

STAT430 Homework #11: Due Saturday, May 11, 2019.

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0. We finish **Chapter 10** on hypothesis testing.
 1. Complete **Exercise 10.23** of the text. For part (c), give a complete **RANTDRC** test of the appropriate hypothesis.
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Answer:

a)

$$H_a : \mu_1 \neq \mu_2$$

$$H_o : \mu_1 = \mu_2$$

b)

This would be a two tailed test because this a difference.

c)

R: Do the ranges of deer populations differ?

A: $H_a : \mu_1 \neq \mu_2$

N: $H_o : \mu_1 = \mu_2$

T: $Z = \frac{\mu_1 - \mu_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = -0.95358$

D: $Z \sim N(0, 1)$ where $\pm Z_{\alpha/2} = \pm 1.645$

R: $Z = -0.95358 > -1.645$

C: We could Fail to Reject H_o , so the deer do not travel significantly different differences.

2. Complete **Exercise 10.24** of the text, giving a complete **RANTDRC** test of the appropriate hypothesis.
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Answer:

R: Is the percentage reported too high?

A: $H_a : p < 0.15$

N: $H_o : p = 0.15$

T: $Z = \frac{\hat{p} - p}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}} = \frac{0.13 - 0.15}{\sqrt{\frac{.13(.87)}{100}}} = -0.5947$

D: $Z \sim N(0, 1)$ where $Z_{\alpha/2} = -1.645$

R: $-0.5947 > -1.645$

C: We would Fail to Reject H_o .

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3. Complete **Exercise 10.33** of the text, giving a complete **RANTDRC** test of the appropriate hypothesis.
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Answer:

R: Republicans support it more.

A: $H_a : p_1 > p_2$

N: $H_o : p_1 = p_2$

T: $Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n} + \frac{\hat{p}_2(1-\hat{p}_2)}{n}}} = 1.504$

D: $Z \sim N(0, 1)$ where $Z_\alpha = 1.645$

R: $1.504 < 1.645$

C: So we would Fail to Reject H_o .

4. Complete **Exercise 10.38** of the text.
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Answer:

$\beta = P(T \notin RR, H_o \text{ is false})$

$$\begin{aligned} P\left(\frac{\bar{X} - \mu_o}{s/\sqrt{n}} \geq -2.33\right) &= P(\bar{X} \geq \mu_o - 2.33(s/\sqrt{n})) = P(\bar{X} \geq 61.3752) = P\left(\frac{\bar{X} - 60}{\sqrt{S^2/n}} \geq \frac{61.37 - 60}{\sqrt{S^2/n}}\right) \\ &= P(Z \geq 1.21) = 0.1132 \end{aligned}$$

5. Complete **Exercise 10.45** of the text.
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Answer:

$$\left[(\mu_1 - \mu_2) \pm Z_{0.005} \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} \right]$$

a)

After computing the interval, 0 is not in the interval.

b)

So H_o would be rejected because the target is $\mu_1 - \mu_2 = 0$.

c)

We have the same conclusion. *****

6. Complete **Exercise 10.50** of the text.
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Answer:

$$\hat{p} = 0.58, sd = .11, p = 0.6$$

$$H_a : p < 0.6$$

$$H_o : p = 0.6$$

$$T = \frac{0.58 - 0.6}{\sqrt{0.11^2 / 120}} = -1.9917 \sim N(0, 1)$$

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pnorm(-1.9917)
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## [1] 0.02320199
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So we would reject H_o at $\alpha = 0.1$ and say it is unprofitable.

7. Complete **Exercise 10.54** of the text.

Answer:

$$p = 0.85, \hat{p} = 0.96, n = 300, \alpha = 0.01$$

$$Z = \frac{\hat{p} - p}{\sqrt{\frac{\hat{p}}{1 - \hat{p}} / n}} \sim N(0, 1)$$

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pnorm(9.7227)
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## [1] 1
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Since the p-value is so small, we would reject H_o in almost every situation. So the reported proportion is too low.

8. Complete **Exercise 10.106** of the text.

Answer:

$$\hat{\theta}_1 = 76/200$$

$$\hat{\theta}_2 = 53/200$$

$$\hat{\theta}_3 = 59/200$$

$$\hat{\theta}_4 = 48/200$$

$$\hat{\theta}_0 = 236/800$$

$$H_o : \theta_1 = \theta_2 = \theta_3 = \theta_4 = \theta_0$$

$$H_a : \theta_1, \theta_2, \theta_3, \theta_4, \text{ at least one is different.}$$

$$T = -2\ln(LR) \sim \chi_3^2$$

$$\text{Max Likelihood under } H_o = \prod_{i=1}^4 \binom{200}{y_i} \theta_0^{y_i} (1 - \theta_0)^{200 - y_i}$$

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ML1 <- dbinom(76,200,236/800)*dbinom(53,200,236/800)*dbinom(59,200,236/800)*dbinom(48,200,236/800)
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$$\text{Max Likelihood} = \prod_{i=1}^4 \binom{200}{y_i} \theta_i^{y_i} (1 - \theta_i)^{200 - y_i}$$

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ML2 <- dbinom(76,200,76/200)*dbinom(53,200,53/200)*dbinom(59,200,59/200)*dbinom(48,200,48/200)
Tstat <- -2*log(ML1/ML2)
Tstat

## [1] 10.53527
(qchisq(0.95, 3)<Tstat)

## [1] TRUE

```

Since T is larger than $\alpha = 0.05$ for a χ^2_3 we reject H_0 and say that at least one ward votes differently.

Optional problems: We have touched on nearly all of Chapter 10, especially sections 10.2, 10.3, 10.4, 10.5, 10.6, and 10.11. Any of the (non-applet) exercises in those sections are good to review. Less emphasis on the exact tests in 10.8 and 10.9, though these are just applications of the normal-theory distributions that we studied in the first part of the course and applied in the confidence interval section.