EIOPA RISK-FREE CURVE JUNE-23 RECALCULATION

The risk-free curve is one of the principal inputs into an economic scenario generator. This notebook recalculates the risk-free curve using the parameters that are claimed to be used. The European Insurance and Occupational Pensions Authority (EIOPA) publishes their own yield curve prediction. To do this they use the Smith & Wilson algorithm.

Summary

The goal of this test is to replicate the EIOPA yield curve. This test will use the methodology that EIOPA claims it is using and the calibration vector that they publish. If the test is passed, the user can be more confident, that EIOPA risk free rate (RFR) curve was generated using the described methodology/calibration and that the process was implemented correctly.

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Note on Smith & Wilson algorithm

To replicate the calculations, this example uses a modified Smith&Wilson implementation (The original implementation is availible on GitHub):

- Python
- Matlab
- JavaScript

Limitations of the implementation

Current implementation only looks at a single currency and with/without Volatility Adjustment (VA). The day count convention assumes that each year has the same number of days.

Data requirements

This script contains the EIOPA risk-free rate publication for June 2023. The publication can be found on the EIOPA RFR website.

The observed maturities `M_Obs` and the calibrated vector `Qb` can be found in the Excel sheet *EIOPA_RFR_20230630_Qb_SW.xlsx*.

The target maturities (`T_Obs`), the additional parameters (`UFR` and `alpha`), and the given curve can be found in the Excel *EIOPA_RFR_20230630_Term_Structures.xlsx*, sheet *RFR_spot_no_VA* if the test looks at the curve without the Volatility Adjustment and the sheet *RFR_spot_with_VA* if the test looks at the curve with the Volatility Adjustment.

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Success criteria

The following success criteria is defined:

- Maximum difference between the calculated curve and the one provided by EIOPA is less than 0.1 bps
- Average difference between the calculated curve and the one provided by EIOPA is less than 0.05 bps

In [219…

```
test_statistics_max_diff_in_bps = 0.1
test_statistics_average_diff_in_bps = 0.05
```

The success function is called at the end of the test to confirm if the success criteria have been met.

In [220…

```
def SuccessTest(TestStatistics, threshold_max, threshold_mean):
    out1 = False
    out2 = False
    if max(TestStatistics)<threshold_max:
        print("Test passed")
        out1 = True
    else:
        print("Test failed")

    if np.mean(TestStatistics)<threshold_mean:
        print("Test passed")
        out2 = True
    else:
        print("Test failed")
    return [out1, out2]
```

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External dependencies

This implementation uses three well established Python packages widely used in the financial industry. Pandas (https://pandas.pydata.org/docs/), Numpy (https://numpy.org/doc/), and Matplotlib (https://matplotlib.org/stable/index.html)

In [221…

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import matplotlib.ticker as mtick
%matplotlib notebook
pd.options.display.max_rows = 150
```

Importing data

In [222…

```
selected_param_file = 'Param_VA.csv'
selected_curves_file = 'Curves_VA.csv'

#selected_param_file = 'Param_no_VA.csv'
#selected_curves_file = 'Curves_no_VA.csv'
```

In [223…

```
param_raw = pd.read_csv(selected_param_file, sep=',',index_col=0)
```

Parameter input

Parameters sheet

In [224...
```
param_raw.head()
```

Out[224...

Country	Euro_Maturities	Euro_Values	Austria_Maturities	Austria_Values	Belgium_Maturities	Be
Coupon_freq	1.000000	1.000000	1.000000	1.000000	1.000000	
LLP	20.000000	20.000000	20.000000	20.000000	20.000000	
Convergence	40.000000	40.000000	40.000000	40.000000	40.000000	
UFR	3.450000	3.450000	3.450000	3.450000	3.450000	
alpha	0.111987	0.111987	0.111987	0.111987	0.111987	

5 rows × 106 columns

The country selected is:

In [225...
```
country = "United Kingdom"
```

In [226...
```
maturities_country_raw = param_raw.loc[:,country+"_Maturities"].iloc[6:]
param_country_raw = param_raw.loc[:,country + "_Values"].iloc[6:]
extra_param = param_raw.loc[:,country + "_Values"].iloc[:6]
```

Extra parameters

Smith-Wilson calibration parameters

In [227...
```
extra_param
```

Out[227...
```
Country
Coupon_freq     1.000000
LLP            50.000000
Convergence    40.000000
UFR             3.450000
alpha           0.093261
CRA             0.000000
Name: United Kingdom_Values, dtype: float64
```

In [228...
```
relevant_positions = pd.notna(maturities_country_raw.values)
```

In [229…
```
maturities_country = maturities_country_raw.iloc[relevant_positions]
```

Maturity vector

Vector of maturities used in the calibration

In [230…
```
maturities_country.head(15)
```

Out[230…
```
Country
1      1.0
2      2.0
3      3.0
4      4.0
5      5.0
6      6.0
7      7.0
8      8.0
9      9.0
10    10.0
11    11.0
12    12.0
13    13.0
14    14.0
15    15.0
Name: United Kingdom_Maturities, dtype: float64
```

In [231…
```
Qb = param_country_raw.iloc[relevant_positions]
```

Calibration vector

Vector **Qb** provided as input

In [232…
```
Qb
```

Out[232…
```
Country
1     11.350192
2    -20.684799
3      7.460112
4     -2.287102
5      2.750946
6     -2.945772
7      3.125840
8     -0.015851
9     -1.781287
10     2.286607
11    -0.037185
12    -0.495368
13    -0.009503
14    -0.009204
15    -0.264215
16     0.003854
```

```
17     0.003449
18     0.003229
19     0.003101
20     0.186594
21    -0.005598
22    -0.005100
23    -0.004767
24    -0.004406
25    -0.004067
26    -0.003728
27    -0.003415
28    -0.003034
29    -0.002921
30    -0.370281
31     0.013681
32     0.012675
33     0.012023
34     0.011337
35     0.010718
36     0.010132
37     0.009584
38     0.009069
39     0.008586
40     0.008125
41     0.007709
42     0.007243
43     0.007069
44     0.005900
45     0.008529
46    -0.002944
47     0.038303
48    -0.117241
49     0.462069
50    -0.245207
Name: United Kingdom Values, dtype: float64
```

In [233…

```
curve_raw = pd.read_csv(selected_curves_file, sep=',',index_col=0)
```

In [234…

```
curve_country = curve_raw.loc[:,country]
```

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Calibration parameters and calibration vector provided by EIOPA

In [235…

```
# Maturity of observations:
M_Obs = np.transpose(np.array(maturities_country.values))

# Ultimate froward rate ufr represents the rate to which the rate curve will converge
ufr = extra_param.iloc[3]/100

# Convergence speed parameter alpha controls the speed at which the curve converges t
alpha = extra_param.iloc[4]

# For which maturities do we want the SW algorithm to calculate the rates. In this ca
M_Target = np.transpose(np.arange(1,151))

# Qb calibration vector published by EIOPA for the curve calibration:
Qb = np.transpose(np.array(Qb.values))
```

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Smith & Wilson calculation functions

In this step, the independent version of the Smith&Wilson algorithm is implemented. To do this, two functions are taken from the publicly available repository and modified to accept the product of `Q*b` instead of the calibration vector `b`.

In [236…

```
def SWExtrapolate(M_Target, M_Obs, Qb, ufr, alpha):
# SWEXTRAPOLATE Interpolate or/and extrapolate rates for targeted maturities using a
# out = SWExtrapolate(M_Target, M_Obs, Qb, ufr, alpha) calculates the rates for matu
#
# Arguments:
#    M_Target = k x 1 ndarray. Each element represents a bond maturity of interest. E
#    M_Obs =    n x 1 ndarray. Observed bond maturities used for calibrating the cal
#    Qb =       n x 1 ndarray. Calibration vector calculated on observed bonds.
#    ufr =      1 x 1 floating number. Representing the ultimate forward rate.
#       Ex. ufr = 0.042
#    alpha =    1 x 1 floating number. Representing the convergence speed parameter o
#
#
# Returns:
#    k x 1 ndarray. Represents the targeted rates for a zero-coupon bond. Each rate E
#
# For more information see https://www.eiopa.europa.eu/sites/default/files/risk_free_

    def SWHeart(u, v, alpha):
    # SWHEART Calculate the heart of the Wilson function.
    # H = SWHeart(u, v, alpha) calculates the matrix H (Heart of the Wilson
    # function) for maturities specified by vectors u and v. The formula is
    # taken from the EIOPA technical specifications paragraph 132.
    #
    # Arguments:
    #    u =     n_1 x 1 vector of maturities. Ex. u = [1; 3]
    #    v =     n_2 x 1 vector of maturities. Ex. v = [1; 2; 3; 5]
    #    alpha = 1 x 1 floating number representing the convergence speed parameter o
    #
    # Returns:
    #    n_1 x n_2 matrix representing the Heart of the Wilson function for selected
    #
    # For more information see https://www.eiopa.europa.eu/sites/default/files/risk_f

        u_Mat = np.tile(u, [v.size, 1]).transpose()
        v_Mat = np.tile(v, [u.size, 1])
        return 0.5 * (alpha * (u_Mat + v_Mat) + np.exp(-alpha * (u_Mat + v_Mat)) - al

    H = SWHeart(M_Target, M_Obs, alpha) # Heart of the Wilson function from paragraph
    p = np.exp(-np.log(1+ufr)* M_Target) + np.diag(np.exp(-np.log(1+ufr) * M_Target))
    return p ** (-1/ M_Target) -1 # Convert obtained prices to rates and return price
```

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Generation of the risk-free curve

The observed maturities, target maturities, and the model parameters provided by EIOPA are used to generate the target curve.

In [237…

```
r_Target = SWExtrapolate(M_Target,M_Obs, Qb, ufr, alpha)
r_Target = pd.DataFrame(r_Target,columns=['Recalculated rates'])
```

Yield curve calculated

Yield curve calculated using the calibration vector **Qb**

In [238...
```
r_Target.head(15)
```

Out[238...

	Recalculated rates
0	0.062420
1	0.061401
2	0.058096
3	0.054889
4	0.052079
5	0.049751
6	0.047814
7	0.046339
8	0.045199
9	0.044302
10	0.043654
11	0.043168
12	0.042763
13	0.042406
14	0.042074

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Test 1; Comparison test

Comparison of the calculated yield curve with the yield curve provided by EIOPA. The test is passed if the success criteria is reached.

The provided yield curve can be found in file *EIOPA_RFR_20230630_Term_Structures.xlsx*, sheet *RFR_spot_no_VA* if the test looks at the curve without the Volatility Adjustment and the sheet *RFR_spot_with_VA* if the test looks at the curve with the Volatility Adjustment.

In [239...
```
target_curve = np.transpose(np.array(curve_country.values))
```

This implementation looks at two kinds of test statistics. The average deviation and the maximum deviation.

The average deviation is defined as:

$$S_{AVERAGE} = \frac{1}{T}\sum_{t=0}^{T}|r_{EIOPA}(t) - r_{EST}(t)|$$

The maximum deviation is defined as:

$$S_{MAX} = \max_{t}|r_{EIOPA}(t) - r_{EST}(t)|$$

Where `T` is the maximum maturity available.

The average difference test is successful if:

$$S_{AVERAGE} < 0.05bps$$

The maximum difference test is successful if:

$$S_{MAX} < 0.1bps$$

In [240…

```
target_curve = pd.DataFrame(target_curve,columns=['Given rates'])
```

EIOPA curve provided

Yield curve provided by EIOPA

In [241…

```
target_curve.head()
```

Out[241…

	Given rates
0	0.06242
1	0.06140
2	0.05810
3	0.05489
4	0.05208

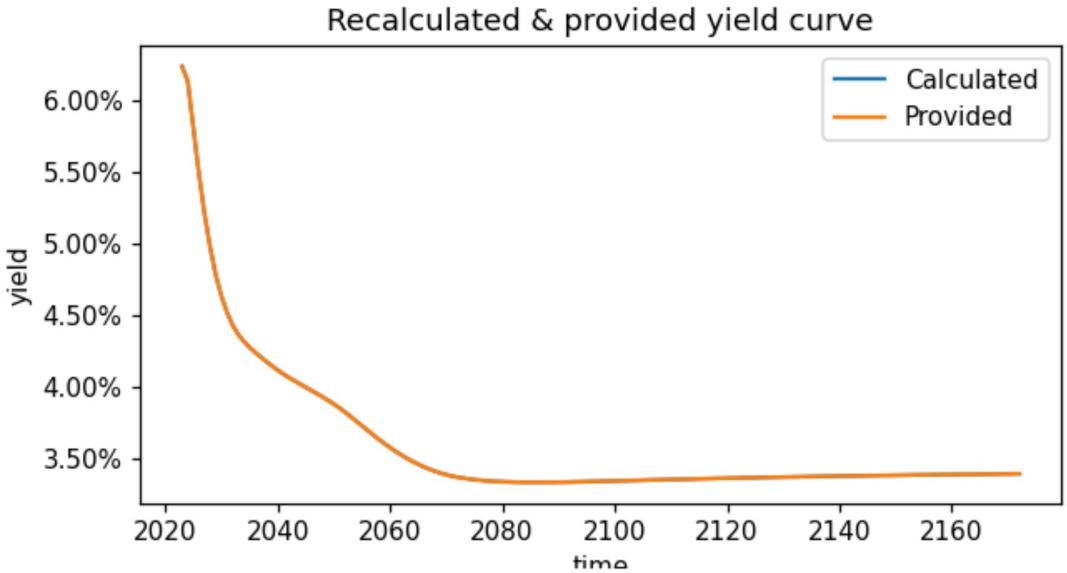
In [242...

```
x_data_label = range(2023,2023+r_Target.shape[0],1)
```

In [243...

```
fig, ax1 = plt.subplots(1,1)
ax1.plot(x_data_label, r_Target.values*100, color='tab:blue',label="Calculated")
ax1.plot(x_data_label, target_curve.values*100, color='tab:orange',label="Provided")

ax1.set_ylabel("yield")
ax1.set_title('Recalculated & provided yield curve')
ax1.set_xlabel("time")
ax1.legend()
ax1.yaxis.set_major_formatter(mtick.PercentFormatter())
fig.set_figwidth(6)
fig.set_figheight(3)
plt.show()
```



In [244...

```
test_statistics_bdp = pd.DataFrame(abs(r_Target.values-target_curve.values)*10000, c
```

EIOPA curve comparison

Absolute difference in bps

In [245...

```
test_statistics_bdp.head()
```

Out[245...

	Abs diff in bps
0	2.391429e-07
1	5.119096e-03
2	4.179371e-02
3	1.362811e-02

Abs diff in bps

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Test 1; Success criteria

The successful application of the success criteria marks the completion of the test.

In [246...

```
result1 = SuccessTest(test_statistics_bdp.values, test_statistics_max_diff_in_bps, te
```

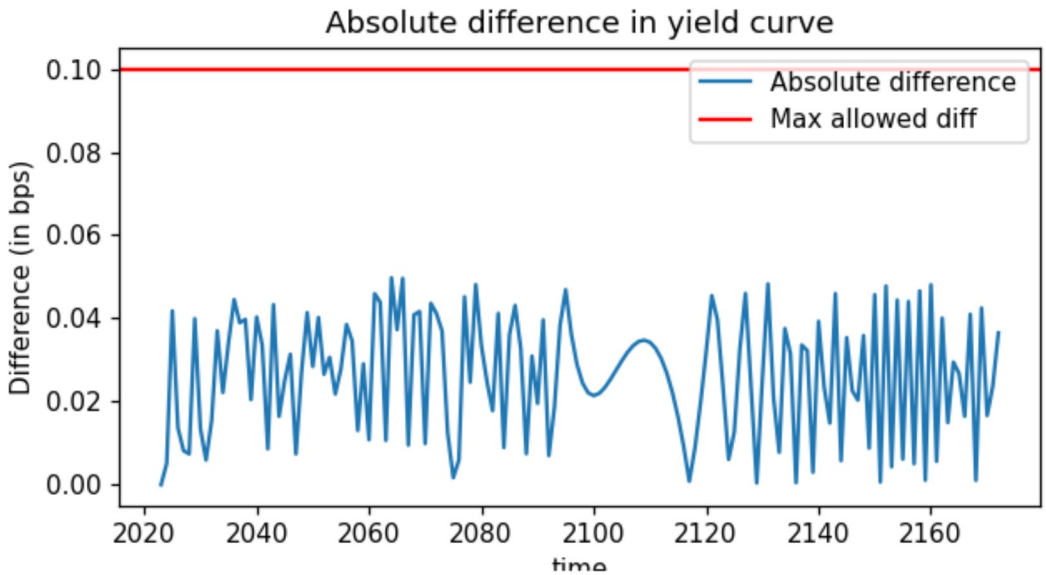
```
Test passed
Test passed
```

In [247...

```
x_data_label = range(2023,2023+r_Target.shape[0],1)
fig, ax1 = plt.subplots(1,1)
ax1.plot(x_data_label, test_statistics_bdp, label= "Absolute difference")
ax1.axhline(y = test_statistics_max_diff_in_bps, color = 'r', linestyle = '-',label='

ax1.set_xlabel("time")
ax1.set_ylabel("Difference (in bps)")
ax1.set_title('Absolute difference in yield curve')
ax1.legend()
fig.set_figwidth(6)
fig.set_figheight(3)

plt.show()
```



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Conclusion

This test checks the success criteria on the EIOPA curve generated for June 2023. If the tests are passed, it is likely that the curve was generated using the Smith & Wilson algorithm with the calibration vector that was provided in the file *EIOPA_RFR_20230630_Qb_SW.xlsx* and the parameters displayed in the file *EIOPA_RFR_20230630_Term_Structures.xlsx*.

In [248...

```
pd.DataFrame(data = [result1], columns = ["Mean test","Max test"],  \
             index= ["Provided vs calculated"])
```

Out[248...

	Mean test	Max test
Provided vs calculated	True	True

Final yield curve

Full yield curve provided by EIOPA in %

In [249...

```
(curve_country*100).head(150)
```

Out[249...

```
Country
1      6.242
2      6.140
3      5.810
4      5.489
5      5.208
6      4.975
7      4.781
8      4.634
9      4.520
10     4.430
11     4.365
12     4.317
13     4.276
14     4.241
15     4.207
16     4.176
17     4.145
18     4.116
19     4.089
20     4.064
21     4.041
22     4.018
23     3.996
24     3.974
25     3.952
26     3.929
27     3.904
28     3.879
```

29	3.851
30	3.822
31	3.791
32	3.759
33	3.727
34	3.694
35	3.663
36	3.632
37	3.602
38	3.574
39	3.547
40	3.522
41	3.499
42	3.477
43	3.458
44	3.439
45	3.423
46	3.408
47	3.395
48	3.384
49	3.374
50	3.366
51	3.360
52	3.354
53	3.349
54	3.345
55	3.342
56	3.339
57	3.336
58	3.335
59	3.333
60	3.332
61	3.331
62	3.331
63	3.331
64	3.331
65	3.331
66	3.331
67	3.331
68	3.332
69	3.332
70	3.333
71	3.334
72	3.335
73	3.335
74	3.336
75	3.337
76	3.338
77	3.339
78	3.340
79	3.341
80	3.342
81	3.343
82	3.344
83	3.345
84	3.346
85	3.347
86	3.348
87	3.349

88	3.350
89	3.351
90	3.352
91	3.353
92	3.354
93	3.355
94	3.356
95	3.357
96	3.358
97	3.359
98	3.360
99	3.361
100	3.361
101	3.362
102	3.363
103	3.364
104	3.365
105	3.365
106	3.366
107	3.367
108	3.368
109	3.368
110	3.369
111	3.370
112	3.371
113	3.371
114	3.372
115	3.373
116	3.373
117	3.374
118	3.375
119	3.375
120	3.376
121	3.376
122	3.377
123	3.378
124	3.378
125	3.379
126	3.379
127	3.380
128	3.380
129	3.381
130	3.382
131	3.382
132	3.383
133	3.383
134	3.384
135	3.384
136	3.385
137	3.385
138	3.385
139	3.386
140	3.386
141	3.387
142	3.387
143	3.388
144	3.388
145	3.389
146	3.389

```
147    3.389
148    3.390
149    3.390
150    3.391
```