Defining quantum channels

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March 15, 2018

1 Pre-defined channels

Below is a table summarizing the pre-defined channels in `chflow`, the names by which they must be addressed, the corresponding number of parameters N_p and their Krauss representations.

Name	Channel	N_p	Parameters	Krauss operators
`ad`	Amplitude damping	1	λ	$K_0 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-\lambda} \end{pmatrix}$, $K_1 = \begin{pmatrix} 0 & \sqrt{\lambda} \\ 0 & 0 \end{pmatrix}$
`bp`	Bit flip	1	p	$K_0 = \sqrt{1-p}\mathbb{I}$, $K_1 = \sqrt{p}X$
`pd`	Dephasing	1	p	$K_0 = \sqrt{1-p}\mathbb{I}$, $K_1 = \sqrt{p}X$
`bpf`	Bit phase flip	1	p	$K_0 = \sqrt{1-p}\mathbb{I}, K_1 = \sqrt{p}Y \quad (1)$

`gd`	Generalized damping	2	λ, p	$$K_0 = \sqrt{1-p}\begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-\lambda} \end{pmatrix}, \quad K_1 = \sqrt{1-p}\begin{pmatrix} 0 & \sqrt{\lambda} \\ 0 & 0 \end{pmatrix}, \quad K_2 = \sqrt{p}\begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-\lambda} \end{pmatrix}, \quad K_3 = \sqrt{p}\begin{pmatrix} 0 & \sqrt{\lambda} \\ 0 & 0 \end{pmatrix} \qquad (2)$$
`gd`	Generalized damping	2	$\frac{t}{T_1}, \frac{T_2}{T_1}$	$$\gamma = 1 - e^{-t/T_1}, \quad p = 1 - \frac{e^{-2\frac{(t/T_1)}{T_2/T_1}}}{1-\lambda} \qquad (3)$$ Krauss operators are identical to Eq. 2.
`gdtx`	Generalized damping	3	t, T_1, T_2	Krauss operators can be derived from Eqs. 3 and 2.
`dp`	Depolarizing	1	p	$$K_0 = \sqrt{1-p}\,\mathbb{I}, \quad K_1 = \sqrt{p}\,X, \quad K_2 = \sqrt{p}\,Y, \quad K_3 = \sqrt{p}\,Z$$
`pauli`	Generic Pauli	3	p_X, p_Y, p_Z	$$K_0 = \sqrt{1-p_X-p_Y-p_Z}\,\mathbb{I}, \quad K_1 = \sqrt{p_X}\,X, \quad K_2 = \sqrt{p_Y}\,Y, \quad K_3 = \sqrt{p_Z}\,Z$$
`rtx`	Rotation about $X-$axis	1	θ	$$K_0 = e^{i\pi\theta X} \qquad (4)$$
`rtxpert`	Inexact rotations about the $X-$axis	1	$\overline{\theta}$	Let $\theta = \mathcal{N}(\overline{\theta}, 1)$, the Krauss operator is indeitcal to Eq. 4.

`rty`	Rotation about $Y-$axis	1	θ	$K_0 = e^{i\pi\theta Y} \qquad (5)$
`rtypert`	Inexact rotations about the $Y-$axis	1	$\overline{\theta}$	Let $\theta = \mathcal{N}(\overline{\theta}, 1)$, the Krauss operator is indeitcal to Eq. 5.
`rtz`	Rotation about $Z-$axis	1	θ	$K_0 = e^{i\pi\theta Z} \qquad (6)$
`rtzpert`	Inexact rotations about the $Z-$axis	1	$\overline{\theta}$	Let $\theta = \mathcal{N}(\overline{\theta}, 1)$, the Krauss operator is indeitcal to Eq. 6.
`rtnp`	Rotation about an arbitrary axis	3	p, θ, ϕ	$K_0 = e^{i\pi p(n_1 X + n_2 Y + n_3 Z)}$, where $n_1 = \sin\theta\cos\phi, n_2 = \sin\theta\sin\phi, n_3 = \cos\phi$.
`strtz`	Stochastic rotation about $Z-$axis	2	p, θ	$K_0 = \sqrt{1-p}\mathbb{I},$ $K_1 = \sqrt{p}\, e^{i\pi\theta X}$
`pl`	Photon loss channel	2	γ, α	$K_0 = \sqrt{\frac{\Gamma_+ + \Gamma_-}{2}}\mathbb{I},$ $K_1 = \begin{pmatrix} 0 & \sqrt{\frac{\Gamma_- - \Gamma_+}{\tanh(\lvert\alpha\rvert^2)}} \\ \sqrt{\tanh(\lvert\alpha\rvert^2)}\sqrt{\Gamma_- - \Gamma_+} & 0 \end{pmatrix}$ where $\Gamma_\pm = (1-\gamma)^{\pm\lvert\alpha\rvert^2}$.
`rand`	Random channel	2	δ, M, r	$\{K_i\}_{i=1}^{2^r}$ are randomly generated following one of a few available recipes, specified by M. These methods are detailed in Sec. 1.1.

Table 1: Pre-defined quantum channels in `chflow`.

1.1 Methods for generating random unitary matrices

1. For $M \in \{1, 2, 3\}$, the Krauss operators of the random CPTP map are derived from a $2^{r+1} \times 2^{r+1}$ unitary matrix U, following the procedure in Eq. **??**. The method of generating U is specified by the value of M.

   - $M = 1$: A random unitary matrix is derived from exponentiating a random Hermitian matrix H, i.e $U = e^{i\delta H}$ for $0 \leq \delta \leq 1$. The Hermitian matrix H is generated from a matrix A with gaussian random entries as: $H = A + A^{\dagger}$.

- $M = 2$: A random unitary matrix U is derived by diagonalising a random Hermitian matrix H, i.e, U is a matrix whose columns are eigenvectors of H. The Hermitian matrix H is generated as in the above ($M = 1$) method.
- $M = 3$: A Haar random unitary matrix is generated.
- $M = 4$: A random unitary matrix U is derived by exponentiating a random Hermitian matrix H, i.e, $U = e^{iH}$. The $2^r \times 2^r$ Hermitian matrix H is constructed by sampling 2^r random numbers in $\{c_1, c_2, \ldots, c_{2^r}\}$, each in $[0, 1]$ such that $\sum_{i=1}^{r} |c_i|^2 = \delta$.

2. In this case of $M = 5$, a random Pauli channel is generated by choosing random numbers $\{c_1, c_2, c_3\}$, each in $[0, 1]$ such that $|c_1|^2 + |c_2|^2 + |c_3|^2 = \delta$. The Krauss operators of the Pauli channel are $K_0 = (1 - \delta)\mathbb{I}, K_1 = c_1 X, K_2 = c_2 X, K_3 = c_3 Z$.

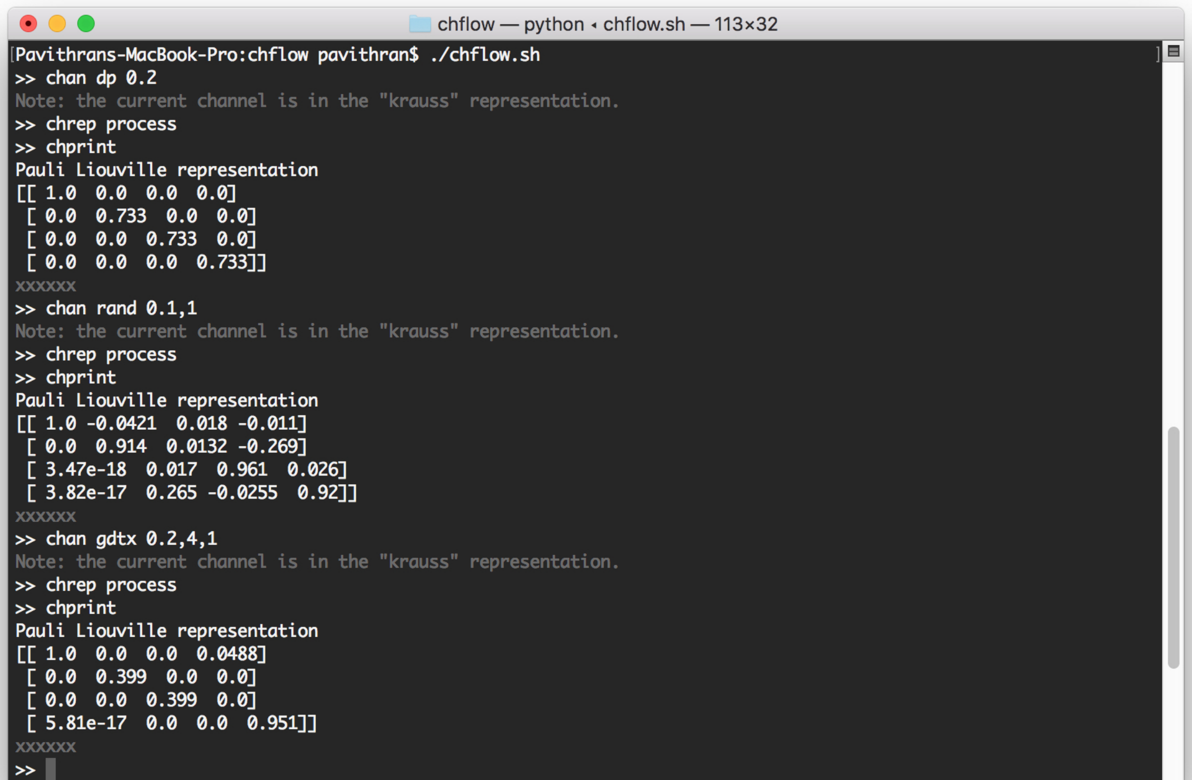


Figure 1: The `chan` command can be used to define quantum channels. For multi-parameter channels, the values of the parameters must be separated by commas.

2 Defining you own quantum channels

In addition to the predefined channels in table 1, one can also define a quantum channel in two distinct ways. First, by supplying the explicit form of the quantum channel in one of the many representations: Krauss operators, Pauli Liouville matrix, Choi matrix, Chi matrix and Stinespring dilation. The corresponding array form must be stored in a numpy formatted (`.npy`) or a text file. For eg. `fname.npy`. This definition can be recalled in `chflow` as: `chan fname.npy`.

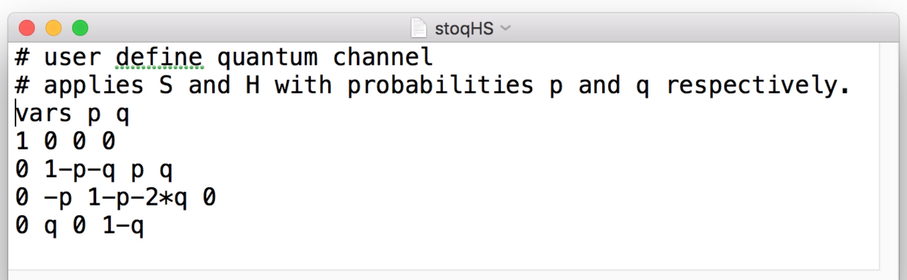
Second, a symbolic definition of a quantum channel. Here again, one can choose any desired representation, except the Krauss operators, of the underlying quantum channel and provide the corresponding matrix as a text file. The list of variables must be first provided, separated by spaces, with the keyword `vars`, see fig. 2 for example. Some of the entries in the channel description can now be represented by variables and symbolic expressions involving the variables. Mathematical expressions involving symbols must be in a Python interpretable format, since the inbuilt command `eval(...)` will be under to interpret the expressions. Finally, the channel definition can be recalled in `chflow` by providing the file-name as in fig. 1, followed by the values of the variables separated by commas (if necessary). We will illustrate the definition using an example of a stochastic quantum channel that performs the Clifford operations H and S with equal probabilities, $p/2$ for some $p \in [0, 1]$. This channel has the following Krauss representation:

$$\mathcal{E}(\rho) = (1 - p - q)\ \rho + p\ H\rho H + q\ S\rho S \tag{7}$$

and the Pauli Liouville matrix

$$\Gamma = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1-p-q & p & q \\ 0 & -p & 1-p-2q & 0 \\ 0 & q & 0 & 1-q \end{pmatrix}. \tag{8}$$

To specify the channel using its Pauli Liouville matrix Γ, we must create a text file with the description of Γ, as shown in the figure below.




```
# user define quantum channel
# applies S and H with probabilities p and q respectively.
vars p q
1 0 0 0
0 1-p-q p q
0 -p 1-p-2*q 0
0 q 0 1-q
```

Figure 2: Description of Γ as mentioned in eq. 8 in a text file. The above file can be found in `examples/stoqHS.txt`.

Note that the order of variables in `vars` must be the same as the order of parameter values, provided while recalling the channel definition using `chan` (see fig. 1). Lines commencing with "#" are only for commenting purposes, they will be ignored by `chflow`.

3 References

1. https://docs.python.org/2/library/functions.html#eval.