Expected Value & Standard Error

What does the sum look like?

Review

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Box Model

… *is like* drawing ____ times from the box _____ with replacement and summing the draws.

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… *is like* drawing ____ times from the box _____ with replacement and summing the draws.

Example #1

Rolling a die 40 times and summing the numbers shown

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Rolling a die 40 times and summing the numbers shown

…is like…

drawing 40 times from the box 1, 2, 3, 4, 5, 6 with replacement and summing the draws.

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Box Model

… *is like* drawing ____ times from the box _____ with replacement and summing the draws.

Example #2

Rolling a die 40 times and counting the number of aces

…is like…

drawing ____ times from the box _____ with replacement and summing the draws.

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Box Model

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Rolling a die 40 times and counting the number of aces

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Box Model

… *is like* drawing ____ times from the box _____ with replacement and summing the draws.

Example #2

Rolling a die 40 times and counting the number of aces

…is like…

drawing 40 times from the box 0, 0, 0, 0, 0, 1 with replacement and summing the draws.

Fact

If we execute the box model, the result is a sum.

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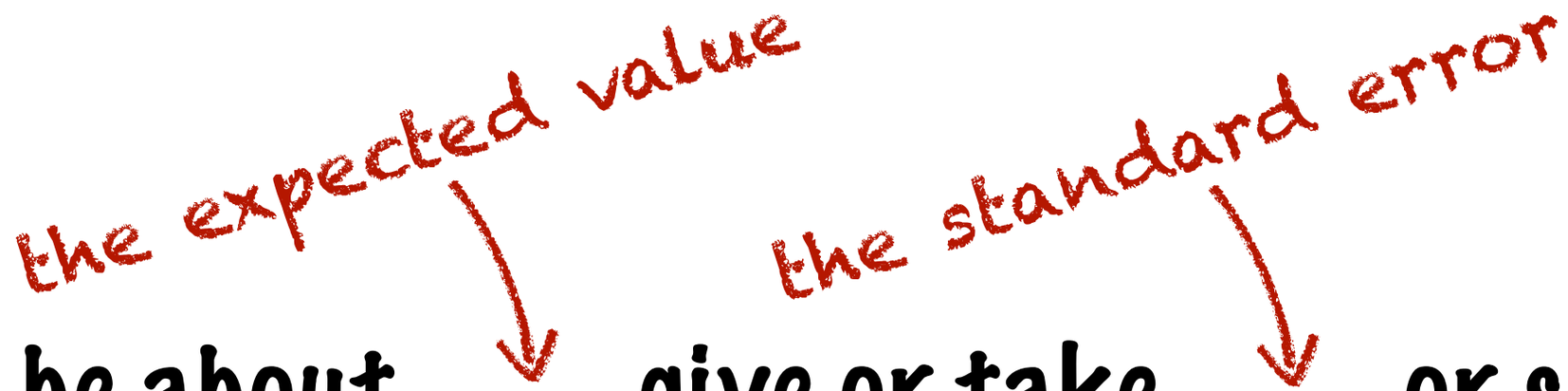

The sum will be about _____ give or take _____ or so.

Question

What can we say about this (yet to be produced) sum?

the expected value

the standard error

The sum will be about _____ give or take _____ or so.

Suppose an actual list of numbers.

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The entries in that list are about [the average] give or take [the SD] or so.

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Suppose a <u>hypothetical list</u> of numbers that we generate by executing the box model an infinite number of times.

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Suppose a hypothetical list of numbers that we generate by executing the box model an infinite number of times.

The (hypothetical) average is the expected value.

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The (hypothetical) SD is the standard error.

We can think of the expected value and standard error as a "long-run" average and SD of a chance process.

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We can think of the expected value and standard error as a "long-run" average and SD of a chance process.

	actual list of numbers	**chance process**
typical value	average	expected value
give or take	SD	standard error

Equations

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expected value for sum $=$ (**number of draws**) $\times$ (**average of box**)

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$$\textbf{expected value for sum} = (\textbf{number of draws}) \times (\textbf{average of box})$$

$$\textbf{SE for sum} = \sqrt{\textbf{number of draws}} \times (\textbf{SD of box})$$

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Helpful Hints

Suppose the box is a "big-small box" that only has big numbers B and small numbers S (e.g., the box 2, 2, 2, 2, 14, 14), then

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Suppose the box is a "big-small box" that only has big numbers B and small numbers S (e.g., the box 2, 2, 2, 2, 14, 14), then

$$\textbf{SD of big-small box} = (\textbf{B} - \textbf{S}) \times \sqrt{(\textbf{fraction that are B}) \times (\textbf{fraction that are S})}$$

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Suppose the box is a "0-1 box" that only has 0s and 1s (e.g., the box 0, 0, 0, 1), then

Equations

expected value for sum $=$ (**number of draws**) $\times$ (**average of box**)

SE for sum $= \sqrt{\textbf{number of draws}} \times$ (**SD of box**)

Helpful Hints

Suppose the box is a "big-small box" that only has big numbers B and small numbers S (e.g., the box 2, 2, 2, 2, 14, 14), then

SD of big-small box $=$ (**B** $-$ **S**) $\times \sqrt{}$(**fraction that are B**) $\times$ (**fraction that are S**)

Suppose the box is a "0-1 box" that only has 0s and 1s (e.g., the box 0, 0, 0, 1), then

SD of 0-1 box $= \sqrt{}$(**fraction that are 0**) $\times$ (**fraction that are 1**)

Example #1

Rolling a die 40 times and summing the numbers shown

…is like…

drawing 40 times from the box 1, 2, 3, 4, 5, 6 with replacement and summing the draws.

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The sum will be about _____ give or take _____ or so.

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the expected value

The sum will be about _____ give or take _____ or so.

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drawing **40** times from the box **1, 2, 3, 4, 5, 6** with replacement and summing the draws.

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the standard error

The sum will be about ____ give or take ____ or so.

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The sum will be about ____ give or take ____ or so.

$$\text{expected value for sum} = (\text{number of draws}) \times (\text{average of box})$$
$$= 40 \times 3.5$$
$$= 140$$

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Rolling a die 40 times and summing the numbers shown

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drawing 40 times from the box 1, 2, 3, 4, 5, 6 with replacement and summing the draws.

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the standard error

The sum will be about 140 give or take ____ or so.

$$\text{expected value for sum} = (\text{number of draws}) \times (\text{average of box})$$
$$= 40 \times 3.5$$
$$= 140$$

Example #1

Rolling a die 40 times and summing the numbers shown

…is like…

drawing 40 times from the box 1, 2, 3, 4, 5, 6 with replacement and summing the draws.

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The sum will be about 140 give or take ____ or so.

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Rolling a die 40 times and summing the numbers shown

…is like…

drawing **40** times from the box **1, 2, 3, 4, 5, 6** with replacement and summing the draws.

the expected value

the standard error

The sum will be about 140 give or take ____ or so.

$$\text{SE for sum} = \sqrt{\text{number of draws}} \times (\text{SD of box})$$

$$= \sqrt{40} \times ???$$

$$= 6.32 \times 1.71$$

$$= 10.81$$

```
> x <- c(1, 2, 3, 4, 5, 6)
> sqrt(mean((x - mean(x))^2))
[1] 1.707825
```

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Rolling a die 40 times and summing the numbers shown

…is like…

drawing 40 times from the box 1, 2, 3, 4, 5, 6 with replacement and summing the draws.

the expected value

the standard error

The sum will be about 140 give or take 11 or so.

$$\text{SE for sum} = \sqrt{\text{number of draws}} \times (\text{SD of box})$$

$$= \sqrt{40} \times ???$$

$$= 6.32 \times 1.71$$

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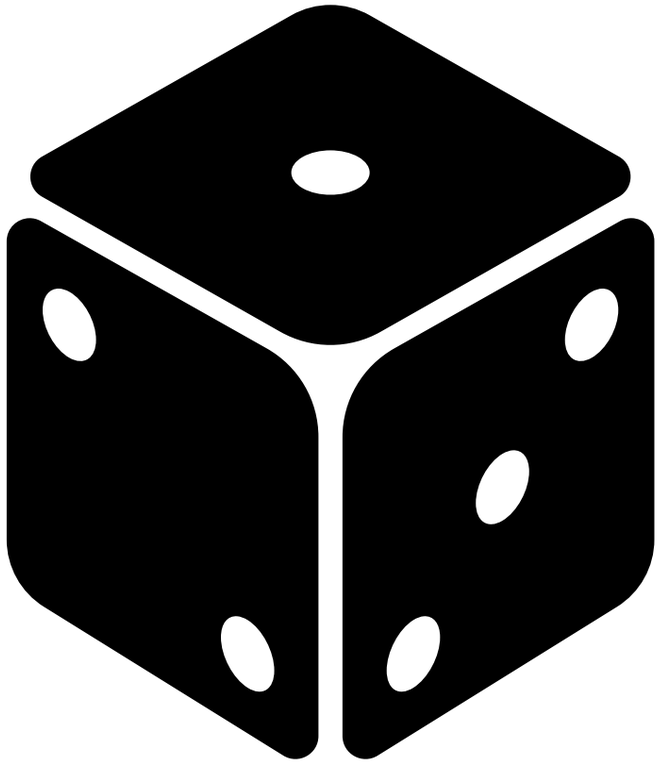
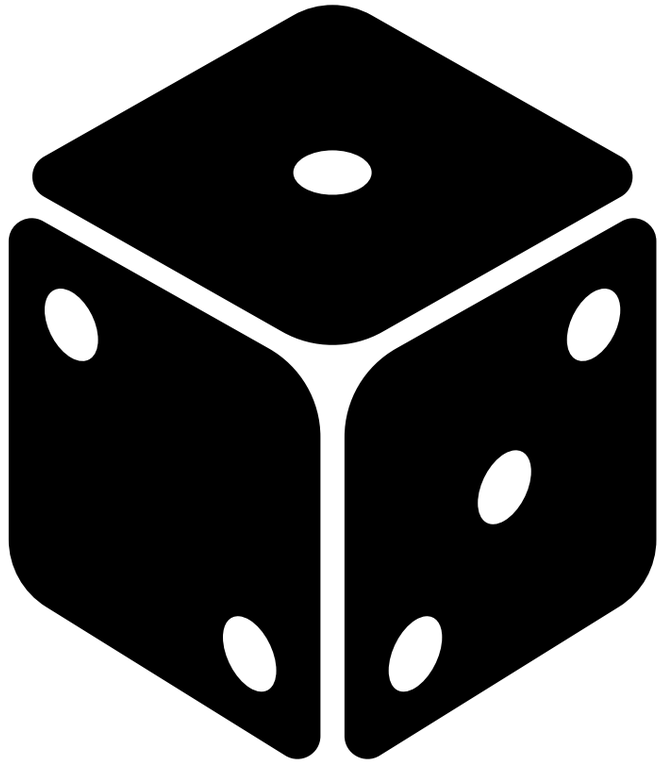
```
> x <- c(1, 2, 3, 4, 5, 6)
> sqrt(mean((x - mean(x))^2))
[1] 1.707825
```

I rolled a die 40 times and got a sum of 141. That's right in line with our claim that the sum will be about 140 give or take 11 or so.



I rolled a die 40 times and got a sum of 141. That's right in line with our claim that the sum will be about 140 give or take 11 or so.



I did it nine more times and got 116, 137, 143, 146, 130, 126, 136, 128, and138. Again, that's right in line with our claim that the sum will be about 140 give or take 11 or so.

Highlights

- If we have a chance process, we can sometimes describe it with a **box model**.

- If we have a box model, then we can say compute the **expected value** and the **standard error**.

$$\textbf{expected value for sum} = (\textbf{number of draws}) \times (\textbf{average of box})$$

$$\textbf{SE for sum} = \sqrt{\textbf{number of draws}} \times (\textbf{SD of box})$$

- With the expected value and standard error, we can fill in the following: **The sum will be about ___ give or take ___ or so.**