

Defining quantum channels

Pavithran Iyer

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1 Pre-defined channels

Below is a table summarizing the pre-defined channels in `chflow`, the names by which they must be addressed, the corresponding number of parameters N_p and their Krauss representations.

| Name | Channel | N_p | Parameters | Krauss operators |
|------|-------------------|-------|------------|--|
| ad | Amplitude damping | 1 | λ | $K_0 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-\lambda} \end{pmatrix},$ $K_1 = \begin{pmatrix} 0 & \sqrt{\lambda} \\ 0 & 0 \end{pmatrix}$ |
| bp | Bit flip | 1 | p | $K_0 = \sqrt{1-p}\mathbb{I},$ $K_1 = \sqrt{p}X$ |
| pd | Dephasing | 1 | p | $K_0 = \sqrt{1-p}\mathbb{I},$ $K_1 = \sqrt{p}X$ |
| bpf | Bit phase flip | 1 | p | $K_0 = \sqrt{1-p}\mathbb{I}, K_1 = \sqrt{p}Y_{(1)}$ |

| | | | | |
|---------|---------------------------------------|---|----------------------------------|--|
| gd | Generalized damping | 2 | λ, p | $K_0 = \sqrt{1-p} \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-\lambda} \end{pmatrix},$ $K_1 = \sqrt{1-p} \begin{pmatrix} 0 & \sqrt{\lambda} \\ 0 & 0 \end{pmatrix},$ $K_2 = \sqrt{p} \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-\lambda} \end{pmatrix},$ $K_3 = \sqrt{p} \begin{pmatrix} 0 & \sqrt{\lambda} \\ 0 & 0 \end{pmatrix} \quad (2)$ |
| gd | Generalized damping | 2 | $\frac{t}{T_1}, \frac{T_2}{T_1}$ | $\gamma = 1 - e^{-t/T_1},$ $p = 1 - \frac{e^{-2\frac{(t/T_1)}{T_2/T_1}}}{1-\lambda} \quad (3)$ <p>Krauss operators are identical to Eq. ??.</p> |
| gdtx | Generalized damping | 3 | t, T_1, T_2 | Krauss operators can be derived from Eqs. ?? and ??. |
| dp | Depolarizing | 1 | p | $K_0 = \sqrt{1-p} \mathbb{I},$ $K_1 = \sqrt{p} X,$ $K_2 = \sqrt{p} Y,$ $K_3 = \sqrt{p} Z$ |
| pauli | Generic Pauli | 3 | p_X, p_Y, p_Z | $K_0 = \sqrt{1-p_X-p_Y-p_Z} \mathbb{I},$ $K_1 = \sqrt{p_X} X,$ $K_2 = \sqrt{p_Y} Y,$ $K_3 = \sqrt{p_Z} Z$ |
| rtx | Rotation about X -axis | 1 | θ | $K_0 = e^{i\pi\theta X} \quad (4)$ |
| rtxpert | Inexact rotations about the X -axis | 1 | $\bar{\theta}$ | Let $\theta = \mathcal{N}(\bar{\theta}, 1)$, the Krauss operator is indeitcal to Eq. ??. |

| | | | | |
|----------------|---------------------------------------|---|-------------------|--|
| rty | Rotation about Y -axis | 1 | θ | $K_0 = e^{i\pi\theta Y} \quad (5)$ |
| rtypert | Inexact rotations about the Y -axis | 1 | $\bar{\theta}$ | Let $\theta = \mathcal{N}(\bar{\theta}, 1)$, the Krauss operator is indeitcal to Eq. ??. |
| rtz | Rotation about Z -axis | 1 | θ | $K_0 = e^{i\pi\theta Z} \quad (6)$ |
| rtzpert | Inexact rotations about the Z -axis | 1 | $\bar{\theta}$ | Let $\theta = \mathcal{N}(\bar{\theta}, 1)$, the Krauss operator is indeitcal to Eq. ??. |
| rtnp | Rotation about an arbitrary axis | 3 | p, θ, ϕ | $K_0 = e^{i\pi p(n_1 X + n_2 Y + n_3 Z)},$ where $n_1 = \sin \theta \cos \phi, n_2 = \sin \theta \sin \phi, n_3 = \cos \phi$. |
| strtz | Stochastic rotation about Z -axis | 2 | p, θ | $K_0 = \sqrt{1-p}\mathbb{I},$ $K_1 = \sqrt{p} e^{i\pi\theta X}$ |
| pl | Photon loss channel | 2 | γ, α | $K_0 = \sqrt{\frac{\Gamma_+ + \Gamma_-}{2}}\mathbb{I},$ $K_1 = \begin{pmatrix} 0 & \sqrt{\frac{\Gamma_- - \Gamma_+}{\tanh(\alpha ^2)}} \\ \sqrt{\tanh(\alpha ^2)}\sqrt{\Gamma_- - \Gamma_+} & 0 \end{pmatrix}$ where $\Gamma_{\pm} = (1 - \gamma)^{\pm \alpha ^2}$. |
| rand | Random channel | 2 | δ, M, r | $\{K_{ij}\}_{i=1}^{2^r}$ are randomly generated following one of a few available recipes, specified by M . These methods are detailed in Sec. ??. |

Table 1: Pre-defined quantum channels in **chflow**.

1.1 Methods for generating random unitary matrices

1. For $M \in \{1, 2, 3\}$, the Krauss operators of the random CPTP map are derived from a $2^{r+1} \times 2^{r+1}$ unitary matrix U , following the procedure in Eq. ?? . The method of generating U is specified by the value of M .

- $M = 1$: A random unitary matrix is derived from exponentiating a random Hermitian

- matrix H , i.e $U = e^{i\delta H}$ for $0 \leq \delta \leq 1$. The Hermitian matrix H is generated from a matrix A with gaussian random entries as: $H = A + A^\dagger$.
- $M = 2$: A random unitary matrix U is derived by diagonalising a random Hermitian matrix H , i.e, U is a matrix whose columns are eigenvectors of H . The Hermitian matrix H is generated as in the above ($M = 1$) method.
 - $M = 3$: A Haar random unitary matrix is generated.
 - $M = 4$: A random unitary matrix U is derived by exponentiating a random Hermitian matrix H , i.e, $U = e^{iH}$. The $2^r \times 2^r$ Hermitian matrix H is constructed by sampling 2^r random numbers in $\{c_1, c_2, \dots, c_{2^r}\}$, each in $[0, 1]$ such that $\sum_{i=1}^r |c_i|^2 = \delta$.
2. In this case of $M = 5$, a random Pauli channel is generated by choosing random numbers $\{c_1, c_2, c_3\}$, each in $[0, 1]$ such that $|c_1|^2 + |c_2|^2 + |c_3|^2 = \delta$. The Krauss operators of the Pauli channel are $K_0 = (1 - \delta)\mathbb{I}, K_1 = c_1X, K_2 = c_2X, K_3 = c_3Z$.

```
Pavithrans-MacBook-Pro:chflow pavithran$ ./chflow.sh
>> chan dp 0.2
Note: the current channel is in the "krauss" representation.
>> chrep process
>> chprint
Pauli Liouville representation
[[ 1.0  0.0  0.0  0.0]
 [ 0.0  0.733  0.0  0.0]
 [ 0.0  0.0  0.733  0.0]
 [ 0.0  0.0  0.0  0.733]]
xxxxxx
>> chan rand 0.1,1
Note: the current channel is in the "krauss" representation.
>> chrep process
>> chprint
Pauli Liouville representation
[[ 1.0 -0.0421  0.018 -0.011]
 [ 0.0  0.914  0.0132 -0.269]
 [ 3.47e-18  0.017  0.961  0.026]
 [ 3.82e-17  0.265 -0.0255  0.92]]
xxxxxx
>> chan gdtx 0.2,4,1
Note: the current channel is in the "krauss" representation.
>> chrep process
>> chprint
Pauli Liouville representation
[[ 1.0  0.0  0.0  0.0488]
 [ 0.0  0.399  0.0  0.0]
 [ 0.0  0.0  0.399  0.0]
 [ 5.81e-17  0.0  0.0  0.951]]
xxxxxx
>>
```

Figure 1: The `chan` command can be used to define quantum channels. For multi-parameter channels, the values of the parameters must be separated by commas.

2 Defining you own quantum channels

In addition to the predefined channels in table ??, one can also define a quantum channel in two distinct ways. First, by supplying the explicit form of the quantum channel in one of the many representations: Krauss operators, Pauli Liouville matrix, Choi matrix, Chi matrix and Stinespring dilation. The corresponding array form must be stored in a numpy formatted (.`numpy`) or a text file. For eg. `fname.npy`. This definition can be recalled in `chflow` as: `chan fname.npy`.

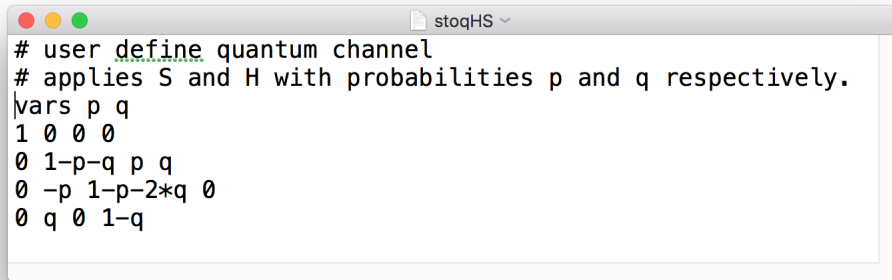
Second, a symbolic definition of a quantum channel. Here again, one can choose any desired representation, except the Krauss operators, of the underlying quantum channel and provide the corresponding matrix as a text file. The list of variables must be first provided, separated by spaces, with the keyword `vars`, see fig. ?? for example. Some of the entries in the channel description can now be represented by variables and symbolic expressions involving the variables. Mathematical expressions involving symbols must be in a Python interpretable format, since the `inbuilt command eval(...)` will be under to interpret the expressions. Finally, the channel definition can be recalled in `chflow` by providing the file-name as in fig. ??, followed by the values of the variables separated by commas (if necessary). We will illustrate the definition using an example of a stochastic quantum channel that performs the Clifford operations H and S with equal probabilities, $p/2$ for some $p \in [0, 1]$. This channel has the following Krauss representation:

$$\mathcal{E}(\rho) = (1 - p - q) \rho + p H \rho H + q S \rho S \quad (7)$$

and the Pauli Liouville matrix

$$\Gamma = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 - p - q & p & q \\ 0 & -p & 1 - p - 2q & 0 \\ 0 & q & 0 & 1 - q \end{pmatrix}. \quad (8)$$

To specify the channel using its Pauli Liouville matrix Γ , we must create a text file with the description of Γ , as shown in the figure below.



```
# user define quantum channel
# applies S and H with probabilities p and q respectively.
vars p q
1 0 0 0
0 1-p-q p q
0 -p 1-p-2*q 0
0 q 0 1-q
```

Figure 2: Description of Γ as mentioned in eq. ?? in a text file. The above file can be found in [examples/stoqHS.txt](#).

Note that the order of variables in `vars` must be the same as the order of parameter values, provided while recalling the channel definition using `chan` (see fig. ??). Lines commencing with “#” are only for commenting purposes, they will be ignored by `chflow`.

3 References

1. <https://docs.python.org/2/library/functions.html#eval>.