ΗΥ-215 ΕΦΑΡΜΟΣΜΕΝΑ ΜΑΘΗΜΑΤΙΚΑ
ΣΕΙΡΑ ΑΣΚΗΣΕΩΝ 1
ΧΡΗΣΤΟΣ ΠΑΠΑΣΤΑΜΟΣ cs04569

Άσκηση 1

α) $z+4-3j/z+2=-4 \Rightarrow z+4-3j=-4(z+2) \Rightarrow z+4-3j=-4z-8$

Αντικαθιστούμε $z=x+jy$:

$x+jy+4-3j=-4(x+jy)-8 \Rightarrow x+jy=-4x-4jy-8-4+3j \Rightarrow$

$x+jy+4x+4jy=-12+3j \Rightarrow 5x+5jy=-12+3j$

Χωρίζουμε πραγματικό από μιγαδικό μέρος, και έχουμε:

$5x=-12$ $\quad$ $5jy=3j$

$\boxed{x=-12/5}$ $\quad$ $5y=3$

$\boxed{y=3/5}$

Τελικά: $\boxed{z=-12/5+3/5\,j}$

β) $w-2z+1=0 \Rightarrow w=2z-1$ ①

$2w+3z=j \xrightarrow{①} 2(2z-1)+3z=j \Rightarrow 4z-2+3z=j \Rightarrow 7z=j+2$

$\Rightarrow \boxed{z=2/7+j/7}$ ②

① $w=2z-1 \xrightarrow{②} w=2(2/7+j/7)-1 \Rightarrow w=4/7+2j/7-1 \Rightarrow$

$\boxed{w=3/7+2/7\,j}$

Άσκηση 2

Έχουμε $z^2+4=0$, με $a=1, b=0, \gamma=4$

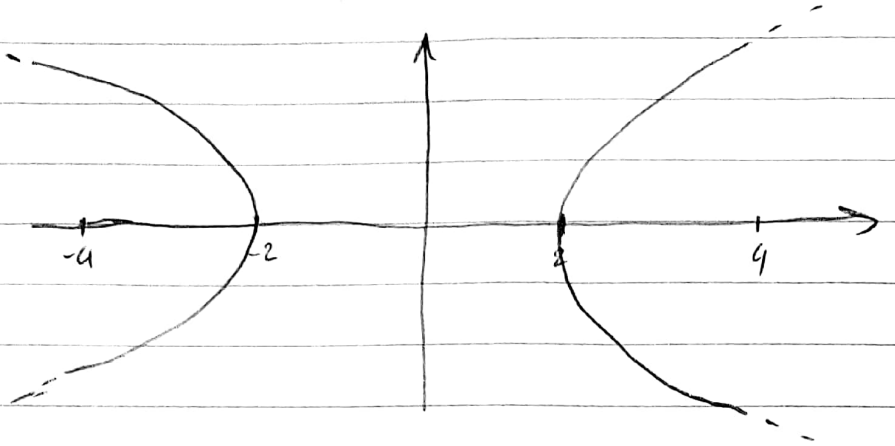
Οπότε $z_{1,2}=\dfrac{0\pm\sqrt{0^2-4\cdot1\cdot4}}{2}=\pm\sqrt{-16}/2=\pm j\sqrt{16}/2$

$z_1=+j\sqrt{16}/2=4j/2=2j$ $\quad$ $z_2=-j\sqrt{16}/2=-4j/2=-2j$

Άσκηση 3

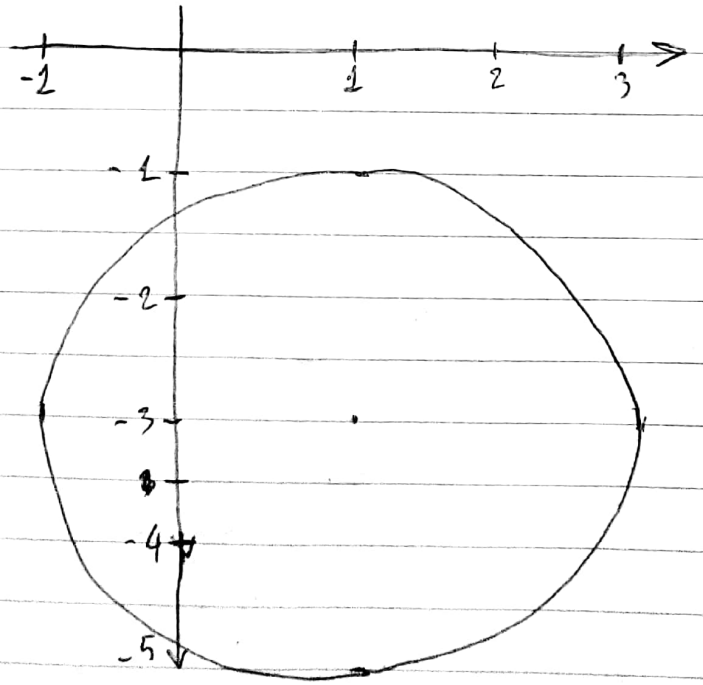
α) $R\{z^2\} = R\{(x+jy)^2\} = R\{x^2+2xjy+j^2y^2\} = R\{x^2+2xjy-y^2\}$
$= x^2-y^2$

Άρα $x^2-y^2=4$



β) $|z-1+3j|=2 \Rightarrow |x+jy-1+3j|=2 \Rightarrow |x-1+(y+3)j|=2$

$\sqrt{(x-1)^2+(y+3)^2}=2 \Leftrightarrow \sqrt{(x-1)^2+(y+3)^2}^2 = 2^2 = (x-1)^2+(y+3)^2 = 4$



Εκδοση 3

α) $\arg(z-1-2j)=\pi/4$

$\arg(x+jy-1-2j)=\pi/4$

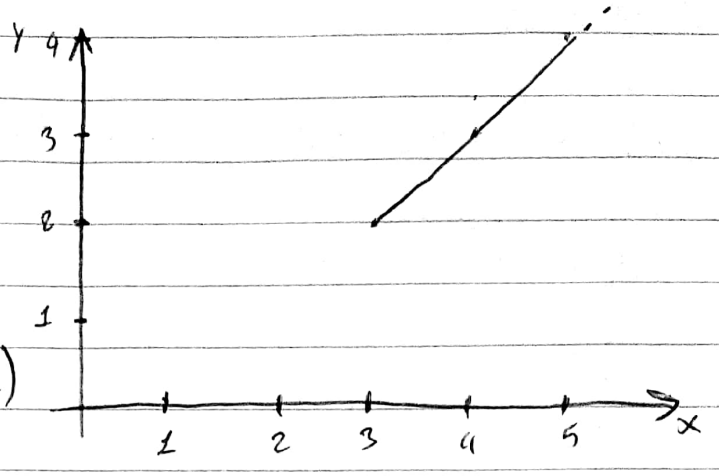
$\tan^{-1}\left(\frac{y-2}{x-1}\right)=\pi/4$

$\frac{y-2}{x-1}=\tan(\pi/4)$

$\frac{y-2}{x-1}=1$

$y-2=x-1$

$y=x+1 \quad (x>1, y>2)$



β) $|z|=|z+1|$

$|x+jy|=|x+jy+1|$

$\sqrt{x^2+y^2}=\sqrt{(x+1)^2+y^2}$

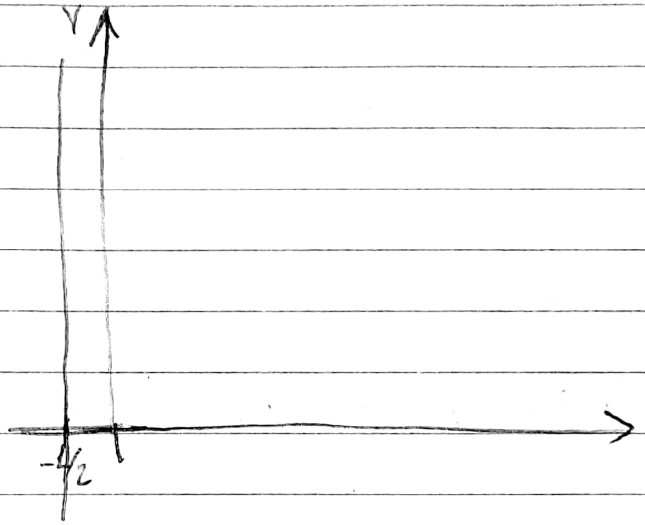
$x^2+y^2=(x+1)^2+y^2$

$x^2=(x+1)^2$

$x^2=x^2+2x+1$

$2x=-1$

$\boxed{x=-1/2}$



Άσκηση 4

$w=x+jy \quad R(z)=0$

$z=a+jb \Rightarrow jb$

$w=\frac{2-z}{z+j}$

$x+jy=\frac{2-jb}{jb+j}$

$x+jy=\frac{2j+b}{-b-1}$

$-bx-x-jby-jy=2j+b$

$-x(b+1)-j(by+y)=2j+b$

$\begin{cases} -x(b+1)=b \Rightarrow -xb-x=b \Rightarrow b+xb=-x \\ -j(by+y)=2j \end{cases}$

$b(1+x)=-x$

$\textcircled{1} \; b=\left(-\frac{x}{1+x}\right)$

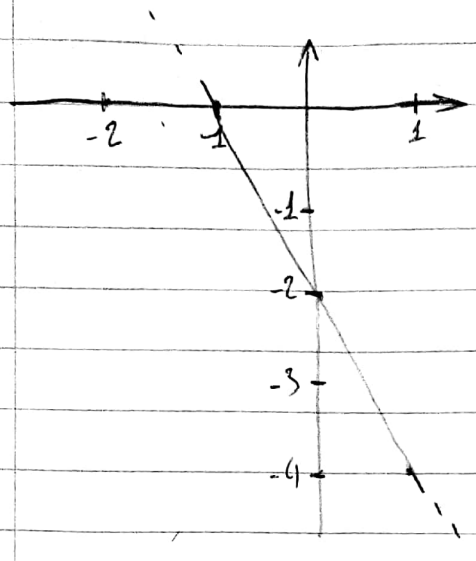
$\Rightarrow yb+y=-2$

$\textcircled{1} \Rightarrow y\left(-\frac{x}{1+x}+1\right)=-2$

$y\left(\frac{-x+1+x}{1+x}\right)=-2$

$y\left(\frac{1}{1+x}\right)=-2$

$\boxed{y=-2-2x}$



Άσκηση 5

α) Αν $x=1$ ρίζα, έχουμε:

$$1^3-5\cdot 1^2+9\cdot 1+k=0 \Rightarrow 1-5+9+k=0 \Rightarrow k+5=0 \Rightarrow \boxed{k=-5}$$

β) Σχήμα Horner

	1	-5	9	-5
1	↓	1	-4	5
	1	-4	5	0

$$(x-1)(x^2-4x+5)=0 \begin{cases} x-1=0 \Rightarrow \boxed{x=1} \\ x^2-4x+5=0 \end{cases}$$

$$\Delta=(-4)^2-4\cdot 5=16-20=-4$$

$$x_{1,2}=\frac{-(-4)\pm\sqrt{-4}}{2\cdot 1}=\frac{4\pm j\sqrt{4}}{2}=\frac{4\pm 2j}{2}=2\pm j$$

$$\boxed{x_1=2+j} \qquad \boxed{x_2=2-j}$$

Άσκηση 6

α) $z^4-1=0 \Rightarrow z^4=1$

$$\begin{cases} |z|=|1|^{1/4}=1 \\ \varphi=\frac{2\pi k}{4}=\frac{\pi k}{2} \end{cases} \qquad z=1\cdot e^{j(\pi k/2)}, \; k=0,1,2,3$$

β) $z^3-(1+j)=0 \Rightarrow z^3=1+j$

$$\begin{cases} |z|=\sqrt{1+1}^{1/3}=2^{\frac{1}{2}\cdot\frac{1}{3}}=2^{1/6} \\ \varphi=\frac{\pi/4+2k\pi}{3}=\pi/12+\frac{2k\pi}{3} \end{cases} \qquad z=2^{1/6}\, e^{j\left(\pi/12+\frac{2k\pi}{3}\right)}$$

$k=0,1,2$

Σελίδα 5

γ) $z^{10} = 100$ $\begin{cases} |z| = 100^{\frac{1}{10}} = 100^{1/2 \cdot 1/5} = 10^{1/5} \\ \varphi = \frac{2k\pi}{10} = \frac{k\pi}{5} \end{cases}$

$z = 10^{1/5} e^{j(k\pi/5)}$, $k = 0, 1, 2, 3, 4 \dots 8, 9$

Άσκηση 7

α) $(1+j)^{20} = (\sqrt{2} e^{j\pi/4})^{20} = \sqrt{2}^{20} e^{j\pi/4 \cdot 20} = 2^{10} e^{j5\pi} = -1024$

β) $\left(\frac{\sqrt{3}}{2} + j\frac{1}{2}\right)^{888} = \left(\sqrt{\frac{3}{4} + \frac{1}{4}}\, e^{j\pi/6}\right)^{888} = \left(1\, e^{j\pi/6 \cdot 888}\right) = e^{j148\pi} = 1$

γ) $(-j)^{888} = \left(1\, e^{-j\pi/2}\right)^{888} = e^{-j444\pi} = 1$