EIOPA RISK-FREE CURVE JULY-23 RECALCULATION

The risk-free curve is one of the principal inputs into an economic scenario generator. This notebook recalculates the risk-free curve using the parameters that are claimed to be used. The European Insurance and Occupational Pensions Authority (EIOPA) publishes their own yield curve prediction. To do this they use the Smith & Wilson algorithm.

Summary

The goal of this test is to replicate the EIOPA yield curve. This test will use the methodology that EIOPA claims it is using and the calibration vector that they publish. If the test is passed, the user can be more confident, that EIOPA risk free rate (RFR) curve was generated using the described methodology/calibration and that the process was implemented correctly.

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Note on Smith & Wilson algorithm

To replicate the calculations, this example uses a modified Smith&Wilson implementation (The original implementation is availible on GitHub):

- Python
- Matlab
- JavaScript

Limitations of the implementation

Current implementation only looks at a single currency and with/without Volatility Adjustment (VA). The day count convention assumes that each year has the same number of days.

Data requirements

This script contains the EIOPA risk-free rate publication for July 2023. The publication can be found on the EIOPA RFR website.

The observed maturities `M_Obs` and the calibrated vector `Qb` can be found in the Excel sheet *EIOPA_RFR_20230731_Qb_SW.xlsx*.

The target maturities (`T_Obs`), the additional parameters (`UFR` and `alpha`), and the given curve can be found in the Excel *EIOPA_RFR_20230731_Term_Structures.xlsx*, sheet *RFR_spot_no_VA* if the test looks at the curve without the Volatility Adjustment and the sheet *RFR_spot_with_VA* if the test looks at the curve with the Volatility Adjustment.

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Success criteria

The following success criteria is defined:

- Maximum difference between the calculated curve and the one provided by EIOPA is less than 0.1 bps
- Average difference between the calculated curve and the one provided by EIOPA is less than 0.05 bps

In [32]:
```
test_statistics_max_diff_in_bps = 0.1
test_statistics_average_diff_in_bps = 0.05
```

The success function is called at the end of the test to confirm if the success criteria have been met.

In [33]:
```
def SuccessTest(TestStatistics, threshold_max, threshold_mean):
    out1 = False
    out2 = False
    if max(TestStatistics)<threshold_max:
        print("Test passed")
        out1 = True
    else:
        print("Test failed")

    if np.mean(TestStatistics)<threshold_mean:
        print("Test passed")
        out2 = True
    else:
        print("Test failed")
    return [out1, out2]
```

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External dependencies

This implementation uses three well established Python packages widely used in the financial industry. Pandas (https://pandas.pydata.org/docs/), Numpy (https://numpy.org/doc/), and Matplotlib (https://matplotlib.org/stable/index.html)

In [34]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import matplotlib.ticker as mtick
%matplotlib notebook
pd.options.display.max_rows = 150
```

Importing data

In [35]:
```
#selected_param_file = 'Param_VA.csv'
#selected_curves_file = 'Curves_VA.csv'

selected_param_file = 'Param_no_VA.csv'
selected_curves_file = 'Curves_no_VA.csv'
```

In [36]:
```
param_raw = pd.read_csv(selected_param_file, sep=',',index_col=0)
```

Parameter input

Parameters sheet

In [37]:
```
param_raw.head()
```

Out[37]:

Country	Euro_Maturities	Euro_Values	Austria_Maturities	Austria_Values	Belgium_Maturities	Be
Coupon_freq	1.000000	1.000000	1.000000	1.000000	1.000000	
LLP	20.000000	20.000000	20.000000	20.000000	20.000000	
Convergence	40.000000	40.000000	40.000000	40.000000	40.000000	
UFR	3.450000	3.450000	3.450000	3.450000	3.450000	
alpha	0.112203	0.112203	0.112203	0.112203	0.112203	

5 rows × 106 columns

The country selected is:

In [38]:
```
country = "Italy"
```

In [39]:
```
maturities_country_raw = param_raw.loc[:,country+"_Maturities"].iloc[6:]
param_country_raw = param_raw.loc[:,country + "_Values"].iloc[6:]
extra_param = param_raw.loc[:,country + "_Values"].iloc[:6]
```

Extra parameters

Smith-Wilson calibration parameters

In [40]:
```
extra_param
```

Out[40]:
```
Country
Coupon_freq     1.000000
LLP            20.000000
Convergence    40.000000
UFR             3.450000
alpha           0.112203
CRA            10.000000
Name: Italy_Values, dtype: float64
```

In [41]:
```
relevant_positions = pd.notna(maturities_country_raw.values)
```

In [42]:
```
maturities_country = maturities_country_raw.iloc[relevant_positions]
```

Maturity vector

Vector of maturities used in the calibration

In [43]:
```
maturities_country.head(15)
```

Out[43]:
```
Country
1      1.0
2      2.0
3      3.0
4      4.0
5      5.0
6      6.0
7      7.0
8      8.0
9      9.0
10    10.0
11    11.0
12    12.0
13    13.0
14    14.0
15    15.0
Name: Italy_Maturities, dtype: float64
```

In [44]:
```
Qb = param_country_raw.iloc[relevant_positions]
```

Calibration vector

Vector **Qb** provided as input

In [45]:
```
Qb
```

Out[45]:
```
Country
1     -8.182191
2      0.658896
3      0.666484
4      0.727548
5      1.361903
6     -2.307430
7      2.493323
8     -2.331474
9      4.696474
10    -9.428838
11    11.003868
12    -4.824905
13    -0.004031
14    -0.003897
15    -0.761741
16     0.018179
```

```
17     0.017572
18     0.016986
19     0.016420
20     0.570069
Name: Italy Values, dtype: float64
```

In [46]:
```
curve_raw = pd.read_csv(selected_curves_file, sep=',',index_col=0)
```

In [47]:
```
curve_country = curve_raw.loc[:,country]
```

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Calibration parameters and calibration vector provided by EIOPA

In [48]:
```
# Maturity of observations:
M_Obs = np.transpose(np.array(maturities_country.values))

# Ultimate froward rate ufr represents the rate to which the rate curve will converge
ufr = extra_param.iloc[3]/100

# Convergence speed parameter alpha controls the speed at which the curve converges t
alpha = extra_param.iloc[4]

# For which maturities do we want the SW algorithm to calculate the rates. In this ca
M_Target = np.transpose(np.arange(1,151))

# Qb calibration vector published by EIOPA for the curve calibration:
Qb = np.transpose(np.array(Qb.values))
```

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Smith & Wilson calculation functions

In this step, the independent version of the Smith&Wilson algorithm is implemented. To do this, two functions are taken from the publicly available repository and modified to accept the product of `Q*b` instead of the calibration vector `b`.

In [49]:

```
def SWExtrapolate(M_Target, M_Obs, Qb, ufr, alpha):
# SWEXTRAPOLATE Interpolate or/and extrapolate rates for targeted maturities using a
# out = SWExtrapolate(M_Target, M_Obs, Qb, ufr, alpha) calculates the rates for matu
#
# Arguments:
#    M_Target = k x 1 ndarray. Each element represents a bond maturity of interest. E
#    M_Obs =    n x 1 ndarray. Observed bond maturities used for calibrating the cal
#    Qb =       n x 1 ndarray. Calibration vector calculated on observed bonds.
#    ufr =      1 x 1 floating number. Representing the ultimate forward rate.
#       Ex. ufr = 0.042
#    alpha =    1 x 1 floating number. Representing the convergence speed parameter o
#
#
# Returns:
#    k x 1 ndarray. Represents the targeted rates for a zero-coupon bond. Each rate E
#
# For more information see https://www.eiopa.europa.eu/sites/default/files/risk_free_

    def SWHeart(u, v, alpha):
    # SWHEART Calculate the heart of the Wilson function.
    # H = SWHeart(u, v, alpha) calculates the matrix H (Heart of the Wilson
    # function) for maturities specified by vectors u and v. The formula is
    # taken from the EIOPA technical specifications paragraph 132.
    #
    # Arguments:
    #    u =     n_1 x 1 vector of maturities. Ex. u = [1; 3]
    #    v =     n_2 x 1 vector of maturities. Ex. v = [1; 2; 3; 5]
    #    alpha = 1 x 1 floating number representing the convergence speed parameter o
    #
    # Returns:
    #    n_1 x n_2 matrix representing the Heart of the Wilson function for selected
    #
    # For more information see https://www.eiopa.europa.eu/sites/default/files/risk_f

        u_Mat = np.tile(u, [v.size, 1]).transpose()
        v_Mat = np.tile(v, [u.size, 1])
        return 0.5 * (alpha * (u_Mat + v_Mat) + np.exp(-alpha * (u_Mat + v_Mat)) - al

    H = SWHeart(M_Target, M_Obs, alpha) # Heart of the Wilson function from paragraph
    p = np.exp(-np.log(1+ufr)* M_Target) + np.diag(np.exp(-np.log(1+ufr) * M_Target))
    return p ** (-1/ M_Target) -1 # Convert obtained prices to rates and return price
```

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Generation of the risk-free curve

The observed maturities, target maturities, and the model parameters provided by EIOPA are used to generate the target curve.

In [50]:

```
r_Target = SWExtrapolate(M_Target,M_Obs, Qb, ufr, alpha)
r_Target = pd.DataFrame(r_Target,columns=['Recalculated rates'])
```

Yield curve calculated

Yield curve calculated using the calibration vector **Qb**

In [51]:
```
r_Target.head(15)
```

Out[51]:

	Recalculated rates
0	0.039190
1	0.036105
2	0.033572
3	0.031876
4	0.030860
5	0.030269
6	0.029863
7	0.029679
8	0.029601
9	0.029608
10	0.029543
11	0.029721
12	0.029837
13	0.029811
14	0.029656

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Test 1; Comparison test

Comparison of the calculated yield curve with the yield curve provided by EIOPA. The test is passed if the success criteria is reached.

The provided yield curve can be found in file *EIOPA_RFR_20230731_Term_Structures.xlsx*, sheet *RFR_spot_no_VA* if the test looks at the curve without the Volatility Adjustment and the sheet *RFR_spot_with_VA* if the test looks at the curve with the Volatility Adjustment.

In [52]:
```
target_curve = np.transpose(np.array(curve_country.values))
```

This implementation looks at two kinds of test statistics. The average deviation and the maximum deviation.

The average deviation is defined as:

$$S_{AVERAGE} = \frac{1}{T}\sum_{t=0}^{T}|r_{EIOPA}(t) - r_{EST}(t)|$$

The maximum deviation is defined as:

$$S_{MAX} = \max_{t}|r_{EIOPA}(t) - r_{EST}(t)|$$

Where `T` is the maximum maturity available.

The average difference test is successful if:

$$S_{AVERAGE} < 0.05bps$$

The maximum difference test is successful if:

$$S_{MAX} < 0.1bps$$

In [53]:
```
target_curve = pd.DataFrame(target_curve,columns=['Given rates'])
```

EIOPA curve provided

Yield curve provided by EIOPA

In [54]:
```
target_curve.head()
```

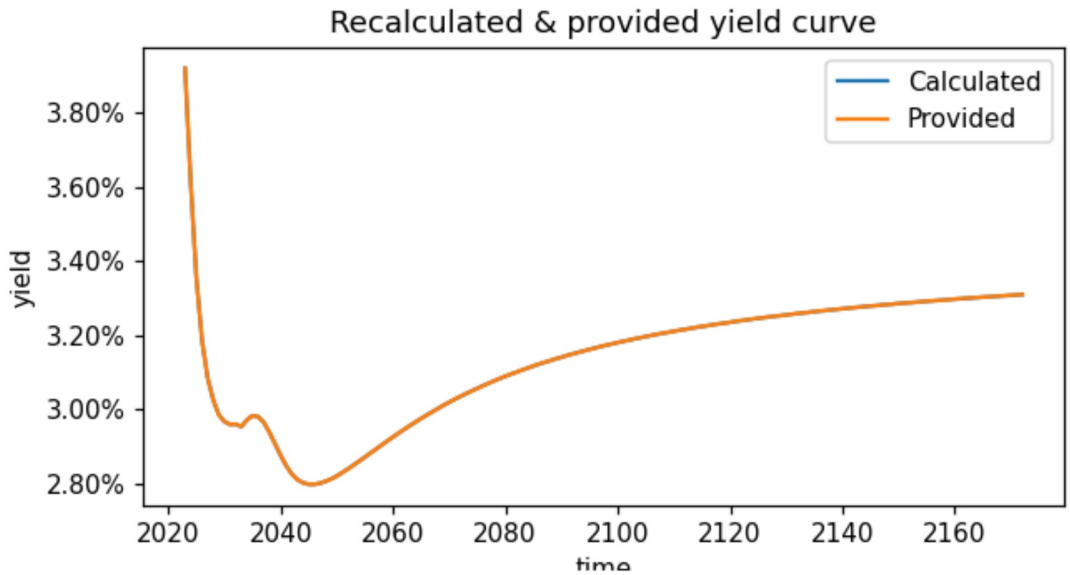
Out[54]:

	Given rates
0	0.03919
1	0.03611
2	0.03357
3	0.03188
4	0.03086

In [55]:
```
x_data_label = range(2023,2023+r_Target.shape[0],1)
```

In [56]:
```
fig, ax1 = plt.subplots(1,1)
ax1.plot(x_data_label, r_Target.values*100, color='tab:blue',label="Calculated")
ax1.plot(x_data_label, target_curve.values*100, color='tab:orange',label="Provided")

ax1.set_ylabel("yield")
ax1.set_title('Recalculated & provided yield curve')
ax1.set_xlabel("time")
ax1.legend()
ax1.yaxis.set_major_formatter(mtick.PercentFormatter())
fig.set_figwidth(6)
fig.set_figheight(3)
plt.show()
```



In [57]:
```
test_statistics_bdp = pd.DataFrame(abs(r_Target.values-target_curve.values)*10000, c
```

EIOPA curve comparison

Absolute difference in bps

In [58]:
```
test_statistics_bdp.head()
```

Out[58]:

	Abs diff in bps
0	0.000005
1	0.046189
2	0.024584
3	0.040507

Abs diff in bps

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Test 1; Success criteria

The successful application of the success criteria marks the completion of the test.

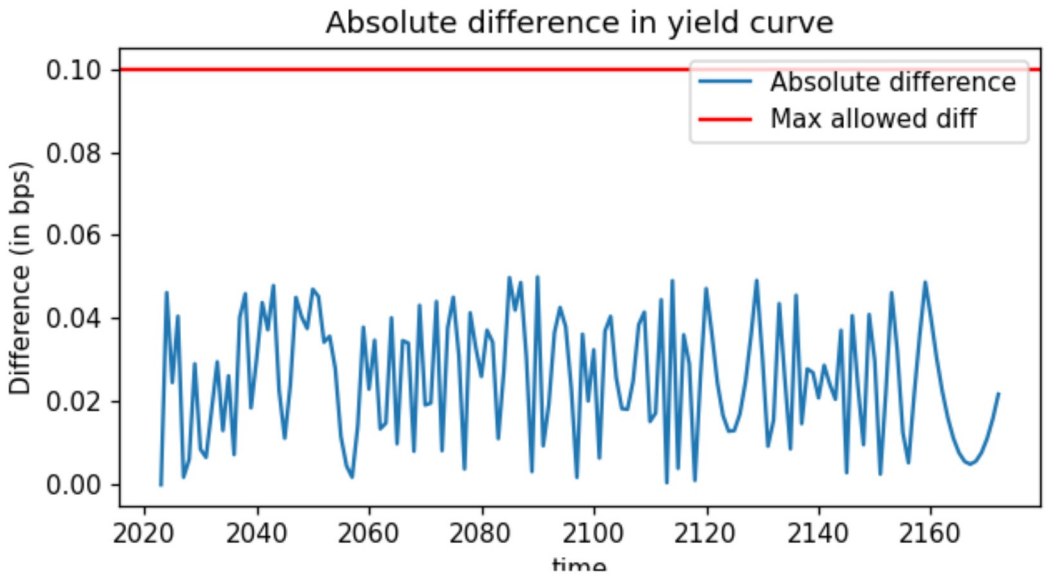
In [59]:
```
result1 = SuccessTest(test_statistics_bdp.values, test_statistics_max_diff_in_bps, te
```

```
Test passed
Test passed
```

In [60]:
```
x_data_label = range(2023,2023+r_Target.shape[0],1)
fig, ax1 = plt.subplots(1,1)
ax1.plot(x_data_label, test_statistics_bdp, label= "Absolute difference")
ax1.axhline(y = test_statistics_max_diff_in_bps, color = 'r', linestyle = '-',label='

ax1.set_xlabel("time")
ax1.set_ylabel("Difference (in bps)")
ax1.set_title('Absolute difference in yield curve')
ax1.legend()
fig.set_figwidth(6)
fig.set_figheight(3)

plt.show()
```



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Conclusion

This test checks the success criteria on the EIOPA curve generated for July 2023. If the tests are passed, it is likely that the curve was generated using the Smith & Wilson algorithm with the calibration vector that was provided in the file *EIOPA_RFR_20230731_Qb_SW.xlsx* and the parameters displayed in the file *EIOPA_RFR_20230731_Term_Structures.xlsx*.

In [61]:
```
pd.DataFrame(data = [result1], columns = ["Mean test","Max test"],  \
             index= ["Provided vs calculated"])
```

Out[61]:

	Mean test	Max test
Provided vs calculated	True	True

Final yield curve

Full yield curve provided by EIOPA in %

In [62]:
```
(curve_country*100).head(150)
```

Out[62]:
```
Country
1      3.919
2      3.611
3      3.357
4      3.188
5      3.086
6      3.027
7      2.986
8      2.968
9      2.960
10     2.961
11     2.954
12     2.972
13     2.984
14     2.981
15     2.966
16     2.939
17     2.908
18     2.877
19     2.849
20     2.826
21     2.811
22     2.803
23     2.799
24     2.799
25     2.802
26     2.807
27     2.813
28     2.821
```

29	2.831
30	2.840
31	2.851
32	2.861
33	2.872
34	2.883
35	2.894
36	2.905
37	2.916
38	2.926
39	2.937
40	2.947
41	2.957
42	2.967
43	2.976
44	2.985
45	2.994
46	3.003
47	3.012
48	3.020
49	3.028
50	3.036
51	3.043
52	3.050
53	3.057
54	3.064
55	3.071
56	3.077
57	3.084
58	3.090
59	3.096
60	3.101
61	3.107
62	3.112
63	3.118
64	3.123
65	3.128
66	3.132
67	3.137
68	3.141
69	3.146
70	3.150
71	3.154
72	3.158
73	3.162
74	3.166
75	3.170
76	3.174
77	3.177
78	3.181
79	3.184
80	3.187
81	3.191
82	3.194
83	3.197
84	3.200
85	3.203
86	3.206
87	3.208

88	3.211
89	3.214
90	3.216
91	3.219
92	3.222
93	3.224
94	3.226
95	3.229
96	3.231
97	3.233
98	3.235
99	3.238
100	3.240
101	3.242
102	3.244
103	3.246
104	3.248
105	3.250
106	3.252
107	3.253
108	3.255
109	3.257
110	3.259
111	3.261
112	3.262
113	3.264
114	3.266
115	3.267
116	3.269
117	3.270
118	3.272
119	3.273
120	3.275
121	3.276
122	3.278
123	3.279
124	3.280
125	3.282
126	3.283
127	3.284
128	3.286
129	3.287
130	3.288
131	3.289
132	3.291
133	3.292
134	3.293
135	3.294
136	3.295
137	3.296
138	3.298
139	3.299
140	3.300
141	3.301
142	3.302
143	3.303
144	3.304
145	3.305
146	3.306

```
147    3.307
148    3.308
149    3.309
150    3.310
```