Robust Focal Length Computation

Bachelor Thesis

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Department of Cybernetics

2017

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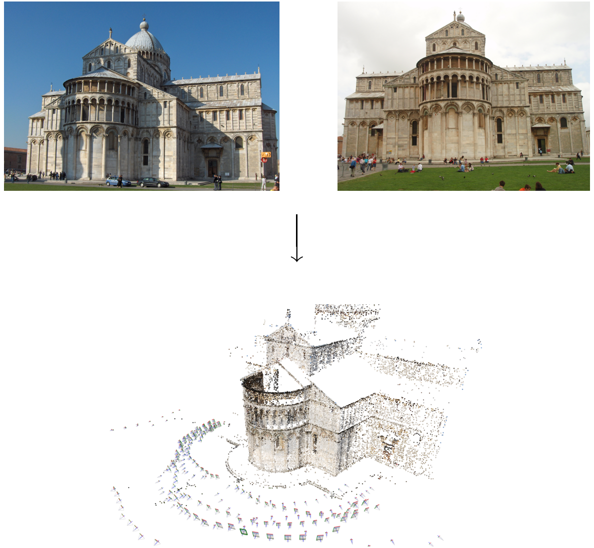
Hello. My name is Oleh Rybkin and the topic of my bachelor thesis is Robust Focal Length Computation.

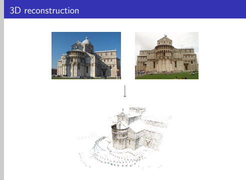
Outline

1. Introduction
2. Theroretical analysis
3. Experimental analysis
4. Improvements

Let's have a brief introduction. Broadly speaking, the field of my work is 3D reconstruction.

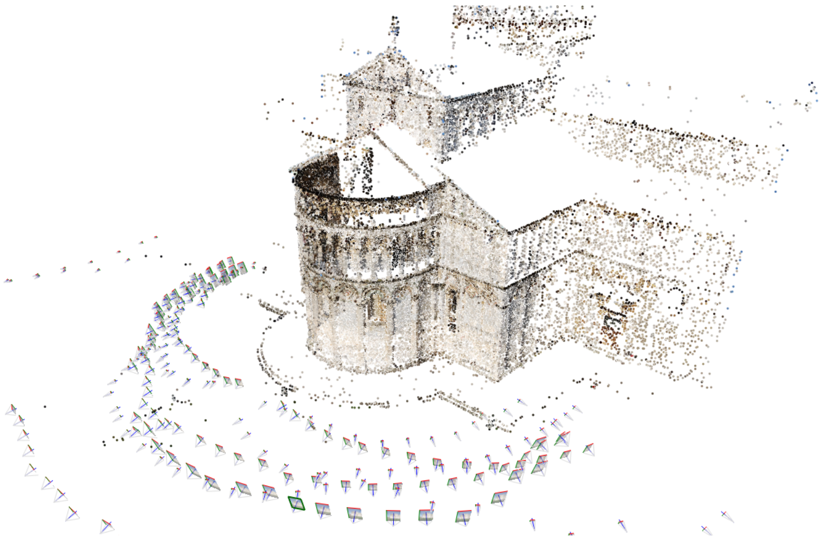
3D reconstruction

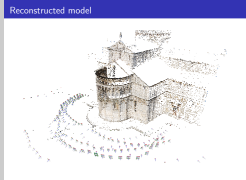




In 3D reconstruction we have some images of an object *shows*, and we want to reconstruct the actual 3D structure of it.

Reconstructed model



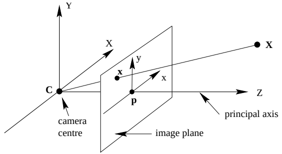


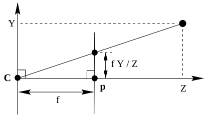
Let's see the reconstruction fullsize. Here *shows* you see some cameras which with which the images were taken. As it is a model reconstructed in 3D, we can do things as rotate it freely, print it on a 3D printer, or take a virtual photo from whatever side. To make a reconstruction, one approach includes first finding focal lengths of the cameras with which the photos were taken.

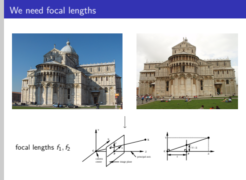
We need focal lengths

↓

focal lengths f_1, f_2



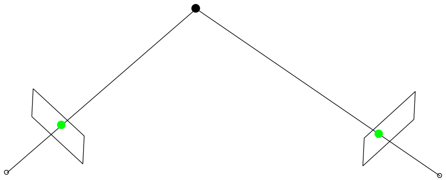




Let's think of this sub-problem of 3D reconstruction. How do we compute the focal lengths?

Correspondences

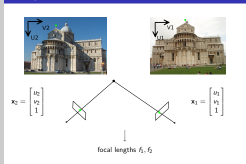
$$\mathbf{x}_2 = \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix}$$



$$\mathbf{x}_1 = \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}$$

$$\downarrow$$

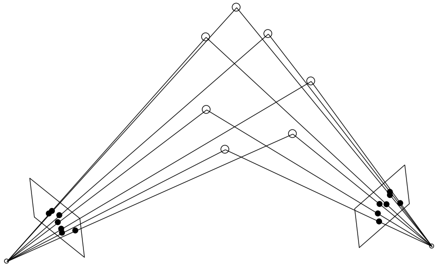
focal lengths f_1, f_2



The mainly used procedure firstly involves finding the corresponding points in images. For instance, this green *shows* point in the left image corresponds to this green *shows* point in the right one. This picture *shows* shows that the points are projections of the same 3D point *shows* to the image planes, schematically shown here *shows* and here *shows*.

Fundamental matrix

$$\mathbf{x}_2 = \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix}$$



$$\mathbf{x}_1 = \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}$$

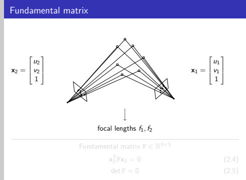
$$\downarrow$$

focal lengths f_1, f_2

Fundamental matrix $\mathrm{F} \in \mathbb{R}^{3\times 3}$

$$\mathbf{x}_2^\top \mathrm{F} \mathbf{x}_1 = 0 \tag{2.4}$$

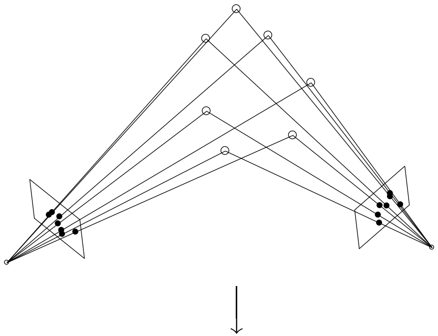
$$\det \mathrm{F} = 0 \tag{2.5}$$



Now we can turn our problem into a geometrical one. Given the correspondences as projective points (that is, coordinates x, y and a 1 added) *shows* we can compute the focal lengths using the Fundamental matrix.

Fundamental matrix

$$\mathbf{x}_2 = \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix}$$



$$\mathbf{x}_1 = \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}$$

focal lengths f_1, f_2

Fundamental matrix $\mathtt{F} \in \mathbb{R}^{3\times3}$

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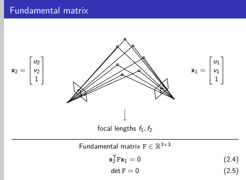

We can understand the Fundamental matrix in terms of constraints it satisfies. Firstly, it satisfies for each correspondence the epipolar constraint *shows*, and, secondly, it is a matrix of rank 2 *shows*. Bear in mind that in all real situations the measured point positions aren't known with absolute precision, but are noisy, which means that the constraint will not be satisfied exactly by the ground truth matrix.

How to compute the fundamental matrix

Algorithm 1: 7pt

Data: $n \geq 7$ corresponding image points $\mathbf{x}_{1,i}$, $\mathbf{x}_{2,i}$

Result: Fundamental matrix $\mathtt{F}$

begin

matrix $\mathtt{B} \in \mathbb{R}^{n\times 9}$ with columns $\mathbf{b}_i = \mathrm{vec}(\mathbf{x}_{2,i} \otimes \mathbf{x}_{1,i})$;

SVD($\mathtt{B}$) $\rightarrow$ singular vectors $\mathbf{f}_1, \mathbf{f}_2$ for the two smallest singular values;

Reshape $\mathbf{f}_1, \mathbf{f}_2 \rightarrow \mathtt{F}_1, \mathtt{F}_2 \in \mathbb{R}^{3\times 3}$;

if $\det(\mathtt{F}_2) = 0$ **then**

return $\mathtt{F}_2$;

else

Solve the 3rd degree polynomial in x: $\det(x\mathtt{F}_1 + \mathtt{F}_2) = 0$;

choose the real roots x_i.;

return $x_i\mathtt{F}_1 + \mathtt{F}_2$;

end

end

How to compute the fundamental matrix
Algorithm 1: 7pt
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SVD($\mathtt{B}$) $\rightarrow$ singular vectors $\mathbf{f}_1, \mathbf{f}_2$ for the two smallest singular values;
Reshape $\mathbf{f}_1, \mathbf{f}_2 \rightarrow \mathtt{F}_1, \mathtt{F}_2 \in \mathbb{R}^{3\times 3}$;
if $\det(\mathtt{F}_2) = 0$ then
return $\mathtt{F}_2$;
else
Solve the 3rd degree polynomial in x: $\det(x\mathtt{F}_1 + \mathtt{F}_2) = 0$;
choose the real roots x_i.;
return $x_i\mathtt{F}_1 + \mathtt{F}_2$;
end
end

The fundamental matrix has 7 degrees of freedom. Thus, we can exhaust all its degrees of freedom given 7 correspondences. Here we formulate the task of computing the matrix so that even more than 7 correspondences can be used. We first find a one-dimensional space of fundamental matrices that minimizes the algebraic error of the epipolar constraints via SVD *shows*,

How to compute the fundamental matrix

Algorithm 2: 7pt

Data: $n \geq 7$ corresponding image points $\mathbf{x}_{1,i}$, $\mathbf{x}_{2,i}$
Result: Fundamental matrix $\mathtt{F}$
begin
  matrix $\mathtt{B} \in \mathbb{R}^{n\times 9}$ with columns $\mathbf{b}_i = \mathrm{vec}(\mathbf{x}_{2,i} \otimes \mathbf{x}_{1,i})$;
  $\mathrm{SVD}(\mathtt{B}) \rightarrow$ singular vectors $\mathbf{f}_1, \mathbf{f}_2$ for the two smallest singular values;
  Reshape $\mathbf{f}_1, \mathbf{f}_2 \rightarrow \mathtt{F}_1, \mathtt{F}_2 \in \mathbb{R}^{3\times 3}$;
  if $\det(\mathtt{F}_2) = 0$ **then**
    return $\mathtt{F}_2$;
  else
    Solve the 3rd degree polynomial in x: $\det(x\mathtt{F}_1 + \mathtt{F}_2) = 0$;
    choose the real roots x_i.;
    return $x_i\mathtt{F}_1 + \mathtt{F}_2$;
  end
end

How to compute the fundamental matrix
Algorithm 2: 7pt
Data: $n \geq 7$ corresponding image points $\mathbf{x}_{1,i}$, $\mathbf{x}_{2,i}$
Result: Fundamental matrix $\mathtt{F}$
begin
matrix $\mathtt{B} \in \mathbb{R}^{n\times 9}$ with columns $\mathbf{b}_i = \mathrm{vec}(\mathbf{x}_{2,i} \otimes \mathbf{x}_{1,i})$;
$\mathrm{SVD}(\mathtt{B}) \rightarrow$ singular vectors $\mathbf{f}_1, \mathbf{f}_2$ for the two smallest singular values;
Reshape $\mathbf{f}_1, \mathbf{f}_2 \rightarrow \mathtt{F}_1, \mathtt{F}_2 \in \mathbb{R}^{3\times 3}$;
if $\det(\mathtt{F}_2) = 0$ then
return $\mathtt{F}_2$;
else
Solve the 3rd degree polynomial in x: $\det(x\mathtt{F}_1 + \mathtt{F}_2) = 0$;
choose the real roots x_i.;
return $x_i\mathtt{F}_1 + \mathtt{F}_2$;
end
end

and then we apply the rank constraint *shows*. This gives us three possible matrices and we choose the one which minimizes the reproduction error.

How to compute the focal lengths

- Use the Bougnoux formula[1]:

$$f_2^2 = -\frac{\mathbf{p}_1^\top [\mathbf{e}_1]_\times \tilde{\mathtt{I}}\mathtt{F}\mathbf{p}_2\mathbf{p}_2^\top \mathtt{F}^\top \mathbf{p}_1}{\mathbf{p}_1^\top [\mathbf{e}_1]_\times \tilde{\mathtt{I}}\mathtt{F}^\top \tilde{\mathtt{I}}\mathtt{F}\mathbf{p}_1} \tag{2.6}$$

- Transpose $\mathtt{F}$ and switch indices to get another focal length.
- SVD($\mathtt{F}$) $\rightarrow \mathbf{e}_1$

[1] Sylvain Bougnoux. "From Projective to Euclidean Space Under any Practical Situation, a Criticism of Self-Calibration.". In: *ICCV*. 1998.

How to compute the focal lengths
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Transpose F and switch indices to get another focal length.
SVD(F) → e1
[1] Sylvain Bougnoux. "From Projective to Euclidean Space Under any Practical Situation, a Criticism of Self-Calibration.". In: ICCV. 1998.

We can now compute the focal lengths using Bougnoux formula. Note that a computed focal length may be imaginary, if the right-hand side is negative due to errors. We will go back to this problem later. The epipole e in this formula is computed via SVD of the fundamental matrix.

Outline

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1 Introduction
2 Theroretical analysis
3 Experimental analysis
4 Improvements

I will now summarize the 4th chapter of the work, in which we use Algebraic Geometry to analyze the problem.

Derivation

$$\mathbb{Q}[F_{1,1}, F_{1,2}, F_{1,3}, F_{2,1}, F_{2,2}, F_{2,3}, F_{3,1}, F_{3,2}, F_{3,3}, f_1, f_2]$$

$$\Big\langle \det \mathrm{F} = 0, \tag{2.5}$$

$$\mathrm{K}_2^\mathsf{T}\mathrm{F}\mathrm{K}_1(\mathrm{K}_2^\mathsf{T}\mathrm{F}\mathrm{K}_1)^\mathsf{T}\mathrm{K}_2^\mathsf{T}\mathrm{F}\mathrm{K}_1 = \operatorname{trace}(\mathrm{K}_2^\mathsf{T}\mathrm{F}\mathrm{K}_1(\mathrm{K}_2^\mathsf{T}\mathrm{F}\mathrm{K}_1)^\mathsf{T})\mathrm{K}_2^\mathsf{T}\mathrm{F}\mathrm{K}_1 \Big\rangle \tag{2.2}$$

$\downarrow$ Eliminate f_1

$$\Big\langle f_2^2 = -\frac{F_{3,3}(F_{1,1}F_{2,3}F_{3,1} + F_{1,2}F_{2,3}F_{3,2} - F_{1,3}F_{2,1}F_{3,1} - F_{1,3}F_{2,2}F_{3,2})}{(F_{1,1}^2F_{1,3}F_{2,3} - F_{1,1}F_{1,3}^2F_{2,1} + F_{1,1}F_{2,1}F_{2,3}^2 + F_{1,2}^2F_{1,3}F_{2,3} - F_{1,2}F_{1,3}^2F_{2,2} + F_{1,2}F_{2,2}F_{2,3}^2 - F_{1,3}F_{2,1}^2F_{2,3} - F_{1,3}F_{2,2}^2F_{2,3})}, \tag{4.1}$$

$$f_2^2 = -\frac{F_{3,3}(F_{1,1}F_{3,1}F_{3,3} + F_{1,2}F_{3,2}F_{3,3} - F_{1,3}F_{3,1}^2 - F_{1,3}F_{3,2}^2)}{(F_{1,1}^2F_{1,3}F_{3,3} - F_{1,1}F_{1,3}^2F_{3,1} + F_{1,1}F_{2,1}F_{2,3}F_{3,3} + F_{1,2}^2F_{1,3}F_{3,3} - F_{1,2}F_{1,3}^2F_{3,2} + F_{1,2}F_{2,2}F_{2,3}F_{3,3} - F_{1,3}F_{2,1}F_{2,3}F_{3,1} - F_{1,3}F_{2,2}F_{2,3}F_{3,2})}, \tag{4.2}$$

$$f_2^2 = -\frac{F_{3,3}(F_{2,1}F_{3,1}F_{3,3} + F_{2,2}F_{3,2}F_{3,3} - F_{2,3}F_{3,1}^2 - F_{2,3}F_{3,2}^2)}{(F_{1,1}F_{1,3}F_{2,1}F_{3,3} - F_{1,1}F_{1,3}F_{2,3}F_{3,1} + F_{2,1}^2F_{2,3}F_{3,3} - F_{2,1}F_{2,3}^2F_{3,1} + F_{1,2}F_{1,3}F_{2,2}F_{3,3} - F_{1,2}F_{1,3}F_{2,3}F_{3,2} + F_{2,2}^2F_{2,3}F_{3,3} - F_{2,2}F_{2,3}^2f_{3,2})} \Big\rangle \tag{4.3}$$

Here we consider the set of constraints on a fundamental matrix, that is, the rank constraint and the demazure constraints, which express how the focal lengths depend on the matrix entries.

Derivation

$$\mathbb{Q}[F_{1,1}, F_{1,2}, F_{1,3}, F_{2,1}, F_{2,2}, F_{2,3}, F_{3,1}, F_{3,2}, F_{3,3}, f_1, f_2]$$

$$\Big\langle \det \mathrm{F} = 0, \Big\rangle \tag{2.5}$$

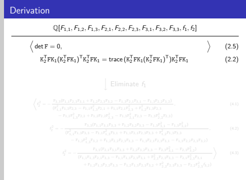
$$\mathrm{K}_2^T \mathrm{F} \mathrm{K}_1 (\mathrm{K}_2^T \mathrm{F} \mathrm{K}_1)^T \mathrm{K}_2^T \mathrm{F} \mathrm{K}_1 = \operatorname{trace}(\mathrm{K}_2^T \mathrm{F} \mathrm{K}_1 (\mathrm{K}_2^T \mathrm{F} \mathrm{K}_1)^T) \mathrm{K}_2^T \mathrm{F} \mathrm{K}_1 \tag{2.2}$$

$\downarrow$ Eliminate f_1

$$\Big\langle f_2^2 = -\frac{F_{3,3}(F_{1,1}F_{2,3}F_{3,1} + F_{1,2}F_{2,3}F_{3,2} - F_{1,3}F_{2,1}F_{3,1} - F_{1,3}F_{2,2}F_{3,2})}{(F_{1,1}^2F_{1,3}F_{2,3} - F_{1,1}F_{1,3}^2F_{2,1} + F_{1,1}F_{2,1}F_{2,3}^2 + F_{1,2}^2F_{1,3}F_{2,3} - F_{1,2}F_{1,3}^2F_{2,2} + F_{1,2}F_{2,2}F_{2,3}^2 - F_{1,3}F_{2,1}^2F_{2,3} - F_{1,3}F_{2,2}^2F_{2,3})}, \tag{4.1}$$

$$f_2^2 = -\frac{F_{3,3}(F_{1,1}F_{3,1}F_{3,3} + F_{1,2}F_{3,2}F_{3,3} - F_{1,3}F_{3,1}^2 - F_{1,3}F_{3,2}^2)}{(F_{1,1}^2F_{1,3}F_{3,3} - F_{1,1}F_{1,3}^2F_{3,1} + F_{1,1}F_{2,1}F_{2,3}F_{3,3} + F_{1,2}^2F_{1,3}F_{3,3} - F_{1,2}F_{1,3}^2F_{3,2} + F_{1,2}F_{2,2}F_{2,3}F_{3,3} - F_{1,3}F_{2,1}F_{2,3}F_{3,1} - F_{1,3}F_{2,2}F_{2,3}F_{3,2})}, \tag{4.2}$$

$$f_2^2 = -\frac{F_{3,3}(F_{2,1}F_{3,1}F_{3,3} + F_{2,2}F_{3,2}F_{3,3} - F_{2,3}F_{3,1}^2 - F_{2,3}F_{3,2}^2)}{(F_{1,1}F_{1,3}F_{2,1}F_{3,3} - F_{1,1}F_{1,3}F_{2,3}F_{3,1} + F_{2,1}^2F_{2,3}F_{3,3} - F_{2,1}F_{2,3}^2F_{3,1} + F_{1,2}F_{1,3}F_{2,2}F_{3,3} - F_{1,2}F_{1,3}F_{2,3}F_{3,2} + F_{2,2}^2F_{2,3}F_{3,3} - F_{2,2}F_{2,3}^2f_{3,2})} \Big\rangle \tag{4.3}$$



We formulate the constraints as matricial polynomial equations over the field of rationals with 11 variables. Nine of it are fundamental matrix entries and two are the calibration matrices entries, which in this case are just focal lengths. Using algebraic geometry techniques we can eliminate one focal length from this system,

Derivation

$$\mathbb{Q}[F_{1,1}, F_{1,2}, F_{1,3}, F_{2,1}, F_{2,2}, F_{2,3}, F_{3,1}, F_{3,2}, F_{3,3}, f_1, f_2]$$

$$\left\langle \det \mathrm{F} = 0, \right. \tag{2.5}$$

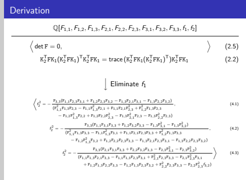
$$\left. \mathrm{K}_2^{\mathrm{T}}\mathrm{F}\mathrm{K}_1(\mathrm{K}_2^{\mathrm{T}}\mathrm{F}\mathrm{K}_1)^{\mathrm{T}}\mathrm{K}_2^{\mathrm{T}}\mathrm{F}\mathrm{K}_1 = \operatorname{trace}(\mathrm{K}_2^{\mathrm{T}}\mathrm{F}\mathrm{K}_1(\mathrm{K}_2^{\mathrm{T}}\mathrm{F}\mathrm{K}_1)^{\mathrm{T}})\mathrm{K}_2^{\mathrm{T}}\mathrm{F}\mathrm{K}_1 \right\rangle \tag{2.2}$$

$$\Big\downarrow \text{ Eliminate } f_1$$

$$\left\langle f_2^2 = -\frac{F_{3,3}(F_{1,1}F_{2,3}F_{3,1} + F_{1,2}F_{2,3}F_{3,2} - F_{1,3}F_{2,1}F_{3,1} - F_{1,3}F_{2,2}F_{3,2})}{(F_{1,1}^2F_{1,3}F_{2,3} - F_{1,1}F_{1,3}^2F_{2,1} + F_{1,1}F_{2,1}F_{2,3}^2 + F_{1,2}^2F_{1,3}F_{2,3} - F_{1,2}F_{1,3}^2F_{2,2} + F_{1,2}F_{2,2}F_{2,3}^2 - F_{1,3}F_{2,1}^2F_{2,3} - F_{1,3}F_{2,2}^2F_{2,3})}, \right. \tag{4.1}$$

$$f_2^2 = -\frac{F_{3,3}(F_{1,1}F_{3,1}F_{3,3} + F_{1,2}F_{3,2}F_{3,3} - F_{1,3}F_{3,1}^2 - F_{1,3}F_{3,2}^2)}{(F_{1,1}^2F_{1,3}F_{3,3} - F_{1,1}F_{1,3}^2F_{3,1} + F_{1,1}F_{2,1}F_{2,3}F_{3,3} + F_{1,2}^2F_{1,3}F_{3,3} - F_{1,2}F_{1,3}^2F_{3,2} + F_{1,2}F_{2,2}F_{2,3}F_{3,3} - F_{1,3}F_{2,1}F_{2,3}F_{3,1} - F_{1,3}F_{2,2}F_{2,3}F_{3,2})}, \tag{4.2}$$

$$\left. f_2^2 = -\frac{F_{3,3}(F_{2,1}F_{3,1}F_{3,3} + F_{2,2}F_{3,2}F_{3,3} - F_{2,3}F_{3,1}^2 - F_{2,3}F_{3,2}^2)}{(F_{1,1}F_{1,3}F_{2,1}F_{3,3} - F_{1,1}F_{1,3}F_{2,3}F_{3,1} + F_{2,1}^2F_{2,3}F_{3,3} - F_{2,1}F_{2,3}^2F_{3,1} + F_{1,2}F_{1,3}F_{2,2}F_{3,3} - F_{1,2}F_{1,3}F_{2,3}F_{3,2} + F_{2,2}^2F_{2,3}F_{3,3} - F_{2,2}F_{2,3}^2f_{3,2})} \right\rangle \tag{4.3}$$



We find that there are three algebraically independent constraints on another focal length, which gives us three formulae for computing the focal length. This interesting fact shows that there is a way to analyze the process purely algebraically. We also can derive algebraical formulae for the focal length computing, without the need to rely on SVD approach.

Outline

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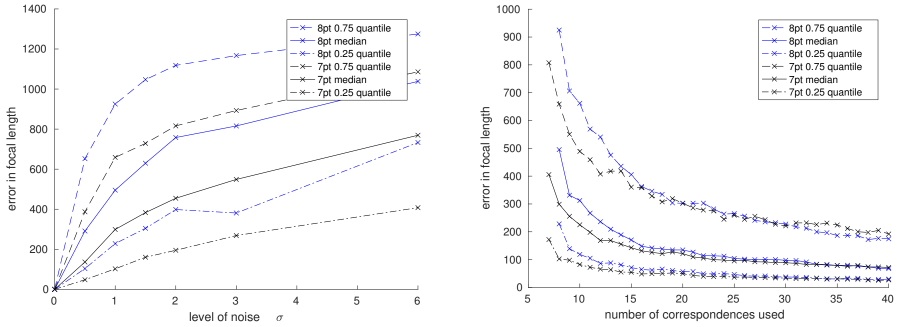
Introduction
Theroretical analysis
Experimental analysis
Improvements

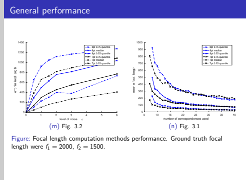
In the next part of the thesis, chapter 3 I analyze how does the computation of the focal lengths behave under image noise.

General performance



(m) Fig. 3.2 (n) Fig. 3.1

Figure: Focal length computation methods performance. Ground truth focal length were $f_1 = 2000$, $f_2 = 1500$.



The median error in a real-life situation may be something around 300 pixels *shows*, which makes some 20 percent error. On the graphs we can see two general trends of the computation. Firstly, the error grows with increasing noise in image measurements. Secondly, the error decreases with increasing number of used correspondences *shows*. This means the effect of noise is somehow cancelled when we use more correspondences. The median error can shrink three times when going from 7 to 40 correspondences *shows,shows*.

General performance

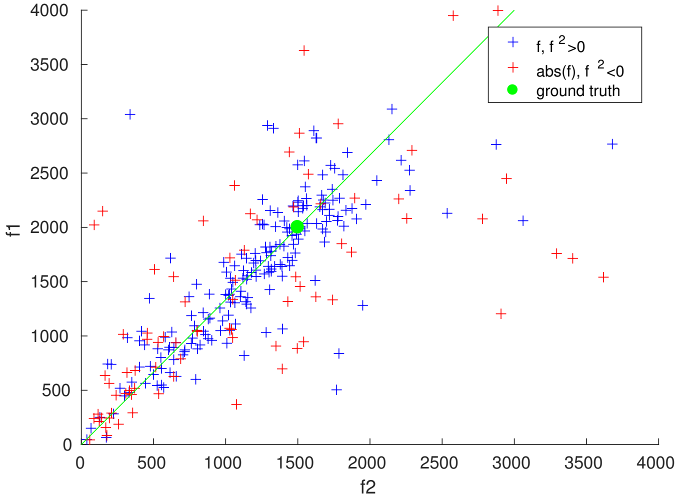
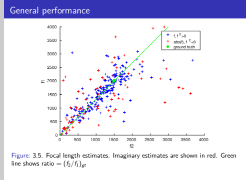


Figure: 3.5. Focal length estimates. Imaginary estimates are shown in red. Green line shows ratio $= (f_2/f_1)_{gt}$



Here you can see the estimates themselves, plotted as dots in the *shows,shows* space of focal lengths of the first and second camera. There are two interesting things to note about this graph. Firstly, the points tend to the line through the ground truth point *shows*. The line represents points with the right ratio of the focal lengths *shows*. This hints us at a possibility that the ratio can be estimated better, which we will use later to produce a more efficient solver. Secondly, imaginary focal lengths estimates, shown here in red in absolute value, surprisingly, also tend to the right ratio line, and also may be used for the ratio estimation. The absolute value of the imaginary estimates, however, cannot be used. At the graph you can see that they tend to center much at much lower value that the ground truth.

Outline

Outline
Introduction
Theroretical analysis
Experimental analysis
Improvements

Lets move on to more sophisticated procedures to recover the focal lengths, described chapter 5 of my work.

Algorithm 3: The cost function of Hartley and Silpa-Anan[a]

Data: Fundamental matrix $\mathtt{F}$, principal points $\mathbf{p}_1$, $\mathbf{p}_2$.
Result: Vector of costs $\mathbf{C}$
begin
From $\mathtt{F}$, $\mathbf{p}_1$, $\mathbf{p}_2$ compute f_1, f_2;

$C_{\mathtt{F}} \leftarrow$ Sampson error $\;S(\mathtt{F}) = \Sigma_i \frac{(\mathbf{x}_{2,i}^{\mathsf{T}}\mathtt{F}\mathbf{x}_{1,i})^2}{(\mathtt{F}\mathbf{x}_{1,i})_1^2 + (\mathtt{F}\mathbf{x}_{1,i})_2^2 + (\mathbf{x}_{2,i}^{\mathsf{T}}\mathtt{F})_1^2 + (\mathbf{x}_{2,i}^{\mathsf{T}}\mathtt{F})_2^2}$;

$C_{\mathbf{p}} \leftarrow w_p^2\|\mathbf{p}_1 - \bar{\mathbf{p}}_1\|^2 + w_p^2\|\mathbf{p}_2 - \bar{\mathbf{p}}_2\|$;

$C_f \leftarrow w_1^2(f_1^2 - \bar{f}_1^2)^2 + w_2^2(f_2^2 - \bar{f}_2^2)^2 + w_d^2(f_1^2 - f_2^2)^2 +$
$w_{z1}^2(f_{min}^2 - f_1^2)^2 + w_{z2}^2(f_{min}^2 - f_2^2)^2$;

return *Costs* $C_{\mathtt{F}}$, $C_{\mathbf{p}}$, C_f;
end

[a] Richard Hartley and Chanop Silpa-anan. "Reconstruction from two views using approximate calibration". In: *ACCV*. 2002.

The algorithm of Hartley and Silpa-Anan uses prior information about the focal lengths and principal points. The priors are shown with bars on them *shows p,f*. The algorithm of Hartley is an optimization procedure for finding the fundamental matrix.

Algorithm 4: The cost function of Hartley and Silpa-Anan[a]

Data: Fundamental matrix $\mathtt{F}$, principal points $\mathbf{p}_1$, $\mathbf{p}_2$.

Result: Vector of costs $\mathbf{C}$

begin

From $\mathtt{F}$, $\mathbf{p}_1$, $\mathbf{p}_2$ compute f_1, f_2;

$C_\mathtt{F} \leftarrow$ Sampson error $S(\mathtt{F}) = \Sigma_i \frac{(\mathbf{x}_{2,i}^\mathsf{T} \mathtt{F} \mathbf{x}_{1,i})^2}{(\mathtt{F}\mathbf{x}_{1,i})_1^2 + (\mathtt{F}\mathbf{x}_{1,i})_2^2 + (\mathbf{x}_{2,i}^\mathsf{T}\mathtt{F})_1^2 + (\mathbf{x}_{2,i}^\mathsf{T}\mathtt{F})_2^2}$;

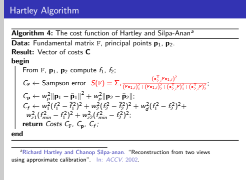
$C_\mathbf{p} \leftarrow w_p^2 \|\mathbf{p}_1 - \bar{\mathbf{p}}_1\|^2 + w_p^2 \|\mathbf{p}_2 - \bar{\mathbf{p}}_2\|$;

$C_f \leftarrow w_1^2(f_1^2 - \bar{f}_1^2)^2 + w_2^2(f_2^2 - \bar{f}_2^2)^2 + w_d^2(f_1^2 - f_2^2)^2 + w_{z1}^2(f_{min}^2 - f_1^2)^2 + w_{z2}^2(f_{min}^2 - f_2^2)^2$;

return *Costs* $C_\mathtt{F}$, $C_\mathbf{p}$, C_f;

end

[a] Richard Hartley and Chanop Silpa-anan. "Reconstruction from two views using approximate calibration". In: *ACCV*. 2002.



It uses Sampson error *shows* on the correspondences.

Algorithm 5: The cost function of Hartley and Silpa-Anan[a]

Data: Fundamental matrix $\mathtt{F}$, principal points $\mathbf{p}_1$, $\mathbf{p}_2$.

Result: Vector of costs $\mathbf{C}$

begin

From $\mathtt{F}$, $\mathbf{p}_1$, $\mathbf{p}_2$ compute f_1, f_2;

$C_{\mathtt{F}} \leftarrow$ Sampson error $S(\mathtt{F}) = \Sigma_i \frac{(\mathbf{x}_{2,i}^{\mathsf{T}} \mathtt{F} \mathbf{x}_{1,i})^2}{(\mathtt{F}\mathbf{x}_{1,i})_1^2 + (\mathtt{F}\mathbf{x}_{1,i})_2^2 + (\mathbf{x}_{2,i}^{\mathsf{T}} \mathtt{F})_1^2 + (\mathbf{x}_{2,i}^{\mathsf{T}} \mathtt{F})_2^2}$;

$C_{\mathbf{p}} \leftarrow w_p^2 \|\mathbf{p}_1 - \bar{\mathbf{p}}_1\|^2 + w_p^2 \|\mathbf{p}_2 - \bar{\mathbf{p}}_2\|$;

$C_f \leftarrow w_1^2 (f_1^2 - \bar{f}_1^2)^2 + w_2^2 (f_2^2 - \bar{f}_2^2)^2 + w_d^2 (f_1^2 - f_2^2)^2 + w_{z1}^2 (f_{min}^2 - f_1^2)^2 + w_{z2}^2 (f_{min}^2 - f_2^2)^2$;

return *Costs* $C_{\mathtt{F}}$, $C_{\mathbf{p}}$, C_f;

end

[a] Richard Hartley and Chanop Silpa-anan. "Reconstruction from two views using approximate calibration". In: *ACCV*. 2002.

Hartley Algorithm
Algorithm 5: The cost function of Hartley and Silpa-Anan
Data: Fundamental matrix F, principal points p1, p2.
Result: Vector of costs C
begin
From F, p1, p2 compute f1, f2;
end
Richard Hartley and Chanop Silpa-anan. "Reconstruction from two views using approximate calibration". In: ACCV. 2002.

Then there are terms which express distance of principal points *shows* and focal lengths *shows* to priors.

Hartley Algorithm

Algorithm 6: The cost function of Hartley and Silpa-Anan[a]

Data: Fundamental matrix $\mathtt{F}$, principal points $\mathbf{p}_1$, $\mathbf{p}_2$.
Result: Vector of costs $\mathbf{C}$
begin
From $\mathtt{F}$, $\mathbf{p}_1$, $\mathbf{p}_2$ compute f_1, f_2;
$C_{\mathtt{F}} \leftarrow$ Sampson error $S(\mathtt{F}) = \Sigma_i \frac{(\mathbf{x}_{2,i}^{\top}\mathtt{F}\mathbf{x}_{1,i})^2}{(\mathtt{F}\mathbf{x}_{1,i})_1^2+(\mathtt{F}\mathbf{x}_{1,i})_2^2+(\mathbf{x}_{2,i}^{\top}\mathtt{F})_1^2+(\mathbf{x}_{2,i}^{\top}\mathtt{F})_2^2}$;
$C_{\mathbf{p}} \leftarrow w_p^2\|\mathbf{p}_1 - \bar{\mathbf{p}}_1\|^2 + w_p^2\|\mathbf{p}_2 - \bar{\mathbf{p}}_2\|$;
$C_f \leftarrow w_1^2(f_1^2 - \bar{f}_1^2)^2 + w_2^2(f_2^2 - \bar{f}_2^2)^2 + w_d^2(f_1^2 - f_2^2)^2 +$
$w_{z1}^2(f_{min}^2 - f_1^2)^2 + w_{z2}^2(f_{min}^2 - f_2^2)^2$;
return *Costs* $C_{\mathtt{F}}$, $C_{\mathbf{p}}$, C_f;
end

[a] Richard Hartley and Chanop Silpa-anan. "Reconstruction from two views using approximate calibration". In: *ACCV*. 2002.

Hartley Algorithm
Algorithm 6: The cost function of Hartley and Silpa-Anan

In addition, the third term here *shows* serves to drive the focal lengths close to each other,

Hartley Algorithm

Algorithm 7: The cost function of Hartley and Silpa-Anan[a]

Data: Fundamental matrix $\mathtt{F}$, principal points $\mathbf{p}_1$, $\mathbf{p}_2$.
Result: Vector of costs $\mathbf{C}$
begin

From $\mathtt{F}$, $\mathbf{p}_1$, $\mathbf{p}_2$ compute f_1, f_2;

$C_{\mathtt{F}} \leftarrow$ Sampson error $S(\mathtt{F}) = \Sigma_i \frac{(\mathbf{x}_{2,i}^{\top}\mathtt{F}\mathbf{x}_{1,i})^2}{(\mathtt{F}\mathbf{x}_{1,i})_1^2+(\mathtt{F}\mathbf{x}_{1,i})_2^2+(\mathbf{x}_{2,i}^{\top}\mathtt{F})_1^2+(\mathbf{x}_{2,i}^{\top}\mathtt{F})_2^2}$;

$C_{\mathbf{p}} \leftarrow w_p^2\|\mathbf{p}_1 - \bar{\mathbf{p}}_1\|^2 + w_p^2\|\mathbf{p}_2 - \bar{\mathbf{p}}_2\|$;

$C_f \leftarrow w_1^2(f_1^2 - \bar{f}_1^2)^2 + w_2^2(f_2^2 - \bar{f}_2^2)^2 + w_d^2(f_1^2 - f_2^2)^2 +$
$w_{z1}^2(f_{min}^2 - f_1^2)^2 + w_{z2}^2(f_{min}^2 - f_2^2)^2$;

return *Costs* $C_{\mathtt{F}}$, $C_{\mathbf{p}}$, C_f;

end

[a] Richard Hartley and Chanop Silpa-anan. "Reconstruction from two views using approximate calibration". In: *ACCV*. 2002.

and the fourth and fifth terms *shows* should prevent the focals lengths from becoming imaginary. Because of the third term it can be said that the algorithm assumes the same, or nearly the same focal lengths.

Our modification

Algorithm 8: Our modification to the optimization of Hartley and Silpa-Anan[a]

Data: Point correspondences $\mathbf{x}_1$, $\mathbf{x}_2$, prior focal lengths $\bar{f}_1$, $\bar{f}_2$,prior principal points $\bar{\mathbf{p}}_1$,$\bar{\mathbf{p}}_2$, minimal focal length f_{min}.

Result: Fundamental matrix $\mathtt{F}$

begin

Compute a $\mathtt{F}$ from $\mathbf{x}_1$, $\mathbf{x}_2$ using the 7pt algorithm;

Estimate the focal lengths ratio $r = f_2/f_1$ from $\mathtt{F}$;

Optimize modified H-S cost;

$C_f = w_1^2(f_1^2 - \bar{f}_1^2)^2 + w_2^2(f_2^2 - \bar{f}_2^2)^2 + w_d^2((rf_1)^2 - f_2^2)^2 + w_{z1}^2(f_{min}^2 - f_1^2)^2 + w_{z2}^2(f_{min}^2 - f_2^2)^2$;

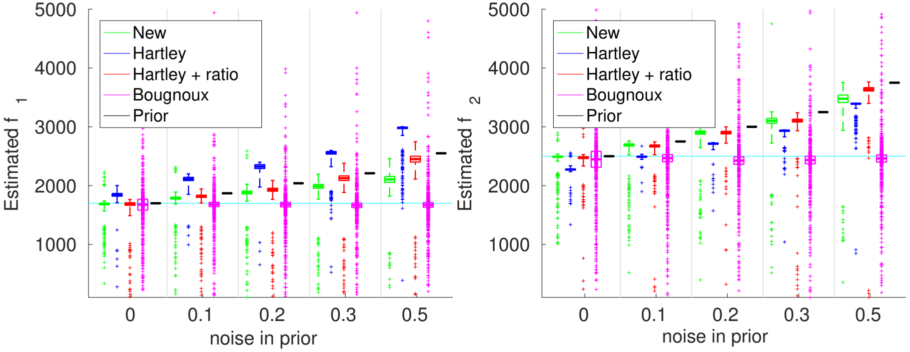
return $\mathtt{F}$;

end

[a] Richard Hartley and Chanop Silpa-anan. "Reconstruction from two views using approximate calibration". In: *ACCV*. 2002.

One possible improvement to this is to use the computed ratio of the focal lengths in the optimization, and this is what I do here. We modify the cost function so that the ratio computed from the correspondences is used instead of plain one *shows*.

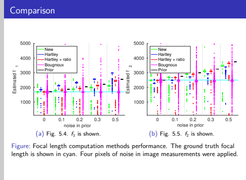
Comparison



(a) Fig. 5.4. f_1 is shown.

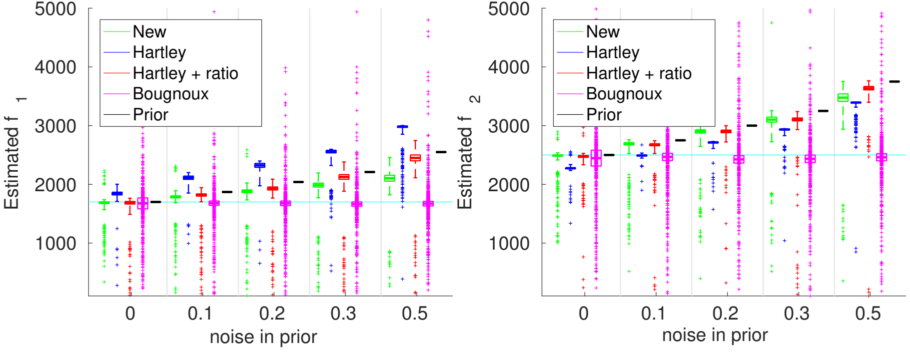
(b) Fig. 5.5. f_2 is shown.

Figure: Focal length computation methods performance. The ground truth focal length is shown in cyan. Four pixels of noise in image measurements were applied.



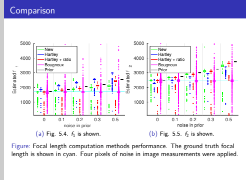
And here are the results which we get. The ground truth is this *shows* line, the Bougnoux formula is shown in pink, and the prior increases along the x axis *shows the axis*, so here *shows the first bin* the prior is equal to the ground truth, an here *shows the last bin* it is 50 percent bigger. For this level of noise, 4 pixels, we can see that the Bougnoux approach is less stable in that it has big variance. However, it's median is centered roughly at ground truth *shows*. The Hartley's method, which is shown in blue, is shifted to the prior *shows*, but, on the contrary, has almost no variance. Our modification to the Hartley's method is shown in red. It differs from the original method in an interesting way. If you look at the first bins, to the left, where the smaller focal length is shown *shows the second and third bin*, the Hartley's original method tends to overestimate it, and to the right we can see that the bigger focal length, on the contrary, is underestimated. This is because, as I have said, the original method assumes that the focal lengths are the same.

Comparison



(a) Fig. 5.4. f_1 is shown. (b) Fig. 5.5. f_2 is shown.

Figure: Focal length computation methods performance. The ground truth focal length is shown in cyan. Four pixels of noise in image measurements were applied.



Our modification doesn't suffer from this behaviour *shows*. However, it has a bigger variance, which is because the ratio which we compute from the correspondences introduces some unstability. Now, remember, that the noise is bigger in this experiment, we have 4 pixels of noise. For more mild levels of noise, Bougnoux formula achieves better results. The conclusion from this graph is that the prior information can be used to efficiently lower the variance introduced by higher levels of noise. The graph features also an algorithm of Chandraker, which we optimize using a procedure by Zuzana Kukelova, CMP CTU. I believe that the graph is a good summary of what is possible with current methods of computing focal lengths.

Summary

- We have good methods for computing focal lengths from images.
- Even more can be done to improve stability against noise or in degenerate situations.

Summary
- We have good methods for computing focal lengths from images.
- Even more can be done to improve stability against noise or in degenerate situations.

That was my message. In general, the analysis of my thesis shows that this method of computing focal lengths, if used properly, may be robust and efficient enough to get usable results. We hope to continue research on this subject and show that indeed this method can be used in practical situations, say, industrial applications. Its big benefit is that the cameras used may have different focal lengths, which indeed happens in many real-world situations. We believe that this method of computing the focal lengths directly from two images has better potential than the methods currently used. Thank you. Your questions.