx*, sheet *RFR_spot_no_VA* if the test looks at the curve without the Volatility Adjustment and the sheet *RFR_spot_with_VA* if the test looks at the curve with the Volatility Adjustment.

Back to the top

Success criteria

The following success criteria is defined:

- Maximum difference between the calculated curve and the one provided by EIOPA is less than 0.1 bps
- Average difference between the calculated curve and the one provided by EIOPA is less than 0.05 bps

In [63]:
```
test_statistics_max_diff_in_bps = 0.1
test_statistics_average_diff_in_bps = 0.05
```

The success function is called at the end of the test to confirm if the success criteria have been met.

In [64]:
```
def SuccessTest(TestStatistics, threshold_max, threshold_mean):
    out1 = False
    out2 = False
    if max(TestStatistics)<threshold_max:
        print("Test passed")
        out1 = True
    else:
        print("Test failed")

    if np.mean(TestStatistics)<threshold_mean:
        print("Test passed")
        out2 = True
    else:
        print("Test failed")
    return [out1, out2]
```

Back to the top

External dependencies

This implementation uses three well established Python packages widely used in the financial industry. Pandas (https://pandas.pydata.org/docs/), Numpy (https://numpy.org/doc/), and Matplotlib (https://matplotlib.org/stable/index.html)

In [65]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import matplotlib.ticker as mtick
%matplotlib notebook
pd.options.display.max_rows = 150
```

Importing data

In [66]:
```
#selected_param_file = 'Param_VA.csv'
#selected_curves_file = 'Curves_VA.csv'

selected_param_file = 'Param_no_VA.csv'
selected_curves_file = 'Curves_no_VA.csv'
```

In [67]:
```
param_raw = pd.read_csv(selected_param_file, sep=',',index_col=0)
```

Parameter input

Parameters sheet

In [68]:
```
param_raw.head()
```

Out[68]:

Country	Euro_Maturities	Euro_Values	Austria_Maturities	Austria_Values	Belgium_Maturities	Be
Coupon_freq	1.000000	1.000000	1.000000	1.000000	1.000000	
LLP	20.000000	20.000000	20.000000	20.000000	20.000000	
Convergence	40.000000	40.000000	40.000000	40.000000	40.000000	
UFR	3.450000	3.450000	3.450000	3.450000	3.450000	
alpha	0.117567	0.117567	0.117567	0.117567	0.117567	

5 rows × 106 columns

The country selected is:

In [69]:
```
country = "United States"
```

In [70]:
```
maturities_country_raw = param_raw.loc[:,country+"_Maturities"].iloc[6:]
param_country_raw = param_raw.loc[:,country + "_Values"].iloc[6:]
extra_param = param_raw.loc[:,country + "_Values"].iloc[:6]
```

Extra parameters

Smith-Wilson calibration parameters

In [71]:
```
extra_param
```

Out[71]:
```
Country
Coupon_freq     1.00000
LLP            30.00000
Convergence    40.00000
UFR             3.45000
alpha           0.11177
CRA             0.00000
Name: United States_Values, dtype: float64
```

In [72]:
```
relevant_positions = pd.notna(maturities_country_raw.values)
```

In [73]:
```
maturities_country = maturities_country_raw.iloc[relevant_positions]
```

Maturity vector

Vector of maturities used in the calibration

In [74]:
```
maturities_country.head(15)
```

Out[74]:
```
Country
1      1.0
2      2.0
3      3.0
4      4.0
5      5.0
6      6.0
7      7.0
8      8.0
9      9.0
10    10.0
11    11.0
12    12.0
13    13.0
14    14.0
15    15.0
Name: United States_Maturities, dtype: float64
```

In [75]:
```
Qb = param_country_raw.iloc[relevant_positions]
```

Calibration vector

Vector Qb provided as input

In [76]:
```
Qb
```

Out[76]:
```
Country
1    -23.814834
2      9.193555
3      0.610216
4      1.367943
5     -0.588287
6      0.007033
7      0.060663
8      0.004887
9      0.678860
10    -0.476508
11    -0.001511
12    -0.001461
13    -0.001412
14    -0.001365
15     0.201034
16    -0.007500
```

```
17    -0.007250
18    -0.007008
19    -0.006774
20    -0.607990
21     0.011914
22     0.011517
23     0.011133
24     0.010761
25     0.010402
26     0.010056
27     0.009720
28     0.009396
29     0.009083
30     0.308534
Name: United States Values, dtype: float64
```

In [77]:
```
curve_raw = pd.read_csv(selected_curves_file, sep=',',index_col=0)
```

In [78]:
```
curve_country = curve_raw.loc[:,country]
```

Back to the top

Calibration parameters and calibration vector provided by EIOPA

In [79]:
```
# Maturity of observations:
M_Obs = np.transpose(np.array(maturities_country.values))

# Ultimate froward rate ufr represents the rate to which the rate curve will converge
ufr = extra_param.iloc[3]/100

# Convergence speed parameter alpha controls the speed at which the curve converges t
alpha = extra_param.iloc[4]

# For which maturities do we want the SW algorithm to calculate the rates. In this ca
M_Target = np.transpose(np.arange(1,151))

# Qb calibration vector published by EIOPA for the curve calibration:
Qb = np.transpose(np.array(Qb.values))
```

Back to the top

Smith & Wilson calculation functions

In this step, the independent version of the Smith&Wilson algorithm is implemented. To do this, two functions are taken from the publicly available repository and modified to accept the product of `Q*b` instead of the calibration vector `b`.

In [80]:

```
def SWExtrapolate(M_Target, M_Obs, Qb, ufr, alpha):
# SWEXTRAPOLATE Interpolate or/and extrapolate rates for targeted maturities using a
# out = SWExtrapolate(M_Target, M_Obs, Qb, ufr, alpha) calculates the rates for matu
#
# Arguments:
#    M_Target = k x 1 ndarray. Each element represents a bond maturity of interest. E
#    M_Obs =    n x 1 ndarray. Observed bond maturities used for calibrating the cal
#    Qb =       n x 1 ndarray. Calibration vector calculated on observed bonds.
#    ufr =      1 x 1 floating number. Representing the ultimate forward rate.
#       Ex. ufr = 0.042
#    alpha =    1 x 1 floating number. Representing the convergence speed parameter o
#
#
# Returns:
#    k x 1 ndarray. Represents the targeted rates for a zero-coupon bond. Each rate E
#
# For more information see https://www.eiopa.europa.eu/sites/default/files/risk_free_

    def SWHeart(u, v, alpha):
    # SWHEART Calculate the heart of the Wilson function.
    # H = SWHeart(u, v, alpha) calculates the matrix H (Heart of the Wilson
    # function) for maturities specified by vectors u and v. The formula is
    # taken from the EIOPA technical specifications paragraph 132.
    #
    # Arguments:
    #    u =     n_1 x 1 vector of maturities. Ex. u = [1; 3]
    #    v =     n_2 x 1 vector of maturities. Ex. v = [1; 2; 3; 5]
    #    alpha = 1 x 1 floating number representing the convergence speed parameter o
    #
    # Returns:
    #    n_1 x n_2 matrix representing the Heart of the Wilson function for selected
    #
    # For more information see https://www.eiopa.europa.eu/sites/default/files/risk_f

        u_Mat = np.tile(u, [v.size, 1]).transpose()
        v_Mat = np.tile(v, [u.size, 1])
        return 0.5 * (alpha * (u_Mat + v_Mat) + np.exp(-alpha * (u_Mat + v_Mat)) - al

    H = SWHeart(M_Target, M_Obs, alpha) # Heart of the Wilson function from paragraph
    p = np.exp(-np.log(1+ufr)* M_Target) + np.diag(np.exp(-np.log(1+ufr) * M_Target))
    return p ** (-1/ M_Target) -1 # Convert obtained prices to rates and return price
```

Back to the top

Generation of the risk-free curve

The observed maturities, target maturities, and the model parameters provided by EIOPA are used to generate the target curve.

In [81]:

```
r_Target = SWExtrapolate(M_Target,M_Obs, Qb, ufr, alpha)
r_Target = pd.DataFrame(r_Target,columns=['Recalculated rates'])
```

Yield curve calculated

Yield curve calculated using the calibration vector Qb

In [82]:
```
r_Target.head(15)
```

Out[82]:

	Recalculated rates
0	0.047188
1	0.040640
2	0.036710
3	0.034543
4	0.033322
5	0.032553
6	0.032058
7	0.031766
8	0.031627
9	0.031598
10	0.031603
11	0.031618
12	0.031632
13	0.031640
14	0.031638

Back to the top

Test 1; Comparison test

Comparison of the calculated yield curve with the yield curve provided by EIOPA. The test is passed if the success criteria is reached.

The provided yield curve can be found in file *EIOPA_RFR_20230331_Term_Structures.xlsx*, sheet *RFR_spot_no_VA* if the test looks at the curve without the Volatility Adjustment and the sheet *RFR_spot_with_VA* if the test looks at the curve with the Volatility Adjustment.

In [83]:
```
target_curve = np.transpose(np.array(curve_country.values))
```

This implementation looks at two kinds of test statistics. The average deviation and the maximum deviation.

The average deviation is defined as:

$$S_{AVERAGE} = \frac{1}{T}\sum_{t=0}^{T}|r_{EIOPA}(t) - r_{EST}(t)|$$

The maximum deviation is defined as:

$$S_{MAX} = \max_{t}|r_{EIOPA}(t) - r_{EST}(t)|$$

Where `T` is the maximum maturity available.

The average difference test is successful if:

$$S_{AVERAGE} < 0.05bps$$

The maximum difference test is successful if:

$$S_{MAX} < 0.1bps$$

In [84]:
```
target_curve = pd.DataFrame(target_curve,columns=['Given rates'])
```

EIOPA curve provided

Yield curve provided by EIOPA

In [85]:
```
target_curve.head()
```

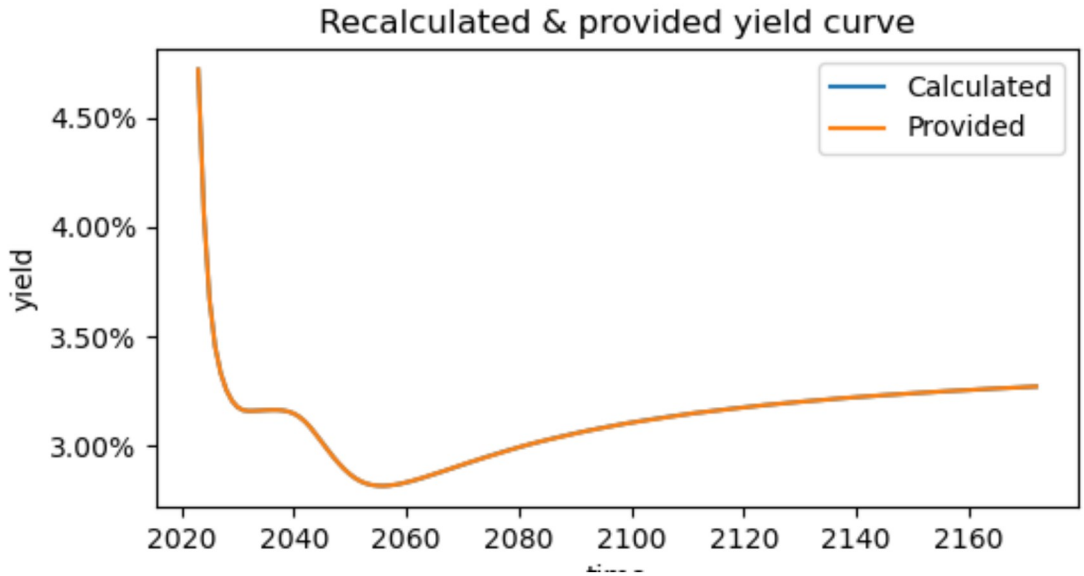
Out[85]:

	Given rates
0	0.04719
1	0.04064
2	0.03671
3	0.03454
4	0.03332

In [86]:
```
x_data_label = range(2023,2023+r_Target.shape[0],1)
```

In [87]:
```
fig, ax1 = plt.subplots(1,1)
ax1.plot(x_data_label, r_Target.values*100, color='tab:blue',label="Calculated")
ax1.plot(x_data_label, target_curve.values*100, color='tab:orange',label="Provided")

ax1.set_ylabel("yield")
ax1.set_title('Recalculated & provided yield curve')
ax1.set_xlabel("time")
ax1.legend()
ax1.yaxis.set_major_formatter(mtick.PercentFormatter())
fig.set_figwidth(6)
fig.set_figheight(3)
plt.show()
```



In [88]:
```
test_statistics_bdp = pd.DataFrame(abs(r_Target.values-target_curve.values)*10000, c
```

EIOPA curve comparison

Absolute difference in bps

In [89]:
```
test_statistics_bdp.head()
```

Out[89]:

	Abs diff in bps
0	0.020000
1	0.000047
2	0.004152
3	0.026523

Abs diff in bps

Back to the top

Test 1; Success criteria

The successful application of the success criteria marks the completion of the test.

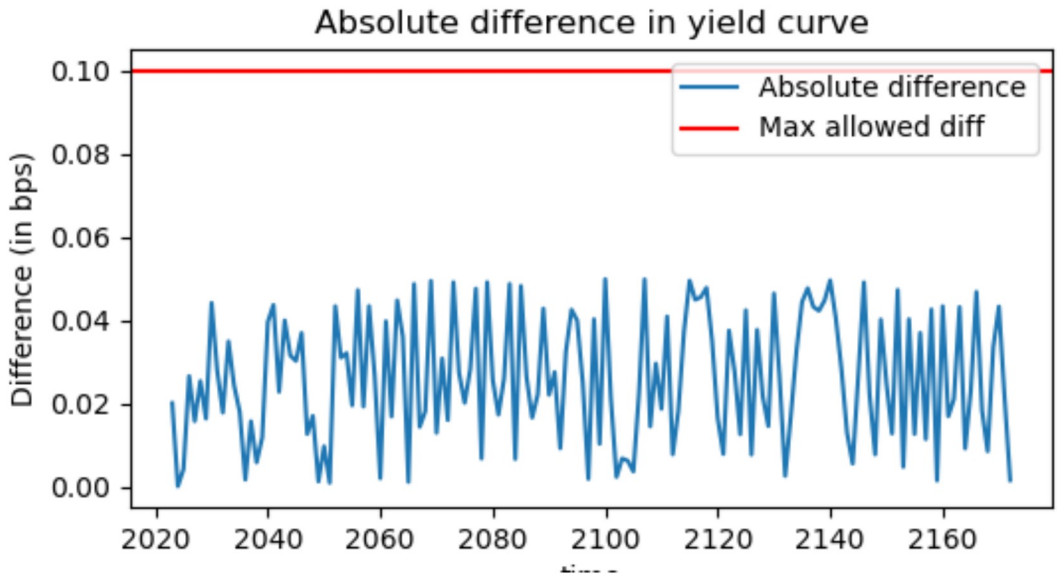
In [90]:
```
result1 = SuccessTest(test_statistics_bdp.values, test_statistics_max_diff_in_bps, te
```

```
Test passed
Test passed
```

In [91]:
```
x_data_label = range(2023,2023+r_Target.shape[0],1)
fig, ax1 = plt.subplots(1,1)
ax1.plot(x_data_label, test_statistics_bdp, label= "Absolute difference")
ax1.axhline(y = test_statistics_max_diff_in_bps, color = 'r', linestyle = '-',label='

ax1.set_xlabel("time")
ax1.set_ylabel("Difference (in bps)")
ax1.set_title('Absolute difference in yield curve')
ax1.legend()
fig.set_figwidth(6)
fig.set_figheight(3)

plt.show()
```



Back to the top

Conclusion

This test checks the success criteria on the EIOPA curve generated for March 2023. If the tests are passed, it is likely that the curve was generated using the Smith & Wilson algorithm with the

In [92]:
```
pd.DataFrame(data = [result1], columns = ["Mean test","Max test"],  \
             index= ["Provided vs calculated"])
```

Out[92]:

	Mean test	Max test
Provided vs calculated	True	True

Final yield curve

Full yield curve provided by EIOPA in %

In [93]:
```
(curve_country*100).head(150)
```

Out[93]:
```
Country
1      4.719
2      4.064
3      3.671
4      3.454
5      3.332
6      3.255
7      3.206
8      3.177
9      3.163
10     3.160
11     3.160
12     3.162
13     3.163
14     3.164
15     3.164
16     3.162
17     3.157
18     3.148
19     3.132
20     3.111
21     3.083
22     3.050
23     3.016
24     2.982
25     2.950
26     2.920
27     2.893
28     2.870
29     2.851
30     2.837
31     2.827
32     2.821
33     2.818
```

34	2.817
35	2.819
36	2.823
37	2.827
38	2.833
39	2.840
40	2.847
41	2.855
42	2.863
43	2.871
44	2.880
45	2.888
46	2.897
47	2.905
48	2.914
49	2.923
50	2.931
51	2.940
52	2.948
53	2.956
54	2.964
55	2.971
56	2.979
57	2.986
58	2.994
59	3.001
60	3.008
61	3.014
62	3.021
63	3.027
64	3.034
65	3.040
66	3.046
67	3.052
68	3.057
69	3.063
70	3.068
71	3.073
72	3.078
73	3.083
74	3.088
75	3.093
76	3.098
77	3.102
78	3.106
79	3.111
80	3.115
81	3.119
82	3.123
83	3.127
84	3.131
85	3.135
86	3.138
87	3.142
88	3.145
89	3.149
90	3.152
91	3.155
92	3.158

```
93     3.161
94     3.165
95     3.168
96     3.170
97     3.173
98     3.176
99     3.179
100    3.182
101    3.184
102    3.187
103    3.189
104    3.192
105    3.194
106    3.197
107    3.199
108    3.201
109    3.204
110    3.206
111    3.208
112    3.210
113    3.212
114    3.215
115    3.217
116    3.219
117    3.221
118    3.222
119    3.224
120    3.226
121    3.228
122    3.230
123    3.232
124    3.233
125    3.235
126    3.237
127    3.239
128    3.240
129    3.242
130    3.243
131    3.245
132    3.247
133    3.248
134    3.250
135    3.251
136    3.253
137    3.254
138    3.255
139    3.257
140    3.258
141    3.260
142    3.261
143    3.262
144    3.264
145    3.265
146    3.266
147    3.267
148    3.269
149    3.270
150    3.271
Name: United States, dtype: float64
```