ΕΦΑΡΜΟΣΜΕΝΑ ΜΑΘΗΜΑΤΙΚΑ
ΣΕΙΡΑ ΑΣΚΗΣΕΩΝ 4
ΧΡΗΣΤΟΣ ΠΑΠΑΣΤΑΜΟΣ csd4669

Εξ. λύση 1

Άσκηση 1

$$x_1(t) = \text{rect}(t) * \cos(\pi t) \xrightarrow{F} X_1(f) = \text{sinc}(f)\cdot\left(\tfrac{1}{2}\delta(f-\tfrac{1}{2}) + \tfrac{1}{2}\delta(f+\tfrac{1}{2})\right)$$

$$\begin{aligned} X_1(f) &= \tfrac{1}{2}\,\text{sinc}(f)\,\delta(f-\tfrac{1}{2}) + \tfrac{1}{2}\,\text{sinc}(f)\,\delta(f+\tfrac{1}{2}) \\ &= \tfrac{1}{2}\,\text{sinc}(\tfrac{1}{2})\,\delta(f-\tfrac{1}{2}) + \tfrac{1}{2}\,\text{sinc}(-\tfrac{1}{2})\,\delta(f+\tfrac{1}{2}) \\ &= \tfrac{1}{2}\cdot\tfrac{2}{\pi}\cdot\delta(f-\tfrac{1}{2}) + \tfrac{1}{2}\cdot\tfrac{2}{\pi}\cdot\delta(f+\tfrac{1}{2}) \\ &= \tfrac{2}{\pi}\left(\tfrac{1}{2}\cdot\delta(f-\tfrac{1}{2}) + \tfrac{1}{2}\cdot\delta(f+\tfrac{1}{2})\right) \xrightarrow{F^{-1}} \end{aligned}$$

$$x_1(t) = \tfrac{2}{\pi}\cos(2\pi\cdot\tfrac{1}{2}t) = \tfrac{2}{\pi}\cos(\pi t)$$

$$x_2(t) = \text{rect}(t) * \cos(2\pi t) \xrightarrow{F} X_2(f) = \text{sinc}(f)\cdot\left(\tfrac{1}{2}\delta(f-1) + \tfrac{1}{2}\delta(f+1)\right)$$

$$\begin{aligned} X_2(f) &= \tfrac{1}{2}\,\text{sinc}(f)\,\delta(f-1) + \tfrac{1}{2}\,\text{sinc}(f)\,\delta(f+1) \\ &= \tfrac{1}{2}\cdot\text{sinc}(1)\,\delta(f-1) + \tfrac{1}{2}\,\text{sinc}(-1)\,\delta(f+1) \\ &= \tfrac{1}{2}\cdot 0\cdot\delta(f-1) + \tfrac{1}{2}\cdot 0\cdot\delta(f+1) \\ &= 0 \xrightarrow{F^{-1}} x_2(t) = 0 \end{aligned}$$

$$x_3(t) = \text{sinc}(t) * \text{sinc}(t/2) \xrightarrow[\text{δυικότητας}]{\text{ιδιότητα}} \text{rect}(-f)\cdot 2\,\text{rect}(-2f) = X_3(f)$$

$$X_3(f) = 2\,\text{rect}(2f) \xrightarrow{F^{-1}} 2\cdot\tfrac{1}{2}\,\text{sinc}(t/2) = \text{sinc}(t/2) = x_3(t)$$

Άσκηση 2

$$Y(f) = 3\,\text{sinc}^2(2f) = \left(\pm\sqrt{3}\right)^2 \text{sinc}^2(2f)$$

$$Y(f) = \pm\sqrt{3}\,\text{sinc}(2f) \cdot \pm\sqrt{3}\,\text{sinc}(2f)$$

$$Y(f) = \pm\sqrt{3}/2 \cdot 2 \cdot \text{sinc}(2f) \cdot \pm\sqrt{3}/2 \cdot 2 \cdot \text{sinc}(2f)$$

$$Y(f) = \left(\pm\sqrt{3}/2 \cdot 2 \cdot \text{sinc}(2f)\, e^{-j2\pi f 2}\right) \cdot \left(\pm\sqrt{3}/2 \cdot 2 \cdot \text{sinc}(2f)\, e^{j2\pi f 2}\right)$$

$$Y(f) \xrightarrow{F^{-1}} y(t) = \sqrt{3}/2\ \text{rect}(t-3/2) * \sqrt{3}/2\ \text{rect}(t+3/2)$$

Όπου αν $X(t) = \sqrt{3}/2\ \text{rect}(t/2)$ προκύπτει η σχέση της εκφώνησης

$$y(t) = x(t-2) * x(t+2)$$

Άσκηση 5

α) $x(2t) \xrightarrow{F} \frac{1}{2}X\left(\frac{f}{2}\right) = \frac{1}{2}\,\frac{j2\pi\frac{f}{2}}{-4\pi^2\frac{f^2}{4} + 5j2\pi\frac{f}{2} + 6} = \frac{j2\pi f/2}{-2\pi^2 f^2 + 5j2\pi f + 6}$

β) $x(3t-6) \xrightarrow{F} \frac{1}{3}X(f/3)e^{-j2\pi f 6} =$

γ) $\frac{d}{dt}x(t) \xrightarrow{F} j2\pi f X(f)$

δ) $x(-t) \xrightarrow{F} X(-f)$

ε) $e^{-j100t}x(t) = e^{-j2\pi\frac{50}{\pi}t}x(t) \xrightarrow{F} X\left(f+\frac{50}{\pi}\right)$

ϛ) $\int_{-\infty}^{t} x(\tau)d\tau \xrightarrow{F} \frac{X(f)}{j2\pi f} + \frac{X(0)\,\delta(f)}{2}$

Άσκηση 6

Συναρτ. από το $x(t) = 5\,\text{sinc}(20(t-\frac{1}{40})) + 5\,\text{sinc}(20(t+\frac{1}{40}))$

$x(t) = \frac{1}{4}(20\,\text{sinc}(20(t-\frac{1}{40})) + 20\,\text{sinc}(20(t+\frac{1}{40}))$

Εκτελούμε Μετασχ. F.: $X(f) = \frac{1}{4}\left(e^{j2\pi f \frac{1}{40}}\,\text{rect}(f/20) + e^{-j2\pi f \frac{1}{40}}\,\text{rect}(f/20)\right)$

$= \left(e^{j2\pi f \frac{1}{40}} + e^{-j2\pi f \frac{1}{40}}\right) \cdot \frac{1}{4} \cdot \text{rect}(f/20)$

$= \frac{1}{2}\cos\left(\frac{\pi f}{20}\right)\,\text{rect}(f/20)$

Άρα $X(f)$ $\begin{cases} \frac{1}{2}\cos\left(\frac{\pi f}{20}\right), & -10 < f < 10 \Rightarrow |f| < 10 \\ 0, & \text{αλλού} \end{cases}$