**EE1330: Digital Signal Processing, Spring 2017**

Indian Institute of Technology Bhilai

HW 0, Assigned: 19.02.2017, Due: 25.02.2017.

# 1 Theory

1. Find the Fourier series of the following $2\pi$-periodic functions: (50)
   (a) $f(t) = t, -\pi < t < \pi.$
   (b) $f(t) = |t|, -\pi < t < \pi.$
   (c) $f(t) = \sin^2 t, -\pi < t < \pi.$
   (d) $f(t) = \cos 5t, -\pi < t < \pi.$
   (e) $f(t) = t^2, -\pi < t < \pi.$
   (f) $f(t) = \begin{cases} 0, -\pi < t < 0, \\ \sin t, 0 < t < \pi. \end{cases}$
   (g) $f(t) = |\cos t|, -\pi < t < \pi.$
   (h) $f(t) = \begin{cases} 1, -\pi/2 \leq t < \pi/2, \\ -1, (-\pi \leq t < -\pi/2) \cup (\pi/2 \leq t < \pi). \end{cases}$
   (i) $f(t) = 1, -\pi < t < \pi.$
   (j) $f(t) = \begin{cases} 0, -\pi < t < 0, \\ t,\ 0 < t < \pi. \end{cases}$

2. Find the Fourier tranform of the following functions: (50)
   (a) $f(t) = \begin{cases} 1, -0.5 \leq t \leq 0.5, \\ 0, \text{elsewhere.} \end{cases}$
   (b) $f(t) = e^{j\Omega_0 t}.$
   (c) $f(t) = e^{-a|t|}.$
   (d) $f(t) = \cos \Omega_0 t.$
   (e) $f(t) = \sin \Omega_0 t.$
   (f) If $f(t) \longleftrightarrow F(j\Omega)$, $f(t - t_0) \longleftrightarrow$ ?
   (g) $f(t) = e^{-t^2/2\sigma^2}.$
   (h) $f(t) = \delta(t).$
   (i) $f(t) = \begin{cases} 1 - t,\ 0 \leq t < 1, \\ 1 + t,\ -1 \leq t < 0, \\ 0,\ \text{elsewhere.} \end{cases}$
   (j) $f(t) = \begin{cases} 0,\ t < 0 \\ e^{-at},\ t \geq 0. \end{cases}$

3. Show that $x(t) \cdot y(t) \longleftrightarrow X(j\Omega) * Y(j\Omega)$, where $*$ corresponds to linear convolution. (10)

4. From first principles, derive the Nyquist sampling condition that guarantees perfect reconstruction. State any assumptions. (10)

5. The continuous time signal $x_c(t) = \sin(20\pi t) + \cos(40\pi t)$ is sampled with a sampling period $T$ to obtain the discrete-time signal $x[n] = sin(\frac{\pi n}{5}) + cos(\frac{2\pi n}{5})$.
   (a) Determine a choice for $T$ consistent with this information. (5)
   (b) Is your choice of $T$ unique? If so, why? If not, find another $T$ that satisfies this information. (5)

## 2 Programming (Matlab)

1. Synthesizing discontinous functions: Recall from class that Fourier series synthesis of functions with discontinuities turned out to be challenging. You will visualize this by writing code to synthesize any three of the functions in the first problem above. Make sure to pick functions that have discontinuities in them. Specifically, show the synthesis as you pick more terms in the Fourier series. Make note of any observations. (30)

2. Fun with sampling: In this problem, you will see the effect of sampling and aliasing. You can work with wav files from the source `http://aacapps.com/lamp/voices`.

   (a) Read a female voice file and a male voice file using *wavread* and play it back. You can use the *sound* function to do the playback. In this problem playback the files at the sampling rate returned by the *wavread* function. (10)

   (b) Now repeat the playback at the following sampling rates: 40,000, 35,000, 25,000, and 8,000 . Note down your observations. Can you explain your observations? You can rewrite the files at these sampling rates using *wavwrite*. This experiment should help you understand how voices in the movie Star Wars may have been generated – especially *Darth Vader*! (10)