**SC 602 Assignment 5**

Name:
Roll no.:

**Question 1**
For each of the following systems, investigate stability of the origin using the center manifold theorem:

1.
$$\dot{x}_1 = x_1 {x_2}^3$$
$$\dot{x}_2 = -x_2 - x_1^2 + 2x_1^8$$

2.
$$\dot{x}_1 = -x_1 + {x_2}^3(x_1 + x_2 - 1)$$
$$\dot{x}_2 = x_2^3(x_1 + x_2 - 1)$$

3.
$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -x_2 + ax_1^3(1 + x_1^2)^{-1}; \qquad a \neq 0.$$

**Question 2**
Consider the system

$$\dot{x}_1 x_1 x_2 + ax_1^3 + bx_1 x_2, \qquad \dot{x}_2 = -x_2 + cx_1^2 + dx_1^2 x_2$$

Investigate the stability of the origin by using the center manifold theorem for each of the following cases:

1. $a + c > 0$
2. $a + c < 0$
3. $a + c = 0$ and $cd + bc^2 < 0$.

**Question 3**
Consider the system

$$\dot{x}_1 = ax_1^3 + x_1^2 x_2, \qquad \dot{x}_2 = -x_2 + x_2^2 + x_1 x_2 - x_1^3$$

Investigate the stability of the origin by using the center manifold theorem for all possible values of the real parameter $a$.