EIOPA RISK-FREE CURVE MARCH-23 RECALCULATION

The risk-free curve is one of the principal inputs into an economic scenario generator. This notebook recalculates the risk-free curve using the parameters that are claimed to be used. The European Insurance and Occupational Pensions Authority (EIOPA) publishes their own yield curve prediction. To do this they use the Smith & Wilson algorithm.

Summary

The goal of this test is to replicate the EIOPA yield curve. This test will use the methodology that EIOPA claims it is using and the calibration vector that they publish. If the test is passed, the user can be more confident, that EIOPA risk free rate (RFR) curve was generated using the described methodology/calibration and that the process was implemented correctly.

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Note on Smith & Wilson algorithm

To replicate the calculations, this example uses a modified Smith&Wilson implementation (The original implementation is availible on GitHub:

- Python
- Matlab
- JavaScript

Limitations of the implementation

Current implementation only looks at a single currency and with/without Volatility Adjustment (VA). The day count convention assumes that each year has the same number of days.

Data requirements

This script contains the EIOPA risk-free rate publication for March 2023. The publication can be found on the EIOPA RFR website.

The observed maturities `M_Obs` and the calibrated vector `Qb` can be found in the Excel sheet *EIOPA_RFR_20230331_Qb_SW.xlsx*.

For this example, the curve without the volatility adjustment (VA) is used. It can be found in the sheet *SW_Qb_no_VA*. This example is focused on the EUR curve, but this example can be easily modified for any other curve.

The target maturities (`T_Obs`), the additional parameters (`UFR` and `alpha`), and the given curve can be found in the Excel *EIOPA_RFR_20230331_Term_Structures.xlsx*, sheet *RFR_spot_no_VA* if the test looks at the curve without the Volatility Adjustment and the sheet *RFR_spot_with_VA* if the test looks at the curve with the Volatility Adjustment.

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Success criteria

The following success criteria is defined:

- Maximum difference between the calculated curve and the one provided by EIOPA is less than 0.1 bps
- Average difference between the calculated curve and the one provided by EIOPA is less than 0.05 bps

In [218...

```
test_statistics_max_diff_in_bps = 0.1
test_statistics_average_diff_in_bps = 0.05
```

The success function is called at the end of the test to confirm if the success criteria have been met.

In [219…

```
def SuccessTest(TestStatistics, threshold_max, threshold_mean):
    out1 = False
    out2 = False
    if max(TestStatistics)<threshold_max:
        print("Test passed")
        out1 = True
    else:
        print("Test failed")

    if np.mean(TestStatistics)<threshold_mean:
        print("Test passed")
        out2 = True
    else:
        print("Test failed")
    return [out1, out2]
```

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External dependencies

This implementation uses three well established Python packages widely used in the financial industry. Pandas (https://pandas.pydata.org/docs/), Numpy (https://numpy.org/doc/), and Matplotlib (https://matplotlib.org/stable/index.html)

In [220…

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import matplotlib.ticker as mtick
%matplotlib notebook
pd.options.display.max_rows = 150
```

Importing data

In [221…

```
#selected_param_file = 'Param_VA.csv'
#selected_curves_file = 'Curves_VA.csv'

selected_param_file = 'Param_no_VA.csv'
selected_curves_file = 'Curves_no_VA.csv'
```

In [222…

```
param_raw = pd.read_csv(selected_param_file, sep=',',index_col=0)
```

Parameter input

Parameters sheet

In [223...
```
param_raw.head()
```

Out[223...

Country	Euro_Maturities	Euro_Values	Austria_Maturities	Austria_Values	Belgium_Maturities	Be
Coupon_freq	1.000000	1.000000	1.000000	1.000000	1.000000	
LLP	20.000000	20.000000	20.000000	20.000000	20.000000	
Convergence	40.000000	40.000000	40.000000	40.000000	40.000000	
UFR	3.450000	3.450000	3.450000	3.450000	3.450000	
alpha	0.117567	0.117567	0.117567	0.117567	0.117567	

5 rows × 106 columns

The country selected is:

In [224...
```
country = "Slovenia"
```

In [225...
```
maturities_country_raw = param_raw.loc[:,country+"_Maturities"].iloc[6:]
param_country_raw = param_raw.loc[:,country + "_Values"].iloc[6:]
extra_param = param_raw.loc[:,country + "_Values"].iloc[:6]
```

Extra parameters

Smith-Wilson calibration parameters

In [226...
```
extra_param
```

Out[226...
```
Country
Coupon_freq     1.000000
LLP            20.000000
Convergence    40.000000
UFR             3.450000
alpha           0.117567
CRA            10.000000
Name: Slovenia_Values, dtype: float64
```

In [227...
```
relevant_positions = pd.notna(maturities_country_raw.values)
```

In [228...
```
maturities_country = maturities_country_raw.iloc[relevant_positions]
```

Maturity vector

Vector of maturities used in the calibration

In [229...
```
maturities_country.head(15)
```

Out[229...
```
Country
1      1.0
2      2.0
3      3.0
4      4.0
5      5.0
6      6.0
7      7.0
8      8.0
9      9.0
10    10.0
11    11.0
12    12.0
13    13.0
14    14.0
15    15.0
Name: Slovenia_Maturities, dtype: float64
```

In [230...
```
Qb = param_country_raw.iloc[relevant_positions]
```

Calibration vector

Vector Qb provided as input

In [231...
```
Qb
```

Out[231...
```
Country
1     -1.792555
2     -1.996019
3      0.519822
4      0.677359
5      0.695391
6     -0.689019
7      0.624097
8     -1.532522
9      6.464371
10   -15.351240
11    18.716887
12    -8.651557
13     0.003832
14     0.003704
15    -0.590794
16     0.019939
```

```
17     0.019274
18     0.018631
19     0.018010
20     0.658523
Name: Slovenia Values, dtype: float64
```

In [232…

```
curve_raw = pd.read_csv(selected_curves_file, sep=',',index_col=0)
```

In [233…

```
curve_country = curve_raw.loc[:,country]
```

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Calibration parameters and calibration vector provided by EIOPA

In [234…

```
# Maturity of observations:
M_Obs = np.transpose(np.array(maturities_country.values))

# Ultimate froward rate ufr represents the rate to which the rate curve will converge
ufr = extra_param.iloc[3]/100

# Convergence speed parameter alpha controls the speed at which the curve converges t
alpha = extra_param.iloc[4]

# For which maturities do we want the SW algorithm to calculate the rates. In this ca
M_Target = np.transpose(np.arange(1,151))

# Qb calibration vector published by EIOPA for the curve calibration:
Qb = np.transpose(np.array(Qb.values))
```

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Smith & Wilson calculation functions

In this step, the independent version of the Smith&Wilson algorithm is implemented. To do this, two functions are taken from the publicly available repository and modified to accept the product of `Q*b` instead of the calibration vector `b`.

In [235…

```
def SWExtrapolate(M_Target, M_Obs, Qb, ufr, alpha):
# SWEXTRAPOLATE Interpolate or/and extrapolate rates for targeted maturities using a
# out = SWExtrapolate(M_Target, M_Obs, Qb, ufr, alpha) calculates the rates for matu
#
# Arguments:
#    M_Target = k x 1 ndarray. Each element represents a bond maturity of interest. E
#    M_Obs =    n x 1 ndarray. Observed bond maturities used for calibrating the cal
#    Qb =       n x 1 ndarray. Calibration vector calculated on observed bonds.
#    ufr =      1 x 1 floating number. Representing the ultimate forward rate.
#       Ex. ufr = 0.042
#    alpha =    1 x 1 floating number. Representing the convergence speed parameter o
#
#
# Returns:
#    k x 1 ndarray. Represents the targeted rates for a zero-coupon bond. Each rate E
#
# For more information see https://www.eiopa.europa.eu/sites/default/files/risk_free_

    def SWHeart(u, v, alpha):
    # SWHEART Calculate the heart of the Wilson function.
    # H = SWHeart(u, v, alpha) calculates the matrix H (Heart of the Wilson
    # function) for maturities specified by vectors u and v. The formula is
    # taken from the EIOPA technical specifications paragraph 132.
    #
    # Arguments:
    #    u =     n_1 x 1 vector of maturities. Ex. u = [1; 3]
    #    v =     n_2 x 1 vector of maturities. Ex. v = [1; 2; 3; 5]
    #    alpha = 1 x 1 floating number representing the convergence speed parameter o
    #
    # Returns:
    #    n_1 x n_2 matrix representing the Heart of the Wilson function for selected
    #
    # For more information see https://www.eiopa.europa.eu/sites/default/files/risk_f

        u_Mat = np.tile(u, [v.size, 1]).transpose()
        v_Mat = np.tile(v, [u.size, 1])
        return 0.5 * (alpha * (u_Mat + v_Mat) + np.exp(-alpha * (u_Mat + v_Mat)) - al

    H = SWHeart(M_Target, M_Obs, alpha) # Heart of the Wilson function from paragraph
    p = np.exp(-np.log(1+ufr)* M_Target) + np.diag(np.exp(-np.log(1+ufr) * M_Target))
    return p ** (-1/ M_Target) -1 # Convert obtained prices to rates and return price
```

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Generation of the risk-free curve

The observed maturities, target maturities, and the model parameters provided by EIOPA are used to generate the target curve.

In [236…

```
r_Target = SWExtrapolate(M_Target,M_Obs, Qb, ufr, alpha)
r_Target = pd.DataFrame(r_Target,columns=['Recalculated rates'])
```

Yield curve calculated

Yield curve calculated using the calibration vector Qb

In [237…
```
r_Target.head(15)
```

Out[237…

	Recalculated rates
0	0.034720
1	0.033154
2	0.031400
3	0.030091
4	0.029299
5	0.028860
6	0.028598
7	0.028473
8	0.028454
9	0.028501
10	0.028373
11	0.028698
12	0.028894
13	0.028842
14	0.028604

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Test 1; Comparison test

Comparison of the calculated yield curve with the yield curve provided by EIOPA. The test is passed if the success criteria is reached.

The provided yield curve can be found in file *EIOPA_RFR_20230331_Term_Structures.xlsx*, sheet *RFR_spot_no_VA* if the test looks at the curve without the Volatility Adjustment and the sheet *RFR_spot_with_VA* if the test looks at the curve with the Volatility Adjustment.

In [238…
```
target_curve = np.transpose(np.array(curve_country.values))
```

This implementation looks at two kinds of test statistics. The average deviation and the maximum deviation.

The average deviation is defined as:

$$S_{AVERAGE} = \frac{1}{T}\sum_{t=0}^{T}|r_{EIOPA}(t) - r_{EST}(t)|$$

The maximum deviation is defined as:

$$S_{MAX} = \max_{t}|r_{EIOPA}(t) - r_{EST}(t)|$$

Where `T` is the maximum maturity available.

The average difference test is successful if:

$$S_{AVERAGE} < 0.05bps$$

The maximum difference test is successful if:

$$S_{MAX} < 0.1bps$$

In [239…]
```
target_curve = pd.DataFrame(target_curve,columns=['Given rates'])
```

EIOPA curve provided

Yield curve provided by EIOPA

In [240…]
```
target_curve.head()
```

Out[240…]

	Given rates
0	0.03472
1	0.03315
2	0.03140
3	0.03009
4	0.02930

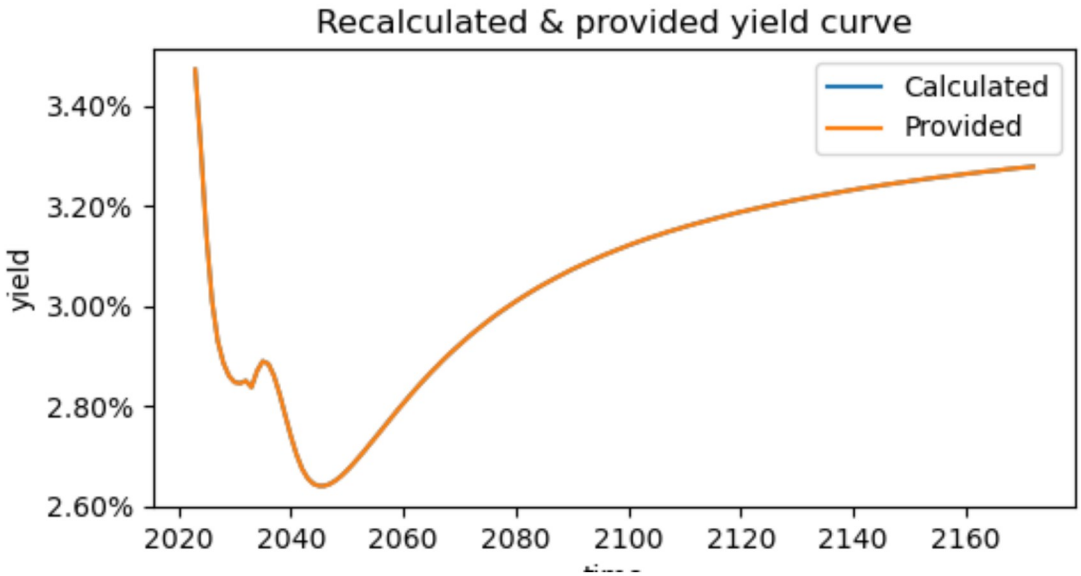
In [241...

```
x_data_label = range(2023,2023+r_Target.shape[0],1)
```

In [242...

```
fig, ax1 = plt.subplots(1,1)
ax1.plot(x_data_label, r_Target.values*100, color='tab:blue',label="Calculated")
ax1.plot(x_data_label, target_curve.values*100, color='tab:orange',label="Provided")

ax1.set_ylabel("yield")
ax1.set_title('Recalculated & provided yield curve')
ax1.set_xlabel("time")
ax1.legend()
ax1.yaxis.set_major_formatter(mtick.PercentFormatter())
fig.set_figwidth(6)
fig.set_figheight(3)
plt.show()
```



In [243...

```
test_statistics_bdp = pd.DataFrame(abs(r_Target.values-target_curve.values)*10000, c
```

EIOPA curve comparison

Absolute difference in bps

In [244...

```
test_statistics_bdp.head()
```

Out[244...

	Abs diff in bps
0	0.000002
1	0.044901
2	0.002998
3	0.010952

Abs diff in bps

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Test 1; Success criteria

The successful application of the success criteria marks the completion of the test.

In [245...

```
result1 = SuccessTest(test_statistics_bdp.values, test_statistics_max_diff_in_bps, te
```

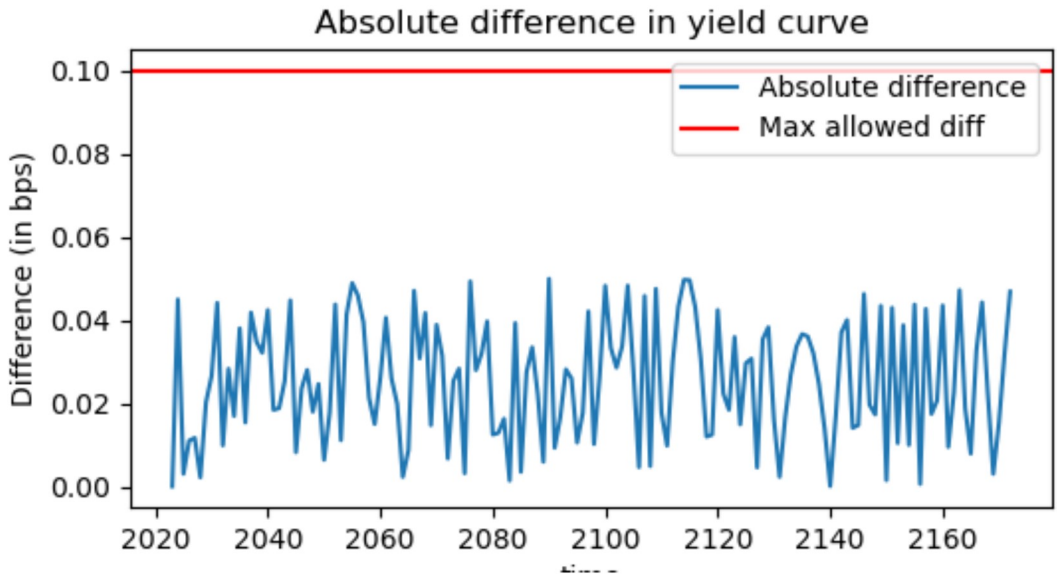
```
Test passed
Test passed
```

In [246...

```
x_data_label = range(2023,2023+r_Target.shape[0],1)
fig, ax1 = plt.subplots(1,1)
ax1.plot(x_data_label, test_statistics_bdp, label= "Absolute difference")
ax1.axhline(y = test_statistics_max_diff_in_bps, color = 'r', linestyle = '-',label='

ax1.set_xlabel("time")
ax1.set_ylabel("Difference (in bps)")
ax1.set_title('Absolute difference in yield curve')
ax1.legend()
fig.set_figwidth(6)
fig.set_figheight(3)

plt.show()
```



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Conclusion

This test checks the success criteria on the EIOPA curve generated for March 2023. If the tests are passed, it is likely that the curve was generated using the Smith & Wilson algorithm with the

In [247...
```
pd.DataFrame(data = [result1], columns = ["Mean test","Max test"],  \
             index= ["Provided vs calculated"])
```

Out[247...

	Mean test	Max test
Provided vs calculated	True	True

Final yield curve

Full yield curve provided by EIOPA in %

In [248...
```
(curve_country*100).head(150)
```

Out[248...
```
Country
1      3.472
2      3.315
3      3.140
4      3.009
5      2.930
6      2.886
7      2.860
8      2.847
9      2.845
10     2.850
11     2.837
12     2.870
13     2.889
14     2.884
15     2.860
16     2.824
17     2.781
18     2.740
19     2.703
20     2.674
21     2.655
22     2.644
23     2.640
24     2.640
25     2.644
26     2.651
27     2.660
28     2.671
29     2.683
30     2.696
31     2.709
32     2.723
33     2.736
```

34	2.750
35	2.764
36	2.778
37	2.792
38	2.805
39	2.818
40	2.831
41	2.844
42	2.856
43	2.868
44	2.879
45	2.891
46	2.902
47	2.912
48	2.922
49	2.932
50	2.942
51	2.951
52	2.960
53	2.969
54	2.978
55	2.986
56	2.994
57	3.001
58	3.009
59	3.016
60	3.023
61	3.030
62	3.037
63	3.043
64	3.049
65	3.055
66	3.061
67	3.067
68	3.073
69	3.078
70	3.083
71	3.088
72	3.093
73	3.098
74	3.103
75	3.107
76	3.112
77	3.116
78	3.121
79	3.125
80	3.129
81	3.133
82	3.137
83	3.140
84	3.144
85	3.148
86	3.151
87	3.154
88	3.158
89	3.161
90	3.164
91	3.167
92	3.170

```
93     3.173
94     3.176
95     3.179
96     3.182
97     3.185
98     3.188
99     3.190
100    3.193
101    3.195
102    3.198
103    3.200
104    3.203
105    3.205
106    3.207
107    3.210
108    3.212
109    3.214
110    3.216
111    3.218
112    3.220
113    3.222
114    3.224
115    3.226
116    3.228
117    3.230
118    3.232
119    3.234
120    3.236
121    3.237
122    3.239
123    3.241
124    3.243
125    3.244
126    3.246
127    3.247
128    3.249
129    3.251
130    3.252
131    3.254
132    3.255
133    3.257
134    3.258
135    3.259
136    3.261
137    3.262
138    3.264
139    3.265
140    3.266
141    3.268
142    3.269
143    3.270
144    3.271
145    3.273
146    3.274
147    3.275
148    3.276
149    3.277
150    3.278
Name: Slovenia, dtype: float64
```