Memory- and Computation-Efficient Statistical Tools for Big Matrices

R-Lille (April 2022)

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Motivation

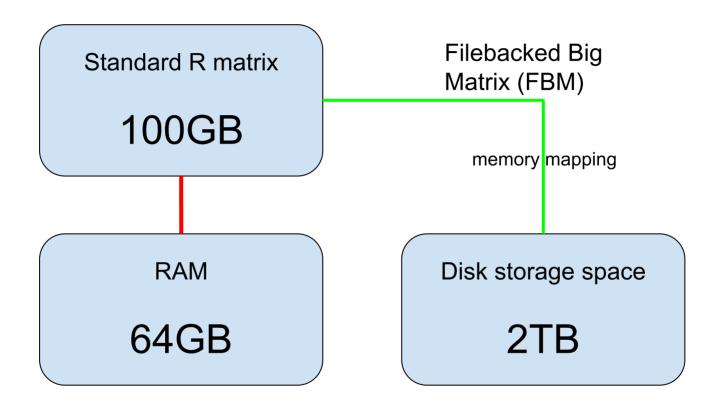
Working with very large genotype matrices

- previously: 15K x 280K, celiac disease (~30GB)
- currently: 500K x 500K, UK Biobank (~2TB)



But I still want to use **Q**..

The solution I found



Format FBM is very similar to format filebacked.big.matrix from package {bigmemory} (details in this vignette).

How memory-mapping works

- when you access the 1st element (1st row, 1st col), it accesses a block (say the first column) from disk into memory (RAM)
- when you access the 2nd element (2nd row, 1st col), it is already in memory so it is accessed very fast
- when you access the second column, you access from disk again (once)
- you can access many columns like that, until you do not have enough memory anymore
- some space is freed automatically so that new columns can be accessed into memory

Everything is seamlessly managed by the operating system (OS).

Simple accessors

Similar accessor as R matrices

```
X <- FBM(2, 5, init = 1:10, backingfile = "test")</pre>
X$backingfile
## [1] "C:\\Users\\au639593\\Desktop\\R-presentation\\test.bk"
X[, 1] ## ok
## [1] 1 2
X[1, ] ## bad
## [1] 1 3 5 7 9
X[] ## super bad
## [,1] [,2] [,3] [,4] [,5]
## [1,] 1 3 5 7 9
## [2,] 2 4 6 8 10
```

Similar accessor as R matrices

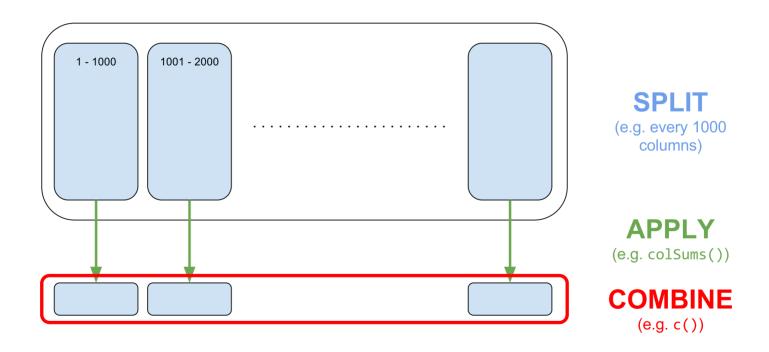
```
colSums(X[]) ## super bad
```

[1] 3 7 11 15 19



Split-(par)Apply-Combine Strategy

Apply standard R functions to big matrices (in parallel)



Implemented in big_apply().

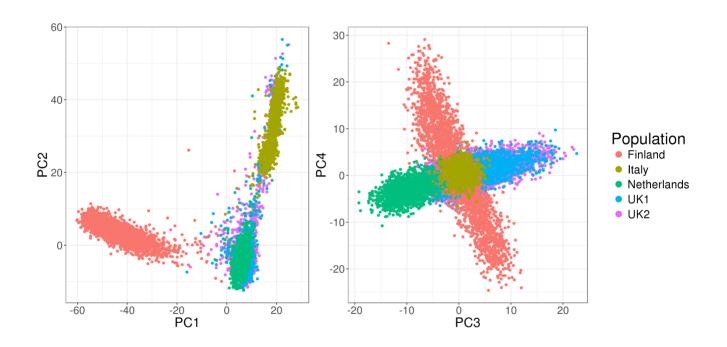
Similar accessor as Rcpp matrices

```
// [[Rcpp::plugins(cpp11)]]
// [[Rcpp::depends(bigstatsr, rmio)]]
#include <bigstatsr/BMAcc.h>
// [[Rcpp::export]]
NumericVector big_colsums(Environment BM) {
  XPtr<FBM> xpBM = BM["address"]; // get the external pointer
  BMAcc<double> macc(xpBM); // create an accessor to the data
  size_t n = macc.nrow();
  size_t m = macc.ncol();
  NumericVector res(m);
  for (size_t j = 0; j < m; j++)</pre>
    for (size_t i = 0; i < n; i++)</pre>
      res[j] += macc(i, j);
  return res;
```

Some examples from my work

Partial Singular Value Decomposition

 $15K \times 100K - 10$ first PCs -- 6 cores -- 1 min (vs 2h in base R)



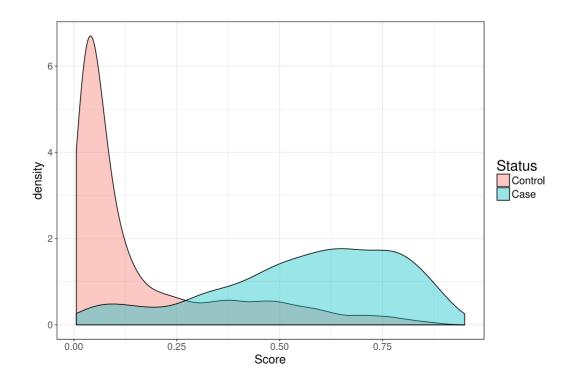
Implemented in big_randomSVD(), powered by R packages {RSpectra} and {Rcpp}.

Sparse linear models

Predicting complex diseases with a penalized logistic regression

 $15K \times 280K - 6$ cores -- 2 min (10x faster than {glmnet})

Automatic (parallel) grid-search for the two hyper-parameters of elastic-net.



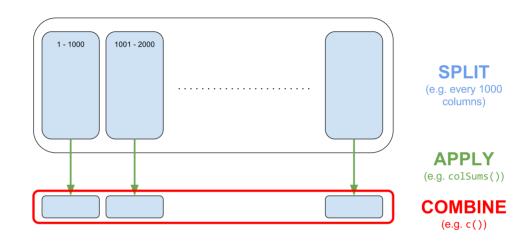
Let us try some functions

from **R** package {bigstatsr}

Create an FBM object

```
X <- FBM(10e3, 1000, backingfile = "test2")$save()</pre>
object.size(X)
## 680 bytes
file.size(X$backingfile) ## 8 x 1e4 x 1e3
## [1] 8e+07
typeof(X)
## [1] "double"
# `$save()` stored the object in an .rds file
# which you can reload in any R session
X <- big_attach("test2.rds")</pre>
```

Fill the FBM with random values



```
big_apply(X, a.FUN = function(X, ind) {
   X[, ind] <- rnorm(nrow(X) * length(ind))
   NULL ## Here, you don't want to return anything
}, a.combine = 'c')</pre>
```

NULL

```
X[1:5, 1]
## [1] -0.63074508 1.55563752 -0.02059371 -0.13974420 -0.73635154
```

Correlation matrix

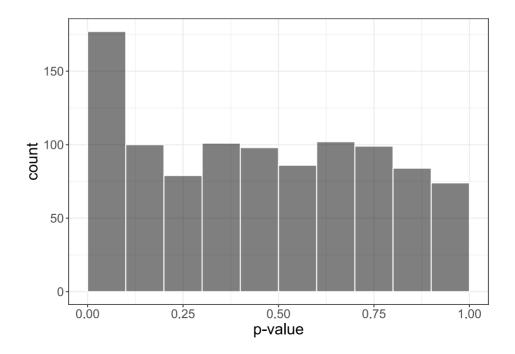
```
mat <- X[]
system.time(corr1 <- cor(mat))</pre>
##
   user system elapsed
## 6.11
             0.00
                     6.15
system.time(corr2 <- big_cor(X))</pre>
## user system elapsed
##
     5.98
             0.04
                     6.04
all.equal(corr1, corr2[])
## [1] TRUE
```

Partial Singular Value Decomposition

```
system.time(svd1 <- svd(scale(mat), nu = 10, nv = 10))</pre>
##
     user system elapsed
##
     24.53
              0.09 25.06
# Quadratic in the smallest dimension, linear in the other one
system.time(svd2 <- big_SVD(X, fun.scaling = big_scale(), k = 10))</pre>
##
     user system elapsed
     6.98
              0.07
                      7.14
##
# Linear in both dimensions
# Extremely useful if both dimensions are very large
system.time(svd3 <- big_randomSVD(X, fun.scaling = big_scale(), k = 1</pre>
     user system elapsed
##
              0.00
##
      2.20
                      2.21
```

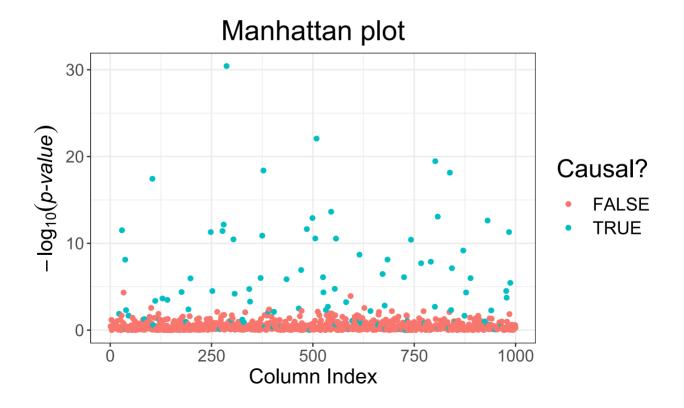
Multiple association

```
M <- 100 # number of causal variables
set <- sample(ncol(X), M)
y <- scale(X[, set]) %*% rnorm(M)
y <- y + rnorm(length(y), sd = 2 * sd(y))
mult_test <- big_univLinReg(X, y, covar.train = svd2$u)
plot(mult_test)</pre>
```



Multiple association

```
library(ggplot2)
plot(mult_test, type = "Manhattan") +
  aes(color = cols_along(X) %in% set) +
  labs(color = "Causal?")
```



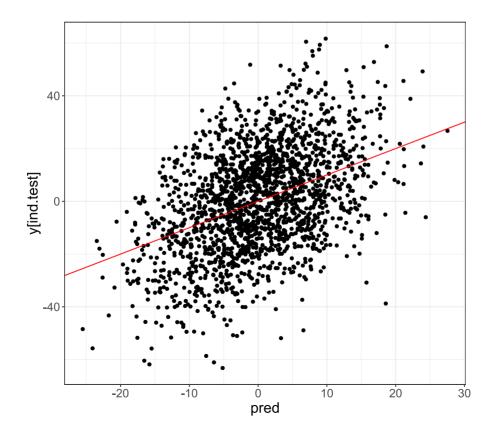
Prediction

```
# Split the indices in train/test sets
ind.train <- sort(sample(nrow(X), size = 0.8 * nrow(X))
ind.test <- setdiff(rows_along(X), ind.train)</pre>
# Train a linear model with elastic-net regularization
# and automatic choice of hyper-parameter lambda
train <- big_spLinReg(</pre>
 X, y[ind.train],
 ind.train = ind.train,
                         # use a subset for training
 covar.train = svd2$u[ind.train, ], # use additional covariables
 pf.covar = rep(0, ncol(svd2$u)), # do not penalize covariables
  alphas = c(1, 0.1, 0.01)
                           # try a grid of values for alpha
# Get predictions for the test set
pred <- predict(train, X = X, ind.row = ind.test,</pre>
                covar.row = svd2$u[ind.test, ])
```

A tutorial on fitting penalized regressions in this vignette.

Prediction

```
# Plot true value vs prediction
qplot(pred, y[ind.test]) +
  geom_abline(intercept = 0, slope = 1, color = "red") +
  theme_bigstatsr()
```



Toy case:

Compute the sum for each column

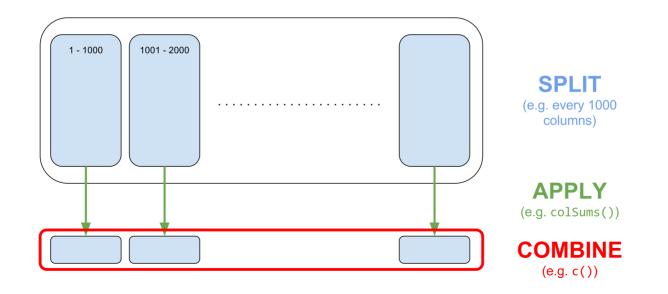
Brute force solution

sums1 <- colSums(X[]) ## /!\ access all the data in memory</pre>



Do it by blocks

[1] TRUE



Using Rcpp (1/3)

```
// [[Rcpp::plugins(cpp11)]]
// [[Rcpp::depends(bigstatsr, rmio)]]
#include <bigstatsr/BMAcc.h>
// [[Rcpp::export]]
NumericVector bigcolsums(Environment BM) {
  XPtr<FBM> xpBM = BM["address"]; // get the external pointer
  BMAcc<double> macc(xpBM); // create an accessor to the data
  size_t n = macc.nrow(), m = macc.ncol();
  NumericVector res(m); // vector of m zeros
  for (size_t j = 0; j < m; j++)</pre>
    for (size_t i = 0; i < n; i++)</pre>
      res[j] += macc(i, j);
  return res;
```

Using Rcpp (1/3)

```
sums3 <- bigcolsums(X)
all.equal(sums3, sums1)</pre>
```

[1] TRUE

Using Rcpp (2/3): the bigstatsr way

```
// [[Rcpp::plugins(cpp11)]]
// [[Rcpp::depends(bigstatsr, rmio)]]
#include <bigstatsr/BMAcc.h>
// [[Rcpp::export]]
NumericVector bigcolsums2(Environment BM,
                           const IntegerVector& rowInd,
                           const IntegerVector& colInd) {
  XPtr<FBM> xpBM = BM["address"];
  SubBMAcc<double> macc(xpBM, rowInd, colInd, 1);
  size_t n = macc.nrow(), m = macc.ncol();
  NumericVector res(m); // vector of m zeros
  for (size_t j = 0; j < m; j++)</pre>
    for (size_t i = 0; i < n; i++)</pre>
      res[j] += macc(i, j);
  return res;
```

Using Rcpp (2/3): the bigstatsr way

```
sums4 <- bigcolsums2(X, rows_along(mat), cols_along(mat))
all.equal(sums4, sums1)

## [1] TRUE

sums5 <- bigcolsums2(X, rows_along(mat), 1:10)
all.equal(sums5, sums1[1:10])

## [1] TRUE</pre>
```

Using Rcpp (3/3): already implemented

```
sums6 <- big_colstats(X)
str(sums6)

## 'data.frame': 1000 obs. of 2 variables:
## $ sum: num -30.5 152.1 148 -102.1 44.6 ...
## $ var: num 1.03 1 1.01 1 0.99 ...

all.equal(sums6$sum, sums1)

## [1] TRUE</pre>
```

Parallelism

Most of the functions are parallelized

```
ind.rep <- rep(cols_along(X), each = 100) ## size: 100,000
 (NCORES <- nb cores())
## [1] 4
system.time(
  mult_test2 <- big_univLinReg(X, y, covar.train = svd2$u,</pre>
                               ind.col = ind.rep)
## user system elapsed
## 9.14 0.00 9.28
system.time(
  mult_test3 <- big_univLinReg(X, y, covar.train = svd2$u,</pre>
                               ind.col = ind.rep, ncores = NCORES)
## user system elapsed
    18.70 0.03
##
                     5.09
```

Parallelize your own functions

```
system.time(
  mult_test4 <- big_parallelize(</pre>
    X, p.FUN = function(X, ind, y, covar) {
      bigstatsr::big_univLinReg(X, y, covar.train = covar,
                                 ind.col = ind)
    }, p.combine = "rbind", ind = ind.rep,
    ncores = NCORES, y = y, covar = svd2$u)
     user system elapsed
##
     0.06
              0.04 7.12
##
all.equal(mult_test4, mult_test3)
## [1] TRUE
```

Conclusion

I'm able to run algorithms on 100GB of data in \bigcirc on my computer

Advantages of using FBM objects

- you can apply algorithms on data larger than your RAM,
- you can easily **parallelize** your algorithms because the data on disk is shared,
- you write **more efficient algorithms** (you do less copies and think more about what you're doing),
- you can use **different types of data**, for example, in my field, I'm storing my data with only 1 byte per element (rather than 8 bytes for a standard R matrix). See the documentation of the FBM class for details.

Two other packages

{bigsparser}

Provides a Sparse matrix format with data on disk with some features:

- convert from a dgCMatrix/dsCMatrix to an SFBM, a Sparse Filebacked Big Matrix
- compute the product and crossproduct of an SFBM with a vector
- solve Ax=b, where A is a symmetric SFBM and b is a vector
- a new *compact* format is available, which is useful when non-zero values in columns are contiguous (or almost).

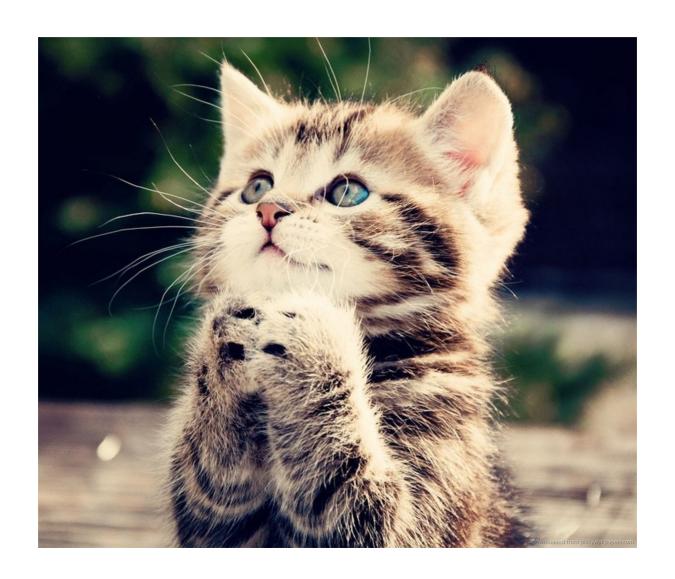
This package is intended for more efficient use of sparse data in C++ and also when parallelizing, since data on disk does not need copying.

{bigsnpr}

Extends {bigstatsr} (and also uses {bigsparser}) with functions specific to genetic SNP data:

- to convert between formats, especially to an FBM
- wrappers around PLINK (e.g. for quality controls)
- special functions for PCA/SVD
- polygenic scores (predictors based on genetic data) methods
- multiple testing
- many utility functions and other algorithms
- functions that work directly on memory-mapped PLINK bed/bim/fam files (often using the same code, just with a different accessor)

Contributions and extensions are welcome!



If we meet in-person someday, make sure to ask for an hex sticker



Thanks!

Presentation available at https://privefl.github.io/R-presentation/bigrverse.html

