Άσκηση 1

α) $\frac{z}{4} + \frac{2-j}{4+j} z - 1 = \frac{1}{2} \Rightarrow \frac{x+jy}{4} + \frac{2-j}{4+j}(x+jy) = 1\frac{3}{2}$

$\Rightarrow \frac{x+jy}{2} + \frac{4x+4jy-2jx+2y}{4+j} = 3$

$\Rightarrow \frac{(x+jy)(2+j/2)}{2\cdot(2+j/2)} + \frac{4x+4jy-2jx+2y}{2\cdot(2+j/2)} = 3$

$\Rightarrow \frac{x+jy+jx/4-y/4+2x+2jy-jx+y}{2+j/2} = 3$

$\Rightarrow 3x + 3jy - \frac{3jx}{4} + \frac{3y}{4} = 6 + 3j/2$

$\Rightarrow x + jy - \frac{jx}{4} + \frac{y}{4} = 2 + \frac{j}{2}$

$$\begin{cases} x + \frac{y}{4} = 2 \Rightarrow x = 2 - \frac{y}{4} \text{ ①} \\ jy - \frac{jx}{4} = \frac{j}{2} \Rightarrow y - \frac{x}{4} = \frac{1}{2} \Rightarrow 4y - x = 2 \end{cases}$$

① $\Rightarrow 4y - 2 + \frac{y}{4} = 2$

$\Rightarrow \frac{16y}{4} + \frac{y}{4} = 4$

$17y = 16$

$y = \frac{16}{17}$

① $\Rightarrow x = 2 - \frac{y}{4}$

$\Rightarrow 4x = 8 - y$

$\Rightarrow 4x = \frac{136}{17} - \frac{16}{17}$

$4x = \frac{120}{17}$

$x = \frac{120}{68}$

$x = \frac{30}{17}$

β) $z = x_1 + y_1 j$

$w = x_2 + y_2 j$

$z - jw + 1 = 2$

$x_1 + y_1 j - x_2 j - y_2 j^2 + 1 = 2$

$x_1 + y_1 j - x_2 j + y_2 = 1$

$\begin{cases} x_1 + y_2 = 1 \ (1) \\ y_1 - x_2 = 0 \Rightarrow y_1 = x_2 \ (2) \end{cases}$

$zj + w + 2(w - z) = 2j$

$x_1 j + y_1 j^2 + x_2 + y_2 j + 2(x_2 + y_2 j - x_1 - y_1 j) = 2j$

$x_1 j - y_1 + 3x_2 + 3y_2 j - 2x_1 - 2y_1 j = 2j$

$\begin{cases} x_1 + 3y_2 - 2y_1 = 2 \overset{(1)}{\Rightarrow} 1 - y_2 + 3y_2 - 2y_1 = 2 \Rightarrow 2y_2 - 2y_1 = 1 \ (3) \\ -y_1 + 3x_2 - 2x_1 = 0 \overset{(2)}{\Rightarrow} -x_2 + 3x_2 - 2x_1 = 0 \Rightarrow 2x_2 - 2x_1 = 0 \overset{(4)}{\Rightarrow} \end{cases}$

$x_2 - 1 + y_2 = 0 \overset{(3)}{\Rightarrow}$

$x_2 - 1 + \frac{1}{2} + y_1 = 0 \overset{(2)}{\Rightarrow}$

$x_2 - 1 + \frac{1}{2} + x_2 = 0$

$2x_2 = \frac{1}{2}$

$\boxed{x_2 = \frac{1}{4}}$

(4) $\boxed{x_1 = \frac{1}{4}}$

(2) $\boxed{y_1 = \frac{1}{4}}$

(3) $y_2 - y_1 = \frac{1}{2}$

$y_2 - \frac{1}{4} = \frac{1}{2}$

$\boxed{y_2 = \frac{3}{4}}$

Άσκηση 2

$z^3 = 8j$

$|z|^3 \cdot (\cos(3\theta) + j\sin(3\theta)) = |8| \cdot [\cos(\frac{\pi}{2}) + j\sin(\frac{\pi}{2})]$

$\begin{cases} z = \sqrt[3]{8} = 2 \\ 3\theta = 2k\pi + \frac{\pi}{2} \Rightarrow \theta = \frac{2}{3}k\pi + \frac{\pi}{6}, \ k = 0,1,2 \end{cases}$

$k=0$ $\quad z_1 = 2 \cdot e^{j\frac{\pi}{6}} = 2(\cos(\frac{\pi}{6}) + j\sin(\frac{\pi}{6})) = 2 \cdot (\frac{\sqrt{3}}{2} + j \cdot \frac{1}{2}) = \sqrt{3} + j$

$k=1$ $\quad z_2 = 2 \cdot e^{j\frac{5\pi}{6}} = 2(\cos(\frac{5\pi}{6}) + j\sin(\frac{5\pi}{6})) = 2(-\frac{\sqrt{3}}{2} + j \cdot \frac{1}{2} = -\sqrt{3} + j$

$k=2$ $\quad z_3 = 2 \cdot e^{j\frac{9\pi}{2}} = 2(\cos(\frac{9\pi}{6}) + j\sin(\frac{9\pi}{6})) = 2(\cos(\frac{3\pi}{2}) + j\sin(\frac{3\pi}{2}))$

$= 2(0 - j)$

$= -2j$

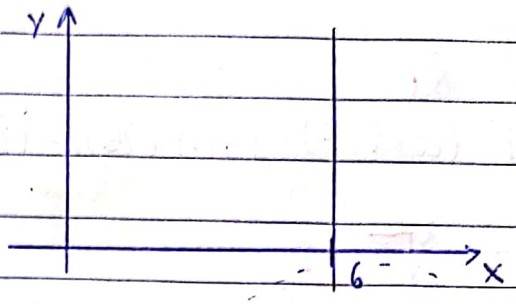
Άσκηση 3

α) $R\{z+1\}=7$

$R\{x+jy+1\}=7$

$x+1=7$

$x=6$

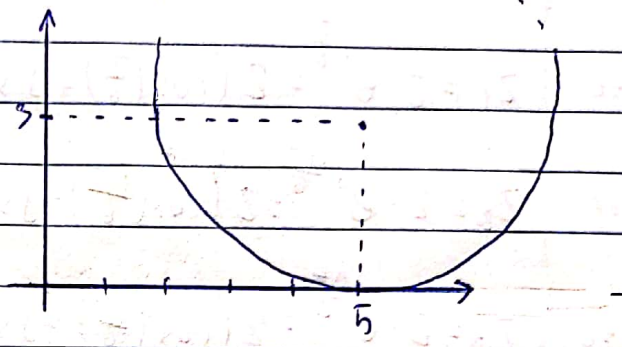


β) $|z-5-3j|=3$

$|x+jy-5-3j|=3$

$|(x-5)+j(y-3)|=3$

$(x-5)^2+(y-3)^2=3^2$



γ) $\arg(z+3+2j)=\frac{3\pi}{4}$

$\arg(x+jy+3+2j)=\frac{3\pi}{4}$

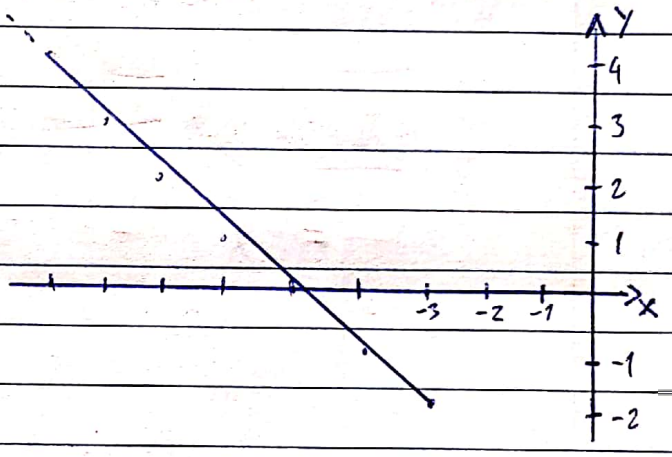
$\tan^{-1}\left(\frac{2+y}{3+x}\right)=\frac{3\pi}{4}$

$\frac{2+y}{3+x}=\tan\left(\frac{3\pi}{4}\right)$

$\frac{2+y}{3+x}=-1$

$y=-x-5$

$(x<-3,\ y>-2)$



δ) $|z|=|z-6j|$

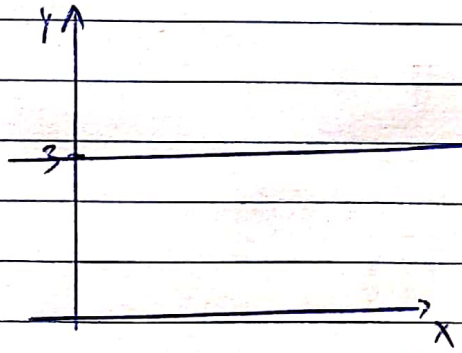
$|x+jy|^2=|x+jy-6j|^2$

$x^2+y^2=|x+(y-6)j|^2$

$x^2+y^2=x^2+y^2-12y+36$

$12y=36$

$y=3$



Άσκηση 4

$$w = \frac{jz-2}{1-z}$$

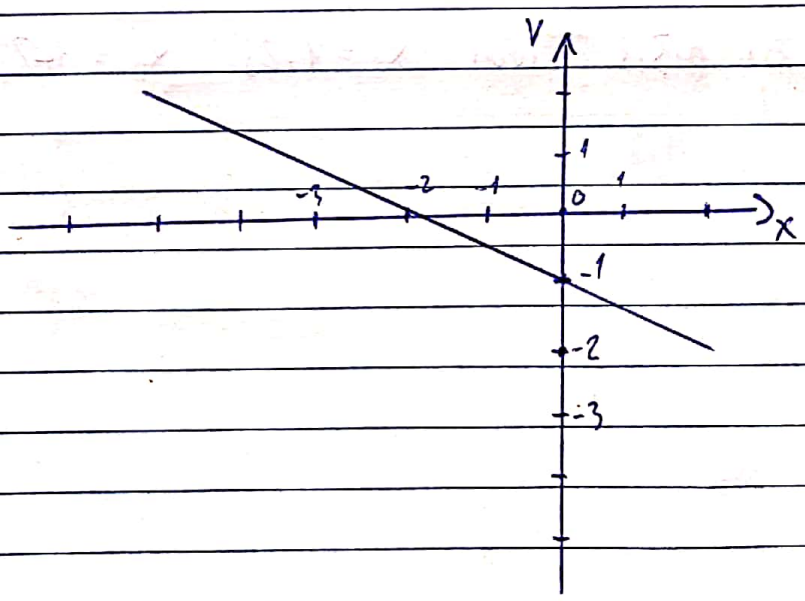
$$x+jy = \frac{jz-2}{1-z}$$

$$x+jy-zx-jyz = jz-2$$
$$x(1-z)+j(y-zy) = jz-2$$

$$\begin{cases} y-zy = z \Rightarrow y(1-z) = z \Rightarrow y = \frac{z}{1-z} \ (1) \\ x(1-z) = -2 \Rightarrow x-xz = -2 \Rightarrow xz = 2+x \Rightarrow z = \frac{2+x}{x} \ (2) \end{cases}$$

$$(1) \Rightarrow y = \frac{z}{1-z} = \frac{\frac{2+x}{x}}{1-\frac{2+x}{x}} = \frac{\frac{2+x}{x}}{\frac{x-2-x}{x}} = \frac{2+x}{-2} \Rightarrow$$

$$y = \frac{2+x}{-2} \Rightarrow y = -\frac{2+x}{2}$$



Ασκηση 5

$x^3 - x^2 + 3x + k$

Για $x = -1$:

$-1^3 - (-1)^2 + 3 \cdot (-1) + k = 0$

$-1 - 1 - 3 + k = 0$

$\boxed{k = 5}$

Σχημα Horner

	1	-1	3	k
-1	↓	-1	2	-5
	1	-2	5	k-5

$x^3 - x^2 + 3x + 5 = (x+1)(x^2 - 2x + 5) = 0$

$x + 1 = 0$ ή $x^2 - 2x + 5 = 0$

$x = -1$

$\Delta = 4 - 20 = -16$

$x_{1,2} = \frac{2 \pm \sqrt{16}}{2}$ $\begin{cases} x_1 = \frac{2+4j}{2} = 1+2j \\ x_2 = \frac{2-4j}{2} = 1-2j \end{cases}$

ΟΠΟΤΕ ΟΙ ΡΙΖΕΣ ΕΙΝΑΙ $x_1 = 1+2j$, $x_2 = 1-2j$, $x_3 = -1$

Άσκηση 6

α) $z^7 = 1$

$$\begin{cases} |z| = |1|^{\frac{1}{7}} = 1 \\ \varphi = \frac{0 + 2k\pi}{7} = \frac{2k\pi}{7} \end{cases}$$

$$\boxed{z = 1 \cdot e^{j\frac{2k\pi}{7}}}$$

β) $z^3 = 2 + 2j$

$$\begin{cases} |z| = \sqrt{2^2 + 2^2}^{\frac{1}{3}} = \sqrt{8}^{\frac{1}{3}} = 8^{\frac{1}{2}\cdot\frac{1}{3}} = 2^{\not{3}\cdot\frac{1}{\not{3}}\cdot\frac{1}{2}} = \sqrt{2} \\ \varphi = \frac{\pi/4 + 2k\pi}{3} = \frac{\pi}{12} + \frac{2k\pi}{3} \end{cases}$$

$$\boxed{z = \sqrt{2}\, e^{j\left(\frac{\pi}{12} + \frac{2k\pi}{3}\right)}}$$

γ) $z^5 = -32$

$$\begin{cases} |z| = \sqrt{(-32)^2}^{\frac{1}{5}} \\ \quad = \sqrt[5]{2^5} = 2 \\ \varphi = \frac{2k\pi}{5} + \frac{\pi}{5} \end{cases}$$

$$\boxed{z = 2\, e^{j\left(\frac{2k\pi}{5} + \frac{\pi}{5}\right)}}$$

Άσκηση 7

α) $(1+j)^{10} = \left(\sqrt{2}\, e^{j\pi/4}\right)^{10} = \sqrt{2}^{10}\, e^{j\pi/4 \cdot 10} = 2^5 \cdot e^{j\frac{5\pi}{2}} = 32j$

β) $\left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)^{333} = \sqrt{\frac{1}{4}+\frac{3}{4}}^{333} \cdot e^{j\pi/3 \cdot 333}$

$= 1 \cdot e^{j111\pi}$

$= 1 \cdot e^{j\pi}$

$= -1$

γ) $\left(\sqrt{2}/2 + \sqrt{2}/2\, j\right)^{100} - j^{100} =$

$\left(\sqrt{\frac{2}{4}+\frac{2}{4}}^{100=1} \cdot e^{j100\pi/4}\right) - 1 =$

$e^{j25\pi} - 1 = -1 - 1 = -2$

δ) $\frac{(1-j)^2}{(j-1)^4} = \frac{(j-1)^2}{(j-1)^{4+2}} = \frac{1}{(j-1)^2} = (-1+j)^{-2} = \sqrt{1+1}^{-2} \cdot e^{j\frac{3\pi}{4}\cdot(-2)}$

$= \sqrt{2}^{-2} \cdot e^{j\frac{-3\pi}{2}}$

$= 2^{\frac{1}{2}(-2)} \cdot e^{j\pi/2}$

$= 2^{-1} e^{j\pi/2}$

$= \frac{1}{2} j$