EOM control of the LFF chaotic behavior in an ECSL – m:n phase synchronization

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Outline

- External-cavity semiconductor laser (ECSL) system
- Some aspects about low-frequency fluctuations (LFFs)
- ECSL system with an electro-optical modulator (EOM)
- The model for ECSL system with phase modulation
- Numerical results and discussion
- Conclusions

The effect of periodic phase modulation of light on a chaotic external-cavity semiconductor laser working in a regime of low-frequency fluctuations is studied numerically

***Methods* used to characterize the degree of synchronization between the lasers' LFFs and the modulator are based on:**

- Evolution of the phase difference
- Shanon's entropy

The ECSL system

- A chaotic behavior can be found in nonlinear dynamics of semiconductor laser diode under external optical feedback

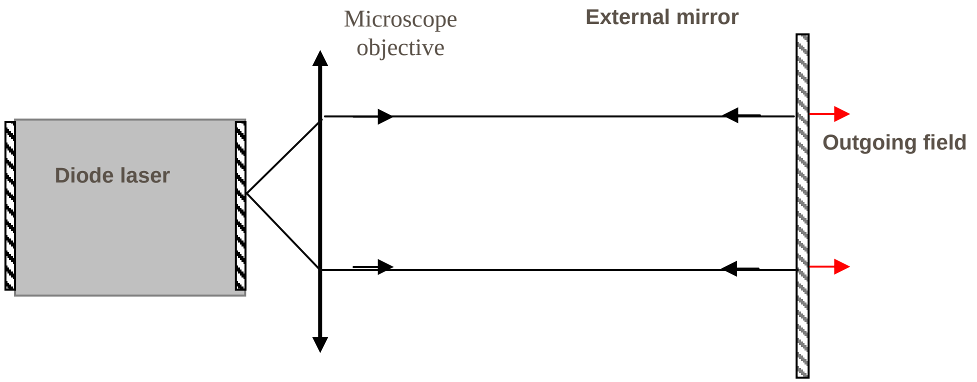


Fig. 1. Scheme of the external-cavity semiconductor laser configuration

ECSL system equations

❑ The electric field inside the external cavity is the complex time-varying amplitude E(t), as introduced by R. Lang & K. Kobayashi (IEEE J. QE-16, 3, 1980):

$$\frac{dE(t)}{dt} = (1-i\alpha)\left(G(t)-\frac{1}{\tau_p}\right)\frac{E(t)}{2} + \gamma E(t-\tau)e^{i\omega\tau} + \sqrt{2\beta N(t)}\xi(t)$$

$$\frac{dN(t)}{dt} = \frac{I}{e} - \frac{N(t)}{\tau_n} - G(t)|E(t)|^2$$

$$with \quad G(t) = \frac{g(N(t)-N_0)}{1+s|E(t)|^2}$$

The rate equations for the complex field E(t) and carrier density *N*

Parameters: α is the linewidth enhancement factor, τ_p is the photon lifetime decay rate, γ is the feedback coefficient, τ_r is the external-cavity roundtrip time, ω_0 is the optical frequency of the laser, *N(t)* is the carrier number density, *I* is the injection current, *e* is the unit charge, τ_n is the carrier lifetime, *g* is the gain parameter, N_0 is the initial carrier number, and *s* is the gain saturation coefficient, β is spontaneous emission rate

Some aspects about low-frequency fluctuations (LFFs)

- At small injected current, below threshold, even with an external coupled cavity, the light emission of the laser diode is spontaneously modeled by Gaussian white noise.

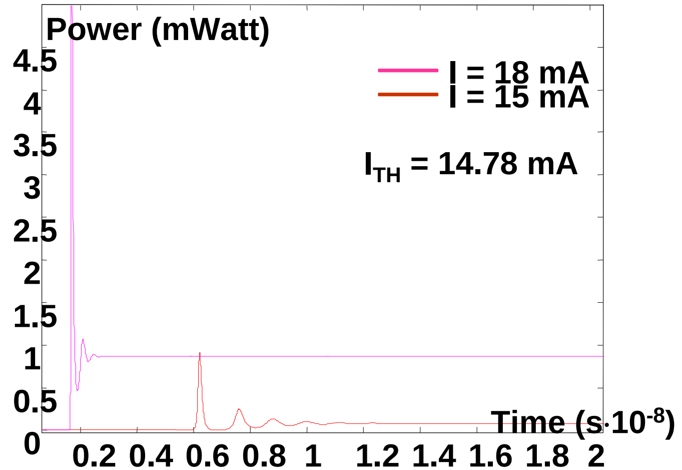


Fig. 2. Neglecting optical feedback (γ=0), a stable steady state solution can be found at a current greater than *threshold current*, I_{th}. The power evolve smoothly to a stationary lavel

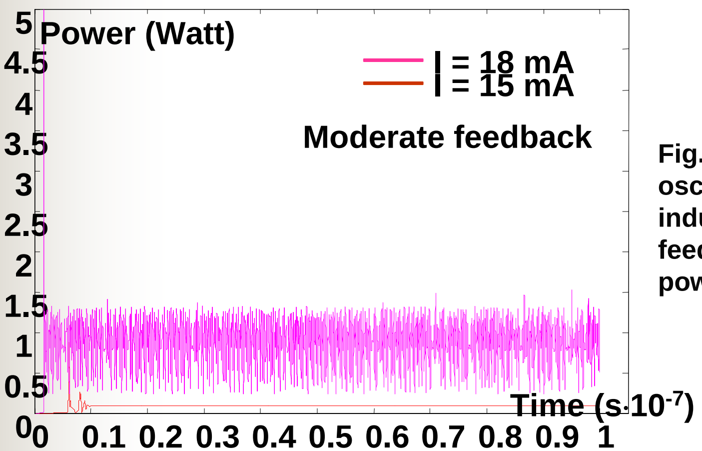


Fig. 3. When the external cavity is coupled, the systems switch to an oscillating state, it follows a chaotic trace. The amplitude of the induced chaotic "noise" seems to be important, dependent on the feedback coefficient, equivalent of approximately 1÷30% of emission power. A low injected current leave the system steady

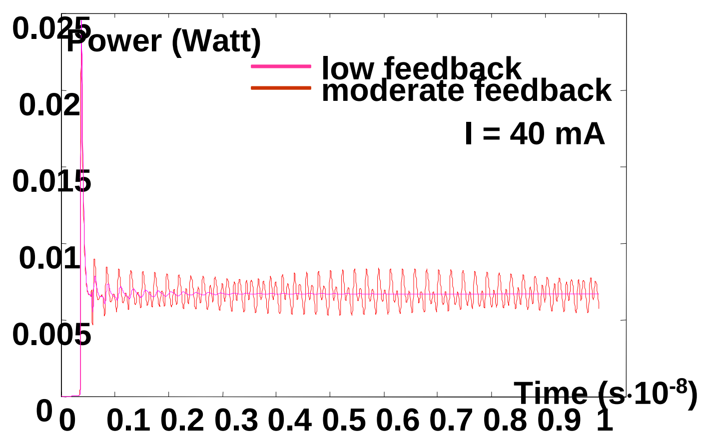
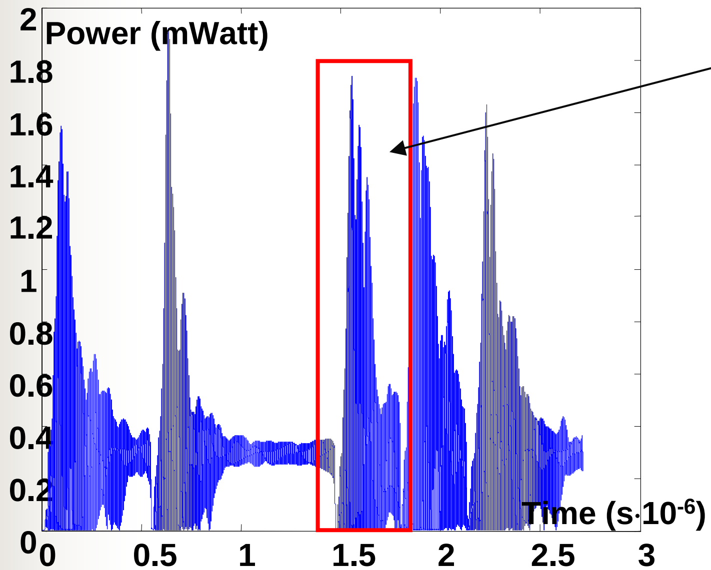
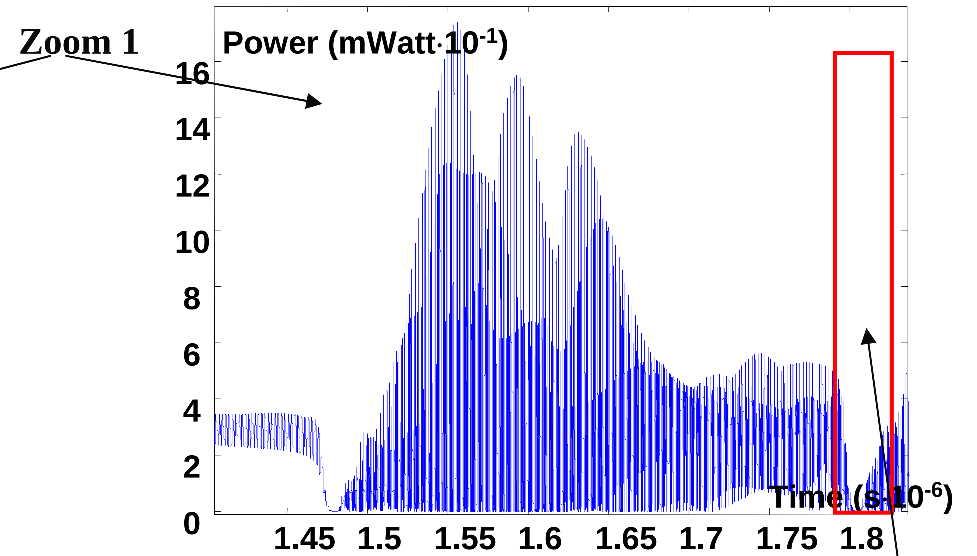


Fig. 4. At a given current it is a low limit for the energy of light re-injected into the laser cavity for chaotic set up. A low feedback causes a damped chaotic oscillation

Some aspects about low-frequency fluctuations (LFFs)

- Slightly operating above the lasing threshold, a typical ECSL, with moderate optical feedback, runs into a temporal, unstable, chaotic regime. Cyclic drops-out to almost zero of the envelope of light intensity are evidently present – LFFs





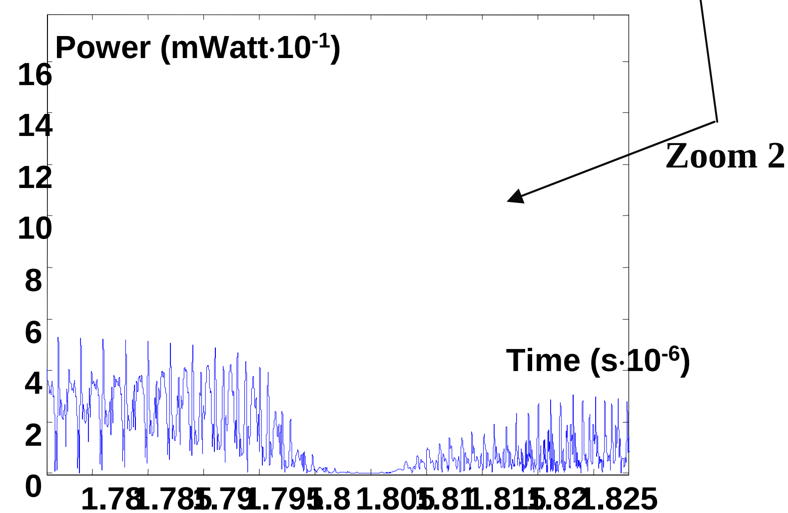


Fig. 5. The system can present short period of low chaotic states. A high dimensional chaos is associated with the *LFF*.

Zoom1 and Zoom 2. Main evolution is represented by short spikes, around ns, with variable amplitude forming the LFF envelopes.

The ECSL system with a electro-optical modulator (EOM)

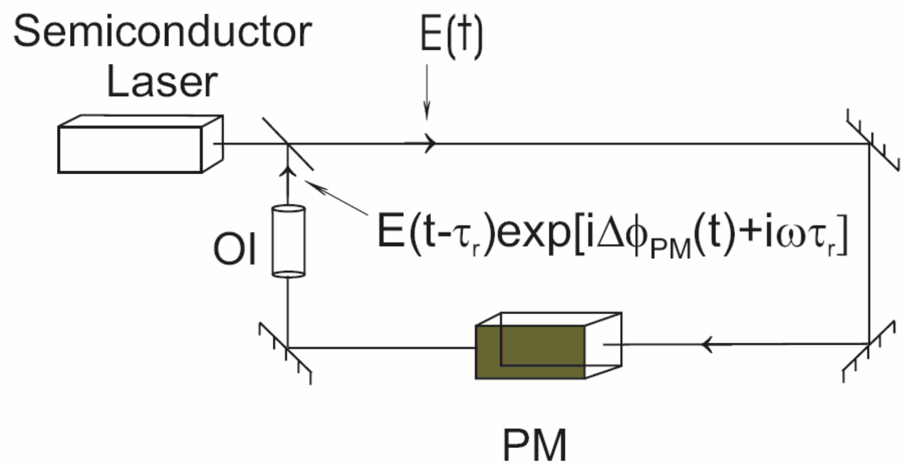


Fig. 6. Schematics of the external-cavity semiconductor laser configuration with a phase modulator (PM) and an unidirectional optical filter (OI).

- The phase modulation of light induces a small time-varying delay in the total optical path inside the cavity influencing the laser dynamics
- The laser variable $\Phi_L(t)$ is defined as the phase of the LFFs
- The phase of the modulator $\Phi_{PM}(t)$ is defined as a variable which increases linearly in time, at a rate given by the modulating frequency
- A correlation between the two phases can be established by calculating their difference $\Delta\Phi(t) = m\Phi_L(t) - n\Phi_{PM}(t)$, where *m* and *n* are integers

The model for the ECSL system with phase modulation

The laser equations

❑ In our case, the effect of the phase modulator on the optical feedback is included in the equations:

$$\frac{dE(t)}{dt} = (1 - i\alpha)\left(G(t) - \frac{1}{\tau_p}\right)\frac{E(t)}{2} + \gamma E(t - \tau_r)\exp[i(\omega_0\tau_r + A\sin(\Omega_{PM} t))] \quad (1)$$

$$\frac{dN(t)}{dt} = \frac{I}{e} - \frac{N(t)}{\tau_n} - G(t)|E(t)|^2 \quad (2)$$

$$G(t) = \frac{g(N(t) - N_0)}{1 + s|E(t)|^2}$$

The rate equations for the complex field E(t) and carrier density *N*

❑ The delay in the electric field introduced by the modulator is considered sinusoidal and is given by , $\Delta\Phi_{PM} = A\sin\Omega_{PM} t$ (3), where *A* is the amplitude

❑ The exponential term $\exp[iA\sin(\Omega_{PM} t)]$ of the first equation accounts for the phase delay caused by the EO PM

❑ The above equations are solved using a fourth-order Runge-Kutta algorithm with a Bogacki-Shampine step control method. We can get time series of the laser intensity $|E(t)|^2$ when low-pass filtered at a cut-off frequency of 3×10^8 Hz are present

❑ The laser parameters used in the simulation are as follows:

$$\alpha = 5; \tau_p = 2ps; \gamma = 0.03ps^{-1}; \tau_r = 1ns; \omega_0 = 1.2x10^6 GHz; I = 15mA;$$

$$\tau_n = 2ns; g = 1.5x10^{-8} ps^{-1}; N_0 = 1.5x10^8; s = 5x10^{-7}$$

The model for the ECSL system with phase modulation

Definition of the phases

- The phase of the power drops-out in the laser intensity at a moment *t* is defined as follows:

$$\Phi_L(t) = 2\pi \frac{t - t_j}{t_{j+1} - t_j} + 2\pi j \qquad (4)$$

with *j=1,2…*, where t_j is the moment at which the j-th drop-out event takes place

for each interval $\Delta t_j = t_{j+1} - t_j$ **we can introduce a single event frequency** $\Omega_L(j) = \frac{2\pi}{\Delta t_j}$

- The phase of the modulator is determined by the modulating frequency Ω_{PM}

$$\Phi_{PM}(t) = \Omega_{PM}(t - t_k) + 2\pi k = \Omega_{PM} t \qquad (5)$$

- Equation $\frac{\Omega_{PM}}{\langle \Omega_L \rangle} = \frac{m}{n}$ is usually used to describe phase locking between two coupled periodic oscillators. We call this state a ‘m:n’ phase synchronized state between the laser power drop-out events and the phase modulator

The model for the ECSL system with phase modulation

Characterization of the coupling

- In the simulations, the ratio between two phases is calculated as the system evolve in time, giving an evolution of the instantaneous coupling

$$r(t)=\frac{\Phi_{PM}(t)}{\Phi_{L}(t)} \quad (6)$$

The degree of time correlation between the two phases and thus of the phase synchronization is indicated by the deviation in time of this ratio from a constant value m/n

- An alternative method - used to characterize the degree of synchronization between the laser and the modulator - is based on Shanon's entropy

$$S=-\sum_{i=1}^{M} p_i \log p_i \quad (7)$$

The entropy of the assembly constituted from time intervals between consecutive drop-out events is evaluated. First these time intervals are plotted on a histogram with M bins. A measure of their distribution in the histogram is inferred by associating a probability where $p_i = N_i / N$ to the number N_i of time intervals found in each bin *I; N* – total number of intervals considered;

- A coefficient $\sigma=\left(S_{\max}-S\right)/S_{\max}$ can be introduced for a particular distribution S to characterize the clustering or spreading of the time-intervals represented in an histogram

Numerical results and discussion

- **The first method compares a newly defined laser variable - namely the phase of the LFFs - with the phase of the modulator**

- **The second technique characterizes the irregularity of the time intervals between consecutive drop-out events, by calculating Shanon's entropy. The synchronization region in the parameters space of the modulator (Ω_{PM} ,A) is mapped. There are also identified the zones of stable, periodic and low amplitude chaotic emission**

Numerical results and discussion

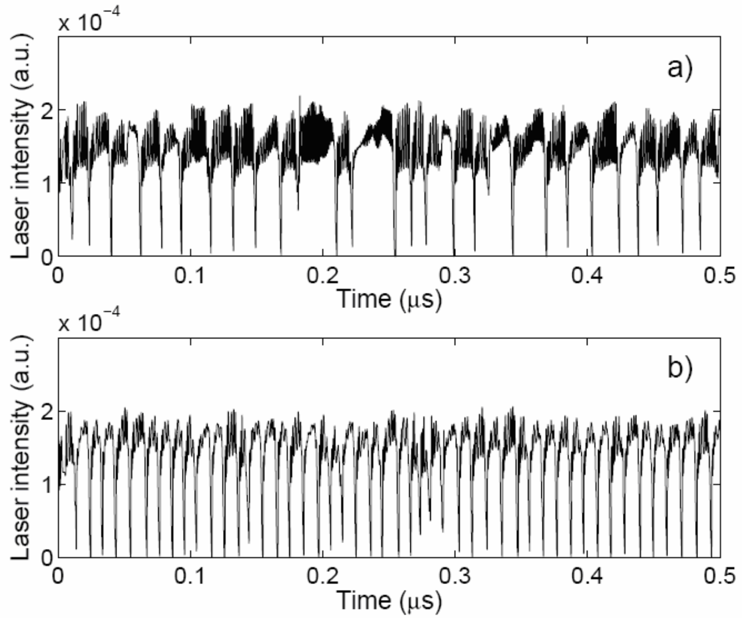


Fig. 7. Drop-out events in the chaotic output intensity of the laser: a) without modulation and b) with modulation at Ω_{PM} =1.97 GHz**, and** A =1**.**

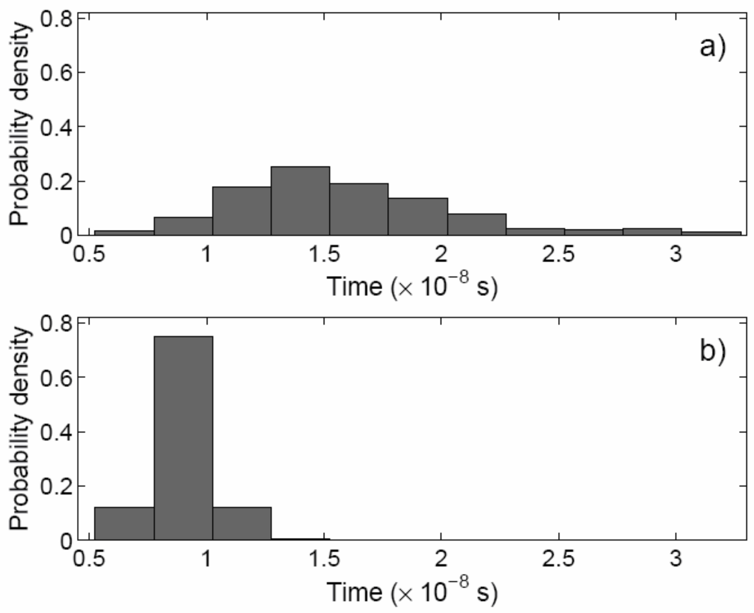


Fig. 8. Histogram of the drop-out events: a) laser without modulation and b) laser modulated at Ω_{PM} =1.97GHz**, and** A =1**.**

Numerical results and discussion

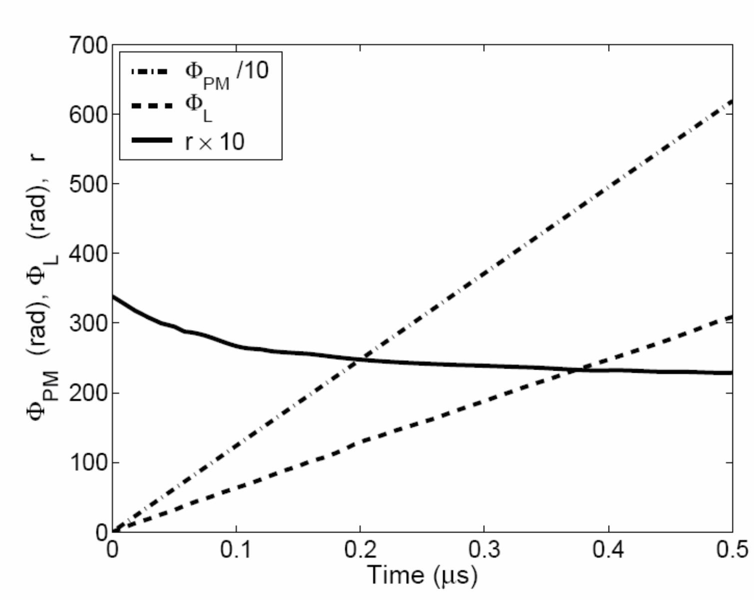
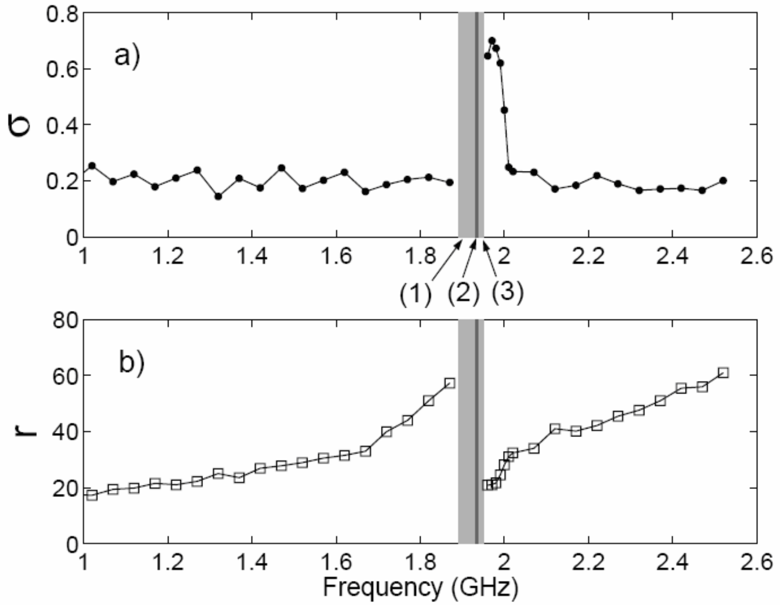


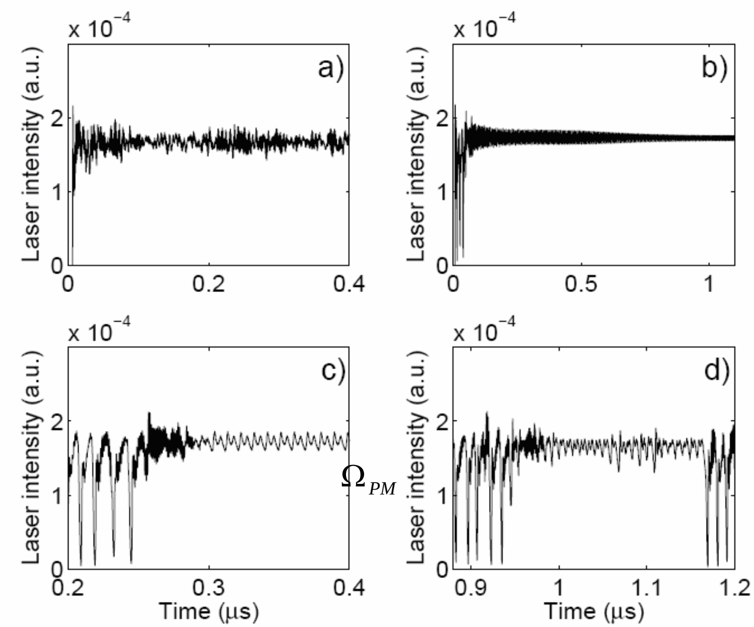
Fig. 9. The phases of the modulator Φ_{PM} and laser Φ_L and their instantaneous ratio $\mathbf{r} = \Phi_{PM} / \Phi_L$, for a modulation frequency Ω_{PM} =1.97 GHz, and A =1.

Fig. 10. σ and *r* versus the modulator frequency in the figures a) and b), respectively, for A = 1. Region (1) is characterized by chaos suppression and stabilization, region (2) by period-2 oscillations and region (3) by chaotic intermittency between the LFFs and the low amplitude chaotic oscillations. For $\sigma \geq 0.6$, the LFFs are steady and well synchronized with the modulating signal.



Numerical results and discussion

Fig. 11. Chaos suppression for Ω_{PM} =1.91 GHz in a), output stabilization for Ω_{PM} =1.93 GHz in b), period-2 oscillations for Ω_{PM}=1.94 GHz in c) and chaos intermittency between the LFFs and the low amplitude oscillations in d). The amplitude of the modulating signal is set to A=1.



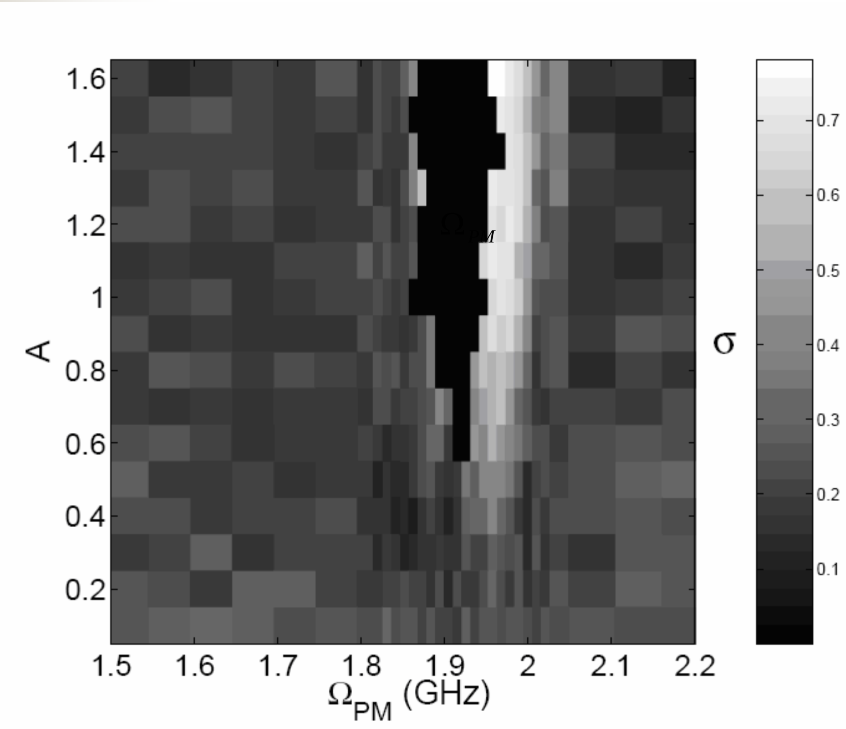
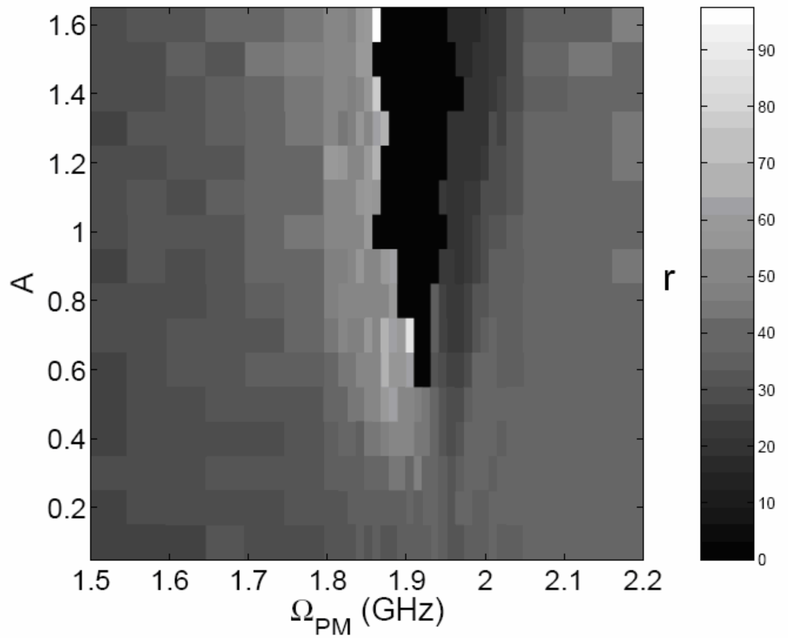


Fig. 12. Map of σ as a function of the modulator parameters Ω_{PM} and *A*. The colorbar indicates the local values of σ . The regions of synchronization between the laser's LFFs and the modulator are marked with white and light grey.

In black ($\sigma = 0$) is the region of non-LFFs states, i.e. states where the emission is stabilized, periodic, or chaotic but with a much lower amplitude.

Numerical results and discussion

Fig. 13. Map of *r* as a function of the modulator parameters Ω_{PM} and *A*. The colorbar indicates the local values of *r*. Notice the region colored uniformly in grey with triangular shape between Ω_{PM}=1.95 and 2.02 GHz and A > 0.7, which corresponds to values of *r* in the range 20 to 30. Within this region the LFFs are synchronized with the PM.



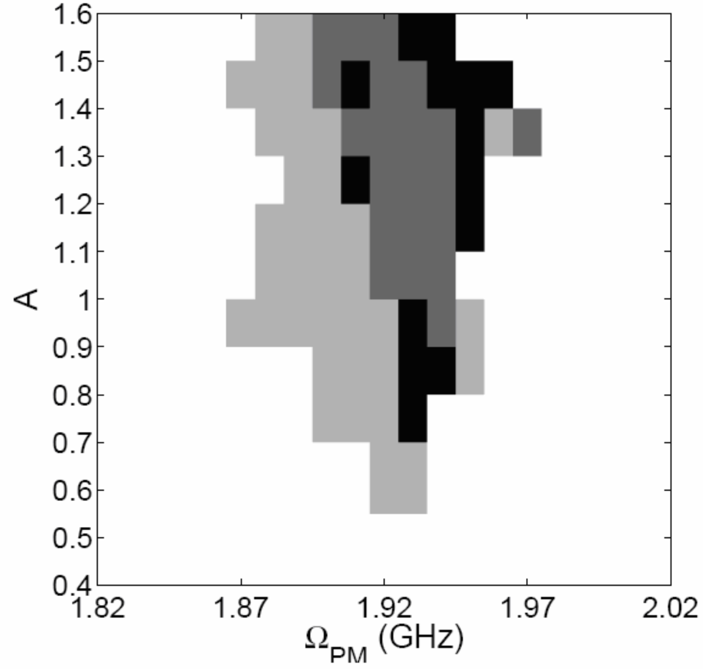


Fig. 14. The region of non-LFFs in the parameters space (Ω_{PM} ,*A*), are represented in black in the figures 12 and 13. The zone of stable emission is marked here in black. The region in dark grey corresponds to periodic laser output while the light grey zones correspond to low amplitude chaos, including the states characterized by intermittency between LFFs and low amplitude chaos.

Conclusions

- Numerical simulations show that the rate of power drops-out, in the emission of an external-cavity semiconductor laser, can be controlled with a phase modulator, placed inside the external cavity

- For some defined values of the modulating frequency, the light emission can be stabilized, from a regime of large amplitude chaotic oscillations (corresponding to the LFFs) to one of low amplitude chaotic or even periodic oscillation

- The two phases are locked to a constant ratio "m:n" - where m and n are integers - when the laser synchronizes with the modulator

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