

## 1 Theory

- Given the following input-output relationships for three different systems, could they be LTI? Why or why not? If LTI, could there be more than one system with the given input-output relationship?
  - System A:  $x[n] = 0.5^n, y[n] = 0.25^n$ . (5)
  - System B:  $x[n] = e^{jn/8}u[n], y[n] = 2e^{jn/8}u[n]$ . (5)
  - System C:  $x[n] = e^{jn/8}, y[n] = 2e^{jn/8}$ . (5)
- Find the frequency response  $H(e^{j\omega})$  of the linear time-invariant system whose input and output satisfy the difference equation  $y[n] - 0.5y[n-1] = x[n] + 2x[n-1] + x[n-2]$ . (10)
- Consider an LTI system with frequency response  $H(e^{j\omega}) = \frac{1-e^{-j\omega}}{1+0.5e^{-j4\omega}}, \pi < \omega \leq \pi$ . Find the output  $y[n]$  for all  $n$  if the input  $x[n]$  for all  $n$  is  $x[n] = \sin(\frac{\pi n}{4})$ . (10)
- Given  $x[n] = -\delta[n+3] + \delta[n+1] + 2\delta[n] + \delta[n-1] + \delta[n-3] + 2\delta[n-4] + \delta[n-5] - \delta[n-7]$ . Answer the following without explicitly evaluating  $X(e^{j\omega})$ .
  - Evaluate  $X(e^{j\omega})|_{\omega=0}$ . (5)
  - Evaluate  $X(e^{j\omega})|_{\omega=\pi}$ . (5)
  - Find  $\angle X(e^{j\omega})$ . (5)
  - Evaluate  $\int_{-\pi}^{\pi} X(e^{j\omega})d\omega$ . (5)

## 2 Programming

- Write a function in Python to implement the DTFT. As you know, the DTFT is a continuous function of  $\omega$ . To see it on a computer there is no option but to discretize  $\omega$ . The input to this function must be a discrete time signal and the number of points  $N$ . Evaluate the DTFT at  $N$  equally spaced frequencies in the interval  $-\pi \leq \omega < \pi$ . (10)

Test your function with the following systems:

- The averaging system of length  $M$ . Plot the magnitude and phase responses. Experiment with different sizes of  $M$  and  $N$  and note your observations. (5)
  - The backward differencing system. Plot the magnitude and phase responses. Experiment with different sizes of  $N$  and note your observations. (5)
- Compare your output to the `fft` function in Python. Use `fftshift` to center your DFT. (5)
  - Go to <http://aacapps.com/lamp/voices> and pick a male and female speech sample (use .wav format). Do the following:
    - Pick 20 milliseconds worth of samples and compute the DTFT using the function you wrote in Problem 1 above. Plot the magnitude and phase spectra for different values of  $N$  (10).
    - Using the convolution function you wrote for the first assignment, filter the speech signals using the averaging filter and the backwards differencing filters. Plot the magnitude and phase spectra for the filtered signals. What inference can you draw? Experiment with different values of  $M$  for the averaging filter. (10)
    - For those of you that are curious: write the filtered signal to a wav file and play it back. How does it sound?