EIOPA RISK-FREE CURVE JULY-23 RECALCULATION

The risk-free curve is one of the principal inputs into an economic scenario generator. This notebook recalculates the risk-free curve using the parameters that are claimed to be used. The European Insurance and Occupational Pensions Authority (EIOPA) publishes their own yield curve prediction. To do this they use the Smith & Wilson algorithm.

Summary

The goal of this test is to replicate the EIOPA yield curve. This test will use the methodology that EIOPA claims it is using and the calibration vector that they publish. If the test is passed, the user can be more confident, that EIOPA risk free rate (RFR) curve was generated using the described methodology/calibration and that the process was implemented correctly.

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Note on Smith & Wilson algorithm

To replicate the calculations, this example uses a modified Smith&Wilson implementation (The original implementation is availible on GitHub):

- Python
- Matlab
- JavaScript

Limitations of the implementation

Current implementation only looks at a single currency and with/without Volatility Adjustment (VA). The day count convention assumes that each year has the same number of days.

Data requirements

This script contains the EIOPA risk-free rate publication for July 2023. The publication can be found on the EIOPA RFR website.

The observed maturities `M_Obs` and the calibrated vector `Qb` can be found in the Excel sheet *EIOPA_RFR_20230731_Qb_SW.xlsx*.

The target maturities (`T_Obs`), the additional parameters (`UFR` and `alpha`), and the given curve can be found in the Excel *EIOPA_RFR_20230731_Term_Structures.xlsx*, sheet *RFR_spot_no_VA* if the test looks at the curve without the Volatility Adjustment and the sheet *RFR_spot_with_VA* if the test looks at the curve with the Volatility Adjustment.

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Success criteria

The following success criteria is defined:

- Maximum difference between the calculated curve and the one provided by EIOPA is less than 0.1 bps
- Average difference between the calculated curve and the one provided by EIOPA is less than 0.05 bps

In [218…

```
test_statistics_max_diff_in_bps = 0.1
test_statistics_average_diff_in_bps = 0.05
```

The success function is called at the end of the test to confirm if the success criteria have been met.

In [219…

```
def SuccessTest(TestStatistics, threshold_max, threshold_mean):
    out1 = False
    out2 = False
    if max(TestStatistics)<threshold_max:
        print("Test passed")
        out1 = True
    else:
        print("Test failed")

    if np.mean(TestStatistics)<threshold_mean:
        print("Test passed")
        out2 = True
    else:
        print("Test failed")
    return [out1, out2]
```

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External dependencies

This implementation uses three well established Python packages widely used in the financial industry. Pandas (https://pandas.pydata.org/docs/), Numpy (https://numpy.org/doc/), and Matplotlib (https://matplotlib.org/stable/index.html)

In [220…

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import matplotlib.ticker as mtick
%matplotlib notebook
pd.options.display.max_rows = 150
```

Importing data

In [221…

```
selected_param_file = 'Param_VA.csv'
selected_curves_file = 'Curves_VA.csv'

#selected_param_file = 'Param_no_VA.csv'
#selected_curves_file = 'Curves_no_VA.csv'
```

In [222…

```
param_raw = pd.read_csv(selected_param_file, sep=',',index_col=0)
```

Parameter input

Parameters sheet

In [223...
```
param_raw.head()
```

Out[223...

Country	Euro_Maturities	Euro_Values	Austria_Maturities	Austria_Values	Belgium_Maturities	Be
Coupon_freq	1.000000	1.000000	1.000000	1.000000	1.000000	
LLP	20.000000	20.000000	20.000000	20.000000	20.000000	
Convergence	40.000000	40.000000	40.000000	40.000000	40.000000	
UFR	3.450000	3.450000	3.450000	3.450000	3.450000	
alpha	0.108242	0.108242	0.108242	0.108242	0.108242	

5 rows × 106 columns

The country selected is:

In [224...
```
country = "Turkey"
```

In [225...
```
maturities_country_raw = param_raw.loc[:,country+"_Maturities"].iloc[6:]
param_country_raw = param_raw.loc[:,country + "_Values"].iloc[6:]
extra_param = param_raw.loc[:,country + "_Values"].iloc[:6]
```

Extra parameters

Smith-Wilson calibration parameters

In [226...
```
extra_param
```

Out[226...
```
Country
Coupon_freq     0.000000
LLP             9.000000
Convergence    51.000000
UFR             5.500000
alpha           0.165581
CRA            10.000000
Name: Turkey_Values, dtype: float64
```

In [227...
```
relevant_positions = pd.notna(maturities_country_raw.values)
```

In [228...
```
maturities_country = maturities_country_raw.iloc[relevant_positions]
```

Maturity vector

Vector of maturities used in the calibration

In [229...
```
maturities_country.head(15)
```

Out[229...
```
Country
1    1.0
2    2.0
3    3.0
4    4.0
5    5.0
6    6.0
7    8.0
8    9.0
Name: Turkey_Maturities, dtype: float64
```

In [230...
```
Qb = param_country_raw.iloc[relevant_positions]
```

Calibration vector

Vector **Qb** provided as input

In [231...
```
Qb
```

Out[231...
```
Country
1    -2.594118
2    10.888107
3    -4.804714
4     0.023312
5    -2.015106
6    -0.304049
7    -0.981962
8     1.036681
Name: Turkey_Values, dtype: float64
```

In [232...
```
curve_raw = pd.read_csv(selected_curves_file, sep=',',index_col=0)
```

In [233...
```
curve_country = curve_raw.loc[:,country]
```

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Calibration parameters and calibration vector provided

by EIOPA

In [234…

```
# Maturity of observations:
M_Obs = np.transpose(np.array(maturities_country.values))

# Ultimate froward rate ufr represents the rate to which the rate curve will converge
ufr = extra_param.iloc[3]/100

# Convergence speed parameter alpha controls the speed at which the curve converges t
alpha = extra_param.iloc[4]

# For which maturities do we want the SW algorithm to calculate the rates. In this ca
M_Target = np.transpose(np.arange(1,151))

# Qb calibration vector published by EIOPA for the curve calibration:
Qb = np.transpose(np.array(Qb.values))
```

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Smith & Wilson calculation functions

In this step, the independent version of the Smith&Wilson algorithm is implemented. To do this, two functions are taken from the publicly available repository and modified to accept the product of `Q*b` instead of the calibration vector `b`.

In [235…

```
def SWExtrapolate(M_Target, M_Obs, Qb, ufr, alpha):
# SWEXTRAPOLATE Interpolate or/and extrapolate rates for targeted maturities using a
# out = SWExtrapolate(M_Target, M_Obs, Qb, ufr, alpha) calculates the rates for matu
#
# Arguments:
#    M_Target = k x 1 ndarray. Each element represents a bond maturity of interest. E
#    M_Obs =    n x 1 ndarray. Observed bond maturities used for calibrating the cal
#    Qb =       n x 1 ndarray. Calibration vector calculated on observed bonds.
#    ufr =      1 x 1 floating number. Representing the ultimate forward rate.
#       Ex. ufr = 0.042
#    alpha =    1 x 1 floating number. Representing the convergence speed parameter o
#
#
# Returns:
#    k x 1 ndarray. Represents the targeted rates for a zero-coupon bond. Each rate E
#
# For more information see https://www.eiopa.europa.eu/sites/default/files/risk_free_

    def SWHeart(u, v, alpha):
    # SWHEART Calculate the heart of the Wilson function.
    # H = SWHeart(u, v, alpha) calculates the matrix H (Heart of the Wilson
    # function) for maturities specified by vectors u and v. The formula is
    # taken from the EIOPA technical specifications paragraph 132.
    #
    # Arguments:
    #    u =     n_1 x 1 vector of maturities. Ex. u = [1; 3]
    #    v =     n_2 x 1 vector of maturities. Ex. v = [1; 2; 3; 5]
    #    alpha = 1 x 1 floating number representing the convergence speed parameter o
    #
    # Returns:
    #    n_1 x n_2 matrix representing the Heart of the Wilson function for selected
    #
    # For more information see https://www.eiopa.europa.eu/sites/default/files/risk_f

        u_Mat = np.tile(u, [v.size, 1]).transpose()
        v_Mat = np.tile(v, [u.size, 1])
        return 0.5 * (alpha * (u_Mat + v_Mat) + np.exp(-alpha * (u_Mat + v_Mat)) - al

    H = SWHeart(M_Target, M_Obs, alpha) # Heart of the Wilson function from paragraph
    p = np.exp(-np.log(1+ufr)* M_Target) + np.diag(np.exp(-np.log(1+ufr) * M_Target))
    return p ** (-1/ M_Target) -1 # Convert obtained prices to rates and return price
```

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Generation of the risk-free curve

The observed maturities, target maturities, and the model parameters provided by EIOPA are used to generate the target curve.

In [236…

```
r_Target = SWExtrapolate(M_Target,M_Obs, Qb, ufr, alpha)
r_Target = pd.DataFrame(r_Target,columns=['Recalculated rates'])
```

Yield curve calculated

Yield curve calculated using the calibration vector **Qb**

In [237…

```
r_Target.head(15)
```

Out[237…

	Recalculated rates
0	0.139454
1	0.149675
2	0.166894
3	0.182276
4	0.194183
5	0.201892
6	0.205980
7	0.207159
8	0.206124
9	0.204380
10	0.202331
11	0.200018
12	0.197477
13	0.194739
14	0.191832

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Test 1; Comparison test

Comparison of the calculated yield curve with the yield curve provided by EIOPA. The test is passed if the success criteria is reached.

The provided yield curve can be found in file *EIOPA_RFR_20230731_Term_Structures.xlsx*, sheet *RFR_spot_no_VA* if the test looks at the curve without the Volatility Adjustment and the sheet *RFR_spot_with_VA* if the test looks at the curve with the Volatility Adjustment.

In [238…

```
target_curve = np.transpose(np.array(curve_country.values))
```

This implementation looks at two kinds of test statistics. The average deviation and the maximum deviation.

The average deviation is defined as:

$$S_{AVERAGE} = \frac{1}{T}\sum_{t=0}^{T}|r_{EIOPA}(t) - r_{EST}(t)|$$

The maximum deviation is defined as:

$$S_{MAX} = \max_{t}|r_{EIOPA}(t) - r_{EST}(t)|$$

Where `T` is the maximum maturity available.

The average difference test is successful if:

$$S_{AVERAGE} < 0.05bps$$

The maximum difference test is successful if:

$$S_{MAX} < 0.1bps$$

In [239…]
```
target_curve = pd.DataFrame(target_curve,columns=['Given rates'])
```

EIOPA curve provided

Yield curve provided by EIOPA

In [240…]
```
target_curve.head()
```

Out[240…]

	Given rates
0	0.13945
1	0.14968
2	0.16689
3	0.18228
4	0.19418

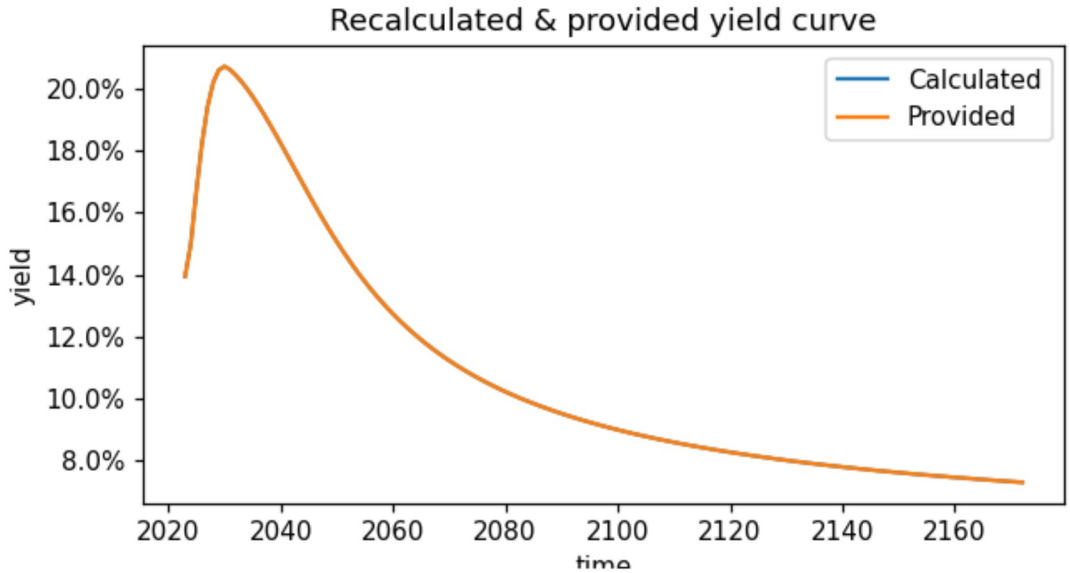
In [241...

```
x_data_label = range(2023,2023+r_Target.shape[0],1)
```

In [242...

```
fig, ax1 = plt.subplots(1,1)
ax1.plot(x_data_label, r_Target.values*100, color='tab:blue',label="Calculated")
ax1.plot(x_data_label, target_curve.values*100, color='tab:orange',label="Provided")

ax1.set_ylabel("yield")
ax1.set_title('Recalculated & provided yield curve')
ax1.set_xlabel("time")
ax1.legend()
ax1.yaxis.set_major_formatter(mtick.PercentFormatter())
fig.set_figwidth(6)
fig.set_figheight(3)
plt.show()
```



In [243...

```
test_statistics_bdp = pd.DataFrame(abs(r_Target.values-target_curve.values)*10000, c
```

EIOPA curve comparison

Absolute difference in bps

In [244...

```
test_statistics_bdp.head()
```

Out[244...

	Abs diff in bps
0	0.040392
1	0.049192
2	0.044102
3	0.037470

Abs diff in bps

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Test 1; Success criteria

The successful application of the success criteria marks the completion of the test.

In [245...

```
result1 = SuccessTest(test_statistics_bdp.values, test_statistics_max_diff_in_bps, te
```

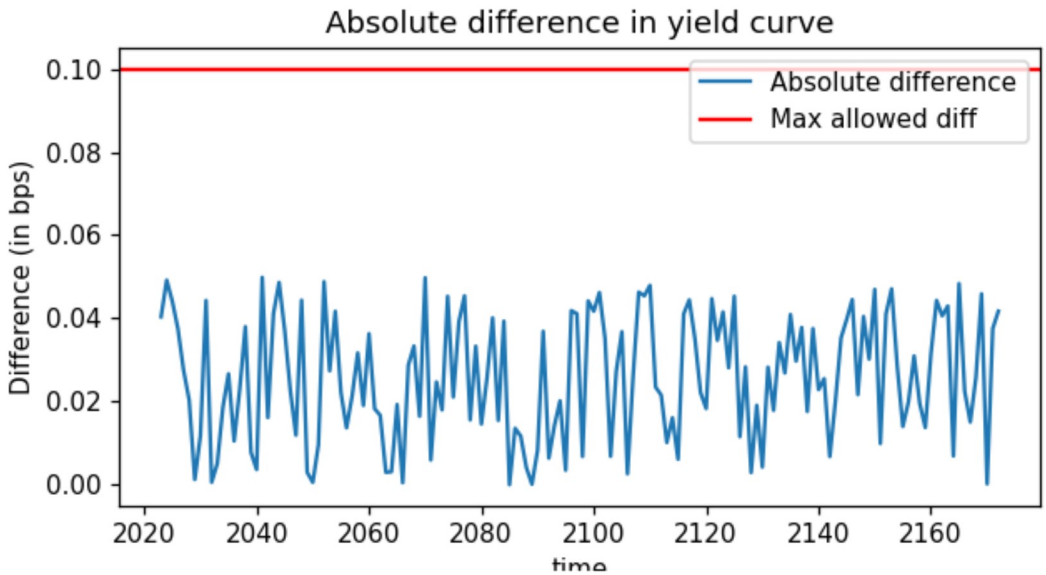
```
Test passed
Test passed
```

In [246...

```
x_data_label = range(2023,2023+r_Target.shape[0],1)
fig, ax1 = plt.subplots(1,1)
ax1.plot(x_data_label, test_statistics_bdp, label= "Absolute difference")
ax1.axhline(y = test_statistics_max_diff_in_bps, color = 'r', linestyle = '-',label='

ax1.set_xlabel("time")
ax1.set_ylabel("Difference (in bps)")
ax1.set_title('Absolute difference in yield curve')
ax1.legend()
fig.set_figwidth(6)
fig.set_figheight(3)

plt.show()
```



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Conclusion

This test checks the success criteria on the EIOPA curve generated for July 2023. If the tests are passed, it is likely that the curve was generated using the Smith & Wilson algorithm with the calibration vector that was provided in the file *EIOPA_RFR_20230731_Qb_SW.xlsx* and the parameters displayed in the file *EIOPA_RFR_20230731_Term_Structures.xlsx*.

In [247...

```
pd.DataFrame(data = [result1], columns = ["Mean test","Max test"],  \
             index= ["Provided vs calculated"])
```

Out[247...

	Mean test	Max test
Provided vs calculated	True	True

Final yield curve

Full yield curve provided by EIOPA in %

In [248...

```
(curve_country*100).head(150)
```

Out[248...

```
Country
1      13.945
2      14.968
3      16.689
4      18.228
5      19.418
6      20.189
7      20.598
8      20.716
9      20.612
10     20.438
11     20.233
12     20.002
13     19.748
14     19.474
15     19.183
16     18.879
17     18.563
18     18.239
19     17.909
20     17.577
21     17.244
22     16.913
23     16.586
24     16.265
25     15.950
26     15.644
27     15.346
28     15.058
```

29	14.780
30	14.512
31	14.255
32	14.008
33	13.772
34	13.545
35	13.328
36	13.121
37	12.922
38	12.732
39	12.551
40	12.377
41	12.211
42	12.052
43	11.900
44	11.754
45	11.614
46	11.480
47	11.352
48	11.229
49	11.110
50	10.996
51	10.887
52	10.782
53	10.680
54	10.583
55	10.489
56	10.398
57	10.310
58	10.226
59	10.144
60	10.065
61	9.989
62	9.915
63	9.844
64	9.775
65	9.708
66	9.643
67	9.580
68	9.519
69	9.460
70	9.402
71	9.346
72	9.292
73	9.239
74	9.188
75	9.138
76	9.089
77	9.042
78	8.995
79	8.950
80	8.907
81	8.864
82	8.822
83	8.782
84	8.742
85	8.703
86	8.665
87	8.629

88	8.592
89	8.557
90	8.523
91	8.489
92	8.456
93	8.424
94	8.392
95	8.362
96	8.331
97	8.302
98	8.273
99	8.244
100	8.217
101	8.189
102	8.163
103	8.137
104	8.111
105	8.086
106	8.061
107	8.037
108	8.013
109	7.990
110	7.967
111	7.944
112	7.922
113	7.901
114	7.879
115	7.858
116	7.838
117	7.818
118	7.798
119	7.778
120	7.759
121	7.740
122	7.722
123	7.703
124	7.685
125	7.668
126	7.650
127	7.633
128	7.616
129	7.600
130	7.584
131	7.568
132	7.552
133	7.536
134	7.521
135	7.506
136	7.491
137	7.476
138	7.462
139	7.447
140	7.433
141	7.420
142	7.406
143	7.392
144	7.379
145	7.366
146	7.353

```
147     7.341
148     7.328
149     7.316
150     7.303
```