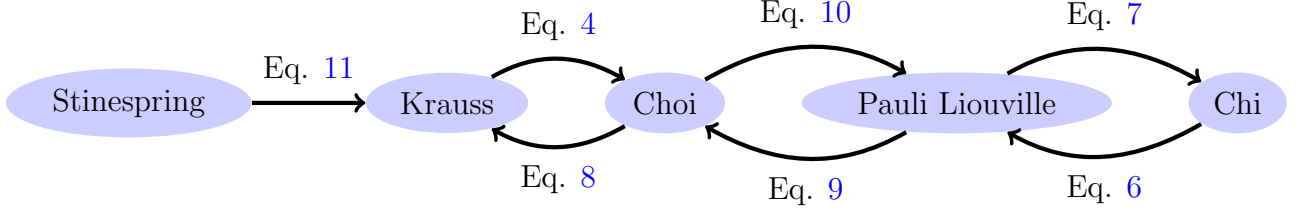


1 Converting between channel representations



1. Krauss operators \rightarrow Pauli Liouville matrix.

Input: $\{K_i\}_{k=1}^r$, where each of K_i are 2×2 matrices. Output: Γ , a 4×4 real matrix.

The entries of Γ are given by the following expression.

$$\Gamma_{i,j} = \text{Tr}(\mathcal{E}(P_i)P_j) \quad (1)$$

where \mathcal{E} is the CPTP map described by the input Krauss operators, as in (Eq. ??) and P_i is one of the Pauli matrices $\{I, X, Y, Z\}$. So, the process matrix can be expressed in terms of the Krauss operators as

$$\Gamma_{i,j} = \sum_{k=1}^r \text{Tr}(K_k P_i K_k^\dagger P_j). \quad (2)$$

2. Krauss operators \rightarrow Choi Matrix

Input: $\{K_i\}_{k=1}^r$, where each of K_i are 2×2 matrices. Output: J , a 4×4 complex matrix.

The Choi matrix of a CPTP map \mathcal{E} (specified by the input Krauss operators) is the result of applying \mathcal{E} on the first qubit of a bell state, i.e,

$$J = \frac{1}{4} \sum_{i,j=0}^1 \mathcal{E}(|i\rangle\langle j|) \otimes |i\rangle\langle j| \quad (3)$$

$$= \frac{1}{4} \sum_{i,j=0}^1 \sum_{k=1}^r K_k |i\rangle\langle j| K_k^\dagger \otimes |i\rangle\langle j| \quad (4)$$

3. χ matrix \rightarrow Pauli Liouville Matrix

Input: χ , a 4×4 complex matrix. Output: Γ , a 4×4 real matrix.

The action of a CPTP map \mathcal{E} on an input state ρ is specified using its χ matrix as

$$\mathcal{E}(\rho) = \sum_{l,m} \chi_{l,m} P_l \rho P_m. \quad (5)$$

Using the definition of the process matrix Γ , corresponding to the CPTP map \mathcal{E} , in (Eq. 1), we have

$$\Gamma_{i,j} = \sum_{l,m} \text{Tr}(P_l P_i P_m P_j) \chi_{l,m}.$$

Defining a 16×16 matrix Ω , such that $\Omega_{4 \times i + j, 4 \times l + m} = \text{Tr}(P_l P_i P_m P_j)$, we have

$$|\Gamma\rangle\rangle = \Omega|\chi\rangle\rangle, \quad (6)$$

where $|\Gamma\rangle\rangle$ and $|\chi\rangle\rangle$ are the (column) vectorized forms of Γ and χ respectively.

4. Pauli Liouville Matrix $\rightarrow \chi$ matrix.

We can easily obtain $|\chi\rangle\rangle$ by inverting the relation in Eq. 6, giving

$$|\chi\rangle\rangle = \Omega^{-1}|\Gamma\rangle\rangle. \quad (7)$$

5. Choi Matrix \rightarrow Krauss operators.

Input: J , a 4×4 complex matrix. Output: $\{K_i\}_{i=1}^r$, where each of K_i are 2×2 matrices.

Eq. 4 can be expressed as

$$J = \sum_{k=1}^r |K_k\rangle\rangle\langle\langle K_k|, \quad (8)$$

where $|K_k\rangle\rangle$ is the (column) vectorized form of the Krauss operator $\frac{1}{2}K_k$. Performing a singular value decomposition of J , we find $J = \sum_k \lambda_k |\lambda_k\rangle\rangle\langle\langle \lambda_k|$. Hence the Krauss operator K_k can be constructed from un-vectorizing $2|\lambda_k\rangle\rangle$.

6. Pauli Liouville Matrix \rightarrow Choi Matrix

Input: Γ , a 4×4 real matrix. Output: J , a 4×4 complex matrix.

Combining the expression for the bell state in the Pauli basis,

$$\frac{1}{4}(|00\rangle + |11\rangle)(\langle 00| + \langle 11|) = \frac{1}{4} \sum_{i=1}^4 P_i \otimes P_i^T,$$

where $P_i \in \{I, X, Y, Z\}$ is a Pauli matrix, with Eq. 3, we find

$$J = \frac{1}{4} \sum_i \mathcal{E}(P_i) \otimes P_i^T.$$

Applying the definition of the Pauli Liouville representation of \mathcal{E} in Eq. 1, we find

$$J = \sum_{i,j} \Gamma_{i,j} P_j \otimes P_i^T. \quad (9)$$

7. Choi Matrix \rightarrow Pauli Liouville Matrix.

Input: J , a 4×4 complex matrix. Output: Γ , a 4×4 real matrix.

Multiplying on either sides of Eq. 9 by $P_a \otimes P_b^T$, we find

$$J \cdot (P_a \otimes P_b^T) = \sum_{i,j} \Gamma_{i,j} P_j P_a \otimes P_i^T P_b^T.$$

Taking the trace of the above equation yields

$$\Gamma_{b,a} = \text{Tr}(J \cdot (P_a \otimes P_b^T)). \quad (10)$$

8. Strinespring dilation \rightarrow Krauss operators.

The action of any CPTP map \mathcal{E} on an input state ρ can be derived from unitary dynamics U on a larger Hilbert space, that comprises of the system ρ and its environment, which is initially in some state $|0\rangle_E\langle 0|_E$. Under the unitary evolution described by U , we find

$$\rho \otimes |0\rangle_E\langle 0|_E \rightarrow U [\rho \otimes |0\rangle_E\langle 0|_E] U^\dagger.$$

Since the state of the environment is essentially unknown, we must discard it resulting in a partial trace over the environment basis, denoted by $\{|e_k\rangle_E\}$, resulting in

$$\begin{aligned} \rho \otimes |0\rangle_E\langle 0|_E &\rightarrow \text{Tr}_E (U [\rho \otimes |0\rangle_E\langle 0|_E] U^\dagger) \\ &= \rho_{ab} \sum_{k=1}^r \langle e_k | U | e_0 \rangle \rho \langle e_0 | U^\dagger | e_k \rangle. \end{aligned} \quad (11)$$

The above expression is the Krauss representation of the channel where the Krauss operators are $\{\langle e_k | U | e_0 \rangle\}_{k=1}^r$.

```
Pavithrans-MacBook-Pro:chflow pavithran$ ./chflow.sh
>> chan rand 0.2
Note: the current channel is in the "krauss" representation.
>> man chrep
"chrep"
Description: Convert from its current representation to another form.
Usage
chrep s1(string)
where s1 must be one of "krauss", "choi", "chi", "process", "stine".
xxxxxx
>> chrep choi
>> chprint
Choi representation
[[ 0.439+0.j    0.106-0.05j  -0.096-0.025j  0.415-0.109j]
 [ 0.106+0.05j  0.046+0.j    -0.008-0.023j  0.108+0.023j]
 [-0.096+0.025j -0.008+0.023j  0.061+0.j    -0.106+0.05j ]
 [ 0.415+0.109j  0.108-0.023j -0.106-0.05j  0.454+0.j   ]]
xxxxxx
>> chrep process
>> chprint
Pauli Liouville representation
[[ 1.0  0.0255  0.00448 -0.0307]
 [ 5.55e-17  0.813  0.263  0.424]
 [-2.78e-17 -0.172  0.846 -0.2]
 [ 2.78e-16 -0.408  0.0961  0.786]]
xxxxxx
>> chrep chi
>> chprint
Chi representation
[[ 0.861+0.j  0.00636+0.074j  0.00112+0.208j  -0.00768-0.109j]
 [ 0.00636-0.074j  0.0452+0.0j  0.0227-0.00768j  0.00396-0.00112j]
 [ 0.00112-0.208j  0.0227+0.00768j  0.0619+0.0j  -0.0259+0.00636j]
 [-0.00768+0.109j  0.00396+0.00112j  -0.0259-0.00636j  0.0319+0.0j]]
xxxxxx
>> exit
Pavithrans-MacBook-Pro:chflow pavithran$
```

Figure 1: The `chrep` command in `chflow` can be used to change channel representations. The usage of the command is `chrep <representation>`, where `<representation>` should be replaced by one of `{krauss, process, choi, chi, stine}`, which results in the current channel being stored in the specified representation.