Data Analysis - 2022

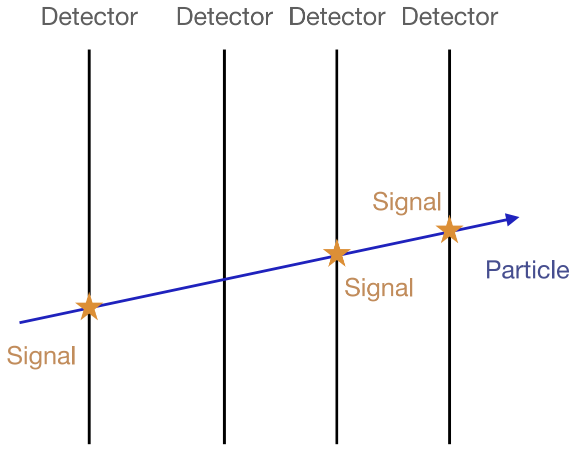
Exercise sheet no 3: 11th October 2022

Useful commands:

- `help(scipy.stats)`

Exercise 1: Particle detection efficiency (6 Points)

You are designing an experiment to detect particles as shown in the figure. When traversing the planes, a particle has a probability of 85% in each detector to induce a signal. Your goal is to decide how many detectors to build for the whole experiment.



(a) Write and execute a function to plot the probability distribution for the number of signals registered if the particle travels through four detectors. (2 points)

(b) Assume that in order to detect a particle (and to avoid detecting "ghost particles"), you must have signals in at least three detectors. How many detectors need to be built to ensure that the particle detection efficiency is above 99%? (2 points)

(c) You run your experiment with four detectors and produce 1000 particles which are sent through the four detector planes. Plot the probability distribution of the number of detected particles (number of particles with more than 3 signals). Does the width of the distribution agree with what one would expect from the Poisson distribution? (2 points)

Exercise 2: Uniform distribution (4 Points)

This exercise should be solved with pen & paper or LaTeX or similar and handed in as a PDF.
The *uniform probability distribution* $p(x|a, b)$ is constant in the interval $a < x < b$ and zero elsewhere.

(a) What is the value of the PDF in the interval $a < x < b$? Why? (1 point)

(b) Calculate the mean of the distribution. (1 point)

(c) Calculate the variance of the distribution. (1 point)

(d) Calculate the cumulative distribution function for $x < a$, $a < x < b$ and $x > b$. (1 point)

Exercise 3: PDFs and CDFs (3 Points)

To build a fence around your house you order a few hundred wood boards from a local carpenter.
Being a fellow physicist, the carpenter reports that the heights of the wood boards are well described by a gaussian distribution with $\mu = 1$m and $\sigma = 1$cm. Using the properties of the Gaussian distribution, answer the following:

(a) What is the probability to observe a wood board with a height within $[0.97, 1.03]$ m? (0.5 points)

(b) What is the probability to observe a wood board with a height within $[0.99, 1.00]$ m? (0.5 points)

(c) What is the probability to observe a wood board with a height within $[0.95, 1.05]$ m? (0.5 points)

(d) What is the probability to observe a wood board with a height less than 1.015 m? (0.5 points)

Exercise 4: Approximation of the binomial distribution (3 Points)

The Z-boson decays with a probability of 82% into charged particles and with about 18% probability into neutrinos, which cannot be detected in regular particle detectors. In some experiment, 500 Z-bosons were produced during a running time of 125 hours. *Hint: this is a new experiment which detects charged particles with a 100% efficiency.*

(a) Write a Python script that uses a binomial distribution to calculate the probability for 390 or more Z-bosons to be detected using charged particles. (2 points)

(b) Knowing the expected value and its standard deviation, use a gaussian approximation of the binomial distribution. Write a Python script that determines the same as in a). Plot both the original distribution and the approximation together. How good is this approximation? (2 points)

(c) Now make a Poisson approximation. Write a Python script to determine the same as in a). Plot both the original distribution and its approximation together. How good is this approximation? (2 points)

(d) Write a Python script to determine the probability that at least one Z-boson was created, but could not be observed because it decayed to neutrinos, during the first hour of running this experiment. You may assume that the rate of Z-bosons being produced is constant.
Use both the binomial distribution and its Poisson approximation to determine this and plot both distributions together. How good is the approximation this time? Why is it different to c)? (2 points)

Deadline for submission: Friday, 21st October 2022 14:00
Form: Please submit your solutions to da@physik.uzh.ch. The solution should be a single python script (i.e. ending in .py, if that fails, .txt is also fine), (if possible) based on the skeleton_sheet3.py and the answers (and justification) to each question as print outs. Plots should be saved when running the script. For exercise "Uniform distribution", hand the answers in as a PDF (pen & paper or LaTeX or similar)). Make sure to adhere to the "exercise rules", the recommended Python snippets and having setup Anaconda and PyCharm according to the instructions (or informed us about it)