# EIOPA RISK-FREE CURVE APRIL-23 RECALCULATION

The risk-free curve is one of the principal inputs into an economic scenario generator. This notebook recalculates the risk-free curve using the parameters that are claimed to be used. The European Insurance and Occupational Pensions Authority (EIOPA) publishes their own yield curve prediction. To do this they use the Smith & Wilson algorithm.

## Summary

The goal of this test is to replicate the EIOPA yield curve. This test will use the methodology that EIOPA claims it is using and the calibration vector that they publish. If the test is passed, the user can be more confident, that EIOPA risk free rate (RFR) curve was generated using the described methodology/calibration and that the process was implemented correctly.

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# Note on Smith & Wilson algorithm

To replicate the calculations, this example uses a modified Smith&Wilson implementation (The original implementation is availible on GitHub:

- Python
- Matlab
- JavaScript

### Limitations of the implementation

Current implementation only looks at a single currency and with/without Volatility Adjustment (VA). The day count convention assumes that each year has the same number of days.

# Data requirements

This script contains the EIOPA risk-free rate publication for April 2023. The publication can be found on the EIOPA RFR website.

The observed maturities `M_Obs` and the calibrated vector `Qb` can be found in the Excel sheet *EIOPA_RFR_20230430_Qb_SW.xlsx*.

The target maturities ( `T_Obs` ), the additional parameters ( `UFR` and `alpha` ), and the given curve can be found in the Excel *EIOPA_RFR_20230430_Term_Structures.xlsx*, sheet *RFR_spot_no_VA* if the test looks at the curve without the Volatility Adjustment and the sheet *RFR_spot_with_VA* if the test looks at the curve with the Volatility Adjustment.

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# Success criteria

The following success criteria is defined:

- Maximum difference between the calculated curve and the one provided by EIOPA is less than 0.1 bps
- Average difference between the calculated curve and the one provided by EIOPA is less than 0.05 bps

In [478...

```
test_statistics_max_diff_in_bps = 0.1
test_statistics_average_diff_in_bps = 0.05
```

The success function is called at the end of the test to confirm if the success criteria have been met.

In [479…

```
def SuccessTest(TestStatistics, threshold_max, threshold_mean):
    out1 = False
    out2 = False
    if max(TestStatistics)<threshold_max:
        print("Test passed")
        out1 = True
    else:
        print("Test failed")

    if np.mean(TestStatistics)<threshold_mean:
        print("Test passed")
        out2 = True
    else:
        print("Test failed")
    return [out1, out2]
```

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# External dependencies

This implementation uses three well established Python packages widely used in the financial industry. Pandas (https://pandas.pydata.org/docs/), Numpy (https://numpy.org/doc/), and Matplotlib (https://matplotlib.org/stable/index.html)

In [480…

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import matplotlib.ticker as mtick
%matplotlib notebook
pd.options.display.max_rows = 150
```

# Importing data

In [481…

```
selected_param_file = 'Param_VA.csv'
selected_curves_file = 'Curves_VA.csv'

#selected_param_file = 'Param_no_VA.csv'
#selected_curves_file = 'Curves_no_VA.csv'
```

In [482…

```
param_raw = pd.read_csv(selected_param_file, sep=',',index_col=0)
```

### Parameter input

Parameters sheet

In [483...]
```
param_raw.head()
```

Out[483...]

| Country | Euro_Maturities | Euro_Values | Austria_Maturities | Austria_Values | Belgium_Maturities | Be |
|---|---|---|---|---|---|---|
| **Coupon_freq** | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | |
| **LLP** | 20.000000 | 20.000000 | 20.000000 | 20.000000 | 20.000000 | |
| **Convergence** | 40.000000 | 40.000000 | 40.000000 | 40.000000 | 40.000000 | |
| **UFR** | 3.450000 | 3.450000 | 3.450000 | 3.450000 | 3.450000 | |
| **alpha** | 0.111906 | 0.111906 | 0.111906 | 0.111906 | 0.111906 | |

5 rows × 106 columns

---

The country selected is:

In [484...]
```
country = "Italy"
```

In [485...]
```
maturities_country_raw = param_raw.loc[:,country+"_Maturities"].iloc[6:]
param_country_raw = param_raw.loc[:,country + "_Values"].iloc[6:]
extra_param = param_raw.loc[:,country + "_Values"].iloc[:6]
```

---

### Extra parameters

Smith-Wilson calibration parameters

In [486...]
```
extra_param
```

Out[486...]
```
Country
Coupon_freq     1.000000
LLP            20.000000
Convergence    40.000000
UFR             3.450000
alpha           0.111906
CRA            10.000000
Name: Italy_Values, dtype: float64
```

---

In [487...]
```
relevant_positions = pd.notna(maturities_country_raw.values)
```

In [488...
```
maturities_country = maturities_country_raw.iloc[relevant_positions]
```

---

### Maturity vector

Vector of maturities used in the calibration

In [489...
```
maturities_country.head(15)
```

Out[489...
```
Country
1      1.0
2      2.0
3      3.0
4      4.0
5      5.0
6      6.0
7      7.0
8      8.0
9      9.0
10    10.0
11    11.0
12    12.0
13    13.0
14    14.0
15    15.0
Name: Italy_Maturities, dtype: float64
```

---

In [490...
```
Qb = param_country_raw.iloc[relevant_positions]
```

---

### Calibration vector

Vector **Qb** provided as input

In [491...
```
Qb
```

Out[491...
```
Country
1    -8.945909
2     0.541504
3     1.925282
4     1.058779
5    -0.475350
6    -0.387083
7     0.712179
8    -0.383027
9    -0.393004
10    2.715405
11   -3.918373
12    2.701016
13   -0.034363
14   -0.031608
15   -1.692104
16    0.023894
```

```
17    0.051802
18   -0.063782
19    0.361738
20    0.440014
Name: Italy Values, dtype: float64
```

---

In [492…

```
curve_raw = pd.read_csv(selected_curves_file, sep=',',index_col=0)
```

In [493…

```
curve_country = curve_raw.loc[:,country]
```

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# Calibration parameters and calibration vector provided by EIOPA

In [494…

```
# Maturity of observations:
M_Obs = np.transpose(np.array(maturities_country.values))

# Ultimate froward rate ufr represents the rate to which the rate curve will converge
ufr = extra_param.iloc[3]/100

# Convergence speed parameter alpha controls the speed at which the curve converges t
alpha = extra_param.iloc[4]

# For which maturities do we want the SW algorithm to calculate the rates. In this ca
M_Target = np.transpose(np.arange(1,151))

# Qb calibration vector published by EIOPA for the curve calibration:
Qb = np.transpose(np.array(Qb.values))
```

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# Smith & Wilson calculation functions

In this step, the independent version of the Smith&Wilson algorithm is implemented. To do this, two functions are taken from the publicly available repository and modified to accept the product of `Q*b` instead of the calibration vector `b`.

In [495…

```
def SWExtrapolate(M_Target, M_Obs, Qb, ufr, alpha):
# SWEXTRAPOLATE Interpolate or/and extrapolate rates for targeted maturities using a
# out = SWExtrapolate(M_Target, M_Obs, Qb, ufr, alpha) calculates the rates for matu
#
# Arguments:
#    M_Target = k x 1 ndarray. Each element represents a bond maturity of interest. E
#    M_Obs =    n x 1 ndarray. Observed bond maturities used for calibrating the cal
#    Qb =       n x 1 ndarray. Calibration vector calculated on observed bonds.
#    ufr =      1 x 1 floating number. Representing the ultimate forward rate.
#       Ex. ufr = 0.042
#    alpha =    1 x 1 floating number. Representing the convergence speed parameter o
#
#
# Returns:
#    k x 1 ndarray. Represents the targeted rates for a zero-coupon bond. Each rate E
#
# For more information see https://www.eiopa.europa.eu/sites/default/files/risk_free_

    def SWHeart(u, v, alpha):
    # SWHEART Calculate the heart of the Wilson function.
    # H = SWHeart(u, v, alpha) calculates the matrix H (Heart of the Wilson
    # function) for maturities specified by vectors u and v. The formula is
    # taken from the EIOPA technical specifications paragraph 132.
    #
    # Arguments:
    #    u =     n_1 x 1 vector of maturities. Ex. u = [1; 3]
    #    v =     n_2 x 1 vector of maturities. Ex. v = [1; 2; 3; 5]
    #    alpha = 1 x 1 floating number representing the convergence speed parameter o
    #
    # Returns:
    #    n_1 x n_2 matrix representing the Heart of the Wilson function for selected
    #
    # For more information see https://www.eiopa.europa.eu/sites/default/files/risk_f

        u_Mat = np.tile(u, [v.size, 1]).transpose()
        v_Mat = np.tile(v, [u.size, 1])
        return 0.5 * (alpha * (u_Mat + v_Mat) + np.exp(-alpha * (u_Mat + v_Mat)) - al

    H = SWHeart(M_Target, M_Obs, alpha) # Heart of the Wilson function from paragraph
    p = np.exp(-np.log(1+ufr)* M_Target) + np.diag(np.exp(-np.log(1+ufr) * M_Target))
    return p ** (-1/ M_Target) -1 # Convert obtained prices to rates and return price
```

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# Generation of the risk-free curve

The observed maturities, target maturities, and the model parameters provided by EIOPA are used to generate the target curve.

In [496…

```
r_Target = SWExtrapolate(M_Target,M_Obs, Qb, ufr, alpha)
r_Target = pd.DataFrame(r_Target,columns=['Recalculated rates'])
```

---

**Yield curve calculated**

Yield curve calculated using the calibration vector **Qb**

In [497…

```
r_Target.head(15)
```

Out[497…

| | Recalculated rates |
|---|---|
| **0** | 0.038530 |
| **1** | 0.035419 |
| **2** | 0.033076 |
| **3** | 0.031777 |
| **4** | 0.031117 |
| **5** | 0.030734 |
| **6** | 0.030516 |
| **7** | 0.030446 |
| **8** | 0.030461 |
| **9** | 0.030552 |
| **10** | 0.030700 |
| **11** | 0.030764 |
| **12** | 0.030832 |
| **13** | 0.030851 |
| **14** | 0.030747 |

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# Test 1; Comparison test

Comparison of the calculated yield curve with the yield curve provided by EIOPA. The test is passed if the success criteria is reached.

The provided yield curve can be found in file *EIOPA_RFR_20230430_Term_Structures.xlsx*, sheet *RFR_spot_no_VA* if the test looks at the curve without the Volatility Adjustment and the sheet *RFR_spot_with_VA* if the test looks at the curve with the Volatility Adjustment.

In [498…

```
target_curve = np.transpose(np.array(curve_country.values))
```

This implementation looks at two kinds of test statistics. The average deviation and the maximum deviation.

The average deviation is defined as:

$$S_{AVERAGE} = \frac{1}{T}\sum_{t=0}^{T}|r_{EIOPA}(t) - r_{EST}(t)|$$

The maximum deviation is defined as:

$$S_{MAX} = \max_{t}|r_{EIOPA}(t) - r_{EST}(t)|$$

Where `T` is the maximum maturity available.

The average difference test is successful if:

$$S_{AVERAGE} < 0.05bps$$

The maximum difference test is successful if:

$$S_{MAX} < 0.1bps$$

In [499…]
```
target_curve = pd.DataFrame(target_curve,columns=['Given rates'])
```

---

**EIOPA curve provided**

Yield curve provided by EIOPA

In [500…]
```
target_curve.head()
```

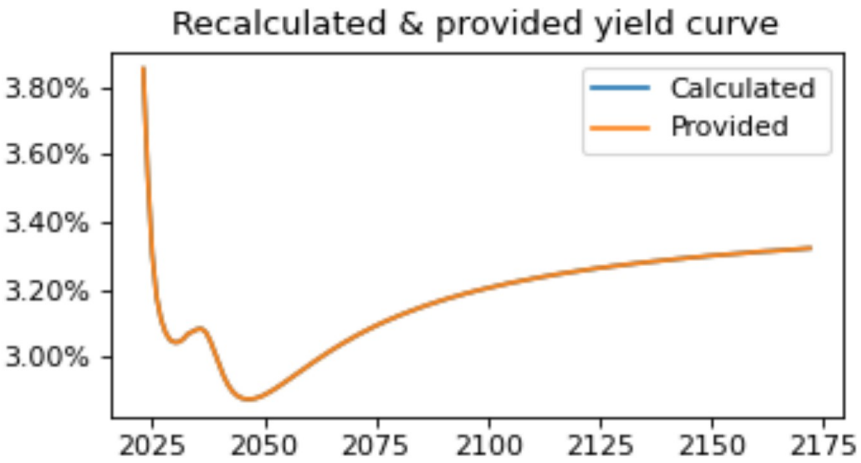
Out[500…]

| | Given rates |
|---|---|
| **0** | 0.03853 |
| **1** | 0.03542 |
| **2** | 0.03308 |
| **3** | 0.03178 |
| **4** | 0.03112 |

In [501...]
```
x_data_label = range(2023,2023+r_Target.shape[0],1)
```

In [502...]
```
fig, ax1 = plt.subplots(1,1)
ax1.plot(x_data_label, r_Target.values*100, color='tab:blue',label="Calculated")
ax1.plot(x_data_label, target_curve.values*100, color='tab:orange',label="Provided")

ax1.set_ylabel("yield")
ax1.set_title('Recalculated & provided yield curve')
ax1.set_xlabel("time")
ax1.legend()
ax1.yaxis.set_major_formatter(mtick.PercentFormatter())
fig.set_figwidth(6)
fig.set_figheight(3)
plt.show()
```




---

In [503...]
```
test_statistics_bdp = pd.DataFrame(abs(r_Target.values-target_curve.values)*10000, c
```

---

**EIOPA curve comparison**

Absolute difference in bps

In [504...]
```
test_statistics_bdp.head()
```

Out[504...]

| | Abs diff in bps |
|---|---|
| **0** | 2.958918e-08 |
| **1** | 1.359217e-02 |
| **2** | 4.078525e-02 |
| **3** | 2.903841e-02 |
| **4** | 3.067402e-02 |

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# Test 1; Success criteria

The successful application of the success criteria marks the completion of the test.

In [505…

```
result1 = SuccessTest(test_statistics_bdp.values, test_statistics_max_diff_in_bps, te
```

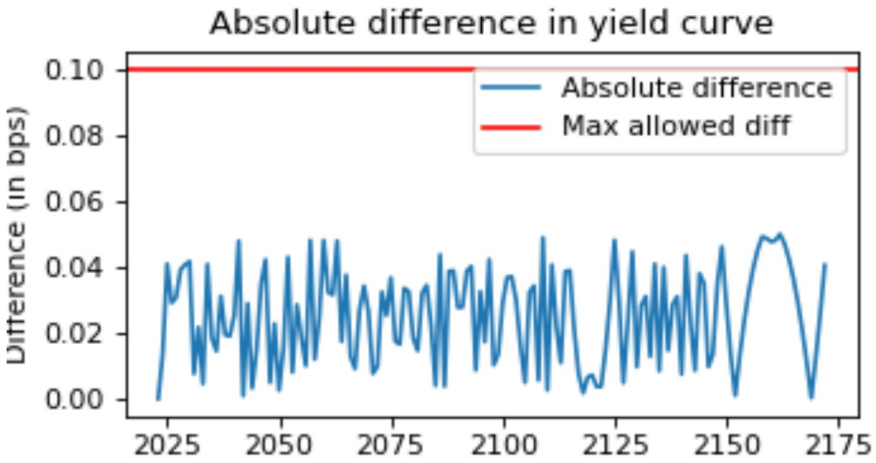
```
Test passed
Test passed
```

In [506…

```
x_data_label = range(2023,2023+r_Target.shape[0],1)
fig, ax1 = plt.subplots(1,1)
ax1.plot(x_data_label, test_statistics_bdp, label= "Absolute difference")
ax1.axhline(y = test_statistics_max_diff_in_bps, color = 'r', linestyle = '-',label='

ax1.set_xlabel("time")
ax1.set_ylabel("Difference (in bps)")
ax1.set_title('Absolute difference in yield curve')
ax1.legend()
fig.set_figwidth(6)
fig.set_figheight(3)

plt.show()
```

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# Conclusion

This test checks the success criteria on the EIOPA curve generated for April 2023. If the tests are passed, it is likely that the curve was generated using the Smith & Wilson algorithm with the

calibration vector that was provided in the file *EIOPA_RFR_20230430_Qb_SW.xlsx* and the parameters displayed in the file *EIOPA_RFR_20230430_Term_Structures.xlsx*.

In [507...

```
pd.DataFrame(data = [result1], columns = ["Mean test","Max test"],  \
             index= ["Provided vs calculated"])
```

Out[507...

| | Mean test | Max test |
|---|---|---|
| **Provided vs calculated** | True | True |

---

**Final yield curve**

Full yield curve provided by EIOPA in %

In [508...

```
(curve_country*100).head(150)
```

Out[508...

```
Country
1      3.853
2      3.542
3      3.308
4      3.178
5      3.112
6      3.073
7      3.052
8      3.045
9      3.046
10     3.055
11     3.070
12     3.076
13     3.083
14     3.085
15     3.075
16     3.049
17     3.014
18     2.978
19     2.944
20     2.918
21     2.899
22     2.887
23     2.880
24     2.877
25     2.877
26     2.879
27     2.883
28     2.889
29     2.896
30     2.904
31     2.912
32     2.921
33     2.930
34     2.939
35     2.948
36     2.958
37     2.967
```

| | |
|---|---|
| 38 | 2.977 |
| 39 | 2.986 |
| 40 | 2.995 |
| 41 | 3.004 |
| 42 | 3.012 |
| 43 | 3.021 |
| 44 | 3.029 |
| 45 | 3.037 |
| 46 | 3.045 |
| 47 | 3.052 |
| 48 | 3.060 |
| 49 | 3.067 |
| 50 | 3.074 |
| 51 | 3.081 |
| 52 | 3.087 |
| 53 | 3.094 |
| 54 | 3.100 |
| 55 | 3.106 |
| 56 | 3.112 |
| 57 | 3.117 |
| 58 | 3.123 |
| 59 | 3.128 |
| 60 | 3.133 |
| 61 | 3.138 |
| 62 | 3.143 |
| 63 | 3.148 |
| 64 | 3.153 |
| 65 | 3.157 |
| 66 | 3.161 |
| 67 | 3.166 |
| 68 | 3.170 |
| 69 | 3.174 |
| 70 | 3.178 |
| 71 | 3.181 |
| 72 | 3.185 |
| 73 | 3.189 |
| 74 | 3.192 |
| 75 | 3.196 |
| 76 | 3.199 |
| 77 | 3.202 |
| 78 | 3.205 |
| 79 | 3.208 |
| 80 | 3.211 |
| 81 | 3.214 |
| 82 | 3.217 |
| 83 | 3.220 |
| 84 | 3.223 |
| 85 | 3.225 |
| 86 | 3.228 |
| 87 | 3.230 |
| 88 | 3.233 |
| 89 | 3.235 |
| 90 | 3.238 |
| 91 | 3.240 |
| 92 | 3.242 |
| 93 | 3.245 |
| 94 | 3.247 |
| 95 | 3.249 |
| 96 | 3.251 |

```
97     3.253
98     3.255
99     3.257
100    3.259
101    3.261
102    3.263
103    3.265
104    3.266
105    3.268
106    3.270
107    3.271
108    3.273
109    3.275
110    3.276
111    3.278
112    3.279
113    3.281
114    3.282
115    3.284
116    3.285
117    3.287
118    3.288
119    3.289
120    3.291
121    3.292
122    3.293
123    3.295
124    3.296
125    3.297
126    3.298
127    3.300
128    3.301
129    3.302
130    3.303
131    3.304
132    3.305
133    3.306
134    3.307
135    3.308
136    3.309
137    3.311
138    3.312
139    3.313
140    3.314
141    3.314
142    3.315
143    3.316
144    3.317
145    3.318
146    3.319
147    3.320
148    3.321
149    3.322
150    3.323
Name: Italy, dtype: float64
```