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#### **Continuations**

All of our non-side-effecting languages (V1 through V6) have relied to implement iterative behavior. See, for example, the mutually cedures even? and odd? shown in Slides 3.89 and 3.93. In our implementations, any eval call requires setting up a Java stack fra arguments to eval. If evaluating the arguments requires additional additional stack frames are required. So if a procedure calls itself underlying Java eval methods call themselves recursively, and it stack frames can build up to exhaust available stack memory. Even a programs can result in stack overflow.

#### **Continuations**

An eval method call is intended to carry out some computation be used, for example, to evaluate an actual parameter expression application or a test expression in an if expression. A stack fran implicitly by the Java Runtime Environment (JRE) upon each eval This stack frame consists of information including the method argur find non-local variables (*i.e.*, an environment), and a "return address where the JRE should execute next when the method finishes. This is be considered as an "execution context". Once the method finishes discarded (popped off the stack) and the execution context of the call

One way to avoid stack overflow is to maintain execution context stead of using the JRE stack to hold this information, we pass alon context to the eval method that is used by the method to determine the executed next when the method finishes. Such an execution con continuation. The idea is that a continuation determines how the overshould continue once the current computation is finished.

### **Continuations** (continued)

We start with language REF, a language with set and call-by-refeter passing. In this language, the eval method for expressions has parameter that gives the environment in which the expression is expressionally adding explicit execution context requires passing rameter to the eval method, namely a continuation. The purpose of tion is to receive the value of the expression and to determine what is

We implement continuations as members of the Cont class, with the ACont, VCont, and RCont.

An ACont continuation has an apply method that takes no parame continuation has an apply method that takes a single Val para RCont continuation has an apply method that takes a single Ref of these continuations return an instance of ACont. Here are the me for each:

```
ACont: public ACont apply();
VCont: public ACont apply(Val val);
RCont: public ACont apply(Ref val);
```

The essential purpose of the apply method for an ACont continua out some action that represents "what to do now". Its return value, ACont, says "what to do next".

For a VCont continuation, the purpose of the apply method is to action that represents "what to do now with the value val". Its retu an instance of ACont, says "what to do next". Similar remarks approximation. You will see that the RCont continuations are used onl call-by-reference parameter passing.

## **Continuations** (continued)

The apply method in the ACont class is the fundamental action evaluation. This method carries out some action and returns another uation whose apply method is invoked, and so on. This proceeds umethod stops the evaluation by throwing an exception – either a run indicating an error, or a special ContException indicating that evaluation has finished.

In the absence of an exception, applying a continuation involves cre ecution context that continues the expression evaluation.

Here is the signature of the abstract eval method in our continuation guage, declared in the Exp class:

```
public abstract ACont eval(Env env, VCont v
```

This method is intended to work as follows:

- evaluate the expression in the environment env, yielding a value val
- call vcont.apply (val), yielding an ACont instance which
- call acont.apply() to continue evaluation

It may appear that we have simply added recursive calls to apply recursive calls to eval. The difference is that once we call app need to preserve the current execution context. Languages that imp elimination (look this up!) do this explicitly: see Lisp, Scheme, Has

If our implementation language supported tail call elimination, we to worry about this. In Java it's not as simple, since Java does 1 elimination – but we can accomplish essentially the same thing throu called *trampolining*. This technique de-couples the recursive calls looping instead. The next slide shows how this works:

# **Continuations** (continued)

```
public abstract class ACont {
    public Val trampoline() {
        ACont acont = this;
        while(true)
            try {
                  acont = acont.apply();
            } catch (ContException e) {
                 return e.val;
            }
        }
}
```

Things start off by creating an initial expression evaluation conting and then jumping onto the trampoline:

```
acont.trampoline();
```

In this code, the trampoline loops until one of the calls to ap ContException. In order for expression evaluation to terminatinuation must therefore throw a ContException that jumps off Since the purpose of expression evaluation is to produce a value, needs to have a Val field that can be used to return a value to the loop.

We choose a special "halt continuation" HaltCont to do this. It tends the VCont class, so its apply method takes a Val parameter method in this class – the one that actually jumps off the trampoline

```
public class HaltCont extends VCont {
    public ACont apply(Val val) {
        throw new ContException(val);
    }
}
```

This continuation is created once, at the top level of expression eval

#### **Continuations** (continued)

One useful continuation, EvalCont, has fields for an expression, an environment continuation. When an EvalCont continuation is applied, the expression is evalenvironment and its value is passed to the saved value continuation.

```
public class EvalCont extends ACont {
   public Exp exp;
   public Env env;
   public VCont cont;

   public EvalCont(Exp exp, Env env, VCont vcont)
        this.exp = exp;
        this.env = env;
        this.vcont = vcont;
   }

   public ACont apply() {
        return exp.eval(env, vcont);
   }
}
```

The continuation field vcont has an apply (Val) method that receives the expectation an ACont continuation that dictates "what to do next".

In the Exp class, we start things out by defining a simple top-level as follows:

```
public Val eval(Env env) {
    ACont acont = new EvalCont(this, env, new HaltCont val val = acont.trampoline();
    return val;
}
```

As described on Slide 8.7, The HaltCont continuation created here behavior of jumping off the trampoline by throwing a ContExcep the top-level evaluation is complete and its value is passed to the ap the HaltCont object, the trampoline loop stops and this value is re

## **Continuations** (continued)

For certain expressions (such as LitExp and VarExp) whose vareful termined directly, the value could be sent to its VCont continuation

```
return vcont.apply(...)
```

However, this would result in building a stack frame to call the a contrary to our objective of using continuations (and the trampoline) ing stack frames.

To get around this, we define a special ValCont continuation that direct call to apply with a value parameter and that passes the respondent class:

```
public class ValCont extends ACont {
   public Val val;
   public VCont vcont;

   public ValCont(Val val, VCont vcont) {
       this.val = val;
       this.vcont = vcont;
   }
   public ACont apply() {
       return vcont.apply(val);
   }
}
```

It may appear that this apply method just postpones the recont.apply(val), but since the apply method in the Val is being handled by the trampoline, this de-couples the direct recur it with iteration.

### **Continuations** (continued)

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As noted before, the continuation-based eval method in the Exp following signature:

```
public abstract ACont eval(Env env, VCont vcont);
```

Let's consider the easiest subclasses of Exp, namely LitExp and eval code for the LitExp class is simple: convert the literal to an and return a ValCont continuation that passes the value to the votion. Similarly, for the VarExp class, look up the variable in the give to get the value it is bound to, and return a ValCont continuation value to the vcont continuation. Here is the code for both:

```
LitExp
%%%

public ACont eval(Env env, VCont vcont) {
    return new ValCont(new IntVal(lit.toString())
}
%%%

VarExp
%%%

public ACont eval(Env env, VCont vcont) {
    return new ValCont(env.applyEnv(var.toString))
```

A letrec simply creates a new environment with bindings of ProcVals. Moreover, evaluating a proc (don't confuse this w proc) requires no more than gathering together the formal paramet cedure body expression, and the captured environment. Thus the which we evaluate the letrec body is obtained by calling the a method in the LetrecDecls class, unchanged from the REF lar that addBindings returns an environment that extends the given a binding the LHS variables to (references to) their corresponding RF. The eval method then returns a EvalCont continuation that eval of the letrec in this extended environment and passes its value continuation. Here is the code for eval in the LetrecExp class:

```
LetrecExp
%%%
    public ACont eval(Env env, VCont vcont) {
        Env nenv = letrecDecls.addBindings(env);
        return new EvalCont(exp, nenv, vcont);
    }
%%%
```

Notice that we don't call the voont apply method directly here pect that the EvalCont continuation will ultimately do so when it trampoline.

### **Continuations** (continued)

The eval method in the ProcExp class is even simpler, since credoes not require any further evaluation.

```
ProcExp
%%%
    public ACont eval(Env env, VCont vcont) {
        return new ValCont(proc.makeClosure(env), vc
}
%%%
```

An if expression requires that the test expression be evaluated be exactly one of trueExp or falseExp. So after evaluating the test IfCont continuation uses the result of the test (it's a VCont!) to do of these two expressions must be evaluated next. The resulting value on to the original value continuation. The IfCont class appears as

```
public class IfCont extends VCont {
   public Exp trueExp;
   public Exp falseExp;
   public Env env;
   public VCont vcont;

public IfCont(Exp trueExp, Exp falseExp, Env env this.trueExp = trueExp;
      this.falseExp = falseExp;
      this.env = env;
      this.vcont = vcont;
   }

// continued on next slide ...
```

### **Continuations** (continued)

```
// ... continued from previous slide

public ACont apply(Val val) {
   if (val.isTrue())
      return new EvalCont(trueExp, env, vcont
   else
      return new EvalCont(falseExp, env, vcont
}
```

In the IfExp class, the eval method creates an EvalCont metho the test expression and passes its value to a suitably constructed continuation.

To evaluate a SeqExp, we evaluate the first expression and save its are more expressions in the list, we evaluate each of them in turn, kee value of the last expression and passing it on to the saved continuation SequenceCont continuation that has fields for the initial express that produces the next expression in the sequence if there is one, the in which the expressions are evaluated, and the original continuation send the final expression value.

The SequenceCont class is on the next slide.

```
SequenceCont
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import java.util.*;
// used with SeqExps
public class SequenceCont extends VCont {
    public Iterator<Exp> expIter; // iterate over the expList
    public Env env;
                                  // the environment
    public VCont vcont;
                                  // apply this to the last se
    public SequenceCont (List<Exp> expList, Env env, VCont vco
        this.env = env;
        this.vcont = vcont;
        this.expIter = expList.iterator();
    public ACont apply(Val val) {
        if (expIter.hasNext()) {
            Exp exp = expIter.next();
            return new EvalCont(exp, env, this);
        return new ValCont(val, vcont); // pass the last Val t
    public String toString() {
        return "SequenceCont";
응응응
```

The eval method in the SeqExp class creates a continuation to every expression in the sequence, which passes this value to a Sequence determines if more expressions need to be evaluated. As an optimexpList is empty, we simply side-step the creation of the Sequence ject and directly arrange to evaluate the first expression explusion continuation.

### **Continuations** (continued)

To evaluate a primitive application in the PrimappExp class, we the operand expressions and then send the arguments to the primit passing the resulting value to the saved continuation.

We can use the evalRands method defined in the Rands class to evaluating each of the operand expressions. This method returns a I Since the apply method for each primitive takes an array of value convert the List into an array before calling apply.

```
PrimappExp
%%%

public ACont eval(Env env, VCont vcont) {
    List<Val> valList = rands.evalRands(env);
    int size = valList.size();
    Val [] valArray = valList.toArray(new Val[s. Val val = prim.apply(valArray);
    return new ValCont(val, vcont);
}
%%%
```

Two more expressions require our attention: let expressions and plications. As we have shown, a let expression can (mostly) be conformation procedure application, so code for both of these should be much caution here is that call-by-reference parameter passing semantics application behaves differently from the value semantics for the RI in a let.

In the LetExp class, its eval method asks its letDecls object to vironment by binding all of the LHS variables to (references to) the This extended environment is used to return an EvalCont object the body of the let in this extended environment and pass its value or continuation.

```
LetExp
%%%

public ACont eval(Env env, VCont vcont) {
    env = letDecls.addBindings(env);
    return new EvalCont(exp, env, vcont);
}
%%%
```

### **Continuations** (continued)

The addBindings method in the LetDecls class evaluates the sions using the evalRands method in a suitably constructed Rand method returns a List of Vals. Since our environments bind string we must convert this list of values to a list of references using the method in the Ref class. These values are then bound to the variable obtaining the environment in which the body of the let is to be evoptimization, if there are no variable bindings in the let, addBin returns the original environment.

```
LetDecls
%%%

public Env addBindings(Env env) {
    if (varList.size() > 0) {
        Rands rands = new Rands(expList);
        List<Val> valList = rands.evalRands(env
        Bindings bindings =
            new Bindings(varList, Ref.valsToRef:
        env = env.extendEnvRef(bindings);
    }
    return env;
}
%%%
```

To evaluate a procedure application, we need to evaluate the proced (it must evaluate to a ProcVal), the actual parameter expressions semantics, bind the actual parameter references to their formal parameters these bindings to extend the environment captured by the proced evaluate the body of the procedure in this extended environment. The in the AppExp class is given here:

The AppCont continuation, shown on the next slide, gets an expre evaluate to a ProcVal, evaluates the reference parameters, and prence parameters along with the vcont continuation to the ProcV the procedure body and pass its value along for further processing.

### **Continuations** (continued)

```
public class AppCont extends VCont {

   public Rands rands; // the actual parameter expressions
   public Env env; // evaluate the params in this env
   public VCont vcont; // who gets the result

public AppCont(Rands rands, Env env, VCont vcont) {
      this.rands = rands;
      this.env = env;
      this.vcont = vcont;
   }

   public ACont apply(Val val) {
      ProcVal procVal = val.procVal();
      List<Ref> refList = rands.evalRandsRef(env);
      return procVal.apply(refList, vcont);
   }

   public String toString() {
      return "AppCont";
   }
}
```

In the apply (Val\_val) method in the AppCont class, val mu ProcVal. Notice that the evalRandsRef method produces a lis not values. Once the list of references to actual parameters is built the apply method of the procVal object, along with the vcor This apply method binds the references to the procedure's forn evaluates the procedure body in the appropriate extended environm this value to the vcont continuation for further disposition.

## **Continuations** (continued)

The code for this apply method is in the ProcVal class:

Finally we handle set expressions. A SetCont object captures t necessary to modify the value of the LHS variable of a set:

## **Continuations** (continued)

The eval method in the SetExp class follows:

```
public ACont eval(Env env, VCont vcont) {
    // don't actually modify the binding yet
    Ref ref = env.applyEnvRef(var.toString());
    return new EvalCont(exp, env, new SetCont(ref))
```

Recall that all of these continuations end up jumping on the tramp out the computations iteratively instead of recursively. In particular that makes a tail call (*i.e.*, the return value of the procedure is the v procedure application) discards its own execution context by passi value to the current continuation instead of saving its current exemple evaluating the tail call. Remember that continuations represe happen now and in the future, not what has happened in the past.

For non-tail calls – for example, the naive recursive implementation function – there is no way to avoid building nested execution conrecursive calls are not in tail position. The basic principle here is actual parameters requires creating a nested execution context, t procedures does not.

The even/odd mutual recursion example clearly shows that, without relatively small arguments to even? result in stack overflow. Using an application such as .even? (100000000) terminates normall the mutually recursive calls in the even/odd example are all in tail p

# **Exception Handling**

Because a continuation holds an execution context, it is possible ecution context of an early part of a computation and to return to case something unusual happens later. This gives us the opportunit *exception handling*: that is, the ability to stop the evaluation of an turning instead to a saved execution context.

We implement exception handling by allowing for named *exception* save the current continuation and that otherwise behave like procedeption handlers are installed in a special *exception environment* from the normal evaluation environment. When a named exception we describe shortly – the most recent exception handler having that up in the exception environment, the handler is applied (just like a path the resulting value is passed to the handler's saved continuation.

Since the saved continuation jumps onto the trampoline by callination's apply method, the program execution continues at the perception handler was installed rather than at the point where the thrown.

Here are the new grammar rules that support our exception handling

The CATCH, THROW, and HANDLER tokens are defined in the obvic

One difference between exception handlers and ordinary procedure an exception is thrown, the exception handler is found and evaluated exception environment rather than in the current evaluation environment, the body of a top-level procedure can throw an exception is not visible at the top level but which is defined and invoked in a net environment when the top-level procedure is applied. To throw all that is required is that the handler must be visible in the chait environments when the exception is thrown.

### **Exception Handling (continued)**

Consider the following example:

```
define p = proc() throw eee(5)
.p() % no binding for eee
catch
    eee = handler(x) add1(x)
in
    .p() % evaluates to 6
.p() % still no binding for eee
```

When .p() is evaluated just after the definition of p, there is no exc for the identifier eee. This because the procedure p captures the to tion environment (which is empty) and there is no binding for eee in environment.

The catch expression, on the other hand, evaluates to 6. Within pression, the exception handler identifier eee is bound to a handler to plus its actual parameter value. This handler is added to the exception of the catch expression, so when the p procedure is called in this sion, the throw\_eee (5) can see the binding for the identifier ee apply the handler with an actual parameter value of 5. A throw identical to macro invocation as compared to procedure invocation.

Since we want to maintain static scope rules in ordinary expression allow dynamic scope rules in exception handling, we maintain two a static environment for expression evaluation – usually called er namic environment for exception handling – usually called xenv. xenv are passed to eval methods, but the xenv environment is t installing handlers (in a catch expression) and when throwing exc

Here is the code for a CatchExp. The code for HandlerDecls page.

# **Exception Handling** (continued)

```
HandlerDecls
%%%

public Env addBindings(Env env, Env xenv, VCont
    List<String> idList = new ArrayList<String>
    List<Val> valList = new ArrayList<Val>();
    for (Handler h : handlerList)
        valList.add(h.makeHandler(env, xenv, vcc
        Bindings bindings =
            new Bindings(varList, Ref.valsToRefs(value));
    return xenv.extendEnvRef(bindings);
}
%%%
```

A HandlerVal behaves much like a ProcVal, except that a Han captures the continuation and the exception environment in which created. When the handler is applied, the handler body is evaluated exception environment, with the result passed on to the saved contant therefore define the HandlerVal class as a subclass of the P with two pieces of additional information: the saved exception environment.

When an exception is thrown – always by name, and never anonymor is looked up in the exception environment and the handler is applie value to its saved continuation instead of the continuation in which t thrown:

Notice that evaluating the handler's actual parameters or its body throwing additional exceptions, which can result in a cascade of dling, as shown on the following slide. Notice, too, that when the l evaluated, its exception environment is the one in which the catch evaluated. This means if an exception is thrown when evaluating a its handler is searched for outside of the catch expression handler expression. If evaluating an expression results in a throw that reference that is not in the current exception environment, the value of is undefined.

### **Exception Handling (continued)**

```
%% throwing an exception while evaluating
%% a handler's actual parameter expressions
catch
    h = handler(x) add1(x)
    k = handler(x) * (x, x)
in
    throw h(throw k(3)) % evaluates to 9; h is not
%% throwing an exception in a handler's body express
catch
    h = handler() 5
in
    catch
        h = handler(x) \{throw h(); x\}
    in
        throw h(21) % evaluates to 5
%% throwing an unbound exception in the handler's be
catch
    h = handler () throw h()
in
    throw h() % no binding for h (in the handler's
```

The exception environment rules when evaluating expressions, def and throwing exceptions are:

- The top-level exception environment is always empty.
- Handlers defined in a catch expression are added to the exception in which the catch expression appears, and this extended exception environment in which the catch exist is evaluated.
- A handler defined in a catch expression saves the evaluation environments and the execution continuation in which the cat appears, in addition to the handler's formal parameters and body
- When evaluating a throw expression, the appropriate handle searching the current exception environment in which the thr appears.

continued on next slide ...

### **Exception Handling (continued)**

... continued from previous slide

- The exception environment in which the actual parameters of a are evaluated is the same as the exception environment in which is thrown.
- The exception environment in which the actual parameters of primitive application are evaluated is the same as the exception which the procedure or primitive is applied.
- The exception environment when evaluating a procedure body (v dure is applied) is the same as the exception environment in which is applied.
- When evaluating the thrown handler exception body, the exception and continuation are those saved by the handler when the handlin the catch expression.

#### Concurrency

Using trampolining, we repeatedly apply continuations until a value the HaltCont continuation, which stops the trampolining and r Since each continuation contains *complete information* about how evaluation is to proceed, it is possible to have multiple threads of pression evaluation by associating each thread with a continuation the thread's current execution context.

Observe that we have used a Rands object to evaluate a collection returning a list of values or references. Such lists are used to pas primitive operators or procedures or to bind values to the LHS va expressions.

Assuming that the order of evaluation of expressions in a Rands of matter, we can carry out these evaluations *concurrently* (or *in paral* able hardware to support real concurrency, this can result in run-time

#### Concurrency

Up to now, we have evaluated the expressions in a Rands object which they appear in the expression list, and we have built the cor of values (or references) to appear in the same order.

In the presence of concurrency, the order in which the expression complete is almost certainly not the same as the order in which tappear in the expression list.

To maintain the value order, we create an array of value slots, wi expression. For each expression in the expression list, we create that knows about the expression, its evaluation environment, and the expression value should go. We can then dispatch all of these cor mechanism that evaluates them in parallel (or at least simulate paral once each expression evaluation completes, its continuation deposition value in the corresponding value slot.

In general, we want to define a mechanism that can carry out a simu lel execution of multiple continuations. First, we build a queue that continuations to be executed in parallel. Then we create a "wrappe that takes a single continuation from the queue, calls the dequeued apply () method, and puts the result back on the end of the queue. continuations have completed, the wrapper continuation returns the tion step in the evaluation (such as primitive application, procedure let body evaluation). The wrapper continuation uses the trampoli each of its dequque steps. A ConcurrentCont class, shown on serves this purpose.

This approach does not achieve true parallelism, since we are still ap tinuation steps one at a time (using the trampoline), but in the prese hardware, it would not be difficult to dispatch the application of the uations to separate threads.

## **Concurrency** (continued)

```
Concurrent.Cont.
import java.util.*;
public class ConcurrentCont extends ACont {
    public Queue<ACont> queue; // apply these in parallel
    public ACont acont; // what to do when the queue is empty
    public ConcurrentCont(Queue<ACont> queue, ACont acont) {
        this.queue = queue;
        this.acont = acont;
    public ACont apply() {
        ACont thread = queue.poll();
        if (thread == null)
            return acont:
        try {
            thread = thread.apply();
            queue.add(thread);
        } catch (NullContException) {
        return this; // bounce me!
응응응
```

When a queued expression evaluation continuation completes, its deposited in the proper place in the array of values. The followir represents what to do with the expression value once the evaluation

```
public class ValIndexCont extends VCont {
    Val [] valArray; // an array of values
    int index; // where to put the result

    public ValIndexCont(Val [] valArray, int index)
        this.valArray = valArray;
        this.index = index;
}

public ACont apply(Val val) {
    valArray[index] = val;
    throw new NullContException(); // all done
}
```

#### **Concurrency** (continued)

Before creating a continuation to carry out concurrent evaluation of pressions in a Rands object, we need to create an array that hold values. The result of evaluating concurrent expressions must affect shared environment, possibly the top-level environment or in lettings. Here's an example of such a situation:

```
let
   count = 0
in
   letrec
   d = proc(t)
      if t
      then {set count=add1(count); .d(sub1(t))}
      else 0
in
   let % the RHS expressions are evaluated in paral
      _ = .d(1000)
      _ = .d(10000)
      _ = .d(10000)
   in
   count
```

In this case, the value of count ends up being 10000 and not 11100 expect. (Note the use of '\_' as a dummy variable place-holder.)

The problem here is an example of the "simultaneous update proble a "race condition". Evaluating an expression like set\_count=a can result in the creation of multiple continuations – several, for e evaluate add1 (count) – and when count is in the process of in one thread, there may be other threads that are in the process o modify it as well, with unpredictable results.

When concurrent expressions use side-effects to do their work, we guard against race conditions. We do this through an atomic exconcrete and abstract syntax of such an expression is given here:

### **Concurrency** (continued)

When evaluating an atomic expression, we circumvent the evaluation expression by evaluating the expression directly – using a non-thread Once the value has been determined, we pass it on to the pending so the threading can continue. Observe that during the evaluation expression, the threaded trampoline stops processing queued continue.

```
AtomicExp
%%%
    public Cont eval(Env env, Cont cont) {
        Val val = exp.eval(env); // don't thread on
        return cont.apply(val);
    }
%%%
```

It's harmless to evaluate an atomic expression in a non-threaded However, an atomic expression must complete before its value to the next continuation, so deeply nested atomic expressions can overflow. Using atomic expressions should be done sparingly.

The race condition in the previous example can now be solved by m ification of count atomic:

```
let
   count = 0
in
   letrec
   d = proc(t)
       if t
       then {atomic set count=add1(count) ; .d(sub1(t) else 0
   in
       let
       _ = .d(1000)
       _ = .d(10000)
       _ = .d(10000)
       in
       count
```

This expression evaluates to 11100, as expected.

# **Concurrency** (continued)

Of course, threads can start other threads, limited only by the memounderlying machine.

See what happens if you omit the atomic modifier in the definition