EE1330: Digital Signal Processing, Spring 2017

Indian Institute of Technology Bhilai

Assignment Solutions

1.Theory

Question 1

(1.) (a) $x[n] = 0.5^n$ $y[n] = 0.25^n$

The system is not LTI because if the system would have been LTI, the output should be in the form of $A(0.5)^n$

(b) $x[n] = e^{jn/8} u[n]$ $y[n] = 2e^{jn/8} u[n]$

The system may be LTI but does not have to be. For example, for any input other than the given one, the system may output zero, making this system non-LTI

$\Rightarrow$ the system can be LTI and there is only one LTI system that satisfies this input-output constraint.

(c) $x[n] = e^{jn/8}$ $y[n] = 2e^{jn/8}$

The information given shows that the system satisfies the eigen-function property of exponential sequences for LTI systems for one particular eigenfunction input.

However, we donot know the system response for any other eigen function input.

$\Rightarrow$ The system can be LTI, but cannot be uniquely determined from the information in this input-output constaint

Question 2

Q. $y[n] - 0.5y[n-1] = x[n] + 2x[n-1] + x[n-2]$

Since,

$$Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega})$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

Shifting property of DTFT,

$$x[n-n_0] \Leftrightarrow X(e^{j\omega})e^{-j\omega n_0}$$

$$\Rightarrow \text{DTFT}\left\{ y(n) - 0.5y[n-1] = x[n] + 2x[n-1] + x[n-2] \right\}$$

$$\Rightarrow Y(e^{j\omega}) - 0.5Y(e^{j\omega})e^{-j\omega} = X(e^{j\omega}) + 2X(e^{j\omega})e^{-j\omega} + X(e^{j\omega})e^{-j\omega 2}$$

$$\Rightarrow \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1 + 2e^{-j\omega} + e^{-2j\omega}}{1 - 0.5e^{-j\omega}}$$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1 + 2e^{-j\omega} + e^{-2j\omega}}{1 - 0.5e^{-j\omega}}$$

Question 3

(3.) $x[n] = \sin\left(\frac{\pi n}{4}\right)$

$$x[n] = \frac{e^{j\pi n/4} - e^{-j\pi n/4}}{2j}$$

Since the input is of the form $e^{j\omega n}$. So the output should be of the form $A\,x[n]$

$\Rightarrow y(n) = (\text{eigenvalue})(\text{eigenfunction})$

$\Rightarrow y[n] = A\,x[n]$ where $A = \text{DTFT of } h(n)$
$= H(e^{j\omega})$

$$\Rightarrow y(n) = \frac{H(e^{j\pi/4})\, e^{j\pi n/4} - H(e^{-j\pi/4})\, e^{-j\pi n/4}}{2j}$$

Now,

$$H(e^{j\pi/4}) = \frac{1 - e^{-j\pi/4}}{1 + 0.5\, e^{-j\pi}} = \frac{1 - (\cos\pi/4 - j\sin\pi/4)}{1 + 0.5[\cos\pi - j\sin\pi]}$$

$$= \frac{1 - (1/\sqrt{2} - j\,1/\sqrt{2})}{1 + 0.5\,(-1-0)} = \frac{1 - 1/\sqrt{2} + j/\sqrt{2}}{0.5} = 2(1 - e^{-j\frac{\pi}{4}})$$

$$H(e^{-j\pi/4}) = \frac{1 - e^{j\pi/4}}{1 + 0.5\, e^{j\pi}} = \frac{1 - (\cos\pi/4 + j\sin\pi/4)}{1 + 0.5(\cos\pi + j\sin\pi)}$$

$$= \frac{1 - (1/\sqrt{2} + j\,1/\sqrt{2})}{1 + 1/2\,(-1+0)} = 2 - \sqrt{2} - j\sqrt{2} = 2(1 - e^{j\pi/4})$$

$$\Rightarrow y[n] = \frac{e^{j\pi n/4} \cdot \cancel{2}(1 - e^{-j\pi/4}) - e^{-j\pi n/4} \cdot \cancel{2}(1 - e^{j\pi/4})}{\cancel{2}j}$$

$$= 2\sin(\pi n/4) - 2\sin(\pi n/4 - \pi/4)$$

$$= 2\left[2\cos\left(\frac{\frac{\pi n}{2} - \frac{\pi}{4}}{2}\right)\sin\left(\frac{\pi/4}{2}\right)\right]$$

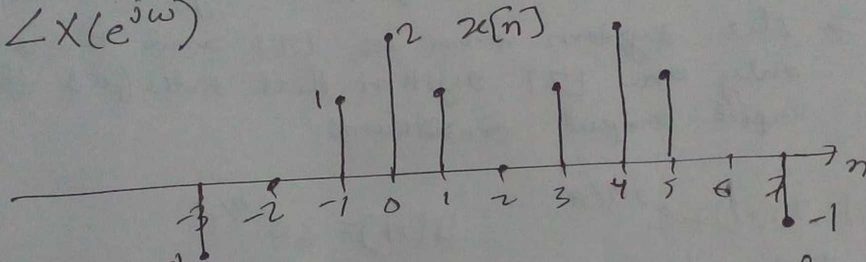
Question 4

(4.) $x[n] = -\delta[n+3] + \delta[n+1] + 2\delta[n] + \delta[n-1] + \delta[n-3] +$
$2\delta[n-4] + \delta[n-5] - \delta[n-7]$

(a.) $X(e^{j\omega})\Big|_{\omega=0} = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}\Big|_{\omega=0} = \sum_{n=-\infty}^{\infty} x[n]$
$= 6$

(b.) $X(e^{j\omega})\Big|_{\omega=\pi} = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\pi n} = \sum_{n=-\infty}^{\infty} x[n](-1)^n = 2$

(c.) $\angle X(e^{j\omega})$



Because $x[n]$ is symmetric about $n=2$, the signal has linear phase,
$X(e^{j\omega}) = A(\omega)e^{-2j\omega}$
$A(\omega)$ is a zero phase(real) function of ω
Hence, $\angle X(e^{j\omega}) = -2\omega \quad , \quad -\pi \leq \omega \leq \pi$

(d.) $\int_{-\pi}^{\pi} X(e^{j\omega})\,d\omega$
We know that $\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}\,d\omega = 2\pi x[n]$
Put $n=0$,
$\Rightarrow \int_{-\pi}^{\pi} X(e^{j\omega})\,d\omega = 2\pi x[0] = 4\pi$