— 第22届全国心理学学术会议 · 会前工作坊 —

多层线性模型：原理、关键议题与R语言实现

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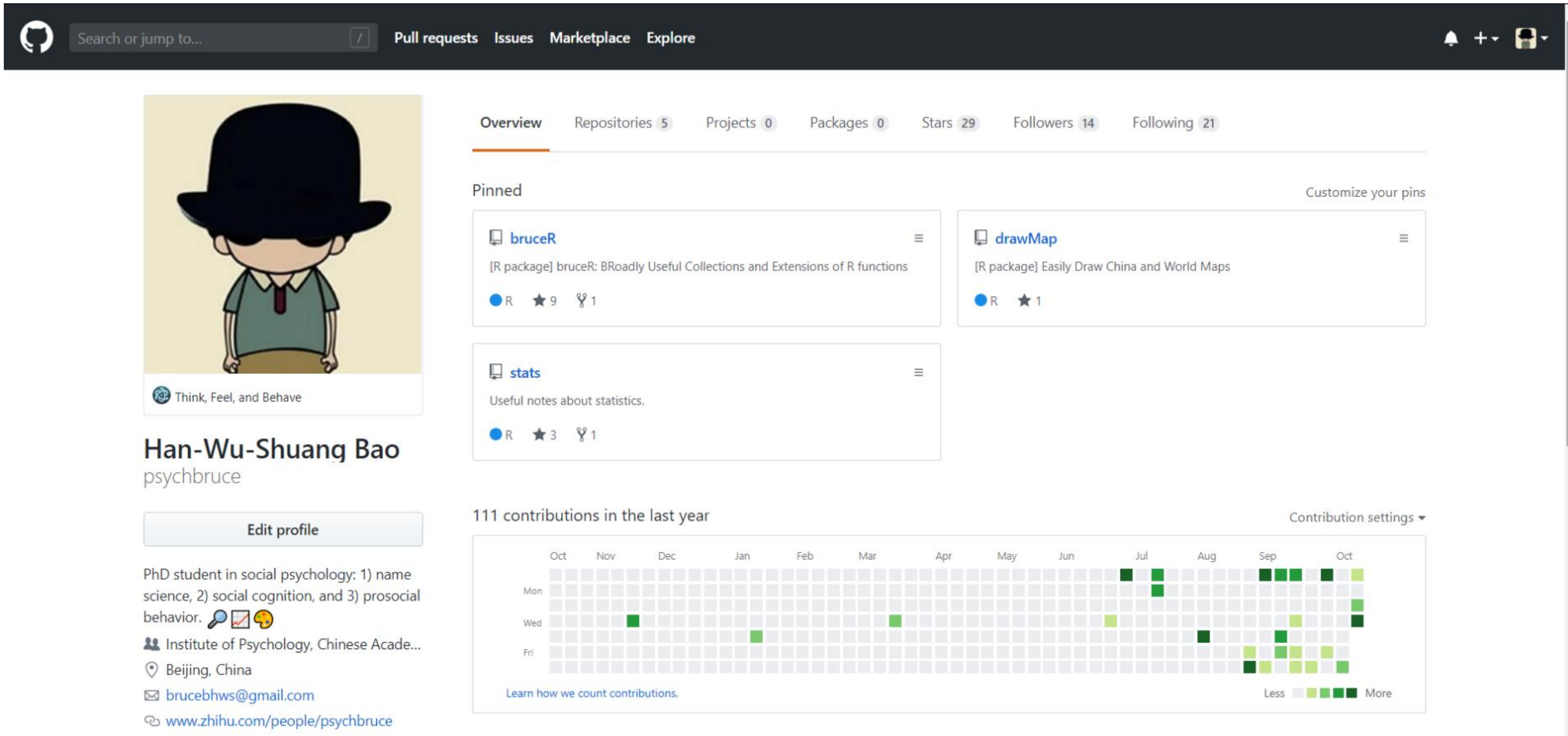
2019-10-18



https://github.com/psychbruce

https://www.zhihu.com/people/psychbruce

多层线性模型的“昵称”们

- 多层线性模型（多水平模型）
  - Multilevel Linear Model (MLM)
- 阶层线性模型（分层线性模型）
  - Hierarchical Linear Model (HLM)
- 线性混合模型（混合线性模型）
  - Linear Mixed Model (LMM)
- 混合效应模型
  - Mixed Effects Model
- 随机效应模型
  - Random Effects Model
- ……

→ 社会心理学、方法学研究者

→ 教育心理学

→ 认知心理学

都一样！

△注意：多层线性模型≠层次(逐步)多元回归

- **Multilevel (Hierarchical) Linear Model** (i.e., MLM, HLM)
  - IVs are **nested** within different groups
  - Focus on the **clustered structure** of IVs

嵌套结构

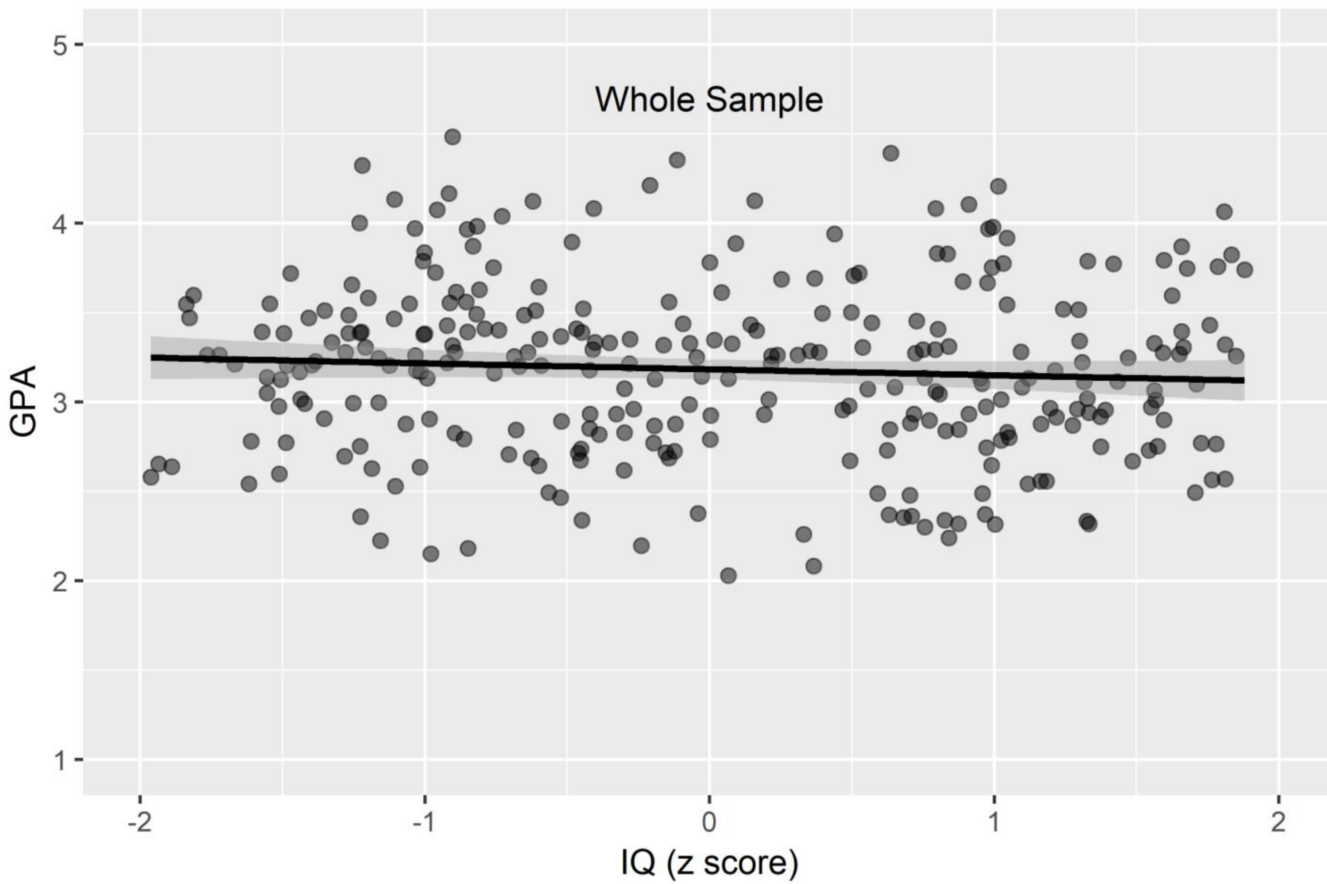
- **Sequential (Hierarchical) Multiple Regression** (i.e., stepwise regression)
  - IVs are **prioritized** at different steps/blocks
  - Focus on the **variable importance** of IVs

变量顺序

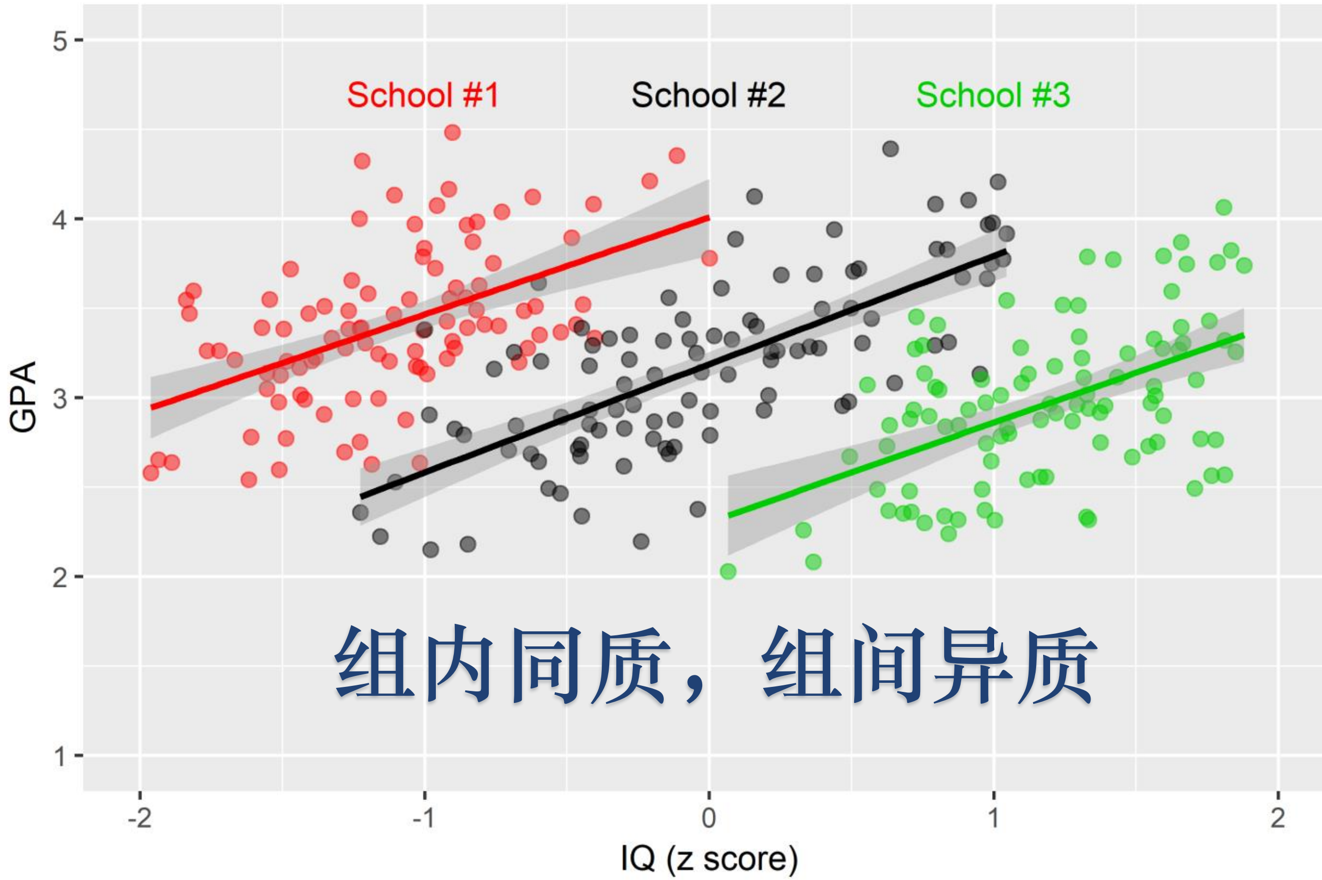
假如你想考察IQ对GPA的影响，

收集了多所学校的数据，

你会怎么统计？


Whole Sample
GPA
5
4
3
2
1
-2
-1
0
1
2
IQ (z score)

“grouping factor”
“clustering variable”



组内同质，组间异质

随机截距

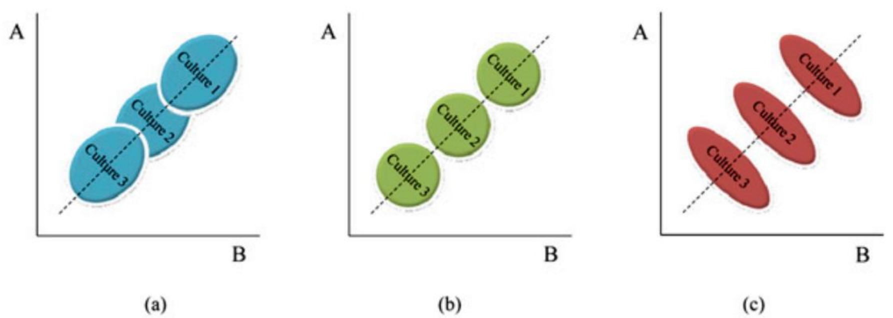
固定斜率

嵌套结构数据：

因变量的总方差 = 组内方差 (Level 1) + 组间方差 (Level 2)

为什么要用HLM?

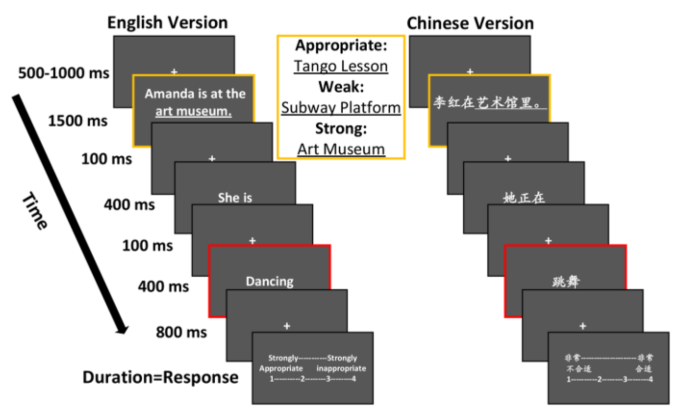
- 数据结构是嵌套的（nested structure）
  - 组内同质、组间异质——违背了OLS回归的误差独立性假设
  - 个体层面的「微观变量」vs. 群体层面的「宏观变量」
- 不平衡的组内样本量（unequal/unbalanced sample sizes）
  - 若只做个体层面的回归，会导致样本量偏大的组具有偏大的回归权重
- 群体差异并不必然等于个体差异（group vs. individual diff.）
  - 若只做群体层面的回归，会损失个体层面的信息



HLM的典型应用

- 社会心理学
  - 个体（Level 1）嵌套于群体（Level 2）
  - 跨地区/跨文化取样
  - 特点：同一地区的个体具有共享环境

- 认知心理学
  - 试次（Level 1）嵌套于被试（Level 2）
  - 重复测量设计
  - 特点：同一被试的反应具有特定模式



HLM的公式表示

Level 1（组内/个体水平，或重复测量/追踪设计中的时间水平；p个自变量，i个样本量）：

$$Y_{ij} = \beta_{0j} + \beta_{1j}X_{1ij} + \beta_{2j}X_{2ij} + \ldots + \beta_{pj}X_{pij} + \varepsilon_{ij}$$

Level 2（组间/群体水平，或重复测量/追踪设计中的个体水平；q个自变量，j个样本量）：

$$\beta_{pj} = \gamma_{p0} + \gamma_{p1}W_{1j} + \gamma_{p2}W_{2j} + \ldots + \gamma_{pq}W_{qj} + u_{pj}$$

- **例如：**
  - Level 1: $(GPA_{ij}) = \beta_{0j} + \beta_{1j}(IQ_{ij}) + \varepsilon_{ij}$ $Var(\varepsilon_{ij}) = \sigma_e^2$ (组内方差/残差)
  - Level 2: $\beta_{0j} = \gamma_{00} + u_{0j}$ (随机截距) $Var(u_{0j}) = \sigma_{u_0}^2$ (组间方差)
    $\beta_{1j} = \gamma_{10}$ (固定斜率) γ表示回归系数Gamma
  - 混合: $(GPA_{ij}) = \gamma_{00} + \gamma_{10}(IQ_{ij}) + u_{0j} + \varepsilon_{ij}$
  - R语言： GPA ~ 1 + IQ + (1 | School) 1表示截距，( )表示随机成分
  - ICC (intra-class correlation):
    * ICC需在零模型中计算

$$ICC = \frac{\sigma_{u_0}^2}{\sigma_{u_0}^2 + \sigma_e^2}$$ ，组间差异占比

HLM的组成部分（R公式写法）

- 普通的OLS回归模型（lm()/glm()函数）
  - Y ~ A*B + C （等价于：Y ~ 1 + A + B + A:B + C）

- 几种典型的HLM模型（lmer()/glmer()函数）
  - Y ~ 1 + (1|Group)   随机截距
  - Y ~ X + (1|Group)   随机截距、固定斜率
  - Y ~ X + (X|Group)   随机截距、随机斜率（此时1可以省略）
  - Y ~ X + (1|Prov/City)   三层嵌套模型
  - Y ~ X + (1|Sub) + (1|Item)   交叉分类模型（crossed）

  - Y：必为第1层结果变量
  - X：可以为第1或第2层预测变量，但最高层预测变量不存在随机斜率
  - 1：在HLM中，随机截距永远存在，否则降级为OLS回归

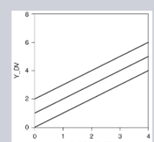
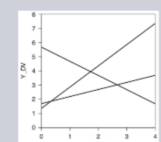
HLM的组成部分（R公式写法）

Formula	Alternative	Meaning
`(1 | g)`	`1 + (1 | g)`	Random intercept with fixed mean.
`0 + offset(o) + (1 | g)`	`-1 + offset(o) + (1 | g)`	Random intercept with *a priori* means.
`(1 | g1/g2)`	`(1 | g1) + (1 | g1:g2)`	Intercept varying among `g1` and `g2` within `g1`.
`(1 | g1) + (1 | g2)`	`1 + (1 | g1) + (1 | g2)`	Intercept varying among `g1` and `g2`.
`x + (x | g)`	`1 + x + (1 + x | g)`	Correlated random intercept and slope.
`x + (x || g)`	`1 + x + (1 | g) + (0 + x | g)`	Uncorrelated random intercept and slope.

Table 2: Examples of the right-hand-sides of mixed-effects model formulas. The names of grouping factors are denoted `g`, `g1`, and `g2`, and covariates and *a priori* known offsets as `x` and `o`.

https://cran.r-project.org/web/packages/lme4/vignettes/lmer.pdf

Q1：固定效应vs.随机效应？随机斜率的取舍问题

	固定效应 （Fixed Effect, FE）	随机效应 （Random Effect, RE）
总框架：回归分析	$Y = \underbrace{b_0 + b_1{*}X_1 + b_2{*}X_2 + \dots}_{\text{[结构项]（固定部分）}} + \underbrace{error}_{\text{[误差项]（随机部分）}}$ • 个体差异 ε_{ij} • 群体差异 u_j [观测项] = [结构项]（固定部分） + [误差项]（随机部分）	
多层线性模型 (HLM)	**固定截距**（非HLM）： Y ~ 1 + X – 实质：GLM（降级为OLS回归）	**随机截距**：Y ~ 1 + X + (1 | group) – 每组有不同“基线”且正态
* 又称：线性混合模型（LMM） * 两大应用情形： – 个体嵌套于群体、组别（横截面数据） – 重复测量嵌套于个体（纵向追踪数据）	**固定斜率**：Y ~ 1 + X + (1 | group) – $X_{Lv.1}$效应各组一致 – 组数过少时$df_{Lv.2}$小，建议使用固定斜率 	**随机斜率**：Y ~ 1 + X + (X | group) – $X_{Lv.1}$效应依组而变 – 有跨层交互作用时需要使用随机斜率 
	* 结果中的“固定效应”是指回归系数（“平均”的截距和斜率）	* 结果中的“随机效应”是指残差方差（不同组、不同个体的“特异”部分）

详见知乎专栏文章：https://zhuanlan.zhihu.com/p/60528092

Q1：固定效应vs.随机效应？随机斜率的取舍问题

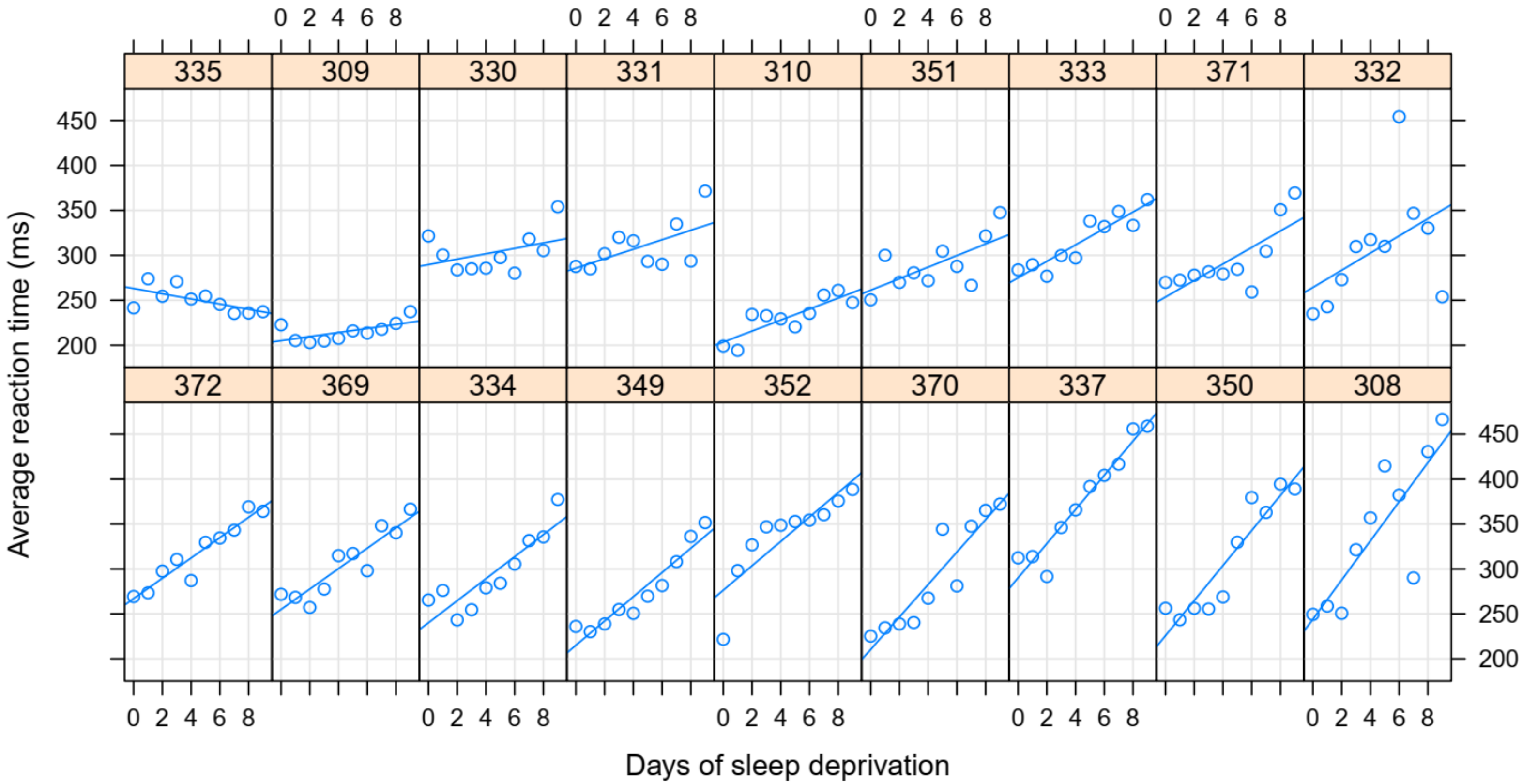


Figure 1: Average reaction time versus days of sleep deprivation by subject. Subjects ordered (from left to right starting on the top row) by increasing slope of subject-specific linear regressions.

Q1：固定效应vs.随机效应？随机斜率的取舍问题

- **社会心理学**
  - **HLM**
    - Level 1：被试
    - Level 2：群组/城市/省份/国家/……
  - **惯例：仅考虑可能存在的随机斜率；跨层交互作用需考虑随机斜率**
  - **原因：个体层面心理变量间的关系往往比较稳定**

- **认知心理学**
  - **HLM**
    - Level 1：重复测量/刺激项目
    - Level 2：被试
  - **惯例：尽量都设为随机斜率（保持最大化，模型不收敛时再简化）**
  - **原因：被试的行为模式往往存在较大的个体差异**

Barr et al. (2013)

Q2：HLM的适用条件问题

1）样本量的要求

Level 2的组别（cluster）也是一种随机取样，

因变量在不同组的均值（截距）需满足正态分布。

如果Level 2的组数量过少（< 10组），

则Level 2的自由度过小，

统计检验力（power）不足，

回归系数的估计也可能有偏，

不建议使用HLM！

（可对组别虚拟编码，进行GLM、ANCOVA分析）

Q2：HLM的适用条件问题

2）ICC的要求

ICC既反映了组间方差占总方差的比例，

也反映了同组内任意两人因变量的相关系数期望值。

一般认为：当ICC ≥ 0.059时，

组间差异（或组内同质性）已不容忽视，

需要使用HLM。

但是，ICC并非金标准！

如果数据在理论层面属于嵌套结构，

则必须使用HLM！

Q3：样本量与统计检验力问题

研究关注点	Level 2组数量（或被试量）	每组内样本量（或观测数量）
固定效应	≥ 30	≥ 30
跨层交互	≥ 50	≥ 20
随机效应	≥ 100	≥ 10
【一般建议】Level 2组数量更关键，不能过少（≥ 20或30组）；如果每组内样本量过少，可通过增加组数量来弥补		

Hox et al. (2017; Chapter 12)

Q4：自由度问题

df (能够独立变化的数据量) = N (总数据量) – k (待估计参数个数)
= $N – p$ (自变量个数) – **1** (截距)

三种估计方法

- 变量类型法（理论驱动，根据变量处于哪一层分别计算）
- Satterthwaite 近似估计法（数据驱动，会出现小数，但建议使用）
- Kenward-Roger 近似估计法（同上）

Predictor	Effect	*df*	Software
1. Large group sample size (*K* >= 30 groups)			
Level-1 predictor (γ_{10})	Fixed slope	$N - p - q - 1$	HLM
	Random slope	$K - q_c - 1$	
Cross-level interaction (γ_{11})	—		
Level-2 predictor (γ_{01})	—	$K - q - 1$	
Intercept (γ_{00})	—		
2. Small group sample size (*K* < 30 groups)			
All predictors	—	Estimated by Satterthwaite's approximation	R (lmerTest), SPSS (MIXED), jamovi (GAMLj)

详见知乎专栏文章：https://zhuanlan.zhihu.com/p/50048784

Q5：效应量问题

American Psychological Association
Psychological Methods

1082-989X/19/$12.00

2019, Vol. 24, No. 3, 309–338
http://dx.doi.org/10.1037/met0000184

Quantifying Explained Variance in Multilevel Models: An Integrative Framework for Defining R-Squared Measures

Jason D. Rights and Sonya K. Sterba
Vanderbilt University

Methods in Ecology and Evolution
British Ecological Society

Methods in Ecology and Evolution 2013, **4,** 133–142
doi: 10.1111/j.2041-210x.2012.00261.x

A general and simple method for obtaining R^2 from generalized linear mixed-effects models

Shinichi Nakagawa[1,2]* and Holger Schielzeth[3]

Q5：效应量问题

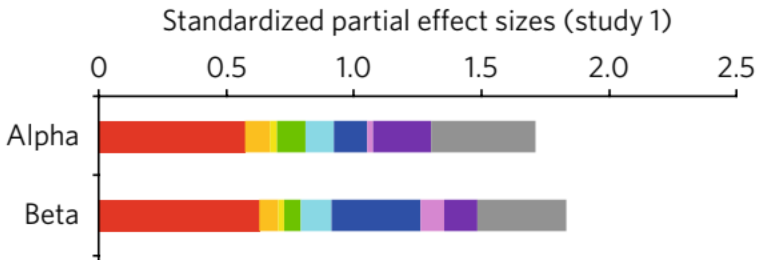
- R^2
  - Marginal R^2 = 固定效应的方差解释率
  - Conditional R^2 = 固定效应 + 随机效应的总方差解释率
  - Ω^2 = 1 – 未被解释的比例
- 标准化回归系数

$$standardized\ coefficient = \frac{unstandardized\ coefficient \times stand.dev.explanatory\ var.}{stand.dev.outcome\ var.}$$

- 基于模型比较的pseudo-R^2
  - 不推荐，易出现负值，难以解释

$$R_1^2 = \left(\frac{\sigma_{e|b}^2 - \sigma_{e|m}^2}{\sigma_{e|b}^2}\right) \quad R_2^2 = \left(\frac{\sigma_{u0|b}^2 - \sigma_{u0|m}^2}{\sigma_{u0|b}^2}\right)$$

- 基于“t-to-r”转换的 r
  - 不推荐，有误导性，夸大效应量



Q6：变量中心化问题

预测变量	总均值中心化（总平减）（grand mean centering）	组均值中心化（组平减）（group mean centering）
Level 1	• 主要关注Level 2预测变量，仅想控制Level 1预测变量时	• 主要关注Level 1预测变量时 • 存在Level 1交互作用时 • 存在跨层交互作用时
Level 2	• 存在Level 2交互作用时 • 存在跨层交互作用时	–

建议把组平均数作为Level 2变量放回方程

Psychological Methods
2007, Vol. 12, No. 2, 121–138

1082-989X/07/$12.00 DOI: 10.1037/1082-989X.12.2.121

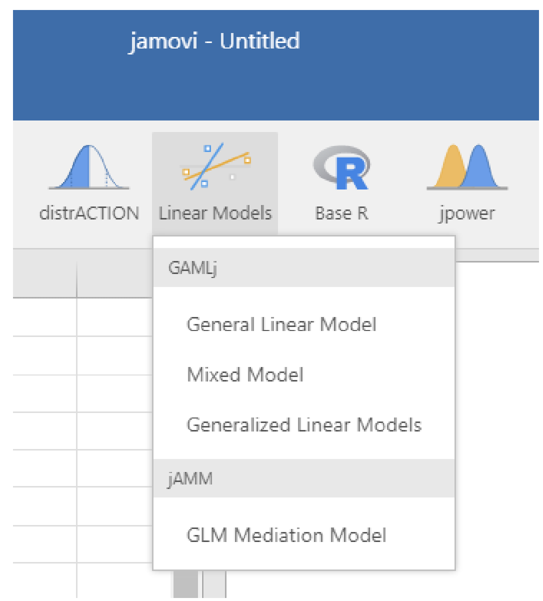
Centering Predictor Variables in Cross-Sectional Multilevel Models: A New Look at an Old Issue

Craig K. Enders and Davood Tofighi
Arizona State University

可以做HLM分析的软件

- SPSS
  - MIXED语句
- HLM
- Mplus
- SAS
- Stata
- MLwiN
- R
  - R语言编程
    - lmerTest包
  - jamovi
    - GAMLj模块

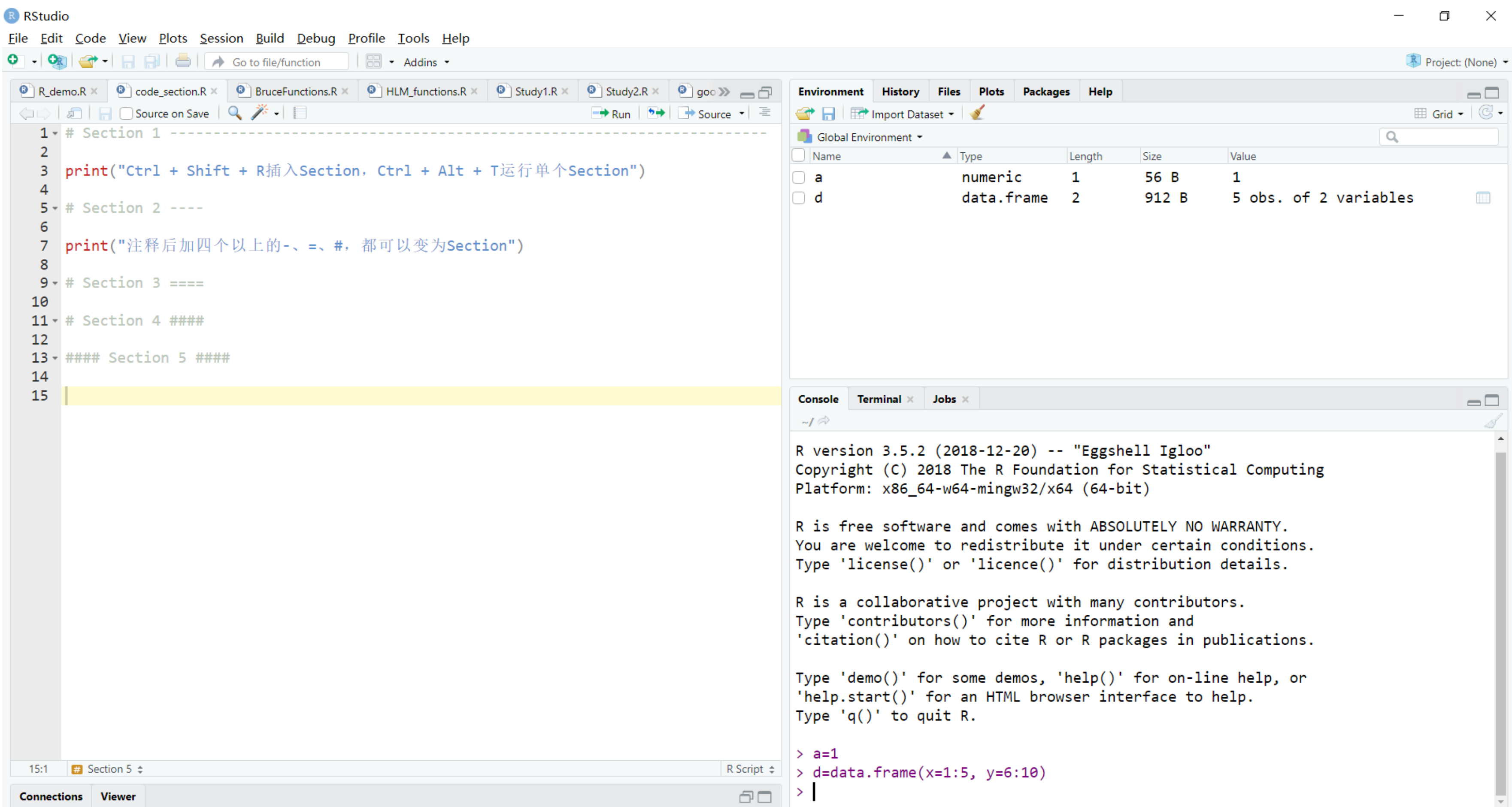
```
* SPSS - 固定斜率.
MIXED GPA WITH Gender Age Edu City_GDP
  /METHOD=REML
  /PRINT=DESCRIPTIVES SOLUTION TESTCOV
  /FIXED=Gender Age Edu City_GDP | SSTYPE(3)
  /RANDOM=INTERCEPT | SUBJECT(city) COVTYPE(VC).

* SPSS - 随机斜率.
MIXED GPA WITH Gender Age Edu City_GDP
  /METHOD=REML
  /PRINT=DESCRIPTIVES SOLUTION TESTCOV
  /FIXED=Gender Age Edu City_GDP | SSTYPE(3)
  /RANDOM=INTERCEPT Edu | SUBJECT(city) COVTYPE(UN).
```



R语言

R语言与RStudio（代码编辑器）

- 请使用RStudio！请使用RStudio！请使用RStudio！
  - 没有人会用“记事本”写论文！



R语言

准备工作：如何导入、合并、转换数据？

	R包	函数	实例
数据导入	rio	import()	`data=import("D:/raw.xlsx")` `data=import("D:/raw.csv")`
数据导出	rio	export()	`export(data, "new.sav")` `export(data, "new.txt")`
合并数据	dplyr	left_join()	`left_join(data, provdata,` `by="province")`

长宽转换	data.table	melt()
		dcast()
			`melt(data, measure=`
			`patterns("A[1-2]B[1-9]"),`
			`var="Cond", value="Y")`

	tidyr	pivot_longer()
		pivot_wider()
			`pivot_longer(dt, A1B1:A2B2,`
			`names_to=c("A", "B"),`
			`names_pattern="A(.)B(.)",`
			`values_to="Y")`

bruceR包：数据分析的“万能工具箱”

Main functions in `bruceR`

- [x] Basic use and analyses (e.g., correlation matrix with plot)
  - `Print()`, `Describe()`, `Freq()`, `Corr()`, ...
  - `set.wd()`, `set.seeds()`, `dtime()`, ...
  - `%notin%`, `%partin%`, ...
  - `LOOKUP()`, ...
- [x] Multivariate computing (e.g., scale mean score with reverse scoring)
  - `RECODE()`, `RESCALE()`
  - `COUNT()`, `MODE()`, `SUM()`, `MEAN()`, `STD()`, `CONSEC()`
- [x] Reliability and validity analyses (e.g., Cronbach's alpha, exploratory/confirmatory factor analysis)
  - `Alpha()`
  - `EFA()`, `CFA()`
- [ ] *t*-test, ANOVA, simple-effect analysis, and multiple comparison
  - *(coming soon...)*
- [x] Advanced toolbox and output for general/generalized ordinary/multilevel linear models
  - `grand_mean_center()`, `group_mean_center()`
  - `GLM_summary()`, `HLM_summary()`, `regress()`, `model_check()`
  - `med_mc()`, `simple_slope()`
- [x] Nice themes of `ggplot2` ready for scientific publication
  - `theme_bruce()`

https://github.com/psychbruce/bruceR

与HLM相关的R包

- 建模
  - nlme（可以处理异方差和自相关，可以设定协方差结构，但有局限）
  - lme4（比nlme更简单灵活，但未提供上述设定）
  - lmerTest（基于lme4的扩展，推荐使用）

- 结果汇总与扩展分析
  - bruceR（更全面的结果输出）
  - performance（模型诊断）

  - simr（基于模拟的Power分析）
  - mediation（中介作用分析）
  - interactions、emmeans（调节作用分析、简单斜率检验）

如何建立HLM模型?

```
#### Multilevel Analysis ####

library(bruceR)
library(lmerTest)

data=sleepstudy
# a demo dataset from 'lme4' package
# see: ?sleepstudy
# see also: ?HLM_summary
str(data)

hlm.0=lmer(Reaction ~ (1 | Subject), data=data)
hlm.1=lmer(Reaction ~ Days + (1 | Subject), data=data)
hlm.2=lmer(Reaction ~ Days + (Days | Subject), data=data)

summary(hlm.0)
summary(hlm.1)
summary(hlm.2)

HLM_summary(hlm.0)
HLM_summary(hlm.1)
HLM_summary(hlm.2)
```

如何查看结果？summary()函数并不够用……

```
> summary(hlm.2)
Linear mixed model fit by REML. t-tests use Satterthwaite's method [
lmerModLmerTest]
Formula: Reaction ~ Days + (Days | Subject)
   Data: data

REML criterion at convergence: 1743.6

Scaled residuals: 
    Min      1Q  Median      3Q     Max 
-3.9536 -0.4634  0.0231  0.4633  5.1793 

Random effects:
 Groups   Name        Variance Std.Dev. Corr
 Subject  (Intercept) 611.90   24.737       
          Days         35.08    5.923   0.07
 Residual             654.94   25.592       
Number of obs: 180, groups:  Subject, 18

Fixed effects:
            Estimate Std. Error      df t value Pr(>|t|)    
(Intercept)  251.405      6.824  17.005  36.843  < 2e-16 ***
Days          10.467      1.546  16.995   6.771 3.27e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Correlation of Fixed Effects:
     (Intr)
Days -0.138
```

R语言

如何查看结果？HLM_summary()函数更强大！

```
> HLM_summary(hlm.0)
MODEL INFO:
Model type: Linear Mixed Model (LMM)
= Hierarchical Linear Model (HLM)
= Multilevel Linear Model (MLM)

Formula: Reaction ~ (1 | Subject)

Level-1 Observations: N = 180
Level-2 Groups/Clusters: Subject, 18

MODEL FIT:
AIC = 1910.327
BIC = 1919.905
R_(m)² = 0.00000  (Marginal R²: fixed effects)
R_(c)² = 0.39489  (Conditional R²: fixed + random effects)
Omega² = 0.43347  (= 1 - proportion of unexplained variance)

FIXED EFFECTS:
Outcome variable: Reaction
──────────────────────────────────────────────────────────────────
               Gamma    S.E.     t   df      p         [95%      CI]
──────────────────────────────────────────────────────────────────
(Intercept)  298.508 (9.050) 32.98 17.0 <.001 *** [279.414, 317.602]
──────────────────────────────────────────────────────────────────
'df' is estimated by Satterthwaite approximation.

RANDOM EFFECTS:
────────────────────────────────────────────────────────────────
 Cluster  K  Parameter   Variance      S.E. Wald Z     p      ICC
────────────────────────────────────────────────────────────────
 Subject  18 (Intercept) 1278.34 (506.123)   2.53  .012 * 0.39489
 Residual                1958.87
────────────────────────────────────────────────────────────────
```

```
> HLM_summary(hlm.2)
MODEL INFO:
Model type: Linear Mixed Model (LMM)
= Hierarchical Linear Model (HLM)
= Multilevel Linear Model (MLM)

Formula: Reaction ~ Days + (Days | Subject)

Level-1 Observations: N = 180
Level-2 Groups/Clusters: Subject, 18

MODEL FIT:
AIC = 1755.628
BIC = 1774.786
R_(m)² = 0.27864  (Marginal R²: fixed effects)
R_(c)² = 0.79923  (Conditional R²: fixed + random effects)
Omega² = 0.82590  (= 1 - proportion of unexplained variance)

FIXED EFFECTS:
Outcome variable: Reaction
──────────────────────────────────────────────────────────────────
               Gamma    S.E.     t   df      p         [95%      CI]
──────────────────────────────────────────────────────────────────
(Intercept)  251.405 (6.824) 36.84 17.0 <.001 *** [237.009, 265.802]
Days          10.467 (1.546)  6.77 17.0 <.001 *** [  7.206,  13.729]
──────────────────────────────────────────────────────────────────
'df' is estimated by Satterthwaite approximation.

Standardized coefficients: Reaction
────────────────────────────────────────────────────
      Gamma*   S.E.*    t* df*    p*       [95%     CI]
────────────────────────────────────────────────────
Days   0.535 (0.079) 6.77  17 <.001 *** [0.368, 0.702]
────────────────────────────────────────────────────

RANDOM EFFECTS:
────────────────────────────────────────────────────────────────
 Cluster  K  Parameter   Variance      S.E. Wald Z     p      ICC
────────────────────────────────────────────────────────────────
 Subject  18 (Intercept)  611.90 (232.457)   2.63  .008 ** 0.48301
             Days          35.08 ( 35.256)   1.00  .320
 Residual                 654.94
────────────────────────────────────────────────────────────────
```

如何进行模型比较?

```
> anova(hlm.0, hlm.1, hlm.2)
refitting model(s) with ML (instead of REML)
Data: data
Models:
hlm.0: Reaction ~ (1 | Subject)
hlm.1: Reaction ~ Days + (1 | Subject)
hlm.2: Reaction ~ Days + (Days | Subject)
      Df    AIC    BIC  logLik deviance   Chisq Chi Df Pr(>Chisq)
hlm.0  3 1916.5 1926.1 -955.27   1910.5
hlm.1  4 1802.1 1814.8 -897.04   1794.1 116.462      1  < 2.2e-16 ***
hlm.2  6 1763.9 1783.1 -875.97   1751.9  42.139      2  7.072e-10 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

anova()函数

```
> texreg::screenreg(list(hlm.0, hlm.1, hlm.2))

====================================================================
                               Model 1       Model 2       Model 3
--------------------------------------------------------------------
(Intercept)                     298.51 ***    251.41 ***    251.41 ***
                                 (9.05)        (9.75)        (6.82)
Days                                           10.47 ***     10.47 ***
                                               (0.80)        (1.55)
--------------------------------------------------------------------
AIC                            1910.33       1794.47       1755.63
BIC                            1919.91       1807.24       1774.79
Log Likelihood                 -952.16       -893.23       -871.81
Num. obs.                       180           180           180
Num. groups: Subject             18            18            18
Var: Subject (Intercept)       1278.34       1378.18        611.90
Var: Residual                  1958.87        960.46        654.94
Var: Subject Days                                            35.08
Cov: Subject (Intercept) Days                                 9.61
====================================================================
*** p < 0.001, ** p < 0.01, * p < 0.05
```

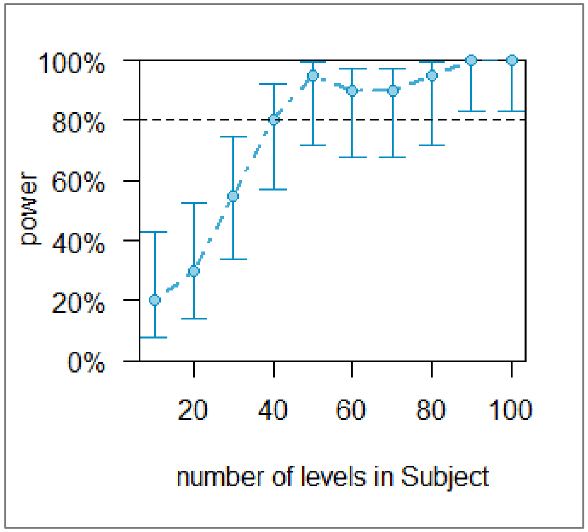
texreg包的screenreg()函数

R语言

如何进行HLM的Power分析?

- simr包：powerSim()、powerCurve()函数

```r
library(simr)

## 'data': can be obtained from a pilot study or by making up
hlm.p=lmer(Reaction ~ Days + (Days | Subject), data=data)
summary(hlm.p)  # b(Days) = 10.467

## 'nsim': number of simulations (default is 1000, much slower)
powerSim(hlm.p, test=fixed("Days"), nsim=20, seed=1)

## Change b to an expected effect size and n to a larger size
hlm.p.n=extend(hlm.p, along="Subject", n=100)
fixef(hlm.p.n)["Days"]=3.0
powerSim(hlm.p.n, test=fixed("Days"), nsim=20, seed=1)

## Estimate power at a range of sample sizes
powers=powerCurve(hlm.p.n, test=fixed("Days"), along="Subject",
                  breaks=seq(10, 100, 10), nsim=20, seed=1)
plot(powers)
```



- sjstats包：samplesize_mixed()　　—不推荐
- powerlmm包：shiny_powerlmm()　　—非常不推荐

R语言

如何进行模型诊断?

- 模型诊断指标
  - 多元正态分布
  - 矩阵满秩（vs.多重共线性）
  - 方差齐性（vs.异方差性）
  - 残差独立性（vs.自相关性）

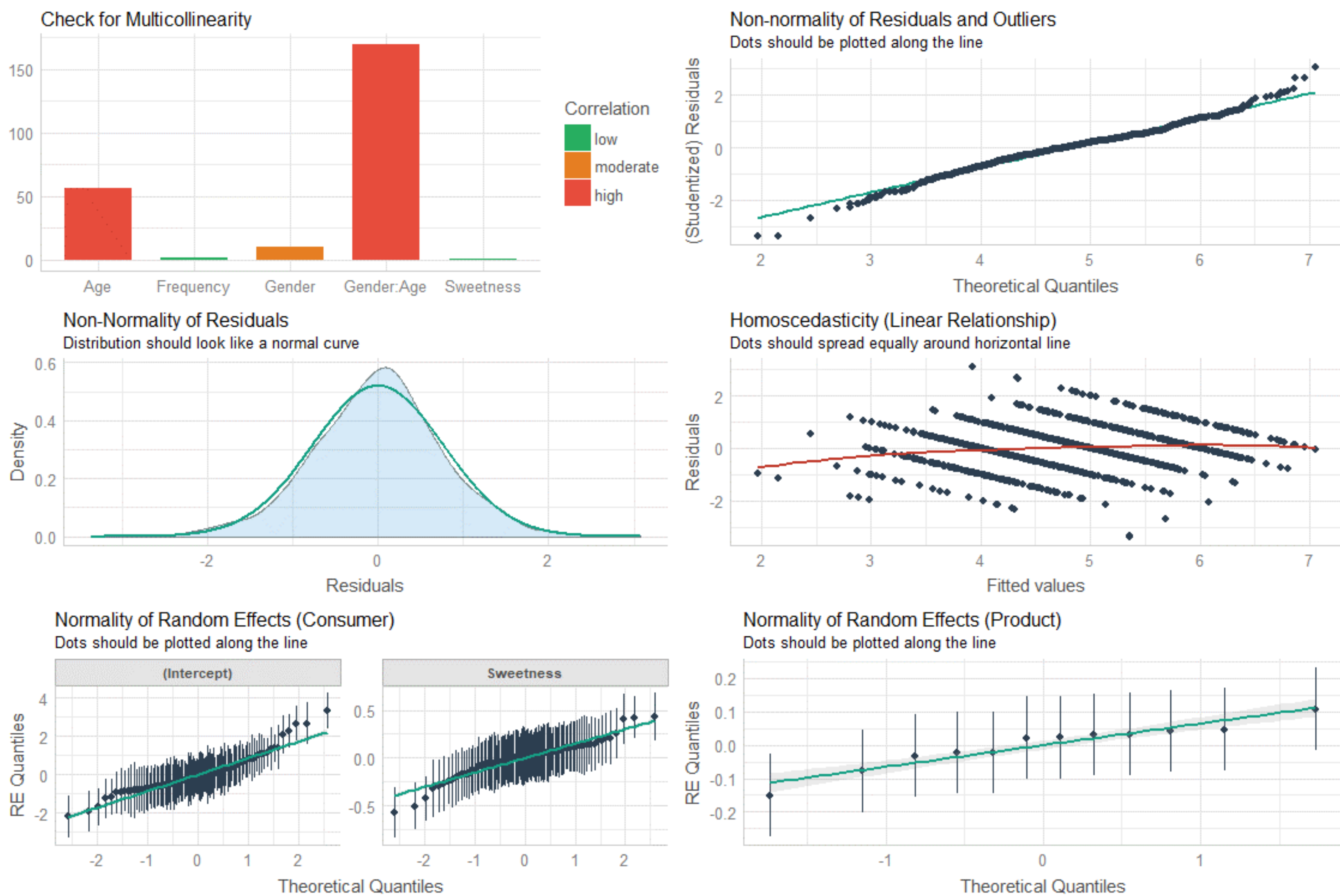
- model_check()函数
  - 来自bruceR包
  - 整合了performance包的一系列函数

```r
hlm=lmer(Preference ~ Sweetness + Frequency +
           Gender * Age + (Sweetness | Consumer) +
           (1 | Product),
         data=carrots)
```

```
> model_check(hlm)
1) Multivariate normality:
(please see plots)

2) Multicollinearity (VIF):
# Check for Multicollinearity

Low Correlation

 Parameter  VIF Increased SE
 Sweetness 1.01         1.00
 Frequency 1.47         1.21

Moderate Correlation

 Parameter  VIF Increased SE
    Gender 9.79         3.13

High Correlation

  Parameter    VIF Increased SE
        Age  56.24         7.50
 Gender:Age 169.27        13.01

3) Homoscedasticity (vs. Heteroscedasticity):
OK: Error variance appears to be homoscedastic (p = 0.293).

4) Independence of residuals (vs. Autocorrelation):
Warning: Autocorrelated residuals detected (p = 0.000).

Plotting...
```

R语言

如何进行模型诊断?



如何整理论文中的HLM结果表?

Table 2. Effects of Socio-Ecological Factors on Individualist Practices and Values

Predictor	Individualist practices		Individualist values	
	b	*t*	*b*	*t*
Socioeconomic development	0.66 (0.04)**	$t(181.22) = 15.46$	0.59 (0.06)**	$t(188.64) = 9.51$
Disaster frequency	0.58 (0.10)**	$t(114.31) = 5.81$	0.19 (0.13)	$t(178.85) = 1.46$
Pathogen prevalence	−0.18 (0.05)*	$t(116.22) = -3.25$	−0.19 (0.07)*	$t(221.32) = -2.71$
Climate × Socioeconomic Development	0.02 (0.007)*	$t(181.79) = 3.13$	0.02 (0.01)	$t(156.49) = 1.58$

Note: The table presents estimates from a multilevel-model analysis with annual data nested within country data. Standard errors for the coefficients are given in parentheses. We calculated these standard errors using the data for all countries and years in the sample.
*$p < .01$. **$p < .001$.

Santos et al. (2017) in *Psych. Sci.*

R语言

如何整理论文中的HLM结果表?

Table 3

Summary of Multilevel Regression Models Predicting Perceived Centrality of Identity Elements (Level 1: n = 958) Nested Within Participants (Level 2: n = 81) With Random Intercept and Fixed Slopes for Motive Ratings

	Null model		Self-esteem model					6-motive model					
Parameter	*B*	*SE*	*B*	*SE*	β	$\Delta\chi^2$ (df = 1)	*p*	*B*	*SE*	β	$\Delta\chi^2$ (df = 1)	*p*	ΔR^2_W (%)
Fixed parameters													
Intercept	4.90	.08	4.90	.08				4.90	.08				
Self-esteem			0.55	.03	.60	343	<.00001	0.13	.03	.14	16	<.00001	0.9
Continuity								0.20	.03	.21	53	<.00001	2.9
Distinctiveness								0.07	.02	.07	8	<.01	0.4
Belonging								0.07	.03	.07	5	.03	0.3
Efficacy								−0.02	.03	−.02	1	.45	0.0
Meaning								0.36	.03	.41	106	<.00001	5.9
Residual variance													
Level 2 (τ^2)	0.34	.08	0.40	.08				0.53	.12				
Level 1 (σ^2)	2.12	.10	1.43	.07				1.01	.06				
Deviance	3,524		3,181					2,847					

Note. Deviance is calculated as $-2 \times$ log likelihood. Values of β for each parameter are derived from *B* weights by multiplying by the standard deviation of the predictor and dividing by the standard deviation of the outcome (Hox, 2002); because between-participant variance was excluded from these analyses by within-participant centering, we used the within-participant standard deviations. Values of $\Delta\chi^2$ and ΔR^2_W for each parameter are derived from comparisons with an alternative model without that parameter.

Vignoles et al. (2006) in *JPSP*

拓展：HLM的替代方法——Cluster-Robust *SE*

Table 3

*Advantages and Disadvantages of HLM, CR-*SE*, and GEE*

McNeish et al. (2017) in *Psych. Methods*

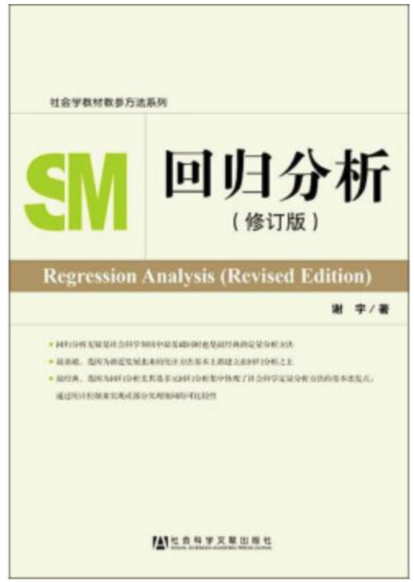
Method	Advantages	Disadvantages
HLM	1. Can directly incorporate substantive multilevel theory into the model	1. Requires many explicit assumptions and is not always robust to violations
	2. Provides information about specific predictors having cluster-level variance, allows for cluster-specific inferences to be made	2. Cluster-specific interpretations and estimation difficult with discrete outcomes; Likelihood does not have a closed form solution with discrete outcomes which requires approximation or linearization
	3. Can more easily partition the variance into more than two levels and allows for full decomposition of cluster-level and within-level effects	3. Difficult to determine if the covariance is modeled correctly
	4. Accommodates either longitudinal or cross-sectionally clustered data well	4. Lacks an overall R^2 for continuous outcomes
CR-*SE*	1. Can output OLS-equivalent R^2 and effect sizes while accounting for clustering	1. Assumes working independence, coefficient estimates may be affected when the ICC is greater than .30
	2. Allow for multiple moderated or blockwise regression with clustered data	2. Less efficient than GEE for longitudinal analyses where the ICC is typically high
	3. Along with GEE, less affected by small cluster sizes (Level-1 sample size)	3. Compared with HLM, more affected by small number of clusters
GEE	1. Straightforward estimation with discrete outcomes; full likelihood not needed	1. Limited ability to compare models or gauge fit
	2. Estimates are robust to misspecifications to the covariance structure of the outcome	2. Compared with CR-*SE*, fewer advantages for cross-sectionally clustered data with continuous outcomes such as no R^2
	3. Along with CR-*SE*, less affected by small cluster sizes (Level-1 sample size)	3. Compared to HLM, more affected by small number of clusters and highly unbalanced clusters
	4. No distributional assumptions concomitant with random effects	4. Cannot fully decompose effects into between-level and within-level components.

Note. GEE = generalized estimating equations; HLM = hierarchical linear model; CR-*SE* = cluster-robust standard errors.

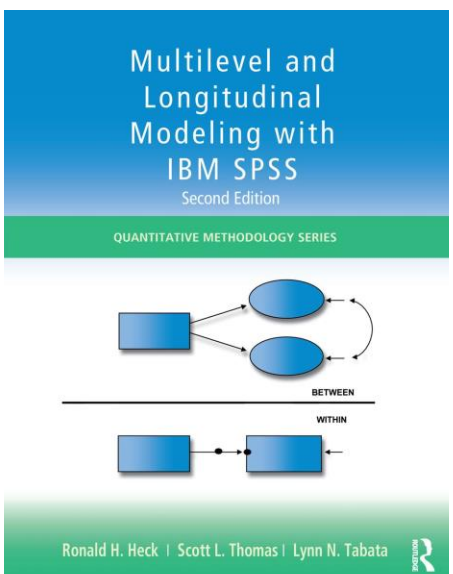
*** bruceR包的GLM_summary()函数可以输出(cluster-)robust SE的结果**

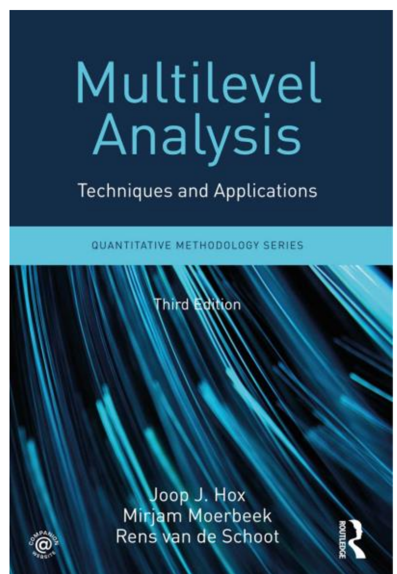
推荐阅读的参考书

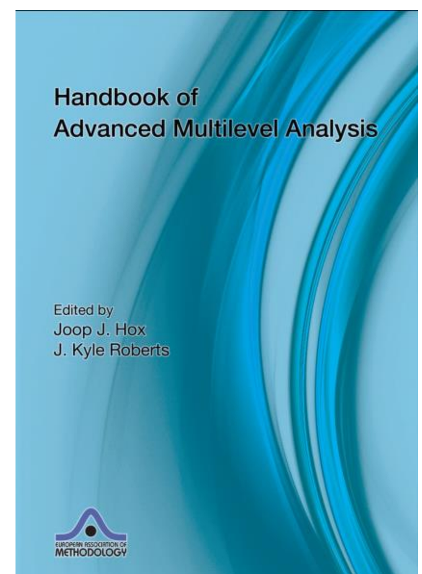

张厚粲先生作序推荐
阶层线性模型
的原理与应用
温福星 ◎著


社会学教材教参方法系列
SM
回归分析
（修订版）
Regression Analysis (Revised Edition)
谢 宇 / 著


Methodology of
Multilevel Modeling
多层次模式方法论
阶层线性模式的
关键问题与试解


Multilevel and
Longitudinal
Modeling with
IBM SPSS
Second Edition
QUANTITATIVE METHODOLOGY SERIES
BETWEEN
WITHIN
Ronald H. Heck | Scott L. Thomas | Lynn N. Tabata


Multilevel
Analysis
Techniques and Applications
QUANTITATIVE METHODOLOGY SERIES
Third Edition
Joop J. Hox
Mirjam Moerbeek
Rens van de Schoot


Handbook of
Advanced Multilevel Analysis
Edited by
Joop J. Hox
J. Kyle Roberts
EUROPEAN ASSOCIATION OF METHODOLOGY

Q & A