پوریا صامتی - پاسخ تمرینات تشریحی سری چهارم

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پاسخ ۲)

a)

$$J(w,b,\alpha,\varepsilon) = \frac{w^T w}{2} + \frac{C}{2}\sum_{i=1}^{m} \varepsilon_i^2 - \sum_{i=1}^{m} \alpha_i \left[y_i (w^T x_i + b) - 1 + \varepsilon_i \right]$$

b)

$$\nabla J = 0 \Rightarrow \frac{\partial J}{\partial w} = w + 0 - \sum_{i=1}^{m} \alpha_i y_i x_i + 0 + 0 \Rightarrow \boxed{w = \sum_{i=1}^{m} \alpha_i y_i x_i}$$

$$\frac{\partial J}{\partial b} = 0 + 0 - \sum_{i=1}^{m} \alpha_i y_i + 0 + 0 \Rightarrow \boxed{\sum_{i=1}^{m} \alpha_i y_i = 0}$$

$$\frac{\partial J}{\partial \varepsilon} = 0 + \frac{C}{2}\sum_{i=1}^{m} 2\varepsilon_i - \sum_{i=1}^{m} \alpha_i \Rightarrow C\sum_{i=1}^{m} \varepsilon_i - \sum_{i=1}^{m} \alpha_i = 0 \Rightarrow \boxed{C\sum_{i=1}^{m} \varepsilon_i = \sum_{i=1}^{m} \alpha_i}$$

$$\Rightarrow \sum_{i=1}^{m} \varepsilon_i = \sum_{i=1}^{m} \frac{\alpha_i}{C}$$

استفاده از فرضیات b که بدست آوردیم.

پاسخ c)

$$j(w,b,\alpha,\varepsilon) = \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m}(\alpha_i y_i x_i)^T(\alpha_j y_j x_j) + \frac{1}{2}\sum_{i=1}^{m}\alpha_i\varepsilon_i$$

$$- \sum_{i=1}^{m}\alpha_i\left[y_i\left[\left[\sum_{j=1}^{m}\alpha_j y_j x_j\right]^T x_i + b\right] - 1 + \varepsilon_i\right]$$

$$= \frac{-1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m}\alpha_i\alpha_j y_i y_j x_i^T x_j + \frac{1}{2}\sum_{i=1}^{m}\alpha_i\varepsilon_i$$

$$0 \leftarrow - \boxed{\left(\sum_{i=1}^{m}\alpha_i y_i\right)} b + \sum_{i=1}^{m}\alpha_i - \sum_{i=1}^{m}\alpha_i\varepsilon_i$$

$$= \sum_{i=1}^{m}\alpha_i - \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m}\alpha_i\alpha_j y_i y_j x_i^T x_j - \frac{1}{2}\sum_{i=1}^{m}\alpha_i\varepsilon_i \quad \varepsilon_i \to \frac{\alpha_i}{C}$$

$\Rightarrow$

$$\text{dual Form} \Rightarrow \sum_{i=1}^{m}\alpha_i - \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m}\alpha_i\alpha_j y_i y_j x_i^T x_j - \frac{1}{2}\sum_{i=1}^{m}\frac{(\alpha_i)^2}{C}$$

پاسخ 3)

	x	y
x_1	4	11
x_2	8	4
x_3	13	5
x_4	7	14

$$\mu_x = \frac{4+8+13+7}{4} = 8$$

$$\mu_y = \frac{11+4+5+14}{4} = 8.5$$

تشکیل ماتریس Cov $\Big\} \Rightarrow \Sigma = \frac{1}{N-1}(x-\mu)(x-\mu)^T = \begin{bmatrix} 14 & -11 \\ -11 & 23 \end{bmatrix}$

محاسبه eigen Values $\Big\}$ $\det(\Sigma - \lambda I) = 0 \Rightarrow \det \begin{vmatrix} 14-\lambda & -11 \\ -11 & 23-\lambda \end{vmatrix} = 0 \Rightarrow (14-\lambda)(23-\lambda) - (-11)(-11)$

$$\Rightarrow \lambda^2 - 37\lambda + 201 = 0$$

$\Rightarrow$ eigen Values $\begin{cases} \lambda_1 = 30.3 \longrightarrow \text{مقدار ویژه دارای ماکزیمم} \\ \lambda_2 = 6.6 \end{cases}$

محاسبه بردار ویژه متناسب با λ_1 $\Big\} \Rightarrow (\Sigma - \lambda_1 I)V = 0 \Rightarrow \begin{bmatrix} 14-30.3 & -11 \\ -11 & 23-30.3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = 0$

$$\Rightarrow \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 11 \\ -16.38 \end{bmatrix}$$

محاسبه داده‌های جدید

$$X_1' = \begin{bmatrix} 4 \\ 8 \end{bmatrix}^T \begin{bmatrix} 11 \\ -16.38 \end{bmatrix} = -87.04$$

$$X_2' = \begin{bmatrix} 8 \\ 4 \end{bmatrix}^T \begin{bmatrix} 11 \\ -16.38 \end{bmatrix} = 22.48$$

$$X_3' = \begin{bmatrix} 13 \\ 5 \end{bmatrix}^T \begin{bmatrix} 11 \\ -16.38 \end{bmatrix} = 61.1$$

$$X_4' = \begin{bmatrix} 7 \\ 14 \end{bmatrix}^T \begin{bmatrix} 11 \\ -16.38 \end{bmatrix} = -152.32$$