Stability Analysis of Rainforest



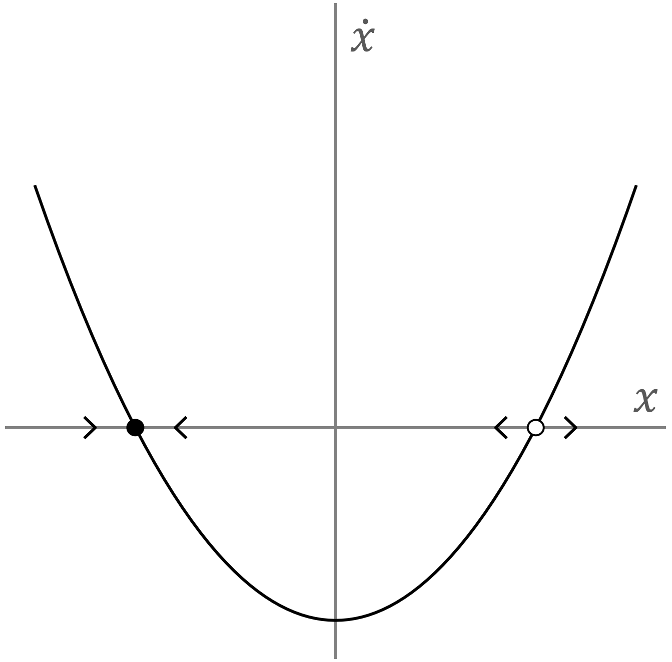
1, 2, 3

Bifurcation Theory

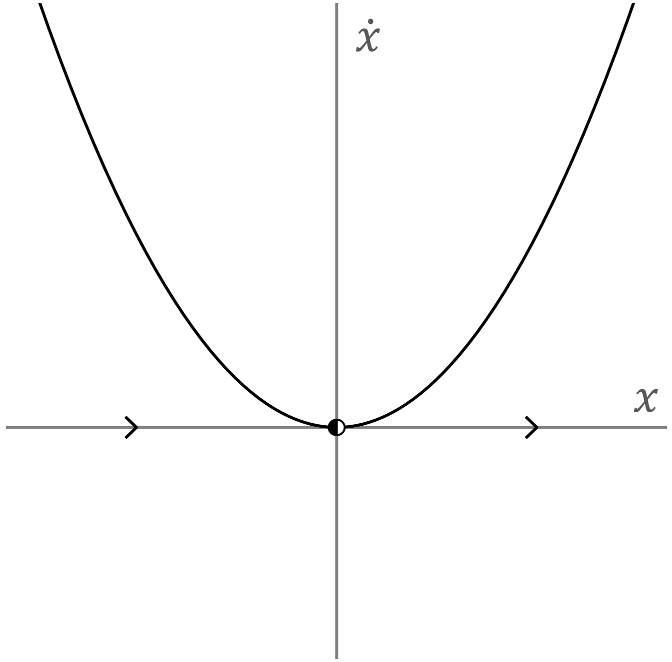
A short reminder

Example:

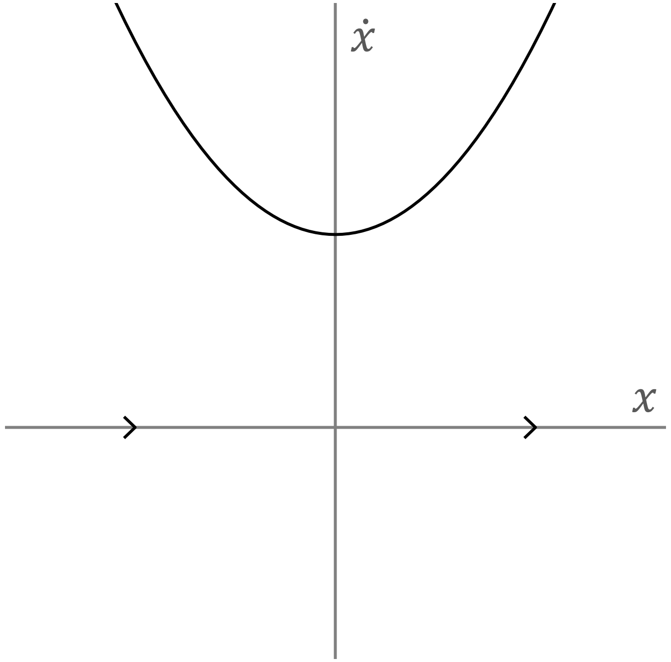
$\dot{x} = r + x^2$



$r < 0$



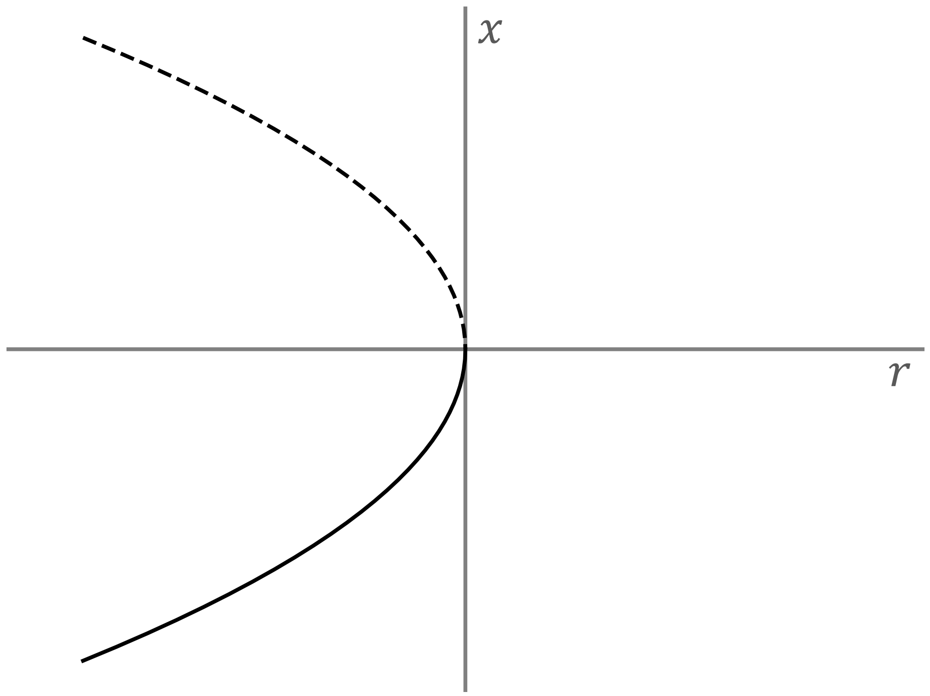
$r = 0$



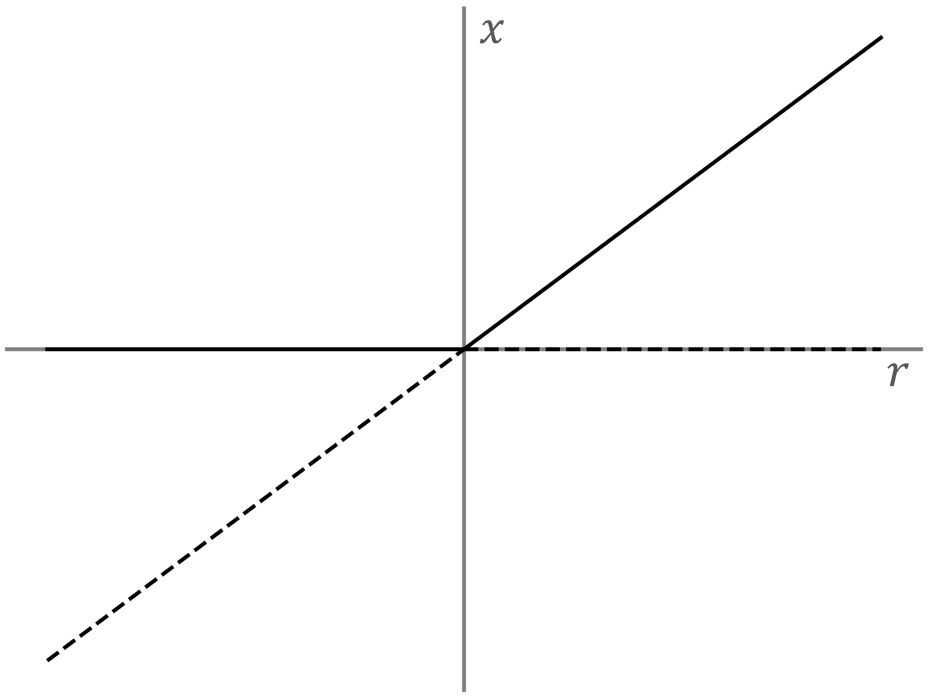
$r > 0$

Bifurcation Diagrams

Critical slowing down, Transcritical Bifurcation



$\dot{x} = r + x^2$



$\dot{x} = rx - x^2$

Tree-Cover Model

Dynamics of Tree-Cover (T in %) in dependence
of Precepitation (P in $\mathrm{mm} \cdot \mathrm{yr}^{-1}$)

- Logistic growth

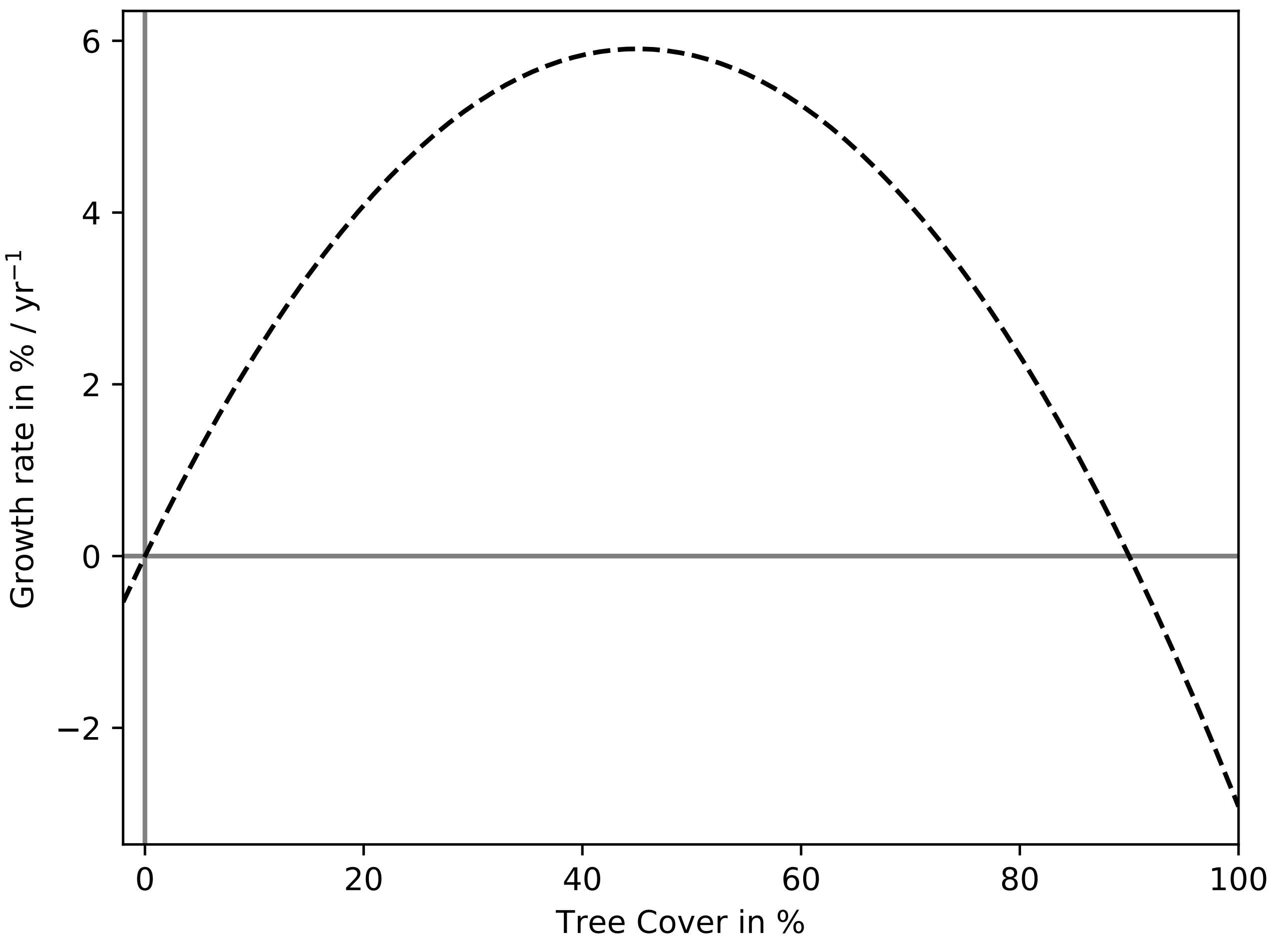
- K … maximum Tree-Cover

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \qquad T\left(1 - \frac{T}{k}\right)$$

Tree-Cover Model

Expansion rate

- $r(P)$ … expansion rate
- $r(P) = r_m \frac{P}{P+h_P}$
- Saturating function / Monod Equation



$$\frac{\mathrm{d}T}{\mathrm{d}t} = r(P)T\left(1 - \frac{T}{k}\right)$$

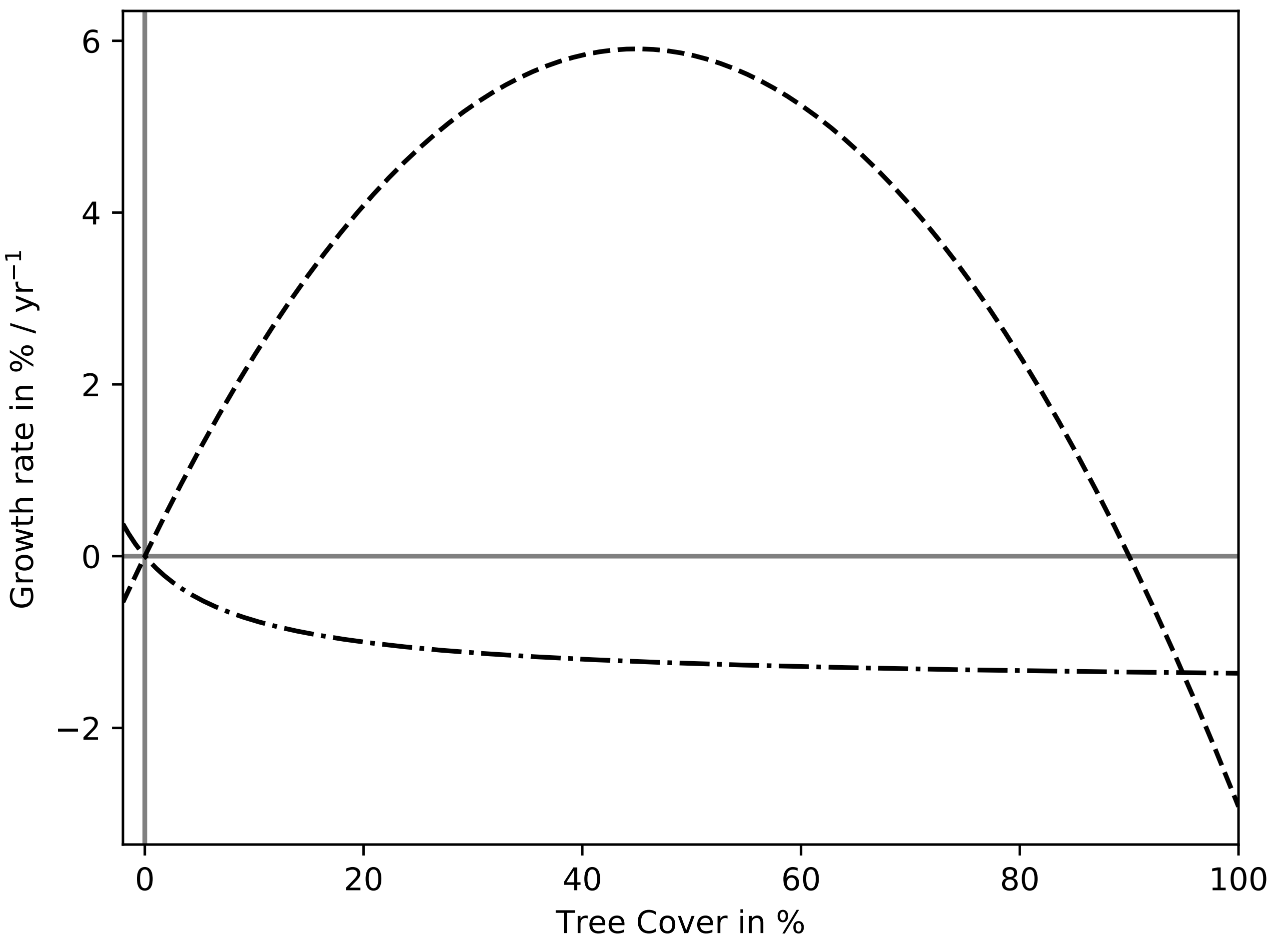
Tree-Cover Model
Allee effect

- m_A … maximum loss rate due to Allee-Effect
- h_A … Tree-cover below which there is an Allee effect

- Saturating function / Monod Equation

- Allee effect: Lower growth rate at low densities.

- Reason: lack of protective covering from „nurse plants"



$$\frac{\mathrm{d}T}{\mathrm{d}t} = r(P)T\left(1 - \frac{T}{k}\right) - m_A T \frac{h_A}{T + h_A}$$

Tree-Cover Model

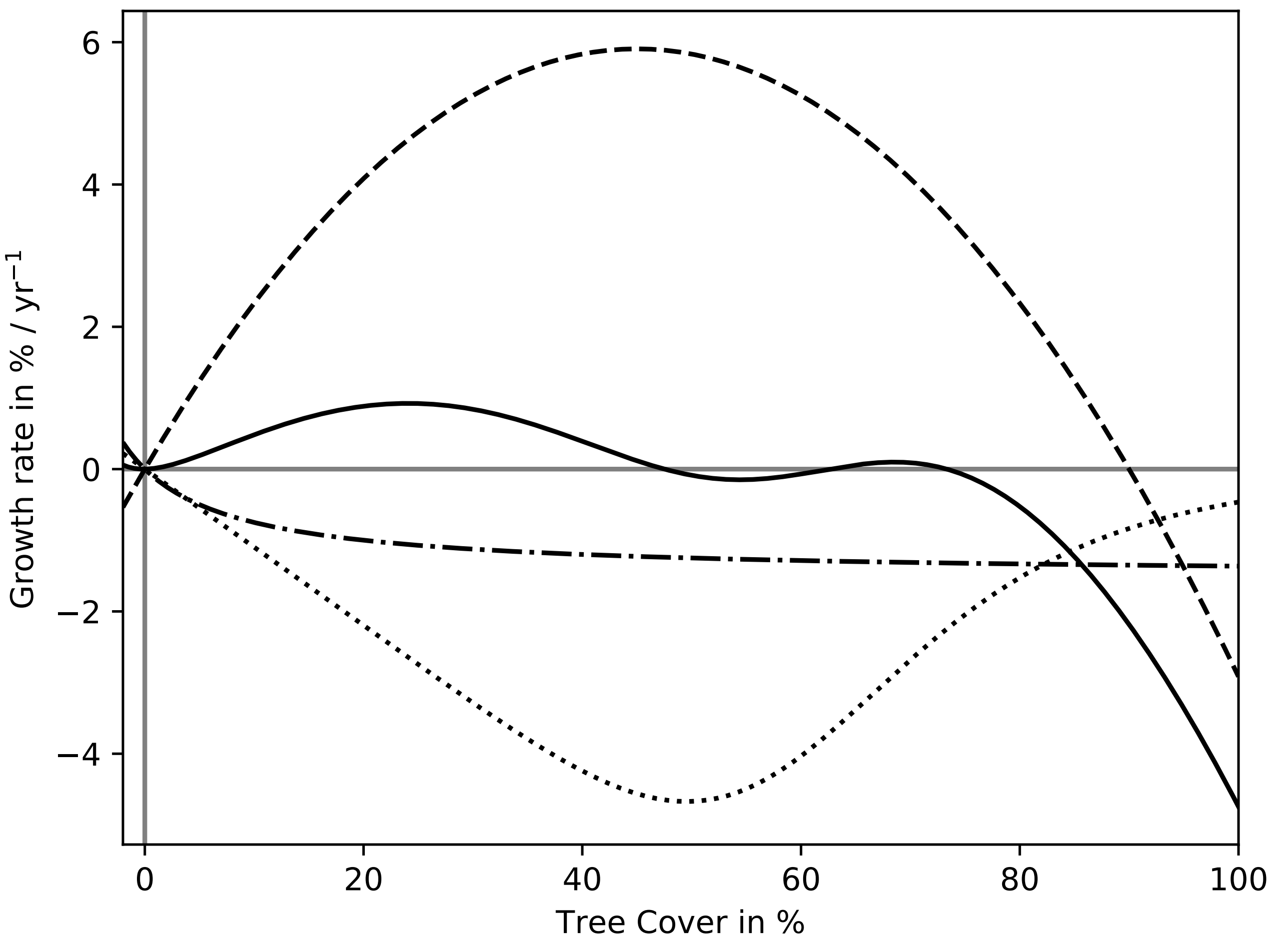
Allee effect

- m_f … maximum loss rate due to fire
- h_A … Tree-cover below which fire extinction rises

- sigmoidal Hill function

- Mortality due to fire declines at high Tree-Covers

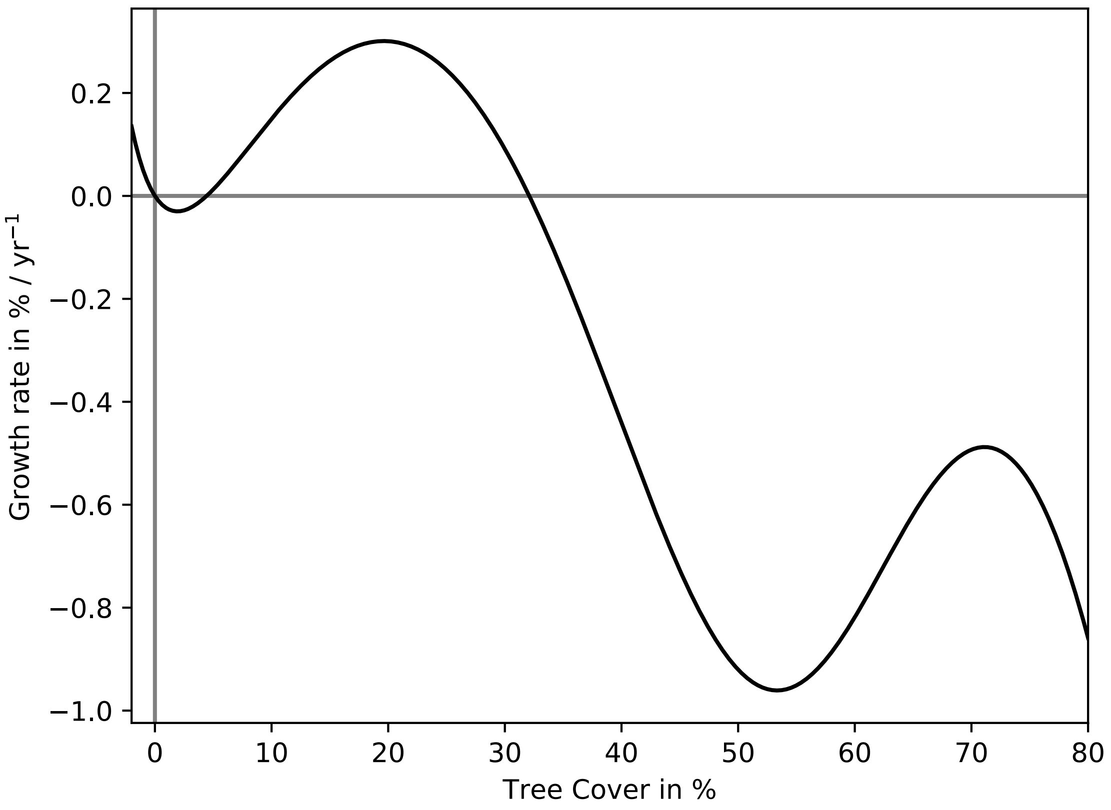
- Reason: High moisture, less grass



$$\frac{\mathrm{d}T}{\mathrm{d}t} = r(P)T\left(1 - \frac{T}{k}\right) - m_A T\,\frac{h_A}{T + h_A} - m_f T\,\frac{h_f^p}{T^p + h_f^p}$$

Tree-Cover Model

Stable solutions

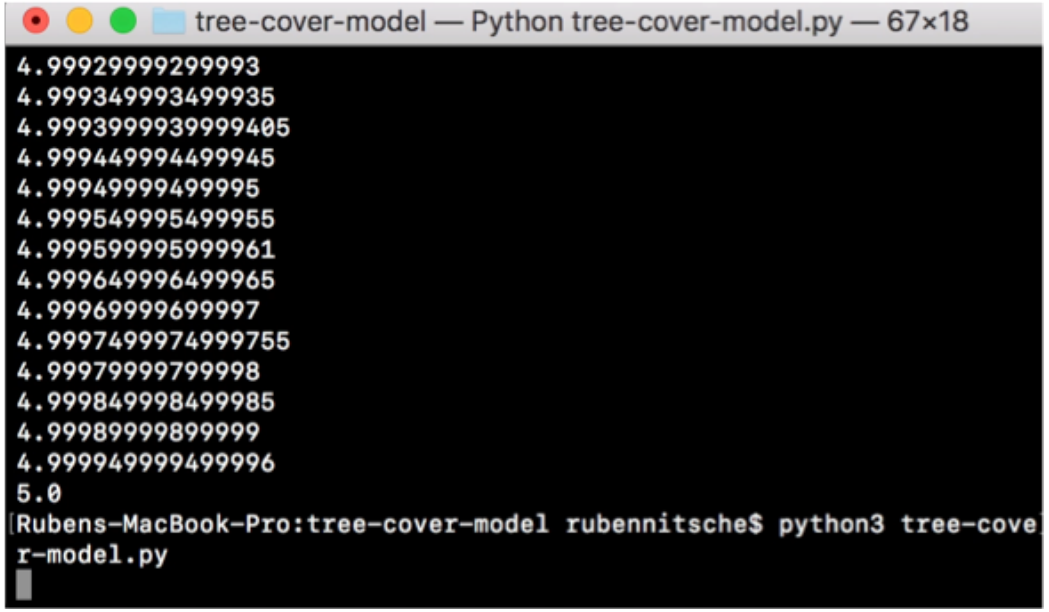


Tree-Cover Model

Solving the differential equation

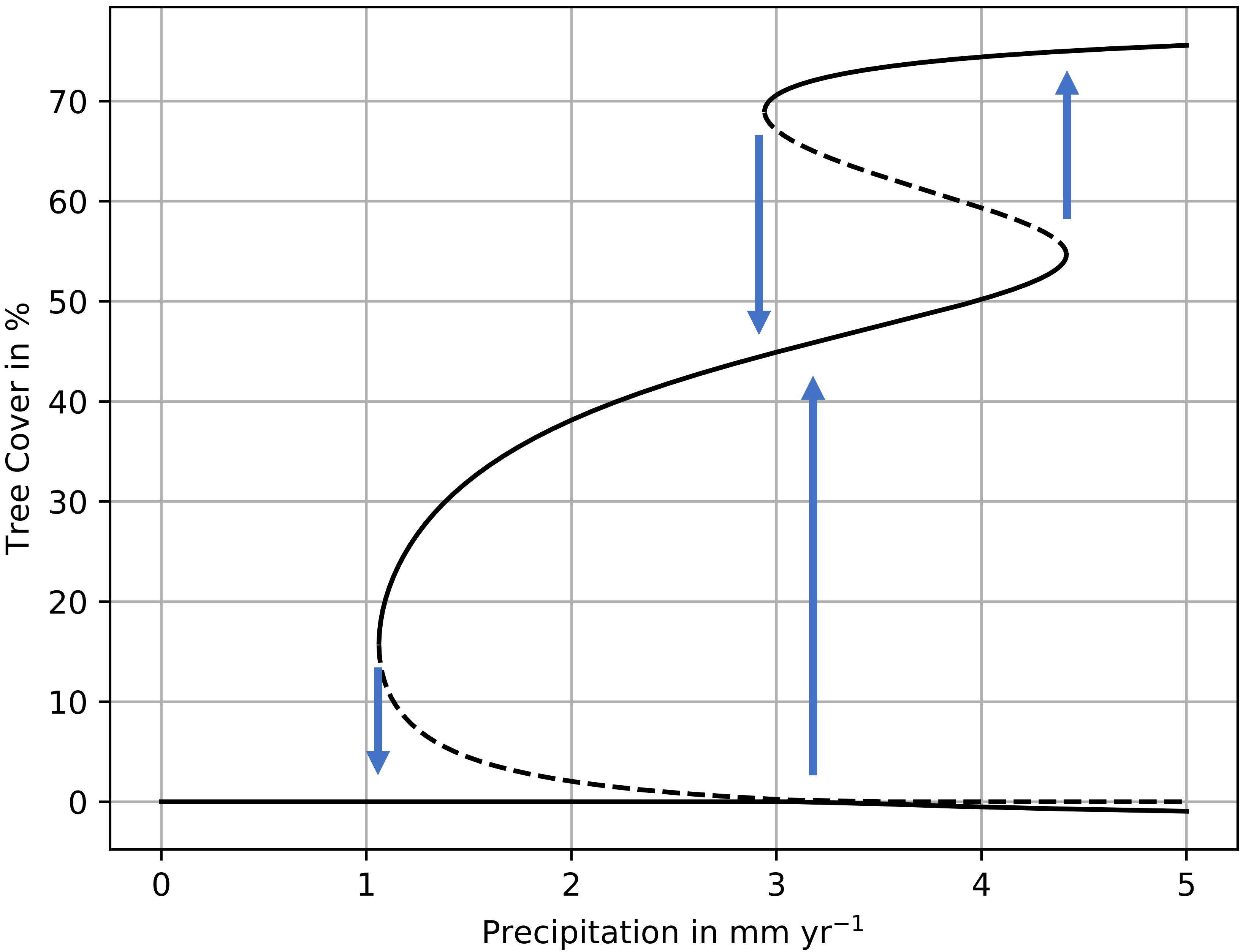
$$\frac{\mathrm{d}T}{\mathrm{d}t} = r(P)T\left(1-\frac{T}{k}\right) - m_A T\frac{h_A}{T+h_A} - m_f T\frac{h_f^p}{T^p+h_f^p} \stackrel{!}{=} 0$$

1. Look roughly for a sign change
2. Use the Newton-Algorithm look for the zero poins
3. Let the Computer do ist magic

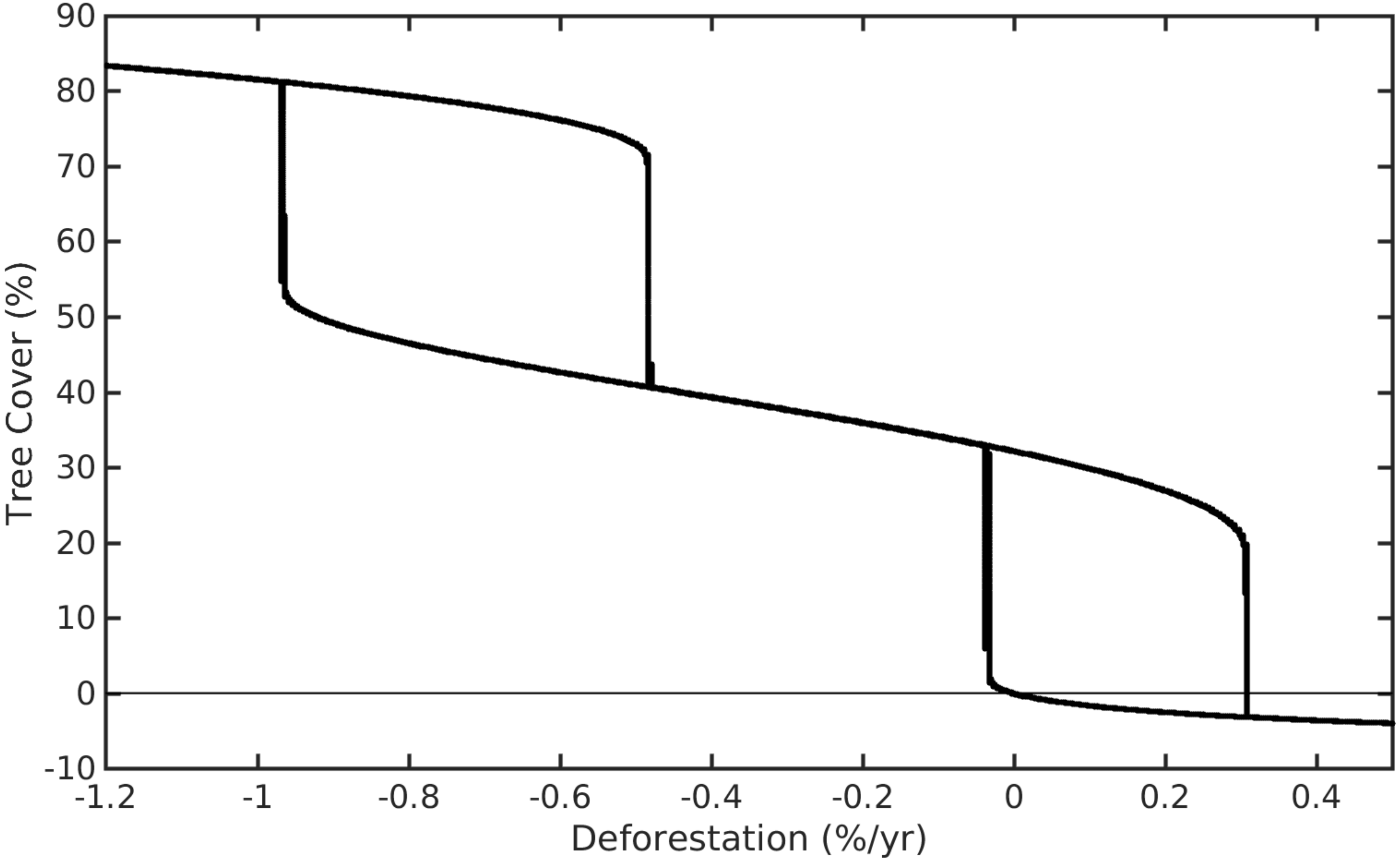


Tree-Cover Model

Bifurcation diagram

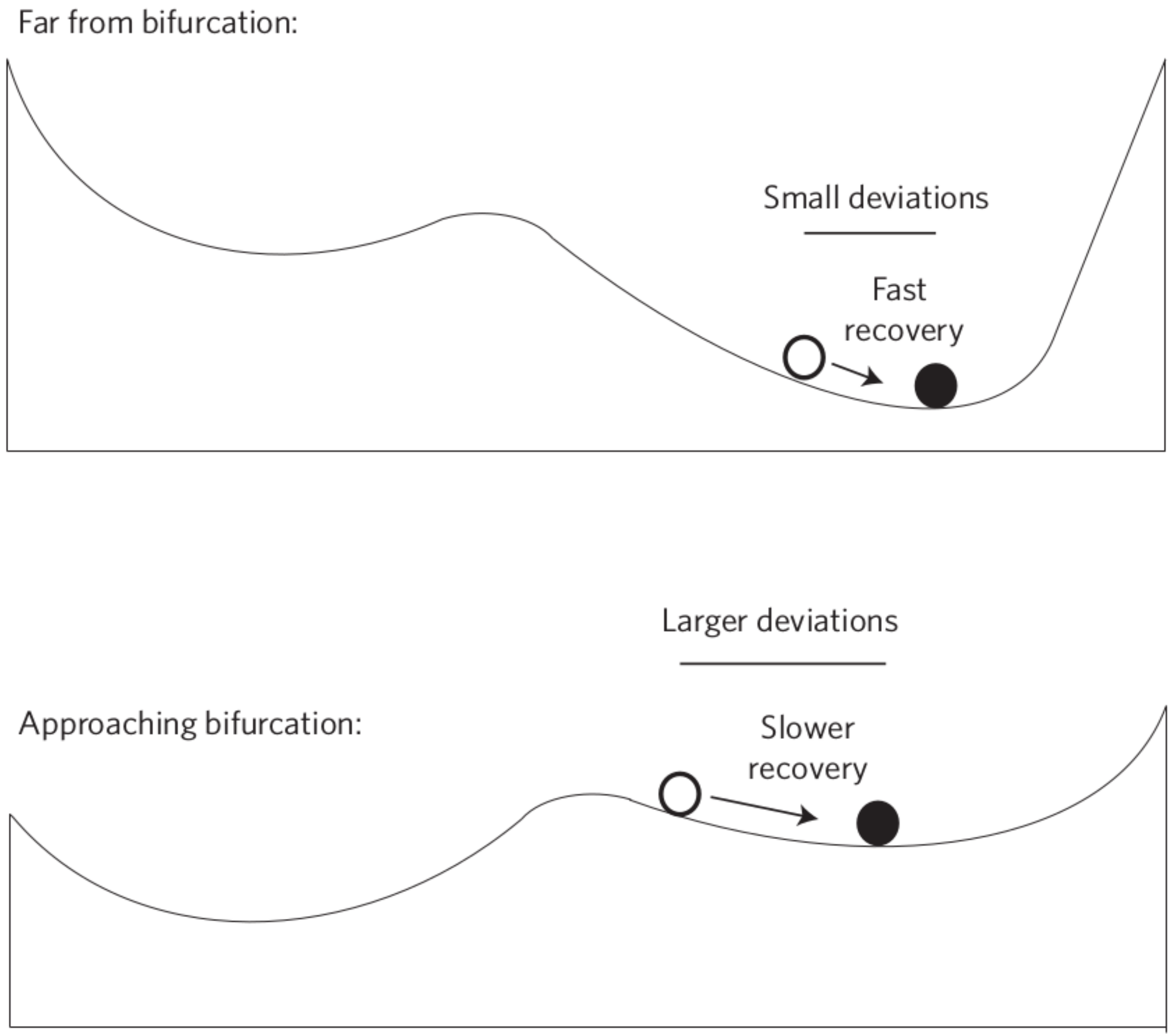


Variation of Deforestation



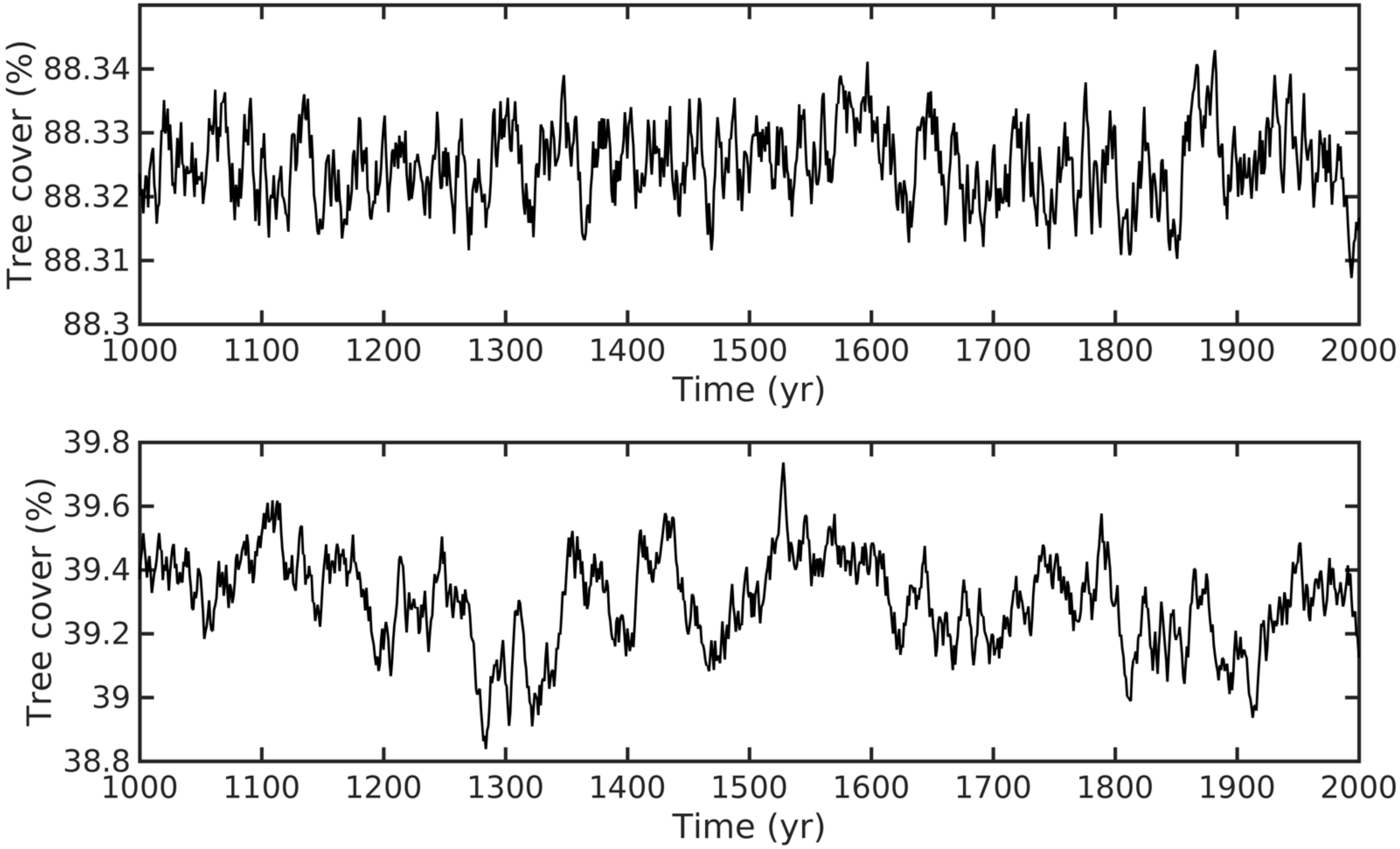
Recovery rate from perturbations

Recovery from small perturbations is slowed down in proximity to a bifurcation



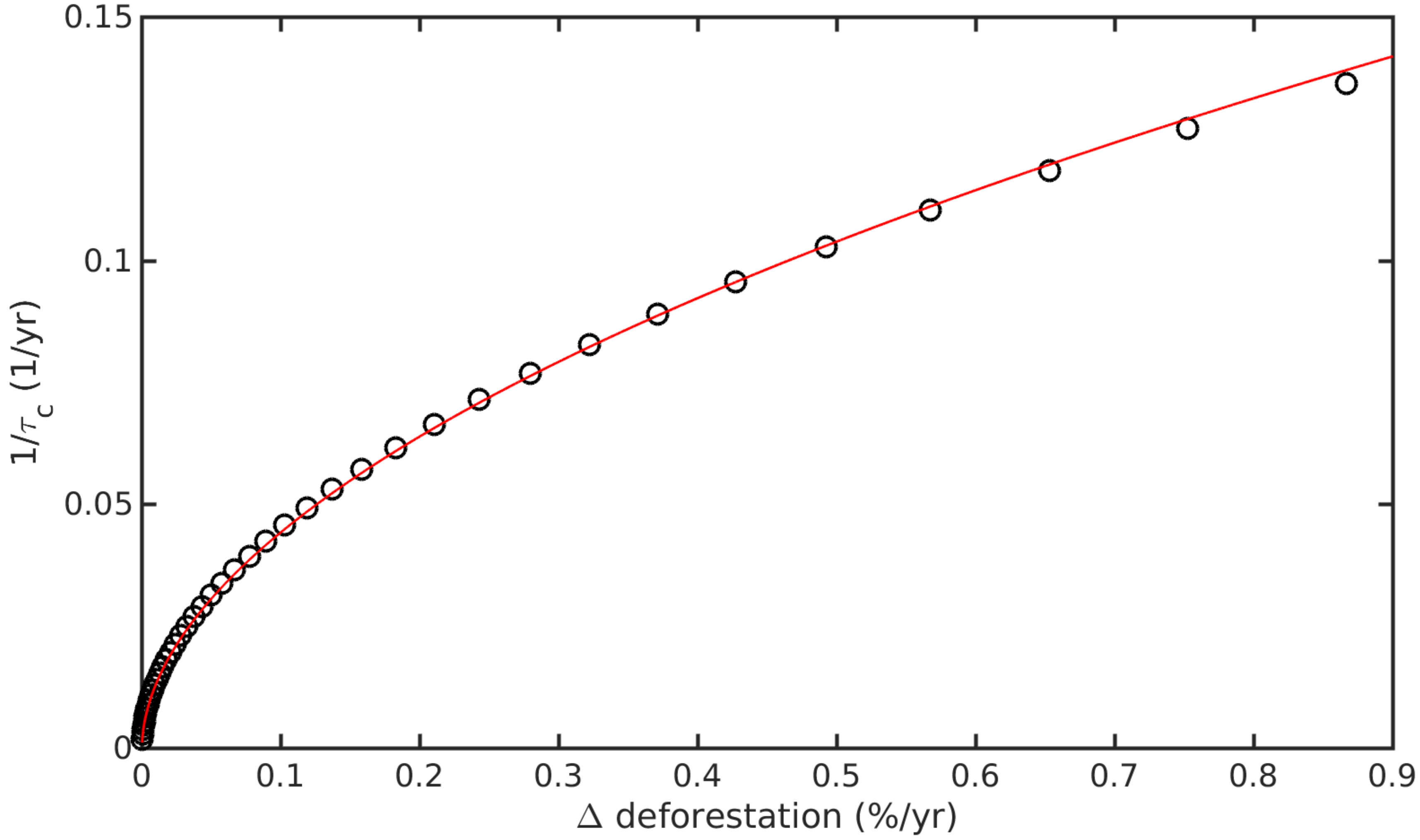
Introducing an artificial perturbation

Much slower recovery rate close to the rainforest
→ savanna bifurcation



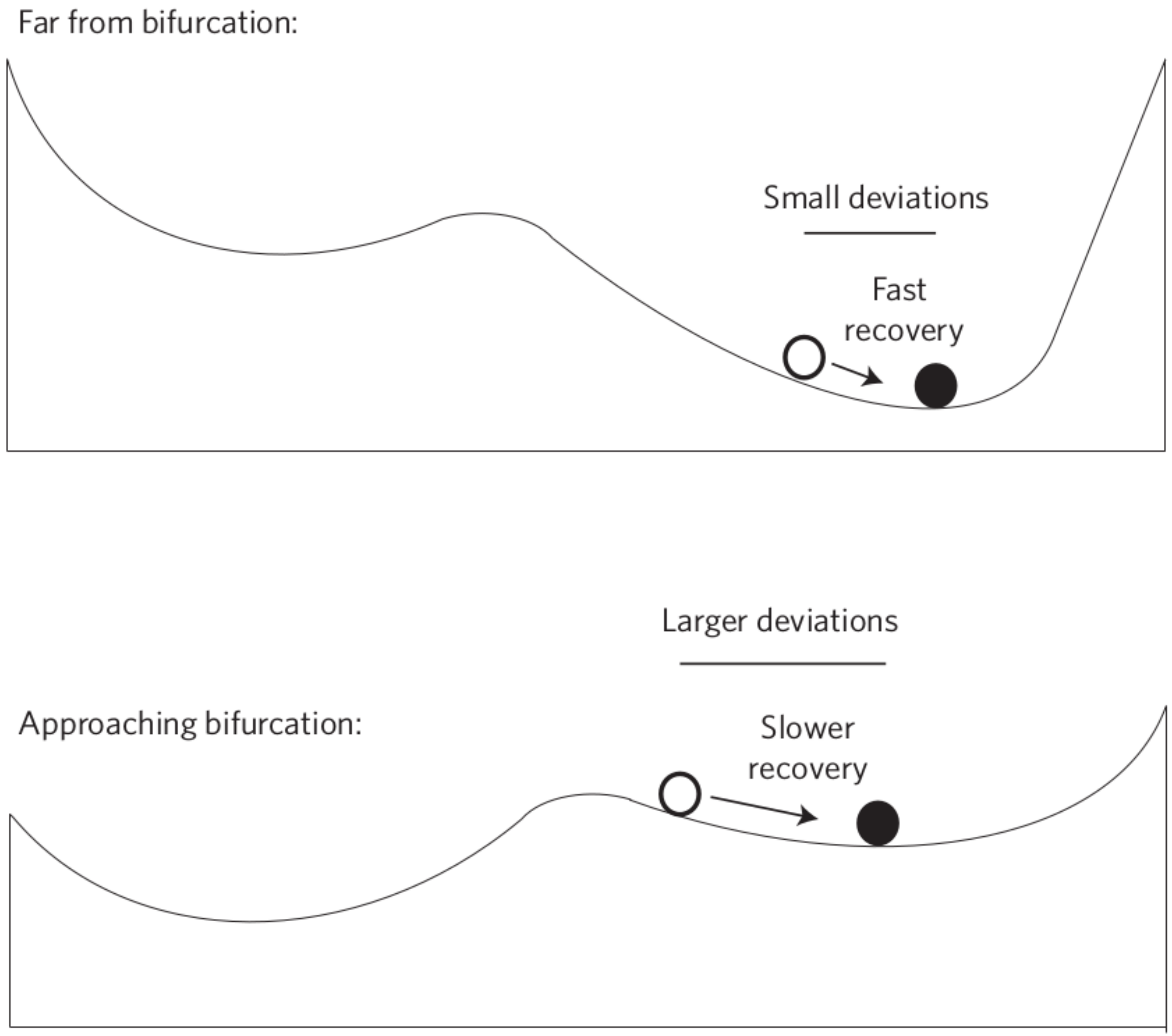
Recovery rate

Variation of the recovery rate as a function of the system state can be used to predict proximity to a bifurcation



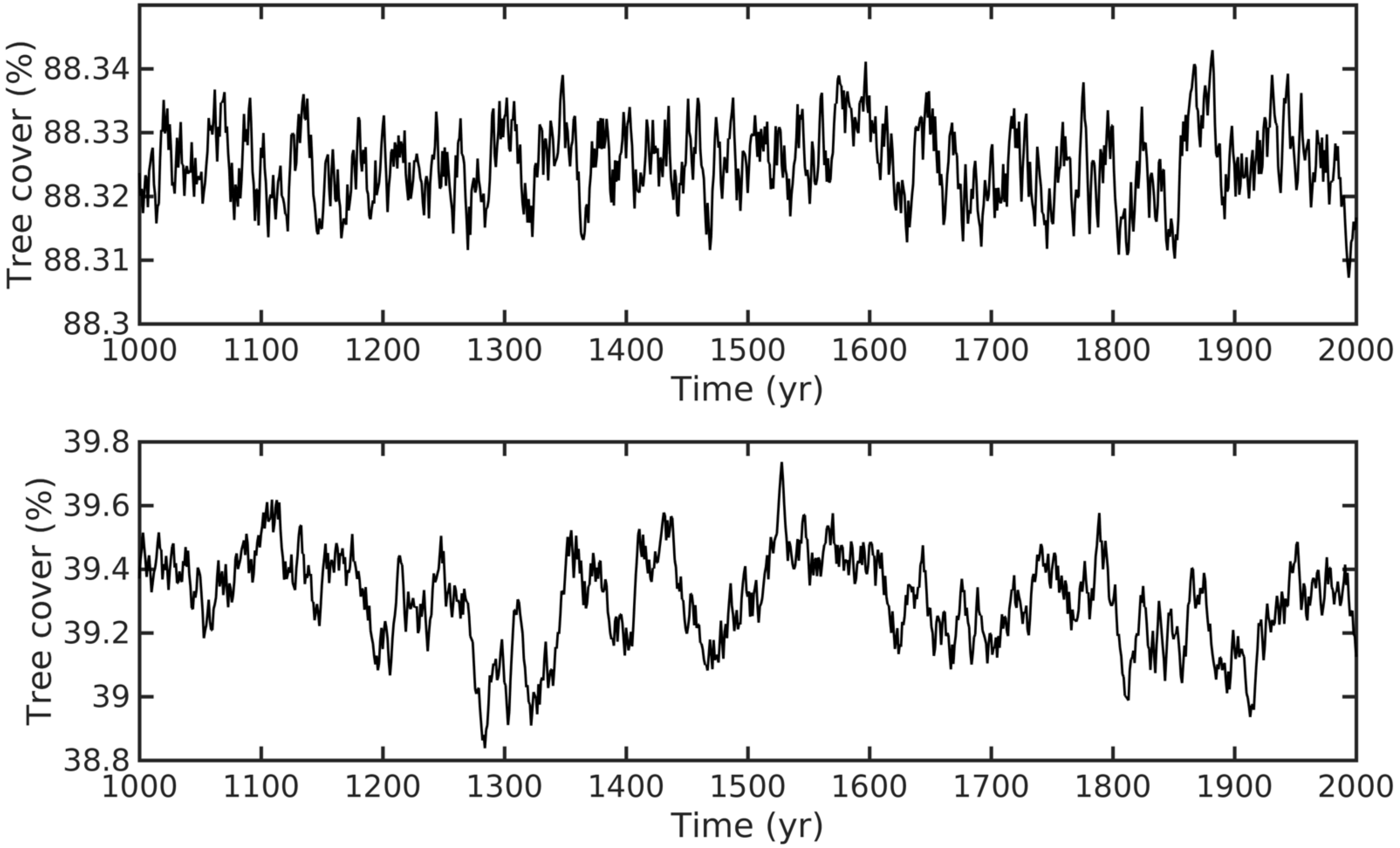
Variation and autocorrelation

Both variation and autocorrelation under random perturbations are increased in proximity to a bifurcation



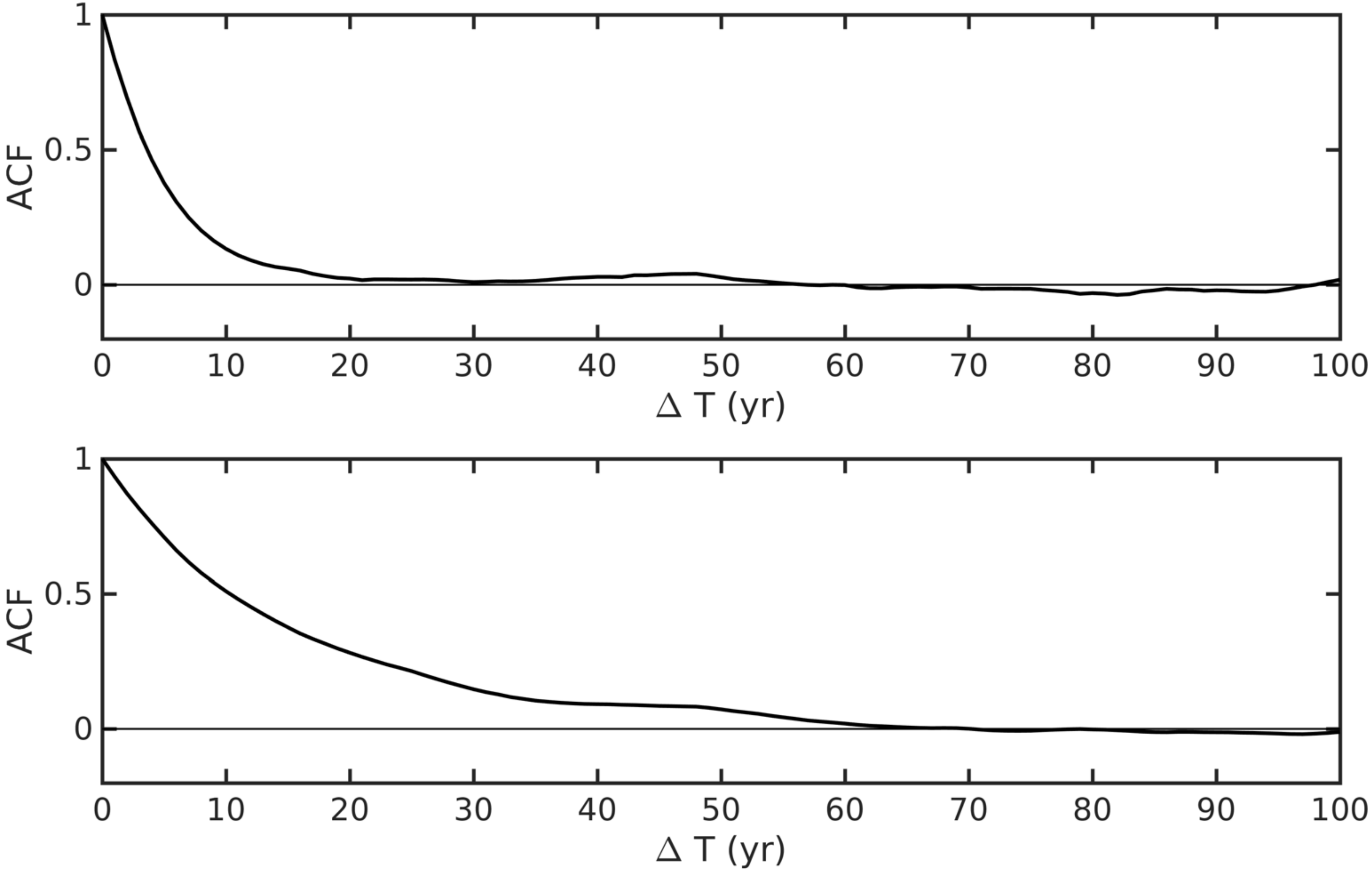
Variation and autocorrelation

Much higher variation and autocorrelation close
to the rainforest
→ savanna bifurcation



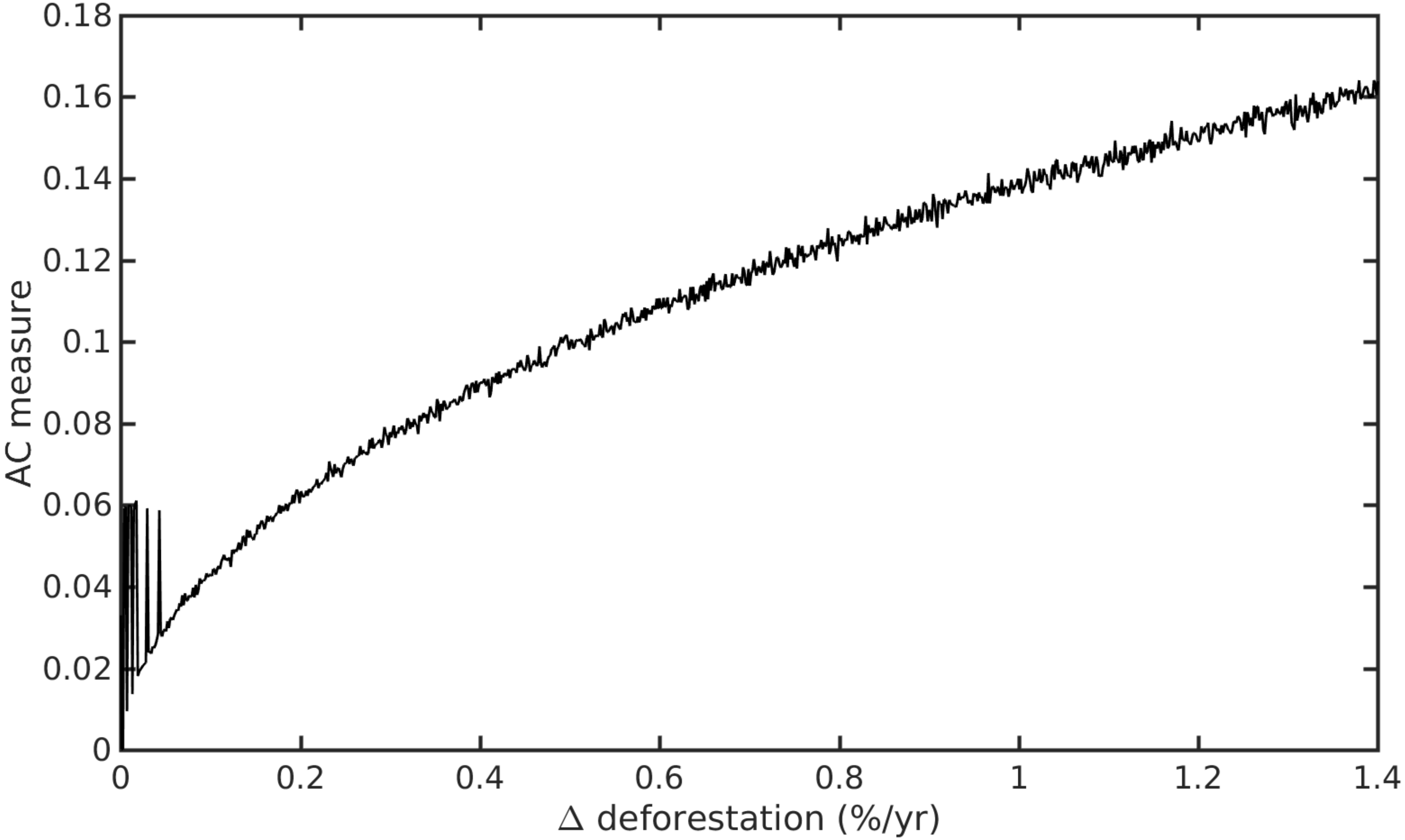
Autocorrelation function

Slope of the autocorrelation function as a
convenaint measure



Autocorrelation

Theoretically, early warning of a bifurcation is possible,
Practical use of straightforward early warning signals is not robust, can be supplemented by detailed modeling effort



Thank you for your attention

References

- *Tipping points in tropical tree cover: linking theory to data*
  Egbert H. van Nes et al., 2014

Pictures / Figures

1 – Ben Britten, *rainforest,* CC BY 2.0
2 – Michele Zanin, *Savanna in the morning,* CC BY-SA 2.0
3 – Gerben van Heijningen, *Desert,* CC BY-NC 2.0
4 – *Early warning of climate tipping points,*
  Timothy M. Lenton, Nature Climate Change volume 1, pages 201–209 (2011),
  Figure 1.