

# Assignment 1

## Question 1

- States: Amount of water in each jug.
- Initial State: Both jugs are empty. Let  $x$  represent the amount of water in the 4-gallon jug, and let  $y$  represent the amount of water in the 3-gallon jug. The initial state is represented by  $x = 0$  gallons and  $y = 0$  gallons.
- Goal State: The 3-gallon jug has exactly 2 gallons of water. Using the same variables from the initial state, this is represented by  $y = 2$  gallons.
- Actions:
  - Completely fill one or both jugs with a pump (if it is not yet already full)
  - Pour water from one jug into the other or onto the ground (pouring stops once the destination jug is full or the source jug is empty)
  - Pour water from one jug onto the ground (if it has water in it)
- Goal Test: The amount of water in the 3-gallon jug is 2 gallons.
- Cost: Filling a jug, pouring water from one jug into the other, and pouring water onto the ground all have a cost of 1. Path cost will equal the number of actions performed from initial state to goal state.
- Solution:

Initially, both jugs are at 0 gallons, represented as (4-gallon jug amount, 3-gallon jug amount)  $\rightarrow (0, 0)$

  1. Completely fill the 3-gallon jug (0, 3)
  2. Pour water from the 3-gallon jug into the 4-gallon jug (3, 0)
  3. Completely fill the 3-gallon jug (3, 3)
  4. Pour water from the 3-gallon jug into the 4-gallon jug (4, 2)

# Assignment 1

## Question 2

- States: Positions of each disc on the poles.
- Initial State: All discs are stacked correctly on one of the poles, with correctly meaning by decreasing diameter from bottom to top.
- Goal State: Correctly stack all of the discs on a different pole.
- Actions:
  - Move a disc from one pole to another (a disc may be stacked only on top of a larger disc or onto an empty pole)
- Goal Test: All of the discs are stacked correctly on another pole.
- Cost: Moving a disc has a cost of 1. Path cost will equal the number of actions performed from initial state to goal state.
- Solution:

Define the three poles as source, destination, and temporary. The source is the pole where all the discs start according to the initial state. The destination is the pole where all the discs are moved to, as defined by the goal state. The temporary is a pole required to perform this task, while being able to follow the constraint that discs are stacked by decreasing diameter. The following algorithm solves the problem:

1. Move a disc from either the source to the temporary, or the temporary to the source
2. Move a disc from either the source to the destination, or the destination to the source
3. Move a disc from either the temporary to the destination, or the destination to the temporary
4. Repeat until complete

The wording in steps 1-3 implies that the correct move is performed each time, meaning that the move follows the decreasing diameter constraint.

# Assignment 1

## Question 3

- States: Locations of each of the four colours on the planar map.
- Initial State: None of the contiguous regions on the map are coloured in.
- Goal State: The map is coloured in such that, with no more than four colours, no two adjacent regions have the same colour.
- Actions:
  - Colour in a region of the map (no two adjacent regions have the same colour)
- Goal Test: The map is coloured in with four colours, with no two adjacent regions having the same colour.
- Cost: Colouring in a region of the map is a cost of 1. Path cost will equal the number of actions performed from initial state to goal state.

# Assignment 1

## Question 4

### Breadth-First Search:

Expanded Node	Open Queue	Closed Queue	Ignored
	{ A <sup>0</sup> }		
A <sup>0</sup>	{ B <sup>1</sup> D <sup>1</sup> E <sup>1</sup> }		
B <sup>1</sup>	{ D <sup>1</sup> E <sup>1</sup> G <sup>2</sup> }	{ A <sup>0</sup> }	
D <sup>1</sup>	{ E <sup>1</sup> G <sup>2</sup> C <sup>2</sup> F <sup>2</sup> }	{ A <sup>0</sup> B <sup>1</sup> }	
E <sup>1</sup>	{ G <sup>2</sup> C <sup>2</sup> F <sup>2</sup> H <sup>2</sup> I <sup>2</sup> }	{ A <sup>0</sup> B <sup>1</sup> D <sup>1</sup> }	G <sup>2</sup>
G <sup>2</sup>	{ C <sup>2</sup> F <sup>2</sup> H <sup>2</sup> I <sup>2</sup> I <sup>3</sup> }	{ A <sup>0</sup> B <sup>1</sup> D <sup>1</sup> E <sup>1</sup> }	
C <sup>2</sup>	{ F <sup>2</sup> H <sup>2</sup> I <sup>2</sup> I <sup>3</sup> B <sup>3</sup> E <sup>3</sup> F <sup>3</sup> }	{ A <sup>0</sup> B <sup>1</sup> D <sup>1</sup> E <sup>1</sup> G <sup>2</sup> }	
F <sup>2</sup>	{ H <sup>2</sup> I <sup>2</sup> I <sup>3</sup> B <sup>3</sup> E <sup>3</sup> F <sup>3</sup> H <sup>3</sup> }	{ A <sup>0</sup> B <sup>1</sup> D <sup>1</sup> E <sup>1</sup> G <sup>2</sup> C <sup>2</sup> }	E <sup>3</sup>
H <sup>2</sup>	{ I <sup>2</sup> I <sup>3</sup> B <sup>3</sup> E <sup>3</sup> F <sup>3</sup> H <sup>3</sup> }	{ A <sup>0</sup> B <sup>1</sup> D <sup>1</sup> E <sup>1</sup> G <sup>2</sup> C <sup>2</sup> F <sup>2</sup> }	I <sup>3</sup>
I <sup>2</sup>	{ I <sup>3</sup> B <sup>3</sup> E <sup>3</sup> F <sup>3</sup> H <sup>3</sup> }	{ A <sup>0</sup> B <sup>1</sup> D <sup>1</sup> E <sup>1</sup> G <sup>2</sup> C <sup>2</sup> F <sup>2</sup> H <sup>2</sup> }	

Solution path found is A E I, cost 2

Number of nodes expanded (including goal node) = 9

### Depth-First Search:

Expanded Node	Open Stack	Closed Stack	Ignored
	{ A <sup>0</sup> }		
A <sup>0</sup>	{ B <sup>1</sup> D <sup>1</sup> E <sup>1</sup> }		
B <sup>1</sup>	{ G <sup>2</sup> D <sup>1</sup> E <sup>1</sup> }	{ A <sup>0</sup> }	
G <sup>2</sup>	{ I <sup>3</sup> D <sup>1</sup> E <sup>1</sup> }	{ A <sup>0</sup> B <sup>1</sup> }	
I <sup>3</sup>	{ D <sup>1</sup> E <sup>1</sup> }	{ A <sup>0</sup> B <sup>1</sup> G <sup>2</sup> }	

Solution path found is A B G I, cost 3

Number of nodes expanded (including goal node) = 4

# Assignment 1

## Question 5

### Breadth-First Search:

Expanded Node	Open Queue	Closed Queue
	{ S <sup>0</sup> }	
S <sup>0</sup>	{ A <sup>3</sup> D <sup>4</sup> }	
A <sup>3</sup>	{ D <sup>4</sup> B <sup>7</sup> D <sup>8</sup> }	{ S <sup>0</sup> }
D <sup>4</sup>	{ B <sup>7</sup> D <sup>8</sup> E <sup>6</sup> }	{ S <sup>0</sup> A <sup>3</sup> }
B <sup>7</sup>	{ D <sup>8</sup> E <sup>6</sup> C <sup>10</sup> H <sup>11</sup> }	{ S <sup>0</sup> A <sup>3</sup> D <sup>4</sup> }
D <sup>8</sup>	{ E <sup>6</sup> C <sup>10</sup> H <sup>11</sup> E <sup>10</sup> }	{ S <sup>0</sup> A <sup>3</sup> D <sup>4</sup> B <sup>7</sup> }
E <sup>6</sup>	{ C <sup>10</sup> H <sup>11</sup> E <sup>10</sup> B <sup>11</sup> C <sup>10</sup> F <sup>10</sup> }	{ S <sup>0</sup> A <sup>3</sup> D <sup>4</sup> B <sup>7</sup> D <sup>8</sup> }
C <sup>10</sup>	{ H <sup>11</sup> E <sup>10</sup> B <sup>11</sup> C <sup>10</sup> F <sup>10</sup> }	{ S <sup>0</sup> A <sup>3</sup> D <sup>4</sup> B <sup>7</sup> D <sup>8</sup> E <sup>6</sup> }
H <sup>11</sup>	{ E <sup>10</sup> B <sup>11</sup> C <sup>10</sup> F <sup>10</sup> G <sup>12</sup> }	{ S <sup>0</sup> A <sup>3</sup> D <sup>4</sup> B <sup>7</sup> D <sup>8</sup> E <sup>6</sup> C <sup>10</sup> }
E <sup>10</sup>	{ B <sup>11</sup> C <sup>10</sup> F <sup>10</sup> G <sup>12</sup> B <sup>15</sup> C <sup>14</sup> F <sup>14</sup> }	{ S <sup>0</sup> A <sup>3</sup> D <sup>4</sup> B <sup>7</sup> D <sup>8</sup> E <sup>6</sup> C <sup>10</sup> H <sup>11</sup> }
B <sup>11</sup>	{ C <sup>10</sup> F <sup>10</sup> G <sup>12</sup> B <sup>15</sup> C <sup>14</sup> F <sup>14</sup> C <sup>14</sup> H <sup>15</sup> }	{ S <sup>0</sup> A <sup>3</sup> D <sup>4</sup> B <sup>7</sup> D <sup>8</sup> E <sup>6</sup> C <sup>10</sup> H <sup>11</sup> E <sup>10</sup> }
C <sup>10</sup>	{ F <sup>10</sup> G <sup>12</sup> B <sup>15</sup> C <sup>14</sup> F <sup>14</sup> C <sup>14</sup> H <sup>15</sup> }	{ S <sup>0</sup> A <sup>3</sup> D <sup>4</sup> B <sup>7</sup> D <sup>8</sup> E <sup>6</sup> C <sup>10</sup> H <sup>11</sup> E <sup>10</sup> B <sup>11</sup> }
F <sup>10</sup>	{ G <sup>12</sup> B <sup>15</sup> C <sup>14</sup> F <sup>14</sup> C <sup>14</sup> H <sup>15</sup> H <sup>15</sup> }	{ S <sup>0</sup> A <sup>3</sup> D <sup>4</sup> B <sup>7</sup> D <sup>8</sup> E <sup>6</sup> C <sup>10</sup> H <sup>11</sup> E <sup>10</sup> B <sup>11</sup> C <sup>10</sup> }
G <sup>12</sup>	{ B <sup>15</sup> C <sup>14</sup> F <sup>14</sup> C <sup>14</sup> H <sup>15</sup> H <sup>15</sup> }	{ S <sup>0</sup> A <sup>3</sup> D <sup>4</sup> B <sup>7</sup> D <sup>8</sup> E <sup>6</sup> C <sup>10</sup> H <sup>11</sup> E <sup>10</sup> B <sup>11</sup> C <sup>10</sup> F <sup>10</sup> }

Solution path found is S A B H G, cost 12

Number of nodes expanded (including goal node) = 13

### Depth-First Search:

Expanded Node	Open Stack	Closed Stack
	{ S <sup>0</sup> }	

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$S^0$	$\{ A^3 D^4 \}$	
$A^3$	$\{ B^7 D^8 D^4 \}$	$\{ S^0 \}$
$B^7$	$\{ C^{10} H^{11} D^8 D^4 \}$	$\{ S^0 A^3 \}$
$C^{10}$	$\{ H^{11} D^8 D^4 \}$	$\{ S^0 A^3 B^7 \}$
$H^{11}$	$\{ G^{12} D^8 D^4 \}$	$\{ S^0 A^3 B^7 C^{10} \}$
$G^{12}$	$\{ D^8 D^4 \}$	$\{ S^0 A^3 B^7 C^{10} H^{11} \}$

Solution path found is S A B H G, cost 12

Number of nodes expanded (including goal node) = 6

## Uniform-Cost Search:

Expanded Node	Open Set	Closed Set
	$\{ S^0 \}$	
$S^0$	$\{ A^3 D^4 \}$	
$A^3$	$\{ D^4 B^7 D^8 \}$	$\{ S^0 \}$
$D^4$	$\{ E^6 B^7 D^8 \}$	$\{ S^0 A^3 \}$
$E^6$	$\{ B^7 D^8 C^{10} F^{10} B^{11} \}$	$\{ S^0 A^3 D^4 \}$
$B^7$	$\{ D^8 C^{10} C^{10} F^{10} B^{11} H^{11} \}$	$\{ S^0 A^3 D^4 E^6 \}$
$D^8$	$\{ C^{10} C^{10} E^{10} F^{10} B^{11} H^{11} \}$	$\{ S^0 A^3 D^4 E^6 B^7 \}$
$C^{10}$	$\{ C^{10} E^{10} F^{10} B^{11} H^{11} \}$	$\{ S^0 A^3 D^4 E^6 B^7 D^8 \}$
$C^{10}$	$\{ E^{10} F^{10} B^{11} H^{11} \}$	$\{ S^0 A^3 D^4 E^6 B^7 D^8 C^{10} \}$
$E^{10}$	$\{ F^{10} B^{11} H^{11} C^{14} F^{14} B^{15} \}$	$\{ S^0 A^3 D^4 E^6 B^7 D^8 C^{10} C^{10} \}$
$F^{10}$	$\{ B^{11} H^{11} C^{14} F^{14} B^{15} H^{15} \}$	$\{ S^0 A^3 D^4 E^6 B^7 D^8 C^{10} C^{10} E^{10} \}$
$B^{11}$	$\{ H^{11} C^{14} C^{14} F^{14} B^{15} H^{15} H^{15} \}$	$\{ S^0 A^3 D^4 E^6 B^7 D^8 C^{10} C^{10} E^{10} F^{10} \}$
$H^{11}$	$\{ G^{12} C^{14} C^{14} F^{14} B^{15} H^{15} H^{15} \}$	$\{ S^0 A^3 D^4 E^6 B^7 D^8 C^{10} C^{10} E^{10} F^{10} B^{11} \}$

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$G^{12}$	$\{ C^{14} C^{14} F^{14} B^{15} H^{15} H^{15} \}$	$\{ S^0 A^3 D^4 E^6 B^7 D^8 C^{10} C^{10} E^{10} F^{10} B^{11} H^{11} \}$
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Solution path found is S A B H G, cost 12

Number of nodes expanded (including goal node) = 13

# Assignment 1

## Question 6

### Breadth-First Search:

Expanded Node	Open Queue	Closed Queue
	{ A <sup>0</sup> }	
A <sup>0</sup>	{ S <sup>140</sup> T <sup>118</sup> Z <sup>75</sup> }	
S <sup>140</sup>	{ T <sup>118</sup> Z <sup>75</sup> F <sup>379</sup> R <sup>360</sup> }	{ A <sup>0</sup> }
T <sup>118</sup>	{ Z <sup>75</sup> F <sup>379</sup> R <sup>360</sup> L <sup>347</sup> }	{ A <sup>0</sup> S <sup>140</sup> }
Z <sup>75</sup>	{ F <sup>379</sup> R <sup>360</sup> L <sup>347</sup> T <sup>221</sup> }	{ A <sup>0</sup> S <sup>140</sup> T <sup>118</sup> }
F <sup>379</sup>	{ R <sup>360</sup> L <sup>347</sup> T <sup>221</sup> }	{ A <sup>0</sup> S <sup>140</sup> T <sup>118</sup> Z <sup>75</sup> }
R <sup>360</sup>	{ L <sup>347</sup> T <sup>221</sup> C <sup>696</sup> P <sup>677</sup> }	{ A <sup>0</sup> S <sup>140</sup> T <sup>118</sup> Z <sup>75</sup> F <sup>379</sup> }
L <sup>347</sup>	{ T <sup>221</sup> C <sup>696</sup> P <sup>677</sup> M <sup>646</sup> }	{ A <sup>0</sup> S <sup>140</sup> T <sup>118</sup> Z <sup>75</sup> F <sup>379</sup> R <sup>360</sup> }
T <sup>221</sup>	{ C <sup>696</sup> P <sup>677</sup> M <sup>646</sup> }	{ A <sup>0</sup> S <sup>140</sup> T <sup>118</sup> Z <sup>75</sup> F <sup>379</sup> R <sup>360</sup> L <sup>347</sup> }
C <sup>696</sup>	{ P <sup>677</sup> M <sup>646</sup> }	{ A <sup>0</sup> S <sup>140</sup> T <sup>118</sup> Z <sup>75</sup> F <sup>379</sup> R <sup>360</sup> L <sup>347</sup> T <sup>221</sup> }

Solution path found is A S R C, cost 696

Number of nodes expanded (including goal node) = 9

### Depth-First Search:

Expanded Node	Open Stack	Closed Stack
	{ A <sup>0</sup> }	
A <sup>0</sup>	{ S <sup>140</sup> T <sup>118</sup> Z <sup>75</sup> }	
S <sup>140</sup>	{ F <sup>379</sup> R <sup>360</sup> T <sup>118</sup> Z <sup>75</sup> }	{ A <sup>0</sup> }
F <sup>379</sup>	{ R <sup>360</sup> T <sup>118</sup> Z <sup>75</sup> }	{ A <sup>0</sup> S <sup>140</sup> }
R <sup>360</sup>	{ C <sup>696</sup> P <sup>677</sup> T <sup>118</sup> Z <sup>75</sup> }	{ A <sup>0</sup> S <sup>140</sup> F <sup>379</sup> }



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$C^{696}$	$\{ P^{677} T^{118} Z^{75} \}$	$\{ A^0 S^{140} F^{379} R^{360} \}$
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Solution path found is A S R C, cost 696

Number of nodes expanded (including goal node) = 5

## Uniform-Cost Search:

Expanded Node	Open Set	Closed Set
	$\{ A^0 \}$	
$A^0$	$\{ Z^{75} T^{118} S^{140} \}$	
$Z^{75}$	$\{ T^{118} S^{140} T^{221} \}$	$\{ A^0 \}$
$T^{118}$	$\{ S^{140} T^{221} L^{347} \}$	$\{ A^0 Z^{75} \}$
$S^{140}$	$\{ T^{221} L^{347} R^{360} F^{379} \}$	$\{ A^0 Z^{75} T^{118} \}$
$T^{221}$	$\{ L^{347} R^{360} F^{379} \}$	$\{ A^0 Z^{75} T^{118} S^{140} \}$
$L^{347}$	$\{ R^{360} F^{379} M^{646} \}$	$\{ A^0 Z^{75} T^{118} S^{140} T^{221} \}$
$R^{360}$	$\{ F^{379} M^{646} P^{677} C^{696} \}$	$\{ A^0 Z^{75} T^{118} S^{140} T^{221} L^{347} \}$
$F^{379}$	$\{ M^{646} P^{677} C^{696} \}$	$\{ A^0 Z^{75} T^{118} S^{140} T^{221} L^{347} R^{360} \}$
$M^{646}$	$\{ P^{677} C^{696} \}$	$\{ A^0 Z^{75} T^{118} S^{140} T^{221} L^{347} R^{360} F^{379} \}$

Solution path found is A T L M, cost 646

Number of nodes expanded (including goal node) = 9

## Depth-Limited Search:

Expanded Node	Open Stack	Closed Stack
	$\{ A^0 \}$	
$A^0$	$\{ S^{140} T^{118} Z^{75} \}$	
$S^{140}$	$\{ F^{379} R^{360} T^{118} Z^{75} \}$	$\{ A^0 \}$
$F^{379}$	$\{ R^{360} T^{118} Z^{75} \}$	$\{ A^0 S^{140} \}$

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$R^{360}$	$\{ T^{118} Z^{75} \}$	$\{ A^0 S^{140} F^{379} \}$
$T^{118}$	$\{ L^{347} Z^{75} \}$	$\{ A^0 S^{140} F^{379} R^{360} \}$
$L^{347}$	$\{ Z^{75} \}$	$\{ A^0 S^{140} F^{379} R^{360} T^{118} \}$
$Z^{75}$	$\{ T^{221} \}$	$\{ A^0 S^{140} F^{379} R^{360} T^{118} L^{347} \}$
$T^{221}$	$\{ \}$	$\{ A^0 S^{140} F^{379} R^{360} T^{118} L^{347} Z^{75} \}$

Solution path not found

Number of nodes expanded (including goal node) = 8

**Iterative-Deepening Search:**

Expanded Node	Open Stack	Limit
	$\{ A^0 \}$	0
$A^0$	$\{ \}$	
	$\{ A^0 \}$	1
$A^0$	$\{ S^{140} T^{118} Z^{75} \}$	
$S^{140}$	$\{ T^{118} Z^{75} \}$	
$T^{118}$	$\{ Z^{75} \}$	
$Z^{75}$	$\{ \}$	
	$\{ A^0 \}$	2
$A^0$	$\{ S^{140} T^{118} Z^{75} \}$	
$S^{140}$	$\{ F^{379} R^{360} T^{118} Z^{75} \}$	
$F^{379}$	$\{ R^{360} T^{118} Z^{75} \}$	
$R^{360}$	$\{ T^{118} Z^{75} \}$	
$T^{118}$	$\{ L^{347} Z^{75} \}$	
$L^{347}$	$\{ Z^{75} \}$	

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$Z^{75}$	$\{ T^{221} \}$	
$T^{221}$	$\{ \}$	
	$\{ A^0 \}$	3
$A^0$	$\{ S^{140} T^{118} Z^{75} \}$	
$S^{140}$	$\{ F^{379} R^{360} T^{118} Z^{75} \}$	
$F^{379}$	$\{ R^{360} T^{118} Z^{75} \}$	
$R^{360}$	$\{ C^{696} P^{677} T^{118} Z^{75} \}$	
$C^{696}$	$\{ P^{677} T^{118} Z^{75} \}$	

Solution path found is A S R C, cost 696

Number of nodes expanded (including goal node) = 18