Graphs

Assignment Project Exam Help

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February 2018

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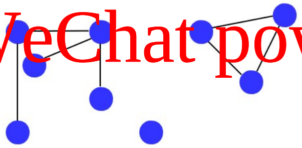

Introduction

Graphs are fundamental to much of computer science

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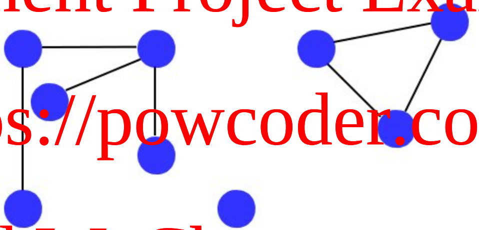

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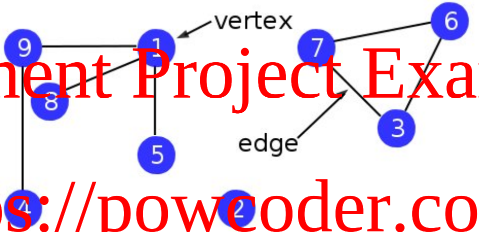
- We have already seen how trees are used as data structures
- All sorts of problems can be modelled using graphs
- Networks, images, programs, anything involving related objects

Graph Terminology



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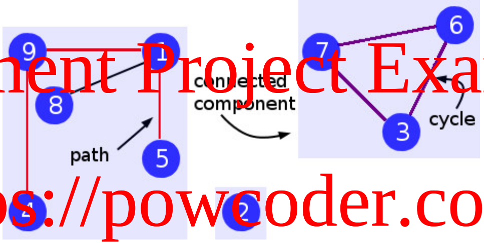
Definition

A graph G is a pair (V, E) where V is a finite set (of objects) and E is a binary relation on V. Elements of V are called vertices and elements of E are called edges.

- E is a set of pairs of vertices: $\{u, v\}$ such that there is an edge between u and v
- Vertices u and v are adjacent if there is an edge $\{u, v\}$

Graph Terminology



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- A path from v_1 to v_n, written $v_1 \rightsquigarrow v_n$, is a sequence $\langle v_1, v_2, \ldots, v_n \rangle$ such that there is an edge $\{v_i, v_{i+1}\}$ for all i, $1 \leq i < n$
- A cycle exists if there is a path from v to v, containing at least 4 vertices, for some vertex v
- Vertex v is reachable from vertex u if $u = v$, or if there is a path $u \rightsquigarrow v$
- A connected component (also just called a component) is a set of vertices all reachable from each other

Graph Representation

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Question

How should a graph be represented as a data structure?

- A graph vertex is connected to 0-to-many other vertices
- Going to assume that $|V|$ is fixed

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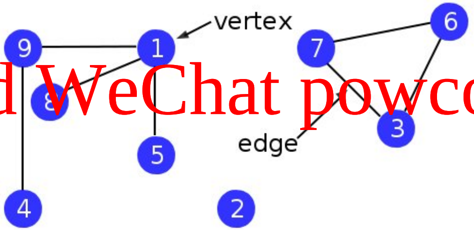

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Graph Representation

Two common ways:

- Adjacency List(s): $adj[u]$ contains v if there is an edge $\{u, v\}$
- Adjacency Matrix: $adj_{uv} = 1$ if there is an edge $\{u, v\}$, else 0

Adjacency lists good for sparse graphs

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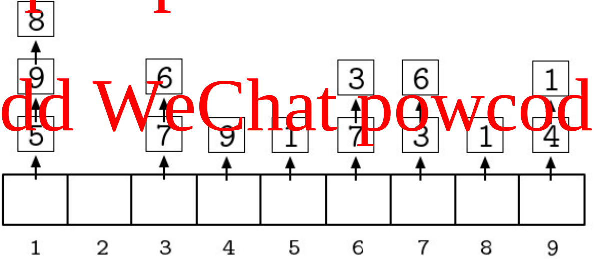

Graph Search

Question

Why search a graph?

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- Searching a graph is like iterating through an ordered structure
- Want to use data in the graph for some computation

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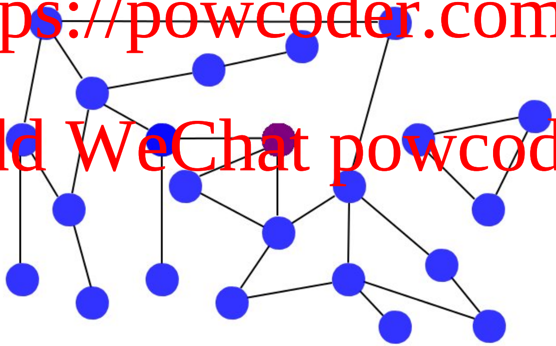

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Graph Search Actions

Searching a graph has two actions:

- Find adjacent vertices
- Visit vertices not found before

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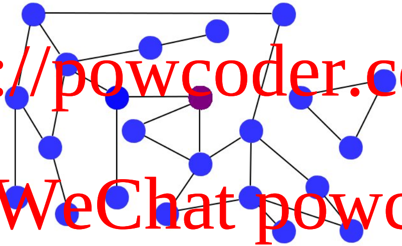

- Visiting means using the vertex: includes finding further vertices
- Vertices are visited in the order they are first found
- Vertices are coloured when they are first found/visited

Breadth-First Search

Question

What is a breadth-first search of a graph?

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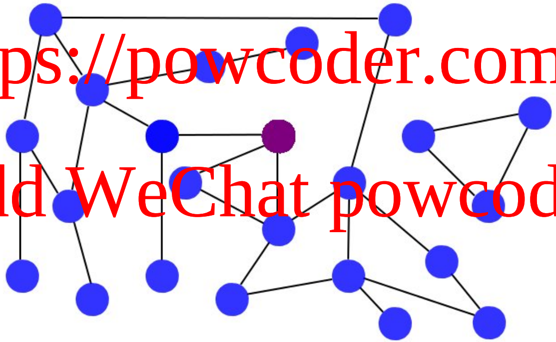

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Breadth-First Search

In breadth-first search

- Visit a vertex v (starting with source vertex s)
- Find all vertices adjacent to v before visiting another
- Result: search proceeds gradually down every path at the same rate

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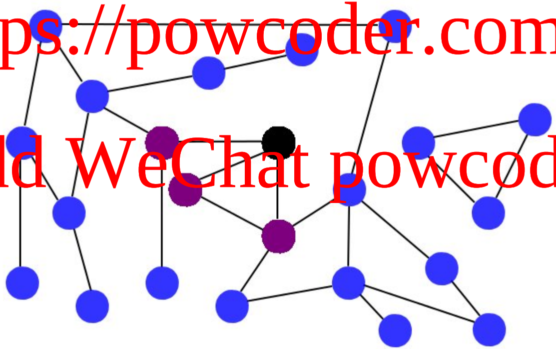

BFS Procedure

Question

How would you implement BFS?

- Inputs are graph g and vertex s
- $g.adj[u]$ returns list of vertices
- $g.vertices$ is number of vertices
- Objective: find all reachable vertices (will add actions later)

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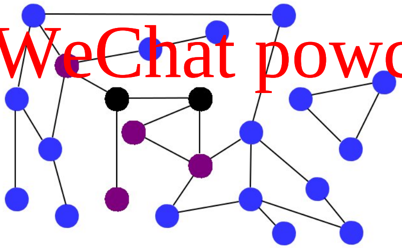

Breadth-First Search

BFS (Input: graph g, vertex s)

```
found = new boolean[g.vertices]
q = new Queue(s)                    // FIFO queue
while q is not empty
  u = q.remove()
  for v in g.adj[u]
    if not found[v]                 // avoid loops
      found[v] = true
      q.add(v)
```

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- The use of a (FIFO) queue is characteristic of BFS
- By convention only search from given s

Shortest Paths

BFS searches all paths at the same rate, so ...

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Question

How would you modify the BFS procedure to find the length (number of edges) of the shortest path from s to every other vertex?

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BFS (Input: graph g, vertex s)

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```
found = new boolean[g.vertices]
q = new Queue(s)              // FIFO queue
while q is not empty
  u = q.remove()
  for v in g.adj[u]
    if not found[v]           // avoid loops
      found[v] = true
      q.add(v)
```

Shortest Paths

BFS (Input: graph g, vertex s)

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```
q = new Queue(s)
dist = new int[g.vertices]
dist.fill(-1)
dist[s] = 0
while q is not empty
  u = q.remove()
  for v in g.adj[u]
    if dist[v] == -1          // not found
      dist[v] = dist[u] + 1
      q.add(v)
```

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- The distance is recorded when a vertex is (first) found
- Arrays of size $|V|$ like *dist* are also common in graph search
- Unreachable vertices have `dist[v] = -1`

Time

Question

For a connected graph with V vertices and E edges, how long does BFS take?

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BFS (Input: graph g, vertex s)

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```
found = new boolean[g.vertices]
q = new Queue(s)
while q is not empty
  u = q.remove()
  for v in g.adj[u]
    if not found[v]
      found[v] = true
      q.add(v)
```

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BFS Time Complexity

- Each vertex is added and removed from the queue exactly once
- Each adjacency list is used exactly once
- Each edge contributes exactly two vertices to the adjacency lists
- Time depends on both variables: $O(V + E)$

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BFS (Input: graph g, vertex s)

```
found = new boolean[g.vertices]
q = new Queue(s)
while q is not empty
  u = q.remove()          runs once per vertex
  for v in g.adj[u]
    if not found[v]        runs twice per edge
      found[v] = true
      q.add(v)
```

Depth-First Search

Question

What is a depth-first search of a graph?

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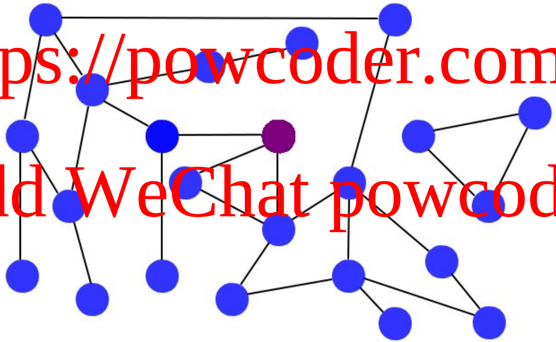

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Depth-First Search

In depth-first search

- Visit every vertex as soon as it is found
- i.e. start the next visit before completing current visit
- Result: search follows a single path as far as possible and then backtracks to the last alternative path

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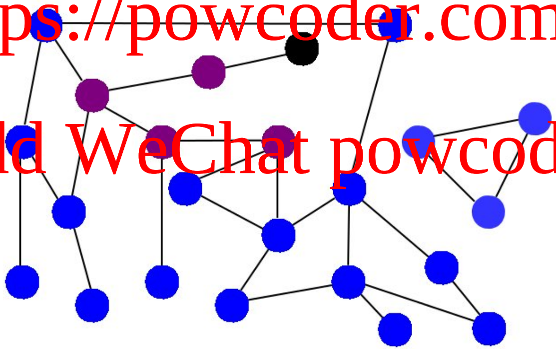

DFS Procedure

Question

How would you implement DFS?

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- Input is graph g
- Assume $g.adj[u]$ returns list of vertices
- Objective: find all vertices

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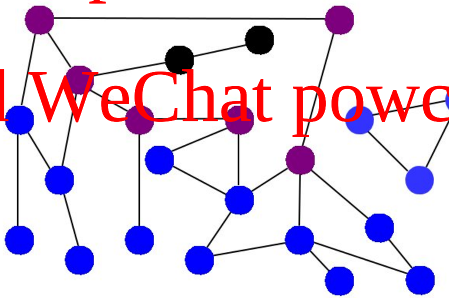

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Depth-First Search

DepthFirstSearch (Input: graph g)

```
found = new boolean[g.vertices]
for v in g
  if not found[v]
    DFS(g, v, found)
```

DFS (Input: graph g, vertex s, array found)

```
found[s] = true
for v in g.adj[s]
  if not found[v]
    DFS(g, v, found)
```

- DFS can use call stack instead of explicit queue
- Restart until whole graph searched (or not)

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An Application

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- Program checks if (whole) graph is acyclic
- Returns true or false

DepthFirstAcyclic (Input: graph g)

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```
parent = new int[g.vertices]
parent.fill(-1)          // nothing found
for v in g
  if parent[v] == -1    // not found
    parent[v] = -2      // found, no parent
    if not DFSAcyclic(g, v, parent)
      return false
return true
```

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Depth-First Search

DFSAcyclic (Input: graph g, vertex u, array parent)

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```
for v in g.adj[u]
  if parent[v] == -1            // not found
    parent[v] = u
    if not DFSAcyclic(g, v, parent)
      return false
  else if parent[u] != v        // cycle detected
    return false
return true
```

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- A cycle exists if `v` was already found, unless it is `u`'s parent
- Since `u` was just found, and not from `v`, the edge {`u`,`v`} completes an alternative path to `u` from the source

Time

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Question

For a connected graph with V vertices and E edges, how long does DFS take?

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DFS (Input: graph g, vertex s, array found)

```
found[s] = true
for v in g.adj[s]
  if not found[v]
    DFS(g, v, found)
```

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DFS Time Complexity

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- DFS is called exactly once per vertex
- Each adjacency list is used exactly once
- Each edge contributes exactly two vertices to the adjacency lists
- Time depends on both variables: $O(V + E)$

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DFS (Input: graph g, vertex s, array found)

```
  found[s] = true        runs V times
  for v in g.adj[s]
    if not found[v]      runs 2E times
      DFS(g, v, found)
```

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