Robust Focal Length Computation

Bachelor Thesis

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Department of Cybernetics

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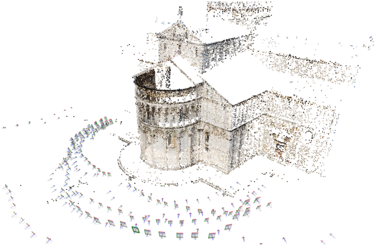
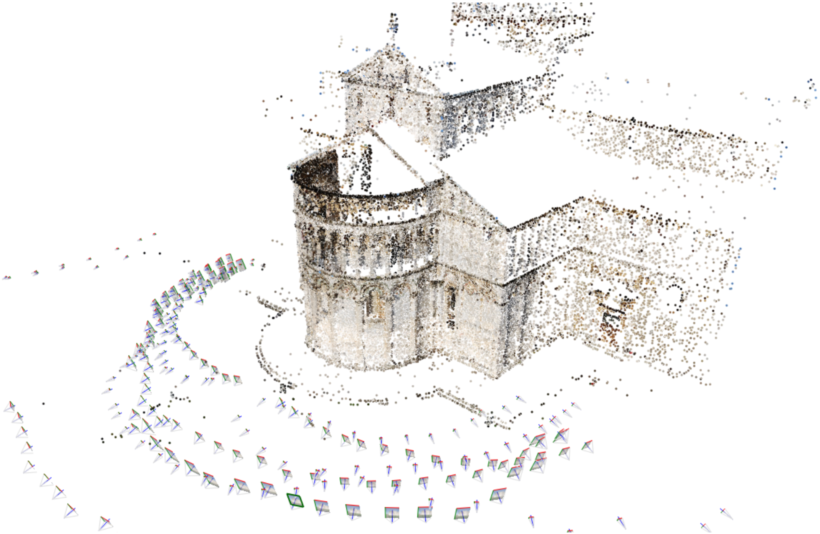
Outline

3D reconstruction

↓



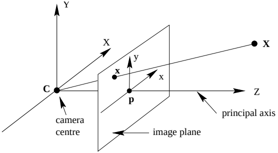
Reconstructed model

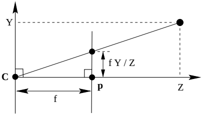


We need focal lengths

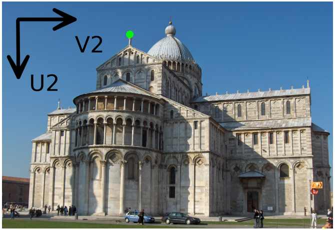
↓

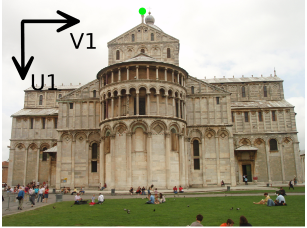
focal lengths f_1, f_2



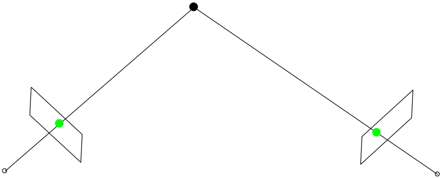


Correspondences





$$\mathbf{x}_2 = \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix}$$



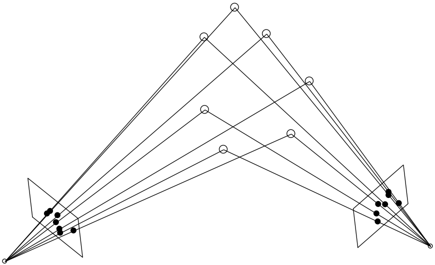
$$\mathbf{x}_1 = \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}$$

$$\downarrow$$

focal lengths f_1, f_2

Fundamental matrix

$$\mathbf{x}_2 = \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix}$$



$$\mathbf{x}_1 = \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}$$

$$\downarrow$$

focal lengths f_1, f_2

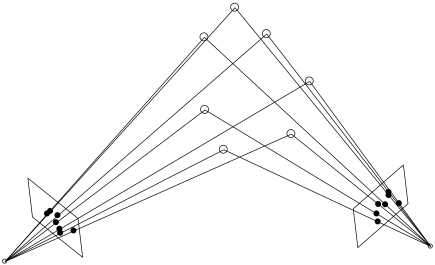
Fundamental matrix $\mathrm{F} \in \mathbb{R}^{3\times3}$

$$\mathbf{x}_2^\top \mathrm{F} \mathbf{x}_1 = 0 \tag{2.4}$$

$$\det \mathrm{F} = 0 \tag{2.5}$$

Fundamental matrix

$$\mathbf{x}_2 = \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix}$$



$$\mathbf{x}_1 = \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}$$

$$\downarrow$$

focal lengths f_1, f_2

Fundamental matrix $\mathtt{F} \in \mathbb{R}^{3\times 3}$

$$\mathbf{x}_2^\top \mathtt{F} \mathbf{x}_1 = 0 \tag{2.4}$$

$$\det \mathtt{F} = 0 \tag{2.5}$$

How to compute the fundamental matrix

Algorithm 1: 7pt

Data: $n \geq 7$ corresponding image points $\mathbf{x}_{1,i}$, $\mathbf{x}_{2,i}$

Result: Fundamental matrix $\mathtt{F}$

begin

matrix $\mathtt{B} \in \mathbb{R}^{n \times 9}$ with columns $\mathbf{b}_i = \text{vec}(\mathbf{x}_{2,i} \otimes \mathbf{x}_{1,i})$;

SVD($\mathtt{B}$) $\rightarrow$ singular vectors $\mathbf{f}_1, \mathbf{f}_2$ for the two smallest singular values;

Reshape $\mathbf{f}_1, \mathbf{f}_2 \rightarrow \mathtt{F}_1, \mathtt{F}_2 \in \mathbb{R}^{3 \times 3}$;

if $\det(\mathtt{F}_2) = 0$ **then**

return $\mathtt{F}_2$;

else

Solve the 3rd degree polynomial in x: $\det(x\mathtt{F}_1 + \mathtt{F}_2) = 0$;

choose the real roots x_i.;

return $x_i\mathtt{F}_1 + \mathtt{F}_2$;

end

end

How to compute the fundamental matrix

Algorithm 2: 7pt

Data: $n \geq 7$ corresponding image points $\mathbf{x}_{1,i}$, $\mathbf{x}_{2,i}$
Result: Fundamental matrix $\mathtt{F}$
begin
  matrix $\mathtt{B} \in \mathbb{R}^{n\times 9}$ with columns $\mathbf{b}_i = \mathrm{vec}(\mathbf{x}_{2,i} \otimes \mathbf{x}_{1,i})$;
  SVD($\mathtt{B}$) $\rightarrow$ singular vectors $\mathbf{f}_1, \mathbf{f}_2$ for the two smallest singular values;
  Reshape $\mathbf{f}_1, \mathbf{f}_2 \rightarrow \mathtt{F}_1, \mathtt{F}_2 \in \mathbb{R}^{3\times 3}$;
  if $\det(\mathtt{F}_2) = 0$ **then**
    return $\mathtt{F}_2$;
  else
    Solve the 3rd degree polynomial in x: $\det(x\mathtt{F}_1 + \mathtt{F}_2) = 0$;
    choose the real roots x_i.;
    return $x_i\mathtt{F}_1 + \mathtt{F}_2$;
  end
end

How to compute the focal lengths

- Use the Bougnoux formula[1]:

$$f_2^2 = -\frac{\mathbf{p}_1^\top [\mathbf{e}_1]_\times \tilde{\mathtt{I}}\mathtt{F}\mathbf{p}_2\mathbf{p}_2^\top \mathtt{F}^\top \mathbf{p}_1}{\mathbf{p}_1^\top [\mathbf{e}_1]_\times \tilde{\mathtt{I}}\mathtt{F}^\top \tilde{\mathtt{I}}\mathtt{F}\mathbf{p}_1} \tag{2.6}$$

- Transpose $\mathtt{F}$ and switch indices to get another focal length.
- $\mathrm{SVD}(\mathtt{F}) \rightarrow \mathbf{e}_1$

[1] Sylvain Bougnoux. "From Projective to Euclidean Space Under any Practical Situation, a Criticism of Self-Calibration.". In: *ICCV*. 1998.

Outline

Derivation

$$\mathbb{Q}[F_{1,1}, F_{1,2}, F_{1,3}, F_{2,1}, F_{2,2}, F_{2,3}, F_{3,1}, F_{3,2}, F_{3,3}, f_1, f_2]$$

$$\Big\langle \det \mathtt{F} = 0, \qquad (2.5)$$

$$\mathtt{K}_2^\mathsf{T}\mathtt{F}\mathtt{K}_1(\mathtt{K}_2^\mathsf{T}\mathtt{F}\mathtt{K}_1)^\mathsf{T}\mathtt{K}_2^\mathsf{T}\mathtt{F}\mathtt{K}_1 = \operatorname{trace}(\mathtt{K}_2^\mathsf{T}\mathtt{F}\mathtt{K}_1(\mathtt{K}_2^\mathsf{T}\mathtt{F}\mathtt{K}_1)^\mathsf{T})\mathtt{K}_2^\mathsf{T}\mathtt{F}\mathtt{K}_1 \Big\rangle \qquad (2.2)$$

$$\Big\downarrow \text{Eliminate } f_1$$

$$\Big\langle f_2^2 = -\frac{F_{3,3}(F_{1,1}F_{2,3}F_{3,1} + F_{1,2}F_{2,3}F_{3,2} - F_{1,3}F_{2,1}F_{3,1} - F_{1,3}F_{2,2}F_{3,2})}{\begin{array}{c}(F_{1,1}^2F_{1,3}F_{2,3} - F_{1,1}F_{1,3}^2F_{2,1} + F_{1,1}F_{2,1}F_{2,3}^2 + F_{1,2}^2F_{1,3}F_{2,3} \\ - F_{1,2}F_{1,3}^2F_{2,2} + F_{1,2}F_{2,2}F_{2,3}^2 - F_{1,3}F_{2,1}^2F_{2,3} - F_{1,3}F_{2,2}^2F_{2,3})\end{array}}, \qquad (4.1)$$

$$f_2^2 = -\frac{F_{3,3}(F_{1,1}F_{3,1}F_{3,3} + F_{1,2}F_{3,2}F_{3,3} - F_{1,3}F_{3,1}^2 - F_{1,3}F_{3,2}^2)}{\begin{array}{c}(F_{1,1}^2F_{1,3}F_{3,3} - F_{1,1}F_{1,3}^2F_{3,1} + F_{1,1}F_{2,1}F_{2,3}F_{3,3} + F_{1,2}^2F_{1,3}F_{3,3} \\ - F_{1,2}F_{1,3}^2F_{3,2} + F_{1,2}F_{2,2}F_{2,3}F_{3,3} - F_{1,3}F_{2,1}F_{2,3}F_{3,1} - F_{1,3}F_{2,2}F_{2,3}F_{3,2})\end{array}}, \qquad (4.2)$$

$$f_2^2 = -\frac{F_{3,3}(F_{2,1}F_{3,1}F_{3,3} + F_{2,2}F_{3,2}F_{3,3} - F_{2,3}F_{3,1}^2 - F_{2,3}F_{3,2}^2)}{\begin{array}{c}(F_{1,1}F_{1,3}F_{2,1}F_{3,3} - F_{1,1}F_{1,3}F_{2,3}F_{3,1} + F_{2,1}^2F_{2,3}F_{3,3} - F_{2,1}F_{2,3}^2F_{3,1} \\ + F_{1,2}F_{1,3}F_{2,2}F_{3,3} - F_{1,2}F_{1,3}F_{2,3}F_{3,2} + F_{2,2}^2F_{2,3}F_{3,3} - F_{2,2}F_{2,3}^2f_{3,2})\end{array}} \Big\rangle \qquad (4.3)$$

Derivation

$$\mathbb{Q}[F_{1,1}, F_{1,2}, F_{1,3}, F_{2,1}, F_{2,2}, F_{2,3}, F_{3,1}, F_{3,2}, F_{3,3}, f_1, f_2]$$

$$\Big\langle \det \mathtt{F} = 0, \qquad \Big\rangle \tag{2.5}$$

$$\mathtt{K}_2^\mathsf{T}\mathtt{F}\mathtt{K}_1(\mathtt{K}_2^\mathsf{T}\mathtt{F}\mathtt{K}_1)^\mathsf{T}\mathtt{K}_2^\mathsf{T}\mathtt{F}\mathtt{K}_1 = \operatorname{trace}(\mathtt{K}_2^\mathsf{T}\mathtt{F}\mathtt{K}_1(\mathtt{K}_2^\mathsf{T}\mathtt{F}\mathtt{K}_1)^\mathsf{T})\mathtt{K}_2^\mathsf{T}\mathtt{F}\mathtt{K}_1 \tag{2.2}$$

$$\Big\downarrow \text{ Eliminate } f_1$$

$$\Big\langle f_2^2 = -\frac{F_{3,3}(F_{1,1}F_{2,3}F_{3,1} + F_{1,2}F_{2,3}F_{3,2} - F_{1,3}F_{2,1}F_{3,1} - F_{1,3}F_{2,2}F_{3,2})}{\begin{gathered}(F_{1,1}^2F_{1,3}F_{2,3} - F_{1,1}F_{1,3}^2F_{2,1} + F_{1,1}F_{2,1}F_{2,3}^2 + F_{1,2}^2F_{1,3}F_{2,3} \\ - F_{1,2}F_{1,3}^2F_{2,2} + F_{1,2}F_{2,2}F_{2,3}^2 - F_{1,3}F_{2,1}^2F_{2,3} - F_{1,3}F_{2,2}^2F_{2,3})\end{gathered}}, \tag{4.1}$$

$$f_2^2 = -\frac{F_{3,3}(F_{1,1}F_{3,1}F_{3,3} + F_{1,2}F_{3,2}F_{3,3} - F_{1,3}F_{3,1}^2 - F_{1,3}F_{3,2}^2)}{\begin{gathered}(F_{1,1}^2F_{1,3}F_{3,3} - F_{1,1}F_{1,3}^2F_{3,1} + F_{1,1}F_{2,1}F_{2,3}F_{3,3} + F_{1,2}^2F_{1,3}F_{3,3} \\ - F_{1,2}F_{1,3}^2F_{3,2} + F_{1,2}F_{2,2}F_{2,3}F_{3,3} - F_{1,3}F_{2,1}F_{2,3}F_{3,1} - F_{1,3}F_{2,2}F_{2,3}F_{3,2})\end{gathered}}, \tag{4.2}$$

$$f_2^2 = -\frac{F_{3,3}(F_{2,1}F_{3,1}F_{3,3} + F_{2,2}F_{3,2}F_{3,3} - F_{2,3}F_{3,1}^2 - F_{2,3}F_{3,2}^2)}{\begin{gathered}(F_{1,1}F_{1,3}F_{2,1}F_{3,3} - F_{1,1}F_{1,3}F_{2,3}F_{3,1} + F_{2,1}^2F_{2,3}F_{3,3} - F_{2,1}F_{2,3}^2F_{3,1} \\ + F_{1,2}F_{1,3}F_{2,2}F_{3,3} - F_{1,2}F_{1,3}F_{2,3}F_{3,2} + F_{2,2}^2F_{2,3}F_{3,3} - F_{2,2}F_{2,3}^2f_{3,2})\end{gathered}} \Big\rangle \tag{4.3}$$

$$\mathbb{Q}[F_{1,1}, F_{1,2}, F_{1,3}, F_{2,1}, F_{2,2}, F_{2,3}, F_{3,1}, F_{3,2}, F_{3,3}, f_1, f_2]$$

$$\Big\langle \det \mathtt{F} = 0, \qquad\qquad \Big\rangle \tag{2.5}$$

$$\mathtt{K}_2^\top \mathtt{F}\mathtt{K}_1 (\mathtt{K}_2^\top \mathtt{F}\mathtt{K}_1)^\top \mathtt{K}_2^\top \mathtt{F}\mathtt{K}_1 = \operatorname{trace}(\mathtt{K}_2^\top \mathtt{F}\mathtt{K}_1 (\mathtt{K}_2^\top \mathtt{F}\mathtt{K}_1)^\top) \mathtt{K}_2^\top \mathtt{F}\mathtt{K}_1 \tag{2.2}$$

$$\Big\downarrow \text{Eliminate } f_1$$

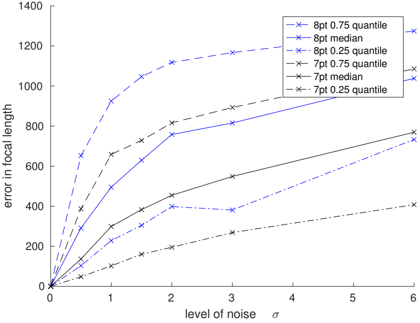
$$\Big\langle f_2^2 = -\frac{F_{3,3}(F_{1,1}F_{2,3}F_{3,1} + F_{1,2}F_{2,3}F_{3,2} - F_{1,3}F_{2,1}F_{3,1} - F_{1,3}F_{2,2}F_{3,2})}{\begin{array}{l}(F_{1,1}^2F_{1,3}F_{2,3} - F_{1,1}F_{1,3}^2F_{2,1} + F_{1,1}F_{2,1}F_{2,3}^2 + F_{1,2}^2F_{1,3}F_{2,3} \\ \quad - F_{1,2}F_{1,3}^2F_{2,2} + F_{1,2}F_{2,2}F_{2,3}^2 - F_{1,3}F_{2,1}^2F_{2,3} - F_{1,3}F_{2,2}^2F_{2,3})\end{array}}, \tag{4.1}$$

$$f_2^2 = -\frac{F_{3,3}(F_{1,1}F_{3,1}F_{3,3} + F_{1,2}F_{3,2}F_{3,3} - F_{1,3}F_{3,1}^2 - F_{1,3}F_{3,2}^2)}{\begin{array}{l}(F_{1,1}^2F_{1,3}F_{3,3} - F_{1,1}F_{1,3}^2F_{3,1} + F_{1,1}F_{2,1}F_{2,3}F_{3,3} + F_{1,2}^2F_{1,3}F_{3,3} \\ \quad - F_{1,2}F_{1,3}^2F_{3,2} + F_{1,2}F_{2,2}F_{2,3}F_{3,3} - F_{1,3}F_{2,1}F_{2,3}F_{3,1} - F_{1,3}F_{2,2}F_{2,3}F_{3,2})\end{array}}, \tag{4.2}$$

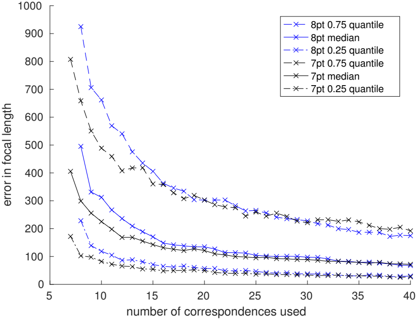
$$f_2^2 = -\frac{F_{3,3}(F_{2,1}F_{3,1}F_{3,3} + F_{2,2}F_{3,2}F_{3,3} - F_{2,3}F_{3,1}^2 - F_{2,3}F_{3,2}^2)}{\begin{array}{l}(F_{1,1}F_{1,3}F_{2,1}F_{3,3} - F_{1,1}F_{1,3}F_{2,3}F_{3,1} + F_{2,1}^2F_{2,3}F_{3,3} - F_{2,1}F_{2,3}^2F_{3,1} \\ \quad + F_{1,2}F_{1,3}F_{2,2}F_{3,3} - F_{1,2}F_{1,3}F_{2,3}F_{3,2} + F_{2,2}^2F_{2,3}F_{3,3} - F_{2,2}F_{2,3}^2 f_{3,2})\end{array}} \Big\rangle \tag{4.3}$$

Outline

General performance



(m) Fig. 3.2



(n) Fig. 3.1

Figure: Focal length computation methods performance. Ground truth focal length were $f_1 = 2000$, $f_2 = 1500$.

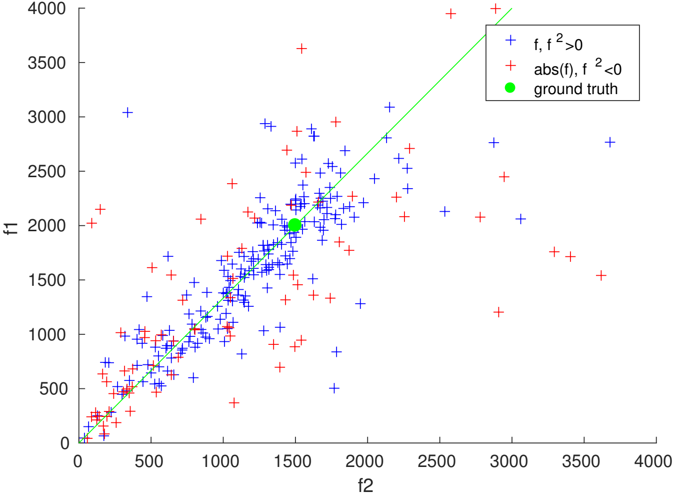


Figure: 3.5. Focal length estimates. Imaginary estimates are shown in red. Green line shows ratio $= (f_2/f_1)_{gt}$

Outline

Hartley Algorithm

Algorithm 3: The cost function of Hartley and Silpa-Anan[a]

Data: Fundamental matrix $\mathtt{F}$, principal points $\mathbf{p}_1$, $\mathbf{p}_2$.

Result: Vector of costs $\mathbf{C}$

begin

From $\mathtt{F}$, $\mathbf{p}_1$, $\mathbf{p}_2$ compute f_1, f_2;

$C_{\mathtt{F}} \leftarrow$ Sampson error $S(\mathtt{F}) = \sum_i \frac{(\mathbf{x}_{2,i}^{\top}\mathtt{F}\mathbf{x}_{1,i})^2}{(\mathtt{F}\mathbf{x}_{1,i})_1^2 + (\mathtt{F}\mathbf{x}_{1,i})_2^2 + (\mathbf{x}_{2,i}^{\top}\mathtt{F})_1^2 + (\mathbf{x}_{2,i}^{\top}\mathtt{F})_2^2}$;

$C_{\mathbf{p}} \leftarrow w_p^2 \|\mathbf{p}_1 - \bar{\mathbf{p}}_1\|^2 + w_p^2 \|\mathbf{p}_2 - \bar{\mathbf{p}}_2\|$;

$C_f \leftarrow w_1^2 (f_1^2 - \bar{f}_1^2)^2 + w_2^2 (f_2^2 - \bar{f}_2^2)^2 + w_d^2 (f_1^2 - f_2^2)^2 + w_{z1}^2 (f_{min}^2 - f_1^2)^2 + w_{z2}^2 (f_{min}^2 - f_2^2)^2$;

return *Costs* $C_{\mathtt{F}}$, $C_{\mathbf{p}}$, C_f;

end

[a] Richard Hartley and Chanop Silpa-anan. "Reconstruction from two views using approximate calibration". In: *ACCV*. 2002.

Hartley Algorithm

Algorithm 4: The cost function of Hartley and Silpa-Anan[a]

Data: Fundamental matrix $\mathtt{F}$, principal points $\mathbf{p}_1$, $\mathbf{p}_2$.

Result: Vector of costs $\mathbf{C}$

begin

From $\mathtt{F}$, $\mathbf{p}_1$, $\mathbf{p}_2$ compute f_1, f_2;

$C_{\mathtt{F}} \leftarrow$ Sampson error $S(\mathtt{F}) = \Sigma_i \frac{(\mathbf{x}_{2,i}^\top \mathtt{F}\mathbf{x}_{1,i})^2}{(\mathtt{F}\mathbf{x}_{1,i})_1^2 + (\mathtt{F}\mathbf{x}_{1,i})_2^2 + (\mathbf{x}_{2,i}^\top \mathtt{F})_1^2 + (\mathbf{x}_{2,i}^\top \mathtt{F})_2^2}$;

$C_{\mathbf{p}} \leftarrow w_p^2 \|\mathbf{p}_1 - \bar{\mathbf{p}}_1\|^2 + w_p^2 \|\mathbf{p}_2 - \bar{\mathbf{p}}_2\|$;

$C_f \leftarrow w_1^2(f_1^2 - \bar{f}_1^2)^2 + w_2^2(f_2^2 - \bar{f}_2^2)^2 + w_d^2(f_1^2 - f_2^2)^2 +$
$w_{z1}^2(f_{min}^2 - f_1^2)^2 + w_{z2}^2(f_{min}^2 - f_2^2)^2$;

return *Costs* $C_{\mathtt{F}}$, $C_{\mathbf{p}}$, C_f;

end

[a] Richard Hartley and Chanop Silpa-anan. "Reconstruction from two views using approximate calibration". In: *ACCV*. 2002.

Hartley Algorithm

Algorithm 5: The cost function of Hartley and Silpa-Anan[a]

Data: Fundamental matrix $\mathtt{F}$, principal points $\mathbf{p}_1$, $\mathbf{p}_2$.

Result: Vector of costs $\mathbf{C}$

begin

From $\mathtt{F}$, $\mathbf{p}_1$, $\mathbf{p}_2$ compute f_1, f_2;

$C_{\mathtt{F}} \leftarrow$ Sampson error $S(\mathtt{F}) = \Sigma_i \frac{(\mathbf{x}_{2,i}^{\top}\mathtt{F}\mathbf{x}_{1,i})^2}{(\mathtt{F}\mathbf{x}_{1,i})_1^2+(\mathtt{F}\mathbf{x}_{1,i})_2^2+(\mathbf{x}_{2,i}^{\top}\mathtt{F})_1^2+(\mathbf{x}_{2,i}^{\top}\mathtt{F})_2^2}$;

$C_{\mathbf{p}} \leftarrow w_p^2\|\mathbf{p}_1 - \bar{\mathbf{p}}_1\|^2 + w_p^2\|\mathbf{p}_2 - \bar{\mathbf{p}}_2\|$;

$C_f \leftarrow w_1^2(f_1^2 - \bar{f}_1^2)^2 + w_2^2(f_2^2 - \bar{f}_2^2)^2 + w_d^2(f_1^2 - f_2^2)^2 +$
$w_{z1}^2(f_{min}^2 - f_1^2)^2 + w_{z2}^2(f_{min}^2 - f_2^2)^2$;

return *Costs* $C_{\mathtt{F}}$, $C_{\mathbf{p}}$, C_f;

end

[a] Richard Hartley and Chanop Silpa-anan. "Reconstruction from two views using approximate calibration". In: *ACCV*. 2002.

Hartley Algorithm

Algorithm 6: The cost function of Hartley and Silpa-Anan[a]

Data: Fundamental matrix $\mathtt{F}$, principal points $\mathbf{p}_1$, $\mathbf{p}_2$.

Result: Vector of costs $\mathbf{C}$

begin

From $\mathtt{F}$, $\mathbf{p}_1$, $\mathbf{p}_2$ compute f_1, f_2;

$C_{\mathtt{F}} \leftarrow$ Sampson error $S(\mathtt{F}) = \Sigma_i \frac{(\mathbf{x}_{2,i}^{\top}\mathtt{F}\mathbf{x}_{1,i})^2}{(\mathtt{F}\mathbf{x}_{1,i})_1^2+(\mathtt{F}\mathbf{x}_{1,i})_2^2+(\mathbf{x}_{2,i}^{\top}\mathtt{F})_1^2+(\mathbf{x}_{2,i}^{\top}\mathtt{F})_2^2}$;

$C_{\mathbf{p}} \leftarrow w_p^2\|\mathbf{p}_1 - \bar{\mathbf{p}}_1\|^2 + w_p^2\|\mathbf{p}_2 - \bar{\mathbf{p}}_2\|$;

$C_f \leftarrow w_1^2(f_1^2 - \bar{f}_1^2)^2 + w_2^2(f_2^2 - \bar{f}_2^2)^2 + w_d^2(f_1^2 - f_2^2)^2 +$
$w_{z1}^2(f_{min}^2 - f_1^2)^2 + w_{z2}^2(f_{min}^2 - f_2^2)^2$;

return *Costs* $C_{\mathtt{F}}$, $C_{\mathbf{p}}$, C_f;

end

[a] Richard Hartley and Chanop Silpa-anan. "Reconstruction from two views using approximate calibration". In: *ACCV*. 2002.

Hartley Algorithm

Algorithm 7: The cost function of Hartley and Silpa-Anan[a]

Data: Fundamental matrix $\mathtt{F}$, principal points $\mathbf{p}_1$, $\mathbf{p}_2$.

Result: Vector of costs $\mathbf{C}$

begin

From $\mathtt{F}$, $\mathbf{p}_1$, $\mathbf{p}_2$ compute f_1, f_2;

$C_{\mathtt{F}} \leftarrow$ Sampson error $S(\mathtt{F}) = \Sigma_i \frac{(\mathbf{x}_{2,i}^{\top}\mathtt{F}\mathbf{x}_{1,i})^2}{(\mathtt{F}\mathbf{x}_{1,i})_1^2+(\mathtt{F}\mathbf{x}_{1,i})_2^2+(\mathbf{x}_{2,i}^{\top}\mathtt{F})_1^2+(\mathbf{x}_{2,i}^{\top}\mathtt{F})_2^2}$;

$C_{\mathbf{p}} \leftarrow w_p^2\|\mathbf{p}_1 - \bar{\mathbf{p}}_1\|^2 + w_p^2\|\mathbf{p}_2 - \bar{\mathbf{p}}_2\|$;

$C_f \leftarrow w_1^2(f_1^2 - \bar{f}_1^2)^2 + w_2^2(f_2^2 - \bar{f}_2^2)^2 + w_d^2(f_1^2 - f_2^2)^2 +$
$w_{z1}^2(f_{min}^2 - f_1^2)^2 + w_{z2}^2(f_{min}^2 - f_2^2)^2$;

return *Costs* $C_{\mathtt{F}}$, $C_{\mathbf{p}}$, C_f;

end

[a] Richard Hartley and Chanop Silpa-anan. "Reconstruction from two views using approximate calibration". In: *ACCV*. 2002.

Our modification

Algorithm 8: Our modification to the optimization of Hartley and Silpa-Anan[a]

Data: Point correspondences $\mathbf{x}_1$, $\mathbf{x}_2$, prior focal lengths $\bar{f}_1$, $\bar{f}_2$,prior principal points $\bar{\mathbf{p}}_1$,$\bar{\mathbf{p}}_2$, minimal focal length f_{min}.

Result: Fundamental matrix $\mathtt{F}$

begin

Compute a $\mathtt{F}$ from $\mathbf{x}_1$, $\mathbf{x}_2$ using the 7pt algorithm;

Estimate the focal lengths ratio $r = f_2/f_1$ from $\mathtt{F}$;

Optimize modified H-S cost;

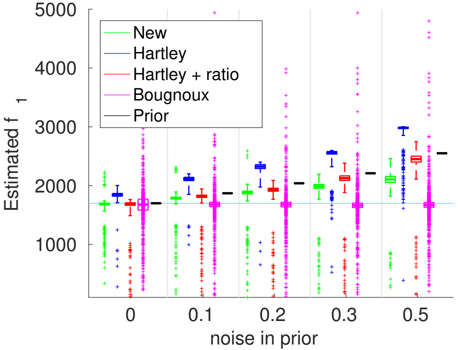
$C_f = w_1^2(f_1^2 - \bar{f}_1^2)^2 + w_2^2(f_2^2 - \bar{f}_2^2)^2 + w_d^2((rf_1)^2 - f_2^2)^2 + w_{z1}^2(f_{min}^2 - f_1^2)^2 + w_{z2}^2(f_{min}^2 - f_2^2)^2$;

return $\mathtt{F}$;

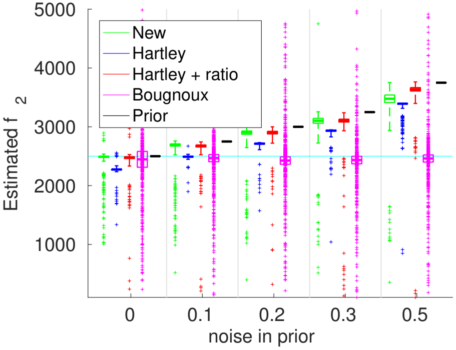
end

[a] Richard Hartley and Chanop Silpa-anan. "Reconstruction from two views using approximate calibration". In: *ACCV*. 2002.

Comparison



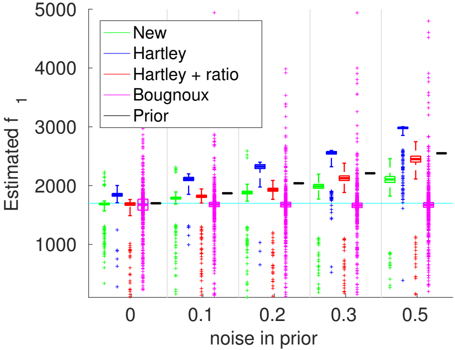
(a) Fig. 5.4. f_1 is shown.



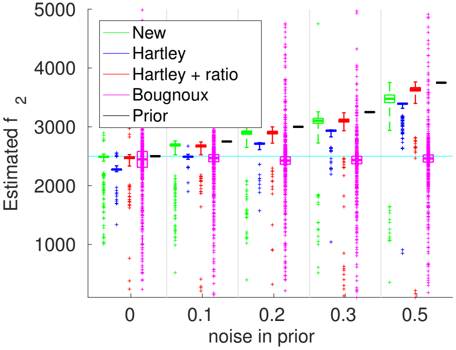
(b) Fig. 5.5. f_2 is shown.

Figure: Focal length computation methods performance. The ground truth focal length is shown in cyan. Four pixels of noise in image measurements were applied.

Comparison



(a) Fig. 5.4. f_1 is shown.



(b) Fig. 5.5. f_2 is shown.

Figure: Focal length computation methods performance. The ground truth focal length is shown in cyan. Four pixels of noise in image measurements were applied.

Summary

- We have good methods for computing focal lengths from images.
- Even more can be done to improve stability against noise or in degenerate situations.