EIOPA RISK-FREE CURVE JANUARY-23 RECALCULATION

The risk-free curve is one of the principal inputs into an economic scenario generator. This notebook recalculates the risk-free curve using the parameters that are claimed to be used. The European Insurance and Occupational Pensions Authority (EIOPA) publishes their own yield curve prediction. To do this they use the Smith & Wilson algorithm.

Summary

The goal of this test is to replicate the EIOPA yield curve. This test will use the methodology that EIOPA claims it is using and the calibration vector that they publish. If the test is passed, the user can be more confident, that EIOPA risk free rate (RFR) curve was generated using the described methodology/calibration and that the process was implemented correctly.

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Note on Smith & Wilson algorithm

To replicate the calculations, this example uses a modified Smith&Wilson implementation (The original implementation is availible on GitHub:

- Python
- Matlab
- JavaScript

Limitations of the implementation

Current implementation only looks at a single currency and with/without Volatility Adjustment (VA). The day count convention assumes that each year has the same number of days.

Data requirements

This script contains the EIOPA risk-free rate publication for January 2023. The publication can be found on the EIOPA RFR website.

The observed maturities `M_Obs` and the calibrated vector `Qb` can be found in the Excel sheet *EIOPA_RFR_20230131_Qb_SW.xlsx*.

For this example, the curve without the volatility adjustment (VA) is used. It can be found in the sheet *SW_Qb_no_VA*. This example is focused on the EUR curve, but this example can be easily modified for any other curve.

The target maturities (`T_Obs`), the additional parameters (`UFR` and `alpha`), and the given curve can be found in the Excel *EIOPA_RFR_20230131_Term_Structures.xlsx*, sheet *RFR_spot_no_VA* if the test looks at the curve without the Volatility Adjustment and the sheet *RFR_spot_with_VA* if the test looks at the curve with the Volatility Adjustment.

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Success criteria

The following success criteria is defined:

- Maximum difference between the calculated curve and the one provided by EIOPA is less than 0.1 bps
- Average difference between the calculated curve and the one provided by EIOPA is less than 0.05 bps

In [218…

```
test_statistics_max_diff_in_bps = 0.1
test_statistics_average_diff_in_bps = 0.05
```

The success function is called at the end of the test to confirm if the success criteria have been met.

In [219…

```
def SuccessTest(TestStatistics, threshold_max, threshold_mean):
    out1 = False
    out2 = False
    if max(TestStatistics)<threshold_max:
        print("Test passed")
        out1 = True
    else:
        print("Test failed")

    if np.mean(TestStatistics)<threshold_mean:
        print("Test passed")
        out2 = True
    else:
        print("Test failed")
    return [out1, out2]
```

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External dependencies

This implementation uses three well established Python packages widely used in the financial industry. Pandas (https://pandas.pydata.org/docs/), Numpy (https://numpy.org/doc/), and Matplotlib (https://matplotlib.org/stable/index.html)

In [220…

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import matplotlib.ticker as mtick
%matplotlib notebook
pd.options.display.max_rows = 150
```

Importing data

In [221…

```
selected_param_file = 'Param_VA.csv'
selected_curves_file = 'Curves_VA.csv'

#selected_param_file = 'Param_no_VA.csv'
#selected_curves_file = 'Curves_no_VA.csv'
```

In [222…

```
param_raw = pd.read_csv(selected_param_file, sep=',',index_col=0)
```

Parameter input

Parameters sheet

In [223…

```
param_raw.head()
```

Out[223…

	Euro_Maturities	Euro_Values	Austria_Maturities	Austria_Values	Belgium_Maturities	Be
Country						
Coupon_freq	1.000000	1.000000	1.000000	1.000000	1.000000	
LLP	20.000000	20.000000	20.000000	20.000000	20.000000	
Convergence	40.000000	40.000000	40.000000	40.000000	40.000000	
UFR	3.450000	3.450000	3.450000	3.450000	3.450000	
alpha	0.116683	0.116683	0.116683	0.116683	0.116683	

5 rows × 106 columns

The country selected is:

In [224…

```
country = "Italy"
```

In [225…

```
maturities_country_raw = param_raw.loc[:,country+"_Maturities"].iloc[6:]
param_country_raw = param_raw.loc[:,country + "_Values"].iloc[6:]
extra_param = param_raw.loc[:,country + "_Values"].iloc[:6]
```

Extra parameters

Smith-Wilson calibration parameters

In [226…

```
extra_param
```

Out[226…

```
Country
Coupon_freq      1.000000
LLP             20.000000
Convergence     40.000000
UFR              3.450000
alpha            0.116683
CRA             10.000000
Name: Italy_Values, dtype: float64
```

In [227…

```
relevant_positions = pd.notna(maturities_country_raw.values)
```

In [228…

```
maturities_country = maturities_country_raw.iloc[relevant_positions]
```

Maturity vector

Vector of maturities used in the calibration

In [229…

```
maturities_country.head(15)
```

Out[229…

```
Country
1      1.0
2      2.0
3      3.0
4      4.0
5      5.0
6      6.0
7      7.0
8      8.0
9      9.0
10    10.0
11    11.0
12    12.0
13    13.0
14    14.0
15    15.0
Name: Italy_Maturities, dtype: float64
```

In [230…

```
Qb = param_country_raw.iloc[relevant_positions]
```

Calibration vector

Vector Qb provided as input

In [231…

```
Qb
```

Out[231…

```
Country
1      4.075974
2     -9.019110
3      5.067062
4     -0.675966
5      0.506904
6     -0.683948
7      0.731517
8     -0.580714
9      2.940751
10    -7.591322
11     9.524673
12    -4.354283
13     0.000398
14    -0.004756
15    -0.896575
16     0.020961
```

```
17    0.047042
18   -0.058291
19    0.329514
20    0.447276
Name: Italy Values, dtype: float64
```

In [232…

```
curve_raw = pd.read_csv(selected_curves_file, sep=',',index_col=0)
```

In [233…

```
curve_country = curve_raw.loc[:,country]
```

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Calibration parameters and calibration vector provided by EIOPA

In [234…

```
# Maturity of observations:
M_Obs = np.transpose(np.array(maturities_country.values))

# Ultimate froward rate ufr represents the rate to which the rate curve will converge
ufr = extra_param.iloc[3]/100

# Convergence speed parameter alpha controls the speed at which the curve converges t
alpha = extra_param.iloc[4]

# For which maturities do we want the SW algorithm to calculate the rates. In this ca
M_Target = np.transpose(np.arange(1,151))

# Qb calibration vector published by EIOPA for the curve calibration:
Qb = np.transpose(np.array(Qb.values))
```

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Smith & Wilson calculation functions

In this step, the independent version of the Smith&Wilson algorithm is implemented. To do this, two functions are taken from the publicly available repository and modified to accept the product of `Q*b` instead of the calibration vector `b`.

In [235…

```
def SWExtrapolate(M_Target, M_Obs, Qb, ufr, alpha):
# SWEXTRAPOLATE Interpolate or/and extrapolate rates for targeted maturities using a
# out = SWExtrapolate(M_Target, M_Obs, Qb, ufr, alpha) calculates the rates for matur
#
# Arguments:
#    M_Target = k x 1 ndarray. Each element represents a bond maturity of interest. E
#    M_Obs =    n x 1 ndarray. Observed bond maturities used for calibrating the cal
#    Qb =       n x 1 ndarray. Calibration vector calculated on observed bonds.
#    ufr =      1 x 1 floating number. Representing the ultimate forward rate.
#       Ex. ufr = 0.042
#    alpha =    1 x 1 floating number. Representing the convergence speed parameter a
#
#
# Returns:
#    k x 1 ndarray. Represents the targeted rates for a zero-coupon bond. Each rate b
#
# For more information see https://www.eiopa.europa.eu/sites/default/files/risk_free_

    def SWHeart(u, v, alpha):
    # SWHEART Calculate the heart of the Wilson function.
    # H = SWHeart(u, v, alpha) calculates the matrix H (Heart of the Wilson
    # function) for maturities specified by vectors u and v. The formula is
    # taken from the EIOPA technical specifications paragraph 132.
    #
    # Arguments:
    #    u =     n_1 x 1 vector of maturities. Ex. u = [1; 3]
    #    v =     n_2 x 1 vector of maturities. Ex. v = [1; 2; 3; 5]
    #    alpha = 1 x 1 floating number representing the convergence speed parameter a
    #
    # Returns:
    #    n_1 x n_2 matrix representing the Heart of the Wilson function for selected
    #
    # For more information see https://www.eiopa.europa.eu/sites/default/files/risk_f

        u_Mat = np.tile(u, [v.size, 1]).transpose()
        v_Mat = np.tile(v, [u.size, 1])
        return 0.5 * (alpha * (u_Mat + v_Mat) + np.exp(-alpha * (u_Mat + v_Mat)) - al

    H = SWHeart(M_Target, M_Obs, alpha) # Heart of the Wilson function from paragraph
    p = np.exp(-np.log(1+ufr)* M_Target) + np.diag(np.exp(-np.log(1+ufr) * M_Target))
    return p ** (-1/ M_Target) -1 # Convert obtained prices to rates and return price
```

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Generation of the risk-free curve

The observed maturities, target maturities, and the model parameters provided by EIOPA are used to generate the target curve.

In [236…

```
r_Target = SWExtrapolate(M_Target,M_Obs, Qb, ufr, alpha)
r_Target = pd.DataFrame(r_Target,columns=['Recalculated rates'])
```

Yield curve calculated

Yield curve calculated using the calibration vector Qb

In [237…

```
r_Target.head(15)
```

Out[237…

	Recalculated rates
0	0.034310
1	0.033609
2	0.031779
3	0.030647
4	0.029990
5	0.029603
6	0.029370
7	0.029286
8	0.029299
9	0.029343
10	0.029294
11	0.029452
12	0.029510
13	0.029394
14	0.029124

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Test 1; Comparison test

Comparison of the calculated yield curve with the yield curve provided by EIOPA. The test is passed if the success criteria is reached.

The provided yield curve can be found in file *EIOPA_RFR_20230131_Term_Structures.xlsx*, sheet *RFR_spot_no_VA* if the test looks at the curve without the Volatility Adjustment and the sheet *RFR_spot_with_VA* if the test looks at the curve with the Volatility Adjustment.

In [238…

```
target_curve = np.transpose(np.array(curve_country.values))
```

This implementation looks at two kinds of test statistics. The average deviation and the maximum deviation.

The average deviation is defined as:

$$S_{AVERAGE} = \frac{1}{T}\sum_{t=0}^{T}|r_{EIOPA}(t) - r_{EST}(t)|$$

The maximum deviation is defined as:

$$S_{MAX} = \max_{t}|r_{EIOPA}(t) - r_{EST}(t)|$$

Where `T` is the maximum maturity available.

The average difference test is successful if:

$$S_{AVERAGE} < 0.05bps$$

The maximum difference test is successful if:

$$S_{MAX} < 0.1bps$$

In [239…

```
target_curve = pd.DataFrame(target_curve,columns=['Given rates'])
```

EIOPA curve provided

Yield curve provided by EIOPA

In [240…

```
target_curve.head()
```

Out[240…

	Given rates
0	0.03431
1	0.03361
2	0.03178
3	0.03065
4	0.02999

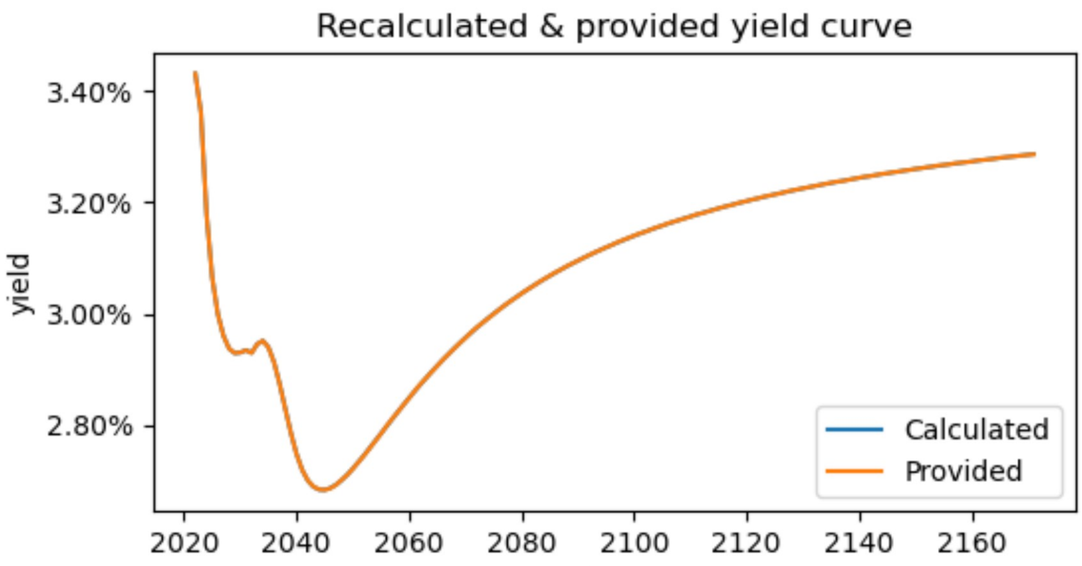
In [241…

```
x_data_label = range(2022,2022+r_Target.shape[0],1)
```

In [242…

```
fig, ax1 = plt.subplots(1,1)
ax1.plot(x_data_label, r_Target.values*100, color='tab:blue',label="Calculated")
ax1.plot(x_data_label, target_curve.values*100, color='tab:orange',label="Provided")

ax1.set_ylabel("yield")
ax1.set_title('Recalculated & provided yield curve')
ax1.set_xlabel("time")
ax1.legend()
ax1.yaxis.set_major_formatter(mtick.PercentFormatter())
fig.set_figwidth(6)
fig.set_figheight(3)
plt.show()
```



In [243…

```
test_statistics_bdp = pd.DataFrame(abs(r_Target.values-target_curve.values)*10000, co
```

EIOPA curve comparison

Absolute difference in bps

In [244…

```
test_statistics_bdp.head()
```

Out[244…

	Abs diff in bps
0	2.754812e-07
1	1.004892e-02
2	9.485243e-03
3	2.603547e-02

Abs diff in bps

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Test 1; Success criteria

The successful application of the success criteria marks the completion of the test.

In [245…

```
result1 = SuccessTest(test_statistics_bdp.values, test_statistics_max_diff_in_bps, te
```

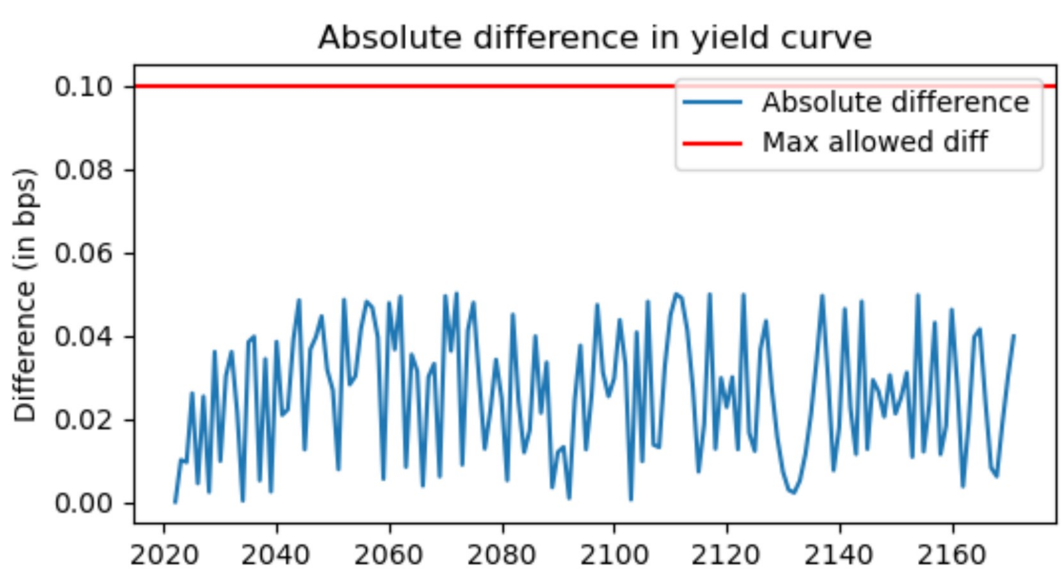
```
Test passed
Test passed
```

In [246…

```
x_data_label = range(2022,2022+r_Target.shape[0],1)
fig, ax1 = plt.subplots(1,1)
ax1.plot(x_data_label, test_statistics_bdp, label= "Absolute difference")
ax1.axhline(y = test_statistics_max_diff_in_bps, color = 'r', linestyle = '-',label='

ax1.set_xlabel("time")
ax1.set_ylabel("Difference (in bps)")
ax1.set_title('Absolute difference in yield curve')
ax1.legend()
fig.set_figwidth(6)
fig.set_figheight(3)

plt.show()
```



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Conclusion

This test checks the success criteria on the EIOPA curve generated for January 2023. If the tests are passed, it is likely that the curve was generated using the Smith & Wilson algorithm with the

In [247…

```
pd.DataFrame(data = [result1], columns = ["Mean test","Max test"],  \
             index= ["Provided vs calculated"])
```

Out[247…

	Mean test	Max test
Provided vs calculated	True	True

Final yield curve

Full yield curve provided by EIOPA in %

In [248…

```
(curve_country*100).head(150)
```

Out[248…

```
Country
1      3.431
2      3.361
3      3.178
4      3.065
5      2.999
6      2.960
7      2.937
8      2.929
9      2.930
10     2.934
11     2.929
12     2.945
13     2.951
14     2.939
15     2.912
16     2.873
17     2.828
18     2.785
19     2.747
20     2.719
21     2.700
22     2.689
23     2.684
24     2.683
25     2.686
26     2.692
27     2.701
28     2.710
29     2.721
30     2.733
31     2.745
32     2.758
33     2.771
```

34	2.784
35	2.798
36	2.811
37	2.823
38	2.836
39	2.848
40	2.861
41	2.873
42	2.884
43	2.895
44	2.906
45	2.917
46	2.927
47	2.937
48	2.947
49	2.956
50	2.966
51	2.975
52	2.983
53	2.991
54	2.999
55	3.007
56	3.015
57	3.022
58	3.029
59	3.036
60	3.043
61	3.049
62	3.056
63	3.062
64	3.068
65	3.074
66	3.079
67	3.085
68	3.090
69	3.095
70	3.100
71	3.105
72	3.110
73	3.114
74	3.119
75	3.123
76	3.128
77	3.132
78	3.136
79	3.140
80	3.144
81	3.147
82	3.151
83	3.155
84	3.158
85	3.162
86	3.165
87	3.168
88	3.171
89	3.174
90	3.178
91	3.180
92	3.183

```
93     3.186
94     3.189
95     3.192
96     3.194
97     3.197
98     3.200
99     3.202
100    3.205
101    3.207
102    3.210
103    3.212
104    3.214
105    3.216
106    3.219
107    3.221
108    3.223
109    3.225
110    3.227
111    3.229
112    3.231
113    3.233
114    3.235
115    3.237
116    3.238
117    3.240
118    3.242
119    3.244
120    3.246
121    3.247
122    3.249
123    3.251
124    3.252
125    3.254
126    3.255
127    3.257
128    3.258
129    3.260
130    3.261
131    3.263
132    3.264
133    3.266
134    3.267
135    3.268
136    3.270
137    3.271
138    3.272
139    3.273
140    3.275
141    3.276
142    3.277
143    3.278
144    3.280
145    3.281
146    3.282
147    3.283
148    3.284
149    3.285
150    3.286
Name: Italy, dtype: float64
```