

# Data Analysis - 2022

Exercise sheet no 4:

25. October 2022

## Systematic uncertainties and error propagation

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### Exercise 1: Systematic uncertainties (7 Points)

Look at the lab experiment Schussgeschwindigkeit ("air gun muzzle velocity"). ([http://www.physik.uzh.ch/%7Ematthias/espace-assistant/manuals/en/anleitung\\_sg\\_e.pdf](http://www.physik.uzh.ch/%7Ematthias/espace-assistant/manuals/en/anleitung_sg_e.pdf)) of the first year. Restrict your attention to setup A and be **specific** about your answers: go more into the detail of an uncertainty and be explicit instead of providing general answers.

- (a) Think of possible **systematic uncertainties and list at least three of them**. (1 point)
- (b) Do you expect the individual uncertainties to have a small or large impact on the final result? Note for each listed uncertainty your answer.  
*Hint: It's asked about the impact on the result. Argue using quantitative measures.*  
(2 points)
- (c) How would you estimate the uncertainties? Just give an idea on how you *would* do it.  
(2 points)
- (d) How would you minimize or avoid your listed systematic uncertainties? (2 points)

### Exercise 2: Common systematic uncertainty (3 Points)

You make three measurements of the electron charge. Each measurement has its own statistical uncertainty. In addition, there is a systematic uncertainty which is common to all three measurements. You can assume that the systematic uncertainty is independent of the central value of the results. cov between two measurements

$$\text{Measurement 1 : } m_e = (1.39 \pm 0.20(\text{stat}) \pm 0.10(\text{syst})) \times 10^{-19} \text{ C}$$

$$\text{Measurement 2 : } m_e = (1.54 \pm 0.04(\text{stat}) \pm 0.10(\text{syst})) \times 10^{-19} \text{ C}$$

$$\text{Measurement 3 : } m_e = (1.47 \pm 0.10(\text{stat}) \pm 0.10(\text{syst})) \times 10^{-19} \text{ C}$$

$$\text{cov}(\text{sys}, \text{sys}) = \text{var}(x)$$

Calculate the three correlation coefficients between the three pairs of measurements.

### Exercise 3: Error propagation (4 Points)

You have been asked to determine the length,  $L$ , of the side of a cube. For simplicity, you assume that all sides have the same length. To do this you measured the volume,  $V$ , such that  $V = 4 \text{ cm}^3$ .

- How big can the absolute uncertainty on your measurement of the volume,  $V$  be such that you obtain a measurement of the length,  $L$ , with a relative precision of 3%?
- How does the answer change depending on whether the uncertainty on the length is statistical or systematic in nature and why?

### Exercise 4: Error propagation with correlated variables (6 Points)

The dataset in `sand.txt` on the course webpage reports the slope of a beach (2<sup>nd</sup> column) as a function of the diameter of its sand granules (in mm) (1<sup>st</sup> column). The third column reports the uncertainty on the slope.

Assuming a simple linear relation between the two variables, the data was fitted with a straight line  $y = m \cdot x + q$ .

The best values for the parameters are:

y is a function of x

- $m = 16.1 \pm 1.0 \text{ mm}^{-1}, \text{std}$
- $q = -2.61 \pm 0.34, \text{std}$

while the covariance matrix for the two is:

$$\begin{pmatrix} \text{var} & \\ \text{cov}(m, q) & \text{var}(q) \end{pmatrix} = \begin{pmatrix} 1.068 & -0.302 \\ -0.302 & 0.118 \end{pmatrix}$$

- Plot the data with errorbars and the fitted line. (2 points)
- Disregarding the correlation between the variables  $m$  and  $q$ , calculate the slope of a beach whose sand grains have the diameter of 1.5 mm. Report the number with the corresponding uncertainty. (2 points)
- Repeat the task of point (b), this time taking into account the correlation between  $m$  and  $q$ . In which case is the uncertainty on the extrapolated value smaller? Argue your answer. (2 points)

**Deadline for submission: Friday, 4. November 2022 14:00**

**Form:** Please submit your solutions to [da@physik.uzh.ch](mailto:da@physik.uzh.ch). Solve questions 1-3 by hand (for calculations, only stating the result is not sufficient!) and attach a PDF version of your answer to the email. The answer to question 4 should be a single python script (i.e. ending in `.py`, if that fails, `.txt` is also fine), and the answers (and justification) to each question as print outs. Plots should be saved when running the script. Make sure to adhere to the "exercise rules", the recommended Python snippets and having setup Anaconda and PyCharm according to the instructions (or informed us about not having done it).