

50 conjuntos

① a) $x(x^2-2)(2x+3)=0 \Leftrightarrow x=0 \vee x^2-2=0 \vee 2x+3=0 \Leftrightarrow$

$\Leftrightarrow x=0 \vee x=\sqrt{2} \vee x=-\sqrt{2} \vee x=-\frac{3}{2} \wedge x \in \mathbb{Q} \quad R: \{-\frac{3}{2}, 0\}$

b) $-1+0=-1 \quad 0+0=0 \quad 1+0=1$

$-1+1=0 \quad 0+1=1 \quad 1+1=2$

$-1+2=1 \quad 0+2=2 \quad 1+2=3$

$R: \{-1, 0, 1, 2, 3\}$

e) $x^2+y^2=25 \quad x=4 \wedge y=3; x=3 \wedge y=4; x=-4 \wedge y=3; x=-3 \wedge y=4; x=0 \wedge y=5$

$R: \{-\frac{4}{3}, -\frac{3}{4}, 0, \frac{3}{4}, \frac{4}{3}\}$

d) $\mathcal{P}(\emptyset) = \{\emptyset\} \quad R: \{\emptyset\}$

e) $R: \{\{a, b, c\}, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}\}$

② a) V

b) V

c) F (moda pertence ao vazio porque o vazio não tem moda).

d) V

e) F

f) V

g) F

h) V

i) F

j) V

k) F

l) F

m) V

n) V

A é um elemento de B, todos os elementos de B são elem de C.

contraex: $A = \{1, 2\} \quad B = \{1, 2, 3\} \quad C = \{5, \{1, 2, 3\}\}$

③

$$A \cap B = A$$

$$x \in A \rightarrow x \in A \cap B \rightarrow x \in A \wedge x \in B$$

$$x \in B$$

$$\text{Logo, } x \in A \rightarrow x \in B, \text{ portanto } A \subseteq B$$

$$A \subseteq B$$

$$\text{Seja } x \in A$$

$$x \in A \rightarrow x \in B$$

$$\text{Por elim. } \rightarrow, x_1 \in B$$

$$\text{Por intro } \wedge, x_1 \in A \wedge x_1 \in B$$

$$\text{Portanto, } x_1 \in A \cap B$$

$$\text{Então } \forall, \forall x (x \in A \rightarrow x \in A \cap B)$$

$$1) \text{ Logo, } A \subseteq A \cap B$$

$$\text{Seja } x_2 \in A \cap B$$

$$x_2 \in A \wedge x_2 \in B$$

$$\text{Portanto, } x_2 \in A$$

$$\text{Logo, } \forall x (x \in A \cap B \rightarrow x \in A)$$

$$2) \text{ Logo, } A \cap B \subseteq A$$

$$\text{Juntando 1 e 2, } A = A \cap B$$

$$\text{Logo, } A \cap B = A \leftrightarrow A \subseteq B$$

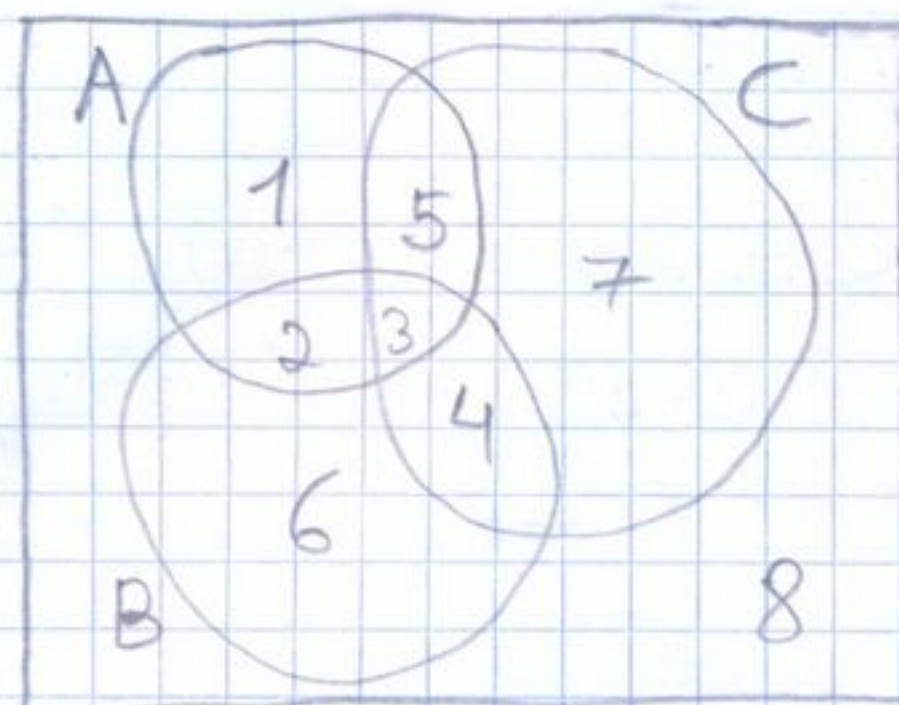
④ a) $[-4, 0[$

b) $]4, 5]$

c) $]-\infty, -4[\cup]4, +\infty[$

⑤ $\neg(x \in A \vee x \in B) \Leftrightarrow x \notin A \wedge x \notin B \Leftrightarrow x \in A^c \wedge x \in B^c \Leftrightarrow$
 $\Leftrightarrow A^c \cap B^c$

⑥



$$A \oplus B = (A \cup B) \setminus (A \cap B)$$

$$(A \oplus B) \oplus C = A \oplus (B \oplus C)$$

$$(A \oplus B) \oplus C = \{1, 4, 5, 6\} \oplus C = \{1, 3, 6, 7\}$$

$$A \oplus (B \oplus C) = A \oplus \{2, 5, 6, 7\} = \{1, 3, 6, 7\}$$

$$\text{Logo, } (A \oplus B) \oplus C = A \oplus (B \oplus C)$$

⑦ $B \setminus C = B \cap C^c$

$$(A \cap (B \setminus C))^c \cap A = (A^c \cup (B \cap C^c)^c) \cap A = (A^c \cup (B^c \cup C)) \cap A = (A^c \cap A) \cup ((B^c \cup C) \cap A) = (B^c \cap A) \cup (A \cap C) = (A \setminus B) \cup (A \cap C)$$

⑧

$$(x, y) \in ((A \times B) \cup (A \times C))$$

$$(x, y) \in (A \times B) \vee (x, y) \in (A \times C)$$

$$(x \in A \wedge y \in B) \vee (x \in A \wedge y \in C)$$

$$x \in A \wedge (y \in B \vee y \in C)$$

$$x \in A \wedge y \in (B \cup C)$$

$$(x, y) \in (A \times (B \cup C))$$

$$(x, y) \in ((A \times B) \cup (A \times C)) \rightarrow (x, y) \in (A \times (B \cup C))$$

$$(A \times B) \cup (A \times C) \subseteq A \times (B \cup C)$$

⑨ a)

$$A \times B \subseteq C \times D$$

$$\text{Seja } (x, y) \in A \times B$$

$$(x, y) \in A \times B \rightarrow (x, y) \in C \times D$$

$$\neg((x, y) \in A \times B) \vee (x, y) \in C \times D$$

$$(x, y) \notin A \times B \vee (x \in C \wedge y \in D)$$

$$(x \notin A \wedge y \notin B) \vee (x \in C \wedge y \in D)$$

$$((x \notin A \wedge y \notin B) \vee x \in C) \wedge ((x \notin A \wedge y \notin B) \vee y \in D)$$

$$x \in A \rightarrow x \in C \wedge (y \in B \rightarrow x \in C) \wedge x \in A \rightarrow y \in D \wedge y \in B \rightarrow y \in D$$

$$* A \subseteq C \wedge (y \in B \rightarrow x \in C) \wedge (x \in A \rightarrow y \in D) \wedge B \subseteq D$$

$$\text{Por elim } \wedge, A \subseteq C \wedge B \subseteq D$$

$$\text{Logo, } A \times B \subseteq C \times D \rightarrow A \subseteq C \wedge B \subseteq D$$

b) Seja $A = \{1, 2, 3\}$ $B = \{2, 3, 4\}$ $C = \{3, 4, 5\}$ $D = \{4, 5, 6\}$

$$(A \setminus B) \times (C \setminus D) = \{1\} \times \{3\} = \{(1, 3)\}$$

$$(A \times C) \setminus (B \times D) = \{(1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5), (3, 3), (3, 4), (3, 5)\} \setminus \{(2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6)\}$$

$$= \{(1, 3), (1, 4), (1, 5), (2, 3), (3, 3)\}$$

Logo, $(A \setminus B) \times (C \setminus D) \neq (A \times C) \setminus (B \times D)$, pelo que a afirmação é falsa.

10) a) $T \subseteq I$

b) $M \cap P = \emptyset$

c) $I \cap T^c \subseteq P$

d) $((I \cup M) \cap P^c) \subseteq T$

11) a) $P \cap E \neq \emptyset$

b) $0 \in \mathbb{Z} \setminus \mathbb{N}$

c) $P \subseteq (\mathbb{N} \cap \mathbb{Z})$

d) $(P \setminus \{2\}) \subseteq E^c$

12) a) $C \subseteq (D \cup I)$

b) $(C \cap I) \subseteq F^c$

c) $(P \setminus C) \cap X \subseteq I$

d) $(P \cap F^c \cap X^c) \subseteq C$

13) a) $(M \cap A^c) \subseteq C$

b) $(Z \setminus S) \subseteq A$

c) $S \cap A \neq \emptyset$

d) $R \subseteq S$