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A *language*, in computer science theory, is a set of strings, where a *string* is a finite ordered sequence of *symbols* chosen from a given *alphabet*. Computer science theory deals principally with how to specify languages using things such as Non-deterministic Finite Automata (NFAs), Context-Free Grammars, and Turing Machines (TMs).

A *programming language* also defines a language in the theory sense, except that the strings in a programming language are called *programs*, and the symbols are called *tokens*. The *syntax* of a programming language is a set of rules used to specify the programs in the language. A programming language also defines the run-time behavior of a program, called its *semantics*, which we discuss at length later.

Our first step is to describe a way in which we can define the syntax of a programming language. We start out with two simple examples of languages that describe familiar data structures.

BNF is a meta-language used to specify a context-free grammar. Almost every modern programming language uses some sort of BNF notation to define its syntax. We work first with examples of languages that define two simple data structures: lists and trees. Remember that the “alphabet” of a programming language is a set of tokens, so any definition of the syntax of the language must first specify its tokens. In the previous section, we showed how to do this in the context of PLCC.

Our first example is to define a language whose “programs” are lists of numbers. Some sample “programs” in this language are:

```
( 3  4  5 )  
(      7 11  )  
( )
```

Backus-Naur Form (BNF) (continued)

1.3

Here is a BNF definition of this language, using three BNF formulas:

`<lon> ::= LPAREN <nums> RPAREN`

`<nums> ::= NUM <nums>`

`<nums> ::=`

In these formulas, the token names LPAREN and RPAREN stand for left parenthesis ‘(’ and right parenthesis ‘)’, respectively. The token name NUM stands for any (unsigned) decimal number. [Think about what regular expressions would match these.] Conforming to PLCC specifications, we use all-uppercase letters for our token names.

The nonterminal `<lon>` stands for “list of numbers”.

```
<lon> ::= LPAREN <nums> RPAREN  
<nums> ::= NUM <nums>  
<nums> ::=
```

Every BNF formula has the form

$$LHS ::= RHS$$

The *LHS* (Left-Hand Side) of a BNF formula always has the form `<nonterm-symbol>` where `nonterm-symbol` is an identifier, normally written in lowercase. A `<nonterm-symbol>` expression is called a *nonterminal*. In the example above, the nonterminals are `<lon>` and `<nums>`.

The *RHS* (Right-Hand Side) of a BNF formula is a (possibly empty) ordered list of token names and nonterminals.

Notes: The term *syntactic category* is sometimes used instead of the term *nonterminal*, and the term *terminal* is sometimes used instead of *token name*. Instead of using a token name such as `LPAREN`, some BNF formulas just use the corresponding actual character string such as `'('`. In the examples on page 1.2, you can see that we have also ignored whitespace.

BNF (continued)

1.5

```
<lon> ::= LPAREN <nums> RPAREN  
<nums> ::= NUM <nums>  
<nums> ::=
```

BNF has some shortcuts **that we do not use** but that you may run into in your reading. These shortcuts are usually called Extended BNF, or simply EBNF. For example, instead of writing two different formulas with `<nums>` on the LHS, one can use *alternation* notation “|”:

$$\langle \text{nums} \rangle ::= \text{NUM } \langle \text{nums} \rangle \mid \epsilon$$

Note: ‘ ϵ ’ means the empty string.

One could also use the *Kleene star* notation to define `<nums>`:

$$\langle \text{nums} \rangle ::= \{ \text{NUM} \}^*$$

Note: We use a variant of the Kleene star notation later.

Parsing (syntactic derivation):

1.6

```
<lon> ::= LPAREN <nums> RPAREN  
<nums> ::= NUM <nums>  
<nums> ::=
```

The purpose of a BNF grammar is to define the set of “legal” token sequences that conform to the grammar rules. This set is the *language* of the grammar. In the context of programming languages, these legal token sequences are called “programs”. We also use the term *syntax rules* to refer to the rules given by a BNF grammar, and we use the term *syntactically correct* to refer to token sequences that conform to the grammar rules. Finally, when there is little chance for confusion, we use the term *sentence* to refer to a finite length token sequence.

If we had a set of grammar rules for the Java programming language, for example, then the set of sentences that conform to this grammar would be the set of all syntactically correct Java programs.

Parsing (syntactic derivation):

1.7

```
<lon> ::= LPAREN <nums> RPAREN  
<nums> ::= NUM <nums>  
<nums> ::=
```

A grammar can be used:

- to construct syntactically correct sentences, or
- to check to see if a particular target sentence belongs to the language (*i.e.*, is syntactically correct).

A programmer's job is to construct syntactically correct programs in a programming language, illustrating the first use of a grammar. This activity is called *programming*.

A compiler's job is, in part, to check a sentence for syntactic correctness, illustrating the second use of a grammar. This activity is called *parsing*.

Parsing (continued):

1.8

Given a BNF grammar, this algorithm parses a target sentence using a *leftmost* derivation. The algorithm returns “success” if the parse is successful, “failure” otherwise.

1. Find the *start symbol* of the grammar, which is usually the first LHS nonterminal in the set of grammar rules. This nonterminal is the initial *sentential form* of the derivation. (We call it a sentential form because it may still have some token names or nonterminal symbols in it. Once all of these have been removed, using the steps given below, the result is a sentence.) Set the *unmatched* sentence to the target sentence.
2. Repeat Step 3 (see the next page) until the sentential form is a sentence – *i.e.*, consists only of strings (no nonterminals, no token names).

3. (a) If the leftmost unmatched term in the sentential form is a **token name**, match it with the leftmost string in the unmatched sentence. [Here, *match* means that the token name describes exactly the string to be matched. For example, the token name LPAREN matches the string ‘ (’ and NUM matches the string ‘ 4 2 ’. The term *lexical analysis* refers to this process.] If there is no match, return failure. If there is a match, replace the leftmost token name in the sentential form with its matching string from the unmatched sentence and remove the matched string from the unmatched sentence.
- (b) If the leftmost unmatched term in the sentential form is a **nonterminal**, choose a rule from the grammar with this nonterminal as its LHS and replace the nonterminal with the (possibly empty) RHS of the chosen rule. [Which rule to choose depends on finding a rule that is most likely to complete the derivation. For some grammars, there is only one choice: these grammars are said to be *predictive*. All of the grammars we use in this course are predictive.] If no rule can apply, return failure.
4. If the unmatched sentence is empty, return success: the target sentence has been successfully parsed. Otherwise, return failure.

It is possible, for some grammars, that Step 3 loops indefinitely. However, for all of the grammars that we encounter in this course, this step exits in a finite number of iterations.

Parsing (continued):

1.10

```
<lon> ::= LPAREN <nums> RPAREN  
<nums> ::= NUM <nums>  
<nums> ::=
```

We perform a *leftmost derivation* of the target string (14 6). Always start with the first nonterminal in the grammar (the *start symbol* – in this case it's <lon>) as the sentential form:

<u>sentential form</u>	<u>unmatched sentence</u>	<u>algorithm step taken</u>
<lon>	(14 6)	1
⇒ LPAREN <nums> RPAREN	(14 6)	3b
⇒ (<nums> RPAREN	14 6)	3a
⇒ (NUM <nums> RPAREN	14 6)	3b
⇒ (14 <nums> RPAREN	6)	3a
⇒ (14 NUM <nums> RPAREN	6)	3b
⇒ (14 6 <nums> RPAREN)	3a
⇒ (14 6 RPAREN)	3b
⇒ (14 6)		3a
⇒ <i>success!</i>		4

In the above derivation, the leftmost unmatched token name or nonterminal is shown in **boldface**.

A derivation ends successfully when there are no token names or nonterminals in the sentential form and no unmatched tokens.

Parsing (continued):

1.11

```
<lon> ::= LPAREN <nums> RPAREN
<nums> ::= NUM <nums>
<nums> ::=
```

Next we attempt a parse of ‘(14 (6)’ using this grammar. As before, we start with the first nonterminal `<lon>` as the sentential form:

sentential form

`<lon>`

⇒ **LPAREN** <nums> RPAREN
⇒ (**<nums>** RPAREN
⇒ (**NUM** <nums> RPAREN
⇒ (14 **<nums>** RPAREN
⇒ (14 **NUM** <nums> RPAREN
⇒ (14 **<nums>** RPAREN
⇒ (14 **RPAREN**
⇒ *failure!*

unmatched sentence

(14 (6)

(14 (6)

14 (6)

14 (6)

(6) – *try first <nums> rule ...*

?? NUM doesn't match " ("

(6) – *try second <nums> rule ...*

?? RPAREN doesn't match " ("

We can conclude that the string ‘(14 (6)’ does not conform to the grammar specifications, so it is *not* a “list of numbers”.

A tree is another data type that can be specified in BNF form:

```
<tree> ::= NUM
```

```
<tree> ::= LPAREN SYMBOL <tree> <tree> RPAREN
```

Here, SYMBOL is a token name that represents a string of alphanumeric characters starting with a letter. The NUM, LPAREN and RPAREN token names are as before.

Here are some examples of trees that you can check for syntactic correctness using the parsing algorithm given above.

```
3
( bar 1 ( foo 1 2 ) )
( bar ( biz 3 4 ) ( foo 1 2 ) )
```

```
<tree> ::= NUM  
<tree> ::= LPAREN SYMBOL <tree> <tree> RPAREN
```

BNF rules are *context-free*. This means that in a given sentential form, the nonterminal `<tree>` can be replaced by the right-hand side of *either* rule having `<tree>` as its left-hand side. For example, parsing the string ‘(bar 1 (foo 1 2))’ gives this:

<tree>

⇒ **LPAREN SYMBOL** <tree> <tree> RPAREN

⇒ (bar **<tree>** <tree> RPAREN [**see below...*]

⇒ (bar **NUM** <tree> RPAREN

⇒ (bar 1 **<tree>** RPAREN

⇒ (bar 1 **LPAREN** SYMBOL <tree> <tree> RPAREN RPAREN

⇒ ...

Note: this line collapses two **match steps into one:*

LPAREN ⇒ ' ('

SYMBOL ⇒ bar

Recall our grammar for a list of numbers:

```
<lon> ::= LPAREN <nums> RPAREN  
<nums> ::= NUM <nums>  
<nums> ::=
```

We have seen how to take a sentence in this language and use a leftmost derivation to parse it. What can we do to automate this algorithm?

A *parser* is a program that takes as input a sentence in a language and that carries out a parse of the sentence, producing either success or failure. Building a parser from the BNF specification of a language is conceptually simple: we only need to write a program to carry out the steps of the parsing algorithm. The difficulty is the algorithm's essential *nondeterminism*: given a nonterminal in a sentential form, how do we replace this nonterminal with the right-hand side of the “correct” grammar rule having this nonterminal as its left-hand side? (For some grammars, there may even be more than one “correct” rule that leads to a successful parse – such grammars are said to be *ambiguous*.)

```
<lon> ::= LPAREN <nums> RPAREN  
<nums> ::= NUM <nums>  
<nums> ::=
```

There are several tool sets that can be used to convert a BNF-like grammar specification into a parser. Many of these tool sets use C or C++ as the target language: input to such a tool set is a grammar specification, and the output is a set of target language programs that, when compiled, produces a parser. Learning how to use some of these tool sets can be a daunting task, and the generated parsers can be obscure. Furthermore, most of these tool sets are split into two separate parts: a lexical analysis (scanning) part and a syntax analysis (parsing) part.

The PLCC tool set is designed to make it easy to build a scanner and parser for a large collection of programming languages. While it is not “industrial strength” like many of the standard tool sets, it is easy to learn. The target language for PLCC is Java, so all of the source code that PLCC generates consists of self-contained Java programs. To use PLCC, you only need to be comfortable reading and writing programs in Java.

To say that a parser returns only success or failure is cold comfort, since in most cases you will want to “run” a program once it parses successfully. So a parser generally does one of two things: it “runs” the program *as it carries out the parse algorithm* (which is called *interpretation on the fly*), or it produces some form of output that can later be used to run the program *once the parse is complete*. PLCC uses the latter approach, since generating a parser is simpler if it is divorced from any attempts to carry out run-time behavior during the parse.

The output of a PLCC parse is a *parse tree* of a program: more specifically, it is a Java object that is the root of the parse tree. We show how this works by elaborating on the list-of-numbers example.

```
<lon> ::= LPAREN <nums> RPAREN  
<nums> ::= NUM <nums>  
<nums> ::=
```

From each BNF grammar rule for a language, PLCC generates Java class. A rule such as

```
<lon> ::= LPAREN <nums> RPAREN
```

generates the Java class `Lon`. PLCC gets the name `Lon` from the LHS nonterminal `<lon>` of this grammar rule by converting the first letter of this nonterminal name to uppercase.

PLCC determines the *fields* of a Java class generated from a BNF grammar rule from the RHS terms of the rule – specifically, the terms that are enclosed in angle brackets `<...>`. The RHS of the `lon` grammar rule given above has one term in angle brackets: `<nums>`. The corresponding field name in the `Lon` class is `nums` (Java field names always begin with a lowercase letter). The *type* of this field is `Nums`.

Since `<nums>` appears as the LHS nonterminal on the second and third grammar rules for this language, PLCC generates a corresponding `Nums` class.

```
<lon> ::= LPAREN <nums> RPAREN  
<nums> ::= NUM <nums>  
<nums> ::=
```

However, there are two grammar rules with `<nums>` as the LHS nonterminal. PLCC generates the class name `Nums` automatically (by converting the first character of the LHS nonterminal name to uppercase), but PLCC must generate a *unique* Java class name for each grammar rule. We accomplish this by annotating the LHS nonterminal on each of these lines with a Java class name that is different from `Nums` and that distinguishes one from the other. We modify these grammar rules with annotations as follows:

```
<nums>:NumsNode ::= NUM <nums>  
<nums>:NumsNull ::=
```

(Any Java class names are OK for these annotations, but good naming conventions should prevail, and the names must be unique among the Java class names that PLCC generates.) A colon is used to separate the nonterminal from its annotated Java class name. The RHS entries of these grammar rules are unchanged.

With these modifications, PLCC generates two new classes, `NumsNode` and `NumsNull`. Both of these classes are declared to extend the `Nums` class, so an instance of a `NumsNode` class, for example, is also automatically an instance of its parent `Nums` class.

Here is a complete text file representation of the language, including its lexical specification and grammar rules, suitable for processing by `plccmk`. Notice that we have included the annotations for the two `<nums>` rules as described on the previous slide:

```
# Language specification for a list of numbers
# Lexical spec
skip WHITESPACE '\s+'
NUM '\d+'
LPAREN '\('
RPAREN '\)'
%
# BNF rules
<lon> ::= LPAREN <nums> RPAREN
<nums>:NumsNode ::= NUM <nums>
<nums>:NumsNull ::=
%
```

```
# Language specification for a list of numbers
# Lexical spec
skip WHITESPACE '\s+'
NUM '\d+'
LPAREN '\('
RPAREN '\)'
%
# BNF rules
<lon>          ::= LPAREN <nums> RPAREN
<nums>:NumsNode ::= NUM <nums>
<nums>:NumsNull ::=
%
```

To summarize, the top lines of the file, up to the first percent (%) line, constitute the *lexical specification* of the language. For this language, these lines say that whitespace (including spaces, tabs, and newlines) should be skipped, that a NUM is a string of one or more decimal digits, and that LPAREN and RPAREN match the characters ‘ (’ and ‘) ’, respectively.

The remaining lines of the file constitute the *syntax specification* of the language, given in BNF form. PLCC takes this file as input and generates a collection of Java source files in a `Java` subdirectory that implement a scanner and parser for the language.

```
# Language specification for a list of numbers
# Lexical spec
skip WHITESPACE '\s+'
NUM '\d+'
LPAREN '\('
RPAREN '\)'
%
# BNF rules
<lon> ::= LPAREN <nums> RPAREN
<nums>:NumsNode ::= NUM <nums>
<nums>:NumsNull ::=
%
```

To complete this example, assume that you have created a directory named `LON`, and that in this directory you have created a file named `grammar` that contains lines appearing above. In your `LON` directory, run the `plccmk` script as follows:

```
plccmk
```

Your script output should appear as follows:

```
Nonterminals (* indicates start symbol):
```

```
*<lon>
```

```
<nums>
```

```
Abstract classes:
```

```
Nums
```

```
Source files created:
```

```
NumsNode.java
```

```
...
```

```
<lon>                ::= LPAREN <nums> RPAREN  
<nums>:NumsNode      ::= NUM <nums>  
<nums>:NumsNull      ::=
```

The above text shows just the grammar rules section of the `grammar` file we have been examining.

In your `LON` directory, change to a subdirectory named `Java`. This subdirectory was created and populated by the `plccmk` command. In this subdirectory you will find (among other things) the following Java source files:

```
Lon.java  
Nums.java  
NumsNode.java  
NumsNull.java
```

Each of these corresponds to one or more of the grammar rule lines. For example, the line beginning with `<lon>` results in the file `Lon.java` being created in the `Java` subdirectory. As you can see from looking at the Java code in the `Java` subdirectory for the `NumsNode` and `NumsNull` classes, both of these classes extend the `Nums` abstract class. This is because the `<nums>` nonterminal appears as the LHS of two grammar rules.

For every grammar rule, PLCC creates a Java class uniquely associated with the rule. For grammar rules that have the same nonterminal appearing on the LHS of multiple rules, PLCC creates an abstract class based on the nonterminal name, and the annotated class names become derived classes of this abstract class.

```
<lon>          ::= LPAREN <nums> RPAREN
<nums>:NumsNode ::= NUM <nums>
<nums>:NumsNull ::=
```

PLCC generates a stand-alone parser with class name `Parser` that uses the Java classes created from processing the lexical and grammar specification sections. To run this parser, change to the Java subdirectory (of the LON directory, in this case) and run the `Parser` class file with command-line arguments consisting of strings that you want to parse. For example:

```
java Parser "(14 6)" "(14 (6))" "(42)" "()"
```

The output produced looks something like this:

```
(14 6) -> Lon@15db9742
(14 (6)) -> java.lang.RuntimeException: Nums cannot begin with LPAREN
(42) -> Lon@7852e922
() -> Lon@6d06d69c
```

For each command-line argument, the parser prints a copy of the argument, then a string ‘->’, and finally a string of the form `Lon@...`

The `Lon@...` strings are simply the `toString()` values of the instances of the `Lon` class that are created for each **successful** parse. For any grammar, an instance of the class associated with the *start symbol* of the language is always the root of the parse tree generated by the parser. If the parse fails for a particular command-line string, the parser prints an error message giving the nature of the error.

```
<lon>          ::= LPAREN <nums> RPAREN  
<nums>:NumsNode ::= NUM <nums>  
<nums>:NumsNull ::=
```

PLCC also generates an interactive parser called `Rep` that resides in the `Java` subdirectory along with the `Parser` program. `Rep` executes a loop that prints a prompt, *Reads* program input from the keyboard, *Evaluates* (parses) the program, and *Prints* the result: a human-readable representation of the root of the program's parse tree. Recall that for the language given above, the root of the parse tree is an instance of the `Lon` class.

Here is a sample interaction with the `Rep` program using this grammar:

```
$ java Rep  
--> (14 6)  
Lon@15db9742  
--> (14 (6)  
java.lang.RuntimeException: Nums cannot begin with LPAREN  
--> (42)  
Lon@6d06d69c  
--> ()  
Lon@7852e922  
-->
```

Observe that the output is similar to that produced by the `Parser` program

```
<lon>          ::= LPAREN <nums> RPAREN
<nums>:NumsNode ::= NUM <nums>
<nums>:NumsNull ::=
```

The Rep program prints only the Java object that represents the root of the program's parse tree. To see how the objects in the parse tree are being constructed as the parse proceeds, you can use the `-t` switch when invoking Rep. The `-t` stands for `trace`. Here's a sample interaction using this feature, with the output edited for the sake of readability:

```
$ java Rep -t
--> (14 6)
<lon>
| LPAREN "("
| <nums>:NumsNode
| | NUM "42"
| | <nums>:NumsNode
| | | NUM "6"
| | | <nums>:NumsNull
| RPAREN ")"
```

In this example, the root of the parse tree is a `Lon` object whose `nums` field is an instance of `NumsNode`. This instance in turn has a `nums` field that is also an instance of `NumsNode`. And finally, this instance has a `nums` field that is an instance of `NumsNull`. A `NumsNull` object has no fields. The trace also shows how each token is matched by the parser, along with the lexeme from the input program that matched the token.

In order to make the connection clear between a grammar rule and its PLCC-generated Java class, we often display the grammar rules in the following way:

<code><lon></code>	<code>::= LPAREN <nums> RPAREN</code>
	<div><code>Lon (Nums nums)</code></div>
<code><nums> : NumsNode</code>	<code>::= NUM <nums></code>
	<div><code>NumsNode (Nums nums)</code></div>
<code><nums> : NumsNull</code>	<code>::=</code>
	<div><code>NumsNull ()</code></div>

The item in the box following a grammar rule is the *Java signature of the Java class constructor* corresponding to the class PLCC generates from the grammar rule. So the box

`Lon (Nums nums)`

means that the constructor for the PLCC-generated class `Lon` has a single parameter `nums` of type `Nums`.

There is a one-to-one correspondence between the types and formal parameters in the constructor and the types and field names in the class. More specifically, when the constructor is invoked, the constructor body simply copies the values of its parameters into the corresponding field names of the instance being constructed.

As you can see from above, the `NumsNull` constructor takes no parameters, and the `NumsNull` class has no fields.

```
<lon>          ::= LPAREN <nums> RPAREN  
<nums>:NumsNode ::= NUM <nums>  
<nums>:NumsNull ::=
```

The parse tree for a program in this language accurately represents the length of the list of numbers given as input – count the number of instances of NUM in the parse trace given on slide 1.27 above – but the actual values of the NUM tokens are not preserved in the parse tree. The problem is that the parse tree instances of NumsNode objects do not have fields corresponding to their token values.

To remedy this situation, we need to have a field in the `NumsNode` class that captures the value of the `NUM` token. We use angle brackets for all nonterminals on the RHS of grammar rules, and these nonterminals automatically become fields in the Java class. To capture values of the tokens on the RHS, we use angle brackets for these token names as well. This means that the `NumsNode` line now looks like this:

```
<nums> : NumsNode ::= <NUM> <nums>
```

The ‘<NUM>’ entry creates a field named `num` (which is the name ‘NUM’ converted to lowercase) of type `Token` (since `NUM` is the name of a token in the `grammar` file).

Observe that it’s unnecessary to capture tokens that always have the same string representation, like `LPAREN`. Every instance of the `LPAREN` token looks like every other instance, so there’s no need to distinguish among them. This is not so with tokens that can take on multiple string values, such as `NUM`. Indeed, for a list of numbers, knowing exactly *what* numbers are in the list can be essential – for example, if you want to find the sum of the numbers in the list. If these items do not appear as fields in the PLCC-generated classes, their values do not appear in the parse tree, and so their values are not retrievable after the parse.

The revised grammar now looks like this:

```
<lon>                ::= LPAREN <nums> RPAREN
<nums>:NumsNode      ::= <NUM> <nums>
<nums>:NumsNull      ::=
```

Since we now have two fields in the `NumsNode` class, we need to modify the signature of the `NumsNode` constructor, as shown here (compare with slide 1.28):

```
<lon>                ::= LPAREN <nums> RPAREN
                        Lon (Nums nums)
<nums>:NumsNode      ::= <NUM> <nums>
                        NumsNode (Token num, Nums nums)
<nums>:NumsNull      ::=
                        NumsNull ()
```


Let's return to our tree example. Here is a modified grammar for a tree as originally given on slide 1.12 that takes into account the PLCC requirements for unique class names and the introduction of fields for the NUM and SYMBOL tokens: The Java signatures for the constructors are also given.

```
<tree>:Leaf      ::= <NUM>  
                        Leaf(Token num)  
<tree>:Interior ::= LPAREN <SYMBOL> <tree> <tree> RPAREN  
                        Interior(Token symbol, Tree tree, Tree tree)
```

But there is a problem with the `Interior` constructor signature. Java does not allow multiple field names or constructor formal parameters with the same name: specifically, in this case, there cannot be two field names with the name `tree`. Here is what PLCC has to say about this when given the above grammar (the line has been folded for clarity):

```
duplicate field name tree in rule  
RHS LPAREN <SYMBOL> <tree> <tree> RPAREN
```

The solution is to use different identifiers for these field names and to their corresponding constructor formal parameters. PLCC allows duplicate RHS fields (in angle brackets) to be annotated – much like we have seen for duplicate LHS nonterminal names – with alternate names that avoid this conflict.

In the case we are considering, we can resolve this issue in the RHS of the `<tree>:Interior` grammar rule by using the identifier `left` for the first `<tree>` field and the identifier `right` for the second `<tree>` field, as shown here:

```
<tree>:Interior ::= LPAREN <SYMBOL> <tree>left <tree>right RPAREN  
Interior(Token symbol, Tree left, Tree right)
```

As a result, the following grammar is acceptable to PLCC:

```
<tree>:Leaf      ::= <NUM>  
                               Leaf(Token num)  
<tree>:Interior ::= LPAREN <SYMBOL> <tree>left <tree>right RPAREN  
                               Interior(Token symbol, Tree left, Tree right)
```

When processed by PLCC, we get the following interaction using the Rep parser loop:

```
$ java Rep  
--> 3  
Leaf@15db9742  
--> (foo 5 8)  
Interior@6d06d69c  
--> (foo (bar 13 23) 8)  
Interior@7852e922  
--> (goo blah blah)  
java.lang.RuntimeException: Tree cannot begin with SYMBOL  
-->
```

Let's return to the list-of-numbers example.

```
<lon>          ::= LPAREN <nums> RPAREN  
<nums>:NumsNode ::= <NUM> <nums>  
<nums>:NumsNull ::=
```

The `Nums` abstract class is extended by both `NumsNode` and `NumsNull` classes.

```
<nums>:NumsNode ::= <NUM> <nums>  
<nums>:NumsNull ::=
```

Clearly, when parsing the `<nums>` grammar rules, you get zero or more `NumsNode` instances but just one final `NumsNull` instance. Since the RHS of the first `<nums>` rule has `<nums>` as its last entry (which means that this rule is *right recursive*), this rule results in a parsing loop, ending only when there are no more `NUM` tokens in the program.

This sort of looping occurs frequently in programming language specifications, and PLCC has a way to encode this. Instead of having two `<nums>` rules, with the first being right-recursive and the second having an empty RHS, we can re-write these rules using a special ‘`**=`’ notation:

```
<nums>   **= <NUM>
```

```
Nums(List<Token> numList)
```

The parser accumulates all of the NUM tokens into a single `numList` field. The `numList` field name is obtained from the NUM token name by converting all of its characters to lowercase and appending the string `List`.

Note: The use of ‘`**`’ in the notation we have just introduced should suggest the *Kleene star* repetition notation used in EBNF as well as in regular expressions.

The modified list-of-numbers grammar is as follows:

```
<lon>    ::= LPAREN <nums> RPAREN  
<nums>   **= <NUM>
```

```
<lon> ::= LPAREN <nums> RPAREN  
<nums> **= <NUM>
```

Here is an (edited) example of a parse trace for the list of numbers (3 5 8 13):

```
--> (3 5 8 13)  
<lon>  
| LPAREN "("  
| <nums>  
| | NUM "3"  
| | NUM "5"  
| | NUM "8"  
| | NUM "13"  
| RPAREN ")"
```

If you compare this with the parse trace using the previous grammar that does not use the `**=` construct, you see that the earlier parse trace drifts to the right as additional NUM entries are encountered. Using the `**=` construct, the parse trace becomes flat.

PLCC grammar rules that use this construct are called *repeating grammar rules*. Repeating rules are useful in specifying most of our languages.

```
<lon> ::= LPAREN <nums> RPAREN  
<nums> ::= <NUM>
```

How exactly can we access the values of the NUM fields from the parse tree? The Rep program first parses the program, yielding an instance of the start symbol class – an instance of `Lon`, in this case. It then prints the `toString()` value of this instance, which as we have seen defaults to something like `'Lon@...'`.

But a `Lon` object has a `nums` field of type `Nums`, and a `Nums` object has a `numList` field of type `List<Token>`, so perhaps we can redefine the `toString()` method in the `Lon` class to print the values in this list.

To do this, we need to modify the `Lon.java` file to incorporate this new `toString()` behavior. We can do this directly by editing the file (once it has been created by PLCC). But every change we make to the language specification involves re-running the `plccmk` script, which will clobber any edits we may have made to the previous version.

Fortunately, PLCC allows us to add methods to PLCC-generated source files by including the added methods in the language specification file. Every time `plccmk` is run, these added methods are incorporated into the Java source files automatically.

```
<lon> ::= LPAREN <nums> RPAREN  
<nums> **= <NUM>
```

For example, if we want to add a `toString()` method to the `Lon.java` source file, we put the following lines into the `grammar` file in the section *following the BNF section*. This section is separated from the BNF section by a line containing a single ‘%’.

```
Lon  
%%  
    public String toString() {  
        ...  
    }  
%%
```

The `Lon` line tells PLCC that we are adding a method to the `Lon.java` file, and the lines containing `%%` bracket the Java code to be added. This technique can be used to add Java code to *any* PLCC-generated Java file arising from the BNF grammar lines. The added code appears at the end of the class definition in the Java code generated automatically by PLCC for the class.


```
<lon> ::= LPAREN <nums> RPAREN  
<nums> **= <NUM>
```

We want to redefine the `toString()` method in the `Lon` class so it prints the values of the tokens in the `numList` field. This field, an object of type `List<Token>`, is accessible from the `Lon` class as follows:

```
nums.numList
```

So we can get the the `NUM` entries by iterating over this list, accumulating the corresponding (`String`) values of the tokens, and returning the resulting string. Here is the completed version of the PLCC specification that adds the modified `toString()` method to the `Lon.java` file. This code also adds back the parentheses to make the output look “pretty”.

```
Lon  
%%%  
    public String toString() {  
        String ret = "(";  
        for (Token tok: nums.numList) {  
            ret += tok + " ";  
        }  
        return ret + ")";  
    }  
%%%
```

Here is the complete grammar file in code directory LON2 with these changes:

```
# Lexical specification
skip WHITESPACE '\s+'
NUM '\d+'
LPAREN '\('
RPAREN '\)'
%
# Grammar
<lon> ::= LPAREN <nums> RPAREN
<nums> **= <NUM>
%

Lon
%%%
    public String toString() {
        String ret = "(" ;
        for (Token tok: nums.numList) {
            ret += tok + " ";
        }
        return ret + ")";
    }
    %%%
```

Continuing with our list-of-numbers example, suppose we want our parser to require that the numbers in our list are separated by commas, like this:

```
(5, 8, 13, 21)
```

How can we devise a grammar that accommodates these separators?

PLCC provides a way to specify a token that serves as a separator between items in a repeating grammar rule. The separator name must appear at the end of the rule, preceded by a '+' character. So if we want to separate the items in our lists by a comma (with token name COMMA), here is what the specification looks like:

```
skip WHITESPACE '\s+'
NUM '\d+'
LPAREN '\('
RPAREN '\)'
COMMA ','
%
<lon> ::= LPAREN <nums> RPAREN
<nums> **= <NUM> +COMMA
%
```

While this grammar requires comma separators between NUM items, the parser generates *exactly the same parse trees* for this language as for the previous specification.

Slide set 1a provides a summary of PLCC features. You should familiarize yourself with this section in preparation for material in subsequent parts of this course.

A *variable* in a program is a symbol that has an associated value at run-time. One of the principal issues in determining the behavior of a program is determining *how* to find the value of a variable at run-time. At any instance in time, the value associated with a variable is called a *binding* of the variable to the value.

An *expression* is a syntactic construct that has a value at run-time. A variable, by itself, is therefore an expression, but other syntactic constructs can also have values: for example $x+y$ is an expression if x and y are numeric-valued variables.

A programming language that is constructed solely for the purpose of evaluating expressions is called an *expression-based language*. Most of the languages we construct in these notes are expression-based. Scheme, ML, and Haskell are examples of expression-based languages used in practice. A programming language that is constructed for the purpose of “doing something” with expressions (such as assigning the value of an expression to a variable or printing the value of an expression to standard output) is called an *imperative language*. C, Java, and Python are examples of imperative languages used in practice.

Expression-based languages get their power from defining and applying *functions*, so another term describing such languages is *functional*.

Determining the value of an expression at run-time is at the heart of executing a program, particularly so in expression-based languages. Since most expressions involve variables, evaluating an expression requires determining the values of its constituent variables – in other words, finding the bindings of these variables.

At run-time, how can you find the binding of a variable? There are two basic approaches: if the binding of a variable can be found by code that is created at compile-time, we call it *static binding*; otherwise we call it *dynamic binding*. Almost all programming languages commonly in use today use static bindings, principally because it is easier to reason (or prove things) about programs that use static bindings.

For a given variable, the *scope* of the variable is the region of code in which that variable's binding can be determined. Consider the following Java program:

```
public class Foo {  
    public static int y;  
    public int z;  
    public static void main(String [] args) {  
        // args is local to main  
        Foo f = new Foo(); // f is local to main  
        int x = 1; // x is local in main  
        Foo.y = 2; // y is static throughout in Foo  
        f.z = 3; // z is known only within instances of Foo  
    }  
}
```

In the above code, the scope of `y` is *global*, from its declaration as a public static variable to the end of the class. The scope of `z` is *instance-global*, known only within (and throughout) instances of the class `Foo`. The scopes of `f` and `x` are *local*, from their declarations to the end of the `main` method body. The scope of `args` is also local, from the beginning of the method body to the end.

Static Properties of Variables (continued)

1.47

It is possible for one symbol to have multiple bindings depending on where it occurs in the program. Consider:

```
public class Bar {  
    public static int x;  
    public static void main(String [] args) {  
        x = 3;  
        System.out.println(x);  
        { // beginning of block  
            int x = 4;  
            System.out.println(x);  
        } // end of block  
        System.out.println(x);  
    }  
}
```

When this program is run, the output will be

```
3  
4  
3
```

This is because the `int x = 4;` line defines a new variable `x` bound to the value 4 whose scope is from its point of declaration to the end of the *block* in which it is defined, which as shown in the program comments. In this case, we say that the definition of `x` in the block puts a *hole* in the scope for the global `int x`.

Static Properties of Variables (continued)

1.48

Here is the `Foo` class given on slide 1.46.

```
public class Foo {  
    public static int y;  
    public int z;  
    public static void main(String [] args) {  
        Foo f = new Foo(); // f is local to main  
        int x = 1; // x is local in main  
        y = 2; // y is static throughout in Foo  
        f.z = 3; // z is known only within instances of Foo  
    }  
}
```

Consider just the `main` procedure in this class:

```
public static void main(String [] args) {  
    Foo f = new Foo(); // f is local to main  
    int x = 1; // x is local in main  
    y = 2; // y is static throughout in Foo  
    f.z = 3; // z is known only within instances of Foo  
}
```

In this method, the identifiers `f` and `x` are explicitly defined. In these cases, we say that these identifiers *occur bound* in the `main` procedure.

However, the variable `y` is not defined anywhere in the procedure `main`. In this case, we say that the identifier `y` *occurs free* in the `main` method, but it *occurs bound* in the class `Foo`.

The following grammar defines a formal language called “the lambda calculus”. This language plays an important role in the foundations of computer science (similar to Turing Machines). The PROC token is the string `proc`, and the SYMBOL, LPAREN, RPAREN, LBRACE, RBRACE, and DOT tokens are straight-forward – see the examples below.

```
<exp> ::= <SYMBOL>
<exp> ::= PROC LPAREN <SYMBOL> RPAREN LBRACE <exp> RBRACE
<exp> ::= DOT <exp> LPAREN <exp> RPAREN
```

Consider the sentential form (remember what that means?) in this language obtained from the second grammar rule, where `s` replaces `<SYMBOL>`:

```
proc(s) { <exp> }
```

The occurrence of the symbol `s` in this expression is called a *variable declaration* that *binds* all occurrences of `s` that appear in `<exp>` unless some intervening declaration of the same symbol `s` occurs in `<exp>`. We say that the expression `<exp>` is the *scope* of the variable declaration for `s`.

Occurs Free, Occurs Bound (informal definitions):

A symbol x *occurs free* in an expression E if x appears somewhere in E in a way that is not bound by any declaration of x in E . A symbol x *occurs bound* in E if x appears in E in such a way that is bound by a declaration of x in E . It is possible for the same symbol to occur both bound and free in different parts of an expression. (Note that the declaration itself is not considered free or bound.)

<code>proc(x) {x}</code>	<code>; x occurs bound</code>
<code>proc(x) {y}</code>	<code>; y occurs free</code>
<code>.proc(x) {x} (x)</code>	<code>; first x is bound,</code>
	<code>; second is free</code>
<code>.proc(x) {x} (y)</code>	<code>; y occurs free</code>
<code>proc(y) {.proc(x) {x} (y)}</code>	<code>; y occurs bound</code>
<code>proc(x) {.proc(y) {x} (y)}</code>	<code>; y occurs free</code>
<code>.t (u)</code>	<code>; t and u occur free</code>

Static Properties of Variables (continued)

1.51

```
<exp> ::= <SYMBOL>
<exp> ::= PROC LPAREN <SYMBOL> RPAREN LBRACE <exp> RBRACE
<exp> ::= DOT <exp> LPAREN <exp> RPAREN
```

Formal definitions of *occurs free* and *occurs bound*:

For a Lambda Calculus expression E, a symbol x *occurs free* in E if

- *Rule 1:*

E is a <SYMBOL> and E is the same as x.

x ; x is free

- *Rule 2:*

E is of the form `proc (y) {E' }` where y is different from x and x occurs free in E'

`proc (y) {x}` ; x is free

- *Rule 3:*

E is of the form `.E1 (E2)` and x occurs free in E1 or E2

`.proc (y) {x} (y)` ; x is free

`.proc (y) {y} (x)` ; x is free

```
<exp> ::= <SYMBOL>
<exp> ::= PROC LPAREN <SYMBOL> RPAREN LBRACE <exp> RBRACE
<exp> ::= DOT <exp> LPAREN <exp> RPAREN
```

For a Lambda Calculus expression E , a symbol x *occurs bound* in E if

- *Rule 1:*

E is of the form $\text{proc } (y) \{E'\}$ where x occurs bound in E' or x and y are the same symbol and y occurs free in E'

$\text{proc } (y) \{\text{proc } (x) \{x\}\}$; x is bound
$\text{proc } (y) \{y\}$; y is bound

- *Rule 2:*

E is of the form $.E1 (E2)$ and x occurs bound in $E1$ or $E2$

$. \text{proc } (y) \{\text{proc } (x) \{x\}\} (y)$; x is bound
$. \text{proc } (y) \{x\} (\text{proc } (y) \{y\})$; y is bound

Lexical and Grammar specification for the Lambda Calculus:

```
# Lexical specification
skip WHITESPACE '\s+'
LPAREN '\('
RPAREN '\)'
LBRACE '\{'
RBRACE '\}'
DOT '\.'
PROC 'proc'
SYM '\w+'
%
# Grammar
<exp>:Var ::= <SYM>
<exp>:Proc ::= PROC LPAREN <SYM> RPAREN LBRACE <exp> RBRACE
<exp>:App ::= DOT <exp>rator LPAREN <exp>rand RPAREN
%
```

In the Lambda Calculus, if a symbol is bound by a declaration, we can easily determine the precise declaration that binds the variable. The Lambda Calculus is of interest theoretically, but it has no practical value as a programming language.

We described the following problem earlier, in the context of commonly used programming languages: for a given variable in an expression, to what value is that variable bound? Most modern programming languages are *block structured* and use *lexical binding*, which is another term for *static scope rules*.

A *block* is a region of code introduced by one or more variable declarations and continuing to the end of the code where this declaration is active. In C, C++, and Java, blocks are delimited by matching pairs of braces ‘{ . . . }’.

In some languages, blocks may be *nested*, in which case variable bindings at outer blocks may be *shadowed* by bindings in inner blocks. Consider, for example the following C++ code fragment:

```
{ int x = 3;
    { int x = 5;
        cout << x << endl;
    }
    cout << x << endl;
}
```

This code prints 5 and then 3.

In block structured languages, a variable in an expression is bound to the variable with the same name in the *innermost* block that defines the variable. (Note that Java does not allow the same variable to be defined both in an outer block and in an inner block.)

Static Properties of Variables (continued)

1.56

Let's return to our C++ example. The following picture shows the blocks of the C++ program fragment given on the previous slide:

```
{ int x = 3;  
  {  
    {  
      cout << x << endl;  
    }  
    cout << x << endl;  
  }  
}
```

To determine the binding of a variable in an expression, cross the boxes textually outwards (up) until a variable declaration with the same variable name is found.

When defining procedures in block structured languages, the formal parameter declarations are considered to be at the same lexical level as local variable declarations in the outermost block of the procedure. In the following C++ example, the formal parameter `x` is at the same lexical level as the local variable `y`:

```
int foo(int x) {  
    int y;  
    ...  
}
```