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In this version of our source language, we allow for the assignment of values to variables. Languages that allow for the mutation of variables are called *side-effecting*. Compared to functional programming (V6 is an example), such languages are inherently more difficult to reason about, which accounts for why functional programming has received so much attention and also for why it is so difficult to produce high-quality software in most side-effecting programming languages.

So far, Languages V1 through V6 have treated denoted values (the things that variables are bound to) as being the same as expressed values (the values that expressions can have). For example, a variable x in one of these languages always evaluates to the same thing no matter where it appears in its scope.

When we add variable mutation (also called “assignment”), such as with

```
set x = add1(x)
```

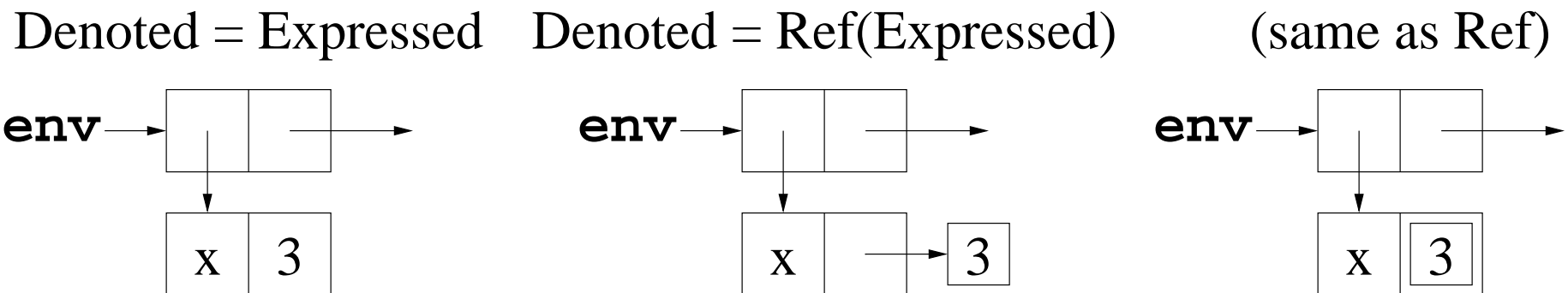
the meaning of x on the LHS is different from its meaning on the RHS. The expression x in the RHS of this “assignment” represents an expressed value, whereas x on the LHS represents a denoted value that can be modified. In order to implement variable assignment, we need to find a way to disconnect denoted values from expressed values.

We introduce the notion of a *reference*, something that *refers to* a mutable location in memory. Instead of binding a variable directly to an expressed value, we bind the variable to a reference containing an expressed value. From a computer architecture point of view, a reference is simply the address of a memory location: the address never changes, but the memory contents at the address can change.

Expressed value = Val = IntVal+ProcVal

Denoted value = Ref(Expressed)

To mutate a variable bound to a reference, we change the *contents* of the reference; the variable is still bound to the same reference.



The two right-hand diagrams depict the same environment. The rightmost one uses a more compact representation.

Language SET (continued)

3a.3

References will also be used to implement various parameter-passing mechanisms as described later in these notes.

We choose the following concrete and abstract syntax for variable mutation:

$$\langle \text{exp} \rangle : \text{SetExp} ::= \text{SET } \langle \text{VAR} \rangle \text{ EQUALS } \langle \text{exp} \rangle$$

`SetExp(Token var, Exp exp)`

We can now write the following program in our newly extended source language:

```
let
  x = 42
in
  { set x = add1(x) ; x }
```

This evaluates to 43.

Language SET (continued)

3a.4

The ability to modify the value bound to a variable allows us to “capture” an environment in a procedure and use the procedure to modify its captured environment. For example, consider:

```
define g = let
    count = 0
in
    proc() set count = add1(count)

.g() % => 1
.g() % => 2
.g() % => 3
```

The value of `count` is captured in the local `let` bindings that defines the `proc()`. Each time we evaluate `.g()`, the procedure increments the value of `count` and returns this newly incremented value. The variable `count` persists from one invocation to the other because the `proc` captures the environment in which it is defined, namely the one with the variable `count`.

In this example, the `count` variable is unbound in the top-level environment, so an attempt to evaluate it throws an exception:

```
count % unbound variable
```

In our Java implementation, we want a reference to be a Java object whose contents can be mutated. When we bind a variable to a reference object (its denoted value), this binding does not change, but the contents of the reference itself – the thing it refers to – can change.

The `Ref` abstract class embodies our notion of a reference – the thing that a variable can be bound to. For now, the only subclass of the `Ref` class is the `ValRef` class.

```
ValRef(Val val)
```

The contents of a `ValRef` object is a `Val`, and we say that such an object is a *reference to a value*. (Recall that a `Val` object is either an `IntVal` or a `ProcVal` – the only two `Val` types that we currently have.)

A `Ref` object has two methods:

```
public abstract Val deRef();  
public abstract Val setRef(Val v);
```

In the `ValRef` class, The `deRef` (dereference) method simply returns the `Val` object stored in the object's `val` field, and the `setRef` (set reference) method modifies the `val` field by changing it to the `Val` parameter `v` (and returning the new `Val` object as well).

Language SET (continued)

3a.6

Ref

%%%

```
public abstract class Ref {
```

```
    public abstract Val deRef();
```

```
    public abstract Val setRef(Val v);
```

```
}
```

%%%

Language SET (continued)

3a.7

ValRef

%%%

```
public class ValRef extends Ref {
```

```
    public Val val;
```

```
    public ValRef(Val val) {  
        this.val = val;  
    }
```

```
    public Val deRef() {  
        return val;  
    }
```

```
    public Val setRef(Val v) {  
        return val = v;  
    }
```

```
}
```

%%%

Our denoted values (the things that variables are bound to) are now references instead of values, so we need to change our `Binding` objects to bind an identifier to a reference. (Notice that we use the terms “variable”, “identifier”, and “symbol” interchangeably.)

```
Binding(String id, Ref ref)
```

In the `Env` class, we want `applyEnv` to continue to return the `Val` object (indirectly) bound to a symbol. Since we now bind identifiers to references, we split up the responsibilities as follows:

```
// returns the reference bound to sym
public abstract Ref applyEnvRef(String sym);

public Val applyEnv(String sym) {
    return applyEnvRef(sym).deRef();
}
```

The `applyEnvRef` method behaves exactly like the `V6` `applyEnv` method, returning the thing (now a `Ref`) bound to the symbol and throwing an exception if there is nothing bound to the given symbol. The `applyEnv` method must still return a `Val`: it simply gets the `Ref` object using `applyEnvRef` and dereferences it to return its corresponding value.

In our semantics code, we need to modify all of the instances of `Binding` or `Bindings` objects so that they use references instead of values. To create a “binding” of a variable to a value, first wrap the value into a new reference and then bind the variable to the newly created reference. Here’s an example of how to create a binding of the variable named `x` to (a reference to) an integer 10:

```
String var = "x";
Val val = new IntVal(10);
Binding b = new Binding(var, new ValRef(val));
```

The `valsToRefs` static method in the `Ref` class takes a list of `Vals` and returns a corresponding list of `Refs`. This is used, for example, in the code for `AppExp` objects (which need to bind formal parameter symbols to references to their actual parameter values) and for `LetExp` objects (which need to bind their LHS variable symbols to reference to their RHS expression values).

```
public static List<Ref> valsToRefs(List<Val> valList) {
    List<Ref> refList = new ArrayList<Ref>(valList.size());
    for (Val v : valList)
        refList.add(new ValRef(v));
    return refList;
}
```

```
<exp>:SetExp ::= SET <VAR> EQUALS <exp>  
                SetExp(Token var, Exp exp)
```

So far, we have dealt only with the implementation details of environments. How do we implement the semantics of `set` expressions? Coding this is now simple:

```
SetExp  
%%%  
    public Val eval(Env env) {  
        Val val = exp.eval(env); // the RHS expression value  
        Ref ref = env.applyEnvRef(var); // the LHS reference  
        return ref.setRef(val); // sets the ref and returns val  
    }  
%%%
```

Remember that an expression (`<exp>`) always evaluates to a value (`Val`), and a `SetExp` is no exception: the value of a `SetExp` is the value of the RHS of the assignment. This means that multiple `set` operations can appear in one expression.

Let's look the three lines in the `eval` method shown on the previous slide:

```
Val val = exp.eval(env); // the RHS expression value
Ref ref = env.applyEnvRef(var); // the LHS reference
return ref.setRef(val); // sets the ref and returns val
```

Notice that the `exp.eval(env)` expression returns a `Val` object (a *value*), whereas the `env.applyEnvRef(var)` expression returns a `Ref` object (a *reference*). We use the terms **value semantics** to refer to obtaining the value of something and **reference semantics** to refer to obtaining a reference to something. The code above shows that *we use value semantics for the RHS of a `set` expression and reference semantics for its LHS*.

Because we need to modify the value denoted by the LHS variable using `setRef` (as shown above), we must use reference semantics for the LHS. Value semantics would have been useless here, since we regard “values” – instances of the `Val` class – as immutable: things that cannot be modified.

In a `set` expression, the LHS variable must exist in the environment where the `set` expression occurs, otherwise we could find no reference to modify. Contrast this to the LHS variables occurring in `let` expressions. A `let` expression creates *new* bindings to the LHS variables: we don't *modify* these variables, we *create* them!

A variable that occurs on the LHS of a set expression is not an expression. In particular, observe that the grammar rule for a set expression uses `<VAR>`, not `VarExp`, to the left of `EQUALS`, and we use reference semantics, not value semantics, for such an occurrence, because we are not treating the LHS variable as an expression.

When we evaluate a `VarExp` (this *is* an expression), either by itself or as part of another expression, we use value semantics in Language SET. This is clear once you examine the `eval` semantics for a `VarExp`:

```
public Val eval(Env env) {  
    return env.applyEnv(var); // value semantics!  
}
```

Even though all variables are bound to references in an `Env`, the `applyEnv` method de-references the reference bound to `var` to return its value semantics.

Observe that expressed values (instances of `Val`) get wrapped into references (instances of `Ref`) when they are bound to variables in creating environments: for example, when evaluating the RHS expressions in a `let/letrec` or when evaluating the actual parameter expressions during a procedure application. We evaluate these expressions using value semantics in Language SET before wrapping them into `ValRef` objects.

Language SET (continued)

3a.13

To illustrate how `set` expressions evaluate to something, consider the following `let` expression in Language SET:

```
let
  t = 3
  u = 42
  v = 0
in
  { set v = set u = set t = add1(t) ; +(t, +(u, v)) }
```

This expression evaluates to 12. The first expression in the body of this `let` (a sequence expression) gets evaluated like this:

```
set v = { set u = { set t = add1(t) } }
```

The innermost `set` expression sets `t` to 4 and evaluates to 4. Proceeding outwards, the next `set` expression sets `u` to 4 (the value of the innermost `set`) and evaluates to 4. Finally, the outermost `set` expression sets `v` to 4 and evaluates to 4. This final value gets discarded by the sequence expression, but the last expression in the `let` body uses the modified values of `t`, `u`, and `v`.

What happens if you try to mutate the value of an identifier that is one of the formal parameters to a procedure? For example, what value is returned by the following program?

```
let
  x = 3
  p = proc (t) set t = add1 (t)
in
  { .p (x) ; x }
```

In our procedure application semantics (see the AppExp code), the formal parameters are bound to (references to) the *values* of the actual parameters. Since the value of the actual parameter x in the expression $.p(x)$ is 3, this means that the variable t in the procedure application is bound to *a new reference* to this value, and evaluating the body of the procedure modifies this reference to the value 4. But it's the reference to variable t , not the variable x , that gets modified. Thus the value of this entire expression is 3.

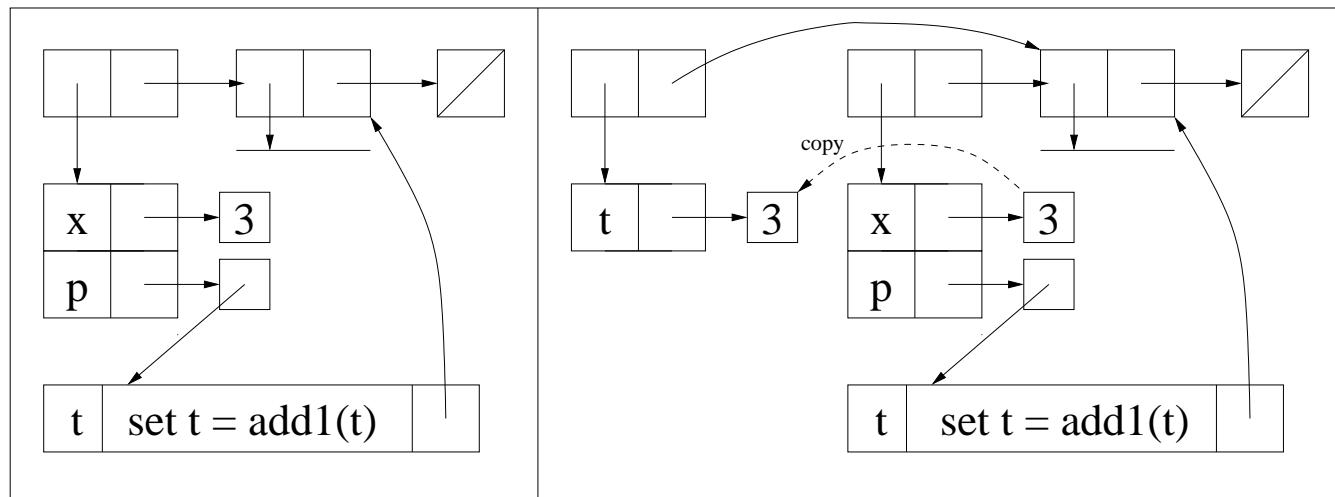
Language SET (continued)

3a.15

```
let
  x = 3
  p = proc(t) set t = add1(t)
in
  { .p(x) ; x }
```

The following illustration shows

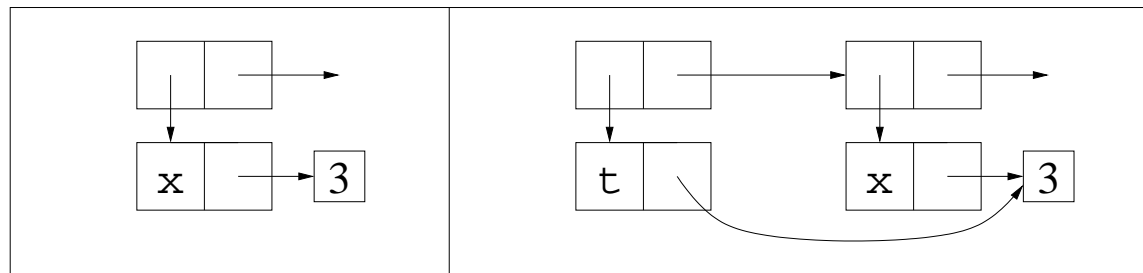
- the environment immediately before the procedure application $.p(x)$ – in particular, the binding of x to a reference to the value 3, and
- the environment during the procedure application $.p(x)$, binding the formal parameter t to a *new* reference containing a copy of the value of x (as shown by the dashed line).



A parameter passing approach that evaluates actual parameters using value semantics and that binds the formal parameters to these actual parameter values is called *call-by-value*. This is what we use in languages V1 to V6.

In the language SET, where bindings are to references instead of values, we wrap the actual parameter values into *new* references, and these references are bound to the formal parameters. Though denoted values are references in Language SET, the parameter passing approach is still call-by-value.

Considering the illustration in the previous slide, Suppose we *want* a behavior that binds the formal parameter t to the *same* reference that is bound to x instead of a new reference containing a copy of the value. The following diagram shows how the bindings in the previous diagram change when t is bound to the same reference as x :



Such a parameter passing semantics is called *call-by-reference*. We explore call-by-reference next, along with variants on this theme.

To repeat:

- The parameter passing semantics that we have been using up to now is called *call-by-value*. In call-by-value semantics – which we have referred to as *value semantics*, when an actual parameter expression in a procedure application is a variable, the procedure's corresponding formal parameter denotes a new reference to the expressed value of the actual parameter.
- In *call-by-reference* semantics – which we have referred to as *reference semantics*, when an actual parameter expression in a procedure application is a variable, the procedure's corresponding formal parameter denotes the *same reference* as the actual parameter.

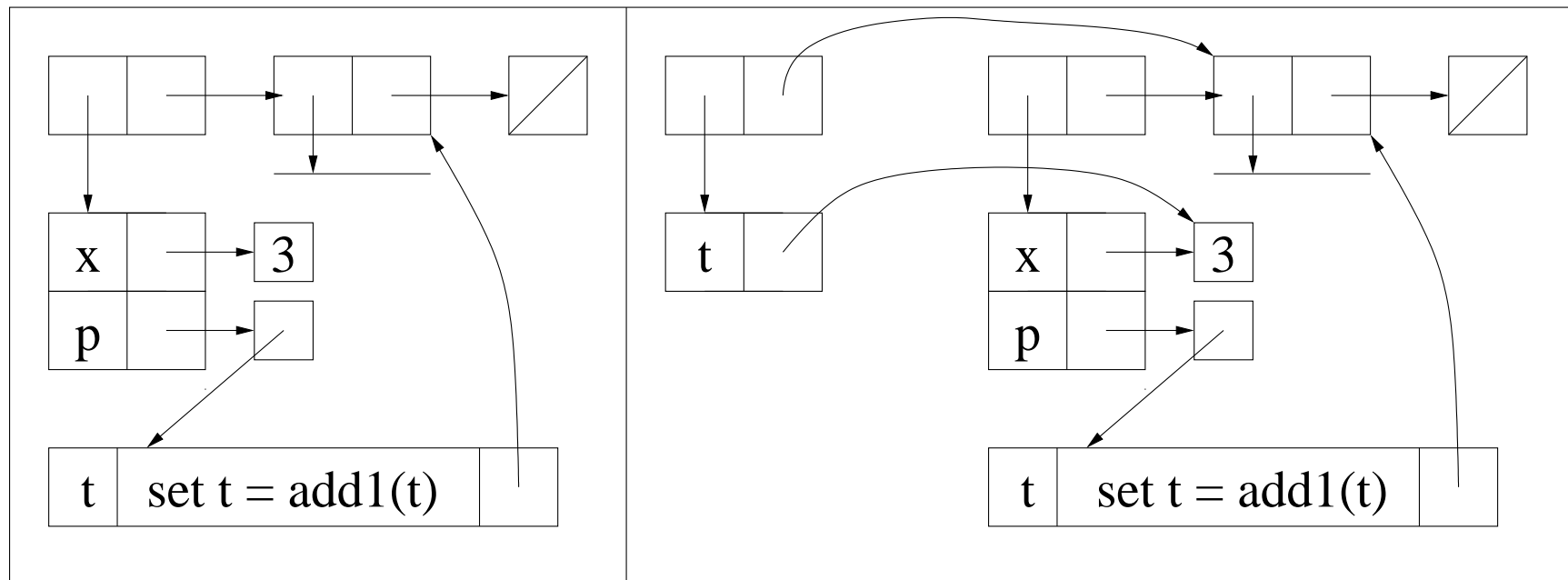
The differences between call-by-value and call-by-reference semantics only apply when the actual parameter expression is a variable. When the actual parameter expression is not a variable, the corresponding formal parameter denotes a *new* reference to the expressed value of the actual parameter, just as in Language SET.

Observe that in `let` and `letrec` expressions, we always use value semantics for the variable bindings. This means that each LHS variable in a `let/letrec` expression always denotes a new reference to the expressed value of its corresponding RHS expression.

Using call-by-reference semantics, the program

```
let
  x = 3
  p = proc(t) set t = add1(t)
in
  { .p(x) ; x }
```

returns the value 4, since t denotes the same reference as x . The environments created by the `let` and then during evaluation of the application `.p(x)` (just prior to evaluating the procedure body) are illustrated in the following figure:



In Language REF, when actual parameter expressions are not themselves variables, we use value semantics. To illustrate this, consider the value returned by the following program:

```
let
  x = 3
  p = proc (t) set t = add1 (t)
in
  { .p (+ (x, 0)) ; x }
```

Clearly the expressed value of the actual parameter $+(x, 0)$ is the same as that of x , but the expression $+(x, 0)$ is not a variable, so value semantics apply to this actual parameter. This means that when we apply the procedure p , the formal parameter t denotes a *new* reference to the value of this expression: the variables t and x have the same expressed values, but they have different denoted values, so modifying t does not affect the value of x . This expression evaluates to 3.

The term *L-value* refers to a semantic entity that can be considered as a reference. It's called an L-value because it is the sort of thing that can appear to the *left* of the '=' in a `set` expression. In Languages SET and REF, a variable like `x` can be considered as an L-value (because variables are always bound to references) but an expression like `+(x, 0)` can only be considered as a value, never a reference.

Whether we consider a semantic entity as an L-value depends on where it occurs. In the expression

```
set x = 3
```

the occurrence of `x` is considered as an L-value. On the other hand, in the expression

```
set y = x
```

the occurrence of `x` is not considered as an L-value.

In Languages SET and REF, only variables can be considered as L-values. In Language SET, actual parameter expressions (even variables) *in procedure applications* are never considered as L-values. In Language REF, only actual parameter expressions that are variables are considered as L-values.

*To summarize: for procedure applications that appear in Language REF, if an actual parameter is a variable (and therefore denotes a reference), then we bind the corresponding formal parameter to the same reference. If an actual parameter is something other than a variable, then the corresponding formal parameter is bound to a **new** temporary reference containing the expressed value of the actual parameter.*

Our REF language has exactly the same grammar rules as our SET language. The *only* differences are in the bindings of formal parameters during procedure application. As the discussion on the previous slides show, we need to handle actual parameters that are variables differently from actual parameters that are expressions. The idea here is to define an `evalRef` method for instances of the `Exp` classes that takes care of how to translate themselves into a reference: for anything but a `VarExp`, `evalRef` evaluates the expression and returns a new reference to the value. For a `VarExp`, `evalRef` returns the same reference that the actual parameter denotes.

So in the `Exp` class, the `evalRef` method has the following *default* behavior:

```
public Ref evalRef(Env env) {  
    return new ValRef(eval(env)); // value semantics  
}
```

For the `VarExp` subclass – *and only for this class*, `evalRef` is implemented as:

```
public Ref evalRef(Env env) {  
    return env.applyEnvRef(var); // reference semantics  
}
```

The `evalRef` method in the `VarExp` class overrides the `evalRef` method in the `Exp` class. In all other classes that extend the `Exp` class, the default definition in the parent `Exp` class is used.

The other change is in the `Rands` code. In the `SET` language, the `evalRands` method was used in the implementation of `eval` for both a `LetExp` object and an `AppExp` object, since both created new bindings to values. In the `REF` language, an `AppExp` object needs new bindings to values except for actual parameters which are variables – a situation that is described in the previous slides. Therefore, to implement the correct `eval` semantics for an `AppExp` object, we need to collect `evalRef` references instead of `eval` values to bind them to the formal parameters. The method `evalRandsRef` in the `Rands` class does this work for us. The `eval` method in the `AppExp` class uses the `evalRandsRef` method to create the bindings of the formal parameters to their appropriate references. The definition for `evalRandsRef` follows:

```
public List<Ref> evalRandsRef(Env env) {
    List<Ref> refList = new ArrayList<Ref>(expList.size());
    for (Exp exp : expList)
        refList.add(exp.evalRef(env));
    return refList;
}
```


Language REF (continued)

3a.24

Remember that we always use value semantics for `let` bindings. This means that a Language REF program such as

```
let
  x = 3
in
  let
    y = x
  in
    { set y = add1(y) ; x }
```

evaluates to 3.

Our observation (see Slide 3.79) that any `let` can be re-written as an equivalent procedure application no longer applies with languages that implement call-by-reference semantics. Specifically, if we attempt to re-write the inner `let` in the above Language REF program as a procedure application using the algorithm given on Slide 3.79, we get

```
let
  x = 3
in
  .proc(y) {set y = add1(y) ; x } (x)
```

which evaluates to 4.

We want literals (LITs) always to have value semantics. For example, a ‘4’ that appears in an expression should always evaluate to the integer 4. Consider the following code in Language REF:

```
define square = proc(x) set x=*(x,x)
define four = 4
{ .square(four) ; four }
```

By the time the variable `four` gets evaluated the second time in the sequence expression, its value has changed to 16, because Language REF uses reference semantics for the actual parameter `four`. Thus the value of the sequence expression is 16. Consider now what happens if we replace the sequence expression above with the following:

```
{ .square(4) ; 4 }
```

Of course, this sequence expression evaluates to 4, because Language REF uses value semantics for everything but variables. However, languages such as FORTRAN IV (in the 1970s) treated numeric literals (like ‘4’) as variables and used reference semantics when passing them to procedures. The equivalent sequence expression, if written FORTRAN IV, would evaluate to 16. Furthermore, subsequent statements such as

```
IF 4 = 16 THEN CALL YIKES
```

(not really a legal FORTRAN IV statement) would end up calling `YIKES`. Yikes!

We now turn to a different parameter passing mechanism, *call-by-name*. In call-by-name procedure application, we bind each procedure's formal parameter to its corresponding *un-evaluated actual parameter expression*. Each time we evaluate a formal parameter in the procedure body, we evaluate its corresponding actual parameter expression *in the environment where the procedure was called* – the *calling environment*, and this value becomes the expressed value of the formal parameter.

Call-by-name has behaviors that differ from call-by-reference: (1) if we never reference the formal parameter in the procedure body, we never evaluate the actual parameter expression; and (2) every time we evaluate the formal parameter in the procedure body, we re-evaluate the actual parameter expression.

In the presence of side-effects, call-by-name has interesting properties that make it very powerful but often difficult to reason about. The language ALGOL 60 had call-by-value and call-by-name as its parameter passing mechanisms. ALGOL 60 had its greatest influence on languages such as Pascal, C/C++, and Java. Although call-by-name has been all but abandoned by modern imperative (side-effecting) programming languages – mostly because of its inefficiency, it still plays a role in functional programming: Scheme supports a variant, *call-by-need*, by means of `promise/force`; Haskell also supports call-by-need. We proceed to implement both call-by-name and call-by-need.

We implement Language NAME with call-by-name semantics. It is based on Language REF.

Two actual parameter expressions that can appear in procedure applications have *constant behavior*: They are `LitExp` and `ProcExp`. Evaluating these expressions do not produce any side-effects, and their expressed values (`IntVal` and `ProcVal`, respectively) can never change. (Don't confuse evaluating the actual parameters of a procedure application with applying the procedure.) Because of this, in call-by-name, we can evaluate these actual parameter expressions directly and can bind their corresponding formal parameters to new references to their values. In other words, we use value semantics for these actual parameter expressions in Language NAME.

Evaluating variables that appear as actual parameter expressions (`VarExp`) also do not produce any side-effects, but such variables may have expressed values that can change, perhaps in `set` expressions. We therefore use reference semantics for actual parameter expressions that are variables in Language NAME.

In the presence of side-effects, call-by-reference and call-by-name may give different results. Consider evaluating the following expression in Language REF:

```
let
  x = 1
  f = proc (t, u)
    { set t = add1(t) ; u }
in
  .f(x, +(x, 5))
```

With call-by-reference, when we evaluate the application `.f(x, +(x, 5))`, the formal parameter `t` in the definition of `f` denotes the same reference that `x` denotes (initially containing 1), whereas the formal parameter `u` denotes a new reference to the value 6 (using value semantics for the `+(x, 5)` expression). Modifying `t` in the body of `f` changes the expressed value of `x` (because `t` and `x` denote the same reference) but does not change the expressed value of `u`. Thus this expression evaluates to 6.

Now consider the same expression in Language NAME.

```
let
  x = 1
  f = proc (t, u)
    { set t = add1 (t) ; u }
in
  .f (x, + (x, 5) )
```

Consider what happens when we evaluate `.f (x, + (x, 5))` using call-by-name: The formal parameter `t` still denotes the same reference that `x` denotes (initially containing 1), but the formal parameter `u` denotes the (un-evaluated) expression `+ (x, 5)`.

The `set` operation in the body of this procedure increments the formal parameter `t`; but since `t` denotes the same reference as `x`, the value of `x` changes too, to two. When we then evaluate the formal parameter `u` at the end of the `proc` body, we evaluate the expression `+ (x, 5)` denoted by `u` *in the environment of the caller*. Since this expression gets evaluated after the `set`, and the value of `x` is now 2, the value of the expression `+ (x, 5)` (and thus the value returned by the procedure application) is `+ (2, 5)` or 7. Thus the entire expression evaluates to 7.

Consider the following definition in Language NAME:

```
define while = proc(test?, do, result)
  letrec loop = proc()
    if test? then {do ; .loop()} else result
  in .loop()
```

Using call-by-name, the expression

```
let x = 0 sum = 0 in
  .while(
    <=? (x, 10),                                % test?
    { set sum=+(sum, * (x, x)) ; set x = add1(x) }, % do
    sum                                           % result
  )
```

evaluates to the sum

$$\sum_{x=0}^{10} x^2 = 385$$

Suppose we were to consider these in Language REF. In this case, the `let` expression evaluation would never terminate. This is because the actual parameter expression `<=? (x, 10)` is evaluated only once, to 1 (true) when `x` is initially 0, so the `test?` parameter is bound permanently to (a reference to) 1. Evaluating `test?` repeatedly always returns 1 (true), so the “loop” never terminates.

We proceed to implement call-by-name. We take our Language REF (call-by-reference) implementation as a starting point.

If an actual parameter is a literal expression (such as 4), we bind the formal parameter to (a reference to) the literal value. If an actual parameter is a procedure, we bind the formal parameter to (a reference to) the procedure's closure in the calling environment. If an actual parameter is an identifier, we bind the formal parameter to the same reference as the actual parameter, using reference semantics.

If an actual parameter is any other kind of expression, we bind the formal parameter to a `Ref` object that captures the expression in the environment in which it was called and that can be evaluated, when needed, by the called procedure. We call such an object a *thunk*.

Following our terminology for defining *value semantics* and *reference semantics* discussed earlier, we call this *name semantics*.

A thunk amounts to a parameterless procedure that consists of an expression and an environment in which the expression is to be evaluated. It looks just like a closure, except that there is no formal parameter list.

ThunkRef (Exp exp, Env env)

A ThunkRef is a Ref, since we want to de-reference (deRef) it whenever we refer to the corresponding actual parameter. We bind a formal parameter to a thunk reference only during procedure application. Thunks will otherwise not play a role in expression semantics.

To change from call-by-reference to call-by-name, we need to change the default `evalRef` behavior of the `Exp` objects so that `evalRef` returns a thunk for most expressions *except for* `LitExp`, `ProcExp`, and `VarExp`. This means that we define `evalRef` in the `Exp` class with its default behavior as follows:

```
public Ref evalRef(Env env) {  
    return new ThunkRef(this, env);  
}
```

For a `LitExp` and a `ProcExp`, a thunk is not necessary, so we return an ordinary `ValRef` as in the REF language:

```
public Ref evalRef(Env env) {  
    return new ValRef(eval(env));  
}
```

Finally, for a `VarExp`, we simply use reference semantics as in the REF language:

```
public Ref evalRef(Env env) {  
    return env.applyEnvRef(var);  
}
```

The `ThunkRef` class is straight-forward:

```
ThunkRef
%%%
public class ThunkRef extends Ref {

    public Exp exp;
    public Env env;

    public ThunkRef(Exp exp, Env env) {
        this.exp = exp;
        this.env = env;
    }

    public Val deRef() {
        return exp.eval(env);
    }

    public Val setRef(Val v) {
        throw new PLCCEException("cannot modify a read-only expression");
    }
}
%%%
```

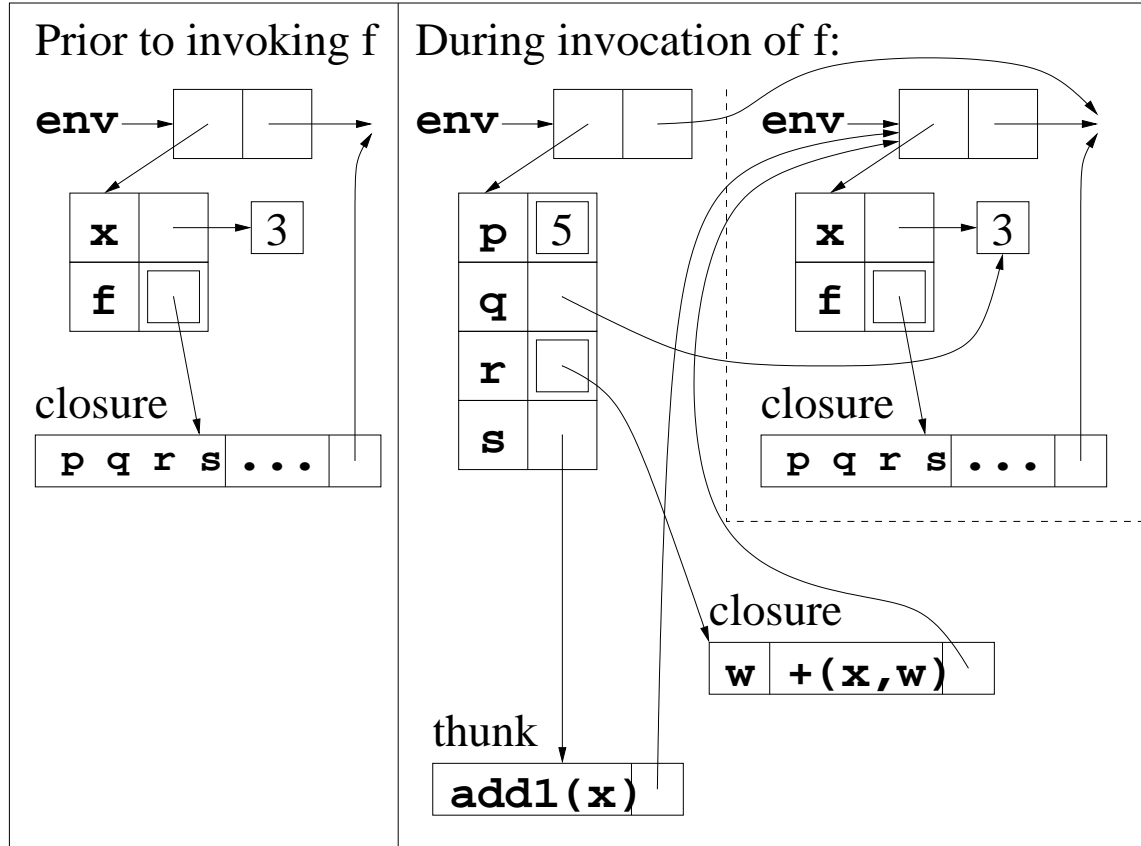
Observe that the `setRef` method throws an exception. The `setRef` method is only used with LHS variable references during evaluation of `set` expressions.

Language NAME (continued)

3a.35

The following illustration may help you to understand how these bindings work. The example shows all four possible cases of actual parameter expressions: literal, variable, procedure, and other:

```
let x = 3
    f = proc(p,q,r,s) ...
in .f(5, x, proc(w) +(x,w), add1(x))
```



The call-by-need parameter passing mechanism is the same as call-by-name, except that a thunk is called at most once, and its value is remembered (*memoized*).

Suppose a procedure with formal parameter x is invoked with actual parameter `set z = add1 (z)`, using call-by-need semantics. As with call-by-name, the formal parameter x is bound to the thunk with `set z = add1 (z)` as its body, in an enclosing environment that we will assume has z bound to (a reference to) the value 8. When x is referenced in the body of calling procedure, its corresponding thunk is dereferenced, producing a result of 9 for `set z = add1 (z)`. The thunk now remembers (*memoizes*) the value 9, and any further references to the formal parameter x in the body of the procedure will continue to evaluate to 9 without making any further changes to the variable z .

If call-by-name had been used in the above example, additional evaluations of x in the body of the procedure would result in evaluating the body of the thunk for each such evaluation, further modifying z and yielding values 10, 11, 12, and so forth.

In both call-by-need and call-by-name (and unlike call-by-reference), if the formal parameter is never referenced in the body of the procedure, the thunk is never evaluated. Compared to call-by-name, call-by-need reduces the overhead of repeatedly evaluating a thunk when evaluating the corresponding formal parameter.

Implementing call-by-need is easy, starting from the call-by-name interpreter. The principal change is to have a `Val` field named `val` in the `ThunkRef` class that is used to memoize the value of the body of the thunk. This field is initialized to `null` when the thunk is created. When the thunk's `deRef` method is invoked, it checks to see if the `val` field has been memoized (*i.e.*, is non-null). If so, the `deRef` method simply returns the memoized value. Otherwise, it evaluates the body of the thunk, saves the value in the `val` field (thereby memoizing it), and then returns that value; subsequent `deRef` calls simply use the resulting memoized value.

In the NEED language, the `ThunkRef` constructor initializes the `val` field to `null`, indicating that the thunk has not been memoized. This field is modified when the thunk's `deRef` method is called.

Language NEED (continued)

3a.38

Here are the appropriate changes to ThunkRef ...

```
ThunkRef
%%%
public class ThunkRef extends Ref {

    public Exp exp;
    public Env env;
    public Val val; // memoized value

    public ThunkRef(Exp exp, Env env) {
        this.exp = exp;
        this.env = env;
        this.val = null;
    }

    public Val deRef() {
        if (val == null)
            val = exp.eval(env);
        return val;
    }

    ...

}
%%%
```

You might have noticed in the `code` file that an instance of the class `ValRORef` is constructed by the `evalRef` methods in the `LitExp` and `ProcExp` classes. This is a slight change from the NAME language, where the instances were `ValRefs`. The `RO` part stands for “Read Only”. We do this because it doesn’t make sense for a literal or procedure to be modified.

Consider, for example, the following code:

```
let
  f = proc (x) set x=add1(x)
in
  .f(3)
```

One’s intuition would be to think of the procedure application `.f(3)` as saying:

```
set 3=add1(3)
```

But of course this doesn’t make any sense.

We have already seen that the `setRef` method in the `ThunkRef` class throws an exception. What we are now doing is to have this same behavior for *any* actual parameter expression except for a variable (where call-by-reference is the default).

The following example illustrates the difference between call-by-name and call-by-need:

```
let
  x = 3
  p = proc (t) {t;t;t}
in
  .p (set x=add1 (x) )
```

With call-by-name, when we apply the procedure p , its formal parameter t is bound to a thunk containing the expression `set x=add1 (x)`. Each time we evaluate the formal parameter t in the body of the procedure p , its thunk is dereferenced, resulting in evaluation of the expression `set x=add1 (x)`. So since we evaluate t three times in the body of p , the expression `set x=add1 (x)` gets evaluated three times, incrementing x from 3 to 6. Consequently, the entire expression evaluates to 6.

With call-by-need, the first time we evaluate t in the body of p , its corresponding actual parameter expression `set x=add1 (x)` is evaluated, which has the side-effect of incrementing the value of x to 4 and evaluates to 4. However, the thunk memoizes the expressed value of 4, so any further references we make to t evaluate to 4. Consequently, the entire expression evaluates to 4.

Here's another example illustrating the difference between call-by-reference and call-by-name/need. Examine the definition of `seq`, which seems to recurse infinitely but doesn't with call-by-name (why?).

```
define pair = proc(x,y)
  proc(t) if t then y else x
define first = proc(p) .p(0)
define rest  = proc(p) .p(1)
define nth   = proc(n,lst)      % zero-based
  if n then .nth(sub1(n),.rest(lst)) else .first(lst)
define seq   = proc(n) .pair(n,.seq(add1(n)))
define natno = .seq(0)          % all the natural numbers!!
%% The above never terminates with call-by-reference.
%% With call-by-name or call-by-need, we get:
.first(natno)                  % => 0
.first(.rest(natno))           % => 1
.first(.rest(.rest(natno)))    % => 2, and so forth ...
.nth(100,natno)                % => 100
```

Order of evaluation

3a.42

Let's examine the following example in Language SET (or REF):

```
let
  x = 3
in
  let
    y = {set x = add1 (x) }
    z = {set x = add1 (x) }
  in
    z
```

Consider the inner `let`. We know that the right-hand side expressions (here written inside curly braces for clarity) are evaluated before their values are bound to the left-hand variables. Languages SET and REF *specify* the order in which the right-hand side expressions are evaluated, namely first to last. (In Language V6, the RHS expressions in a `let` cannot produce side effects, so the order of evaluation of RHS expressions wouldn't matter. However, when side-effects are possible, the order of evaluation does matter.)

Suppose we did not specify this order of evaluation. In the above example, if the second `set` were to be evaluated first, then `z` becomes 4 and `y` becomes 5, so the entire expression evaluates to 4 – the value of `z`. If the order of evaluation were reversed, the entire expression evaluates to 5. Since Languages SET and REF specify the order of evaluation, we have an unambiguous interpretation of the value of the above expression. In the absence of such a specification, the value of this expression is ambiguous. *Do you know what your favorite language does?*

Order of evaluation (continued)

3a.43

A similar situation exists when evaluating actual parameter expressions, as shown by this example, assuming call-by-value semantics.

```
let
  x = 3
  p = proc (t,u) t
in
  .p(set x = add1(x), set x = add1(x))
```

When the actual parameters are evaluated left-to-right as specified in our languages, `t` would be bound to 4 and `u` to 5, so the entire expression evaluates to 4. If the evaluation order had been right-to-left, the entire expression would evaluate to 5.

You can see that both `evalRands` and `evalRandsRef` use `for-each` loops (also called enhanced `for` loops) to traverse and evaluate the expressions in the list of actual parameters. The traversal is guaranteed by the Java API specification to be “natural”, in the sense that the elements of the list are visited in ascending item number order. Here is the code for `evalRands` in the `Rands` class:

```
public List<Val> evalRands(Env env) {  
    List<Val> valList = new ArrayList<Val>();  
    for (Exp e : expList)  
        valList.add(e.eval(env));  
    return valList;  
}
```

Our order of evaluation semantics depends on the behavior of Java’s `for-each` order of evaluation mechanism, which is guaranteed to be left-to-right. If we had chosen right-to-left semantics, we could, instead, explicitly traverse the `expList` from last to first if we wished.

The point is that, to make any language semantics well-defined and unambiguous, it is necessary to specify the order of evaluation. Unless the language specification clearly addresses the issue of order of evaluation, the language implementor can choose any evaluation order. *Let the buyer beware!*

Order of evaluation (continued)

3a.45

Both Java and Python specify that actual parameter expressions are evaluated left-to-right. However, the C language specification explicitly states that the order in which actual parameter expressions are evaluated is *undefined* – which means that the evaluation order is implementation dependent. In the following C program, the output is 3 (using the GCC compiler on the date this file was last modified), which shows that the operand expressions `foo(3)` and `foo(5)` are evaluated right-to-left. Your mileage may vary!

```
#include <stdio.h>
#include <stdlib.h>
static int xx = 0;
void foo2(int x1, int x2) {
    return;
}
int foo(int x) {
    xx = x;
    return x;
}
int main(int argc, char ** argv) {
    foo2(foo(3), foo(5));
    printf("xx=%d\n", xx);
    return 0;
}
```

Order of evaluation does not matter in languages without side-effects, which makes functional languages immune to order of evaluation issues. See http://en.wikipedia.org/wiki/Evaluation_strategy for more information about order of evaluation.

Another way to avoid order of evaluation problems is to require that all procedures have at most one formal parameter. In languages that use this approach, coupled with call-by-need, there is never an “order of evaluation” issue because there is never more than one actual parameter to evaluate.

While you may think that a language with procedures having only one formal parameter might be limited, it's possible for such a language to behave like having multiple formal parameters using an approach called “Currying”, as employed in the Haskell programming language – named after Haskell Curry. The following slide gives an example.

Order of evaluation (continued)

3a.47

Here is an example without currying:

```
let
  x = 3
  y = 5
  p = proc (t, u) + (t, u)
in
  .p (x, y) % => 8
```

Here is semantically equivalent code that has exactly one formal parameter per `proc`:

```
let
  x = 3
  y = 5
  p = proc (t) proc (u) + (t, u)
in
  ..p (x) (y)
```

In the second example above, `x` must be evaluated first, so that the procedure `.p (x)` can then be applied to the value of `y`.

Side-effecting languages that use call-by-reference suffer from another danger. Consider, for example, the following program using the REF language semantics:

```
let
  addplus1 = proc(x,y) {set x = add1(x) ; +(x,y)}
in
  .addplus1(3,3)
```

It's clear that this program returns 7. But what about the following program?

```
let
  a = 3
  addplus1 = proc(x,y) {set x = add1(x) ; +(x,y)}
in
  .addplus1(a,a)
```

```
let
  a = 3
  addplus1 = proc(x,y) {set x = add1(x) ; +(x,y) }
in
  .addplus1(a,a)
```

Using call-by-reference, when `addplus1` is applied to the actual parameters `a` and `a`, both formal parameters `x` and `y` of `addplus1` refer to the *same cell* as `a`. Therefore the `set x = add1(x)` expression is equivalent to the expression `set a = add1(a)` which increments `a` to 4, and the next expression `+(x,y)` is essentially equivalent to the expression `+(a,a)` which now evaluates to 8. Thus the value of the program is 8.

Aliasing occurs when two different formal parameters refer to the same actual parameter. As this example shows, aliasing can lead to unexpected side-effects and should be avoided. **Of course, the best way to avoid problems such as order of evaluation ambiguities and aliasing is to avoid using languages with side-effects!**