EE1330: Digital Signal Processing, Spring 2017

Indian Institute of Technology Bhilai

<u>Assignment Solutions</u>

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1. Solution

(a) and (b)

(1.) (a.) $x[n] = e^{(j\pi n/6)}$

$\omega = \frac{\pi}{6} = 2\pi f$

$f = \frac{1}{12}$ (for a discrete signal to be periodic, f must be rational number.

$\Rightarrow f =$ rational no.

$\therefore x[n]$ is periodic

$T = \frac{1}{f} = 12$ sec

(b.) $x[n] = e^{j(3\pi n/4)}$

$\omega = \frac{3\pi}{4} = 2\pi f$

$\Rightarrow f = \frac{3}{8} =$ Rational no.

$\therefore x[n]$ is periodic signal.

$T = \frac{K}{f} = \frac{8K}{3}$ sec $= 8$; K = multiple of 3.

1. (c)

1.(c) $x[n] = [\sin(\frac{\pi n}{5})]/(\pi n)$

Let N be the period of $x[n]$

$$\frac{-1}{\pi n} \le \frac{\sin(\frac{\pi n}{5})}{\pi n} \le \frac{1}{\pi n}$$

Let this interval be

$$I_1 = \left[\frac{-1}{\pi n}, \frac{1}{\pi n}\right]$$

Since, $x[n]$ is periodic with period N

$$\Rightarrow \quad \frac{\sin(\pi(n+N))}{\pi(n+N)} = \frac{\sin(\frac{\pi n}{5})}{\pi n}$$

$$\frac{-1}{\pi(n+N)} \le \frac{\sin(\pi(n+N))}{\pi(n+N)} \le \frac{1}{\pi(n+N)}$$

$$I_2 = \left[\frac{-1}{\pi(n+N)}, \frac{1}{\pi(n+N)}\right]$$

Case I: if n is positive integer

then $I_2 \subset I_1$

$\Rightarrow$ all values of original signal $x[n]$ is not lying in the range of I_2

$\therefore$ contradiction

Hence, N is <u>not the period</u> of $x[n]$

$\therefore$ $x[n]$ is <u>non-periodic</u>

its amplitude is decreasing.

(2.) Solution

(a.)

(2.) (a) $T\{x[n]\} = \sum_{k=n_0}^{n} x[k]$

(1) Let $B_x \in \mathbb{R}^+$ & $|x[n]| < B_x \quad \forall\, n \in N$

$$|y[n]| = |T\{x[n]\}| = \left|\sum_{k=n_0}^{n} x[k]\right| < (n-n_0)B_x = B_y$$

Since $(n-n_0)B_x$ is a function of n

$\Rightarrow$ B_y may goes on increasing for increasing n.

$\therefore$ ~~follows~~ Given system is <u>not stable</u>.

(2.) A system is causal if its output depends on present and past inputs.

$$y[n] = T\{x[n]\} = \sum_{k=n_0}^{n} x[k] =$$

$$= x[n_0] + x[n_0+1] + \dots + x[n-1] + x[n]$$

it is obvious that $y[n]$ depends only on the summation of past ~~output~~ input and present ~~output~~ input $x[n]$.

$\therefore$ Given system is ~~cas~~ <u>causal</u> (if $n > n_0$)

if $n < n_0$ then system depends on future value. So <u>not casusal</u>

Q. (a) (3.) Let $T\{ax_1[n]\} = \sum_{K=n_0}^{n} a x_1[K] = a y_1[n]$

$$T\{bx_2[n]\} = \sum_{K=n_0}^{n} b x_2[K] = b y_2[n]$$

$$T\{ax_1[n] + bx_2[n]\} = \sum_{K=n_0}^{n} \left(a x_1[K] + b x_2[K]\right)$$

$$= \sum_{K=n_0}^{n} a x_1[K] + \sum_{K=n_0}^{n} b x_2[K]$$

$$= a y_1[n] + b y_2[n]$$

$\Rightarrow$ Given system is linear.

Q. (a)

(4.) $T\{x[n-n_0]\} = \sum_{k=n_0}^{n} x[k-n_0]$ — ①

$y[n-n_0] = \sum_{k=n_0}^{n-n_0} x[k]$ — ②

① & ② are not equal.

⇒ System is not time invariant.

(5.) Not memoryless system
because $y[n]$ is depending on the past inputs if $n > n_0$.
and if $n < n_0$ then $y[n]$ is depending on the future input values.

(2) (b)

(2.) (b) $T\{x[n]\} = ax[n] + b$

(1.) Let $B_x \in \mathbb{R}^+$ and $|x[n]| < B_x$

$$|y[n]| = |ax[n] + b| < |a||x[n]| + |b|$$
$$< |a| B_x + |b|$$
$$|y[n]| < B_y$$

$\Rightarrow$ System is <u>stable</u> ($\because$ BIBO)

(2.) System is <u>causal</u>

Output $y[n]$ is depending only on the present input $x[n]$

(3.) $T\{tx_1[n]\} = tax_1[n] + b$

$T\{px_2[n]\} = pax_1[n] + b$

Let $x_3[n] = tx_1[n] + px_2[n]$

$$T\{tx_1[n] + px_2[n]\} = a(tx_1[n] + px_2[n]) + b$$
$$\neq tT\{x_1[n]\} + pT\{x_2[n]\}$$

$\therefore$ System is <u>not -linear</u>

(2.) (b) (4.) $T\{x[n-n_0]\} = ax[n-n_0] + b$ — (i)

$y[n-n_0] = ax[n-n_0] + b$ — (ii)

(i) = (ii)

$\Rightarrow$ System is <u>time-invariant</u>

(5.) $y[n] = ax[n] + b$

Present output ~~is~~ depends on present input

$\Rightarrow$ System is <u>memoryless</u>

(2) (c)

(2) (c)

(1.) $T\{x[n]\} = x[n] + 3u[n+1]$

Let $B_x \in \mathbb{R}^+$ & $|x[n]| < B_x$

$\Rightarrow |y[n]| = |x[n] + 3u[n+1]|$

$< |x[n]| + 3|u[n+1]|$

$< B_x + 3$ $\quad (\because 3|u[n+1]| \leq 3)$ unit step function

$\Rightarrow |y[n]|$ is bounded.

$\therefore$ System is stable.

(2.) System is causal.
Since output $y[n]$ depends only on the present ~~and~~ values of input $x[n]$. ($u[n+1]$ is not the input signal)

(3.) $T\{ax_1[n]\} = ax_1[n] + 3u[n+1]$

$T\{bx_2[n]\} = bx_2[n] + 3u[n+1]$

Let $x_3[n] = (ax_1[n] + bx_2[n])$ be third input

$T\{x_3[n]\} = T\{ax_1[n] + bx_2[n]\} = ax_1[n] + bx_2[n] + 3u[n+1]$

$\neq aT\{x_1[n]\} + bT\{x_1[n]\}$

$\therefore$ Not linear

(Q) (C)

(4.) Delaying input;

$T\{x[n-n_0]\} = x[n-n_0] + 3u[n+1]$ — (i)

$y[n] = x[n] + 3u[n+1]$

Delaying output;

$y[n-n_0] = x[n-n_0] + 3u[n-n_0+1]$ — (ii)

(i) $\neq$ (ii)

$\Rightarrow$ System is <u>not time invariant</u>

(5.) <u>Memoryless system</u>

output just depends on current input value of $x[n]$

(3.) Solution

(3) If $x[n]$ is the input sequence to a LTI system then the output sequence is given by convolution sum as,

$$y[n] = \sum_{k=-\infty}^{\infty} (h[k] \cdot x[n-k])$$

where $h[k]$ is the unit impulse response of the system.

$$y[n] = \sum_{k=-\infty}^{-1} (h[k]\, x[n-k]) + \sum_{k=0}^{\infty} (h[k] \cdot x[n-k])$$

Now the first summation represents a weighted sum of future values of input, which must be zero for this system to be causal.

ie. $\sum_{k=-\infty}^{-1} (h[k]\, x[n-k]) = 0$

which can be guaranted only if
$h[k] = 0 \quad \forall\, k < 0$

$\therefore\, h[n] = 0 \quad \forall\, n < 0$

Hence Proved.

(4.) Solution

(a)

(4.) Discrete Convolution Sum:

(a.) $x[n] = 2\delta[n] - 1\delta[n-1]$

$h[n] = -\delta[n] + 2\delta[n-1] + \delta[n-2]$

$$y[n] = \sum_{k=-\infty}^{\infty} x[n-k]\, h[k]$$

$$= \sum_{k=-\infty}^{\infty} x[n-k]\left(-\delta[k] + 2\delta[k-1] + \delta[k-2]\right)$$

$$= -x[n-k]\Big|_{k=0} + 2x[n-k]\Big|_{k=1} + x[n-k]\Big|_{k=2}$$

$$= -x[n] + 2x[n-1] + x[n-2]$$

$$= -2\delta[n] + \delta[n-1] + 4\delta[n-1] - 2\cancel{\delta[n-2]} + 2\cancel{\delta[n-2]} - 1\delta[n-3]$$

$$= -2\delta[n] + 5\delta[n-1] + \cancel{\delta[n-2]} - \delta[n-3]$$

$$y[n] = -2\delta[n] + 5\delta[n-1] - \delta[n-3]$$

(4,) (b)

(b) $x[n] = \delta[n+2] + 2\delta[n+1] + \delta[n] + \delta[n-1]$

$h[n] = \delta[n] - \delta[n-1] + \delta[n-4] + \delta[n-5]$

$$y[n] = \sum_{k=-\infty}^{\infty} x[n-k]\, h[k]$$

$$= \sum_{k=-\infty}^{\infty} x[n-k]\big(\delta[k] - \delta[k-1] + \delta[k-4] + \delta[k-5]\big)$$

$$= x[n-k]\big|_{k=0} - x[n-k]\big|_{k=1} + x[n-k]\big|_{k=4} + x[n-k]\big|_{k=5}$$

$$= x[n] - x[n-1] + x[n-4] + x[n-5]$$

$$= \delta[n+2] + 2\delta[n+1] + \delta[n] + \delta[n-1] - \big(\delta[n+1] + 2\delta[n] + \delta[n-1] + \delta[n-2]\big) + \delta[n-2] + 2\delta[n-3] + \delta[n-4] + \delta[n-5] + \delta[n-3] + 2\delta[n-4] + \delta[n-5] + \delta[n-6]$$

$$y[n] = -\delta[n] + \delta[n+2] + \delta[n+1] + 3\delta[n-3] + 3\delta[n-4] + 2\delta[n-5] + \delta[n-6]$$

(5.) Solution

(5) (a) $h[n] = 4^n u[n]$

$$\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=-\infty}^{\infty} |4^n u[n]| = \sum_{n=-\infty}^{\infty} |4^n| = \sum_{n=0}^{\infty} |4^n| = \infty$$

$h[n]$ is <u>not stable</u> as it is not absolutely summable. It goes to ∞ as $n \to \infty$

(b) $h[n] = u[n] - u[n-10]$

$$= \begin{cases} 0 & n<0 \\ 1 & 0 \le n < 10 \\ 0 & n \geqslant 10 \end{cases}$$

$$\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=0}^{9} 1 = 10 < \infty$$

Hence, $h[n]$ is absolutely summable
$\Rightarrow$ $h[n]$ is <u>stable</u>

(c) $h[n] = 3^n u[-n-1] = \begin{cases} 3^n & , n \le -1 \\ 0 & , n > -1 \end{cases}$

$$\sum_{n=-\infty}^{\infty} |h[n]| = |3^n u[-n-1]| = \sum_{n=-\infty}^{-1} 3^n$$

$$= \sum_{K=1}^{\infty} \frac{1}{3^K} = \frac{1}{2} < \infty$$

$\Rightarrow$ $h[n]$ is absolutely summable.
Hence $h[n]$ is <u>stable</u>.

(d) $h[n] = \left(\frac{3}{4}\right)^{|n|} \cos(\pi n/4 + \pi/4)$

$$|h[n]| < \left(\frac{3}{4}\right)^{|n|} < 1$$

$$\Rightarrow \sum_{n=-\infty}^{\infty} |h[n]| < \sum_{n=-\infty}^{\infty} \left(\frac{3}{4}\right)^{|n|} = 2\sum_{n=1}^{\infty}\left(\frac{3}{4}\right)^{n} + 1$$

$$\Rightarrow \sum_{n=-\infty}^{\infty} |h[n]| < 7 \quad (\text{using G.P})$$

$\therefore$ $h[n]$ is absolutely summable and hence LTI system is stable

(e) $h[n] = 2u[n+5] - u[n] - u[n-5]$

$$= \begin{cases} 0 & , n < -5 \\ 2 & , -5 \le n < 0 \\ 1 & , 0 \le n < 5 \\ 0 & , 5 \le n \end{cases}$$

$$\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=-\infty}^{\infty} |2u[n+5] - u[n] - u[n-5]|$$

$$= \sum_{n=-5}^{-1} 2 + \sum_{n=0}^{4} 1 = 10 + 5 = 15$$

$\Rightarrow$ $h[n]$ is absolutely summable and Hence, $h[n]$ is stable

(6.) Solution

(a)

(6.) Given;

$$x_0[n] = \delta[n+1] + 2\delta[n] + \delta[n-1]$$

$$y_0[n] = -\delta[n+2] - 2\delta[n+1] + 2\delta[n-1] + \delta[n-2]$$

(a)

$$x_1[n] = \delta[n-1] + 2\delta[n-2] + 3\delta[n-3] + 4\delta[n-4] + 3\delta[n-5] + 2\delta[n-6] + \delta[n-7]$$

$$\Rightarrow x_1[n] = x_0[n-2] + 2x_0[n-4] + x_0[n-6]$$

$$T\{x_1[n]\} = y_1[n] = T\{x_0[n-2] + 2x_0[n-4] + x_0[n-6]\}$$

$$= y_0[n-2] + 2y_0[n-4] + y_0[n-6]$$

(Since, x_0 is input of LTI system)

$$= -\delta[n] - 2\delta[n-1] - 2\delta[n-2] - 2\delta[n-3] + 2\delta[n-5] + 2\delta[n-6] + 2\delta[n-7] + \delta[n-8]$$

$$\Rightarrow \boxed{T\{x_1[n]\} = y[n] = -\delta[n] + 2\left(-\delta[n-1] - \delta[n-2] - \delta[n-3] + \delta[n-5] + \delta[n-6] + \delta[n-7]\right) + \delta[n-8]}$$

(b)

6(b) $y[n] = x[n] * h[n]$

Taking fourier Transform

$$\mathcal{F}\{y[n]\} = \mathcal{F}\{x[n]\}\,\mathcal{F}\{h[n]\}$$

$$\mathcal{F}\{h[n]\} = \frac{\mathcal{F}\{y[n]\}}{\mathcal{F}\{x[n]\}}$$

$$h[n] = \mathcal{F}^{-1}\left\{\frac{\mathcal{F}\{y[n]\}}{\mathcal{F}\{x[n]\}}\right\}$$

$$y_0[n] = -\delta[n+2] - 2\delta[n+1] + 2\delta[n-1] + \delta[n-2]$$

Since $\mathcal{F}\{\delta[n-n_0]\} = e^{-j\omega n_0}$

$$\mathcal{F}\{y_0[n]\} = -e^{2j\omega} - 2e^{j\omega} + 2e^{-j\omega} + e^{-2j\omega}$$

$$\{x_0[n]\} = \delta[n+1] + 2\delta[n] + \delta[n-1]$$

$$\mathcal{F}\{x_0[n]\} = e^{-j\omega} + 2 + e^{j\omega}$$

$$h[n] = \mathcal{F}^{-1}\left\{\frac{2(e^{-j\omega} - e^{j\omega}) + (e^{-j\omega} - e^{j\omega})(e^{-j\omega} + e^{j\omega})}{e^{j\omega} + 2 + e^{-j\omega}}\right\}$$

$$h[n] = \mathcal{F}^{-1}\left\{\frac{(e^{-j\omega} - e^{j\omega})(e^{-j\omega} + e^{j\omega} + 2)}{(e^{-j\omega} + e^{j\omega} + 2)}\right\}$$

$$h[n] = \mathcal{F}^{-1}(e^{-j\omega} - e^{j\omega})$$

$$\boxed{h[n] = \delta[n-1] - \delta[n+1]}$$