EIOPA RISK-FREE CURVE JANUARY-23 RECALCULATION

The risk-free curve is one of the principal inputs into an economic scenario generator. This notebook recalculates the risk-free curve using the parameters that are claimed to be used. The European Insurance and Occupational Pensions Authority (EIOPA) publishes their own yield curve prediction. To do this they use the Smith & Wilson algorithm.

Summary

The goal of this test is to replicate the EIOPA yield curve. This test will use the methodology that EIOPA claims it is using and the calibration vector that they publish. If the test is passed, the user can be more confident, that EIOPA risk free rate (RFR) curve was generated using the described methodology/calibration and that the process was implemented correctly.

Table of Contents

Note on Smith & Wilson algorithm

To replicate the calculations, this example uses a modified Smith&Wilson implementation (The original implementation is availible on GitHub:

- Python
- Matlab
- JavaScript

Limitations of the implementation

Current implementation only looks at a single currency and with/without Volatility Adjustment (VA). The day count convention assumes that each year has the same number of days.

Data requirements

This script contains the EIOPA risk-free rate publication for January 2023. The publication can be found on the EIOPA RFR website.

The observed maturities `M_Obs` and the calibrated vector `Qb` can be found in the Excel sheet *EIOPA_RFR_20230131_Qb_SW.xlsx*.

For this example, the curve without the volatility adjustment (VA) is used. It can be found in the sheet *SW_Qb_no_VA*. This example is focused on the EUR curve, but this example can be easily modified for any other curve.

The target maturities (`T_Obs`), the additional parameters (`UFR` and `alpha`), and the given curve can be found in the Excel *EIOPA_RFR_20230131_Term_Structures.xlsx*, sheet *RFR_spot_no_VA* if the test looks at the curve without the Volatility Adjustment and the sheet *RFR_spot_with_VA* if the test looks at the curve with the Volatility Adjustment.

Back to the top

Success criteria

The following success criteria is defined:

- Maximum difference between the calculated curve and the one provided by EIOPA is less than 0.1 bps
- Average difference between the calculated curve and the one provided by EIOPA is less than 0.05 bps

In [311…

```
test_statistics_max_diff_in_bps = 0.1
test_statistics_average_diff_in_bps = 0.05
```

The success function is called at the end of the test to confirm if the success criteria have been met.

In [312…

```
def SuccessTest(TestStatistics, threshold_max, threshold_mean):
    out1 = False
    out2 = False
    if max(TestStatistics)<threshold_max:
        print("Test passed")
        out1 = True
    else:
        print("Test failed")

    if np.mean(TestStatistics)<threshold_mean:
        print("Test passed")
        out2 = True
    else:
        print("Test failed")
    return [out1, out2]
```

Back to the top

External dependencies

This implementation uses three well established Python packages widely used in the financial industry. Pandas (https://pandas.pydata.org/docs/), Numpy (https://numpy.org/doc/), and Matplotlib (https://matplotlib.org/stable/index.html)

In [313…

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import matplotlib.ticker as mtick
%matplotlib notebook
pd.options.display.max_rows = 150
```

Importing data

In [314…

```
#selected_param_file = 'Param_VA.csv'
#selected_curves_file = 'Curves_VA.csv'

selected_param_file = 'Param_no_VA.csv'
selected_curves_file = 'Curves_no_VA.csv'
```

In [315…

```
param_raw = pd.read_csv(selected_param_file, sep=',',index_col=0)
```

Parameter input

Parameters sheet

In [316...
```
param_raw.head()
```

Out[316...

Country	Euro_Maturities	Euro_Values	Austria_Maturities	Austria_Values	Belgium_Maturities	Be
Coupon_freq	1.000000	1.000000	1.000000	1.000000	1.000000	
LLP	20.000000	20.000000	20.000000	20.000000	20.000000	
Convergence	40.000000	40.000000	40.000000	40.000000	40.000000	
UFR	3.450000	3.450000	3.450000	3.450000	3.450000	
alpha	0.119621	0.119621	0.119621	0.119621	0.119621	

5 rows × 106 columns

The country selected is:

In [317...
```
country = "United States"
```

In [318...
```
maturities_country_raw = param_raw.loc[:,country+"_Maturities"].iloc[6:]
param_country_raw = param_raw.loc[:,country + "_Values"].iloc[6:]
extra_param = param_raw.loc[:,country + "_Values"].iloc[:6]
```

Extra parameters

Smith-Wilson calibration parameters

In [319...
```
extra_param
```

Out[319...
```
Country
Coupon_freq     1.00000
LLP            30.00000
Convergence    40.00000
UFR             3.45000
alpha           0.11084
CRA             0.00000
Name: United States_Values, dtype: float64
```

In [320...
```
relevant_positions = pd.notna(maturities_country_raw.values)
```

In [321...
```
maturities_country = maturities_country_raw.iloc[relevant_positions]
```

Maturity vector

Vector of maturities used in the calibration

In [322...
```
maturities_country.head(15)
```

Out[322...
```
Country
1      1.0
2      2.0
3      3.0
4      4.0
5      5.0
6      6.0
7      7.0
8      8.0
9      9.0
10    10.0
11    11.0
12    12.0
13    13.0
14    14.0
15    15.0
Name: United States_Maturities, dtype: float64
```

In [323...
```
Qb = param_country_raw.iloc[relevant_positions]
```

Calibration vector

Vector Qb provided as input

In [324...
```
Qb
```

Out[324...
```
Country
1     -18.850272
2       1.264961
3       5.989860
4      -0.756806
5       0.380267
6      -0.001075
7      -0.297055
8       0.008381
9       0.624547
10     -0.260572
11     -0.003261
12     -0.003152
13     -0.003047
14     -0.002946
15      0.156027
16     -0.007698
```

```
17     -0.007441
18     -0.007193
19     -0.006953
20     -0.600741
NaN     0.011775
NaN     0.011382
NaN     0.011002
NaN     0.010635
NaN     0.010281
NaN     0.009938
NaN     0.009606
NaN     0.009286
NaN     0.008976
NaN     0.300048
Name: United States Values, dtype: float64
```

In [325…

```
curve_raw = pd.read_csv(selected_curves_file, sep=',',index_col=0)
```

In [326…

```
curve_country = curve_raw.loc[:,country]
```

Back to the top

Calibration parameters and calibration vector provided by EIOPA

In [327…

```
# Maturity of observations:
M_Obs = np.transpose(np.array(maturities_country.values))

# Ultimate froward rate ufr represents the rate to which the rate curve will converge
ufr = extra_param.iloc[3]/100

# Convergence speed parameter alpha controls the speed at which the curve converges t
alpha = extra_param.iloc[4]

# For which maturities do we want the SW algorithm to calculate the rates. In this ca
M_Target = np.transpose(np.arange(1,151))

# Qb calibration vector published by EIOPA for the curve calibration:
Qb = np.transpose(np.array(Qb.values))
```

Back to the top

Smith & Wilson calculation functions

In this step, the independent version of the Smith&Wilson algorithm is implemented. To do this, two functions are taken from the publicly available repository and modified to accept the product of `Q*b` instead of the calibration vector `b`.

In [328…

```
def SWExtrapolate(M_Target, M_Obs, Qb, ufr, alpha):
# SWEXTRAPOLATE Interpolate or/and extrapolate rates for targeted maturities using a
# out = SWExtrapolate(M_Target, M_Obs, Qb, ufr, alpha) calculates the rates for matu
#
# Arguments:
#    M_Target = k x 1 ndarray. Each element represents a bond maturity of interest. E
#    M_Obs =    n x 1 ndarray. Observed bond maturities used for calibrating the cal
#    Qb =       n x 1 ndarray. Calibration vector calculated on observed bonds.
#    ufr =      1 x 1 floating number. Representing the ultimate forward rate.
#       Ex. ufr = 0.042
#    alpha =    1 x 1 floating number. Representing the convergence speed parameter o
#
#
# Returns:
#    k x 1 ndarray. Represents the targeted rates for a zero-coupon bond. Each rate E
#
# For more information see https://www.eiopa.europa.eu/sites/default/files/risk_free_

    def SWHeart(u, v, alpha):
    # SWHEART Calculate the heart of the Wilson function.
    # H = SWHeart(u, v, alpha) calculates the matrix H (Heart of the Wilson
    # function) for maturities specified by vectors u and v. The formula is
    # taken from the EIOPA technical specifications paragraph 132.
    #
    # Arguments:
    #    u =     n_1 x 1 vector of maturities. Ex. u = [1; 3]
    #    v =     n_2 x 1 vector of maturities. Ex. v = [1; 2; 3; 5]
    #    alpha = 1 x 1 floating number representing the convergence speed parameter o
    #
    # Returns:
    #    n_1 x n_2 matrix representing the Heart of the Wilson function for selected
    #
    # For more information see https://www.eiopa.europa.eu/sites/default/files/risk_f

        u_Mat = np.tile(u, [v.size, 1]).transpose()
        v_Mat = np.tile(v, [u.size, 1])
        return 0.5 * (alpha * (u_Mat + v_Mat) + np.exp(-alpha * (u_Mat + v_Mat)) - al

    H = SWHeart(M_Target, M_Obs, alpha) # Heart of the Wilson function from paragraph
    p = np.exp(-np.log(1+ufr)* M_Target) + np.diag(np.exp(-np.log(1+ufr) * M_Target))
    return p ** (-1/ M_Target) -1 # Convert obtained prices to rates and return price
```

Back to the top

Generation of the risk-free curve

The observed maturities, target maturities, and the model parameters provided by EIOPA are used to generate the target curve.

In [329…

```
r_Target = SWExtrapolate(M_Target,M_Obs, Qb, ufr, alpha)
r_Target = pd.DataFrame(r_Target,columns=['Recalculated rates'])
```

Yield curve calculated

Yield curve calculated using the calibration vector Qb

In [330…

```
r_Target.head(15)
```

Out[330…

	Recalculated rates
0	0.048468
1	0.042212
2	0.037644
3	0.035216
4	0.033831
5	0.032993
6	0.032464
7	0.032125
8	0.031940
9	0.031880
10	0.031882
11	0.031910
12	0.031948
13	0.031983
14	0.032006

Back to the top

Test 1; Comparison test

Comparison of the calculated yield curve with the yield curve provided by EIOPA. The test is passed if the success criteria is reached.

The provided yield curve can be found in file *EIOPA_RFR_20230131_Term_Structures.xlsx*, sheet *RFR_spot_no_VA* if the test looks at the curve without the Volatility Adjustment and the sheet *RFR_spot_with_VA* if the test looks at the curve with the Volatility Adjustment.

In [331…

```
target_curve = np.transpose(np.array(curve_country.values))
```

This implementation looks at two kinds of test statistics. The average deviation and the maximum deviation.

The average deviation is defined as:

$$S_{AVERAGE} = \frac{1}{T}\sum_{t=0}^{T}|r_{EIOPA}(t) - r_{EST}(t)|$$

The maximum deviation is defined as:

$$S_{MAX} = \max_{t}|r_{EIOPA}(t) - r_{EST}(t)|$$

Where `T` is the maximum maturity available.

The average difference test is successful if:

$$S_{AVERAGE} < 0.05bps$$

The maximum difference test is successful if:

$$S_{MAX} < 0.1bps$$

In [332...]
```
target_curve = pd.DataFrame(target_curve,columns=['Given rates'])
```

EIOPA curve provided

Yield curve provided by EIOPA

In [333...]
```
target_curve.head()
```

Out[333...]

	Given rates
0	0.04847
1	0.04221
2	0.03764
3	0.03522
4	0.03383

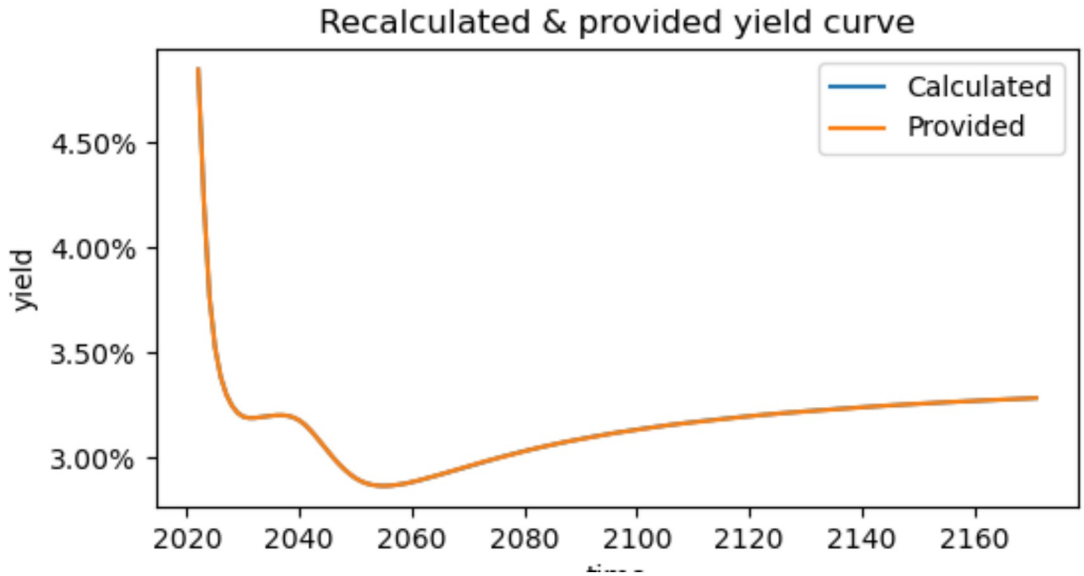
In [334...

```
x_data_label = range(2022,2022+r_Target.shape[0],1)
```

In [335...

```
fig, ax1 = plt.subplots(1,1)
ax1.plot(x_data_label, r_Target.values*100, color='tab:blue',label="Calculated")
ax1.plot(x_data_label, target_curve.values*100, color='tab:orange',label="Provided")

ax1.set_ylabel("yield")
ax1.set_title('Recalculated & provided yield curve')
ax1.set_xlabel("time")
ax1.legend()
ax1.yaxis.set_major_formatter(mtick.PercentFormatter())
fig.set_figwidth(6)
fig.set_figheight(3)
plt.show()
```



In [336...

```
test_statistics_bdp = pd.DataFrame(abs(r_Target.values-target_curve.values)*10000, c
```

EIOPA curve comparison

Absolute difference in bps

In [337...

```
test_statistics_bdp.head()
```

Out[337...

	Abs diff in bps
0	0.020002
1	0.015611
2	0.043682
3	0.039739

Abs diff in bps

Back to the top

Test 1; Success criteria

The successful application of the success criteria marks the completion of the test.

In [338...

```
result1 = SuccessTest(test_statistics_bdp.values, test_statistics_max_diff_in_bps, te
```

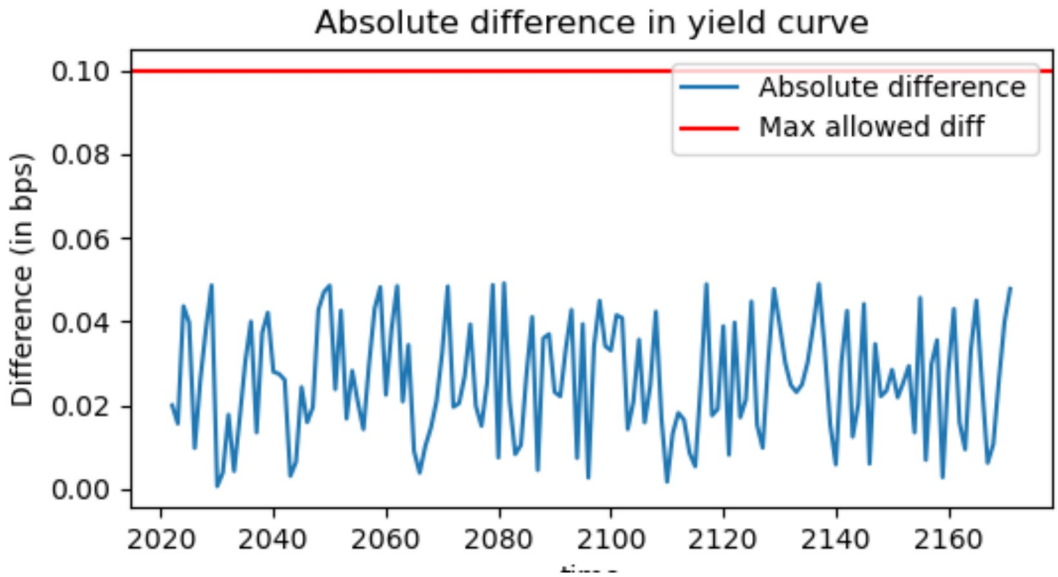
```
Test passed
Test passed
```

In [339...

```
x_data_label = range(2022,2022+r_Target.shape[0],1)
fig, ax1 = plt.subplots(1,1)
ax1.plot(x_data_label, test_statistics_bdp, label= "Absolute difference")
ax1.axhline(y = test_statistics_max_diff_in_bps, color = 'r', linestyle = '-',label='

ax1.set_xlabel("time")
ax1.set_ylabel("Difference (in bps)")
ax1.set_title('Absolute difference in yield curve')
ax1.legend()
fig.set_figwidth(6)
fig.set_figheight(3)

plt.show()
```



Back to the top

Conclusion

This test checks the success criteria on the EIOPA curve generated for January 2023. If the tests are passed, it is likely that the curve was generated using the Smith & Wilson algorithm with the

In [340...

```
pd.DataFrame(data = [result1], columns = ["Mean test","Max test"],  \
             index= ["Provided vs calculated"])
```

Out[340...

	Mean test	Max test
Provided vs calculated	True	True

Final yield curve

Full yield curve provided by EIOPA in %

In [341...

```
(curve_country*100).head(150)
```

Out[341...

```
Country
1      4.847
2      4.221
3      3.764
4      3.522
5      3.383
6      3.299
7      3.246
8      3.212
9      3.194
10     3.188
11     3.188
12     3.191
13     3.195
14     3.198
15     3.201
16     3.201
17     3.198
18     3.190
19     3.176
20     3.155
21     3.128
22     3.097
23     3.064
24     3.031
25     2.999
26     2.969
27     2.943
28     2.920
29     2.900
30     2.886
31     2.875
32     2.869
33     2.865
```

34	2.864
35	2.865
36	2.868
37	2.872
38	2.876
39	2.882
40	2.889
41	2.896
42	2.903
43	2.910
44	2.918
45	2.926
46	2.934
47	2.942
48	2.950
49	2.958
50	2.965
51	2.973
52	2.981
53	2.988
54	2.996
55	3.003
56	3.010
57	3.017
58	3.023
59	3.030
60	3.037
61	3.043
62	3.049
63	3.055
64	3.061
65	3.066
66	3.072
67	3.077
68	3.083
69	3.088
70	3.093
71	3.098
72	3.102
73	3.107
74	3.112
75	3.116
76	3.120
77	3.125
78	3.129
79	3.133
80	3.137
81	3.140
82	3.144
83	3.148
84	3.151
85	3.155
86	3.158
87	3.162
88	3.165
89	3.168
90	3.171
91	3.174
92	3.177

```
93     3.180
94     3.183
95     3.186
96     3.188
97     3.191
98     3.194
99     3.196
100    3.199
101    3.201
102    3.204
103    3.206
104    3.209
105    3.211
106    3.213
107    3.215
108    3.217
109    3.220
110    3.222
111    3.224
112    3.226
113    3.228
114    3.230
115    3.232
116    3.233
117    3.235
118    3.237
119    3.239
120    3.241
121    3.242
122    3.244
123    3.246
124    3.247
125    3.249
126    3.251
127    3.252
128    3.254
129    3.255
130    3.257
131    3.258
132    3.260
133    3.261
134    3.263
135    3.264
136    3.265
137    3.267
138    3.268
139    3.269
140    3.271
141    3.272
142    3.273
143    3.274
144    3.276
145    3.277
146    3.278
147    3.279
148    3.280
149    3.281
150    3.283
Name: United States, dtype: float64
```