Distance dependent bond percolation in a geographical network model

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Structure of talk

- Many networks are geographical
- Methods of studying robustness
- Intuitive effects of geography on robustness
- A simple model that provides some insight

Geographical networks

- Similar concepts: spatial networks, Euclidian networks, lattice networks, etc.
- Usually characterised by a distance-dependent link probability
- Examples: Internet, trade, commuting, Most social networks, distribution networks (oil, gas, electricity)

Robustness

- Bond- and site percolation
  - Formation or destruction of a giant component
- Average network distance
- Performance of specific processes, like packet transport

- Factors usually studied: degree distribution, degree correlations

Robustness of geographical networks

- Network distance (perceived distance) is often much shorter than physical distance
  - Means that “distance is hidden” or “compressed”
- Long range links can be vital for the connectivity of the network

Distance dependence

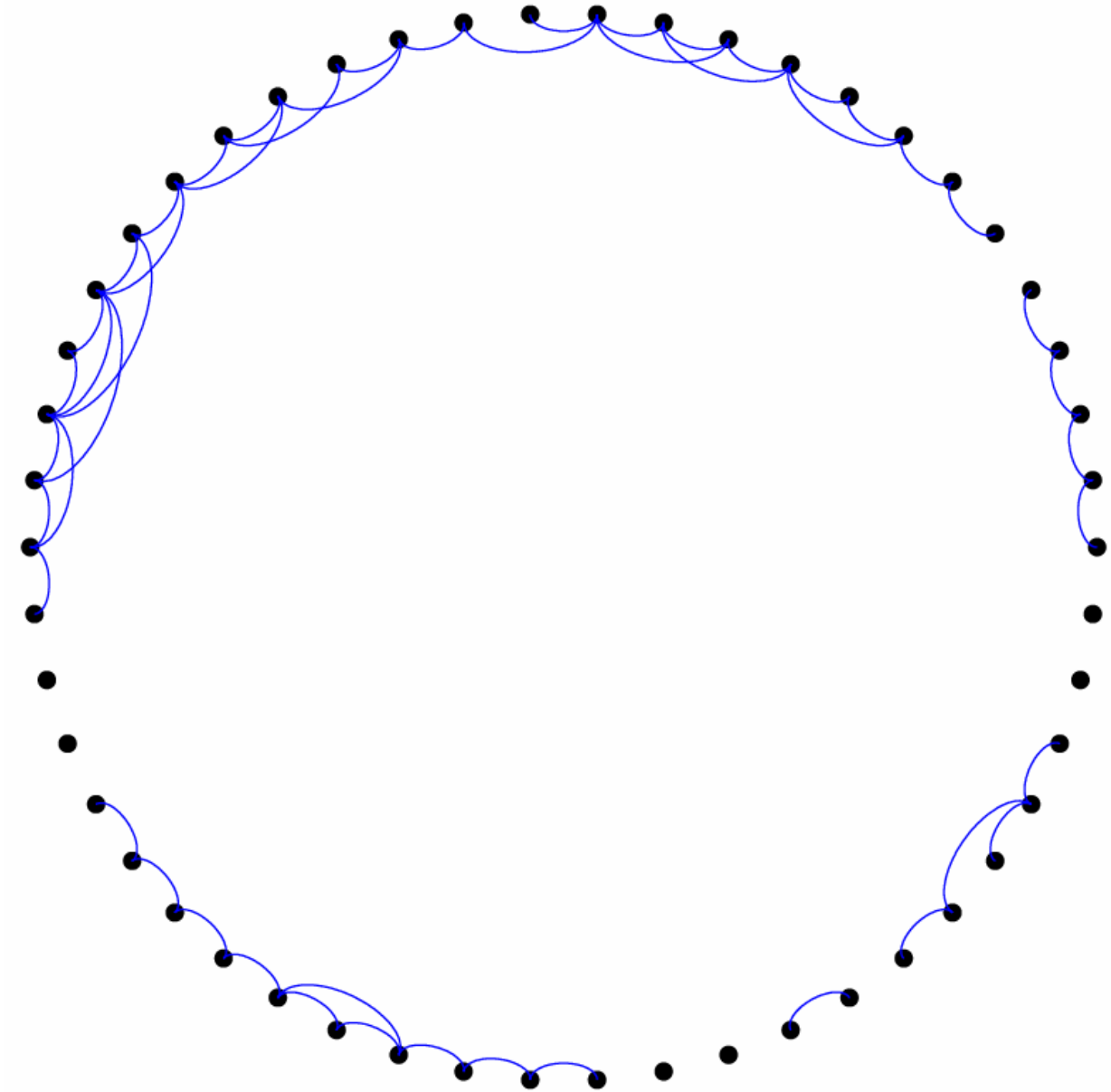
- Long links are more vulnerable in many networks
  - More expensive
  - More points of failure
- At the same time they can have high importance for connectivity

How can this be modelled?

- These things should be given:
  - Network topology
  - Geographical structure of the network
  - A removal mechanism dependent on geographical link length
- Study the effect of these factors on a suitable robustness metric

Simplifications

- Poissonian degree distribution
- 1 dimensional lattice
- Consider addition of links instead of removal
- Links are added in a strict sequence with shortest first



Formation of a giant component

- The probability that a gap between two nodes has no crossing links:

$$\prod_{i=0}^{N}\left(\frac{1}{2}P(2i+1)+\sum_{j=0}^{2i}P(2j)\right),$$

where $P(i)$ is the degree distribution.

Size of the giant component

- Number of components equal to number of holes
- Number of holes proportional to network size
- No giant component is formed in finite time (i.e. for a number of links in the order of the network size)
- In an Erdös-Rényi network, a giant component is formed

Addition of small-world links

- Just a small proportion of long-range links are needed to introduce the possibility for a giant component into the geographical model
- The network will be vulnerable to removal of these links, since that would destroy the giant component

Summary

- Many networks are geographical
- Geographical structure can intuitively affect robustness
- A simple 1d-model was used to study the effect of distance dependent bond percolation
- The model shows that geographical structure can affect robustness

Preprint

http://halex.mailcan.com/lattice_model_paper.pdf