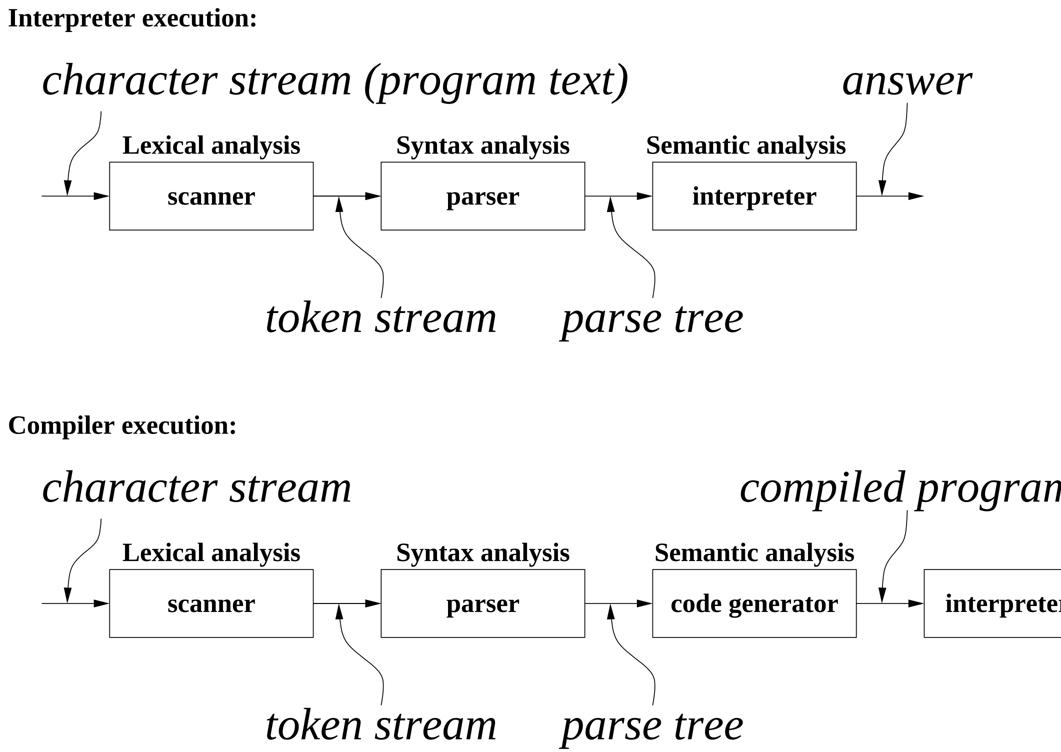
Environment-Passing Interpreters

Interpretation vs. compilation can be illustrated by a picture:



Environment-Passing Interpreters (continued)

Most programming languages have grammar rules defining an *expression*. In
ple, an expression typically involves values (like variables, integers, and the
calls) and operators (like addition and multiplication). Example Java expressic
'`foo(11) && toggle`'. In all of the languages we discuss in this class, every
of evaluating expressions. Such languages are called "expression-based language

An *expressed value* is the value of an expression as specified by the language
ample, the expressed value of the Java expression '`2+3`' is 5. A *denoted value* i
to a symbol. Denoted values are internal to the interpreter, whereas expressed va
expressions that can be seen "from the outside".

For a symbol, say `x`, you normally think that the value of the expression `x` is the sa
value of `x`. But what about a language such as Java? In Java, the denoted value
variable is a *reference* to an object, whereas the expressed value of the variable i
This may seem like a subtle distinction, but you will see its importance later.

In summary, for a symbol, its expressed value is what gets displayed when you
`toString` representation, for example), and its denoted value is the value boun
its environment. In our early languages, the denoted values and expressed values
In our later languages, we will see why we need to separate denoted values from
to implement language features such as mutation.

Environment-Passing Interpreters (continued)

You should also distinguish between a *source language* and its *imple*
guage. A source language is a language to be interpreted, and its i
language is the language in which the interpreter is written. (Th
language is often used to refer to a source language. Similarly, th
language is often used to refer to its implementation language.)

In the rest of this course, our source languages will be a collecti
languages used to illustrate the various stages of language design,
mentation language will be Java. Don't be disappointed by the term
the languages we define have significant computational power, and t
lustrate a number of core ideas that are present in all programming l

We start with a language we call "Language V0" – think of this as "
sion Zero". Its `grammar` specification file appears on the next slide

Language V0

```
# Language V0
skip WHITESPACE '\s+'
LIT '\d+'
ADDOP '\+'
SUBOP '\-'
ADD1OP 'add1'
SUB1OP 'sub1'
LPAREN '\('
RPAREN '\)'
COMMA ','
VAR '[A-Za-z]\w*'
%
<program>          ::= <exp>
# these three grammar rules define what it means to
<exp>:LitExp       ::= <LIT>
<exp>:VarExp       ::= <VAR>
<exp>:PrimAppExp ::= <prim> LPAREN <rands> RPAREN
<rands>            **= <exp> +COMMA
<prim>:AddPrim     ::= ADDOP
<prim>:SubPrim     ::= SUBOP
<prim>:Add1Prim    ::= ADD1OP
<prim>:Sub1Prim    ::= SUB1OP
%
#include code
```

Language V0 (continued)

Observe that this PLCC language specification has all three of the in Slide Set 1a: the lexical specification section (token specificatio specification section (given as BNF rules), and the semantic speci (just an `#include` line).

You can consider a grammar rule like

```
<program>          ::= <exp>
```

to mean that "a `program` consists of an `exp`" (where `exp` stands sion"). Similarly, you can consider a grammar rule like

```
<exp>:LitExp       ::= <LIT>
```

to mean that "a `LitExp` is (an instance of) an expression (an `exp`) a `LIT`." Similar remarks apply to all the grammar rules.

The '`#include code`' line at the end of this file means that the file named `code` should be inserted into the `grammar` file input point to be processed by PLCC.

Language V0 (continued)

Example "programs" in this language:

```
3
x
+(3, x)
add1( +(3,x) )
+(4, -(5,2))
```

Observe that in Language V0 – and in most of the other languages these class notes – we write arithmetic expressions in *prefix form*, v metic operator (such as '+' or '−') precedes its operands. (A prefix f like '`+(3,x)`' would normally be written mathematically as '`3+` *form*. It turns out that prefix form expressions are easier to parse an infix form expressions, which is why we use prefix form in our lang `INFIX` language in Slide Set 6 for a further discussion of infix forn

Prefix form is not entirely unusual: languages in the Lisp family (incl use prefix form. Contrast this to languages such as C, Java, and Pytho metic operators appear principally in infix form.

Language V0 (continued)

Here is a mapping from the concrete (BNF) syntax of Language V0 resentation as an abstract syntax. The Java class files are created au PLCC. Each item in a `Box` is the signature of the corresponding cla

```
<program>            ::= <exp>
                     Program(Exp exp)
<exp>:LitExp         ::= <LIT>
                     LitExp(Token lit)
<exp>:VarExp         ::= <VAR>
                     VarExp(Token var)
<exp>:PrimappExp     ::= <prim> LPAREN <rands> RPAREN
                     PrimappExp(Prim prim, Rands rands)
<rands>              **= <exp> +COMMA
                     Rands(List<Exp> expList)
<prim>:AddPrim       ::= ADDOP
                     AddPrim()
<prim>:SubPrim       ::= SUBOP
                     SubPrim()
<prim>:Add1Prim      ::= ADD1OP
                     Add1Prim()
<prim>:Sub1Prim      ::= SUB1OP
                     Sub1Prim()
```

Language V0 (continued)

The term *abstract syntax* might seem odd because it refers to a col explicit Java classes. Instead, the term *abstract* here means tha keep only the information on the right-hand-side (RHS) of the that can change, principally by ignoring certain RHS tokens. Fo `<exp>:PrimappExp` grammar rule has tokens `LPAREN` and `F` RHS, but the generated `PrimappExp` class does not have fields to these tokens: they are "abstracted away".

Because the `<exp>` and `<prim>` nonterminals appear on the LHS grammar rules, we must disambiguate these grammar rules by annot nonterminals with appropriate class names. For these grammar rules terminal corresponds to the name of an abstract (base) Java class obtained by capitalizing the first letter of the nonterminal name. classes `LitExp`, `VarExp`, and `PrimappExp` extend the abstract b Similarly, the `AddPrim`, `SubPrim`, `Add1Prim` and `Sub1Prim` the abstract base class `Prim`.

Once you create the `grammar` specification file and run `plccmk`, ine the Java code in the `Java` subdirectory. Here you can see, fo the `LitExp` class extends the `Exp` class and that the `AddPrim` cl `Prim` class.

Language V0 (continued)

The `Program` class has one instance variable named `exp` of type `E` is an abstract class, an object of type `Exp` must be an instance of a cl `Exp`: namely, an instance of `LitExp`, `VarExp`, or `PrimappExp`. does not have a constructor, so you can't instantiate an object of typ

The directory `/usr/local/pub/plcc/Code/V0` contains th file, named `grammar`, for this language.

The `grammar` file in Language V0 has three parts, separated by li gle '`%`': the **lexical specification section**, the **syntax specification s** **code (semantics) section**.

Recall that if your `grammar` file has only the lexical specificati `plccmk` tool produces Java code for a scanner (`Scan`), but nothin `grammar` file has only the lexical specification and syntax specific the `plccmk` tool produces Java code for a scanner (`Scan`) and a p for the grammar, but nothing else.

Language V0 (continued)

The code section defines the language semantics. In this section, t
defined by the grammar rules are given life by defining their behavio
by defining the `$run()` method in the start symbol class (`Progr`
of language `V0`).

In the absence of a redefined `$run()` method – for example, if the
omitted – the default `$run()` behavior in the `_Start` class is to
of the start symbol in a format illustrated in Slide 1.29.

In later versions of this language (`V1` and beyond), we see how su
method can be used to print the arithmetic value of an expression. B
`V0` we will be content with simply printing a copy of the expression

In the code section of a PLCC language specification, the behavior
method in the start symbol class defines the language semantics.

Language V0 (continued)

Assuming that we have created the `grammar` file in a directo
running the `plccmk` tool creates a `Java` subdirectory with sour
`Program.java`, `LitExp.java`, and so forth, that correspond
syntax classes shown in Slide 3.7. In the `Java` directory, you ca
source files named `Token.java`, `Scan.java`, `Parse.java`, ar

The `Rep` program repeatedly prompts you for input (with '`-->`'), p
and prints the result: a `String` representation of the expression wi
space removed. If you want to run this program from the directo
`grammar` file – `V0` in this case – you can run it as follows:

```
$ java -cp Java Rep
--> add1( + (2
  , 3))
add1(+(2,3))
--> ...
```

As we discussed in Chapter 1, parsing is the process by which a sequ
(a *program*) can be determined to belong to the language defined by
We showed examples of leftmost derivations and how the derivati
detect whether or not the program is syntactically correct.

Language V0 (continued)

We get more than a success or failure response from our parser: *th returns a Java object that is an instance of the class determine grammar start symbol.* This object is the root of the *parse tree* of th captures all of the elements of the parsed program.

In our Language V0 grammar, the start symbol is <program>, so parse tree is an instance of the Program class.

Language V0 (continued)

We have seen that the RHS of a grammar rule determines what ins belong to the class defined by its LHS. Only those entries on the angle brackets <...> appear as instance variables; *any other RHS token names that are used in the parse but that are abstracted away ing the objects in the parse tree.*

For example, consider the following grammar rule in Language V0:

```
<exp>:PrimappExp ::= <prim> LPAREN <rands>
```

This rule says that PrimappExp is a class that extends the Exp cl instance variables in this class are

```
Prim prim;
Rands rands;
```

Language V0 (continued)

Consider the following grammar rule in Language V0:

```
<exp>:LitExp ::= <LIT>
```

This rule creates a Java class `LitExp` having a single field name `Token`. The lexical specification defines a `LIT` token to be a sequ more decimal digits.

Continuing in this way, each of the BNF grammar rules of Langu 3.7) defines a class given by its LHS with a well-defined set of ins corresponding to the <*angle bracket*> entries in its RHS.

As we have already observed, when we encounter situations where tw have the same (Token or nonterminal) name in angle brackets, we di entries by providing different instance variable names.

Language V0 (continued)

Recall that repeating grammar rules have fields that are Java lists. Fo Language V0 grammar has the following repeating rule:

```
<rands> **= <exp> +COMMA
```

This rule says that the `<rands>` nonterminal can derive zero or mo tries, separated by commas. The following sentences would match nonterminal:

```
a,b,c      <-- <exp> COMMA <exp> COMMA <exp>
1, +(2,3) <-- <exp> COMMA <exp>
add1(x)    <-- <exp>
           <-- empty string
```

The class defined by this rule is named `Rands`. Its RHS shows only nal `<exp>`, so its corresponding Java class `Rands` has a field `ex` `List<Exp>`.

You might wonder how we chose a name like “rands”. It’s actual form of the term “operands”. In mathematics and in programming the things being operated on. For example, given the expression + (erator is ‘+’ and its operands are `2` and `3`. (Similarly, some langua the term “rator” as a shortened form of the term “operator”.)

Language V0 (continued)

When we parse a program such as

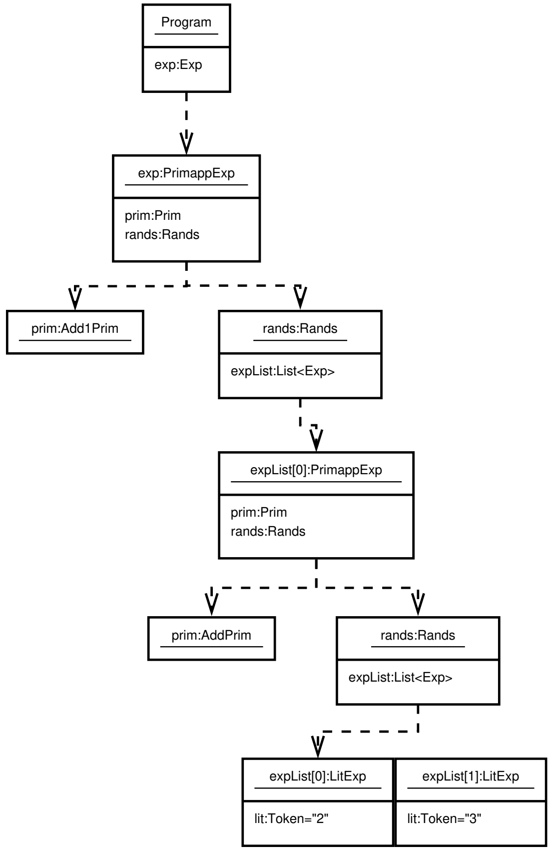
```
add1( +(2,3))
```

the parser returns an object of type `Program`. The `Program` obj
stance variable: `exp` of type `Exp`. The value of the `exp` instance
type `PrimappExp` (which extends the `Exp` class) that has two inst
`prim` of type `Prim` and `rands` of type `Rands`. The value of the `p`
an object of type `Add1Prim` (which extends the `Prim` class) that l
variables. And so forth ...

On the following slide we show the entire parse tree of this expressi

Language V0 (continued)

Parse tree for `add1( +(2,3))` in UML format:



Language V0 (continued)

The Rep program defaults to running the $run() method in the P
which prints the Program object as a String. (Actually, it prin
object as a String, but since the Program class extends the _
these behaviors are the same.) The default $run behavior – which
if $run is not redefined in the Program class – displays a string
this:

```
Program@....
```

We want to show how to override the default $run() method in
class so that it prints the same text as the program input with extra
moved!

This means that we should see the following when interacting wit
gram from Language V0:

```
--> add1( +(2, 3   ))
add1(+(2,3))
--> x
x
--> +  ( p  ,
- ( q,r)  )
+(p,-(q,r))
--> ...
```

Language V0 (continued)

We follow the method described in Slide Set 1 to redefine the defa
the $run() method in the Program class. *In all of our languages,*
semantics *of a program is the output produced by the* $run() *me*
the root of the parse tree of the program.

Recall that to add Java code (fields and method definitions) to a
Program, use the following template:

```
Program
%%%
...Java code...
%%%
```

Since PLCC inserts this code verbatim into the body of the Prog
methods defined in this code can access all of the instance variabl
So for the Program object, these methods can refer to the exp in
of type Exp. Since the RHS of the <program> grammar rule is ju
$run() method of the Program class is simple:

```
Program
%%%
    public void $run() {
        System.out.println(exp.toString());
    }
%%%
```

Language V0 (continued)

To finish our implementation, we show how to implement the method for an `Exp` object so that it returns a `String` representatio

There are three `Exp` classes: `LitExp`, `VarExp`, and `PrimappE` `toString()` method for the first two classes is particularly easy, have right-hand sides that are just token strings: their `toString` return the `String` value of the corresponding `Token` instance vari 3.7):

```
LitExp
%%%
    public String toString() {
        return lit.toString();
    }
%%%

VarExp
%%%
    public String toString() {
        return var.toString();
    }
%%%
```

Language V0 (continued)

Examine the rule for a `PrimappExp`:

```
<exp>:PrimappExp ::= <prim> LPAREN <rands> RPAREN
                     PrimappExp(Prim prim, Rands rands)
```

A `PrimappExp` object has just two instance variables:

```
Prim prim;
Rands rands;
```

There aren't any instance variables corresponding to `LPAREN` and cause the `PrimappExp` class abstracts away these tokens. The only to do, then, is to re-insert them back into the `toString` result, in as they appear on the RHS of the grammar rule. The `toString` `prim` and `rands` are called implicitly.

```
PrimappExp
%%%
    public String toString() {
        return prim + "(" + rands + ")";
    }
%%%
```

Language V0 (continued)

Each of the <prim> rules has an RHS that corresponds to a `Toke`
by the parser. Just as we re-inserted the `LPAREN` and `RPAREN` to
defined the `toString` method in the `PrimappExp` class, each o
classes simply returns the corresponding string token:

```
AddPrim
%%%
    public String toString() {
        return "+";
    }
%%%

SubPrim
%%%
    public String toString() {
        return "-";
    }
%%%

Add1Prim
%%%
    public String toString() {
        return "add1";
    }
%%%
```

The `Sub1Prim` code is similar and has been omitted.

Language V0 (continued)

We have covered all of the `plcc`-generated classes except for `Rand`
bit more attention since a `Rands` object has an `expList` instance
is a `List` of expressions. First examine the <rands> grammar rul

```
<rands> **= <exp> +COMMA
        Rands(List<Exp> expList)
```

To build a `toString` method for this class, we call the `toString`
each of the `expList` entries and construct a `String` that puts co
them. Here is the code:

```
Rands
%%%
public String toString() {
    String s = "";   // the string to return
    String sep = ""; // no separator for the first e
    // get all of the expressions in the operand lis
    for (Exp exp : expList) {
        s += sep + exp; // exp.toString() is called
        sep = ",";   // commas separate the remainin
    }
    return s;
}
%%%
```

Language V0 (continued)

We can now re-build the Java code for this grammar using the `plc`
Assuming that everything compiles correctly, we should get the de
from the `Rep` program: `Rep` parses each syntactically correct input
displays the resulting `Program` object as a `String`, which appea
the input with whitespace removed.

Language V1

Now that you see how a parse tree for Language V0 can *print* itself,
a parse tree can *evaluate itself*.

The term *evaluate* can have many meanings (one of which is to prod
representation of itself), but for our purposes, to evaluate an arithm
such as `add1( + (2,3))` means to produce the integer valu
words, the value of an arithmetic expression is its numeric value us
for arithmetic.

(Remember that we are abstracting the notion of *value* to refer to an
`Val` class. In this setting, a numeric value is an instance of the `In`
of `Val`.)

If an expression involves an identifier (symbol), we need to deter
bound to that identifier in order to evaluate the expression. For exa
the identifier `"x"` is bound to the integer value 10: then the express
would evaluate to 9.

The interpreter evaluates every expression in some environment. Th
determines how to obtain the values bound to the identifiers tha
expression

Language V1

The `Exp` class is the appropriate place to declare evaluation beha implement using a method called `eval`. Here is how we declare the method in the (abstract) `Exp` class. Every class that extends `Exp` define this method:

```
Exp
%%%
    public abstract Val eval(Env env);
%%%
```

Language V1 is the same as Language V0, except for adding `eval` classes that extend the `Exp` class. We continue to consider the only be an `IntVal` that holds an integer value.

Language V1 has three new files included in its `grammar` language envRN, `val`, and `prim`.

- The `envRN` file contains Java class definitions to implement er discussed in Slide Set 2.
- The `val` file defines the `Val` class and its `IntVal` subclass – a s wrapper for Java `ints`.
- The `prim` file defines the semantics of the seven `<prim>` BNF in Language V1.

Language V1 (continued)

Three classes extend the `Exp` class: they are `LitExp`, PrimappExp. We'll start with `LitExp`. Here is the code part file that defines the `eval` behavior of a `LitExp` object. (The eva coexist with the `toString` behavior that we defined in Language not show this here.)

```
LitExp
%%%
    public Val eval(Env env) {
        return new IntVal(lit.toString());
    }
%%%
```

Remember that a `LitExp` has a `Token` field named `lit`. Wher `toString()` method to this field, we get the string of decimal d from the part of the program text we are parsing. The `IntVal` const this into a real Java `int` that becomes part of the `IntVal` instance environment doesn't have anything to do with the value of a num literal `10` evaluates to the integer value 10 no matter what environm so the `eval` routine for a `LitExp` simply returns the appropriate I

Language V1 (continued)

Next we consider VarExp. Here is the code part of the grammar f
the eval behavior of a VarExp object.

```
VarExp
%%%
    public Val eval(Env env) {
        return env.applyEnv(var);
    }
%%%
```

A VarExp object has a var instance variable of type Token. Giv
ment, the value bound to var is precisely the value returned by app
in turn is the value of the expression.

The value of an expression consisting of a symbol is the value bound in the environment in which the expression is evaluated, as determi lication of applyEnv.

Language V1 (continued)

Finally we consider PrimappExp. A PrimappExp object has tw
ables: a Prim object named prim and a Rands object named ran
such an expression, we need to apply the given primitive operation
ject) to the values of the expressions in the rands object.

An object of type Rands has a List<Exp> instance variable nan
In order to perform the operation determined by the prim object, w
uate each of the expressions in expList. A utility method named
in the Rands class does the work for us. Of course, this method
what environment is being used to evaluate the expressions, so an
parameter to this method.

```
Rands
%%%
    public List<Val> evalRands(Env env) {
        List<Val> args = new ArrayList<Val>(expList
        for (Exp exp : expList)
            args.add(exp.eval(env));
        return args;
    }
%%%
```

Language V1 (continued)

We *specify* that the expressions in an `evalRands` method call be ev to-last (or left-to-right, depending on how you are looking at it) or necessarily the case for all programming languages. In particular, *et seq.* for a more complete discussion of order of evaluation.

The `evalRands` method returns a *list* of `Vals`. In order to acce easily and to apply normal arithmetic operations to them, we conve *array* of `Val` objects. The utility method named `toArray` in the complishes this.

The expressions appearing in an application of a primitive are calle also called ***actual parameters****; the values of these expressions are* ***ments***.

As a careful reader of these notes, you will have observed that `Rands` is derived from the word **operands**, and that the name `evalRands` method is derived from the word **arguments**.

Language V1 (continued)

We now have the pieces necessary to define the `eval` method in the class:

```
PrimappExp
%%%
    public Val eval(Env env) {
        // evaluate the terms in the expression list
        // and apply the prim to the array of Vals
        List<Val> args = rands.evalRands(env);
        Val [] va = Val.toArray(args);
        return prim.apply(va);
    }
%%%
```

In summary, to evaluate a primitive application expression (a `Pri` evaluate the operands (a `Rands` object) in the given environment a sulting argument array to the `apply` method of the primitive (a which returns the appropriate value.

We are left to define the behavior of the `apply` methods in the classes. Observe that by the time a `Prim` object gets its array of arg vironment no longer plays a role, since we have already evaluated all expressions: only values remain.

Language V1 (continued)

Since we are using the `apply` method with a `Prim` object, we nee
laration for this method to the (abstract) `Prim` class. Here is how w

```
Prim
%%%
    // apply the primitive to the passed values
    public abstract Val apply(Val [] va);
%%%
```

We use the parameter name '`va`' to suggest the idea of a **v**alue **a**rray

A `Prim` object (there are seven instances of this class) has no inst
However, we can endow these objects with behavior, so that an `Ac`
knows how to add things, a `SubPrim` object knows how to subtrac
forth.

Language V1 (continued)

Four of the `Prim` objects need two arguments ('+', '−', '$\star$', and
of them need one argument (`add1`, `sub1`, and `zerop`). Since `va`
`Val` arguments, we can grab the appropriate items from this array
them, depending on the operation – to evaluate the result. Here is t
`AddPrim` class:

```
AddPrim
%%%
    public Val apply(Val [] va) {
        if (va.length != 2)
            throw new PLCCException("two arguments e
        int i0 = va[0].intVal().val;
        int i1 = va[1].intVal().val;
        return new IntVal(i0 + i1);
    }
%%%
```

The `intVal()` method calls shown in this code convert `Val` ol
`Va[0]`) into `IntVal` objects – essentially like "downcasting". Th
turn, have Java `int` fields named `val`. So both `i0` and `i1` are
`ints` that can be added together to return the resulting `IntVal` ob
class defines the `intVal()` method behavior: an attempt to apply t
method to a `Val` object that is *not* an `IntVal` throws an exception.

Language V1 (continued)

The definitions of apply for the classes SubPrim, MulPrim, and
latter two are added in V1) have obvious implementations, except tha
the apply method throws an exception if it detects an attempt to
this code is not shown here.

For the Add1Prim class, the apply method expects only one
passed as element zero of the va array.

```
Add1Prim
%%%
    public Val apply(Val [] va) {
        if (va.length != 1)
            throw new PLCCException("one argument e
        int i0 = va[0].intVal().val;
        return new IntVal(i0 + 1);
    }
%%%
```

Again, the definition of apply for the Sub1Prim class is entirel
definition of apply for the ZeropPrim class returns an IntVal
a zero argument and an IntVal of 0 (false) for a nonzero argumen

Language V1 (continued)

In our final implementation step, we will define the $run() method
object that displays the string representation of the *value* of its expr
next slide for how we implement this.

An empty environment would only allow for integer expressions wi
since every variable would be unbound. To test Language V1, we
environment initEnv specific to this language that has the foll
bindings (think Roman numerals!):

```
i => 1
v => 5
x => 10
l => 50
c => 100
d => 500
m => 1000
```

For Language V1, this environment can be obtained by a call to Env
In the Program class, we set the static initEnv variable to thi
bindings give us some variables to play with, though, as you wi
dispense with them later.

Language V1 (continued)

Here is the new `$run()` method for the `Program` object, along wit
of the initial environment `initEnv` in the `Program` class:

```
Program
%%%
    public static Env initEnv = Env.initEnv();

    public void $run() {
        System.out.println(exp.eval(initEnv).toStri
    }
%%%
```

To test this, run the `Rep` program defined in the `Java` subdirector
pressions at the prompts:

```
java -cp Java Rep
```

Language V2

Language V2 is the same as Language V1, with the addition of
semantics of an `if` expression. The relevant grammar rule and
representation are shown here:

```
<exp>:IfExp ::= IF <exp>testExp THEN <exp>trueExp El
                IfExp(Exp testExp, Exp trueExp, Exp fal
```

Notice that we need to add token names `IF`, `THEN`, and `ELSE` to ou
fication, along with their obvious regular expressions.

The RHS of the `IfExp` grammar rule has three occurrences of th
terminal. Since the `<exp>` items on the RHS of this grammar rul
stance variables of the class, we have named these instance variab
`trueExp`, and `falseExp`, respectively. Each of these objects
stance of the `Exp` class. The `IfExp` class has three instance variab

```
Exp testExp;
Exp trueExp;
Exp falseExp;
```

[**Exercise** (not to hand in): See what would happen if you used '<I
of this grammar rule instead of '`IF`'.]

Language V2

To evaluate an `if` expression with a given environment, we fir `testExp` expression. If this evaluates to true, we evaluate the `tru` sion and return its result as the value of the entire expression. If th false, we evaluate the `falseExp` expression and return its result. expressions is evaluated in the given environment.

Since all instances of `Val` are really `IntVals` (for the time being) `IntVal` object corresponding to `0` to be false and *all others* to be t

Language V2 (continued)

We define an `isTrue()` method for an `IntVal` object as follow part of the `IntVal` class that is defined in the `val` file – only th `isTrue` is given here:

```
public boolean isTrue() {
    return val != 0; // nonzero is true, zero is fal
}
```

Observe that the `eval()` method in the `IfExp` class applies th method to a `Val` object, so we must include a declaration for th method in the `Val` base class. *Since we currently treat any* `Val` *if it's not an* `IntVal` *of zero, our default* `isTrue()` *method in defaults to returning* `true`.

```
Val
%%%
...
    public boolean isTrue() {
        return true;
    }
...
%%%
```

Language V2 (continued)

```
<exp>:IfExp ::= IF <exp>testExp THEN <exp>trueExp ELSE
            IfExp(Exp testExp, Exp trueExp, Exp falseE
```

Here is the `eval` code for the `IfExp` class:

```
IfExp
%%%
    public Val eval(Env env) {
        Val v = testExp.eval(env);
        if (v.isTrue())
            return trueExp.eval(env);
        else
            return falseExp.eval(env);
    }
%%%
```

The `isTrue()` boolean method applies to any instance of `Val`. It
method used only to implement the semantics of the `if...ther`
pression; it is *not* part of the source language. On the other han
primitive in the source language (starting with Language V1), not a
The `zero?` primitive applies only to integer values in the source
define the *semantics* of the `zero?` primitive using the `apply` Java
`ZeropPrim` class. You may find this somewhat confusing.

Language V2 (continued)

```
<exp>:IfExp ::= IF <exp>testExp THEN <exp>trueExp ELSE
            IfExp(Exp testExp, Exp trueExp, Exp falseE
```

Observe that the `eval` method in the `IfExp` class evaluates *o*
`trueExp` or `falseExp` expressions, never both. This is a sem
not a syntax feature – of the definition of `eval` for an `if` express
special form refers to semantic structures that behave like expres
when evaluated, don't evaluate all of their constituent parts. An `i`
an example of a special form.

Some examples of `if` expressions are on the next slide.

Language V2 (continued)

```
if 1 then 3 else 4
  % => 3

if 0 then 3 else 4
  % => 4

if
  if 1 then 0 else 11
then
  42
else
  15
  % => 15

+(3, if -(x,x) then /(5,0) else 8)
  % => 11 (note that the /(5,0) expression is not ev
```

You must understand that *an* `if` *expression is an expression and the* *ates to a value.* It is entirely unlike `if` statements in imperative lan Java and C++, where the purpose of an `if` statement is to *do* one th not to return a value. Also observe that an `if` expression in our so must have both a `then` part and an `else` part, even though only o pressions ends up being evaluated.

Language V3

Language V3 is the same as Language V2 with the addition of a `l` Here are the relevant grammar rules and abstract syntax representati

```
<exp>:LetExp ::= LET <letDecls> IN <exp>
                 LetExp(LetDecls letDecls, Exp exp)
<letDecls>     **= <VAR> EQUALS <exp>
                 LetDecls(List<Token> varList, List<Exp
```

Notice that we need to change our lexical specification to allow fo `LET`, `IN`, and `EQUALS`. Here are the relevant lexical specifications:

```
LET    'let'
IN     'in'
EQUALS '='
```

Here is an example program in Language V3 that evaluates to `7`:

```
let
  three = 2
  four  = 5
in
  +(three, four)
```

The purpose of a `let` expression is to create an environment wit bindings and to evaluate an expression using these variable bindings

Language V3 (continued)

```
<exp>:LetExp ::= LET <letDecls> IN <exp>
                 LetExp(LetDecls letDecls, Exp exp)
<letDecls>   **= <VAR> EQUALS <exp>
                 LetDecls(List<Token> varList, List<Exp
```

To evaluate a `LetExp`, we perform the following steps:

0. create a set of local bindings (a `Bindings` object) by bindi `<VAR>` symbols to the values of their corresponding `<exp>` ex `<letDecls>` part, where the `<exp>` expressions to the right are all evaluated in the enclosing environment;
1. extend the enclosing environment with these local bindings to c vironment; and
2. use this new environment to evaluate the `<exp>` expression i and return this value as the value of the `letExp` expression.

The `<exp>` part of a `let` expression is called the *body* of the `let`

Examples of `let` expressions are on the next slides, where => me to". Remember: *a* `let` *expression is an expression, and as such, something*!

Language V3 (continued)

In an instance of the `LetDecls` class, we require that no `varLis` curs twice. Slide 3.48 shows how we can achieve this.

Now that we can define our own environments, we will remove our ment with bindings for Roman numerals.

In the first example, a new environment is created binding `x` to 3 and the `+(x,y)` expression evaluates to 11.

```
let x = 3 y = 8
in +(x,y) % => 11
```

In the second `let` expression example, a new environment is create 10. The body of this `let` expression is itself a `let` expression w `z` to 3 and `y` to 8. The environment of the inner `let` extends the the outer `let`, so that the `x` in the expression `+(x,y)` is bound to expression therefore evaluates to 18.

```
let x = 10
in
  let z = 3 y = 8
  in +(x,y) % => 18
```

Language V3 (continued)

In the third example, two new environments are created. The outer
3. The inner `let` binds `x` to the value of `add1(x)` and `y` to the valu
Both of these `add1(x)` RHS expressions in the inner `let`, are eval
outer [enclosing] environment which has `x` bound to 3. Thus `add1`
to 4 *in both cases*. Thus, in the inner environment, `x` is bound to 4
to 4, so that the `+(x,y)` expression evaluates to 8.

```
let x = 3
in
  let
    x = add1(x)
    y = add1(x)
  in
    +(x,y)
  % => 8
```

Observe also that the `add1` primitive is *not* side-effecting. This
expression `add1(x)` does *not* modify the value bound to `x`: `ac`
languages behaves like `x+1` and *not* like `++x` as you would find in
as C++ and Java.

Language V3 (continued)

```
<exp>:LetExp  ::= LET <letDecls> IN <exp>
                  LetExp(LetDecls letDecls, Exp exp)
<letDecls>    **= <VAR> EQUALS <exp>
                  LetDecls(List<Token> varList, List<Exp
```

The code for `eval` in the `LetExp` class is straight-forward:

```
LetExp
%%%
    public Val eval(Env env) {
        Env nenv = letDecls.addBindings(env);
        return exp.eval(nenv);
    }
%%%
```

As we show on Slide 3.49, the `addBindings` method returns an
extends the `env` environment parameter by adding the bindings gi
declarations. We use this extended environment to evaluate the bo
expression.

Language V3 (continued)

```
<exp>:LetExp ::= LET <letDecls> IN <exp>
                 LetExp(LetDecls letDecls, Exp exp)
<letDecls>   **= <VAR> EQUALS <exp>
                 LetDecls(List<Token> varList, List<Exp
```

A `LetDecls` object has two instance variables: `varList` is a list
jects representing the `<VAR>` part of the BNF grammar rule, and `ex`
of expressions representing the `<exp>` part of the BNF grammar ru
that these are `Lists` is because the `letDecls` BNF grammar rul
Our plan for defining the `addBindings` method in the `LetDecls`
evaluating each of the expressions in `expList` in the enclosing er
binding these values to their corresponding token strings in `varL`
use these bindings to extend the enclosing environment given by th
ter, and we return this new environment to the `eval` method in the

The `LetDecls` constructor throws an exception if it finds duplica
its `varList`. This means that a `let` expression cannot have two i
same LHS idenfier. The code to check for duplicates is inserted into t
class constructor using the (*context-sensitive*) `:init` hook, so the c
cates occurs during parsing, not during expression evaluation.

Language V3 (continued)

By coincidence, the `Rands` object already has an `evalRands` met
ates each of the expressions in its `expList` instance variable, so w
the `Rands` class and its `evalRands` method here.

```
LetDecls
%%%
    public Env addBindings(Env env) {
        Rands rands = new Rands(expList);
        List<Val> valList = rands.evalRands(env);
        Bindings bindings = new Bindings(varList, va
        return env.extendEnv(bindings);
    }
%%%
```

Language V3 (continued)

The languages we have discussed do not allow mutation of variable: might be tempted to think that this Language V3 program is doing : to mutation:

```
let
  x = 3
in
  let
    x = add1(x)
  in
    +(x, x)
```

This program evaluates to 8 (which is not surprising), but in the scc `let`, the variable `x` is still bound to 3. To see this, consider the fo of this program:

```
let
  x = 3
in
  +(let x = add1(x) in x, x)
```

The last occurrence of `x` in this expression evaluates to 3 because tl the inner `let` has scope only through the inner `let` expression bc the inner `let` expression body, the binding of `x` to 3 remains uncha entire expression evaluates to 7.

Language V3 (continued)

```
<exp>:LetExp  ::= LET <letDecls> IN <exp>
                  LetExp(LetDecls letDecls, Exp exp)
<letDecls>    **= <VAR> EQUALS <exp>
                  LetDecls(List<Token> varList, List<Exp> expList)
```

Here is another observation you should pay attention to. In the rule, each `<VAR>` symbol is called the *left-hand side* (LHS) of tl the corresponding `<exp>` is called its *right-hand side* (RHS). (Do with the LHS and RHS of the grammar rule itself.) All of the RHS e `LetDecls` are evaluated in the enclosing environment. *The LHS* <\ *become bound to their corresponding RHS expression values* ***after*** *expressions have been evaluated*. Thus the following expression

```
let p = 4
in
  let
    p = 42
    x = p
  in
    x
```

evaluates to 4.

Language V4

So far our languages do not allow for anything like repetition. In
based language (ours fall into this category), repetition is typically
by recursion, and recursion depends on the ability to apply procedu
So we need to build the capability to define procedures.

In Language V4, we add procedure definitions and procedure applic
procedure is synonomous with *function*.

Think of a procedure as a "black box" that, when given zero or mo
returns a single result value. The number of inputs that a procedure a
its *arity*.

To *define* a procedure means to describe how it behaves. To *app*
means to give the procedure the proper number of inputs and to rece

Using mathematical notation, we can *define* a function f by

$$f(x) = x + 3$$

and we can *apply* the function f by

$$f(5)$$

The result of this particular application is 8.

Language V4 (continued)

In Language V4, procedures are treated as values just like integers. I
create a `ProcVal` class that extends the `Val` class. This means th
object can occur anywhere a `Val` object is expected.

Here is an example of a Language V4 program that includes a proce
and application.

```
let
  f = proc(x) +(x,3)
in
  .f(5)
```

In Language V4, a procedure definition starts with the `PROC` toke
dure application starts with a `DOT`. It is possible that you can defi
procedure in one expression, such as

```
.proc(x) +(x,3) (5)
```

Both of these expressions return the same value, namely the integer
that

```
proc(x) +(x,3)
```

also returns a value, but the value is a procedure, not an integer. (On
defining a procedure is eventually to apply it, although this is not a

Language V4 (continued)

Here are some examples of Language V4 programs using procedure

```
let
  f = proc(x,y) +(x,y)
in
  .f(3,8)
  % => 11

let
  f = proc(z,y) +(10,y)
in
  .f(3,8)
  % => 18

let x = 10
in
  let
    x = 7
    f = proc(y) +(x,y)
  in
    .f(8)
  % => 18
```

In the third example, the `x` in the `proc` definition refers to the encl is bound to `10`), not to the inner `x` (which is bound to `7`). Rememb evaluating the `letDecls`!

Language V4 (continued)

Now consider the following examples, all of which evaluate to 5:

```
let
  app = proc(f,x) .f(x)
  add2 = proc(y) add1(add1(y))
in
  .app(add2,3)

let
  app = proc(f,x) .f(x)
in
  .app(proc(y) add1(add1(y)), 3)

.proc(f,x) .f(x) (proc(y) add1(add1(y)), 3)
```

In the first example, observe that we can pass a procedure (in this a parameter to another procedure. This `app` procedure takes two returns the result of applying the first actual parameter to the second first parameter had better be bound to a procedure for this to work attempt to apply it throws an exception.)

In the second example, we have eliminated the identifier `add2` and replaced `add2` in the application `.app(add2,3)` with the name `proc(y) add1(add1(y))` that used to be called `add2`.

In the third example, we have even eliminated the identifier `app`.

Language V4 (continued)

Finally consider the following example, which evaluates to `120`:

```
let
  fact = proc(f,x)
    if x
    then *(x,.f(f,sub1(x)))
    else 1
in
  .fact(fact, 5)
```

This example, quite a bit more subtle than the previous ones, show achieve recursion – factorial, in this case – using our simple langua not yet support direct recursion!).

One final observation: `add1` is a *primitive*, not a *procedure*. Pri expressions, and they do not evaluate to anything, so you can't pas an actual parameter to a procedure. In particular, the following will

```
let
  app = proc(f,x) .f(x)
in
  .app(add1,3)
```

Be aware that the syntax for *applying* a primitive looks somewhat procedure, except that applying a primitive does *not* use a DOT.

Language V4 (continued)

We are now prepared to add syntax and semantics to support pro we add grammar rules for procedure definition and application an corresponding abstract syntax classes:

```
<exp>:ProcExp  ::= <proc>
                   ProcExp(Proc proc)
<proc>         ::= PROC LPAREN <formals> RPAREN <exp>
                   Proc(Formals formals, Exp exp)
<formals>      **= <VAR> +COMMA
                   Formals(List<Token> varList)
<exp>:AppExp   ::= DOT <exp> LPAREN <rands> RPAREN
                   AppExp(Exp exp, Rands rands)
```

Before we can go any further, we need to tackle the definition o which is what we should get when we evaluate a `ProcExp` express

Language V4 (continued)

A `ProcVal` object must capture the formal parameters as an i
`Formals` class, and it must remember its procedure *body* as an in
But what environment should we use to evaluate the procedure body
cedure is applied? In order to conform to our notion of *static scope*
to evaluate the procedure body *using the environment in which th*
defined. So any variables in the procedure body which are *not* am
parameters – in other words, the variables that *occur free* in the pro
are bound to their values in the environment in which the procedure

In Programming Languages terminology, the term *closure* refers to
captures all of the ingredients necessary to apply a procedure. In
`ProcVal` objects are closures.

Language V4 (continued)

The fields of the `ProcVal` class appear here:

```
public class ProcVal extends Val {

    Formals formals;
    Exp body;
    Env env;

    public ProcVal(Formals formals, Exp body, Env en
        this.formals = formals;
        this.body = body;
        this.env = env;
    }

    ...
}
```

Recall that we can do two things with a procedure: *define* it and *ap*
discuss procedure definition shortly, but first we give the semantic
application.

Language V4 (continued)

```
<exp>:AppExp ::= DOT <exp> LPAREN <rands> RPAREN
                 AppExp(Exp exp, Rands rands)
```

Here are the steps to evaluate a procedure *application* – in other wo an AppExp expression:

0. Evaluate exp in the current environment; this must evaluate t object (a closure) with fields formals, body, and env.

1. Evaluate rands (the *actual parameter* [a.k.a. *operand*] expr current environment to get a list of Vals (the *arguments*). [Note: the same thing when evaluating the rands of a PrimappExp.

2. a. Create bindings of the procedure's list of formal parameters the list of values obtained in step 1, and

   b. use these bindings to extend the environment (env) capture dure.

3. Evaluate the body of the procedure in the (extended) environm step 2.

Steps 2 and 3 are carried out by the apply method in the ProcV value obtained in step 3 is the value of the AppExp expression eval

Language V4 (continued)

Let's now examine the detailed semantics of a ProcExp, which is procedure.

```
<exp>:ProcExp ::= <proc>
                  ProcExp(Proc proc)
<proc>        ::= PROC LPAREN <formals> RPAREN <exp
                  Proc(Formals formals, Exp exp)
<formals>     **= <VAR> +COMMA
                  Formals(List<Token> varList)
```

As noted in Slide 3.59, a ProcVal closure is constructed with ins consisting of the list of formal parameters (a Formals object), the (an Exp object), and the environment in which the procedure is d object).

Language V4 (continued)

```
<exp>:ProcExp ::= <proc>
              ProcExp(Proc proc)
<proc>        ::= PROC LPAREN <formals> RPAREN <exp
              Proc(Formals formals, Exp exp)
<formals>     **= <VAR> +COMMA
              Formals(List<Token> varList)
```

The `makeClosure` method in the `Proc` class creates a `ProcVa` an environment.

```
Proc
%%%
    public Val makeClosure(Env env) {
        return new ProcVal(formals, exp, env);
    }
%%%
```

The semantics of the `eval` method in the `ProcExp` class is now tr

```
ProcExp
%%%
    public Val eval(Env env) {
        return proc.makeClosure(env);
    }
%%%
```

Language V4 (continued)

```
<exp>:AppExp ::= DOT <exp> LPAREN <rands> RPAREN
                 AppExp(Exp exp, Rands rands)
```

We provided the structure of the fields in the `ProcVal` class on Slic described the semantics of a procedure application on Slide 3.60. W to give Java code to *implement* application semantics.

We start with the `eval` method in the `AppExp` class. As shown on this method carries out steps 0 and 1 of procedure application sema Slide 3.60: it evaluates the `exp` expression – which should evaluate – and then it evaluates the operand expressions to get a list of `Vals`.

It then passes these arguments along to the `apply` method in the P to carry out steps 2 and 3 of application semantics. This method retu the `AppExp` expression. (As we noted earlier, the operand express the *operands* or *actual parameters*, and their corresponding values *arguments*.)

You can find the code on the following two slides.

Language V4 (continued)

```
<exp>:AppExp ::= DOT <exp> LPAREN <rands> RPAREN
              AppExp(Exp exp, Rands rands)
```

```
AppExp
%%%
    public Val eval(Env env) {
        // evaluate exp in the current environment
        Val v = exp.eval(env); // should be a ProcVa
        // evaluate rands in the current environment
        // to get the arguments (step 1)
        List<Val> args = rands.evalRands(env);
        // let v (step 0) determine what to do next
        v.apply(args, env);
    }
%%%
```

Notice that *the operand expressions (the* `rands`*) are evaluated in t*
in which the expression is applied. Also, the current environment e
the second parameter to the `apply` method in the `Val` class, even t
see that the `apply` method in the `ProcVal` class does not acually

Language V4 (continued)

The only thing we have left is to implement the behavior of the ap
the `Val` class. Since we want `apply` only to be meaningful for a Pr
we define a default behavior in the (abstract) `Val` class to throw a
anything but a `ProcVal`:

```
public Val apply(List<Val> args, Env e) {
    throw new PLCCException("Cannot apply " + this)
}
```

For a `ProcVal`, here's the implementation of `apply`. Notice that
tation carries out steps 2a, 2b, and 3 in the semantics for evaluati
application (Slide 3.60).

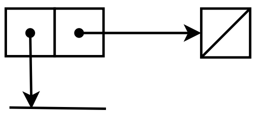
```
public Val apply(List<Val> args, Env e) {
    // bind the formals to the arguments (step 2a)
    Bindings bindings = new Bindings(formals.varList
    // extend the captured environment with these b
    Env nenv = env.extendEnv(bindings);
    // and evaluate the body in this new environment
    return body.eval(nenv);
}
```

Language V4 also checks for duplicate identifiers while parsing th
a procedure definition. This code is inserted into the `Formals` cl
using the `:init` hook.

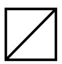
Drawing Envinronments

On Slide 2.20, we showed how to display environments as a linked l
in the list is a pair (`EnvNode`) consisting of a reference to a `Bir`
(which we have called *local bindings*) and a reference to the next r
The end of the list is an empty environment, an `EnvNull` object, w
a box with a slash through it. We usually display the linked list no
right, with the head of the list at the left and the empty environment a
display the local bindings as an array (it's actually an `ArrayLis`
stacked vertically. Each binding is a pair consisting of an identifi
value.

The *initial environment* in Language V4 is a linked list co
`EnvNode` with an empty local environment (no bindings) an
to an `EnvNull` object. Here is how we display the initial



To simplify things in Languages V4 and V5, we omit displaying the empty local environment, so we display the initial environmen



Drawing Envinronments (continued)

There are exactly two ways in which programs in Language V4 cr
ronments using the `extendEnv` method:

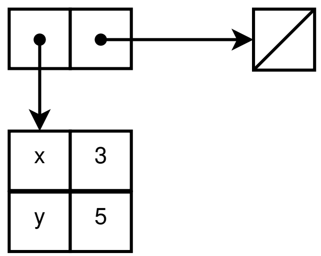
- evaluating a `let` expression
- evaluating a procedure application

A `let` expression, creates a list of local bindings: each binding uses
as its `id` field and the value of the RHS expression as its `val` fie
that the RHS expressions are evaluated *in the enclosing environm*
environment being created. An example expression is given on the r

Drawing Envinronments (continued)

```
let
  x=3
  y=5
in
  +(x,y)
```

This `let` expression creates an environment that extends the inital ronment with bindings for `x` and `y`. This extended environment is th the body expression `+(x,y)` is evaluated. The environment diagra extended environment is shown here:

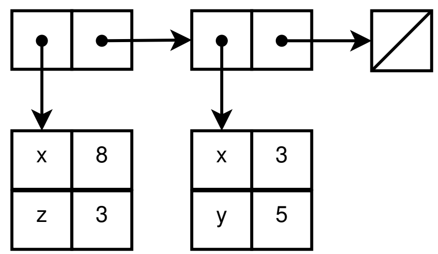


Drawing Envinronments (continued)

Now consider the following expression, with nested `lets`. The inne the environment defined by the outer let (with one node, as shown o page), so the envinroment of the inner `let` is a linked list with two

```
let
  x = 3
  y = 5
in
  let
    x = +(x,y) % the RHS evaluates to 8
    z = x      % the RHS evaluates to 3 (why?)
  in
    +(x,y)
```

In the following diagram, the leftmost node is the environment creat `let`:



The inner `let` body expression evaluates to 13 (why?).

Drawing Envinronments (continued)

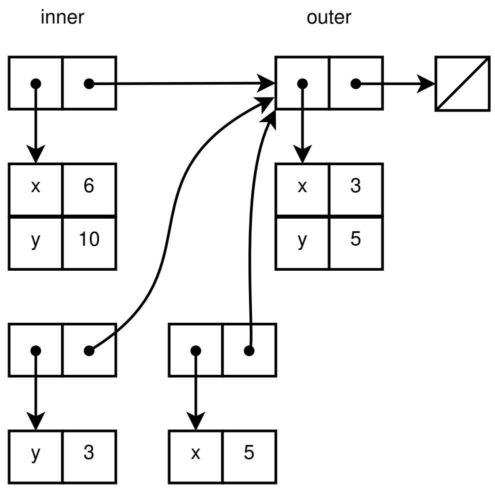
Since a `let` expression is an expression, it must evaluate to a va expression can occur as the RHS of a binding in another `let` expres this example:

```
let % outer
  x = 3
  y = 5
in
  let % inner
    x = let y=x in +(x,y) % LHS x is bound to 6
    y = let x=y in +(x,y) % LHS y is bound to 10
  in
    +(x,y) % evaluates to 16
```

The environment defined by the outer `let` has one node with bindi (to 3 and 5, respectively). The inner let extends the environment of t the inner environment has two nodes. The RHS expressions for the inner `let` are evaluated in the environment defined by the outer `let` these RHS expressions are themselves `let` expressions, each of th environment defined by the outer `let`. A total of four environme the outer `let` (extending the initial null environment), the inner `let` outer `let`), and one for each of the RHS expressions in the inner `l` the outer `let`). The next slide shows all of these environments.

Drawing Envinronments (continued)

```
let % outer
  x = 3
  y = 5
in
  let % inner
    x = let y=x in +(x,y) % LHS x is bound to 6
    y = let x=y in +(x,y) % LHS y is bound to 10
  in
    +(x,y) % evaluates to 16
```



Drawing Envinronments (continued)

In the definition of the `ProcVal` class, a `ProcVal` object has thre

```
public Formals formals; // list of formal parameters
public Exp body;        // procedure body
public Env env;         // captured environment
```

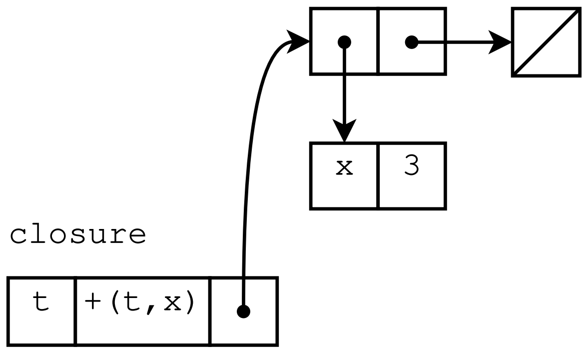
Here, the *captured environnment* is the environment in which the p
fined. For example, consider the following expression:

```
let
  x = 3
in
  proc(t) +(t,x)
```

This expression evaluates to a `ProcVal`: its `formals` field consis ing a single string, '`t`', its `body` is the expression '`+(t,x)`', and i vironment is the one defined by the `let`, having a single binding o `3`. (Recall that we also use the term *closure* to refer to a a `ProcVa` normally display a `ProcVal` object as a rectangle with its three co this order: formals, body, and captured environment. We show the comma-separated list of identifiers, the `body` as an (un-evaluated) e the captured environment (`env`) as an arrow pointing to the appropr environment in which the procedure definition occurs. The followi the `ProcVal` that results from the evaluation of the above expressi

Drawing Envinronments (continued)

```
let
  x = 3
in
  proc(t) +(t,x)
```



Drawing Envinronments (continued)

We described the rules for *applying* a procedure on Slide 3.60.
method in the `ProcVal` class, the key steps are: (2a) bind the p
mal parameters to the values of the actual parameter expressions
object); (2b) extend the captured environment with these bindings t
environment; (3) evaluate the procedure body using the new envi
value obtained in step (3) is the value of the procedure application.

Consider the following expression, which is the same as the previ
except that we apply the procedure to the actual parameter 5:

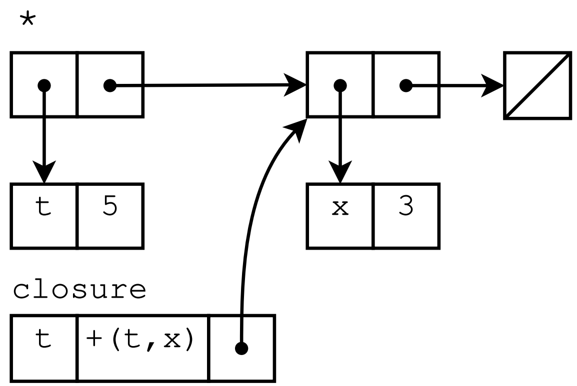
```
let
  x = 3
in
  .proc(t) +(t,x) (5)
```

From the above discussion, this procedure application creates a loca
formal parameter `t` to the value 5 (the actual parameter expression
binding is used to extend the environment captured by the procedu
tended environment is used to evaluate the body of the procedure. T
application is 8.

The following page displays the environment created by this appli
with an asterisk '⋆'.

Drawing Envinronments (continued)

```
let
  x = 3
in
  .proc(t) +(t,x) (5)
```



Drawing Envinronments (continued)

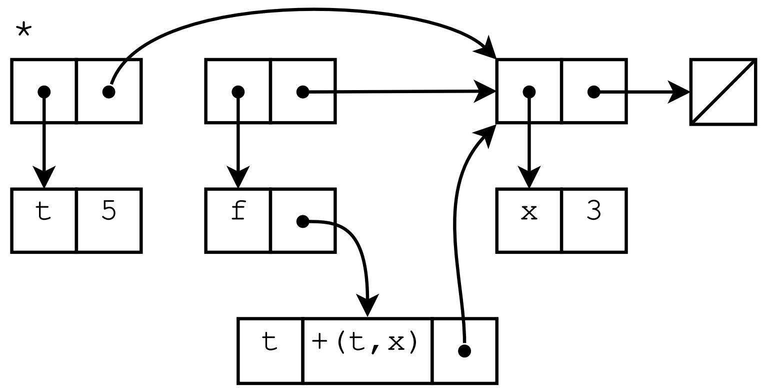
Finally, consider this example:

```
let
  x = 3
in
  let
    f = proc(t) +(t,x)
  in
    .f(5)
```

The value of this expression is also 8. Evaluating this expression cre
ronments: one with a binding of `x` to 3, another with a binding of `f`
and a third created by applying `f` to the argument 5. The resultir
diagram is shown on the following page. The environment in whi
the body of `f` is marked with an asterisk ‘$\star$’.

Drawing Envinronments (continued)

```
let
  x = 3
in
  let
    f = proc(t) +(t,x)
  in
    .f(5)
```



Language V4 (continued)

Once we have procedures, we can entirely eliminate the `let` const
example:

```
let
  p = 3
  q = 5
in
  +(p,q)
```

This can be re-written as an application of an anonymous (un-name
follows:

```
.proc(p,q) +(p,q) (3,5)
```

Language V4 (continued)

In general, a `let` expression

```
let
  v1 = e1
  v2 = e2
  ...
in
  e
```

can be re-written as an equivalent procedure application expression

```
.proc(v1, v2, ...) e (e1, e2, ...)
```

This conversion is *algorithmic*: it can be carried out unambiguous
ditch the `let` construct? The reason is simple: it's easier to think a
with a `let` in it than one without. The `let` construct aligns the LH
`v2`, *etc.* physically close to their corresponding RHS expressions e
it's cognitively easy for the reader to see how these LHS variables
to the values of their RHS expressions. In the equivalent procedure a
formal parameters `v1`, `v2`, *etc.* are physically distant from their corre
expressions, making it difficult to visualize these bindings. A `let` e
example of *syntactic sugar*: a syntactic and semantic construct th
equivalent way of expressing it in the language but that programme
read, understand, and use.

Language V4 (continued)

Though Language V4 does not support direct recursion, its suppor as first-class entities – that is, they are values that are treated the san values, so they can be passed as parameters and returned as values - as direct recursion. Here is another example that recursively com using an "accumulator" and tail recursion (we will return to this top

```
let
  fact = proc(x)
    let
      factx = proc(f, x, acc)
        if zero?(x)
        then acc
        else .f(f, sub1(x), *(x, acc))
    in
      .factx(factx, x, 1)
  in
    .fact(5)
```

Observe that the identifier `f` that appears in the `proc(f, x, acc` *formal parameter* name that binds all occurrences of `f` that appear i body. You may find it instructive to display all of the environments during the evaluation of this expression. (Replace '5' by '2' to make

Language V4 (continued)

Finally, Language V4 includes the ability to evaluate a sequence returning the value of the last expression. The component expression expression are always evaluated left-to-right. Sequence expressions particular usefulness now because our language is not *side-effectin* turn out to be useful in later languages that do support side-effects.

```
<exp>:SeqExp  ::= LBRACE <exp> <seqExps> RBRACE
                  SeqExp(Exp exp, SeqExps seqExps)
<seqExps>     **= SEMI <exp>
                  SeqExps(List<Exp> expList)
```

The semantics of evaluating a `SeqExp` are given here:

```
SeqExp
%%%
    public Val eval(env) {
        Val v = exp.eval(env);
        for (Exp e : seqExps.expList)
            v = e.eval(env);
        return v;
    }
%%%
```

Language V4 (continued)

Observe that we evaluate every expression in the list but only return

```
{1;3;5}
  % => 5

{42}
  % => 42
```

We can use the sequence construct to enclose a single expression th
wise look too unwieldy. Here's an example:

```
.{proc(t,u) +(t,u)} (3,4)
```

The braces in this expression are not required, but they may help t
extent of the `proc` definition. Don't get into the habit of doing th
basis, however: throwing in extra braces can result in an expressior
essarily *noisy* and that is actually more difficult to read than one wit

Language V5

We normally prefer to use direct recursion instead of using the contr
able) tricks on Slides 3.56 and 3.80. For example, we would like to

```
let
  fact = proc(x) if zero?(x) then 1 else *(x,.fact(s
in
  .fact(5)
```

But this does not work! Why??

Remember that in a `let`, the RHS expressions (the expressions to
'=' tokens) are all evaluated in the environment that encloses the l
all the RHS expressions have been evaluated do we bind each of the
to their RHS values.

In the definition of the `proc` above, the `proc` body refers to the i
which is free in the procedure definition – there is no `fact` in its
parameters – but this identifier is not bound to a value in the enclosin
Thus an attempt to apply the `proc` fails because of an unbound ider

Language V5 (continued)

In order to solve this problem, we create a new let-like environme direct recursion. Called letrec, it allows us to define procedur direct recursion.

This is what we want:

```
letrec
  fact = proc(x) if zero?(x) then 1 else *(x,.fact(s
in
  .fact(5)
  % => 120
```

Language V5 (continued)

Here is the grammar rule and associated abstract syntax class:

```
<exp>:LetrecExp ::= LETREC <letDecls> IN <exp>
                   LetrecExp(LetDecls letDecls, Exp exp)
```

The RHS expressions in a letrec are evaluated in the order in pear, using an envionment where *all* of the previous (LHS, RHS) letrec are acesssible. In addition, if the RHS is is a procedure, entire environment created by *all* of the (LHS, RHS) bindings in the means that procedures defined in a letrec can refer to each othe they can call themselves recursively.

This is unlike a normal let, in which the RHS expressions are all e *enclosing* environment, and the (LHS, RHS) bindings created by th accessible in the body of the let. Moreover, in a let, the RHS e be evaluated in any order, which is sometimes called *parallel evalu*

Notice that the syntax of a letrec is the same as the syntax of a difference between the semantics of let and letrec is in the wa build the environment in which the letDecls bindings are create

We proceed to describe how letrec evaluation is handled.

Language V5 (continued)

To implement the recursive behavior of a `letrec` as described abo
new method `addLetrecBindings` in the `LetDecls` class. Th
is passed the environment in which the `letrec` expression appear
a new environment as described in the following steps.

0. Extend the environment actual parameter with an empty `Bind`
sufficient size to hold all of the variable bindings. Assign this ne
to the `env` parameter.

1. The two fields in the `LetDecls` class are `List<Token>`
`List<Exp> expList`. Create iterators for these two lists and
them together, in order. For each step in the iteration, get the next
from the `varList`, and save its `String` representation in a
Also, get the next expression `exp` from the `expList`, evaluate
ronment `env` obtained in Step 0, and save its value in a varial
create a `new Binding(str, val)` and add it to the `env` e
tained in Step 0. This binding now becomes part of the local bi
together with the other local bindings previously added during t

2. Once all of the new bindings have been added to `env`, return e
of this method.

Language V5 (continued)

The implementation of `addLetrecBindings` in the `LetDecls`
here:

```
LetDecls
%%%
    public Env addLetrecBindings(Env env) {
        // Step 0
        env = env.extendEnv(new Bindings(varList.si
        // Step 1
        Iterator<Token> varIter = varList.iterator(
        Iterator<Exp> expIter = expList.iterator();
        while (varIter.hasNext()) {
            String str = varIter.next().toString();
            Val val = expIter.next().eval(env);
            env.add(new Binding(str, val));
        }
        return env; // Step 2
    }
%%%
```

Notice that we have previously defined an `addBindings` r
`LetDecls` class (see Language V3) used to implement the `eva`
a `let` expression. The `addLetrecBindings` method simply be
part of the `LetDecls` class.

Language V5 (continued)

```
<exp>:LetrecExp ::= LETREC <letDecls> IN <exp>
                    LetrecExp(LetDecls letDecls, Exp ex
```

Recall that the `LetDecls` constructor checks for duplicate LHS id
parsing. Since the `LetrecExp` grammar rule uses `LetDecls`,
pression also makes this check.

We can now evaluate a `LetrecExp` object in exactly the same wa
object:

```
LetrecExp
%%%
    public Val eval(Env env) {
        Env nenv = letDecls.addLetrecBindings(env);
        return exp.eval(nenv);
    }
%%%
```

The principal idea, then, is to evaluate the RHS expressions of a
environment that (self-referentially) includes all of the bindings in t

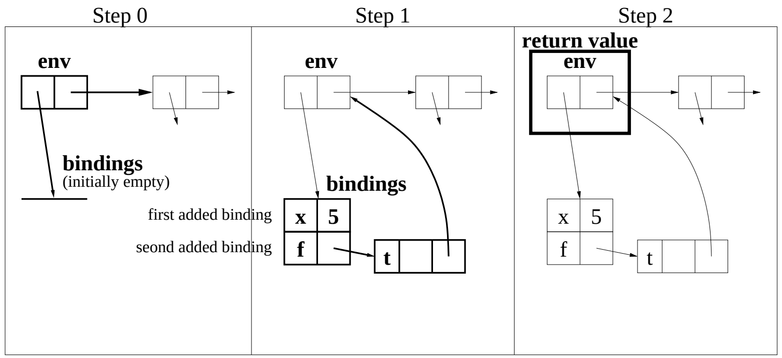
Language V5 (continued)

This picture illustrates the three steps carried out in `addLetrecBindings`, for the following l

```
letrec
    x = 5
    f = proc(t) *(t,x)
in
    .f(42)
```

0. Create an extended environment by extending the enclosing environment with an empty list
1. In the order in which the LHS identifiers appear, create a `Binding` of the LHS identifier
   example) to the value of its corresponding RHS expression (5 and then `proc(t)...` in t
   each RHS expression is evaluated in the extended environment – and add this binding to the e
2. Once all of the (LHS, RHS) bindings have been added to the extended environment, return
   ment as the value of `addLetrecBindings`.

Observe that the environment captured by procedure `f` knows about the binding of `x` to `5`, so the
evaluates to 210. If this had been a `let` instead of a `letrec`, the `x` in the body of `f` would be un



Language V5 (continued)

The `letrec` construct allows us to define *mutually recursive pro* or more procedures that call each other in a recursive fashion. F example:

```
letrec
  even? = proc(x) if zero?(x) then 1 else .odd?(sub
  odd? = proc(x) if zero?(x) then 0 else .even?(sub
in
  .even?(11) % => 0 (false)
```

[**Exercise** (not to hand in): See if you can define the `odd?()/eve` recursive procedures in Language V4 without `letrec`.]

Notice that we have used `?` in the variable names for the `even?` a cedures to suggest that these procedures should be considered as return true (1) or false (0). This is a lexical feature we have added to and beyond.

Language V6

So far, our source language has no capability to define top-level var sist from one expression evaluation to another. A *top-level* variable a binding in the initial ("top-level") environment. We would like to guages to allow for such definitions. All we need to do is to add *initial environment*, the environment that all expressions in the sourc tend.

The initial environment for our languages is the `static Env e` the `Program` class, obtained by calling the `initEnv()` static met class. Notice that this initial (top-level) environment starts out havin of bindings.

Our strategy for making top-level definitions is to take advantage of t in the `Env` class. To add a new top-level definition, we create a `Bi` and add it to the top-level `Env` object. Once we add bindings in bindings will be known in any subsequent expression evaluation that environment.

Language V6 (continued)

Since a "program" can now have two forms – a top-level "define" o
evaluation, we need to have two grammar rules for the `<program`
Here are their grammar rules and corresponding abstract syntax clas

```
<program>:Define ::= DEFINE <VAR> EQUALS <e
                     Define(Token var, Exp exp)
<program>:Eval   ::= <exp>
                     Eval(Exp exp)
```

Language V6 (continued)

Here is an example of expressions that use the `define` feature in
guage:

```
define i = 1
define ii = add1(i)
define iii = add1(ii)
define v = 5
define x = 10
define f = proc(x) if zero?(x) then 1 else *(x,.f(.
.f(v)   % ERROR: g is unbound
define g = proc(x) sub1(x)
.f(v)   % => 120 -- g is now bound
.f(iii) % => 6
```

As long as you stay in the `Rep` loop, the `define`d variable bindi
bered.

Notice that, in the definition for `f`, the body of the procedure refers
named `g`, but `g` hasn't been defined yet. The attempt, in the next l
`.f(v)` fails. After defining `g` on the following line, evaluating `.f(`
is because by the time you attempt to apply `f` the second time, the
been defined, and the body of `f` now recognizes its definition.

Language V6 (continued)

Notice that for top-level procedure definitions, `define` works simi
in terms of being able to support direct recursion. This is because
procedure definition captures (in a closure) the initial environment, w
ified every time another top-level definition is encountered. When
binding to the top-level environment, the binding gets added to the
instead of extending the top-level environment. In this way, all of th
sures can access this binding, as well as any others that may crop up
following works:

```
define even? = proc(x)
  if zero?(x) then 1 else .odd?(sub1(x))
.even?(11) % => Error: unbound procedure od
define odd? = proc(x)
  if zero?(x) then 0 else .even?(sub1(x))
.even?(11) % => 0
.odd?(11)  % => 1
```

Observe that a top-level define can *redefine* a previous definition.
looking up the LHS identifier in the top-level environment. If a
identifier already exists in the top-level environment, we replace the
with the value of the new RHS.

Language V6 (continued)

Since a `define` evaluates its RHS in the *current* environment, we c
save) the value of a variable using a `let`, even though a subsequent
redefine the variable. Consider this example:

```
define x = 2
define f = proc() x % x is the top-level x
.f()                % evaluates to 2
define x = 3        % redefine top-level x
.f()                % now evaluates to 3
```

Compare this to the following:

```
define x = 2
define f =
  let
    x = x           % the RHS is the curren
                    % and the LHS is a loca
  in
    proc() x        % the proc captures the
.f()                % evaluates to 2
define x = 3        % redefine top-level x
.f()                % local copy still eval
```