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Relative acoustic localization with USBL (Ultra-Short Baseline)

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WORKING VERSION



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September 10, 2020

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Resumo

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Dispositivos robóticos programáveis como *Autonomous Underwater Vehicles* (AUVs) são excelentes meios para exploração subaquática, já que são capazes de executar missões de longa duração com variadas possibilidades de aplicação e objetivos. Neste sentido, o conceito de mula AUV surgiu como mecanismo útil que periodicamente recolhe dados dos AUVs em missão. Para que tal seja possível, é necessário implementar um sistema de localização e posicionamento robusto que permite aos AUVs encontrarem outros veículos de forma a aproximarem-se deles eficientemente.

A presente dissertação foca-se na implementação de um sistema que estima a posição relativa entre AUVs através do método *Ultra-Short Baseline* (USBL). Esta técnica baseia-se na determinação da diferença de fases entre sinais recebidos por um vetor de hidrofones.

Após a implementação e validação do sistema referido, este foi integrado num mecanismo existente que adquire e processa dados de quatro hidrofones. Na fase final, serão executados testes de campo e experiências num ambiente enclausurado, como o tanque do DEEC, seguido de um teste em ambiente real, em mar aberto.

Abstract

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Robotic programmable devices such as Autonomous Underwater Vehicles (AUVs) are great means for underwater exploration, as they are capable of executing long term missions with many possible applications and goals. In this regard, the concept of mule AUVs arises as a valuable mechanism to periodically collect data from survey AUVs during the missions. In order to achieve this, a robust localization and positioning system needs to be implemented allowing the mule AUV to find the other vehicle and draw near it efficiently.

The present dissertation focuses on the implementation of a system which estimates the relative position between AUVs through the Ultra-Short Baseline (USBL) method. The technique relies on accurate estimation of the phase difference between signals received in a hydrophone array.

After implementation and validation of the mentioned system, it will be integrated in an existing mechanism which was specifically designed to acquire and process data from four hydrophones. In the final stage, field tests and experiments will be executed in a closed environment such as DEEC's tank, followed by a real environment test in open sea.

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Abbreviations

AUV	Autonomous Underwater Vehicle
BPSK	Binary Phase Shift Keying
CC	Cross-Correlation
DEEC	Departamento de Engenharia Electrotécnica e de Computadores
FIM	Fisher Information Matrix
FPGA	Field-Programmable Gate Array
FSK	Frequency-Shift Keying
GCC	Generalized Cross-Correlation
HDL	Hardware Description Language
LBL	Long Baseline
LOS	Line-of-sight
MF	Medium Frequency
ML	Maximum Likelihood
RMS	Root Mean Square
RSSI	Received Signal Strength Indicator
SBL	Short Baseline
SNR	Signal-Noise Ratio
TDE	Time Delay Estimation
TDoA	Time Difference of Arrival
ToA	Time of Arrival
ToF	Time of Flight
USBL	Ultra-Short Baseline

Chapter 1

Introduction



1.1 Context and Motivation

Today, the deep blue ocean still represents a relevant topic of research in the scientific community as it constantly rises new unexplained mysteries. Up to now, only 15% of the entire ocean floor is mapped based on collected data [1]. As such, it seems essential to create efficient research tools to improve the discovery of information.

Robotic autonomous underwater vehicles (AUVs) are great means for diverse applications in underwater exploration using variable resource requirements and duration, such as monitoring structures installed in shallow waters or exploring the deep ocean floor for scientific purposes. Particularly in long-term missions, the AUV is usually deployed using a docking system and it navigates underwater until the end of the mission, when it returns to the base station. Thus far, the data that is being collected is typically not accessible by any processing system or researchers.

A method that is used to resolve this limitation is employing additional mule AUVs whose goal is to travel near the survey AUV, collect its data during the mission's term and return in a relatively short time period. This allows the data to be periodically processed during the mission, which facilitates the definition of future courses for the mission, such as shortening its duration or sending additional commands. In the mentioned localization system, high accuracy key as it lowers the resource consumption , saves up time in the inherently slow global process and avoids missing the AUV's underwater localization.

The described process can only be achieved if the mule AUV is able to locate the other vehicle and draw near it. For that reason, a USBL (Ultra-Short Baseline) system is used to receive the transmitted signals and calculate the angle of arrival of the acoustic signal, thus the direction that the mule AUV should navigate. Additionally, using a synchronization mechanism, the mule is also able to determine the distance to the acoustic source and thus the vehicles' relative positions.

In such scenario, the USBL system needs to meet specific requirements to assure a reliable localization. Since the acoustic source can be located anywhere, it is essential that the estimation is accurate for both short and long range distances. Additionally, the system needs to have line of sight in any direction, which is compromised from the start by deploying the sensors on an

open AUV. Typically, the available USBL commercial solutions have a limited range or these characteristics, so the development of such system constitutes a technological challenge.

Therefore, this dissertation intends to explore a method that assumes the deployment of multiple hydrophones in a vehicle, from which only four are used simultaneously for the angle of arrival estimation. This allows to dynamically optimize the used sensor configuration according to which returns the lowest estimation error, so it is possible to always have line of sight to the target, independently of navigation maneuvers, and improving the estimation accuracy. Overall, this mechanism intends to rectify issues that arise from classic USBL systems, such as the before mentioned.

In the course of this document, the dynamically reconfigurable configuration method is detailed and refined, determining its limitations and capabilities. Additionally, all the contemplated tools and complementary modules are carefully explained.

This research work falls under the scope of activities developed by the Center of Robotics and Autonomous Systems of INESC TEC. It is integrated in the GROW project which focuses on exploring the use of AUVs as data mules for long duration missions.

1.2 Objectives

The goal of the present work is to study and propose a dynamically reconfigurable configuration method, which assumes the integration of several hydrophones in a USBL system to allow selecting the set of sensors that minimizes the estimation error. This aims to achieve high estimate accuracy for both short and long range distances and provide full line of sight from the chosen hydrophones to the target that can be located anywhere. In order to attain this, a comparative study is developed on tools that allow to compare the performance of sensors configurations in order to select the most reliable option. Then, the proposed system is presented in detail and validated with comprehensive simulations.

In addition to the main objective is intended to implement and validate a digital signal processing system for FPGA technology, which calculates the difference between the times of arrival of an encoded acoustic signal to four hydrophones. Thereafter, a estimator is developed which is able to take the time differences of arrival in order to estimate the intended angle of arrival.

1.3 Document Structure

The present document is partitioned into six chapters, which are summarized in this section.

Chapter 2 offers an overview on background concepts about underwater acoustics, localization estimation and positioning systems, followed by USBL available commercial solutions and developed technology for a similar purpose. Then it focuses on angle of arrival determination methods and optimization mechanisms that are typically employed.

After ~~getting in touch with the terms and~~ reviewing the literature, chapter 3 intends to clarify the problem that is being resolved in this thesis. The research hypothesis is stated as well as the research questions that are discussed and ~~indented~~ to be further explored. The chapter ends with the clarification of the used validation methods for the work.

Chapter 4 presents and explains the developed hardware design for the phase difference calculation. Then, three different approaches are presented for systematic comparison between the performance of a sensor configuration. These are supported with simulation experiments which allow to draw conclusions on the preferred approach.

Chapter 5 details the developed dynamically reconfigurable configuration method. The theoretical specifics and thought process are laid out and the mechanism is then validated through simulations.

Lastly, chapter 6 gives the final remarks about the developed work and mentions research work which could be further developed in the future.

Chapter 2

State of the Art

This chapter presents the fundamental concepts of underwater acoustics engineering for localization and positioning of aquatic autonomous vehicles.

2.1 Underwater acoustic channel

Although satellite based navigation systems are the most commonly used for positioning and localization at the air, the used radio signals are highly absorbed by the water and thus inappropriate for underwater localization and also for communications. Therefore, the state of the art solutions for long range localization and communications rely on the propagation of acoustic signals.

The natural limitations of acoustic channels combined with the properties of an underwater environment, result in challenges and limitations in developing communication and localization systems [2]:

- Long propagation delays;
- Variable speed of the acoustic signals;
- Reference nodes may have different drifting rates from each other due to water currents, which leads to uncertainties on the definition of absolute times and synchronization;
- Limited bandwidth
- Signals are bended due to sound speed variation along the water column and shadowed in many different surfaces, which may lead to the incorrect detection of the line-of-sight (LOS) signal;
- Attenuation and asymmetric signal-to-noise ratio, which arises from SNR depending on depth and frequency with complex behaviors that depend on the characteristics of the environment;



2.1.1 Speed of sound

The oceanic environment has a complex sound propagation model, as it comprises many variants in order to realistically represent underwater acoustics.

Acoustic signals' propagation speed is mainly related with two factors: compressibility and density. The water density can be characterized by the temperature, salinity and pressure, which is associated with depth. Figure 2.1 exhibits a generic sound speed profile in relation to depth. The water surface is commonly a mixed layer which results in an approximately constant sound speed. After this layer, it suffers a significant decrease, usually reaching the lower tangible speed, which results from the variation of temperature that characterizes the thermocline layer. From that point forward, pressure is the greatest influencer on speed of sound, so it increases relatively proportionally to depth .

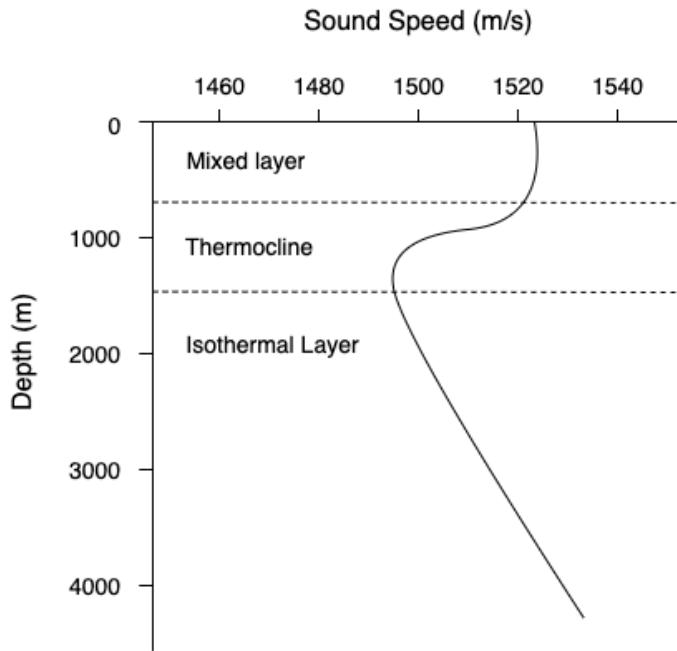


Figure 2.1: Generic sound speed profile

The empirical equation 2.1 [3] is a simplified translation of the behavior of the sound speed c in meters per second, with relation to the temperature T in $^{\circ}\text{C}$, the salinity S in parts per thousand and the depth z in meters.

$$c = 1449.2 + 4.6T - 0.055T^2 + 0.00029T^3 + (1.34 - 0.01T)(S - 35) + 0.016z \quad (2.1)$$

2.1.2 Multipath

Multipath occurs when signals suffer distortion which originate multiple propagation paths, leading to a change in their original characteristics. This phenomenon is originated by diverse factors that cause distortion in the underwater channels, typically affecting the water composition, such as water temperature and depth. The signal distortions caused by multipath include signal

fading, which is usually modeled by Rayleigh fading channel theory, varying operation frequency, time-variant propagation delays, among others.

The multipath behavior can be distinguished depending on depth, in shallow water paths and deep water paths since they demonstrate distinct propagation effects:



Shallow water paths In shallow water, the acoustic signals can be reflected or refracted on the surface, where attenuation is generally little, and on the bottom of the ocean, where it suffers a higher attenuation depending on the soil material, frequency and incidence angle. Figure 2.2 represent a typical multipath caused by reflection on the sea surface and bottom.

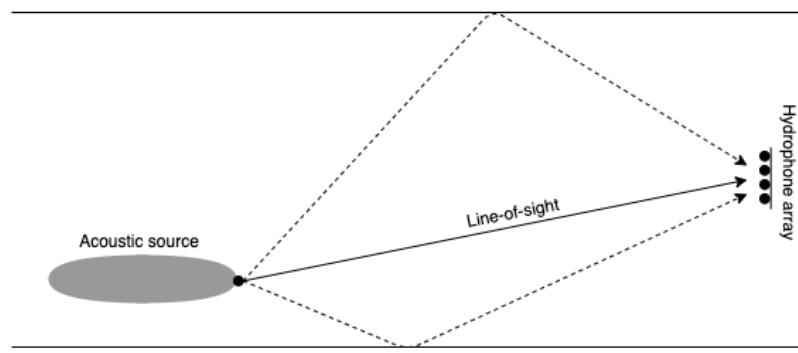


Figure 2.2: Illustrative example of shallow water multipath

Deep water paths In deep water, there are essentially six types of propagation paths, detailed bellow, whose channel model equations can be consulted in [4]. These rely on the notion that signals are bent towards the characteristics that lead to a slower propagation speed, such as a temperature and depth decrease.

- *Surface reflection*: the signal is reflected on the sea surface, where the attenuation depends on its acoustic roughness.
- *Surface duct*: the signal gets confined in a surface layer, where the sound velocity increases with depth, due to varied thermal conditions in deeper layers, which causes it to reflect on the surface and bounce back when reaching the subsequent layer.
- *Bottom bounce*: the signal is reflected on the bottom of the ocean, suffering attenuation dependent on the reached material.
- *Convergence zone*: it depends on the sound speed profile and water depth that characterizes the channel. When the temperature decreases the signal has a tendency to bend downwards, however when an increase in pressure is reached the signal tends to bend upwards again, originating convergence locations.



- *Deep sound channel:* it is originated in levels which are surrounded by layers with characteristics highly dependent on depth. So, the signal is constantly bent according to the depth that leads to a minimum speed.
- *Reliable acoustic path:* this path occurs when the transmitter is positioned in very deep water and the receiver is near the surface. Although the signal is bent downwards due to decreased temperature, when reaches a certain depth it is bent upwards making the communication channel reliable.

2.1.3 Doppler Effect

In a communication and localization system between two entities moving with non-zero relative velocity, if a transmitter sends a signal with a certain operation frequency to the receiver, then the perceived frequency by the receiver will suffer a shift from the original signal. This frequency difference is expressed as a Doppler shift and explained by the Doppler Effect.

The magnitude of the generated frequency shift can be expressed as a ratio 2.2, where the transmitter-receiver velocity is compared to c , the speed of sound [5].

$$a = \frac{v}{c} \quad (2.2)$$

Autonomous Underwater Vehicles (AUVs) usually move with velocities in the order of few meters per second. Therefore, the a factor mentioned above has a significant value and needs to be considered when implementing synchronization systems, as well as developing estimation algorithms.

In certain localization and communication systems, it is critical to correct the Doppler effect because data can be compromised (e.g. FSK modulated signals, in which information is codified into frequency changes). A simple Doppler compensation process was proposed in [6], in a system to detect phase-modulated binary sequences using cross-correlation.

This phenomenon can also be explored to determine the relative velocity between two devices, by measuring the frequency deviation with respect to the frequency expected to be received.

2.1.4 Attenuation and signal-to-noise ratio

When considering underwater communication systems, it is essential to quantify the attenuation of the channel, i.e. the part of the signal's energy which is absorbed by the involving surrounding. In underwater channels, this absorbance is frequency variable and it is also dependent on physical characteristics of the water, as salinity and temperature.

The underwater acoustic channel has a particular model that describes its attenuation path loss $A(d, f)$, given in logarithmic scale by equation 2.3 [7].

$$10 \log(A(d, f)) = 10 k \log(d) + d \log(a(f)) \quad (2.3)$$



From the equation, d is the distance from the transmitter to the receiver in kilometers (Km), f is the operating frequency in kilohertz (KHz), $10k\log(d)$ represents the spreading loss which describes how the sound level (in decibel, dB) decreases as the sound wave spreads, $d10\log(a(f))$ is the absorption loss that a signal suffers during its propagation path, k is the spreading factor which is related with the considered configuration (e.g. cylindrical, spheric, etc.), $a(f)$ is the absorption coefficient that can be obtained through the equation in [7].

Noise is another factor that is considered when analyzing a real underwater acoustic channel, as it defines the signal-to-noise ratio (SNR) that characterizes the channel. The SNR is dependent on the attenuation level which increases with frequency, and the noise which decays with frequency. Consequently, the SNR varies over the signal bandwidth and it is asymmetric. The equation 2.4 [5] expresses this relationship, where $S_d(f)$ represents the power spectral density of the transmitted signal.

$$SNR(d, f) = \frac{S_d(f)}{(A(d, f)) N(f)} \quad (2.4)$$

2.2 Range estimation for underwater localization

Underwater localization takes into consideration the distance between the target object to track and the reference point. As consequence, it is always relevant to apply methods which easily and effectively determine this range.

There are two main types of techniques that are used to achieve such objective: the Received Signal Strength Indicator (RSSI) and the Time Delay Estimation (TDE).

2.2.1 Received Signal Strength Indicator

The Received Signal Strength Indicator (RSSI) method is based on the strength of the signal that reaches the target. It determines the distance between the target and the reference node by analyzing the received signal strength and comparing it with an underwater attenuation model which is range dependent [3].

Since the underwater acoustic channel suffers from multipath, time variance and high overall path loss, the RSSI technique is not adequate for underwater applications.

2.2.2 Time Delay Estimation

Time Delay Estimation (TDE) mechanisms use a pair of nodes, the target and the reference point, to measure the range between them. This distance is based on the time that it takes for a signal to travel from the reference point to the target.

There are three main categories that devide TDE methods, which are Time Difference of Arrival (TDoA), Time of Arrival (ToA) and Time of Flight (ToF).

2.2.2.1 Time of Arrival

Time of Arrival (ToA) is interpreted as the time delay between the transmission of a signal in the reference node until its reception on the target node. Although this is the conceptually simplest method to employ, it requires synchronization between the nodes since the target entity needs to know the instance when the signal was sent to be able to calculate the difference.

Considering a generic transmitted signal $s(t)$, the received signal can be expressed as 2.5, where τ represents the time of arrival and $n(t)$ is white noise with zero mean [8].

$$r(t) = s(t - \tau) + n(t) \quad (2.5)$$

2.2.2.2 Time of Flight

Time of Flight (ToF) measures essentially the round-trip time communication between two nodes. The target node sends a signal to the reference node, which has an integrated transponder that responds transmitting a signal back to the target. The ToF is then estimated as the time interval from the moment the first signal is transmitted until the moment the second signal is received by the same node.

This method may be used without additional synchronization systems as it assumes that the response signal is sent immediately after the received one and the intrinsic transmitting delays are known.

The accuracy of this technique depends mainly on the environment conditions, which include the water properties and the surrounding reflection surfaces which cause multipath. Therefore, the mechanism is susceptible of variable errors according to the location and characteristics of its employment.

2.2.2.3 Time Difference of Arrival

The Time Difference of Arrival (TDoA) is a technique that compares the time of arrival of a signal to different hydrophones in order to estimate the angle of arrival of the acoustic signal. The array of reception hydrophones have known determined positions among them so that it is possible to compare the different times of arrival or phase differences. This method can be employed using a uni-directional signal or a round trip communication.

There are several algorithms and mathematical models that can be employed to execute the TDoA method, such as Cross-Correlation and Maximum Likelihood.

- **Generalized Cross-Correlation**

The Generalized Cross-Correlation (GCC) method is used to generically represent the relationship strength between two signals.

Considering two distanced hydrophones in the same environment and an acoustic source $s(t)$, $x_1(t)$ and $x_2(t)$ are the signals received by each of the two hydrophones. The equations

[2.6](#) and [2.7](#) [9] express the mentioned signals in relation to $w1(t)$ and $w2(t)$ which are Gaussian noise coefficients uncorrelated with the source, τ that represent the delay and α which is an attenuation function.

$$x1(t) = s(t) + w1(t) \quad (2.6)$$

$$x2(t) = \alpha s(t - \tau) + w2(t) \quad (2.7)$$

(2.8)

From these expressions, the generalized cross-correlation function between signals $x1(t)$ and $x2(t)$ is given by [2.9](#). The $G_{x1x2}(f)$ is the spectrum of the cross-correlation. The $\psi(f)$ represents a prefilter and it is essentially the distinctive parameter that originate various different methods of cross-correlation, since it should depend on different environments and properties as SNR.

$$R_{x1x2}(\tau) = \int_{-\infty}^{\infty} \psi(f) G_{x1x2}(f) e^{i2\pi f \tau} df \quad (2.9)$$

$$T = \tau_{max}[R_{x1x2}(\tau)] \quad (2.10)$$

Finally, the maximum value of $R_{x1x2}(\tau)$, expressed in [2.10](#), is the so called correlation peak and provides information about the time delay τ which is the main matter of Time Delay Estimation.

- **Cross-Correlation**

After approaching the generalized method of cross-correlation, it is possible to better understand the Cross-Correlation (CC) method. There are two main variations of CC [9], which are the slow cross-correlation in the time domain and the fast cross-correlation in the frequency domain. The second approach is based on the Fast Fourier Transform as it locates the peak by analyzing frequency similarities between the signals.

The Cross-Correlation technique uses a prefilter $\psi(f)$ equal to 1, as it is the simplest method of its kind.

- **Maximum Likelihood**

The Maximum Likelihood (ML) method is a variation of Cross-Correlation which uses the prefilter $\psi(f)$ represented mathematically by [2.11](#), where $\gamma_{12}(f)$ is a function of spectrum of cross-correlation $G_{x1x2}(f)$ and spectrum of auto-correlations $G_{x1x1}(f)$, $G_{x2x2}(f)$ as expressed in [2.12](#) [9].

$$\psi(f) = \frac{|\gamma_{12}(f)|^2}{|G_{x1x2}(f)|[1 - |\gamma_{12}(f)|^2]} \quad (2.11)$$

$$|\gamma_{12}(f)|^2 = \frac{|G_{x1x2}(f)|^2}{G_{x1x1}(f).G_{x2x2}(f)} \quad (2.12)$$

There is also a version of ML that uses the power spectral densities of the signals, which can be helpful for calculations in various applications.

2.3 Localization estimation

In networks with multiple nodes it is typical to use localization estimation to establish position relationships between elements. The operation principal is usually to have a set of reference nodes with known positions so that it is possible to determine the relative positions between each reference node and the target.

An extensive comparison of different localization schemes for underwater sensors networks can be consulted in [10].

2.3.1 Triangulation

Triangulation is a method of localization based on the measurement of angles which are related to the reference beacons and the target object.

2.3.1.1 Three-Object Triangulation

The simplest method of this category is the Three-Object Triangulation, which considers a configuration as illustrated in figure 2.3. It is assumed that the location of the beacons is pre-configured and the environment is obstacle-free. λ_{12} is the angle formed by the intersection of the straight lines $[O,1]$ and $[O,2]$. Similarly, λ_{31} is the angle formed by the intersection of the straight lines $[O,1]$ and $[O,3]$. Using these two sets of nodes, we can trace circumferences that include their coordinates and as a consequence their intersection will correspond to the location of the target.

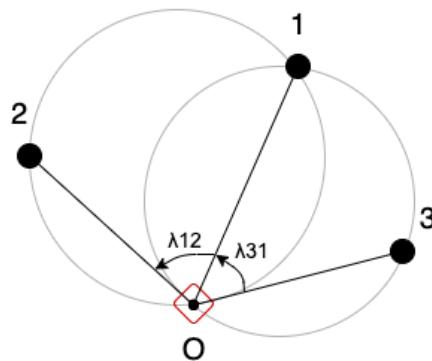


Figure 2.3: Three-Object Triangulation

Although this is a very straightforward technique to implement, it does not cover all possible scenarios, namely when the three beacons and the object are all placed in the same circumference or when the environment has obstacles between nodes.

2.3.1.2 Geometric Triangulation algorithm



A more complex method relies on the Geometric Triangulation algorithm.

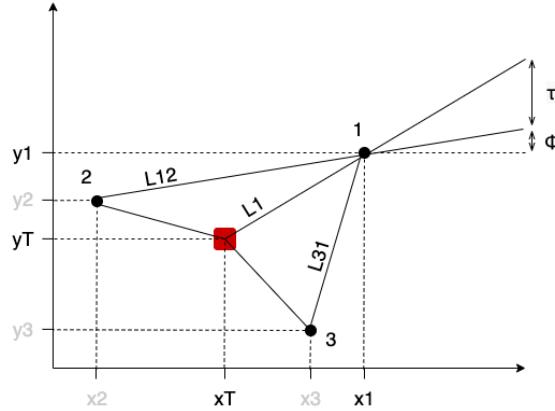


Figure 2.4: Geometric Triangulation algorithm

Considering a Cartesian plane with defined lengths L_1 , L_{12} and L_{31} , as shown in image 2.4, it is possible to establish trigonometrical relationships that estimate the location of the object within the created triangular areas. The position of the target is given by coordinates (x_T, y_T) and can be calculated through equations 2.13 and 2.14. (x_1, y_1) represents the location of beacon 1 and L_1 is the distance between this beacon and the object. The trigonometric relationships for calculating the mentioned variables can be consulted in [11].

$$x_T = x_1 - L_1 * \cos(\phi + \tau) \quad (2.13)$$

$$y_T = y_1 - L_1 * \sin(\phi + \tau) \quad (2.14)$$

2.3.2 Trilateration

Trilateration is a technique that does not rely on calculations using angles but instead it uses distances to locate an object.

Considering a scenario with three reference beacons, the distance between the target and each one of the beacons is taken as the radius of a circumference. By doing this, it is possible to obtain three circumferences that intersect each other. With only two circumferences, there are two possible locations for the object, however, when added the third circumference the exact location is obtained. The 2D coordinates are obtained by solving systems of equations with the circle equation 2.15 [12], where (x_i, y_i) is the beacon coordinates and r_i is the distance between the beacon and the object.

$$(x - x_i)^2 + (y - y_i)^2 = r_i^2 \quad (2.15)$$

Trilateration is commonly used in underwater acoustic localization, as it used to find a relative position of the target in two dimensions and additionally determines the depth as third dimension, by using a pressure sensor with high accuracy.

2.3.3 Multilateration

Multilateration is a generalization of the trilateration technique, as it uses the same conceptual principal with multiple reference beacons instead of exactly three. In this method, the employment of $n+1$ nodes will allow to determine n coordinates [13]. For example, determining the position (x,y,z) of a target, would require to resolve a system of equations using 2.16. (x_i, y_i, z_i) is the coordinates of the beacon and d_i is the distance between the beacon and the target.

$$(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2 = d_i^2 \quad (2.16)$$

Distributed mechanisms, such as multilateration, are usually divided in three phases of positioning [10]:

- Distance estimation between the reference nodes and target object, usually using TDoA or ToF mechanisms;
- Position estimation, usually obtained by solving a system of linear equations through mathematical efficient techniques;
- Final refinement of the measurement in order to improve accuracy.

As an alternative to solve localization issues using circumferences, multilateration can also take advantage of a hyperbola-based localization method. Considering a target at (x,y) and three reference beacon with coordinates (x_i, y_i) , (x_j, y_j) and (x_k, y_k) , we have that the difference between times of arrival t_i and t_j to nodes i and j , respectively, can be related to the distance between nodes, as expressed in 2.17 [13]. d_i and d_j are the distance from node i and j , respectively, to the target object.

$$d_i - d_j = c * (t_i - t_j) = \sqrt{(x - xi)^2 + (y - yi)^2} - \sqrt{(x - xk)^2 + (y - yk)^2} \quad (2.17)$$

2.4 Positioning Systems

Positioning systems are used to track the underwater position of a vehicle or other object, in relation to reference structures of transponders called *baseline stations*. These systems are classified based on the distance between the baseline stations. The configurations that will be explained are Long Baseline (LBL), Short Baseline (SBL), Ultra-Short Baseline (USBL) and the inverted versions of all above.

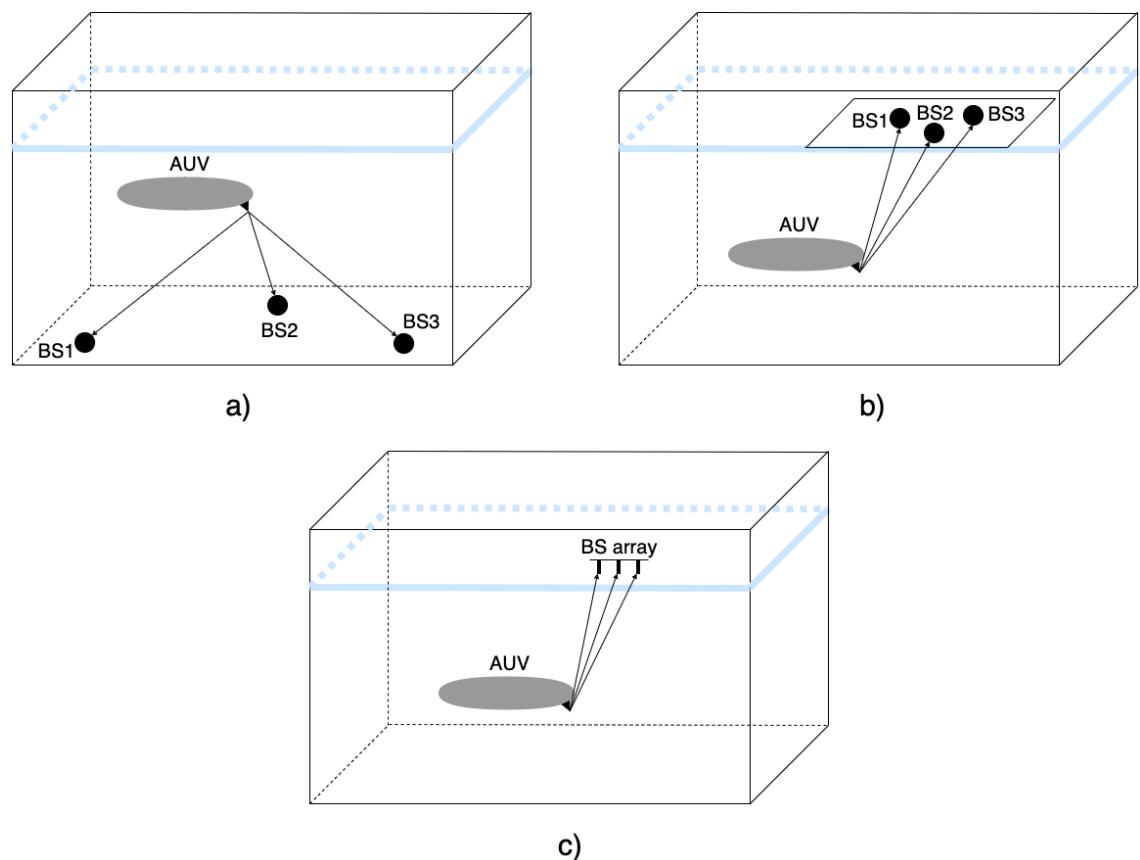


Figure 2.5: Generic configuration of: a) LBL; b) SBL; c) USBL

2.4.1 Long Baseline (LBL)

Long Baseline systems use a positioning method with large distances between baseline stations, with range typically from 50m to more than 2000m and usually similar to the distance between object and transponders [9]. A typical LBL configuration is represented in figure a) 2.5.

The LBL method uses at least three transponder stations deployed usually on the sea floor, allowing to execute trilateration. Additionally, a transducer is integrated on the object to be tracked.

A complete localization procedure starts with the vehicle sending an acoustic signal which is received by the transponders. Thereafter the transponders transmit a response and, by analyzing the Time of Flight of the communication, the system can determine the distance between the vehicle and each base station. Then the relative position of the vehicle is determined through trilateration. Additionally, if the transponders have known geographic positions, it is possible to infer the vehicle geographic position.

~~As this technique presents large distances between the object and the base stations,~~ the typical 1m to few centimeters accuracy is considered to be high because it will not compromise the localization of the vehicle.

2.4.2 Short Baseline (SBL)

Short Baseline systems are characterized by having distances around 20m to 50m between baseline stations [2] and use an operation procedure similar to the LBL method. However, the transponders are usually placed in a moving platform, which assures a fixed relative position between them. A typical SBL configuration is represented in figure b) 2.5.

The position of the vehicle to be tracked can be determined by translating the Time of Flight between the multiple transponders and the object into a distance value, which is achieved by equation 2.18 [14]. The t_i corresponds to the propagation time of the signal from the vehicle to the i th transponder, c is the speed of sound, $[x_{b_i}, y_{b_i}, z_{b_i}]$ is the coordinate position of the transponder.

$$\sqrt{(x_{b_i} - x)^2 + (y_{b_i} - y)^2 + (z_{b_i} - z)^2} = c * t_i \quad (2.18)$$

In a SBL system, when the distance between baseline stations is increased the accuracy improves and, contrarily, when the mentioned distance decreases the accuracy deteriorates, which can raise some deployment challenges.

2.4.3 Ultra-Short Baseline (USBL)

Ultra-short baseline systems are composed essentially by one baseline station, with an array consisting of several traducers distanced typically less than the wavelength [15], and a transponder

integrated on the object to be tracked. It is usually used in underwater positioning in shallow areas of the sea, as represented in figure c) 2.5.

Similarly to the previously mentioned procedures, the USBL positioning method relies on the Time of Flight of the exchanged signals. However, the traducers are too spatially close from each other to execute an accurate trilateration. Instead, it is measured the phase difference or time-delay difference of the received signal between every traducer, in order to estimate the azimuth and distance to the acoustic source.

Assuming a three dimensional scenario for the positioning system, as represented in figure 2.6, the object's coordinates are given by equations 2.20, 2.21 and 2.22 [16]. The λ corresponds to the wavelength of the transmitted signal which depends on its operation frequency, f , and it is affected by the speed of sound c , as represented equation 2.19. The d represents the distance between hydrophones, ψ_{12} and ψ_{22} are the phase difference between H2 and the other two hydrophones, H is the height of the target object, X is the distance of the target along the x-axis direction, Y is the distance of the target along the y-axis direction and l is the slant distance of the target to the hydrophone.

$$c = f * \lambda \quad (2.19)$$

$$l^2 = X^2 + Y^2 + H^2 \quad (2.20)$$

$$\psi_{12} = \frac{2\pi}{\lambda} [\sqrt{l^2} - \sqrt{(d-X)^2 + d^2 + H^2}] \quad (2.21)$$

$$\psi_{22} = \frac{2\pi}{\lambda} [\sqrt{l^2} - \sqrt{X^2 + (d-Y)^2 + H^2}] \quad (2.22)$$

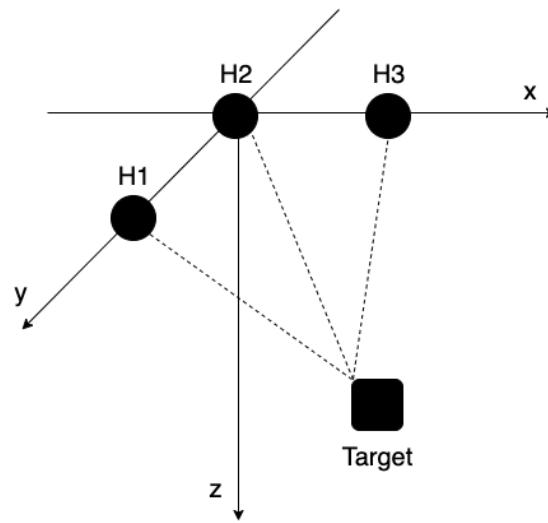


Figure 2.6: USBL system configuration

This is a broadly used technique due to its convenient set up, which allows to have predefined measurements in the order of tens of centimeters and does not require AUV navigation area for the deployment. However it presents the lowest accuracy, comparatively with LBL and SBL, since an

error of few centimeters can be realistically corresponding to an inaccuracy of several meters in the position of the object to be tracked.

2.4.4 Inverted Systems

All the previously mentioned positioning techniques use a configuration in which the vehicle to be tracked has a single transducer and there is an external set of transponder to determine the positioning of the said object. However, there is the possibility to benefit from the inverse configuration in some applications. Therefore, there are also the iLBL, iSBL and iUSBL methods, which have the same operation principals as LBL, SBL and USBL, respectively.

2.5 Commercial Solutions

There are several commercial solutions for underwater positioning using the ultra-short baseline method. In this section, it will be presented some of the available devices in the market, indicating their main properties and capabilities. Table 2.1 summarizes the systems with most relevance to the present work. The Medium Frequency (MF) bandwidth is attributed to devices whose manufacturer did not specified the actual frequency range.

Evologics produces the S2C R USBL series of acoustic modems [17], with Sweep Spread Carrier (S2C) technology [18] which uses a broad frequency range to propagate over large distances with reduced noise. The devices have a fixed 0.01m slant range accuracy and a 0.1 degree bearing resolution. These are essentially divided into two groups:

- High speed mid-range devices: contains the 18/34 transceivers family [19], which presents various options for the USBL antenna beam pattern and it is optimal for transmission in horizontal channels.
- Depth rated long-range devices: includes the 12/24 transceiver [20], which have a directional (70 degrees) USBL antenna and it is optimal for transmission in vertical channels.

Sonardyne markets the Ranger 2 systems. The Micro-Ranger 2 [21] is very easy to use without previous experience and it is appropriate for shallow waters, achieving accuracy of 0.2%. The Mini-Ranger 2 is ideal for nearshore missions and it is used for simultaneous tracking of various mobile targets, whose position is updated every 3 seconds.

Applied Acoustics offers the Easytrak USBL Systems, which includes the processing software for estimating the position. The Alpha Portable 2655 consists in a very compact structure that includes an array transducer and is capable of reaching a 10cm slant range resolution and a 2 degree RMS.

Kongsberg produces the HiPAP family of transducers [22], which can use the Cymbal acoustic protocol (PSK) or the frequency shift (FSK) modulation technique. Particularly the HiPAP 352 is the model with higher number of active transducers and is able to reaches 0.02m of range accuracy.

Company	System	Bandwidth(kHz)	Connection(kbps)	Range(m)
Evologics	S2C R 18/34D USBL	18-34	up to 13.9	3500
	S2C R 12/24 USBL	12-24	up to 9.2	6000
Sonardyne	Micro-Ranger 2	MF	0.2-9	995
	Mini-Ranger 2	MF	0.2-9	995
Applied Acoustics	Easytrak Alpha Portable 2655	MF	n.d.	500
Kongsberg	HiPAP 352	21-31	n.d.	5000

Table 2.1: Overview of commercial solutions

2.6 Angle of arrival determination

methods for calculating signal's angle of arrival

2.7 Optimization of sensor configurations

When dealing with a localization system which integrates a multiple sensor configuration, it is essential to understand how can the sensor layout be designed so that the estimation is optimized. This section is dedicated to explore some commonly employed methodologies which evaluate the performance of sensor layouts considering determined parameters.

2.7.1 Crámer-Rao lower bound

The Crámer-Rao bound is a tool which analyzes the variance of a sensor configuration and determines the minimum bound it can achieve independently from the used estimator. This method assumes the usage of an efficient estimator, which is an optimal estimator for the chosen parameter, and an unbiased estimator, i.e. the real value of the parameter is equal to the expected value.

In this thesis, it is conducted a study based on the Crámer-Rao lower bound, which is generally used to generate a *so-called uncertainty ellipse* [23] that represents the spatial variance distribution of the estimated position. The overall desired result is to find the minimum variance that is related to the chosen configuration geometry, which indicates that it is the optimal solution for estimating a certain position. This method utilizes the Fisher Information matrix (FIM), which measures the quantity of information that can be extracted from an observation vector about a certain parameter.

In order to avoid loss of generality, it is considered a set of N sensors and a settled position for the acoustic source, defined by $s = [x_s, y_s, z_s]^T$. In addition, the position of each sensor is defined as $r_i = [x_{r_i}, y_{r_i}, z_{r_i}]$ and, consequently, the measurement of distance between each sensor and the source is defined as $d_i = \|s - r_i\|$.

Thereafter, the observations vector will be formulated containing the observed times-of-arrival (ToA) of the signal from the acoustic source to each one of the hydrophones, considering their geometric position. These times contain a noise vector component, which can be approximated to a Gaussian distribution $n_i \sim \mathcal{N}(\mu, \sigma^2)$. The samples can be calculated through the expression

[2.23](#), where it is considered an initial time of arrival t_0 . Additionally, c represents the sound speed underwater.

$$t_i = t_0 + \frac{\|d_i\|}{c} + n_i \quad (2.23)$$

After having the observations matrix, ~~it is established~~ the condition to formulate the Fisher Information matrix  (d) , which results into equation [2.24](#).

$$I(d) = \nabla_d t(d)^T \Sigma^{-1} \nabla_d t(d) \quad (2.24)$$

$\nabla_d t(d)$ is the gradient matrix of the observations vector regarding d_i , whereas Σ is the covariance matrix, in which the diagonal contains the standard deviation of the components of each noise vector, construed as $(\sigma_1^2, \sigma_2^2, \dots, \sigma_N^2)$.

~~After formulated~~, the Fisher Information Matrix ~~it will indicate~~ the quantity of information that a certain sensor configuration can give about a position in space. Hence the goal is to obtain the maximum achievable information. By calculating the determinant of FIM it is possible to deduce the minimum *uncertainty ellipsoid* and therefore the configuration's best possible performance. Therefore, the optimal solution is given by the maximum output of the determinant of FIM.

Additionally, it is possible to detail this information by calculating the actual size of the axis that compose the *uncertainty ellipsoid*. This is achieved by calculating the square mean root of the eigenvalues of $I(d)$, which correspond to each axis size.

Further explanation about the methods used in a deeper exploration of the Crámer-Rao lower bound can be consulted in [\[23\]](#), which serves as guide to investigate other scenarios of application  of this theorem. However, the mentioned concepts were all the necessary for the approach on this dissertation .

This same process is adopted in this dissertation. All steps specifically taken for this study are declared in section [4.2.4](#) of the present document.

2.7.2 Optimal design and optimality criteria

When contemplating system designs, the optimal solution for a problem is generally a subjective matter which depends on the chosen principal. Following this idea, we can define optimal designs as experimentally generated designs of various types of systems which are usually optimal for a targeted statistical model and are modelled by a specific optimality criterion. These criteria can be organized in four distinct groups [\[24\]](#):

- **Information-based** : comprehends all criteria that are related to the Fisher information matrix $X'X$. Some of the criteria which fits into this category are A, D, E and G-optimality.
- **Distance-based** : includes criteria which depends on the distance $d(x, A)$ from a point x in the Euclidean space \mathbb{R}^p to a set $A \subset \mathbb{R}^p$. U and S-optimality are integrated in this category.

- **Compound design** : combine different adjusted criteria in different weighted proportions in order to meet a desired statistical function.
- Other : all criteria which do not fit in the previous three sets can be encompassed in a fourth general set.

The most relevant category for the present work is the information-based criteria, since we want to maximize the information that can be extracted from a certain design and we do not need to minimize the distances between the receptors and the source. Therefore some of the most commonly used criteria will be better explained in the following subsections.

2.7.2.1 D-optimality

2.7.2.2 E-optimality

2.7.2.3 A-optimality

2.7.2.4 G-optimality

2.7.3 Particle Swarm Optimization

PAPER: Optimal-receiver-config-short-baseline-localization-systems-using-particle-swarm-opt

Optimal-receiver-config-short-baseline-localization-systems-using-particle-swarm-opt : [25]

Chapter 3

Research Problem

This chapter intends to clarify the problem addressed by the present dissertation. Section 3.1 presents details and motive behind the research work as well as the problems it intends to solve. Having this clear, the dissertation hypothesis is stated in section 3.2 along with the research questions that are the main issues that are being explained with the present document. Lastly, the used validation methods are specified in section 3.3.

3.1 Problem Statement

The Ultra-Short Baseline system is among the most deployed positioning methods using underwater acoustics. There is a vast knowledge of its function and capabilities, therefore its implementation does not constitute a technological innovation nowadays. In the considered scenario, previously mentioned in chapter 1, an AUV is taking part on a long-term underwater mission in which it periodically sends known signals to the surface with a pinger. In such case, the mule AUV needs to be provided with an USBL system to receive the signal and estimate the position of the other AUV to navigate near it. The simplified communication system is illustrated in figure 3.1.

This partial USBL system was developed in previous dissertations and research work, which can be better understood in [26]. Briefly, the system consists on a transducer of four hydrophones forming a 3D array deployed on the mule AUV. The distance between AUVs is given by the cross-correlation between the received and expected signals. Since the operation frequency range is around hundreds of kHz, then the signal's wavelength is expected to be smaller than the hydrophones' baseline but not sufficient to accurately estimate the angle of arrival. Therefore, this measurement is intended to be refined with an additional phase difference calculation of the arriving signals to each hydrophone.

Considering that the survey AUV navigates freely through undisclosed locations, the USBL system to be employed needs to fulfill particular requirements that common commercial solutions do not comply.

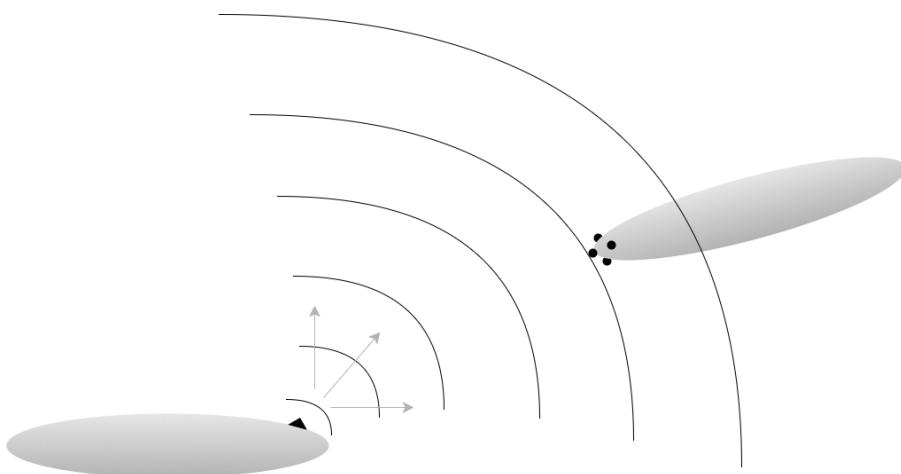


Figure 3.1: Communication System

Firstly, the system needs to be able to cover both short and long range distances, going from few centimeters to tens of kilometers between the receiver and the transmitter, with the best estimation accuracy possible. For long range positions, the accuracy of the estimation affects how direct is the path for the mule AUV to reach the acoustic source. This influences the overall resource consumption, duration of navigation search and can affect the reliability of the process. For short range position, an accurate estimate allows to avoid collisions and correctly establish chosen relative positions between vehicles. Additionally, a sampling frequency increase would also facilitate the achievement of more robust estimations. The available market systems usually offer multiple solutions with different limited operation ranges, which would force to employ more than one system to achieve the mentioned range requirement.

Secondly, the USBL system needs to be capable of detecting incoming signals from any position in space, since the localization detection is solely based on the received signals. Since the system is composed by various sensors, it is expected that they are arranged in varied positions and with different viewing angles so they can cover a wider area. However, considering they are supposed to be employed on an AUV, the vehicle's body represents an opaque obstacle to signals. Therefore, with only four fixed hydrophones, it is not possible to detect positions with full viewing angle, as intended, since not all hydrophones would have line of sight to the transmitter at all times.

The system that is proposed by this dissertation, it intended to resolve this technological gap with a system that satisfies the described requirements. Since using only four hydrophones constitutes a limitation for the viewing angle of the system, the suggested method implies deploying multiple sensors in the vehicle. From the available sensors, only four would be used simultaneously to receive the signals and feed them to the processing system. By adopting this concept, a main issue that arises is where to place the hydrophones within the vehicle. This constitutes the main research topic conducted in the present thesis.

Considering that the mentioned mechanism is meant to be applied in moving vehicles with changing environment conditions, it is useful to integrate it in a system which is responsive in real time. Accordingly, the process that selects four hydrophones among the available set

can be transformed into a dynamically reconfigurable system which enables the hydrophones commutation according to the sensors' configuration that minimizes the estimation error.

The study conducted in the scope of this dissertation intends to prove the functionality of the developed method, validate the hypothesis declared in 3.2 and draw conclusions on the research questions.

3.2 Hypothesis and Research Questions

This dissertation intends to complement previous research work and respond to a core research hypothesis which serves as fundamental investigation purpose. This research hypothesis can be stated as:

"Using a USBL system which dynamically reconfigures the hydrophone selection leads to an improvement on the underwater localization accuracy, allows to have a wider viewing angle and makes it suitable for both short and long range estimation."

Attending the proposed hypothesis, the topics that are intended to be explored and discussed in this thesis's work can be summarized in the following research questions:

- **RQ1:** *What method should be adopted in order to efficiently compare the performance of hydrophone configurations?*
- **RQ2:** *What decision metric(s) should be used to evaluate the optimal hydrophone configuration for a specific angle of arrival?*
- **RQ3:** *How should the system be developed in order to have a broad viewing angle of the space?*
- **RQ2:** *Are there distinct best hydrophone configurations for short and long range estimation?*

These questions summarize the main topic points which are explored in the scope of this thesis and are the essential inquiries that it intends to answer.

3.3 Validation Methods

The validation of scientific work is a key factor to demonstrate how reliable and effective it is. In this thesis, three essential methods are used to validate the functionality of the developed techniques:

- **Simulation**

The considered immediate approach to evaluate the functionality and behavior of the system consists in creating a set of simulation procedures which are as close as possible to the real

environment and the physical system. These simulations were made as MATLAB scripts carefully designed to integrate realistic parameters, such as expected environment noise and other limitations.

- **Scientifically recognized methods**

When composing a system, it can be useful comparing the studied approach with widely used methods which are recognized in the scientific community as **robust** and trustworthy. By doing this, we can gain a level of confidence in the developed system and in the obtained results.

- **Field experiments**

After having the analytical methods and simulations coherent, it is essential then to test the system in a real environment in order to **understand if the system still works correctly** when real conditions are added. By testing it in a real application it is possible to take conclusions about its robustness and consider improvements or refinements for the system. In the present work, only the developed hardware design was subjected to field tests.

Chapter 4

Ultra-Short Baseline System

This chapter is dedicated to the presentation and overall explanation of the developed system, highlighting its capabilities, the used methodologies and overall design strategies. The system will be presented in two distinct sections. The first component is the HDL module, which falls into the spectrum of hardware design and requires insight on hardware development and good practices. The second section relies on software development to complement the functionality of the mentioned module, so that is possible to deliver the desired result.

4.1 HDL Module Architecture

The system which is proposed to be implemented in this research work has as input 4 signals which are received by each hydrophone of the array, and outputs an average phase difference between all combinations of pairs of hydrophones.

- regras basicas de hardware development
- sistema sincrono, available clock cycles globais
- hardware limitations
- tamanho das entradas arg

1. Hilbert Filter
2. Cordic
3. phasediff
4. phasemean

4.1.1 Module components

apresentar esquema global menos pormenorizado

4.1.1.1 Hilbert Filter

$$H(f)(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{f(\tau)}{t - \tau} d\tau \quad (4.1)$$

$$Imag_0 = x_{-1} * c_1 + x_{-3} * c_3 + x_{-5} * c_5 + x_{-7} * c_7 \quad (4.2)$$

$$Real_0 = x_{-4} \quad (4.3)$$

- matematica brevemente, equação base, resposta impulsional, ganho, coeficientes e ordem usada
- schematics
- explicar design decisions
- descrever brevemente flow do sinal no hardware

4.1.1.2 Cordic

- descrição do que faz
- entradas e saídas, clocks, ROM

4.1.1.3 phasediff

- pequeno esquema
- 1 sub

4.1.1.4 phasemean

- pequeno esquema
- N accumulated

4.1.2 Analysis

- present used resources
- overall achievement

4.2 Methods for configuration performance evaluation

As previously declared, the present dissertation ultimately intends to develop a reliable method that determines the optimal configuration to minimize the estimation error. For this purpose, it is necessary to employ a complementary mechanism which is able to perform experiments on sensor configurations in order to quantify their performance based on chosen decision parameters.

Therefore, three different methods will be presented in this section as potential tools to be used. Firstly, a theoretical approach will be detailed, followed by a functional exemplification of each process. Lastly, a practical comparison between them will be presented in order to disclose the preferred option for this application.

The chosen methods are a Monte Carlo estimator based on TDoA, a Monte Carlo estimator based on a plane wavefront and the Fisher Information Matrix.

4.2.1 Preliminary considerations

In order to better understand the used mechanisms for estimating positions, some preliminary considerations are laid out. These contemplate system definitions, deductions about system's phenomena and considered approximations. The following topics are discussed:

- the number of used sensors for the estimation;
- a phase ambiguity issue that affects the ToA determination;
- the relation between the phase differences of hydrophones in known relative positions and the possible location of the transmitter;
- an approximation used for the ToA measurement;
- the influence of the Doppler effect in the system.



4.2.1.1 Number of sensors

For the estimation of the position in 3D space, a multilateration approach was used. As explained in [2.3.3](#), the concept of multilateration combines the information of the relative distances between multiple sensors and a target in order to locate it.

In the present case, a total of four sensors are needed so that it is possible to define the position of target. Using only two sensors, two possibility spheres are formed around these sensors whose intersection originates a circle that contains the location possible solutions. By adding a third sensor, this circle is intersected by another sphere which originates only two location possibilities. Finally, a fourth sensor is added so that it is possible to exactly differentiate which one of the two final solutions is the accurate location solution.

4.2.1.2 Phase Ambiguity

When the information about the time of arrival of a signal is available, it is relatively easy to estimate the range of the communication since there can be a direct conversion between them. However, when dealing with phase differences, there is no exact time notion, so it is necessary to start by defining a reference point.

Considering sinusoidal signals, when we have an array with four hydrophones spatially placed to form a 3D layout, the signal that is arriving to each hydrophone in different times consequently have different phases. However, since sinusoidal signals are periodic, this means that for different signal periods the same phase value is observed, i.e. the phase is ambiguous. It is possible to observe this phenomenon in figure 4.1. In this illustration, α represents the observable phase difference of hydrophone H_4 to the reference point H_1 . However, the actual phase difference which is intended to obtain, Δp_4 , is one period of the signal, λ , added to the observable phase α .

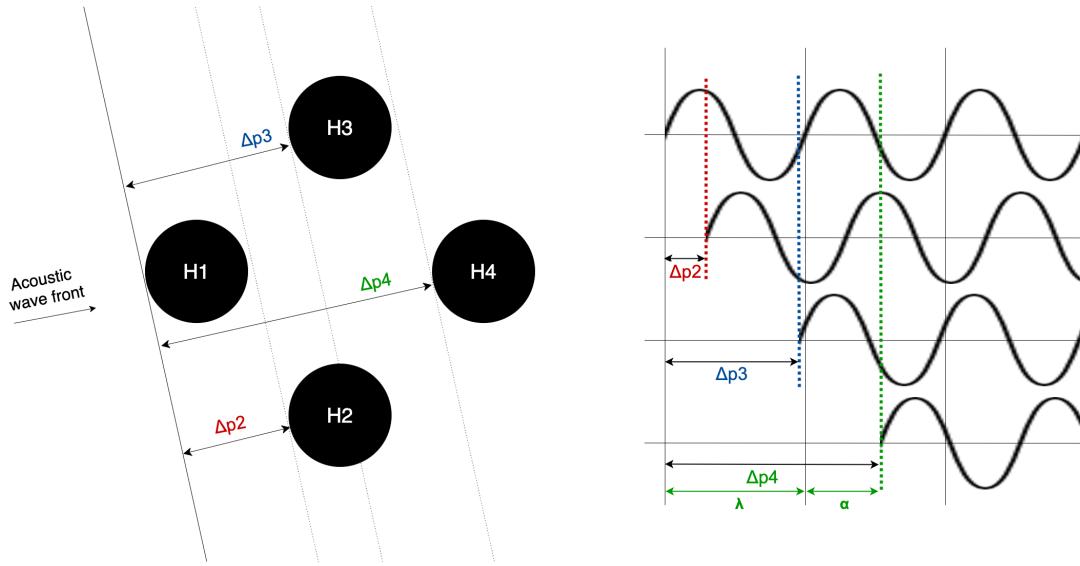


Figure 4.1: Phase difference to reference point and phase ambiguity

For this reason, it is crucial to consider that the phase difference is given by the obtained phase value added by the number of periods ahead from the considered reference period.

In the system under study, the sent signals work with a operation frequency of 24.4 kHz. The corresponding signal period is $T = \frac{1}{24400}$ seconds which, considering the underwater acoustic speed c equal to a standard 1500 m/s, the wavelength is approximately equal to $\lambda = \frac{T}{c} = 6.1\text{cm}$. Having this into consideration, after obtaining the time of arrival to each hydrophone given by the cross correlation instances, besides the reference one, it is possible to conclude if the phase shift is superior to one period by analyzing if the time difference is greater than the duration of one period T . In figure 4.1, each mentioned time difference between H_1 and H_2 , H_3 and H_4 is converted to the corresponding phase differences Δp_2 , Δp_3 and Δp_4 .

One possibility to solve phase ambiguity in this system would be to place the four hydrophones with a baseline spacing inferior to $\frac{1}{2}$ of a wavelength, since the maximum reached

by phase difference is 180 degrees. This way it would be possible to immediately deduce the phase difference since it would always be contained in one period. However, positioning the hydrophones closer together leads to smaller values, causing a consequent increase on the estimation error due to varying environment conditions (briefly enumerated in 2.1). Additionally, since the hydrophones to be used in this system have a corresponding diameter of roughly half of a wavelength, they would not allow to execute the mentioned configuration and so this possibility will not be contemplated.

In order to compensate this phase ambiguity, a simple relation was developed which allows to calculate the absolute time difference between the moment a signal is received by hydrophone A, T_A , and when the same signal is received by a further hydrophone B, T_B . Figure 4.2 illustrates this association, where the represented sinusoidal waves correspond to the same signal arriving at hydrophones A and B. This correspondence uses the time stamps obtained by the correlation peaks combined with the calculated phase difference, that is determined in parallel, so that the measurement is more accurate. Equation 4.4 translates this relation, where t_1 and t_2 are the correlation peaks obtained from the signal arriving at hydrophone A and B, respectively, and so by rounding for the next integer number the difference between the correlation peaks, $t_2 - t_1$, we will obtain in which period, T , of signal in A will the signal in B arrive. Then the measurement is improved by subtracting a phase difference, $\theta_B - \theta_A$, so that the instant in which the signal is detected in hydrophone B can be defined.

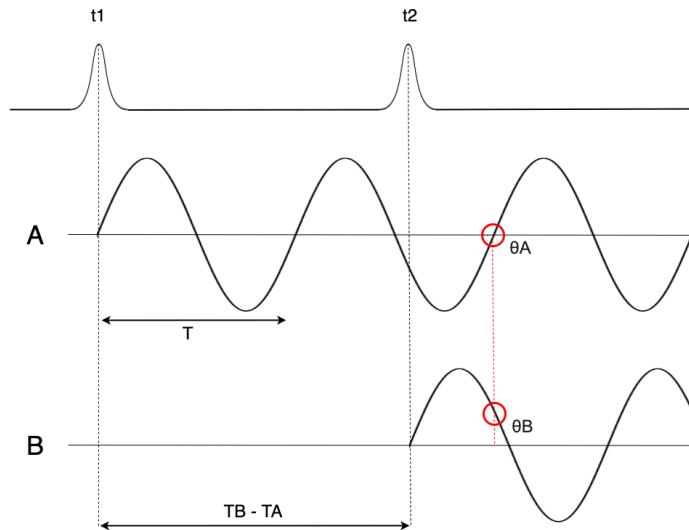


Figure 4.2: Ambiguity correction through correlation and phase difference

$$T_B - T_A = \text{round}\left(\frac{t_2 - t_1}{T}\right) - (\theta_B - \theta_A) \quad (4.4)$$



4.2.1.3 Hydrophone position in relation to ToA

To better understand the location estimation of an acoustic source in relation to the position of a pair of hydrophones, we can initially adopt the two dimensional scenario of figure 4.3.

Considering two hydrophones at known relative positions $(-f, 0)$ and $(f, 0)$, we can model all possible acoustic source locations for a specific ToA through hyperbolas. This is due to the fact that, by definition, the sum of the distances from the focus of each hyperbole, where each hydrophone is placed, to any point of the hyperbolic geometry corresponds to a constant value. This means that, in figure 4.3, any point (x, y) that is contained in the hyperbole corresponds to a constant $|d_2 - d_1|$ value which, after some formulation, is in fact equal to $2 * v$ or the distance between the vertexes of each hydrophone's hyperbola. Therefore, it is possible to trace a hyperbole that represents the positions of the target in space both based on their distance and the signal's ToA. In the exceptional case where $d_3 = d_4$, we can observe that the possible positions are represented by an equidistant straight line to each hydrophone, such as the y axis.

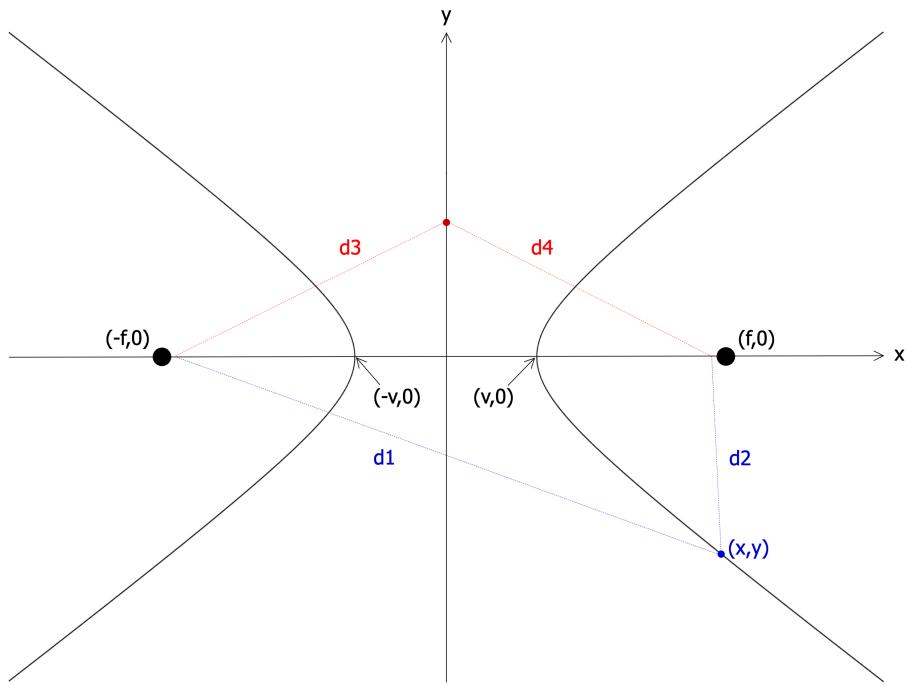


Figure 4.3: Hyperbolic representation of acoustic source position possibilities in relation to ToA to two hydrophones

4.2.1.4 ToA approximation

In order to estimate the location of an acoustic source we take into account the phase differences between each pair of hydrophones, carefully explained in section 4.1. These phase differences can be translated into periods of the signal which combined with the ToA obtained from correlation of arriving acoustic signals are equivalent to relative distances.

Following the previous idea, it is possible to model the distance of one sensor to the target based on the known distance of a second sensor to the same target. This is to say that for two sensors with known relative positions where hydrophone 1 is the closer to the target, the distance from hydrophone i to the target, D_i , can be expressed as the distance of hydrophone j to the target, D_j , added by the time difference of arrival, Δt_{ij} , multiplied by the propagation velocity, c_s . Overall, this relationship is declared in equation 4.5.

$$D_i = D_j + \Delta t_{ij} * c_s \quad (4.5)$$

Therefore, the same logic can be applied for multiple hydrophones. In the present work, in which it is considered a system with four hydrophones, a synchronization mechanism allows to determine the signals' ToA between the transceiver and the hydrophones. However, in order to simplify the synchronicity and decrease errors that arise from it, the module that precisely computes the phase differences of the received signal in the hydrophones is used so that is possible to apply the relationship in 4.5. Consequently, a better angle of arrival estimation can be achieved when using this approximation than if all four times of arrival are used for the same purpose.

4.2.1.5 Doppler Effect

The implemented process uses the information of the transmitted signal's operating frequency in order to compute the phase differences and determine the overall times of arrival. However, in real scenarios the environmental conditions can distort this frequency between the source and the receiver. In the present study, since the vehicle is predominantly moving, then the Doppler effect could influence the signal's frequency, leading to erroneous calculations.

In order to evaluate if the Doppler effect influences the system, it is possible to calculate which is the frequency deviation observed for a known relative speed between vehicles. Considering that the relative speed between the transmitter and the receiver is denoted as *relative_speed* and using a fixed sound speed, c_s , with a determined frequency of the transmitted signal, the relation 4.6 can be established.

$$freq_deviation = \frac{relative_speed}{c_s} \times signal_freq \quad (4.6)$$

Therefore, considering the frequency of the signal equal to 24.4kHz and a c_s of 1500m/s, it is observable that the frequency deviation will be dependent on the relative speed. Considering a transmitter that is static and a typical value for the navigation velocity of an AUV around 2m/s, which results in a frequency deviation of approximately 32.5Hz. This allows to consider the Doppler effect negligible in this case.

A way to prevent this deviation is to integrate a frequency detector which senses the relative navigation speed in real time and adapts the used frequency. This mechanism is not integrated in the present research work.

4.2.2 Monte Carlo estimator based on TDoA

The goal of the first proposed method is to estimate the position of an acoustic source in relation to known positions of a configuration of sensors, in a system of geometric axes with a defined origin. For this purpose, the logic employed is based on vector algebra with other physical considerations, detailed in the present subsection.

Figure 4.4 represents the schematic of a considered scenario, where four hydrophones are placed in known relative positions in space and the origin of the axis is set on the body of the AUV or an alternative fixed structure. Then r_i is defined as the vector that connects the origin of the axis to hydrophone i and rr_i defines the vector that connects hydrophone i to the acoustic source. The black cross represents the acoustic source which is located somewhere in space. At last, the subtraction of the mentioned vectors is equal to r , according to 4.7, which corresponds to the position of the acoustic source in relation to the origin of the axis and, overall, it is the variable that the method aims to determine.

$$r_i = r + rr_i \quad (4.7)$$

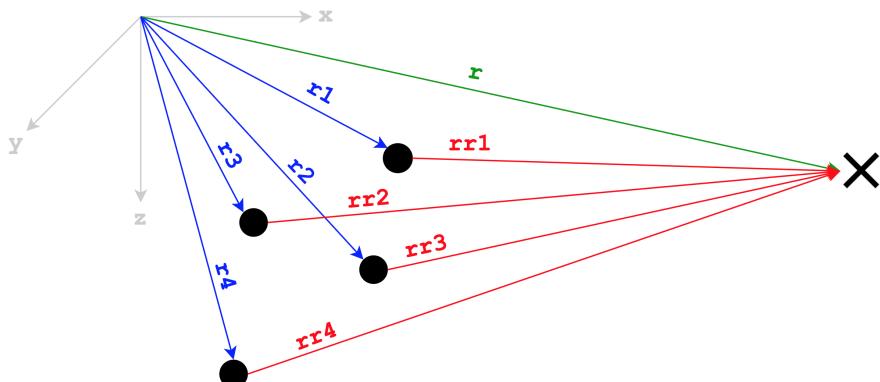


Figure 4.4: Considered scheme for angle of arrival estimation

Then we can define the times of arrival to each hydrophone as 4.8, where t_0 is the absolute time of emission, c_s is the underwater sound speed and ρ_i is the norm of rr_i , 4.12, which translates to the distance from hydrophone i to the acoustic source.

$$t_i = t_0 + \frac{\rho_i}{c_s} \quad (4.8)$$

However, as explained previously, instead of using the absolute ToA in each hydrophone by computing the expression 4.8 for each of them, it can be expressed as a function of a single reference ToA. A simple logic was applied in order to determine this reference hydrophone, which starts by identifying the closest to the acoustic source. This allows to obtain all relative times of arrival by adding the defined reference time to each between a hydrophone and the reference one. This is achieved by analyzing the of each pair, Δt_{ij} , for all possible combinations of two among four hydrophones, making up a total of six combinations. Considering each hydrophone pair ij with $i, j = \{1, 2, 3, 4\}$:

- if Δt_{ij} is positive, then hydrophone i is closer to the acoustic source
- if Δt_{ij} is negative, then hydrophone j is closer to the acoustic source
- if Δt_{ij} is zero, then i and j hydrophones are equidistant to the acoustic source

Considering these relations, it is possible to compose a vector that accumulates the closer hydrophone between each pair for a certain position of the acoustic source. Extracting the mode of this vector will then return the chosen hydrophone in most cases and therefore the overall closer to the acoustic source. If the closer hydrophones are the equidistant to the source, then it is indifferent which one is selected.

Thereafter, recalling expression 4.5, it is possible to write 4.9, 4.10 and 4.11 which translate the used relations, where the chose reference sensor is hydrophone 1, for the purpose of exemplification.

$$T_2 = T_1 + \Delta t_{12} * c_s \quad (4.9)$$

$$T_3 = T_1 + \Delta t_{13} * c_s \quad (4.10)$$

$$T_4 = T_1 + \Delta t_{14} * c_s \quad (4.11)$$

If then the distance ρ_i is raised to the power of two, we know that $||rr_i||^2 = r_i^T r_i$, which allows to deduce equation 4.13 after some mathematical manipulation. Considering ρ_i a physical distance, it is also possible to express it trough equation 4.14, which uses the speed of propagation underwater multiplied by the ToA of the signal from the acoustic source to hydrophone i .

$$\rho_i = ||rr_i|| \quad (4.12)$$

$$\rho_i^2 = r^T r + 2r^T r_i + r_i^T r_i \quad (4.13)$$

$$\rho_i^2 = c_s^2(t_i - t_0)^2 \quad (4.14)$$

Since two distinct relations are defined for ρ_i^2 , then it is possible to consider the algebraic expressions as equivalent, thus forming a single equation to be resolved with only one unknown

variable. After some mathematical manipulation, the matrix relation 4.15 is achieved, where r is isolated and can be estimated.

$$\begin{bmatrix} 1 & 2r_i^T \end{bmatrix} \begin{bmatrix} r^T r \\ r \end{bmatrix} = \begin{bmatrix} c_s^2(t_i - t_0) - r_i^T r_i \end{bmatrix} \quad (4.15)$$

In order to resolve this system of equations and isolate r , the least squares method is applied. If 4.15 is extended to the four considered hydrophones, we obtain matrix A represented as 4.16 and Y equivalent to 4.17. It is important to notice that the A matrix has to be invertible, thus the rows which contain the chosen hydrophone configuration have to be linearly independent. The least squares method is then expressed as 4.18, where $X \in \mathbb{R}^4$ holds the Cartesian result of r . As the method formulates four equations that are meant to calculate only three coordinates, X will contain a fourth element that consists on a nonlinear component equivalent to $\|r\|^2$.

$$A = \begin{bmatrix} 1 & 2r_1^T \\ 1 & 2r_2^T \\ 1 & 2r_3^T \\ 1 & 2r_4^T \end{bmatrix} \quad (4.16)$$

$$Y = \begin{bmatrix} c_s^2(t_1 - t_0)^2 - r_1^T r_1 \\ c_s^2(t_2 - t_0)^2 - r_2^T r_2 \\ c_s^2(t_3 - t_0)^2 - r_3^T r_3 \\ c_s^2(t_4 - t_0)^2 - r_4^T r_4 \end{bmatrix} \quad (4.17)$$

$$X = (A^T * A)^{-1} * A^T * Y \quad \text{4.18}$$

After inferring the Euclidean vector r , it is possible to obtain both the bearing through its direction, \hat{r} , and the range through its magnitude, $\|r\|$.

4.2.3 Monte Carlo estimator based on a plane wavefront

The second explored method for evaluating sensor configurations' performance is a simpler approach than the previously proposed. It consists on an estimator which assumes that the signals' propagation can be approximated by a plane wavefront. When considering a planar wavefront approximation, it is assumed that the acoustic source is at a distance which is sufficiently larger than the relative distances between the hydrophones, for the signal to be considered to have a planar wavefront, thus the angle of arrival to each hydrophone being the same. This estimator

is posteriorly integrated in a Monte Carlo approach to evaluate the configurations based on the obtained variance given different acoustic source positions.

Using a plane wavefront approximation means that, for signals arriving to the USBL system, it is assumed that the wavefront is coincident with a plane perpendicular to the direction of propagation, thus perpendicular to the angle of arrival as well. Figure 4.5 is illustrative of a plane wavefront arriving at two sensors. As represented, the angle of arrival is considered the same for every sensor, since the wavefront is the same as well, and it is possible to deduce the time of arrival to an hydrophone 1 by using the ToA to hydrophone 2 added by the time difference of arrival between 1 and 2. Therefore, it is also possible to obtain the estimation of the range, as it will be presented next.

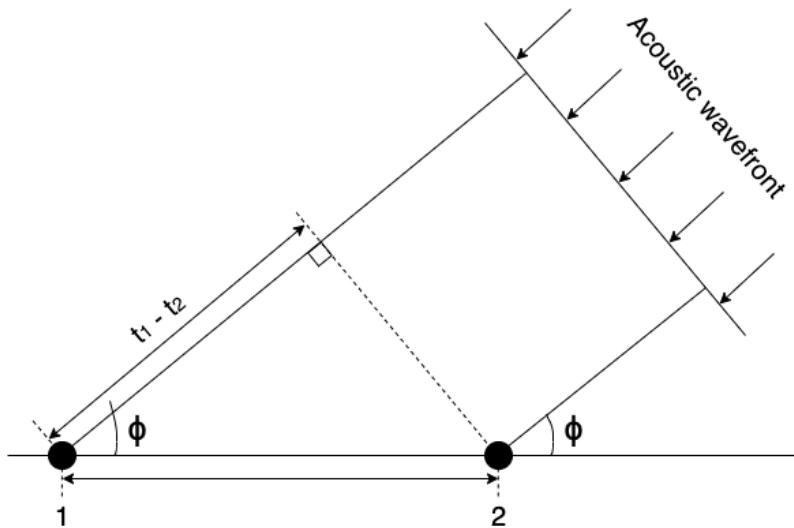


Figure 4.5: Angle of arrival relation considering a plane wavefront

The estimator considers a USBL system composed by four sensors whose positions are previously defined. The time of arrival, t_i , to each hydrophone, i , is represented by expression 4.19. The considered t_0 is the absolute time of emission of the signal, which is acquired through a synchronization mechanism, the s is the position of the transmitter, a is referent to the mass center of the USBL's position, r_i is the Cartesian coordinates of the hydrophone's position, c_s is the sound speed and e_i is an injected error that have the same characteristics expressed previously in ??.

$$t_i = t_0 + \frac{\|s(a-r_i)\|}{c_s} + e_i \quad (4.19)$$

Due to the considered approximation to the plane wavefront, it is possible to calculate the range estimation, ρ , as a mean all propagation times multiplied by the sound speed, c_s , as in equation 4.20. The N represents the number of hydrophones.

$$\rho = c_s \frac{1}{N} \sum_{i=1}^N t_i - t_0 \quad (4.20)$$

Then a matrix S is formulated as 4.21, whose rows are the difference between the position of hydrophone 1 and the remaining three, and a Δ vector is expressed as 4.22, containing all combinations of TDOA between hydrophone 1 and the remaining.

$$S = \begin{bmatrix} r_1 - r_2 \\ r_1 - r_3 \\ r_1 - r_4 \end{bmatrix} \quad (4.21)$$

$$\Delta = \begin{bmatrix} t_1 - t_2 \\ t_1 - t_3 \\ t_1 - t_4 \end{bmatrix} \quad (4.22)$$

Taking into account the defined S and Δ , the signal's angle of arrival can be calculated from the direction of the signal, d , obtained from the least squares expression 4.23 [27]. Since there are only three equations to estimate three coordinates, then the expression is linear.

$$d = c_s (S^T S)^{-1} S^T \Delta \quad (4.23)$$

Having calculated the range ρ and the direction d , the estimate of the transmitter position is given by the multiplication of these variables.

Finally, the plane wavefront estimator was integrated in a Monte Carlo method in order to execute multiple estimations of various transmitter positions, s , for a specific sensor configuration, so that it is possible to draw conclusions on its performance.

4.2.4 Fisher Information Matrix

The Fisher Information Matrix (FIM) serves to quantify the information that an observable variable is capable of returning to an estimator. This concept is then integrated in the Crámer-Rao lower bound, which is a method that expresses the variance lower bound that a sensor configuration is capable of achieving. Unlike the two previously presented processes, the Crámer-Rao bound is independent from the used estimator, as it assumes a linear estimator that is efficient and unbiased.

The fundamental thought process and mathematical notation previously used in 2.7.1 are applied in the procedure that will be explained next.

The implementation of the Crámer-Rao bound method can be essentially partitioned in four steps:

1. Formulate the observations vector
2. Calculate the Fisher Information Matrix (FIM)
3. Calculate the determinant of FIM
4. Apply optimality criteria to draw conclusions

In this thesis, the number of used hydrophones per estimation is four, so all expressions will be presented for this case.

Firstly, the observation vectors are formulated based on an initial time of arrival t_0 , which is given by a synchrony mechanism integrated in the global communication system, and the time of arrival based on the vectors that connect the hydrophone positions, r_i , to the transmitter, s . For a more realistic approach, it is also considered an added noise component that can be approximated to a Gaussian distribution $n_i \sim \mathcal{N}(\mu, \sigma^2)$.

The four observations vectors are then expressed as 4.25 to 4.27.

$$t_1 = t_0 + \frac{s - r_1}{c} + n_1 \quad (4.24)$$

$$t_2 = t_0 + \frac{s - r_2}{c} + n_2 \quad (4.25)$$

$$t_3 = t_0 + \frac{s - r_3}{c} + n_3 \quad (4.26)$$

$$t_4 = t_0 + \frac{s - r_4}{c} + n_4 \quad (4.27)$$

Then, addressing the second step, all conditions are set to calculate the FIM, $I(d) \in \mathbb{R}^{3 \times 3}$. In order to do so, if $d_i = \|s - r_i\|$ is the distance between each sensor and the source, the gradient of the observations vector can be expressed as shown in 4.28.

$$\nabla_d t(d) = \frac{1}{c} \begin{bmatrix} \frac{d_1^T}{\|d_1\|} \\ \frac{d_2^T}{\|d_2\|} \\ \vdots \\ \frac{d_N^T}{\|d_N\|} \end{bmatrix} \quad (4.28)$$

Additionally, the added noise component is present in the covariance matrix, represented as 4.29, which integrates the FIM equation.

$$\Sigma = \begin{bmatrix} \sigma_1^2 & 0 & 0 & 0 \\ 0 & \sigma_2^2 & 0 & 0 \\ 0 & 0 & \sigma_3^2 & 0 \\ 0 & 0 & 0 & \sigma_4^2 \end{bmatrix} \quad (4.29)$$

Overall, the conditions to obtain the FIM matrix are established and, after some mathematical formulation, it can be expressed as 4.30. This function can be validated by a similar study made on TOA based optimal positioning [28].

$$I(d) = \frac{1}{c^2} \left[\sum_{i=1}^N \frac{d_i d_i^T}{\|d_i\|^2} \frac{1}{\sigma_i^2} \right] \quad (4.30)$$

The final step is to calculate the determinant and find its relation to the volume of the *uncertainty ellipsoid*. As mentioned before, in 2.7.1, the Fisher Information Matrix determinant returns a deterministic value that represents the quantity of information which can be obtained and, therefore, the objective is to maximize it by respecting the condition $\text{argmax } \det(I(d))$.

4.2.4.1 Optimality criteria

In order to evaluate the obtained values, several different optimality criteria can be applied. Considering similar applications in literature, the criterion that the most relevant for the present case is the E-optimality, which minimizes the largest eigenvalue of the inverted FIM.

The FIM can be associated with a physical meaning, more specifically an uncertainty volume that characterizes the variance that a specific configuration can achieve for an estimate. In an initial approach the ellipsoid was not considered and instead an uncertainty sphere was analyzed. The radius of the uncertainty sphere, u_{sphere} , is expressed as 4.31, which relates the three ellipsoid axis into a single mean radius that originates a figure of the same volume.

$$u_{sphere}(d) = \sqrt[3]{\det(I(d)^{-1})} \quad (4.31)$$

In this situation, the optimization objective is expressed as $\text{argmin } u_{sphere}(d)$, which indicates that the error that originates a certain uncertainty radius is minimal.

In a further analysis, the uncertainty ellipsoid is calculated using 4.32. The squared eigenvalues of the inverse of the FIM indicate the length of each of the three ellipsoid axis and the eigenvectors specify the direction of each axis. An eigenvector that is associated with the smaller eigenvalue indicates the direction in which there is less uncertainty and, equivalently, an eigenvector whose eigenvalue is the larger represents the direction in which there is the biggest uncertainty. Consequently, the goal in this case is to minimize the maximum squared eigenvalue of the FIM inverse, $\text{argmin } \max(\sqrt{\text{eigenvalue}})$.

$$u_{ellipsoid}(d) = \sqrt{\text{eig}(I(d)^{-1})} \quad (4.32)$$

Besides the E-optimality, several more criteria can be used in order to evaluate the performance of a specific hydrophone configuration, using the Fisher Information Matrix. The most common alternatives would be:

- A-optimality: minimizes the trace of the inverse of the FIM, therefore seeks the minimum average variance of the estimates.
- D-optimality: minimizes the determinant of the inverse of the FIM, therefore the volume of the uncertainty sphere. However, this can be misleading since the information obtained for one direction can be much larger than the other directions, constituting a large FIM determinant which is not representative of the full estimation.

4.2.5 Performance comparison between methods

Upon presenting the theoretical details behind the three considered methods for evaluating configurations' performance, a further functional study will be presented through simulation results.

Throughout the functional demonstrations, three different hydrophone configurations are considered, A, B and C defined in 4.1, where the columns r_{Ai} , r_{Bi} and r_{Ci} contain the position's coordinates of each hydrophone i .

	r_{A1}	r_{A2}	r_{A3}	r_{A4}	r_{B1}	r_{B2}	r_{B3}	r_{B4}	r_{C1}	r_{C2}	r_{C3}	r_{C4}
x	0.02	0.02	0	0	0.1	0.1	0	0	0.1	0	0	0
y	0	0	0.1	-0.1	0	0	0.1	-0.1	0	0	-0.0707	0.0707
z	0.1	-0.1	0	0	0.1	-0.1	0	0	0	0.1	-0.0707	-0.0707

Table 4.1: Hydrophone configurations used for accuracy tests

For testing both estimators, a methodology was formulated in order to evaluate the accuracy that they can achieve in defined circumstances. This approach is a Monte Carlo algorithm which allows to reiterate the estimation process according to the number of positions that are intended to be tested for a specific configuration, as well as create a series of repetitive calculations that allow to deduce the estimation error and turn the overall process more robust. For this initial study, the following conditions are considered:

a) Sensor Configuration

Each hydrophone configuration is analyzed individually. It is a parameter to be always defined and known from the beginning of each simulation.

b) Reference axis

The origin of the reference axis is defined at the center of mass of the structure where the hydrophones are fixed, which in this case is the AUV.

c) Injected error

In order to make the study more realistic, an e_i error is added to the time differences of arrival, Δt_{ij} . These errors are mutually independent and follow a Gaussian distribution with zero mean and a configurable variance of σ^2 , i.e., $e_i \sim \mathcal{N}(0, \sigma^2)$.

For the simulations performed in this project, a deviation of 5° , or a window of $[-2.5^\circ, 2.5^\circ]$, in phase difference estimation of incoming signals was considered to be reasonable for an underwater navigation scenario. Therefore, since the specified period of the signal is $T = \frac{1}{24400}$ and one period corresponds to a 360° phase shift, then the 5° will be equivalent to $\frac{5^\circ}{360^\circ} * T$ which is approximately a deviation of $0.5\mu s$. Hence the considered standard deviation σ of the error e_i in the computed time differences of arrival is equal to $0.5\mu s$.

Overall, the considered total time of arrival is equal to the measured time between the transmitter and the receiver through correlation, added by the computed time difference of arrival for a specific pair of hydrophones and the injected error e_i in seconds.

d) Acoustic source position

The considered positions for the acoustic source, s , are originally defined in spherical coordinates, s_{sph} . Thus the norm, n , corresponds to the source's range in meters, whereas the azimuth, ϕ , and elevation, θ , define the angle of arrival of the received signal in degrees. Additionally, when these positions are mentioned in Cartesian coordinates throughout the document, they will be referenced as s_{cart} .

Recalling the definition of spherical coordinates, it is known that for elevations of -90° or 90° , the azimuth angle is meaningless and should not be considered. Since this system is affected by a Gaussian error, then the estimated azimuth angle is expected to return large errors not only for the absolute mentioned elevation values but for a considerable interval around it, dependent on the injected deviation. For that reason, the elevation values are limited to an interval between -80° and 80° so that the evaluated metrics present a result that is not so reflective of the errors originated from this phenomenon.

The positions to be estimated are contained in a matrix with a number of columns equal to the number of positions and three rows consisting of its spherical coordinates. The matrix is arranged so that for each defined norm, the elevation component covers the interval $[-80^\circ$ to $80^\circ]$ in steps of one and, for each elevation value, the azimuth component covers the interval $[-180^\circ, 180^\circ]$ in steps of one, forming partial spheres around the reference axis' origin.

e) Propagation speed

In all performed simulations, the considered speed of sound is $1500 m/s$, which corresponds to the underwater propagation velocity of waves in typical conditions.

Having the conditions enumerated, the logic of the algorithm occurs as follows. For every defined position of the acoustic source, s , a function that consists on the estimator is called, receiving as input the s , the positions of the hydrophone configuration, r_i , and an injected error in the . It then returns the estimated position of the source in Cartesian coordinates, $[x, y, z]$, and in spherical coordinates, $[n, \phi, \theta]$. As the position s in Cartesian corresponds to the real value that is intended to be estimated, we can also obtain the real spherical coordinates by directly converting s using the Cartesian to spherical relations in 4.33.

$$\begin{cases} n = \sqrt{x^2 + y^2 + z^2} \\ \phi = \arctan \frac{y}{x} \\ \theta = \arctan \frac{\sqrt{x^2 + y^2}}{z} \end{cases} \quad (4.33)$$

Consequently all conditions are met to analyze the achieved error in each coordinate by comparing the real position to the estimated values as 4.34, where the tested coordinates are x, y, z, n, ϕ and θ .

$$\text{error}_{\text{coordinate}} = |\text{estimated}_{\text{coordinate}} - \text{real}_{\text{coordinate}}| \quad (4.34)$$

The metrics used to evaluate the quality of the estimator were :

- **Mean squared error (MSE)**: Incorporates both the variance and the bias of the estimator, indicating its overall **quality**
- **Standard deviation of the error (σ)** : Indicates how disperse are the estimates from the expected value
- **Minimum error ($\min(e_i)$)** : Indicates the minimum error that is obtained by the estimator, thus the best absolute precision achieved

4.2.5.1 Functional analyzes of Monte Carlo TDoA estimator

Since this Monte Carlo algorithm was developed for this specific application, then it is possible to test diverse parameters of the system in order to characterize it. Therefore, a series of detailed simulations were performed using the TDoA estimator to understand the behavior, capabilities and the impact of some design decisions on the overall accuracy.

To illustrate a scenario where this estimator is applicable, we can consider that a vehicle is moving towards an acoustic signal transmitter whose position is unknown. Imagining that the target is at a **considerable** distance, then the main focus is to achieve an optimal bearing estimation which provides a more direct path and saves resources. The range estimation serves as secondary measurement that indicates how near the vehicle is from the destination, so that it is possible to make control decisions such as moderate the navigation speed in the proximity of the target. For the reasons outlined, the study that follows presents a more thorough analysis of the azimuth and elevation errors.

For the data visualization, two essential types of representation were developed :

- A 2D static representation of the error per position s . The errors to be tested are coordinates x, y, z , norm, azimuth angle or elevation angle.

- A 3D map of the obtained error. The x-axis holds the azimuth angle in degrees, the y-axis holds the elevation angle in degrees and the z-axis represents the measurement of a chosen parametric error. This allows to visualize, for a chosen configuration, how the estimation quality evolves in space.

For the first simulation, configuration A is tested for the acoustic source positions s previously described, in a total of $\boxed{21}$, with a norm equal to 10 meters. Upon simulation, it is possible to observe that the geometric symmetrical nature of the configuration leads to estimation error patterns. Accordingly, it is possible to characterize the tendency of the azimuth and elevation errors according to the range of spherical coordinates angles. Table 4.2 summarizes these relations, where the arrows indicates if the error tendency is ascendant \nearrow or descendant \searrow .

	Range	Azimuth error	Elevation error
Azimuth angle	-180° to -90°	\nearrow	\searrow
	-90° to 0°	\searrow	\nearrow
	0° to 90°	\nearrow	\searrow
	90° to 180°	\searrow	\nearrow
Elevation angle	-90° to 0°	exponential \searrow	\searrow
	0° to 90°	exponential \nearrow	\nearrow

Table 4.2: Azimuth and elevation errors slope tendency for configuration A



By analyzing the collected information, it is possible to deduce that since the configuration's geometric center is located along the x-axis, then for elevation angles increasingly closer to 0° the errors have a tendency to decrease. Moreover, adding the fact that the depth of the baseline along the x-axis is shorter than the remaining, it results in a better azimuth estimation and a worst elevation estimation near an azimuth angle of 180° or 0° . Overall, the maximum achieved errors are: for azimuth error $\approx 7.55^\circ$, for elevation error $\approx 1.41^\circ$ and for norm error $\approx 0.25\text{m}$, which corresponds to 2.5% of error in norm estimation.

For the same conditions, the maximum resulting x , y and z errors are: for $x \approx 0.28\text{m}$, for $y \approx 0.037\text{m}$ and for $z \approx 0.038\text{m}$. It should be pointed out that there is a higher error in the x coordinate, which corresponds to the direction of the shorter baseline that consequently originates a greater uncertainty.

From this experiment, it was possible to collect the standard deviation and the minimum estimation error achieved for the azimuth and elevation angles, which are present further ahead in **??**. It should be noted that the chosen configuration A for the former analysis is by no means an optimal solution for the estimation. This particular layout was selected because it emphasizes several system responses to distinct configuration characteristics, allowing to understand how should them be adapt to achieve better results.

In order to further analyze the influence of the hydrophones placement in the estimation and which design choices lead to better estimations, two more scenarios are considered to test different aspects:

- **Influence of range in estimation:** In order to compare the range influence on the estimation, a second simulation intends to test the same configuration for a norm of 100 meters. The azimuth and elevation estimations do not demonstrate any visible changes in terms of range and the norm error increases for a maximum of $2.65m$, which still corresponds to approximately 2.6% error from the original norm. However, the x, y and z estimation demonstrate an error increase of about 10 times, since a small deviation in angle can correspond to a large difference in Cartesian coordinates. Nonetheless, for increasing ranges these errors stay proportional, so the error percentage is similar in any range. This phenomenon is illustrated in 4.6, where the error in x is represented for all the tested positions that form spheres with norms equal to $10m$, $100m$ and $1000m$. As it is observed, the errors increase proportionally with the norm.

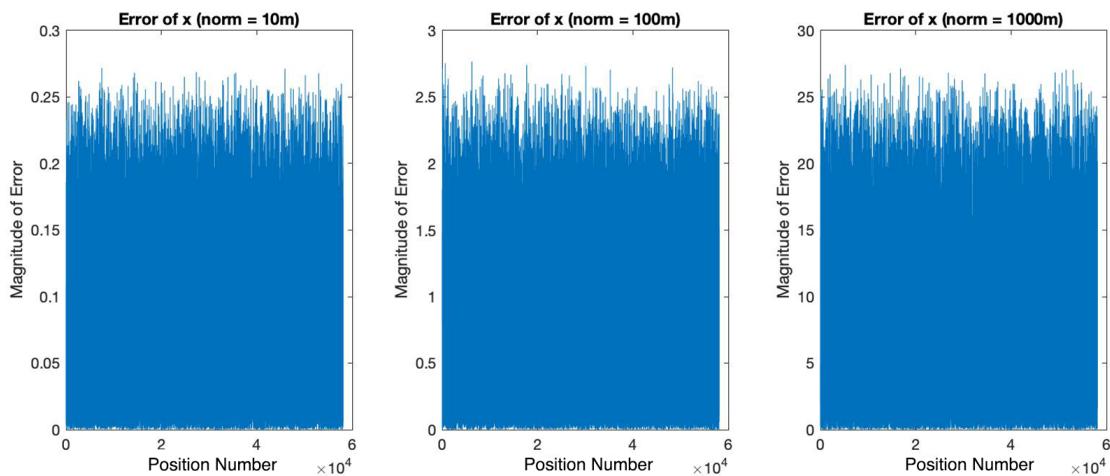


Figure 4.6: Evolution of error in Cartesian coordinates for increasing norm

- **Increasing the shorter baseline:** Since there are some issues that can be observed due to the short baseline along the x-axis, a third simulation serves to understand the influence of increasing this baseline in the estimation. Therefore, the baseline is increased by employing configuration B. After the simulation, the errors decreased visibly, which are indicated in 4.3.

After analyzing a specific configuration which demonstrate various limitation due to its arrangement design, it is desirable to compare it with different hydrophone placements. Therefore, table 4.3 contain the achieved results of MSE, standard deviation and minimum error for azimuth and elevation estimation in degrees, using norms equal to 10 and $1000 m$ for configurations A, B and C. Configuration A represents an almost flat structure, configuration B is

a variation of A with an increased depth and configuration C resembles an AUV shape. By inspection it is possible to conclude that configuration B returns the lowest standard deviation and MSE, as it is also the one with the larger baselines between sensors.

Configuration	Norm	MSE	Azimuth		Elevation	
			Standard Dev	Minimum	Standard Dev	Minimum
A	10	0.544	0.605	8.743×10^{-7}	0.186	3.103×10^{-6}
	1000	0.542	0.599	8.378×10^{-6}	0.185	5.059×10^{-6}
B	10	0.161	0.143	1.21×10^{-6}	0.050	3.456×10^{-6}
	1000	0.161	0.144	1.494×10^{-6}	0.050	4.910×10^{-7}
C	10	0.230	0.213	1.305×10^{-5}	0.067	2.790×10^{-5}
	1000	0.223	0.212	4.096×10^{-6}	0.066	4.394×10^{-6}

Table 4.3: Obtained errors for configurations A,B and C by TDoA estimator

Having explored the behavior of the estimator in specific conditions, there are still factors which were not discussed that may influence the system's performance or lead to an improvement. Therefore, four main ideas will be explored regarding the influence of the numeric quantization on the estimation accuracy, whether the absolute ToA is absolutely necessary for the position estimation, how is the estimates' dispersion for a specific position due to the injected error and the influence of increasing the baseline to the estimation accuracy.

a) Influence of numeric quantization on accuracy The first term to be analyzed is how much does the quantization of the calculations influence the obtained accuracy of the estimator. In order to analyze this, a simple adaptation was made to the numeric precision of the TDoA values that are input of the system. Instead of using the MATLAB precision of fifteen decimal places, the value was truncated to a specified number of decimal places, κ . Since the time differences of arrival have magnitudes around microseconds, then initially they are multiplied by 10^6 to avoid missing information. Then the relation 4.35 is applied resulting in a truncated value with κ decimal places. Finally after the truncation, the value is converted again to seconds to be used in the algorithm.

$$\text{truncated} = \frac{\text{round}(*2^\kappa)}{2^\kappa}; \quad (4.35)$$

To evaluate the influence of the truncation in the estimation, configuration C is used to test a norm equal to 10. The original measurements are already represented previously. For a κ of one decimal place, the azimuth standard deviation is 0.261 degrees, the elevation standard deviation is 0.087 degrees, the norm standard deviation is $0.014m$ and the MSE is 0.272. Overall, the estimation errors increase but, in practical terms, the truncation causes near to any difference in the estimation.

Since one decimal place does not bring too much discrepancy, the limit case is tested where zero decimal places are considered. In this case, the azimuth standard deviation is 0.416° , the elevation standard deviation is 0.136° , the norm standard deviation is $0.0209m$ and the MSE is 0.439. As observed, the estimations are more influenced, however they still do not compromise the reliability of the estimation.

b) Impact of ToA measurement on position estimation As previously mentioned, the estimator uses the approximation explained in 4.2.1.4 which allows all hydrophones to use the same reference ToA added by a TDoA, representing the full distance between each hydrophone and the transmitter. However, for long range positions the ToA is consequently much larger in relation to the time differences of arrival. So, the hypothesis is that for these cases the ToA is irrelevant for the calculations and the TDoAs can be used alone to estimate the angle of arrival.

In order to simulate this, the calculated reference time is substituted by $\frac{10^4 m}{1500 m/s}$, which corresponds to a distance of $10km$. The results demonstrated no visible change in the error of the estimate. However, for distances bellow $100m$ the impact of this approximation starts to be noticed. Therefore, it is considered that the hypothesis is valid.

c) Analysis of estimates' dispersion due to injected error So far, the errors that have been analyzed correspond to a random executed simulation, which is influenced by an injected error equivalent to 5° . If the exact same experiment is executed several times, although the returned values have the same magnitude, they are only similar and rarely the same. Therefore, if each estimation is repeated a defined number of times, then the estimates can be analyzed so that it is possible to extract a single mean value which translates the estimation error.

In order to simulate this mechanism, an adaption was introduced in the previous algorithm. For each transmitter position s , the estimation is reiterated a specified number of times and, in each round, the azimuth and elevation errors are accumulated. After all samples are collected for a specific position, the azimuth/elevation deviation is calculated as the standard deviation of the accumulated azimuth/elevation errors and the azimuth/elevation errors are considered to be the mean of the accumulated azimuth/elevation errors. After the measurements are executed for every position in space, it is computed the overall azimuth and elevation errors and deviations which characterize the specific chosen configuration, as well as the final MSE.

For exemplification purposes, configuration C was tested 100 consecutive times with no reiterations for each position, 4.7, and with 100 accumulated samples, 4.8 in order to verify the dispersion of the obtained results. As it is observed, when the error is averaged with accumulated samples, there is a smaller discrepancy among the results.

This approach allows to achieve more coherent results and to characterize the configurations in a more methodical manner.

d) Influence of increasing the configuration baseline Until this point, there are several mentions to the baseline of the used configuration and its numeric impact on the estimation

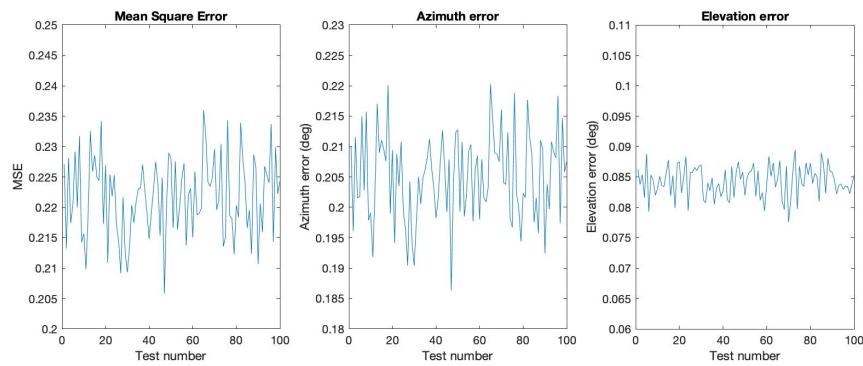


Figure 4.7: Obtained error for a single configuration with no accumulated samples

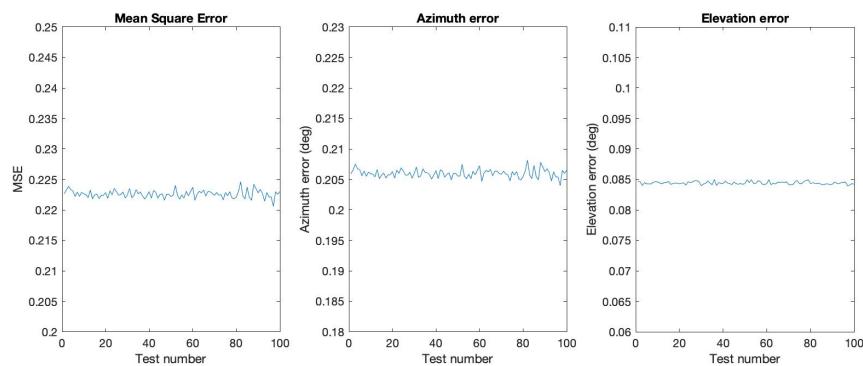


Figure 4.8: Obtained error for a single configuration with 100 accumulated samples

performance. However, there are no conclusions about the optimal baseline that should be used. The goal of this test is to delineate the influence of increasing the hydrophones' baseline in the obtained error.

In order to execute this test, configuration C is used as well as the method previously explained that reiterates each estimation a defined number of times to achieve average dispersion values. To create an increasing baseline along the tests, in each iteration the x coordinate of r_{C1} increases $0.01m$ in a total of 200 times, which makes up a displacement range between 0.1 and $2m$. Each position generates 100 accumulated samples that result in a single measurement per configuration.

Plot 4.9 represents the collected errors for each of the described configurations, whose baseline is progressively increasing in a constant pace. As illustrated, the obtained errors decrease visibly in the first 100 tests, corresponding to a r_{C1} position between 0.1 and $1m$, becoming considerably constant for further distances.

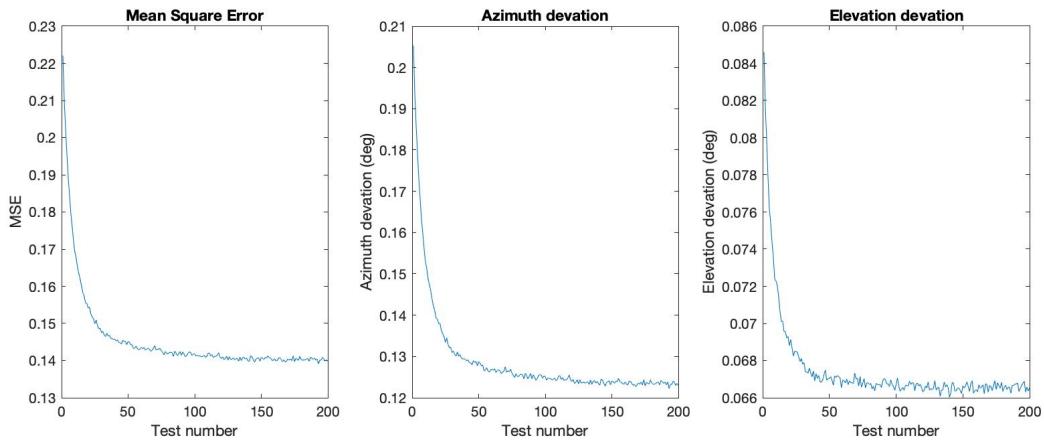


Figure 4.9: Error evolution with increasing baseline for r_{C1}

Additionally, the same experiment was done on hydrophone r_{C2} of the same structure, since its position on the configuration gets a different exposure than r_{C1} and a different outcome is expected. Having considered the same conditions as explained for the previous simulation, figure 4.10 illustrates the obtained results. It can be observed that the displacement of this specific hydrophone only causes an estimation improvement in the elevation deviation, it does not affect the estimation of the azimuth deviation and slightly increases the azimuth deviation. Therefore, an estimation improvement may not be achieved by distancing a random hydrophone, a study should be conducted for each particular configuration to determine which displacements lead to an enhancement.

In conclusion, it is proved that increasing the baseline of a configuration in specific cases may result in a decrement of the overall estimation error. However, this only occurs for a maximum distance after which the error becomes constant.

e) Conclusions on TDoA based estimator The present chapter focused on detailing the developed estimator, including the involved mechanisms and algorithm, a behavioral analysis and

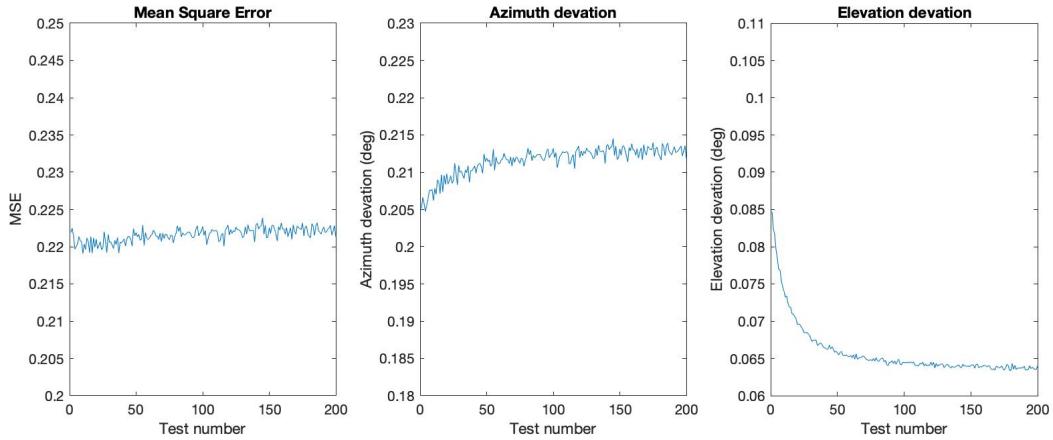


Figure 4.10: Error evolution with increasing baseline for r_{C2}

characterization based on simulated results. Therefore, by way of summary, some main conclusions can be taken:

- The azimuth and elevation errors do not vary with the range of the transmitter's position, in contrast to errors in Cartesian coordinates which increase proportionally with the range;
- The ToA measurement is not essential for long range distances. So, in those cases there is no need for synchronization since a random large ToA can be used instead;
- A lower numeric precision in the calculations affects the obtained results, however since in the explored situations the errors are inherently small, the error increase does not have an impact on the system from a practical point of view;
- Increasing the baseline of a hydrophone configuration can result in a better estimation, until reaching a certain distance after which the error becomes constant. Therefore, when choosing an hydrophone layout which can be limited to the dimensions of a physical structure, it does not have to be sought the maximum baseline possible but the length that leads to the error becoming constant;
- Applying a reiteration process to create averaged errors for each configuration, creates results which are more consistent thus more capable of characterizing a specific process. Additionally, the random nature of the Monte Carlo approach is attenuated, resulting in more coherent results.
- The hydrophone configuration is a main factor on the estimation performance for any position in space. Although the characteristics that a configuration should meet to be optimal are still not clear, some aspects can be pointed out:
 - It is mandatory to ensure a sensor layout which covers three dimensions, so it is possible to estimate coordinates in 3D;

- The positions of the sensors must be linearly independent to allow the application of the least squares method;
- It is fundamental to have an adequate baseline which can be determined with the tool previously explained;
- Bearing in mind that the configuration may be employed in real scenarios, it is useful to create schematics which require achievable distances between hydrophones and respect logical shapes to install in vehicles such as AUVs.

4.2.5.2 Functional analyzes of Monte Carlo plane wavefront estimator

In order to understand the basic behavior of the plane wavefront estimator, it was tested at first for the estimation of a single position. An injected error is considered according to the initial defined circumstances in 4.2.5, which is added to the calculated ToA to each sensor. So, a series of 1000 iterations are performed to the estimation of position $s_{cart}(0, 100, 0)$ using configuration A and the obtained estimations are represented in 4.11.

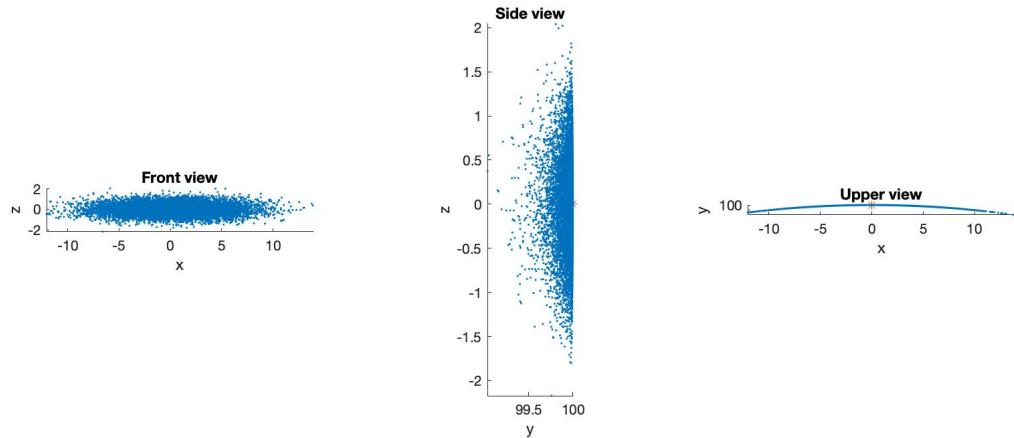


Figure 4.11: Estimation of position $s_{cart}(0, 100, 0)$ using the plane wavefront estimator with configuration A

As can be observed, the upper view reveals that the estimates are placed along a sphere surface, which corresponds to the expected result since the norm is calculated as the average ToA while the direction estimation is influenced by the error that affects the TDoAs. Additionally, the front view, which represents the view from the origin to the estimated position in the y-axis, reveals a narrow estimate cloud with larger error in azimuth. However, by inspection it is possible to conclude that this relation varies according to the position that is being estimated and is not a constant proportion.

After analyzing this specific case, a second simulation intends to achieve a more comprehensive study on the performance sensor configurations. Therefore, using a specific configuration, all s positions that form a sphere of defined norm were estimated a total of 1000 times, in order for the errors to be averaged, and the relevant data is retrieved, namely the

standard deviation and minimum error of both azimuth and elevation estimations. The results are summarized in table 4.4, where configurations A, B and C are tested for norms equal to 10 and 1000 meters.

Configuration	Norm	MSE	Azimuth		Elevation	
			Standard Dev	Minimum	Standard Dev	Minimum
A	10	0.3428	0.1725	0.2355	0.1713	0.2258
	1000	0.3305	0.1779	0.2334	0.1759	0.2301
B	10	0.3375	0.1792	0.2349	0.1739	0.2306
	1000	0.3322	0.1729	0.2336	0.1701	0.2221
C	10	0.4172	0.2420	0.3378	0.1797	0.2358
	1000	0.4093	0.2416	0.3218	0.1691	0.2314

Table 4.4: Obtained errors for configurations A,B and C by plane wavefront estimator

table interpretation

4.2.5.3 Functional analyzes of Fisher Information Matrix

In order to demonstrate the functionality of the described method, a first simulation was conducted in which a chosen configuration is tested for various positions s , returning the uncertainty sphere radius, in meters, that is obtained for each position. Similarly to what was considered before in 4.2.5, for each defined norm, the elevation component covers the interval [-90° to 90°] in steps of one and, for each elevation value, the azimuth component covers the interval [-180°, 180°] in steps of one, forming partial spheres around the reference axis' origin. The simulation was conducted using configuration C previously defined.

When estimating positions correspondent to a sphere of norm equal to 100m, the obtained results are illustrated in figure 4.12, which show the uncertainty sphere radius for each of the s positions, with respect to azimuth and elevation angles. The first plot represents the 3D map of each radius in relation to both angles, the second plot is a 2D view of the uncertainty with respect to the azimuth angle in degrees and the third plot is the 2D view of the uncertainty with respect to elevation angle in degrees.

In this specific case, it is noticeable that this configuration is capable of better estimations for low elevation angles and has an ascendant error for increasing elevation angles. Additionally, the uncertainty has a tendency to increase for azimuth angles around -90° and 90°. Overall, for a norm equal to 100m, the standard deviation of all estimations is 0.0029m and the best estimation occurred for position $s_{sph}(100, -180, 3)$ in spherical coordinates, with an uncertainty radius of 0.0532m.

This tool also makes it possible to analyze the specific uncertainty ellipsoid of a position estimate, i.e. each of the eigenvectors of the FIM, for a specific configuration. Then, resorting to the same configuration C, it is observed that for any estimated position, the eigenvector parallel to

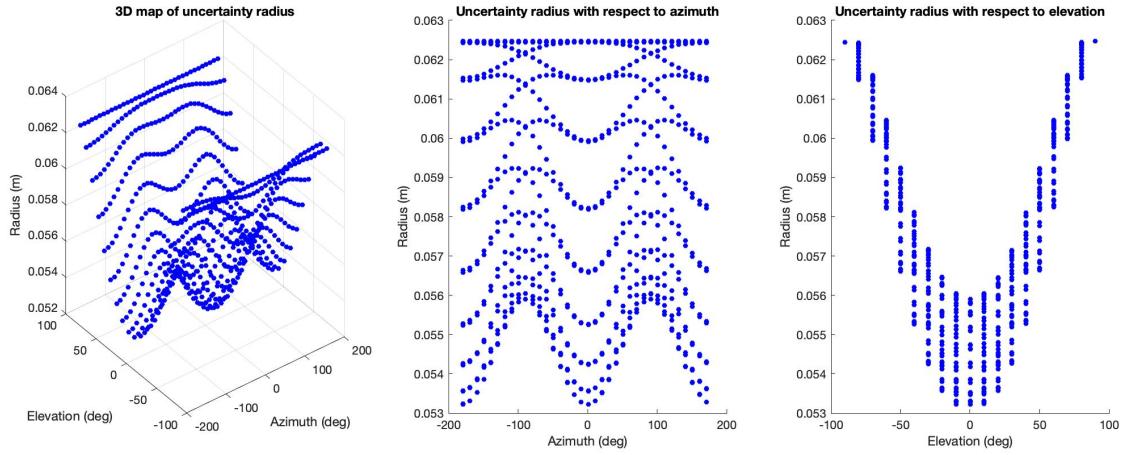


Figure 4.12: Uncertainty radius obtained by configuration C for all sphere positions

the direction of the angle of arrival always demonstrate the smaller uncertainty. This corresponds to what is expected since the injected error affects the TDoA which has a direct influence on the angle of arrival while the ToA is estimated using absolute distances. Additionally, for any estimated position it is also observed that, if it is considered a vector connecting the origin to the transmitter position, this vector forms angles with the eigenvectors that have an approximated uniform pattern: there is always two angles which are close to 90° , correspondent to errors in azimuth and elevation angles, and one angle that is close to 0° , correspondent to the norm error.

Figure 4.13 illustrates this concept for a estimation of position $s_{cart}(10, 0, 0)$. As indicated the first plot represents the side view, plane zx, and the third plot illustrates the view from above, plane yx, which illustrate to the norm error, the second plot represents the USBL front view, plane zy, which corresponds to the error in azimuth and elevation angle.

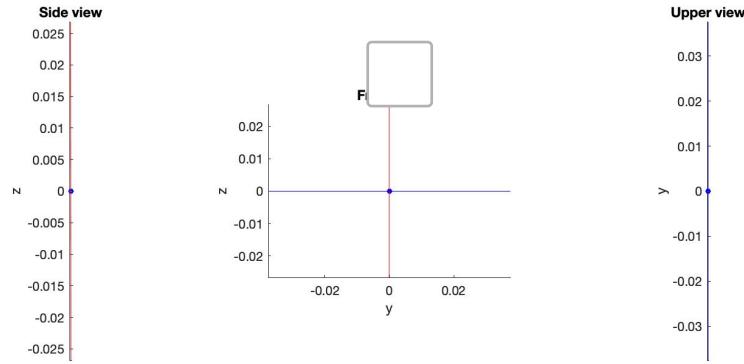


Figure 4.13: Eigenvectors obtained for configuration C when estimation position $s_{cart}(10, 0, 0)$

Finally, similarly to the previous methods a final simulation was ran on configurations A, B and C in order to evaluate their performance on the estimation of several defined positions

along a sphere with norms of 10 and 1000 meters. Since the Crámer-Rao lower bound does not contemplate the azimuth and elevation errors as an optimality criteria, then other metrics are used in this case. Table 4.5 contains the obtained data, including the criteria with the following meaning:

- **Determinant of the inverse of the FIM:** reflects the uncertainty **sphere radius**
 - Minimum: corresponds to the smallest uncertainty **sphere** radius achieved by the configuration. Represents the most accurate estimate among all estimated positions, thus an optimistic case;
 - Maximum: corresponds to the **biggest** uncertainty **sphere** radius achieved by the configuration. Represents the estimate with lower accuracy among all estimated positions, thus a pessimistic case;
 - Standard deviation: corresponds to the standard deviation of the obtained determinants of the inverse of the FIM for all estimated positions. Reflects the variance of the estimates.
- **Maximum Eigenvalue of the inverse of the FIM:** considering the uncertainty ellipsoid, which presents three uncertainty axis with distinct magnitudes, this parameter reflects the maximum uncertainty that is achieved for each position
 - Minimum: considering that each configuration obtains a maximum eigenvalue for each estimated position, this parameters corresponds to the position whose largest uncertainty is minimized.
- **Trace of the inverse of the FIM:** reflects the sum of the magnitudes of the uncertainty axis for each estimated position
 - Minimum: corresponds to the minimum of the uncertainty average variance, thus the positions that reveals a the lowest summed uncertainty magnitudes

Configuration	Norm	Determinant			Max Eigenvalue	Trace
		Min	Max	Std		
A	10	0.010	0.0200	0.003	0.053	5.6×10^{-3}
	1000	0.220	0.421	0.067	5.310	56.325
B	10	0.010	0.011	3.8×10^{-4}	0.0530	5.6×10^{-3}
	1000	0.2193	0.2462	0.008	5.306	56.283
C	10	0.012	0.014	6.3×10^{-4}	0.075	8.5×10^{-3}
	1000	0.247	0.290	0.014	7.410	84.882

Table 4.5: Obtained errors for configurations A,B and C by Crámer-Rao lower bound

4.2.6 Final remarks

This section was dedicated to the implemented hardware design for FPGA technology and a comprehensive study on evaluation tools for the performance of sensor configurations. From this, we may draw some conclusions.

compare tables 4.3, 4.4, 4.5

draw conclusions

The plane wavefront estimator demonstrates better performance for long range estimation, since the considered approximation is only valid for large distances.

For short range, the TDoA estimator demonstrates better performance than the latter, since the TDoA values, which are very precise, are more relevant for the calculation.

Overall, the Crámer-Rao lower bound is a method widely used for performance evaluation. The Fisher Information Matrix reflects the quantity of information that a configuration is capable of acquiring in relation to a specific position, which is directly associated with the accuracy of the estimation. Therefore, it is considered the most reliable mechanism for this study.

Chapter 5

Dynamic reconfigurable configuration method

Having studied the estimator's behavior for several different configurations and conditions, there is still uncertainty about the best performance it can achieve. In the considered system, the hydrophone configuration has a decisive role on the achieved precision, as demonstrated. Therefore, a way to optimize the system's estimation would be to choose, for each position of the acoustic source, the configuration that returns the best estimation and thus the one that should be employed.

In a field scenario where a vehicle is searching for an acoustic transmitter, as it is navigating and readjusting its trajectory, the relative direction that is being estimated in real time is changing. Therefore, the system's performance can vary and arises the necessity of having different angles of vision from the hydrophones to the target. To resolve this issue, the proposed method assumes that the used USBL system integrates more than four hydrophones placed in known positions. This way, it is possible to dynamically reconfigure which four hydrophones are used at a time leading to an estimation that is optimal for the available sensors.

Another possibility is to determine through the same techniques which configuration of four hydrophones, tested in various positions along the vehicle, is the overall best for short and long range estimation. This can lead to a moderate compromise of the estimate accuracy, however decreases the number sensors that are employed and therefore the cost of the system.

This chapter is dedicated to explaining the methodological approach, the main findings and conclusions that were driven from the formulated dynamic reconfigurable configuration method.

descrever mais detalhadamente os temas que vao ser abordados no capitulo

5.1 Monte Carlo Approach

The developed algorithm serves as a tool to determine which is the best available hydrophone configuration for a certain target position. This approach uses a Monte Carlo method which is useful to solve problems that are deterministic in nature through repetition and application of

random parameters. Additionally, it makes use of the developed estimator, which is comprehensively explained in 4.2.2, to estimate the target position for each sensor configuration.

For this experiment, it is considered a total of nine hydrophones, whose positions are contained in matrix_{r_i} , defined by table 5.1. Each column expresses the coordinates of each hydrophone, r_i , where the value of x_i is in the first row, the value of y_i in the second row and the value of z_i in the third row. Additionally, it is assumed $q = 0.1$, $w = 0.1$ and $e = \frac{\sqrt{2}}{2} * w$.

	r1	r2	r3	r4	r5	r6	r7	r8	r9
x	q	0	0	0	0	0	0	0	0
y	0	0	0	w	-w	e	e	-e	-e
z	0	w	-w	0	0	e	-e	e	-e

Table 5.1: Position coordinates for the implementation using 9 hydrophones

These positions are arranged so that they can mimic a possible deployment in an AUV, as represented in figure 5.1, where hydrophone r_1 is placed in front of the vehicle and hydrophones r_2 to r_9 form a circle with a 10cm radius around the vehicle.

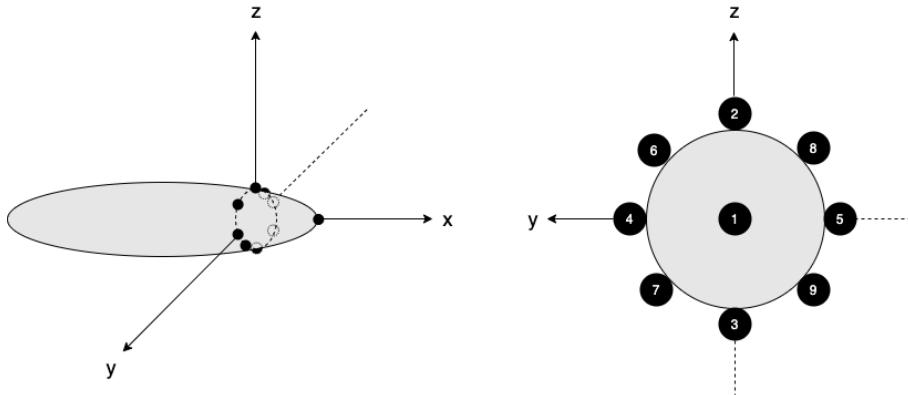


Figure 5.1: Hydrophone positions for the implementation using 9 hydrophones

Since the used configuration has to be three dimensional, it is defined that hydrophone r_1 always integrates the configuration as it is the only one that covers a third dimension. Accordingly, the number of possibilities is combinations of three out of eight, $C(8, 3)$, making up a total of 56 combinations. Each of these configurations are associated with a number from 1 to 56 and the hydrophones that integrate each of them are outlined in table A.1 for consultation when necessary.

Having the system presented, the algorithm will be explained next. For the sake of clarity, the algorithm was outlined in pseudo code and separated into two main parts, where 1 is integrated in 2.

Algorithm 1 is dedicated to computing the average azimuth error, elevation error and MSE for each of the $k = 56$ hydrophone configuration, in order to understand which of them achieves

Algorithm 1 Determines the average azimuth errors, elevation errors and MSE for a set of hydrophone configurations

```

1: for all k configurations do
2:   for all i estimation repetition do
3:     estimator( $s, config(k), error$ ) {returns estimate in Cartesian and spherical coordinates}
4:     accum_estimate(i)  $\leftarrow$  result of the estimator in each repetition
5:     accum_error_azimuth(i)  $\leftarrow$  azimuth error in each repetition
6:     accum_error_elevation(i)  $\leftarrow$  elevation error in each repetition
7:   end for
8:   mean_estimate  $\leftarrow \frac{\text{accum\_estimate}}{\text{accum\_samples}}$ 
9:
10:  deviation_azimuth(config) =  $\text{std}(\text{accum\_error\_azimuth})$ ;
11:  deviation_elevation(config) =  $\text{std}(\text{accum\_error\_elevation})$ ;
12:  error_azimuth(config) =  $\text{mean}(\text{accum\_error\_azimuth})$ ;
13:  error_elevation(config) =  $\text{mean}(\text{accum\_error\_elevation})$ ;
14:
15:  mse(config) =  $\sqrt{\text{error\_azimuth}(\text{config})^2 + \text{error\_elevation}(\text{config})^2}$ ;
16: end for

```

the minimum deviations when estimating a specific position. In order to do so, for each possible hydrophone configuration, the chosen acoustic source position s was estimated $i = 1000$ times (line 3), using the developed estimator. This includes an injected error to the TDoA that follows a Gaussian distribution with zero mean and a configurable variance of σ^2 , i.e., $e_i \sim \mathcal{N}(0, \sigma^2)$. The result of this repetition would be an estimate cloud around the absolute s position, which indicates the estimation variation achieved by a certain configuration for a specific position in space. At each stage of the repetition, the estimate is accumulated and the errors of azimuth and elevation are calculated, similarly to the process described in the accuracy analysis ?? of the estimator. Therefore, after computing all estimation repetitions, it is possible to extract four essential parameters that define the quality of the estimation for each configuration: a mean estimate (line 8); the azimuth and elevation standard deviations (lines 10 and 11); the azimuth and elevation estimation errors (lines 12 and 13); the MSE (line 15).

Finally, using the obtained parameters it is possible to determine three configurations that lead to the best estimation regarding MSE, azimuth deviation and elevation deviation. However, there are two main issues that this simple algorithm does not take into account:

1. For the considered system conditions, the error that is introduced is sufficient to originate different results every time a position s is tested with the same injected error.
2. Assuming that the hydrophone system is deployed in an AUV, it is expected that every hydrophone has a blind spot, where the acoustic source can be located. Even though the transmitted signals could still be received by these hydrophones, they would be distorted and could lead to misinformation so they should not be considered. Consequently the hydrophones that do not have line of sight to the transmitter should be disregarded as well.

In order to resolve both these problems, a second part of logic was developed, which is translated in pseudo code 2.

In order to turn this mechanism more robust and solve the first issue, it is considered that the experiment of algorithm 1 has to be reiterated a defined number of times to obtain coherent and conclusive answers. Having said this, algorithm 2 begins with a loop that reiterates $j = 10$ times the logic previously explained. Addressing the second issue, the *line_of_sight* function (line 7) is called, serving as filter to determine which hydrophones have line of sight to the estimated position. The mathematical definitions and conditions included in this function as well as the inputs and outputs are better clarified in the next subsection 5.1.1. Thereafter, all configurations that have full line of sight to the transmitter are extracted. Meanwhile, the azimuth deviations, elevation deviations and MSE are accumulated in each experiment reiteration (line 8 to 10) so that it is possible to obtain the definitive mean of these parameters for each configuration (line 12 to 14).

Algorithm 2 Determines the overall best configuration for a specific position estimation considering:

- multiple full experiments (Algorithm 1);
 - only hydrophones with line of sight to the target.
-

```

1: for all  $j$  experiment reiterations do
2:   *****
3:   *** INSERT ALGORITHM 1 ***
4:   *****
5:
6:   line_of_sight(mean_estimate, matrixri) {returns which hydrophones have line of sight to the target}
7:
8:   reit_mse = reit_mse + mse
9:   reit_dev_azimuth = reit_error_azimuth + deviation_azimuth
10:  reit_dev_elevation = reit_error_elevation + deviation_elevation
11: end for
12: mean_MSE = reit_mse ÷  $j$ 
13: mean_dev_azimuth = reit_dev_azimuth ÷  $j$ 
14: mean_dev_elevation = reit_dev_elevation ÷  $j$ 
15:
16: Extract the configurations that contain only hydrophones with line of sight
17: Form matrix with errors of only the configurations with full line of sight
18:
19: [best_config_for_mse, overall_min_mse] = min(overall_mse)
20: [best_config_for_azimuth, overall_min_dev_azimuth] = min(overall_dev_azimuth)
21: [best_config_for_elevation, overall_min_dev_elevation] = min(overall_dev_elevation)

```

At this stage, it is possible to know already which are the configurations that are considered to achieve the minimum errors in each j reiteration and how many times each of them are chosen. However, these are still not filtered, thus they can contain hydrophones that have not line of sight to the acoustic source. Consequently, the next step is to extract only the configuration with full LOS and form three matrices with the azimuth deviations, elevation deviations and MSE of these configurations. Having the final parameters calculated and filtered, the overall best configurations for each of the chosen parameters are given by the minimum of the matrices that contain said parameter (line 19 to 21), i.e. for all configurations with full LOS:

- The minimum obtained MSE corresponds to the configuration that more precisely estimates the position s in terms of MSE;
- The minimum obtained azimuth deviation corresponds to the configuration that more precisely estimates the position s in terms of azimuth;
- The minimum obtained elevation deviation corresponds to the configuration that more precisely estimates the position s in terms of elevation.

Additionally, it is possible to obtain the best configuration in terms of azimuth and elevation simultaneously, by computing the mean between the deviation of azimuth and elevation in each of the selected configurations. The minimum value obtained corresponds to the configuration which achieve the minimum the deviation in both parameters simultaneously.

5.1.1 Line of sight definition

As briefly explained before, when considering a set of hydrophones placed in the surface of an AUV, there will be blind regions for each of the hydrophones. Nonetheless, when an acoustic source is positioned in a blind region of an hydrophone, it still can receive a transmitted signal through reflections on path objects or reverberation in the AUV's surface. Since these signals would be distorted from the original, they could lead to misinformation after the processing if they were to be considered. For this reason, it is essential to exclusively consider configurations whose hydrophones have line of sight (LOS) to the transmitter.

In the present application, this feature is executed through function *line_of_sight* of algorithm 2, which outputs a vector containing all the hydrophones that have line of sight to the inputted transmitter position, *mean_estimate*, from the considered set *matrix_{r_i}*.

In order to define which hydrophones have LOS to a specific position in space, a region was defined for each hydrophone as its LOS region, *ls_i*. Thus, three simplifications were initially considered:

- The model for the vehicle is an approximation to a typical shape of an AUV using geometric shapes, as represented in 5.2, composed by a cylinder as the body with $2 * w$ of diameter and a cone in the front with height equal to q ;

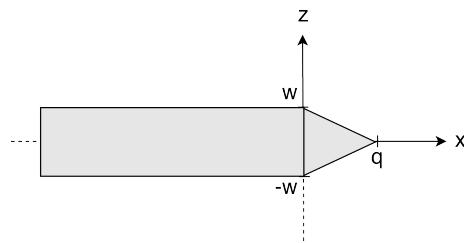


Figure 5.2: Model of AUV used to calculate the LOS region

- The regions for a $x \leq 0$ are defined as if the hydrophones were flat in the vehicle's surface, which leads to a simplified definition of the LOS region;

- Since hydrophone r_1 is integrated in every configuration, there is no necessity of defining its LOS region.

Having these relations into account, the LOS regions are then defined separately for $x \leq 0$ and $x > 0$. When $x \leq 0$, the second simplification previously mentioned is applied so all ls_i are defined as the region greater/less or equal than the tangential to the position of hydrophone i in plane yz . These tangential equations are defined in 5.2 to 5.8.

$$ls_2 \leftarrow x \leq 0 \wedge z > r_{2_z} \quad (5.1)$$

$$ls_3 \leftarrow x \leq 0 \wedge z < r_{3_z} \quad (5.2)$$

$$ls_4 \leftarrow x \leq 0 \wedge y > r_{4_y} \quad (5.3)$$

$$ls_5 \leftarrow x \leq 0 \wedge y < r_{5_y} \quad (5.4)$$

$$ls_6 \leftarrow x \leq 0 \wedge z \geq -y + w\sqrt{2} \quad (5.5)$$

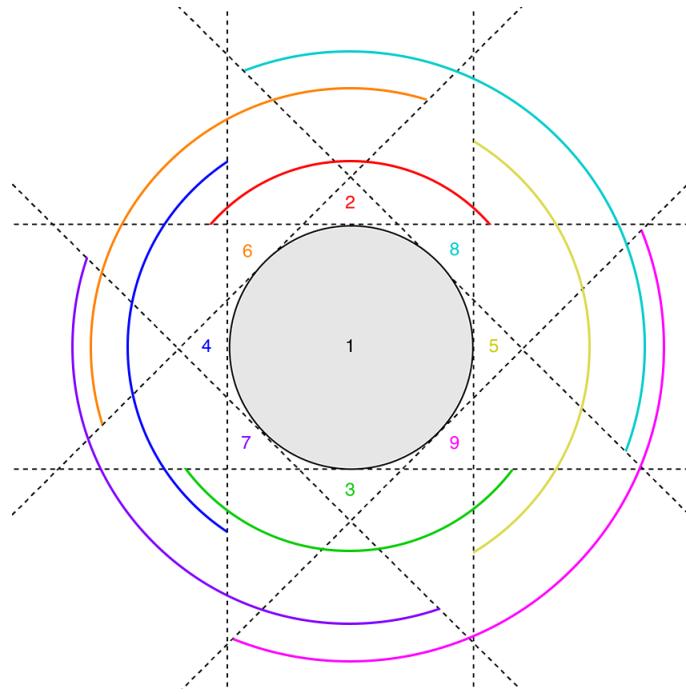
$$ls_7 \leftarrow x \leq 0 \wedge z \leq y - w\sqrt{2} \quad (5.6)$$

$$ls_8 \leftarrow x \leq 0 \wedge z \geq y + w\sqrt{2} \quad (5.7)$$

$$ls_9 \leftarrow x \leq 0 \wedge z \leq -y - w\sqrt{2} \quad (5.8)$$

By evaluating these equations, it is possible to infer that the LOS regions for $x \leq 0$ intersect each other, as illustrated in figure 5.3. The projection is correspondent to the yz plane with an inverted y -axis, in accordance with the previously presented model of the USBL system. Additionally, each colored line corresponds to the LOS region covered by the hydrophone with the same color. For instance, if a transmitter is located in $s_{cart}(-10, -10, -10)$, by analysis of the schematic it is observable that this position is covered by ls_3 , ls_5 and ls_9 , thus in line of sight of hydrophones 3,5 and 9.

Analogously, when $x > 0$, all ls_i are defined as the region greater/less or equal than the tangential to the position of hydrophone i in planes xz or xy , depending on the hydrophone's location. Dealing with the hydrophones positioned in the y and z -axis, r_2 , r_3 , r_4 and r_5 , it is possible to directly formulate equations that help defining ls_2 , ls_3 , ls_4 and ls_5 since the tangent plane to these hydrophones is perpendicular to referential planes. However, when considering hydrophones r_6 , r_7 , r_8 and r_9 , which are not positioned in any referential axis, the tangential plane which passes through the hydrophone and the front limit of the cone is not parallel to any of the referential planes xy , xz or yz . Therefore, the equations that could define these planes require a rotation of the referential axis. In order to counter this issue, since the shape of the model projected in the yz plane is a circle, if hydrophones r_6 , r_7 , r_8 and r_9 are rotated a 45° angle around the x -axis, they can become coincident with the positions of r_2 , r_4 , r_5 and r_3 respectively. Additionally, when considering this rotation, the position of the transmitter would also be rotated the same angle to maintain their relative positions. Overall, the used equations are defined by relations 5.10 to 5.13, where the regarded x, y, z coordinates are the original transmitter position for ls_2 , ls_3 , ls_4 , ls_5 , and the rotated transmitter position for ls_6 , ls_7 , ls_8 , ls_9 .

Figure 5.3: Line of sight regions in plane yz for $x < 0$

$$ls_2, ls_6 \leftarrow x > 0 \wedge z \geq -\frac{w}{q}x + w \quad (5.9)$$

$$ls_3, ls_9 \leftarrow x > 0 \wedge z \leq \frac{w}{q}x - w \quad (5.10)$$

$$ls_4, ls_7 \leftarrow x > 0 \wedge y \geq -\frac{w}{q}x + w \quad (5.11)$$

$$ls_5, ls_8 \leftarrow x > 0 \wedge y \leq \frac{w}{q}x - w \quad (5.12)$$

$$(5.13)$$

After evaluating these equations and the limiting planes that they form, the projected regions of LOS for r_2 , r_3 , r_4 and r_5 are illustrated in 5.4, where their intersection is evident. The regions ls_2 , ls_3 , ls_4 and ls_5 are naturally similar to these, with an additional rotation of 45° around the x-axis.

In summary, it is possible to deduce that there is a larger quantity of different configurations that cover the space for $x > 0$ than for $x \leq 0$ due to the model of the AUV and the considerations described in the course of this subsection. Therefore it is expected a better estimation for positions of the acoustic source with an azimuth angle between -90° and 90° .

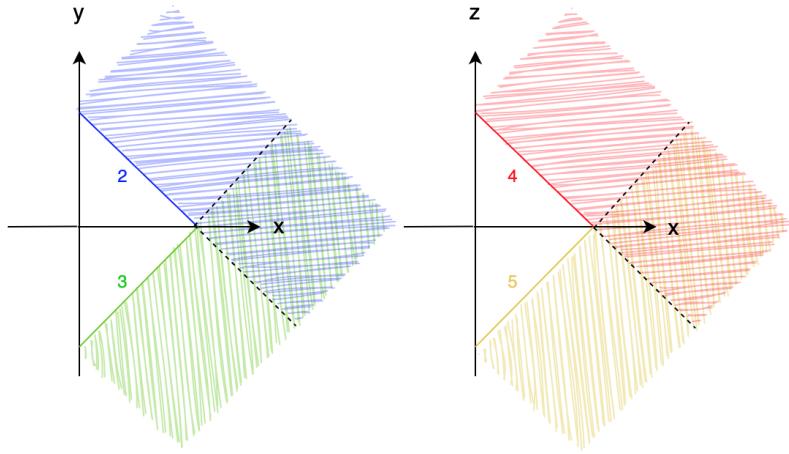


Figure 5.4: Line of sight regions in plane yx and zx for $x \geq 0$

5.2 Comprehensive study of geometric configurations performance

For the purpose of demonstrating the functionality of the implemented system, various acoustic source positions were tested and the results analyzed.

The first demonstration evaluates the achieved MSE, azimuth deviation and elevation deviation for $s_{cart}(10, 10, 10)$. After simulating, plot 5.5 demonstrate the results, where the obtained parametric errors are illustrated for each of the 56 formulated configurations. Additionally, the configurations which are marked with a red circle correspond to those whose hydrophones all have full line of sight to the transmitter and the green star indicates which is the configuration that achieves the lowest error. The results of this experiment are aggregated in table 5.2. For the present case, the hydrophones considered to have line of sight to the target are r_2 , r_4 , r_6 , r_7 and r_8 . The configuration with lowest MSE and azimuth deviation is number 8, composed by hydrophones r_1 , r_2 , r_4 and r_6 , which are the directly closer to the target. The configuration that originates lowest elevation deviation is number 19, composed by r_1 , r_2 , r_7 and r_8 , which maximizes the baseline of the sensors with line of sight.

Additionally, if the azimuth and elevation deviations are evaluated simultaneously, then the obtained plots can be overlaid to a clear comparison between results. Figure 5.6 represents this situation where, as the label indicates, the azimuth deviation in degrees is represented by the blue line, the pink line is the elevation deviation in degrees, the red and green circles represent the configurations with line of sight to the target and the green star indicates, once again, the configurations which achieve the minimum deviations in azimuth and elevation. Additionally, the cyan diamond reveals which of the configurations leads to the minimum mean between the azimuth and the elevation deviations. In this case, this corresponds to configuration number 53 which reaches a mean deviation of 0.385 degrees.

In a second experiment, position $s_{cart}(100, 100, 100)$ is tested, which has the same azimuth and elevation angles as the previous position and a norm 10 times larger. For this case, the error range of azimuth and elevation estimation is approximately the same and the norm estimation presents

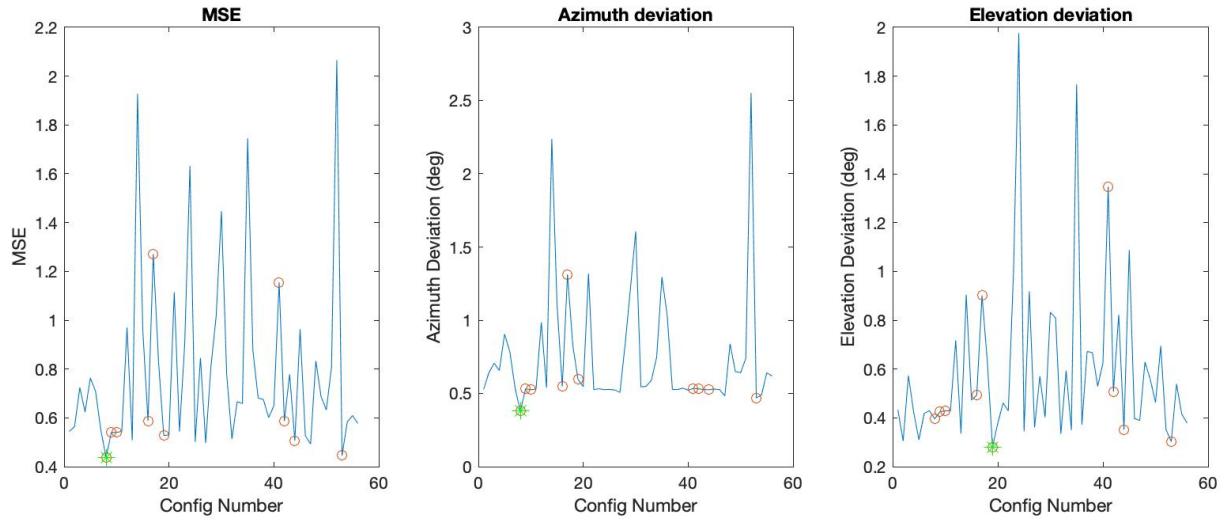


Figure 5.5: Errors obtained for all configurations when estimating position $s_{cart}(10, 10, 10)$

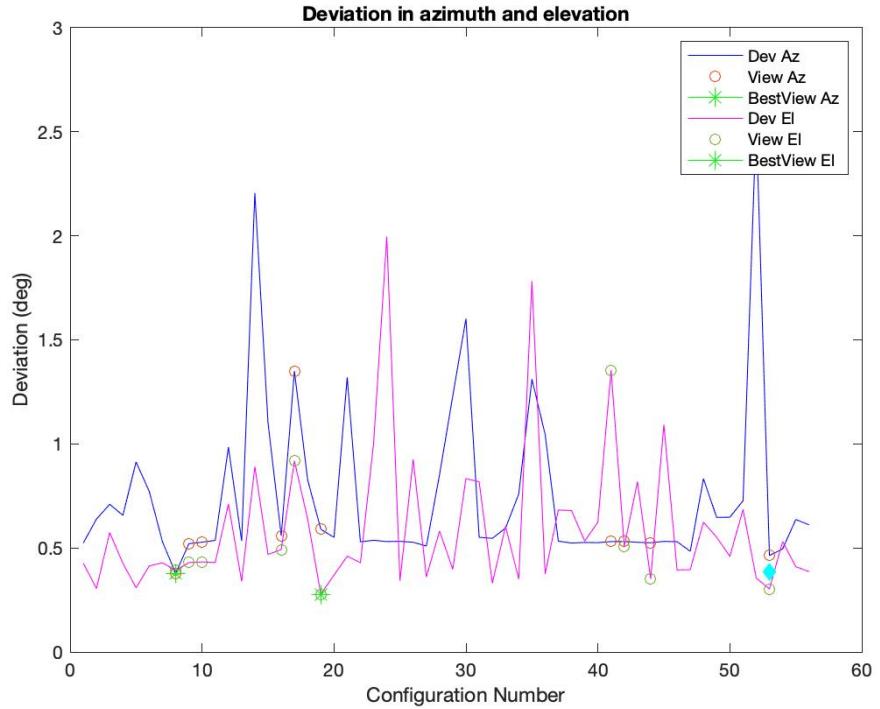


Figure 5.6: Overlaid azimuth and elevation deviations for all configurations when estimating position $s_{cart}(10, 10, 10)$

an error around 10 times larger. Additionally, as expected the configurations that lead to the lowest errors are the same as for the first experiment.

A set of tests were executed in order to compare the performance of various positions estimation and to extract behavior patterns for the chosen configurations. The collected data is present in table 5.2, where the position of the transmitter are given in Cartesian coordinates by (s_x , s_y , s_z) and for each of the performance parameters it is presented the obtained best configuration number and the minimum error achieved by it. It should also be noted that the notation 0_+ / 0_- indicates that for a theoretical position with a coordinate equal to zero it was returned a slightly positive/negative estimate of that same coordinate, due to the injected error.

s_{cart}			Azimuth		Elevation		Azimuth+Elevation		MSE	
x	y	z	Config	Min	Config	Min	Config	Min	Config	Min
10	10	10	8	0.377	19	0.278	53	0.385	8	0.434
-10	-10	-10	30	2.412	30	1.196	30	1.804	30	2.142
0_+	-10	-10	14	2.058	14	0.431	14	1.244	14	1.672
100	0_-	0_-	47	0.177	32	0.177	11	0.284	11	0.330
0_+	100	0_+	9	0.596	16	0.425	16	0.512	16	0.586
0_+	0_-	100	37	52.572	39	0.381	37	26.479	39	32.602
15	-10	5	48	0.364	3	0.218	27	0.346	27	0.399

Table 5.2: Summary of best configurations obtained by Monte Carlo simulation

Some conclusions can be taken upon the analysis of these results:

- Configurations that achieve the overall minimum errors tend to integrate the LOS hydrophones that attain the larger baseline between them;
- Transmitter positions that return a higher number of LOS hydrophones, have more possible configurations to choose from and therefore can be more optimized, presenting the lowest errors;
- The points directly behind the AUV that cover positions with y and z in the range $[-w, w]$, are not contained in any line of sight region;
- When the transmitter positions have $x \leq 0$, they are in line of sight of less hydrophones than for $x > 0$, so there are fewer possible configurations and the returned best option for the various parameters are more consistent. Consequently, the returned minimum errors are usually higher than for $x > 0$;
- Transmitter positions that are located near the y-axis, almost in parallel with the circle of hydrophones, are usually covered by only four LOS regions, which correspond to the sensors that are directly closer to the target;

- For elevation angles close to 90° and -90° , the azimuth deviation is much larger than at any other position as expected, since in that region the azimuth angle is more influenced by the injected errors leading to increased deviations.

5.2.0.1 Evaluation of the line of sight regions

Upon inspecting the overall obtained results and the whole function of the system, a particular analysis is conducted to the mechanism that defines the hydrophones' LOS regions. Therefore, after evaluating the position $s(10, 10, 10)$ that is expected to have an intermediate amount of hydrophones, two additional positions are tested.

The first position to be tested is $s(-10, -10, -10)$ which inherently has less hydrophones with line of sight, due to the method's limitation for $x \leq 0$. Additionally, since it is located in the seventh octant it is expected to be only in LOS with exactly three hydrophones, thus having only one available configuration with line of sight. After simulating, figure 5.7 is obtained which verifies the concepts described. The only configuration with line of sight is number 30, composed by r_1, r_3, r_5 and r_9 .

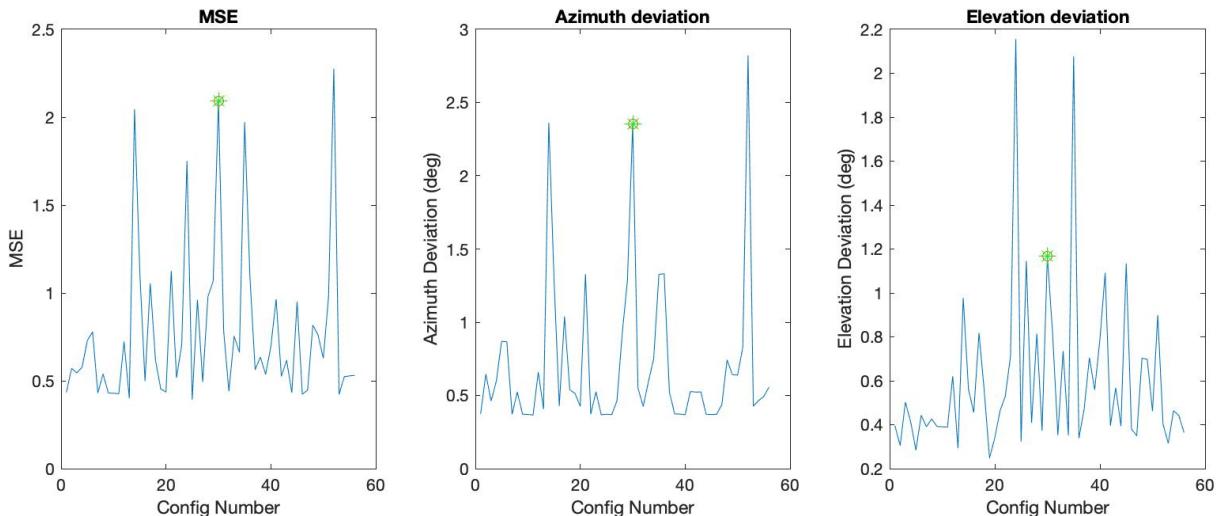


Figure 5.7: Errors obtained for all configurations when estimating position $s_{cart}(-10, -10, -10)$

Then, position $s(100, 0, 0)$ is tested. Due to its long range location and the fact that it is assumed a conic shape at the front of the vehicle, it is expected that this point has line of sight for every employed hydrophone. Figure 5.8 illustrates the simulated results which prove the theoretical expectation.

Lastly, an extensive verification process was held in order to ensure that the returned LOS hydrophones were plausible for many other positions. Table 5.3 summarizes this search, containing a total of eleven positions spread over each octant and the reference axis. By consulting the hydrophone map illustrated in 5.1, it is possible to confirm that for every selected position the returned hydrophones are the logically expected.

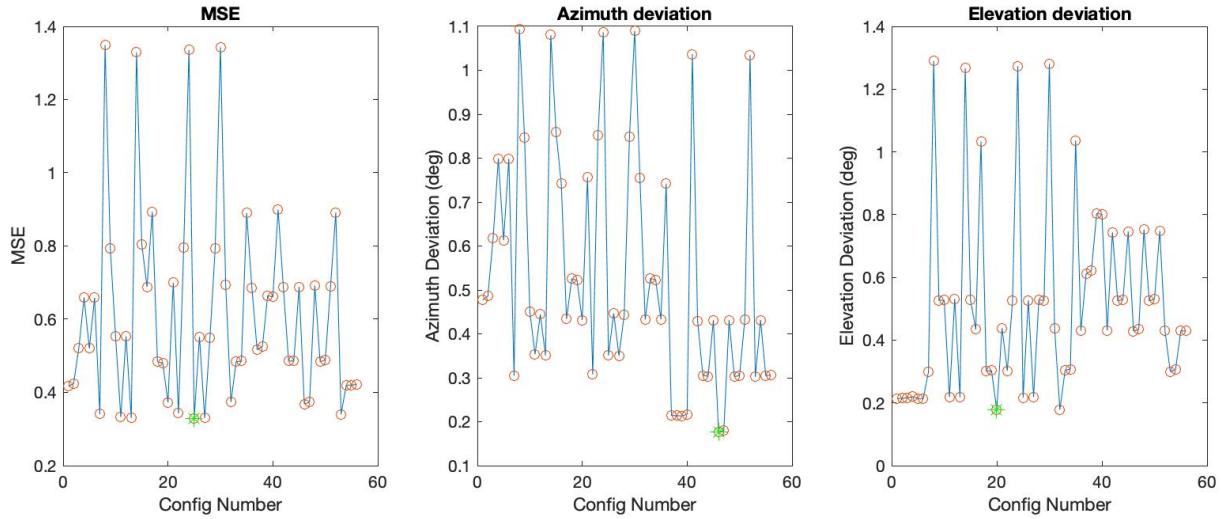


Figure 5.8: Errors obtained for all configurations when estimating position (100,0,0)

	s_x	s_y	s_z	Hydrophones with LOS
x-axis	100	0	0	2, 3, 4, 5, 6, 7, 8, 9
y-axis	0	100	0	4, 6, 7
z-axis	0	0	100	2, 6, 8
1 st octant	10	10	10	2, 4, 6, 7, 8
2 nd octant	5	5	-1	2, 3, 4, 6, 7
3 rd octant	1	-1	0	2, 3, 5, 8, 9
4 th octant	1	-10	-10	3, 5, 7, 8, 9
5 th octant	-100	10	10	2, 4, 6
6 th octant	-1	10	-5	3, 4, 6, 7
7 th octant	-1	-1	1	2, 5, 8
8 th octant	-1	-100	-10	3, 5, 8, 9

Table 5.3: Hydrophones with line of sight for several s positions

5.2.1 Comparison with Crámer-Rao lower bound

Although the Monte Carlo approach and the Crámer-Rao bound method use distinct decision metrics, it is still worth to understand the practical differences between the implementations regarding the developed dynamic reconfigurable method.

A simulation was performed in which position $s_{cart}(0, 1000, 0)$ is estimated by both methods, obtaining the estimate cloud by the Monte Carlo approach and the uncertainty axis in Crámer-Rao bound. Figure 5.9 illustrates both data overlaid, using configuration C.

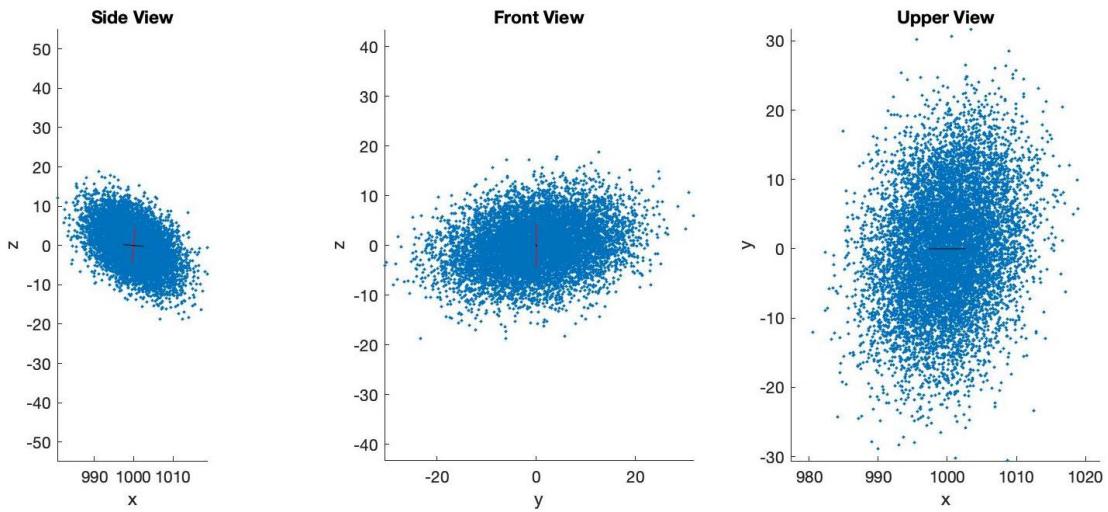


Figure 5.9: Errors obtained for all configurations when estimating position (100,0,0)

The plot reveals that the results are clearly not coherent, which is justified by the fact that the Crámer-Rao bound assumes any linear and unbiased estimator for the configuration's evaluation, while the Monte Carlo method is a non-linear estimator, due to the least squares equation that leads to the estimation in 4.2.2.

Therefore, to avoid lack of generalization, once more it is considered that the preferable mechanism to be used for analyzing the estimation accuracy is the FIM along with the desired optimality criteria.

5.3 Optimal solution based on range

After having a comprehensive understanding on the system that is capable of determining which is the best configuration of hydrophones to estimate a specific transmitter position, an alternate scenario can be explored. We now consider that the USBL system does not reconfigure which sensor configuration it should use in order to optimize the estimation in real time. Instead, a performance study is conducted beforehand so that only two configurations are chosen to be deployed, corresponding to the ones which return the average best estimation of short and long range positions. This way, it would be possible to alternate which configuration is used

depending solely on the distance to the target. Alternatively, it is also possible to define just one overall best configuration, in cases where there is the necessity of reducing the number of deployed hydrophones or if the two obtained configurations do not have a significant difference in performance.

In the development of this mechanism, there are some adjustments and similarities from the conditions established for the previous system. Firstly, it is assumed that the hydrophones are implemented in a structure shaped like an AUV with no physical form, thus all hydrophone positions have line of sight to any point in space. Additionally, the possibilities for the hydrophone locations are increased for a more extensive study. Once again, hydrophone r_1 is integrated in all configurations and a total of 24 possibilities are considered for the remnant three. These include the same 8 positions r_2 to r_9 , established for the initial setup, to which are added positions r_{10} to r_{25} defined in table 5.4. This addition corresponds to two replicated circles from the initial one, that assume the same y and z coordinates but are deviated from each other a d_x equal to 20 centimeters. Figure 5.10 illustrates all the considered possible positions for the hydrophone's deployment. Accordingly, the number of possibilities is not correspondent to combinations of three out of 24, $C(24, 3)$, since these contain linearly dependent configurations, which correspond to hydrophones' positions that only differ in the x coordinate. Therefore, the total number of available combinations is 1512.

	r_{10}	r_{11}	r_{12}	r_{13}	r_{14}	r_{15}	r_{16}	r_{17}	r_{18}	r_{19}	r_{20}	r_{21}	r_{22}	r_{23}	r_{24}	r_{25}
x	0	0	0	0	0	0	0	0	d_x							
y	0	0	w	-w	e	e	-e	-e	0	0	w	-w	e	e	-e	-e
z	w	-w	0	0	e	-e	e	-e	w	-w	0	0	e	-e	e	-e

Table 5.4: Additional coordinates for an implementation with 25 hydrophones

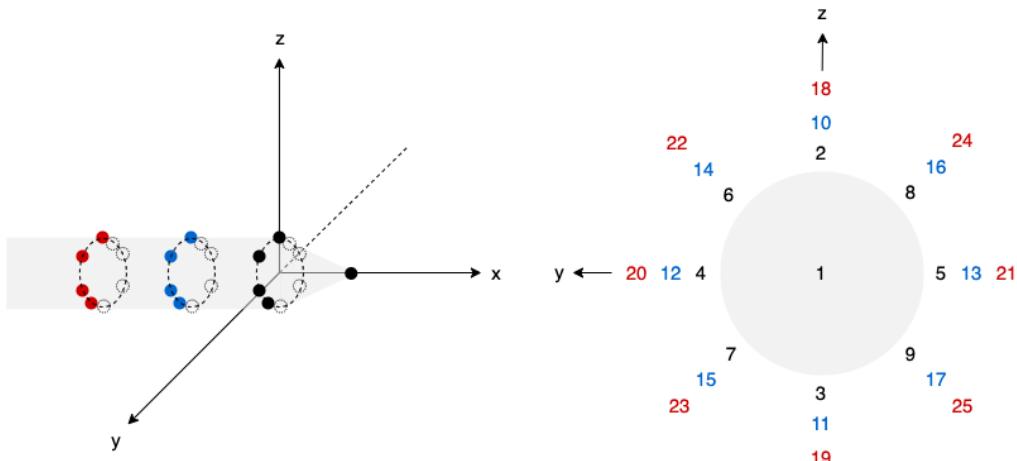


Figure 5.10: Hydrophone possible positions for optimality study based on range

The applied algorithm is similar to the previously explained algorithms 1 and 2, with the following variations:

1. Similarly to the condition described in 4.2.5, the acoustic source positions, s , to be estimated are defined in spherical coordinates, where for each defined norm, the elevation component covers the interval $[-90^\circ \text{ to } 90^\circ]$ in steps of one and, for each elevation value, the azimuth component covers the interval $[-180^\circ, 180^\circ]$ in steps of one, forming partial spheres around the reference axis' origin. The norm covers a range between 1 and 10000 meters.
2. An additional logical loop is added in algorithm 1 between lines 1 and 2, so that for each possible configuration all s are estimated a number of $i = 1000$ iterations.
3. the *line_of_sight* function is no longer used, as it is considered that all hydrophones have line of sight to every position s .

From here it is possible to analyze which are the best returned configurations, according to the chosen parameters MSE, azimuth deviation, elevation deviation and combined azimuth and elevation deviations. According to the numeric errors that are obtained, it is then possible to decide whether there is a single configuration that has evidently a better performance than the others or if there is a pair of configurations that can be used to obtain individually better estimations for short and long range.

5.3.1 Simulation results

The previously defined method was tested through a series of simulations, in which all 1512 defined configurations repeatedly estimate all positions that compose spheres of defined norms. Then, the configurations that reveals lower estimation errors are selected as the most accurate in terms of azimuth angle, elevation angle, both angles simultaneously or MSE.

The overall results do not demonstrate a specific configuration to be the clear best in short or long range for the chosen parameters. In order to associate the obtained configuration numbers to the actual hydrophone placement, figure 5.11 gathers the most relevant representations for the simulations based on range.

For performance based on azimuth, configuration number 792 shows best results for norms between 1 and 10 meters. However, for longer range the results do not tends to a specific configuration, alternating between three options.

Regarding performance based on elevation, configuration 220 is indicated for 1 to 3 meters, while for ranges going from 10 to 10000 meters, configuration 260 is the clear preferred. As can be observed in the illustration, these are structurally similar.

When evaluating azimuth and elevation accuracy simultaneously or MSE, the results are not clear and many configurations are alternated, where number 827 is the most frequent.

Additionally, it should be noted that throughout all simulated norms, for each evaluated metrics the worse performance for every position corresponded to the same configurations.

	Azimuth		Elevation		Azimuth+Elevation		MSE	
Norm	Config	Min	Config	Min	Config	Min	Config	Min
1	792	0.4730	220	0.1758	862	0.3554	862	0.4654
2	792	0.4771	220	0.1738	827	0.3905	827	0.4549
3	792	0.4820	220	0.1752	827	0.3921	860	0.4571
10	792	0.4796	260	0.1702	827	0.3918	827	0.4572
20	827	0.4893	260	0.1702	827	0.3905	827	0.4537
50	792	0.4940	220	0.1705	860	0.3947	860	0.4615
100	860	0.4881	260	0.1707	827	0.3913	860	0.4538
200	827	0.4853	260	0.1706	827	0.3889	827	0.4511
500	792	0.4886	260	0.1722	929	0.3935	862	0.4606
1000	827	0.4846	260	0.1709	827	0.3899	827	0.4524
10000	792	0.4810	260	0.1724	862	0.3920	827	0.4554

Table 5.5: Results of Monte Carlo simulation for range based estimation

To conclude, the obtained data do not show a clear tendency of two specific configurations being overall best for short and long range. This can be a result of the Monte Carlo simulation nature which originates results that do not have a clear sequence as expected. Therefore, this same experiment is repeated in the next subsection using the Fisher Information Matrix integrated in the Crámer-Rao bound method.

5.3.2 FIM simulation

Since the Fisher Information Matrix is a reliable and well known tool for evaluating a configuration's performance, then a simulation was executed for determining the best configuration for short and long range estimations. Therefore, the method was integrated in a Monte Carlo approach that tests all 1512 configurations defined previously described in this section, which integrate a total of 25 different hydrophones. For each configuration, all s positions that make up a sphere with a defined norm are used to calculate the FIM and evaluate the desired criteria. Among all, the configurations that demonstrate the preferred results correspond to the best performance in each specific criteria.

Table 5.6 summarizes the obtained results for a norm range between 1 and 10000 meters. A standard metric and three optimality criteria were observed and for each of them, the table indicates the configuration number that proved to have the best performance and the respective achieved values:

- In E-optimality, the Eigenvalue column contains the minimum larger eigenvalue found among all configurations

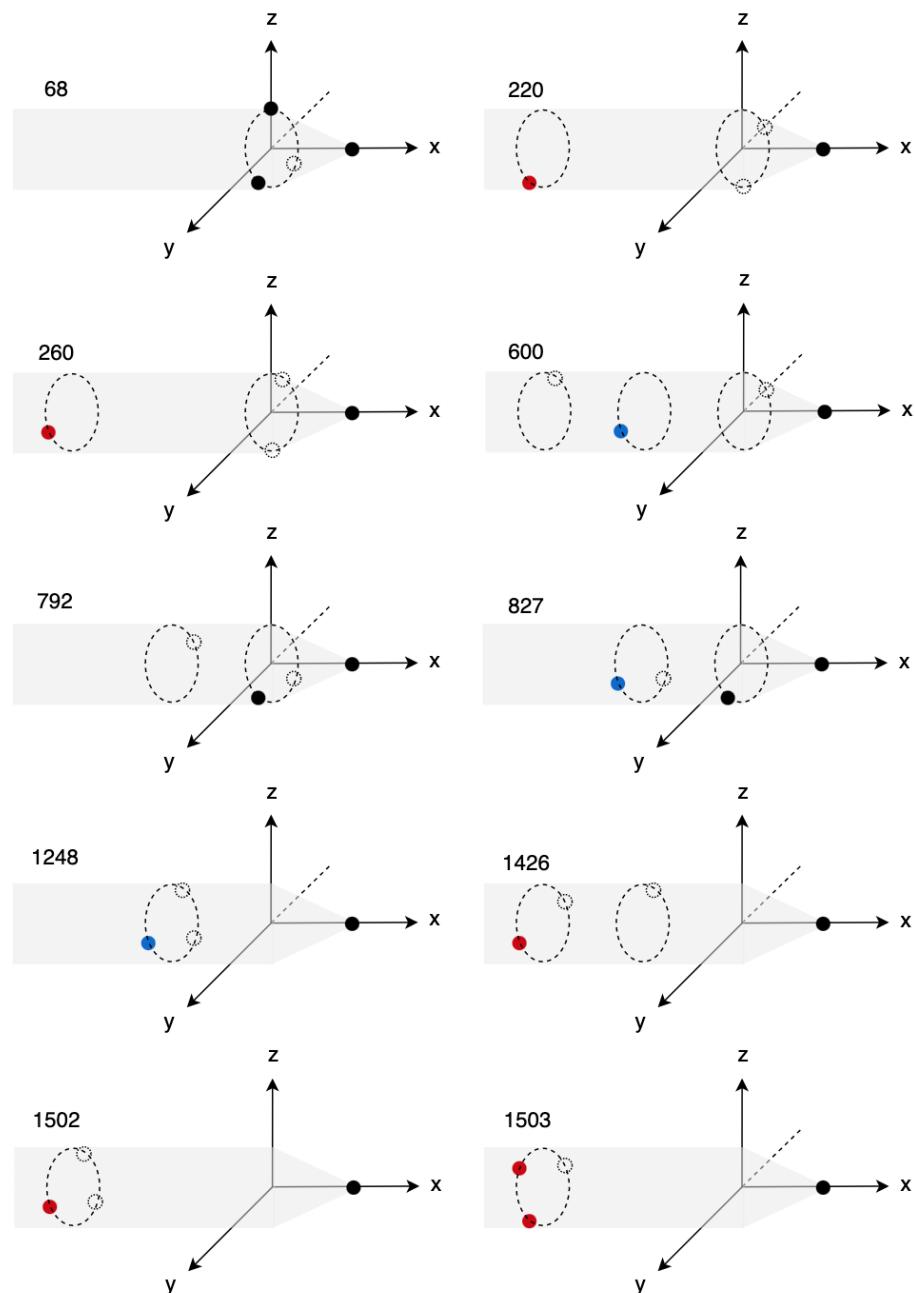


Figure 5.11: Illustration of relevant hydrophone configurations for range based estimation

- In D-optimality, the Determinant column includes the minimum computed determinant of the inverse of the FIM for all configurations
- In A-optimality, the Trace column comprises the minimum sum of eigenvalues of the FIM among all configurations

The overall results do not demonstrate a specific configuration to be the clear best in short or long range for the chosen parameters. In order to associate the obtained configuration numbers

Norm	E-optimality		D-Optimality		A-Optimality	
	Config	Eigenvalue	Config	Determinant	Config	Trace
1	1248	6.2×10^{-3}	1503	1.6×10^{-3}	600	2.3×10^{-5}
2	1248	0.012	1503	2.5×10^{-3}	1426	9.2×10^{-5}
3	1248	0.018	1503	3.3×10^{-3}	1426	2.1×10^{-4}
10	1502	0.061	1503	7.4×10^{-3}	1426	2.4×10^{-3}
20	1502	0.121	1503	0.012	1426	9.6×10^{-3}
50	1502	0.303	1503	0.022	1426	0.061
100	1502	0.061	1503	0.034	1426	0.242
200	1502	1.212	1503	0.054	1426	0.968
500	1502	3.029	1503	0.100	1426	6.055
1000	1502	6.059	1503	0.159	1426	24.226
10000	1502	60.587	1503	0.737	1426	2.42×10^3

Table 5.6: Results of FIM simulation for range based estimation

to the actual hydrophone placement, figure 5.11 gathers the most relevant representations for the simulations based on range.

By inspection, it is possible to understand that for each criterion there is a clear best configuration for short and long range. The hydrophone placement of the mentioned configuration numbers are illustrated in figure 5.11 for a more practical interpretation.

Therefore, in terms of E-optimality configuration, an approximately constant error of 0.6% is visible, where number 1248 shows the best performance for range between 1 and 3 meters, while from 10 to 10000 meters configuration 1502 is considered the best.

For D-optimality, a single configuration, number 1503, was considered the best for all defined ranges, which corresponds to an hydrophone placement that maximizes the baseline between the fixed sensor and the other three.

Finally, in terms of A-optimality configuration number 600 is indicated for norm equal to 1 meters and, for the remaining ranges, number 1426 is chosen as the preferable option.

5.4 Summary and Discussion

escrever melhor, reunir mais ideias

Considering that the E-optimality is the most relevant criterion for this application, two configurations could be deployed, 1248 for short range and 1502 for long range, achieving an approximately constant error of 0.6%.

This system can be applied to a dynamic reconfigurable configuration mechanism that allows the USBL system to be optimized in real time .

Chapter 6

Conclusions

6.1 Summary

6.2 Contributions

The main contributions of this dissertation are:

- A HDL system design with strict area constraints that receives signals from four different sensors and calculates the phase differences between them
- An estimation algorithm for the angle of arrival of an acoustic signal to a configuration of four sensors
- An algorithm that chooses dynamically the best configuration of hydrophones, among a set of possible combinations, for a received signal with a certain angle of arrival

6.3 Future Work

The developed work arose various innovative ideas and topics according to the state of the art on localization optimization methods. Therefore, there is the intent to produce a scientific article on the subject to be submitted in the OCEANS Conference 2021.

The adopted optimization techniques in the present work are the starting point of a topic which is not very explored in the current literature. Consequently, there are several areas that can be improved as follows:

- Implementing the developed algorithm for AoA estimation as a real-time system which can be used for decision making during underwater navigation
- Using Machine Learning techniques in order to find the optimal hydrophone array configuration, to be positioned in an underwater vehicle
- Integrating the estimator with a Kalman Filter or a Particle Swarm Optimization approach

Appendix A

Complementary Information

A.1 Hydrophone configurations numeration

In table A.1 it is possible to consult the composition of the configurations which are mentioned throughout the document. It includes the configuration number and the respective integrated hydrophones.

Table A.1: Configurations for the Monte Carlo approach with 9 employed hydrophones

Configuration number	Hydrophones
1	1 2 3 4
2	1 2 3 5
3	1 2 3 6
4	1 2 3 7
5	1 2 3 8
6	1 2 3 9
7	1 2 4 5
8	1 2 4 6
9	1 2 4 7
10	1 2 4 8
11	1 2 4 9
12	1 2 5 6
13	1 2 5 7
14	1 2 5 8
15	1 2 5 9

Table A.1

Configuration number	Hydrophones
16	1 2 6 7
17	1 2 6 8
18	1 2 6 9
19	1 2 7 8
20	1 2 7 9
21	1 2 8 9
22	1 3 4 5
23	1 3 4 6
24	1 3 4 7
25	1 3 4 8
26	1 3 4 9
27	1 3 5 6
28	1 3 5 7
29	1 3 5 8
30	1 3 5 9
31	1 3 6 7
32	1 3 6 8
33	1 3 6 9
34	1 3 7 8
35	1 3 7 9
36	1 3 8 9
37	1 4 5 6
38	1 4 5 7
39	1 4 5 8
40	1 4 5 9
41	1 4 6 7
42	1 4 6 8
43	1 4 6 9

Table A.1

Configuration number	Hydrophones
44	1 4 7 8
45	1 4 7 9
46	1 4 8 9
47	1 5 6 7
48	1 5 6 8
49	1 5 6 9
50	1 5 7 8
51	1 5 7 9
52	1 5 8 9
53	1 6 7 8
54	1 6 7 9
55	1 6 8 9
56	1 7 8 9

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