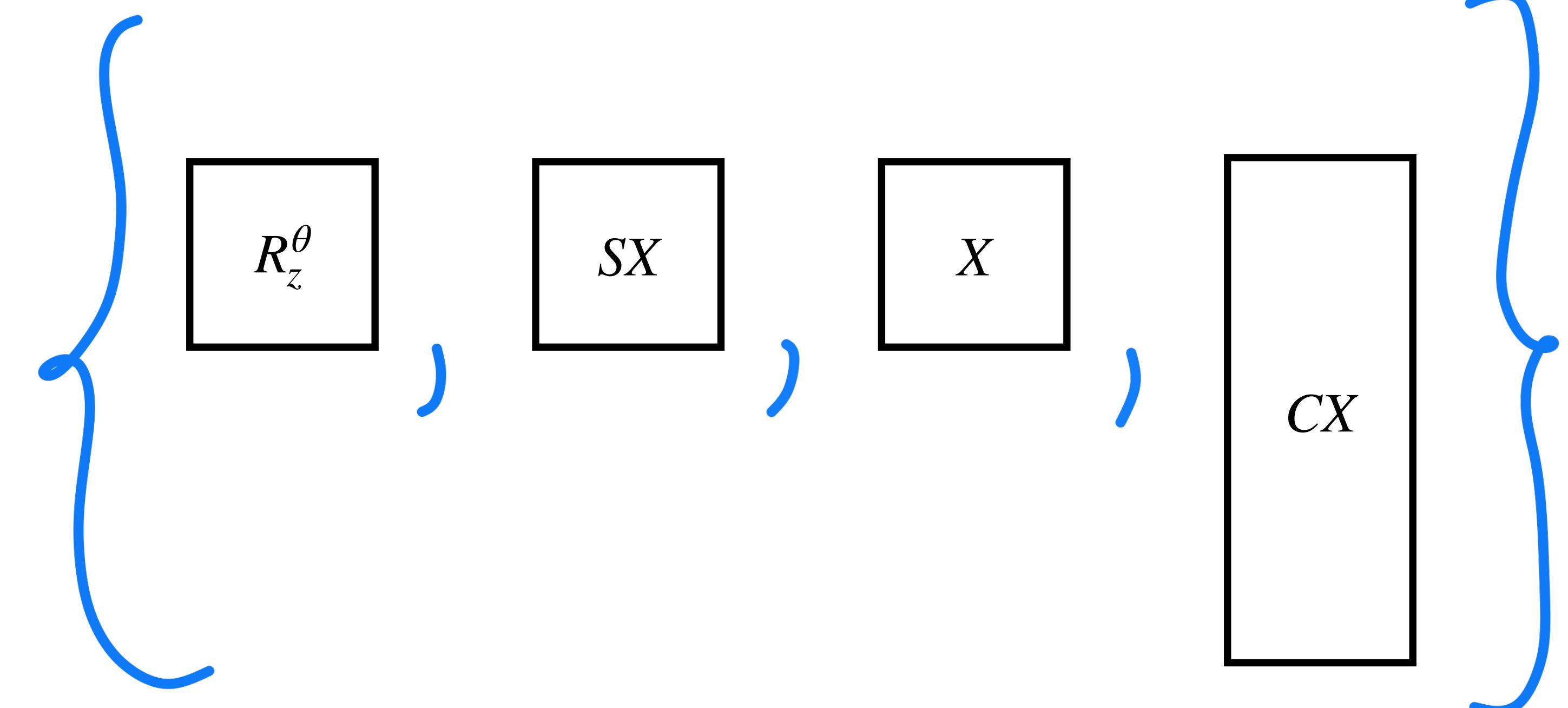


# Quantum-Circuit Optimization

# Native Basis Gate Set



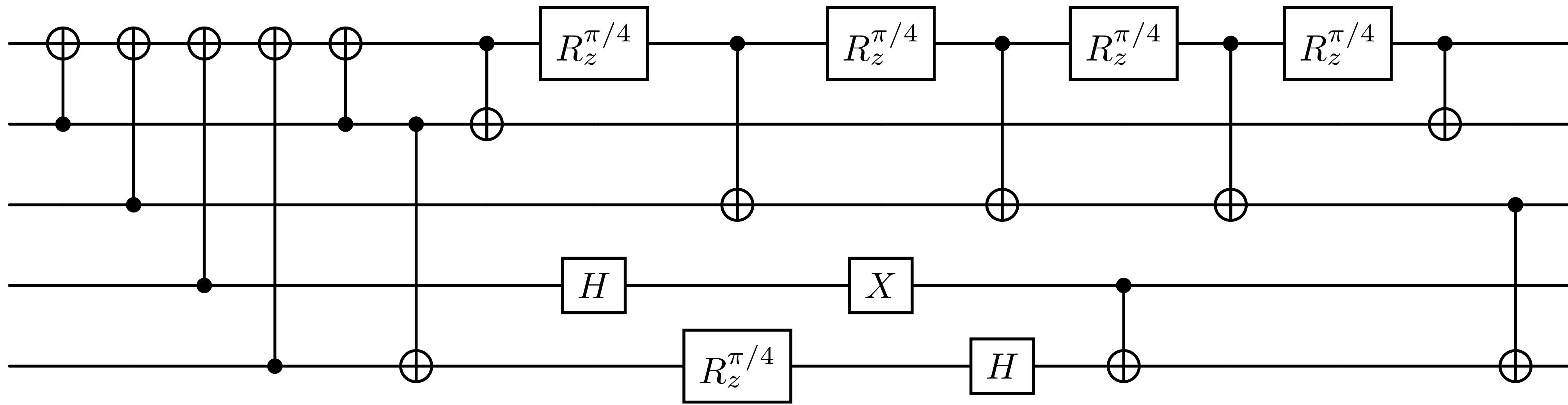
IBM



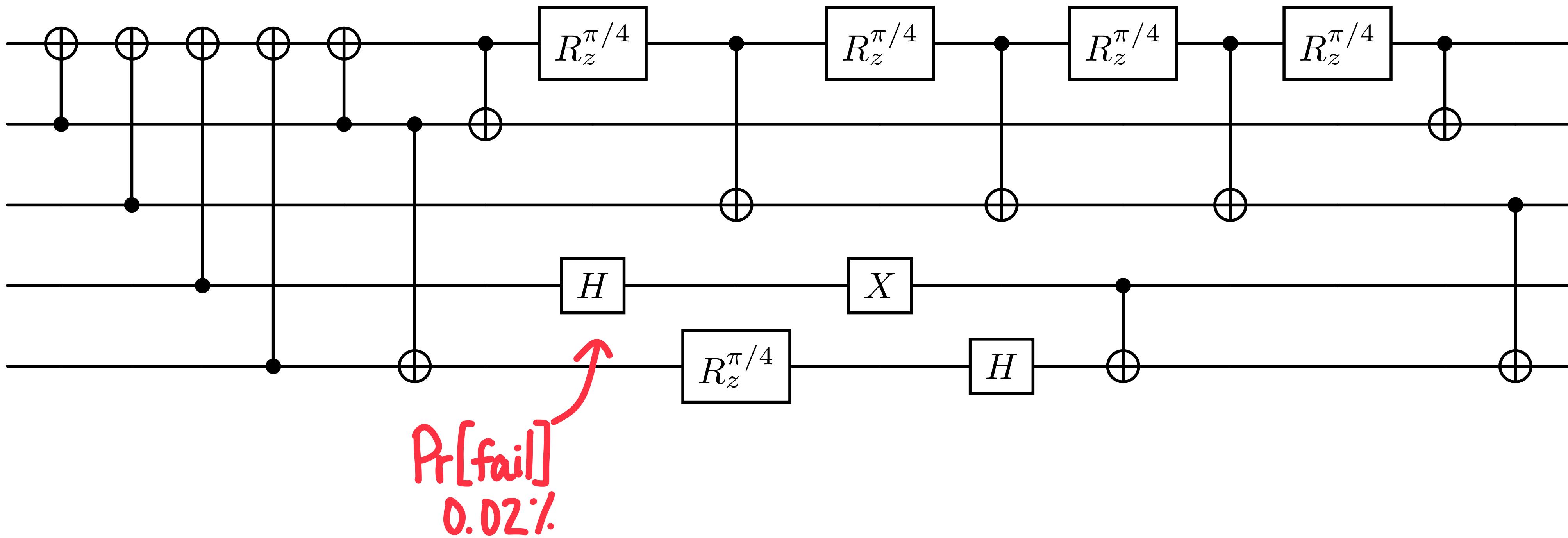
"universal"

i.e. approximate any computation to arbitrary precision

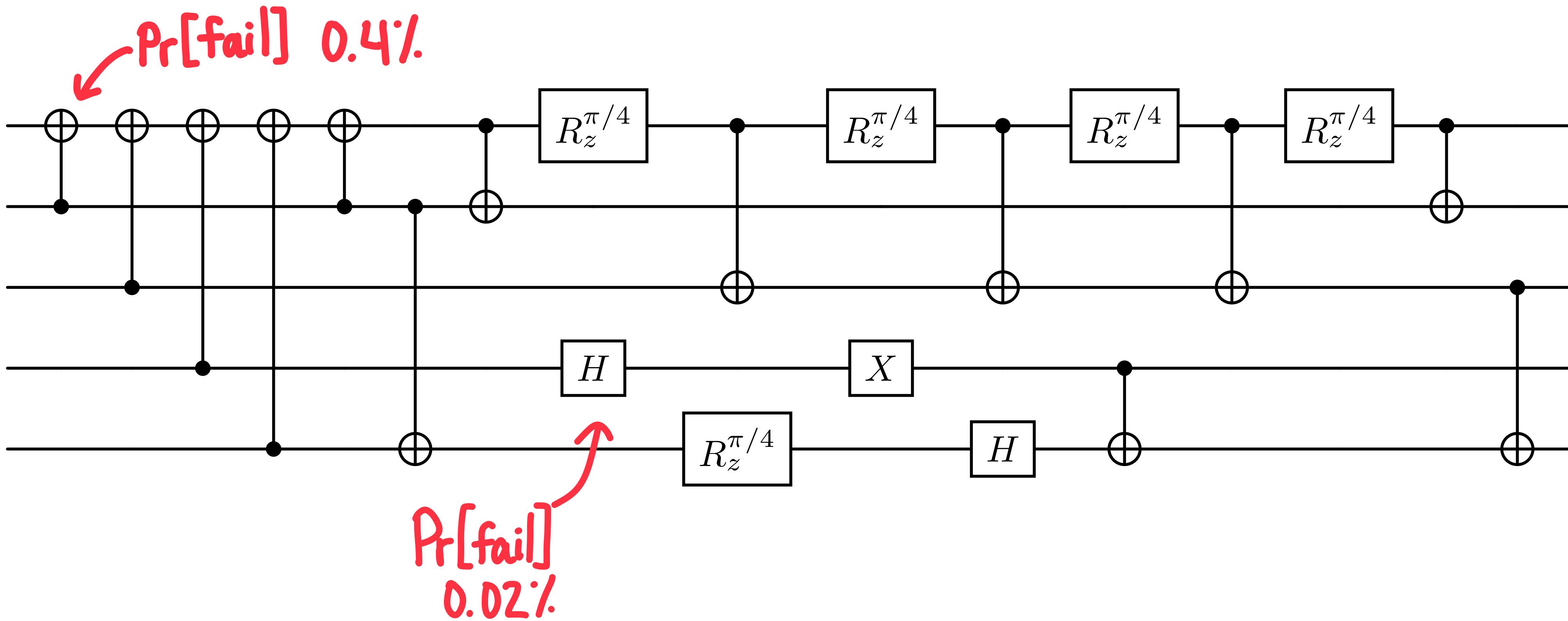
# Optimization Objectives



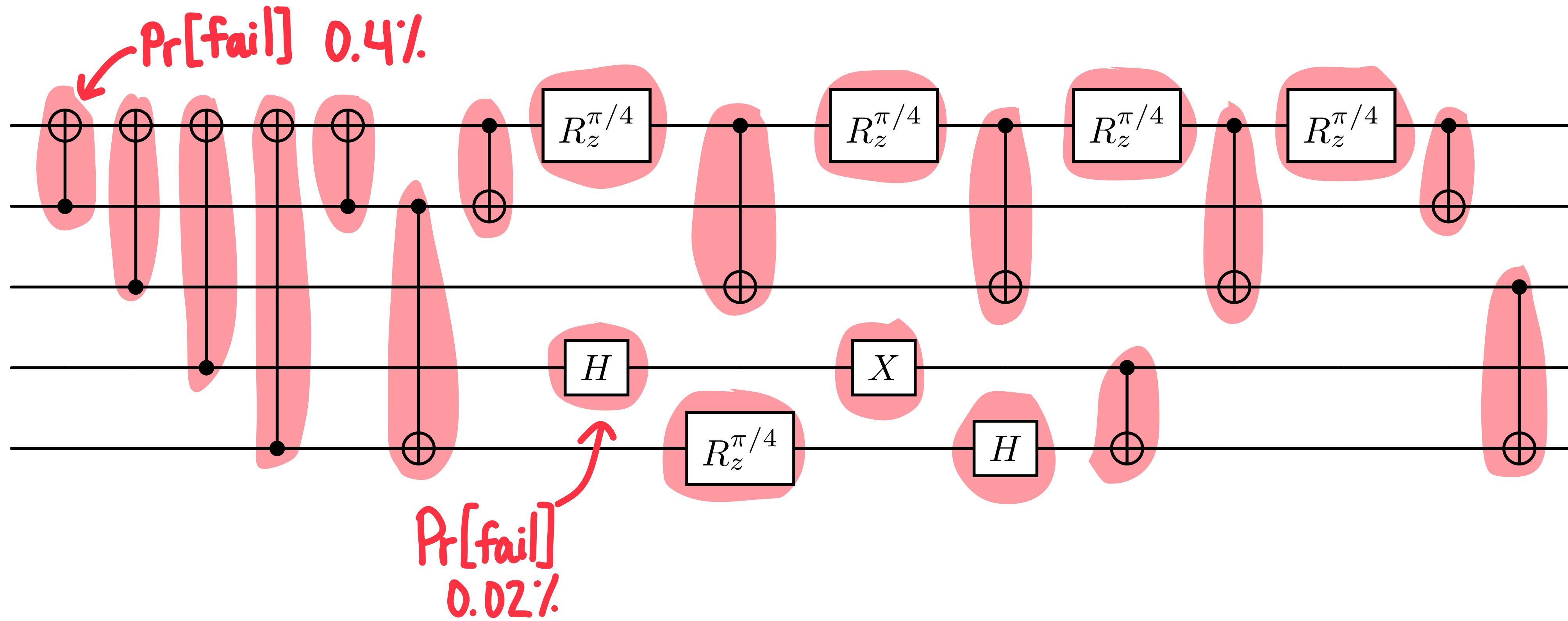
# Optimization Objectives



# Optimization Objectives



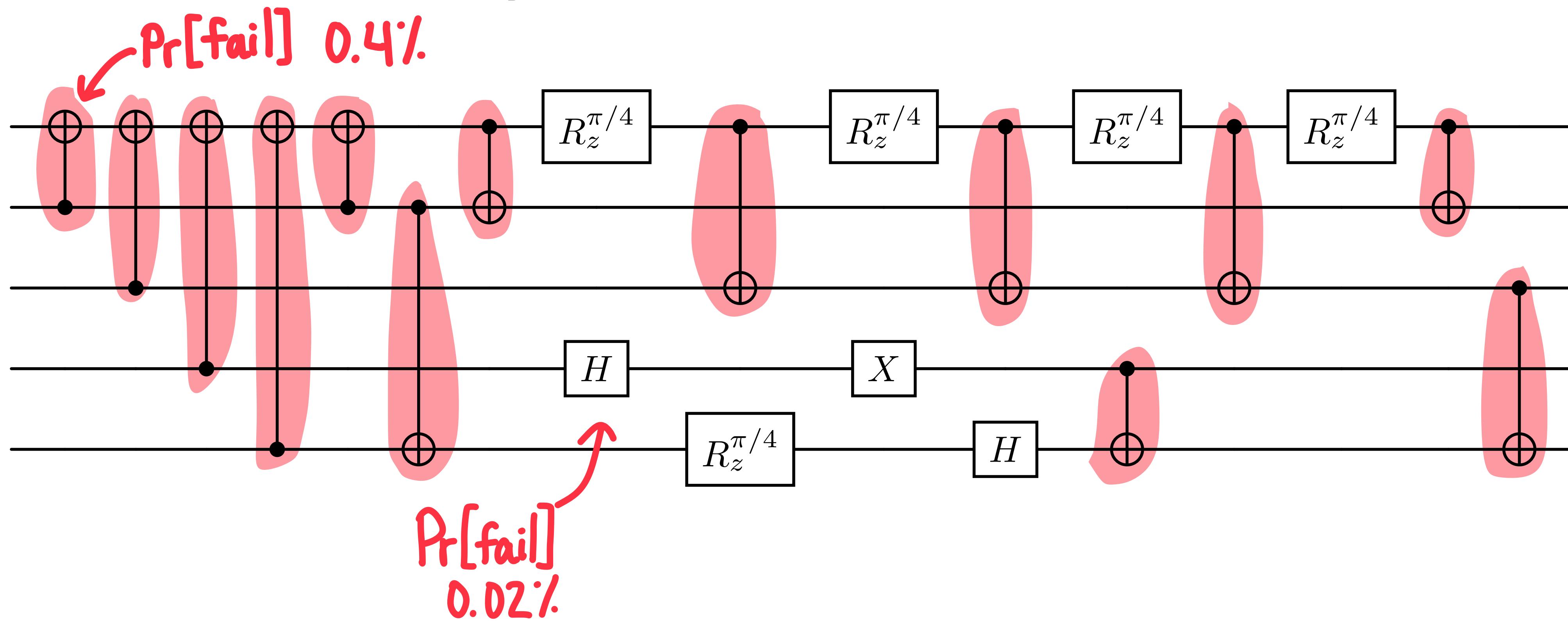
# Optimization Objectives



NISQ

- total gate count

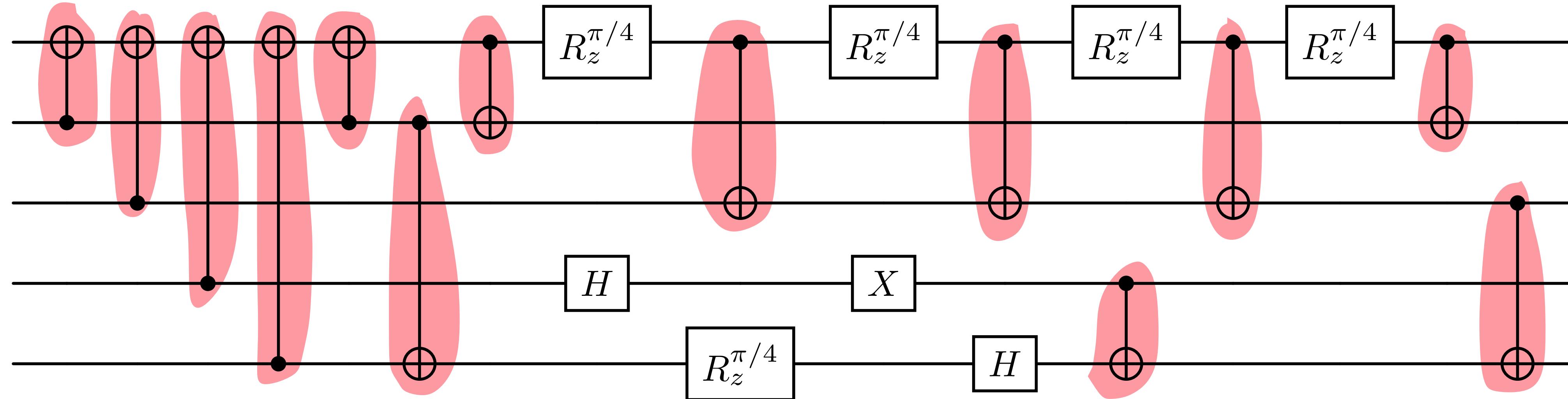
# Optimization Objectives



NISQ

- total gate count
- 2q gate count

# Optimization Objectives



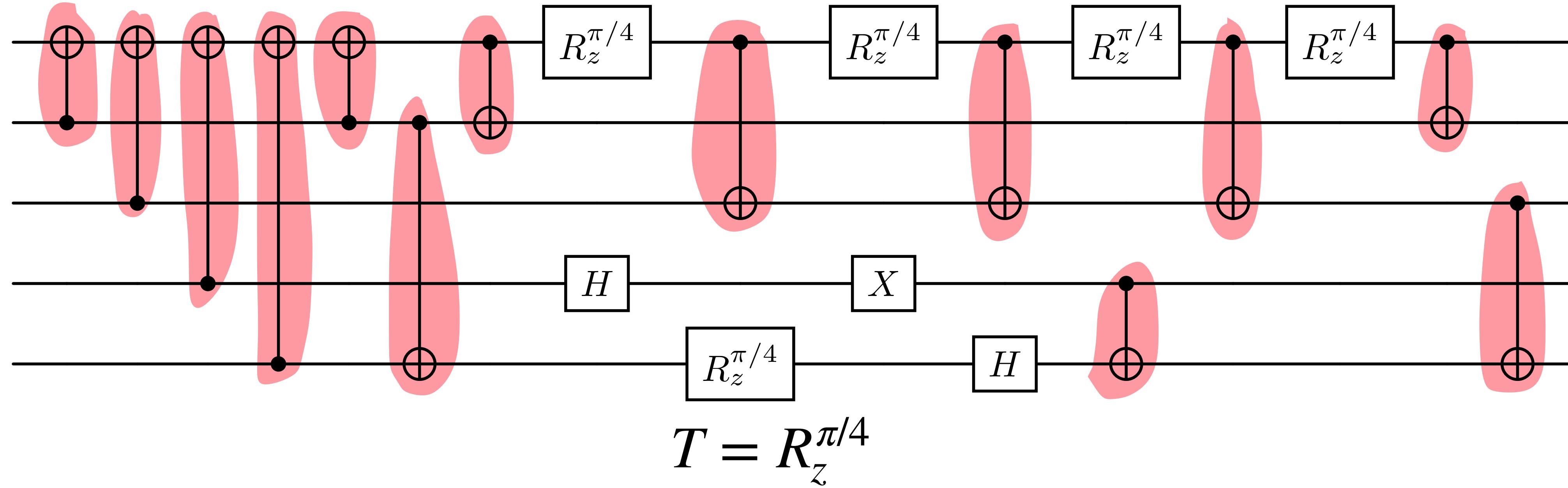
## NISQ

- total gate count
- 2q gate count

## FTQC

- T count and 2q gate count

# Optimization Objectives



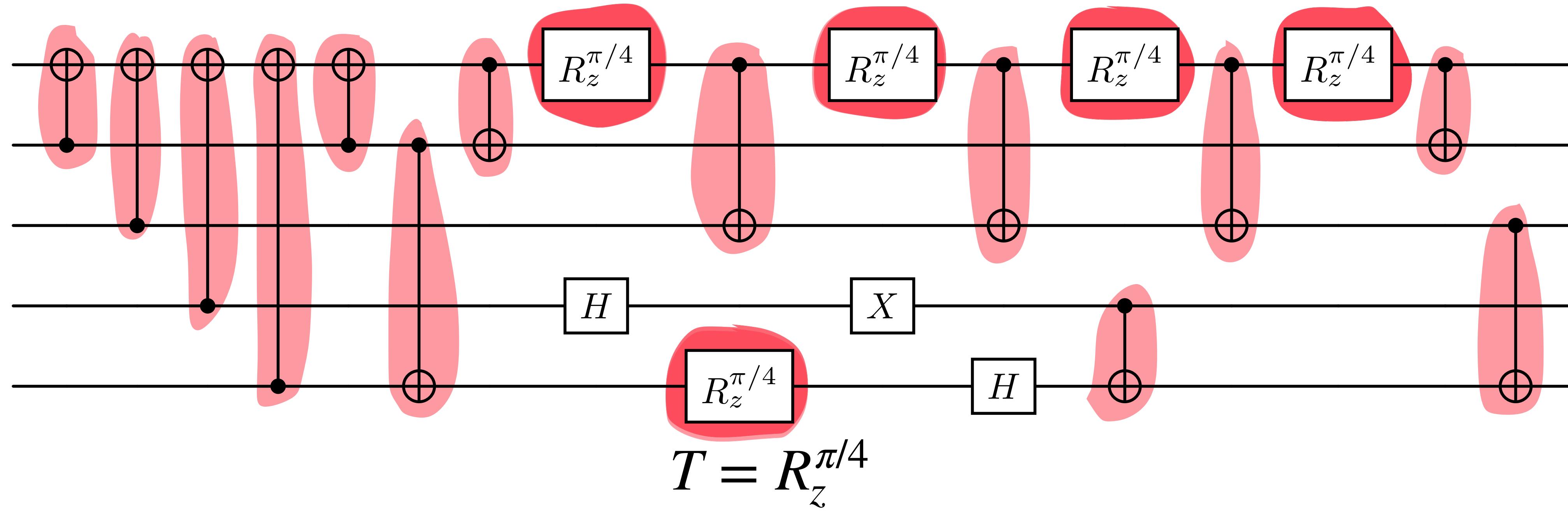
## NISQ

- total gate count
- 2q gate count

## FTQC

- T count and 2q gate count

# Optimization Objectives



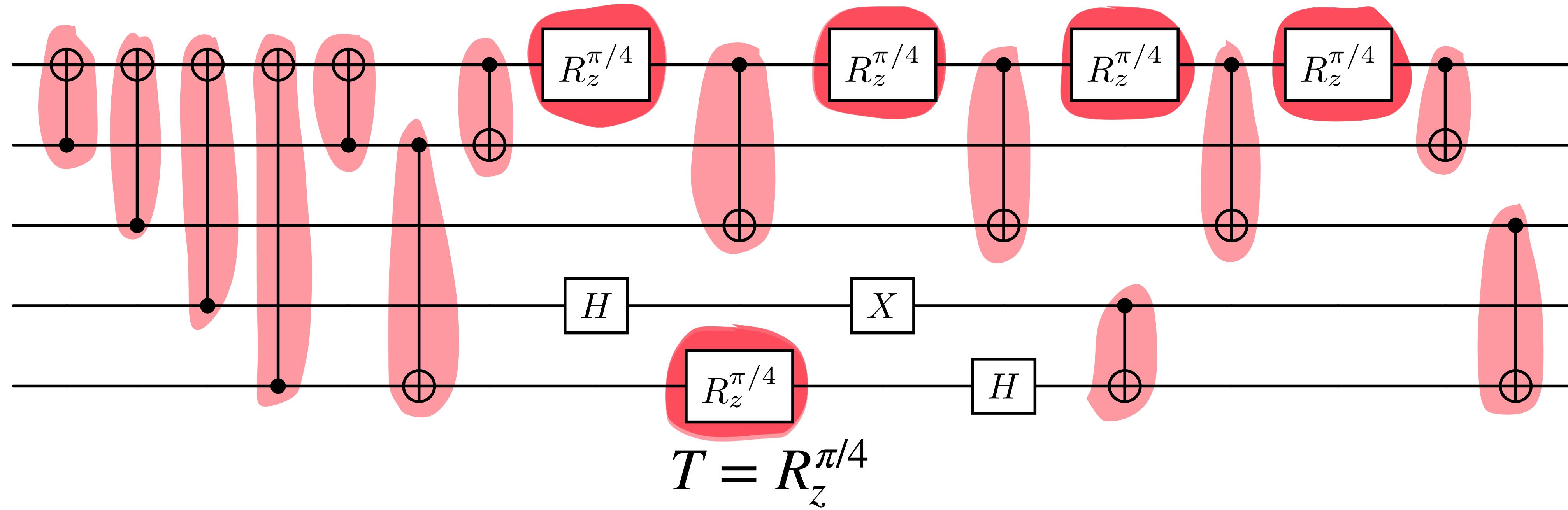
## NISQ

- total gate count
- 2q gate count

## FTQC

- T count and 2q gate count

# Optimization Objectives



NISQ

- total gate count
- 2q gate count

and many more...

FTQC

- T count and 2q gate count

# High-level problem

Given a circuit and optimization objective, output an equivalent circuit that minimizes the optimization objective.

Optimising quantum circuits is generally hard

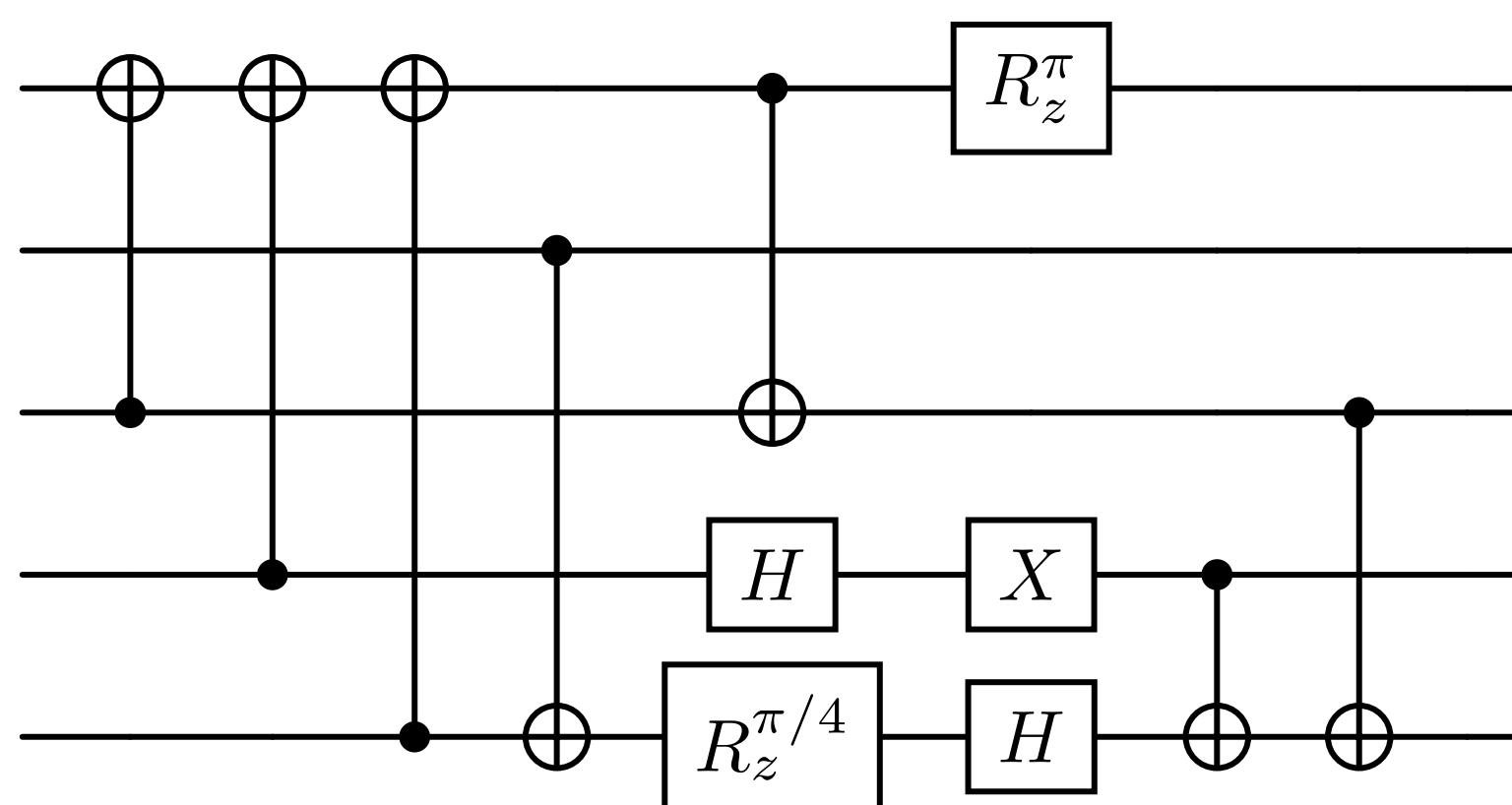
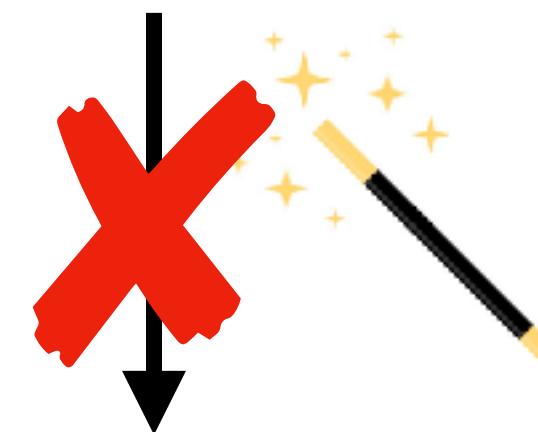
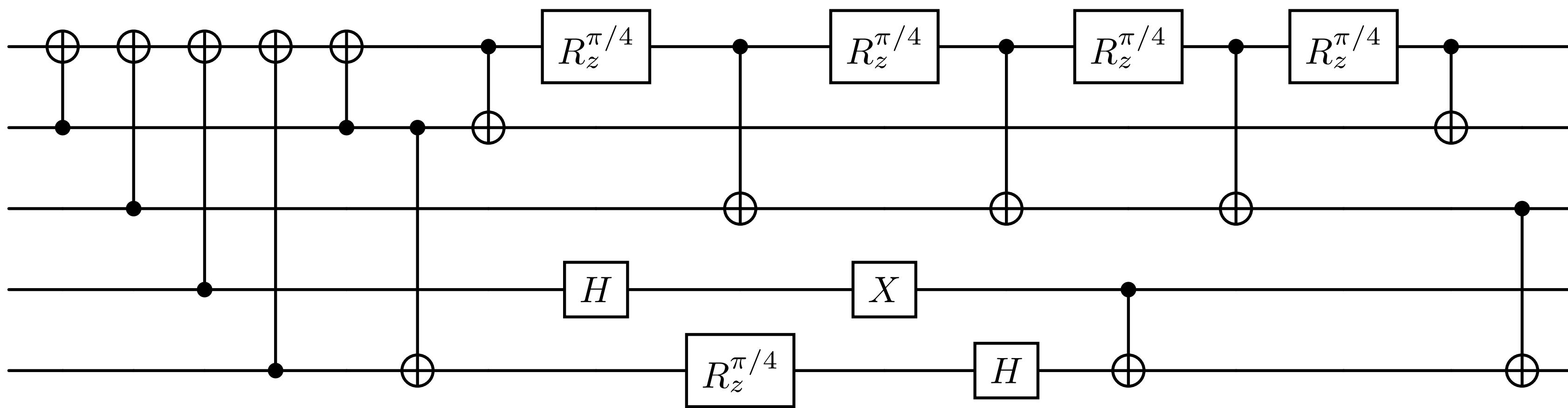
John van de Wetering<sup>1</sup> and Matthew Amy<sup>2</sup>

<sup>1</sup>University of Amsterdam

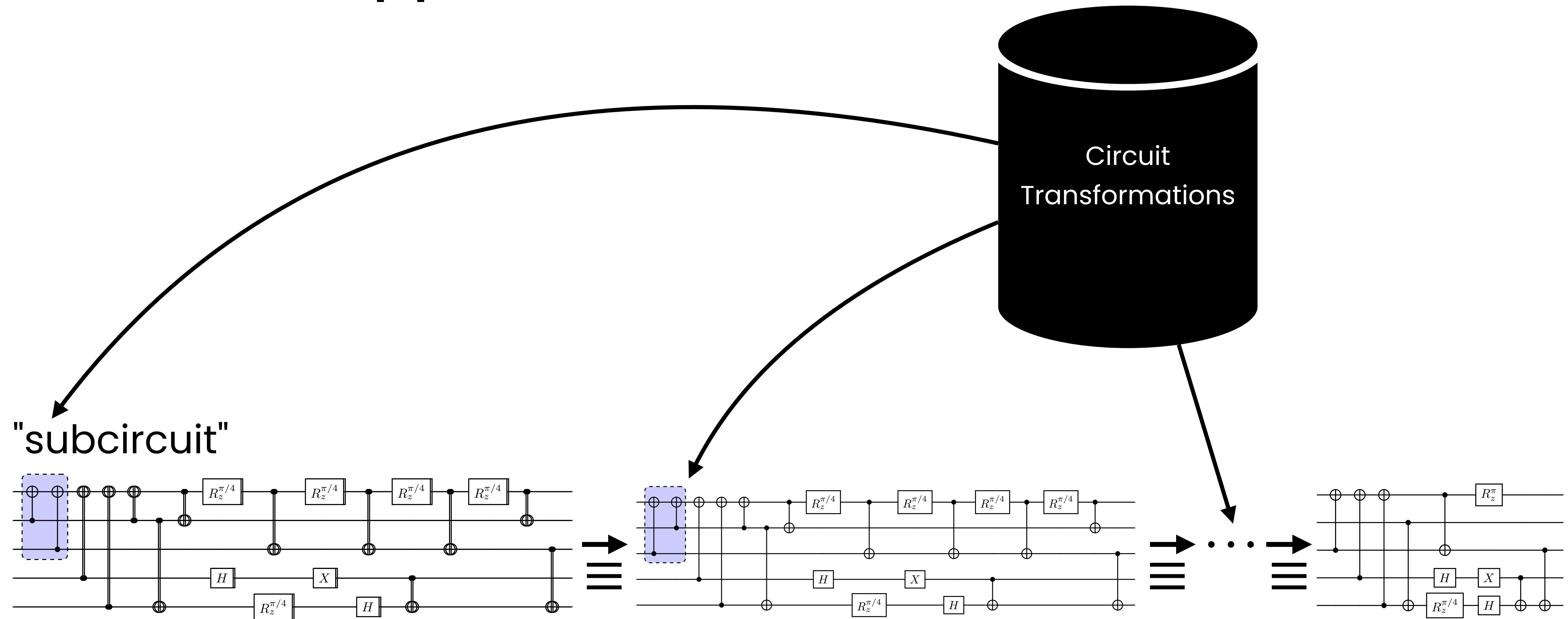
<sup>2</sup>Simon Fraser University

July 28th 2024

# General Approach



# General Approach



# Circuit Equivalence

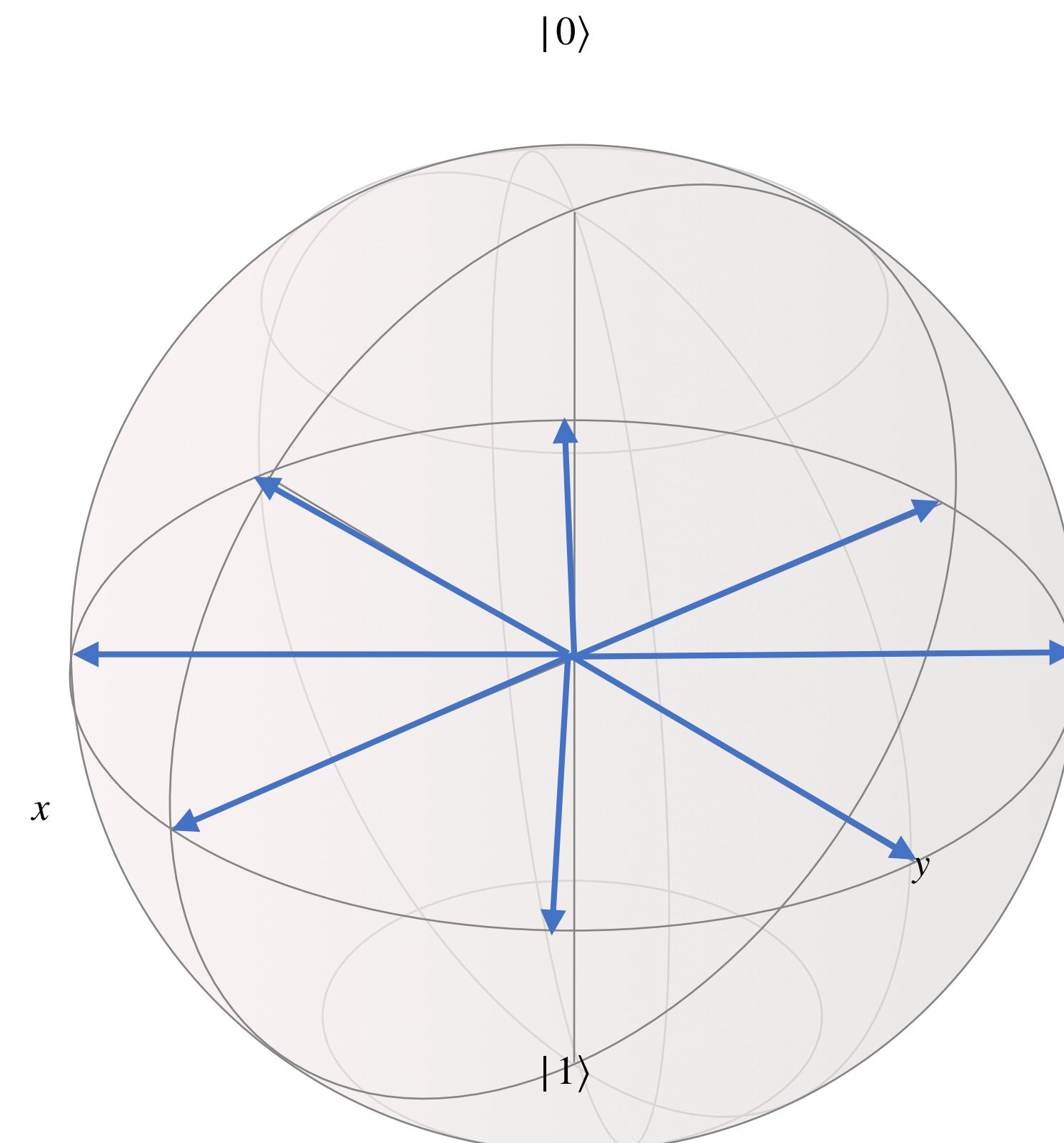
Circuits are unitary matrices:  $U^\dagger U = UU^\dagger = I$

$$C_1 \equiv C_2 \iff U_{C_1} = U_{C_2}$$

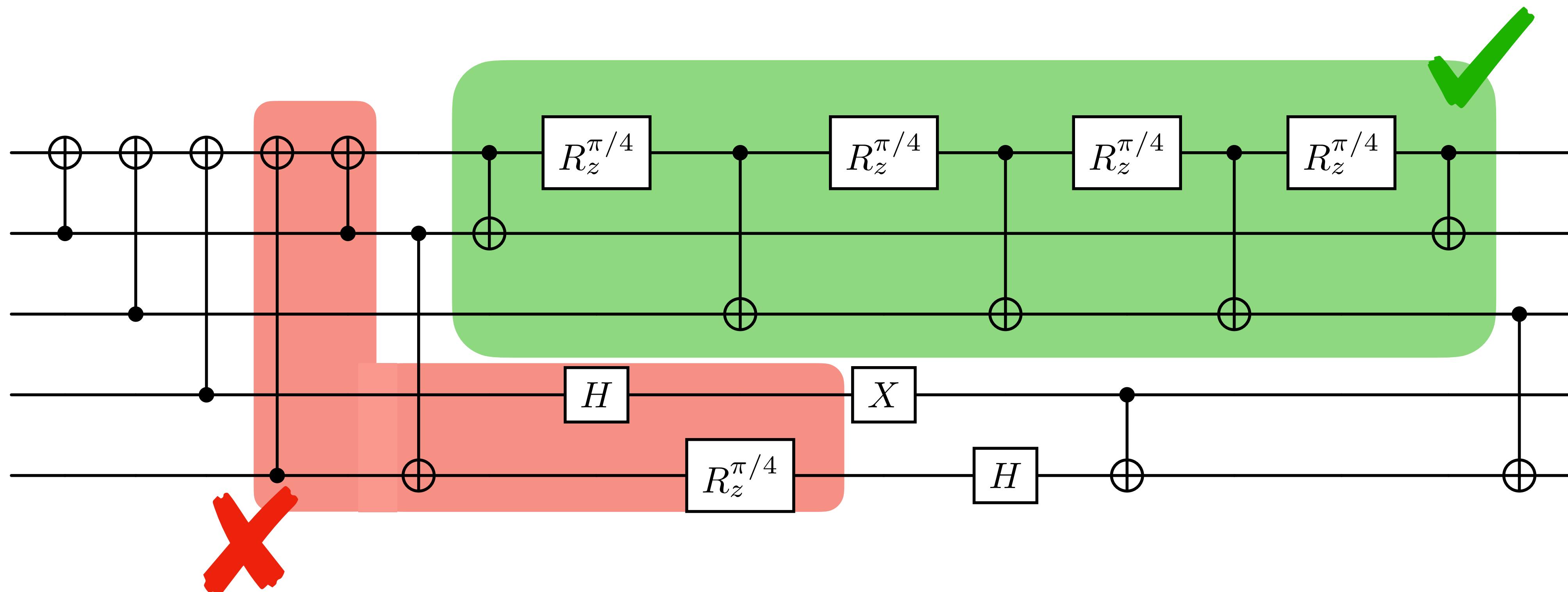
"global phase"

$$C_1 \equiv C_2 \iff \exists \varphi . U_{C_1} = e^{i\varphi} U_{C_2}$$

Hard for even a quantum computer to verify  
(QMA-Complete)



# Subcircuit



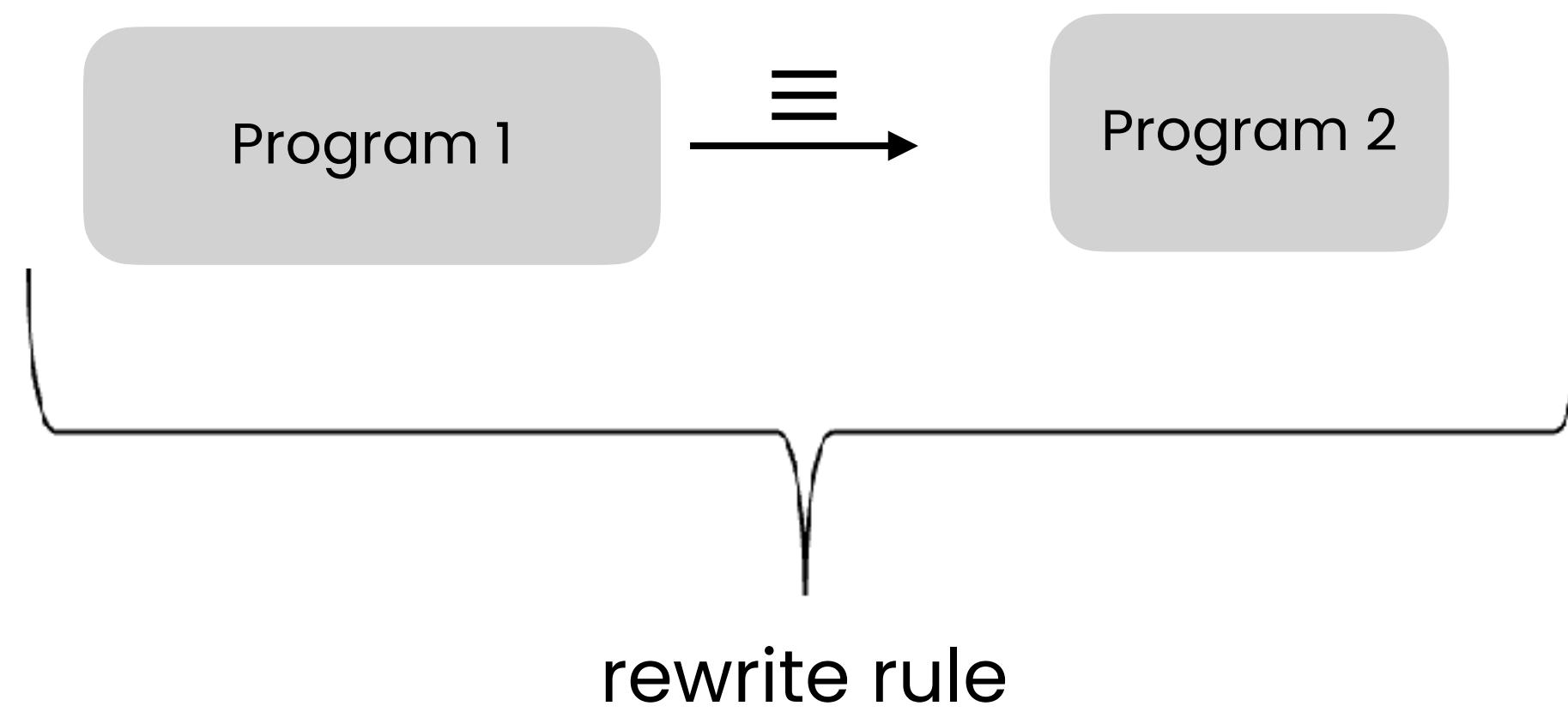
"convex subgraph"

i.e. any path between two vertices in the DAG also in subgraph

# Rewrite Rules

# Rewrite Rules

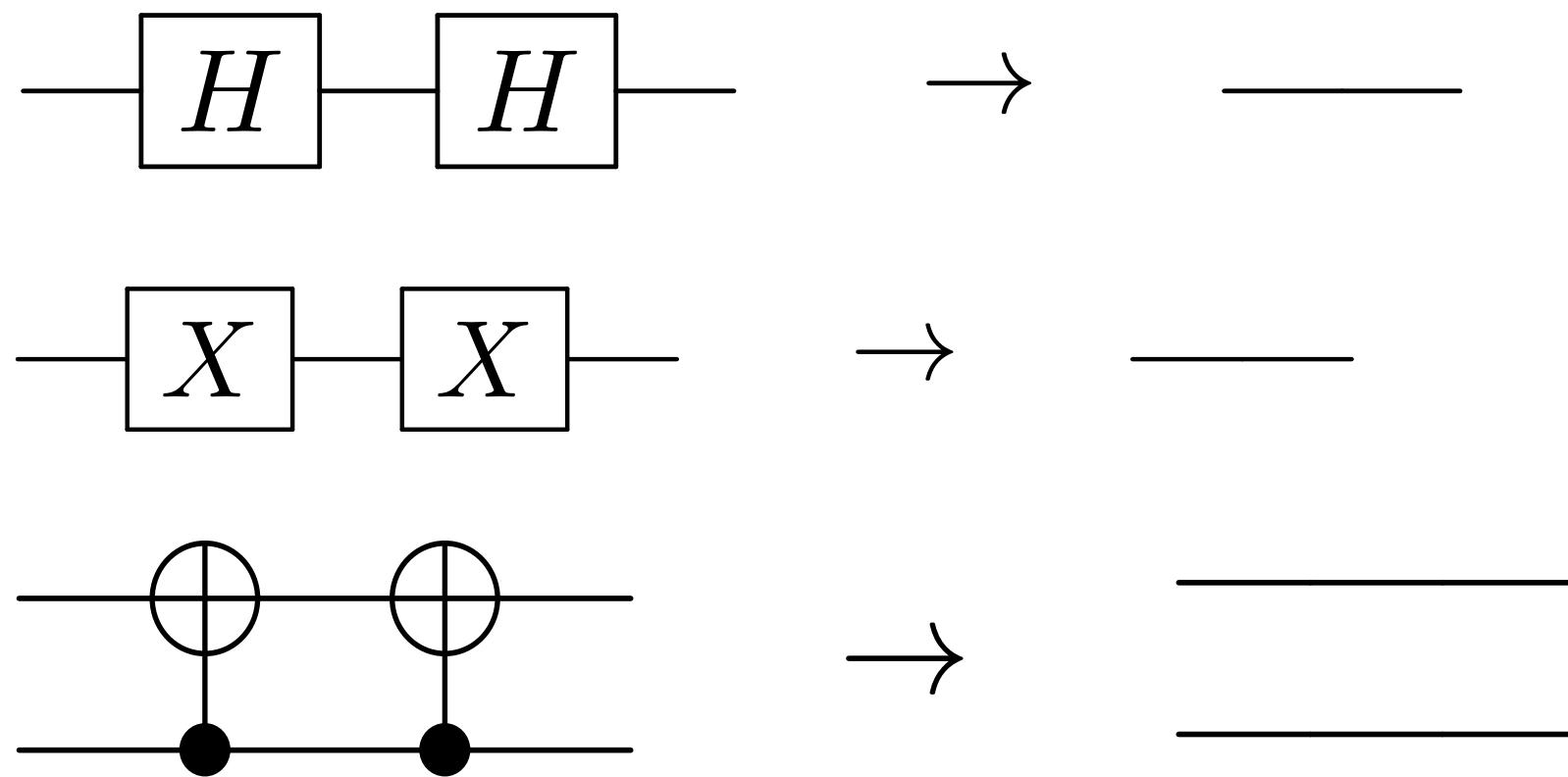
"peephole optimization"



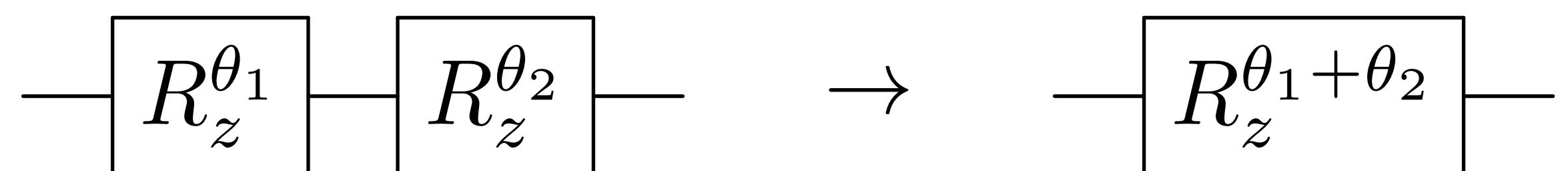
- Optimization: LHS and RHS same gate set
- Decomposition: LHS higher level than RHS

# Simple Rewrite Rules

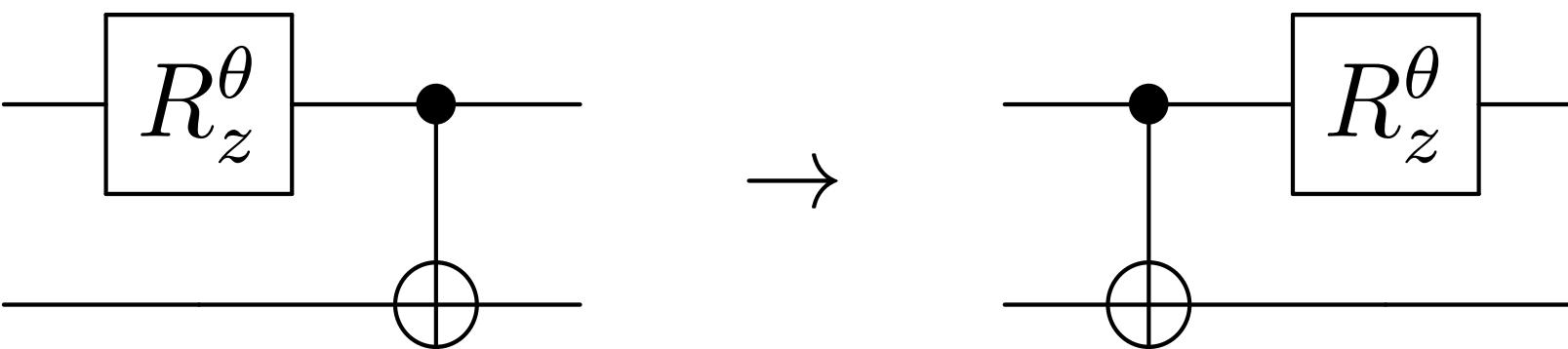
Cancel



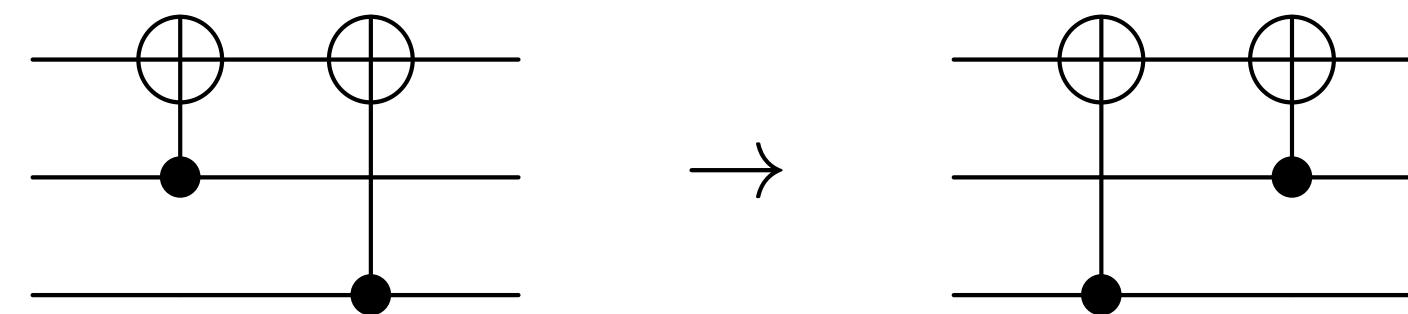
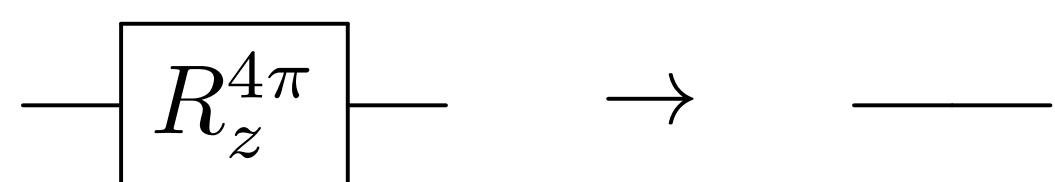
Merge



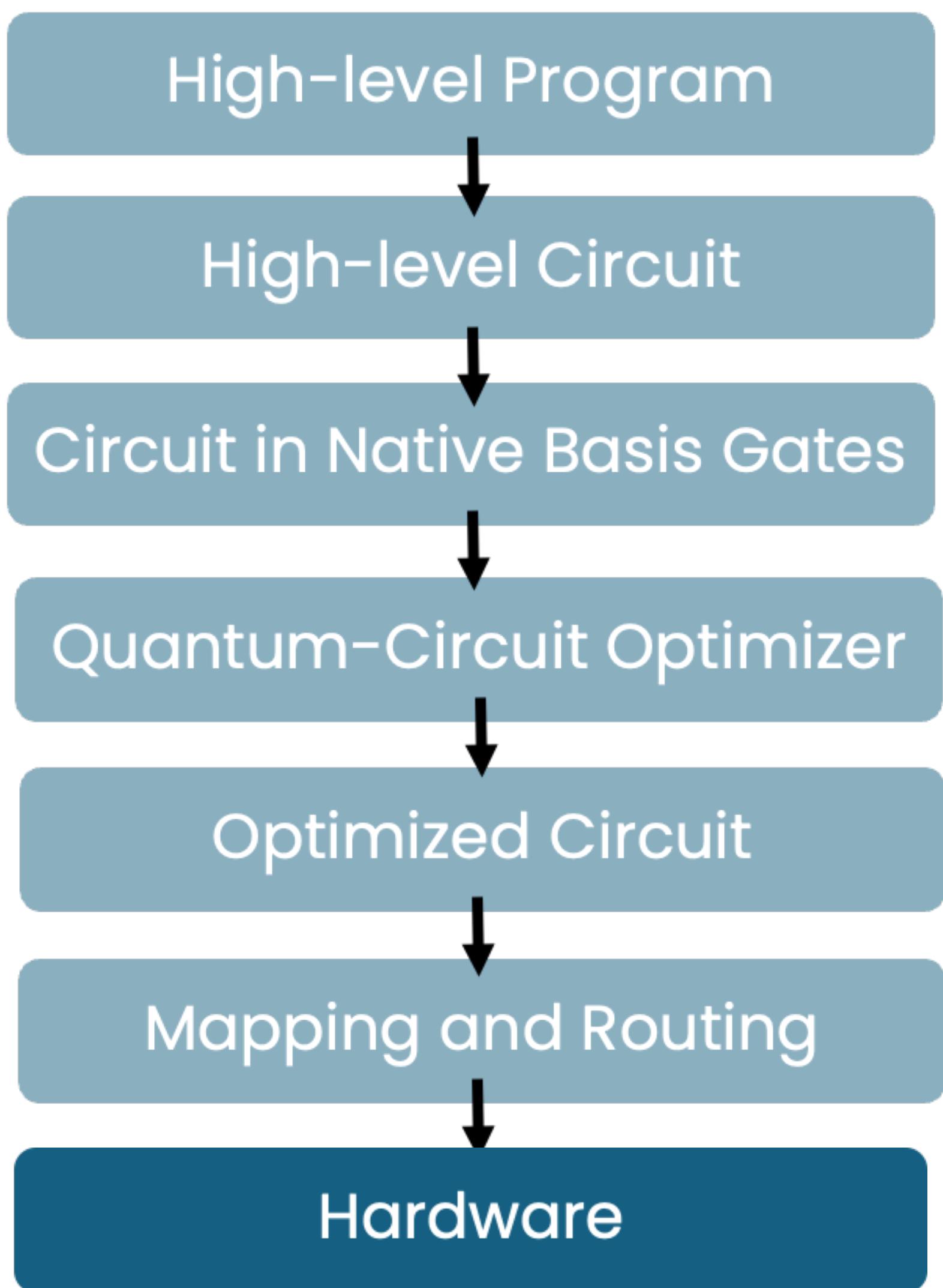
Commute



Global Phase



# Diverse Hardware

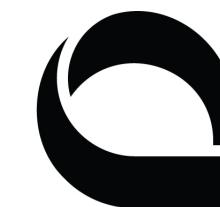


# Diverse Hardware

## Ion Trap

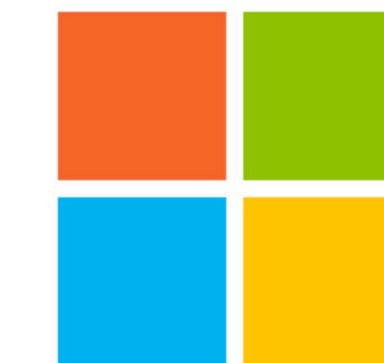


IONQ



QUANTINUUM

## Topological

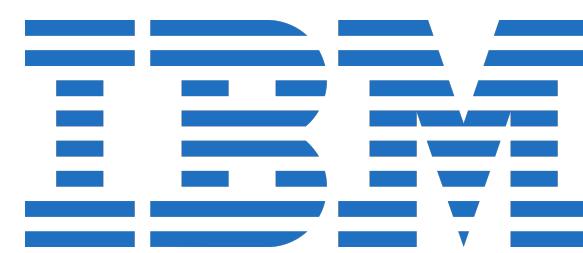


Hardware

## Superconducting



Google  
Quantum AI



U1, U2, U3, CX

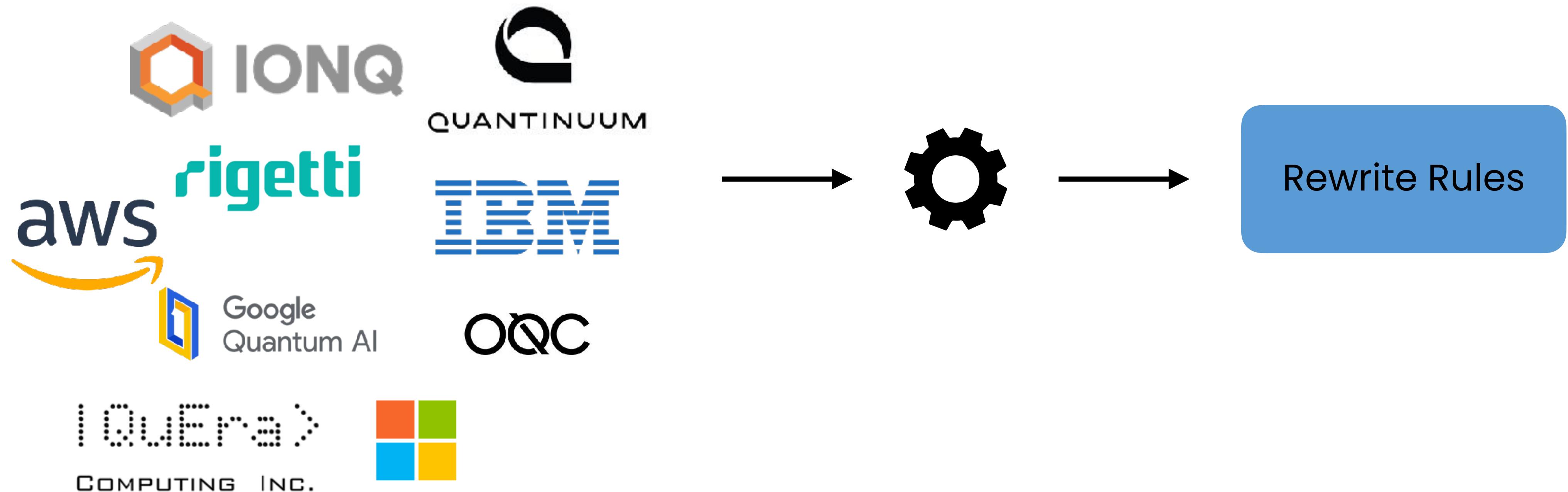
$\rightarrow$  Rz, SX, X, CX

$\rightarrow$  Rz, Rx, SX, X, CZ, Rzz



# Synthesis!

Arbitrary gate set



# Naïve Synthesis

```
rules = []
circuits = enumerate(max_qubits, max_size)
for c1 in circuits:
    for c2 in circuits:
        if verify(c1, c2):
            rules.append(c1 → c2)
```

*expensive*

*big*

*uh oh...*

# QUESO

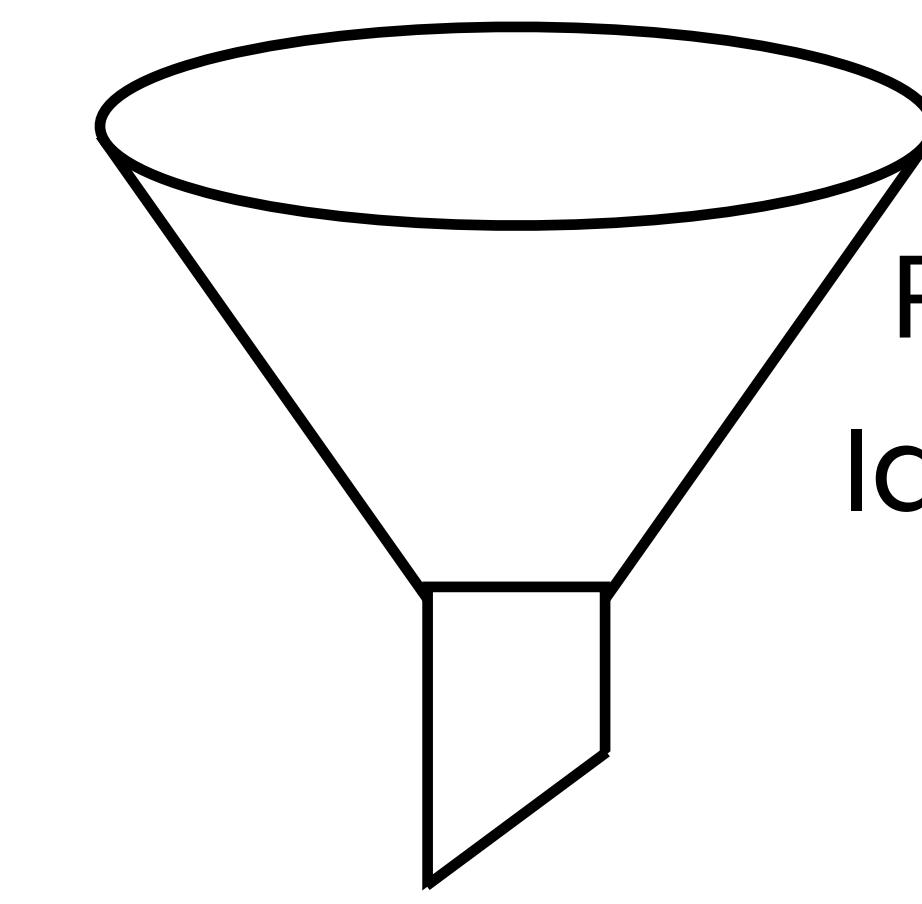
Schwartz-Zippel Lemma for  
polynomial identity testing (PIT)

Feynman's path integral formulation  
for quantum mechanics

(symbolic)  
Circuit



Polynomial



Polynomial  
Identity Filter  
(PIF)

## Synthesizing Quantum-Circuit Optimizers

AMANDA XU, University of Wisconsin-Madison, USA  
ABTIN MOLAVI, University of Wisconsin-Madison, USA  
LAUREN PICK, University of Wisconsin-Madison, USA  
SWAMIT TANNU, University of Wisconsin-Madison, USA  
AWS ALBARGHOUTHI, University of Wisconsin-Madison, USA

Circuit Equivalence  
Classes

# Path-sum Semantics

Algebraically represent semantics of quantum gates

$$R_z(\theta) : |x\rangle \rightarrow e^{i(2x-1)\theta} |x\rangle$$

$$CNOT : |x_1 x_2\rangle \rightarrow |x_1(x_1 \oplus x_2)\rangle$$

More generally:  $|x\rangle \rightarrow \phi(x, \rho) |f(x)\rangle$

amplitude    state

# Simple Circuit as Polynomial

Circuit

$R_z(\theta) \quad q_1 ;$

Polynomial

$e^{i(2(0)-1)\theta} |0\rangle$

# Simple Circuit as Polynomial

Circuit

$R_z(\theta) \quad q_1 ;$

Polynomial

$e^{-i\theta} |0\rangle$

# Simple Circuit as Polynomial

Circuit

$R_z(\theta) \quad q_1 ;$

Polynomial

$$e^{-i\theta} |0\rangle + e^{i\theta} |1\rangle$$

# Polynomial Identity Testing (PIT)

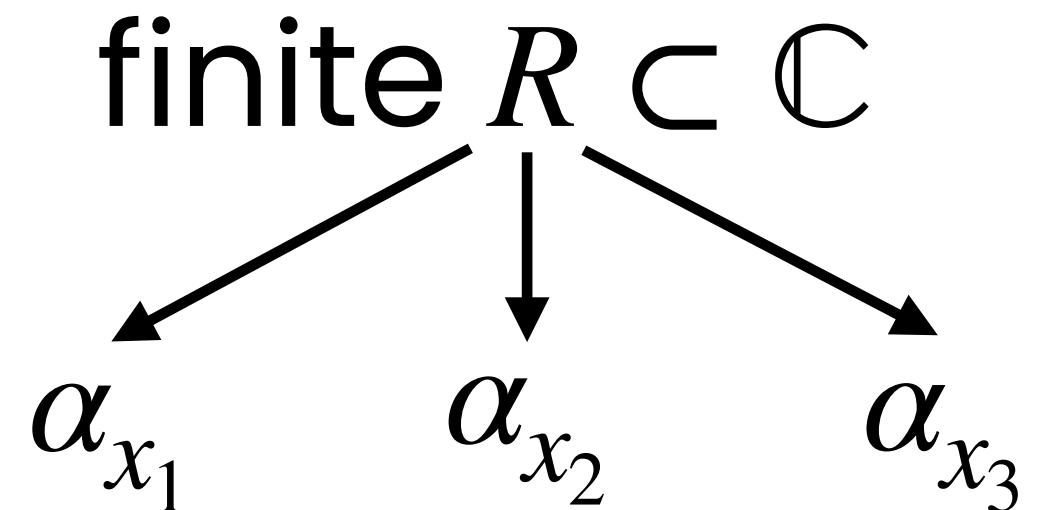
$$\mathbf{x} \in \mathbb{C}^3 \quad p_1(\mathbf{x}) = x_1x_2$$
$$p_2(\mathbf{x}) = x_1x_2 + x_3x_2$$

How can we check  $p_1 \equiv p_2$ ?

# Schwartz-Zippel Lemma

$$p_1(\mathbf{x}) = x_1x_2$$

$$p_2(\mathbf{x}) = x_1x_2 + x_3x_2$$



$$p_1(\boldsymbol{\alpha}) = \alpha_{x_1}\alpha_{x_2}$$

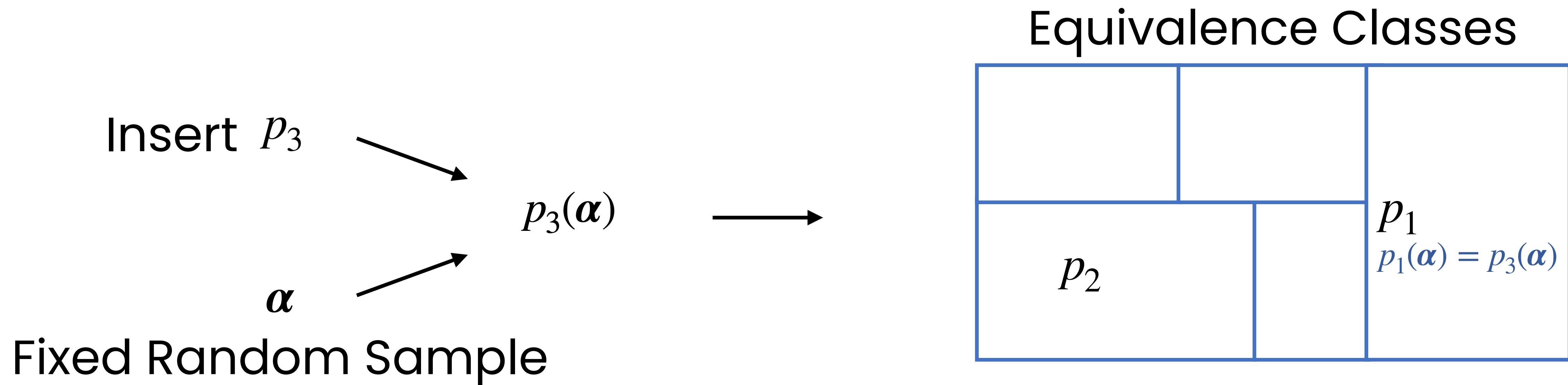
$$p_2(\boldsymbol{\alpha}) = \alpha_{x_1}\alpha_{x_2} + \alpha_{x_3}\alpha_{x_2}$$

Check  $p_1(\boldsymbol{\alpha}) = p_2(\boldsymbol{\alpha})$

If  $p_1 \neq p_2$ , then the probability that  $p_1(\boldsymbol{\alpha}) = p_2(\boldsymbol{\alpha})$  is at most  $\frac{d}{|R|}$

small  $\xrightarrow{\hspace{1cm}}$   $d$  (max total degree)  
very large  $\xrightarrow{\hspace{1cm}}$   $|R|$

# Polynomial Identity Filter (PIF)



# QUESO's Improvements

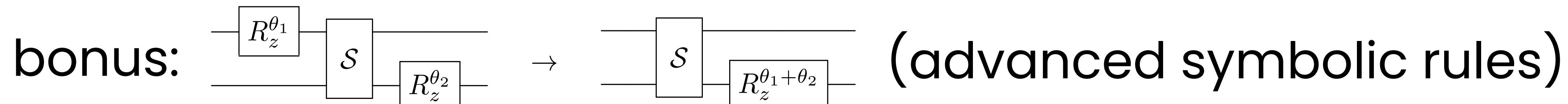
```
rules = []
circuits = enumerate(max_qubits, max_size)
for c1 in circuits:
    for c2 in circuits:
        if verify(c1, c2):
            insert into PIF
            rules.append(c1 → c2)
expensive
```

big

uh oh...

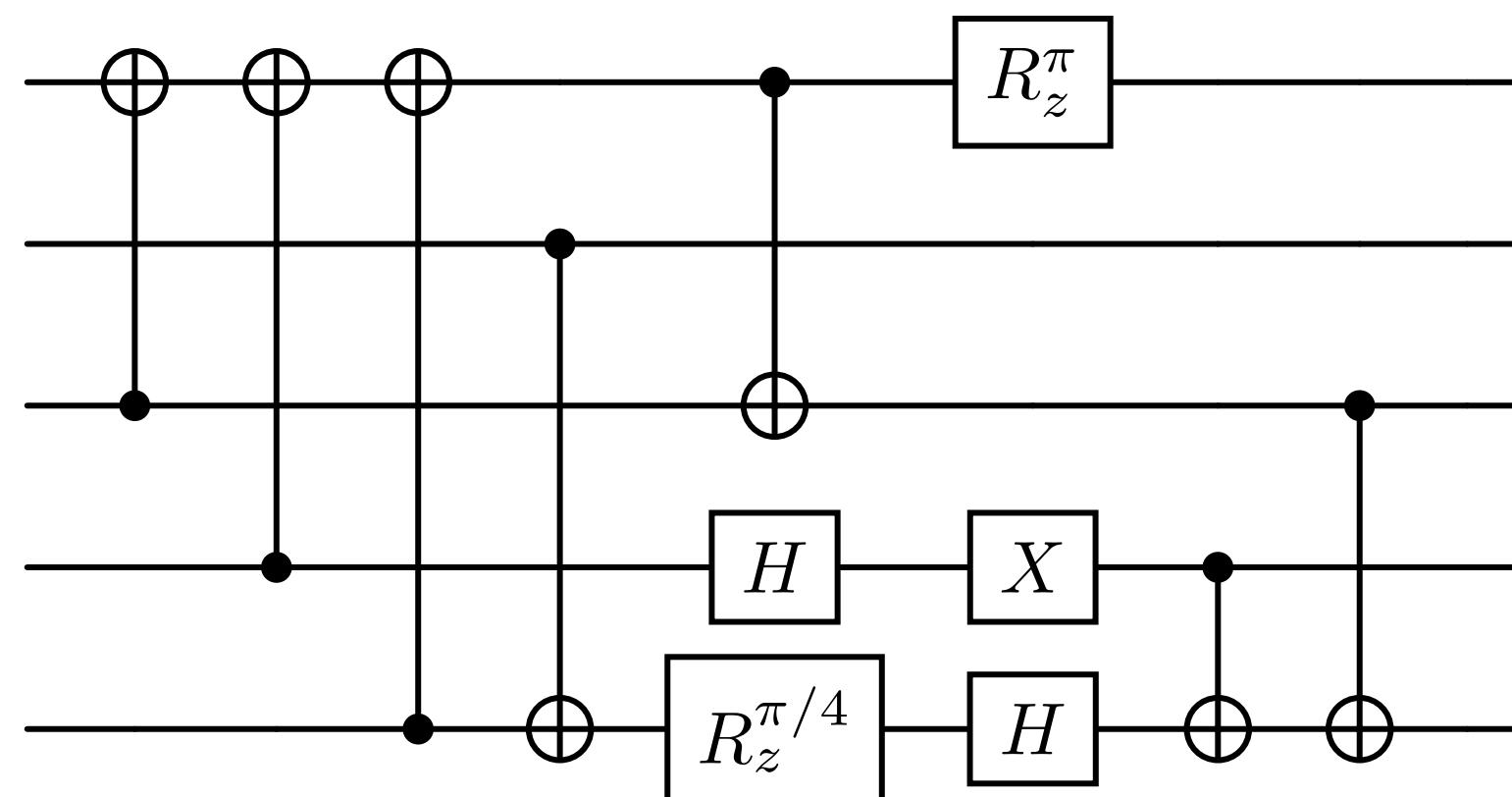
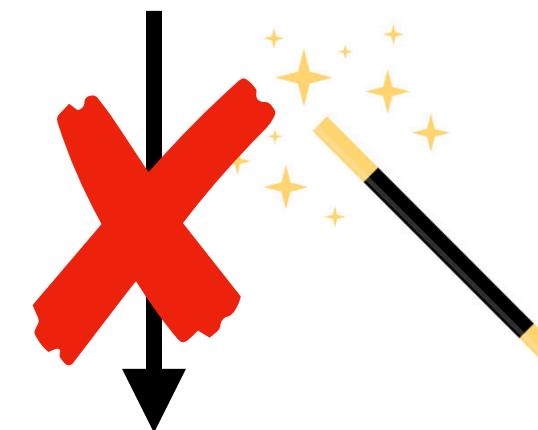
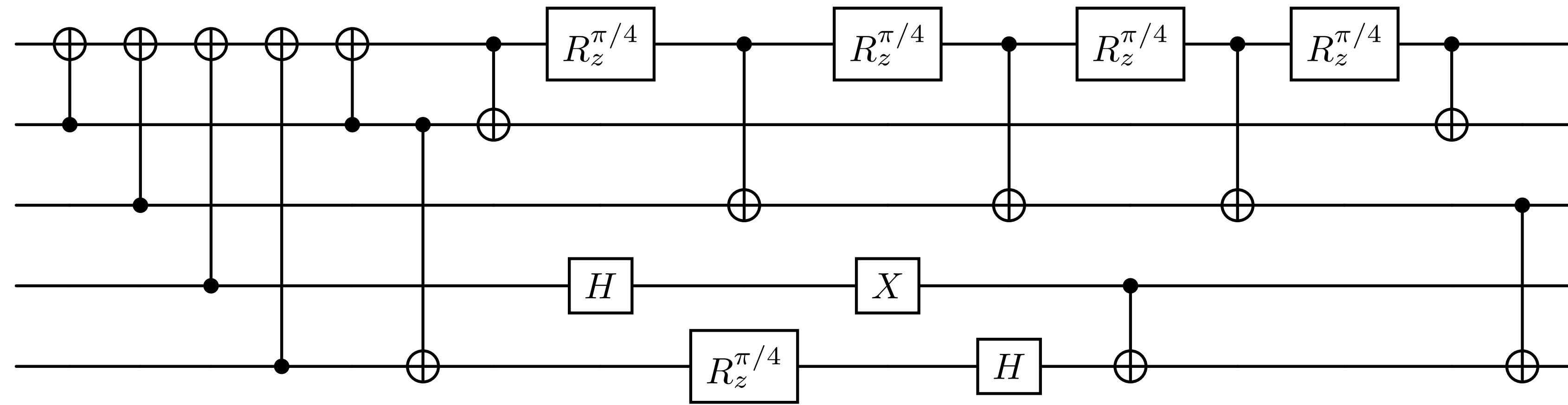
expensive

Efficient equivalence check between circuit pairs!

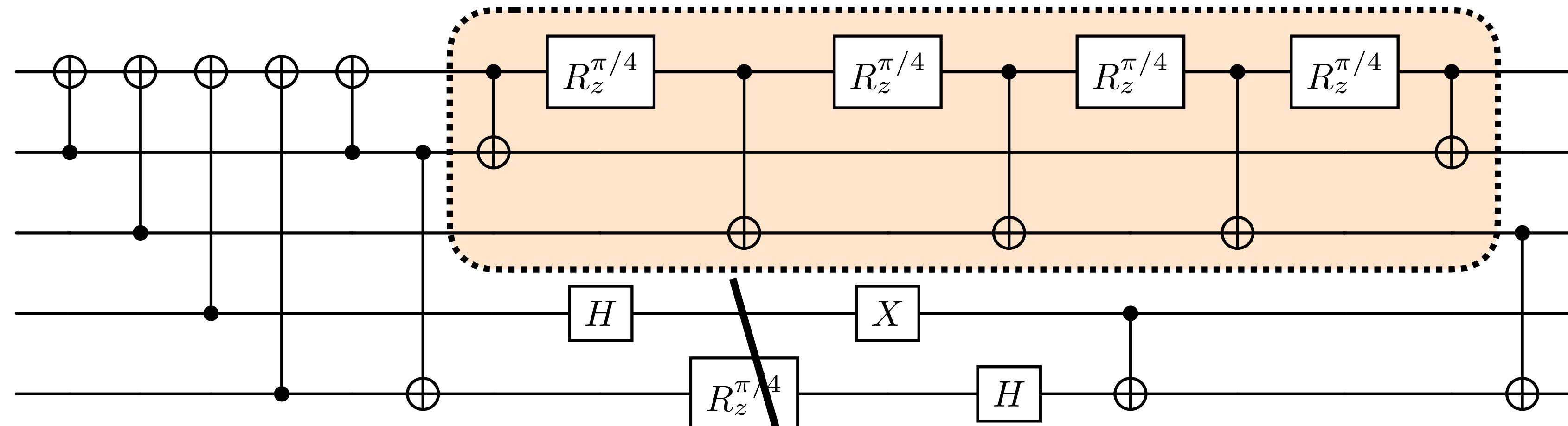


# Circuit Resynthesis

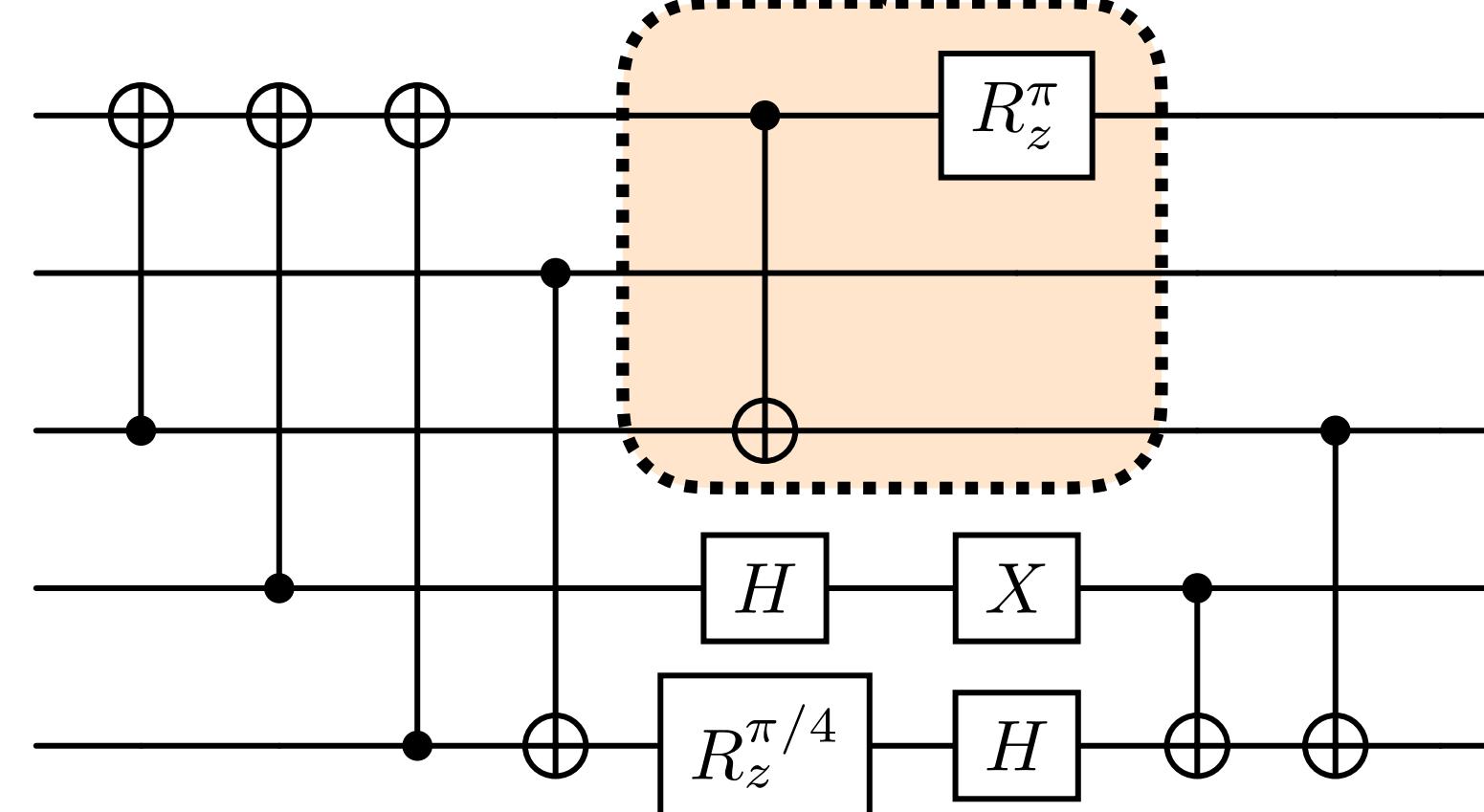
# Circuit Resynthesis



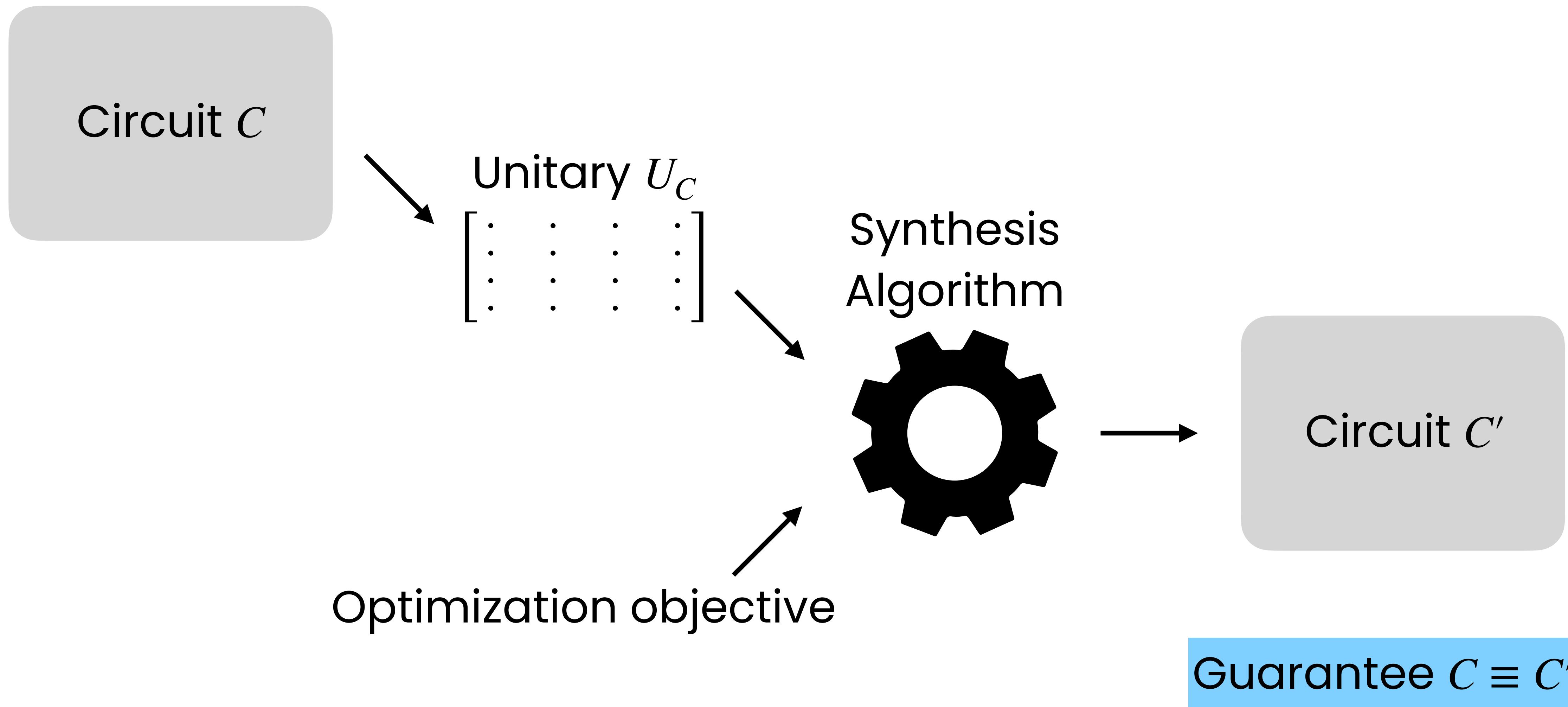
# Circuit Resynthesis



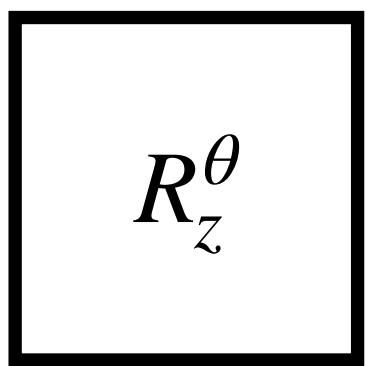
Unitary Synthesis



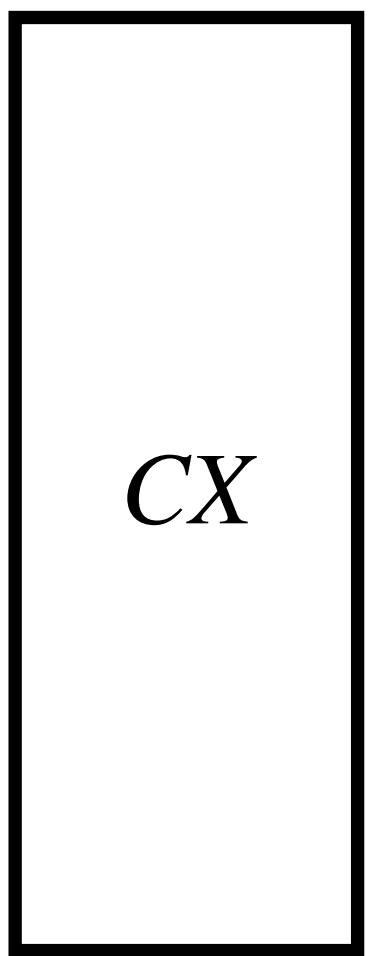
# Unitary Synthesis



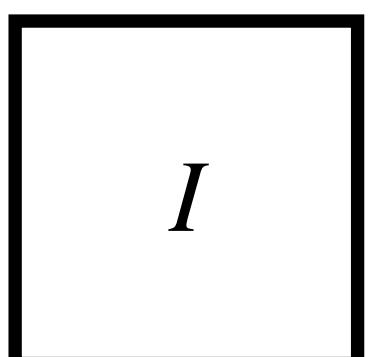
# Circuit to Unitary



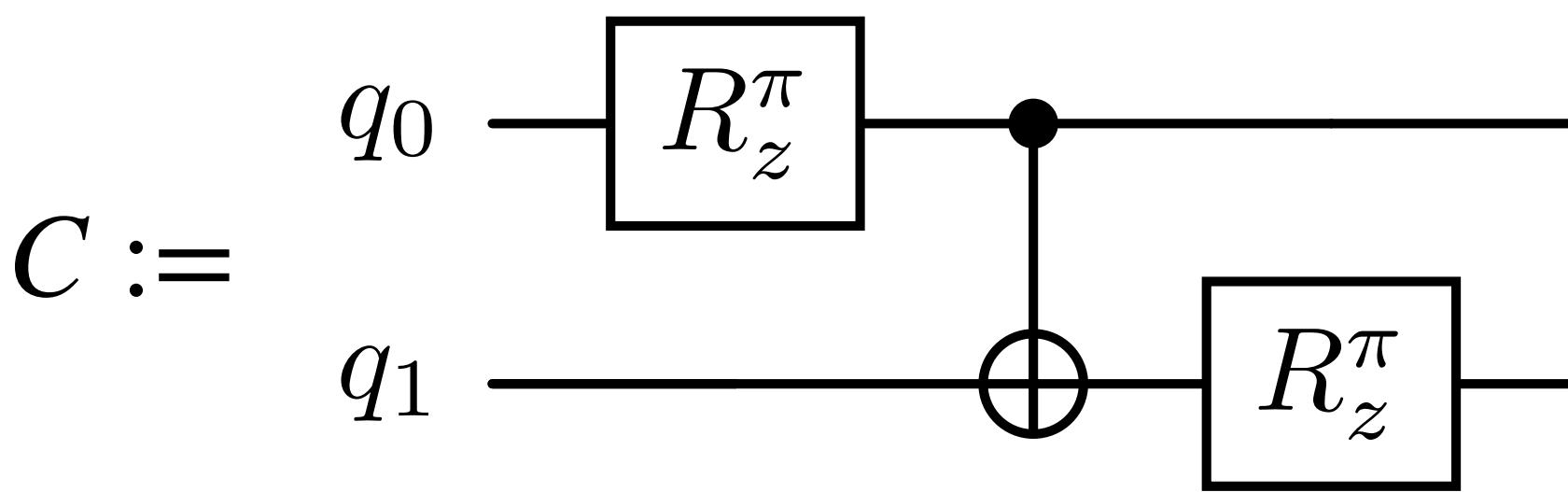
$$U_{R_z^\theta} := \begin{bmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{bmatrix}$$



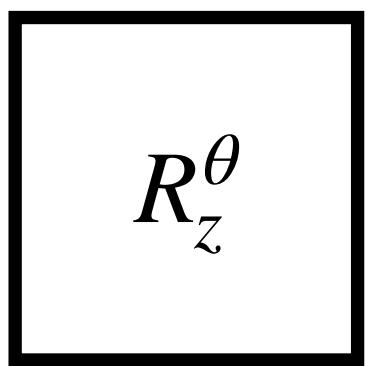
$$U_{CX} := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



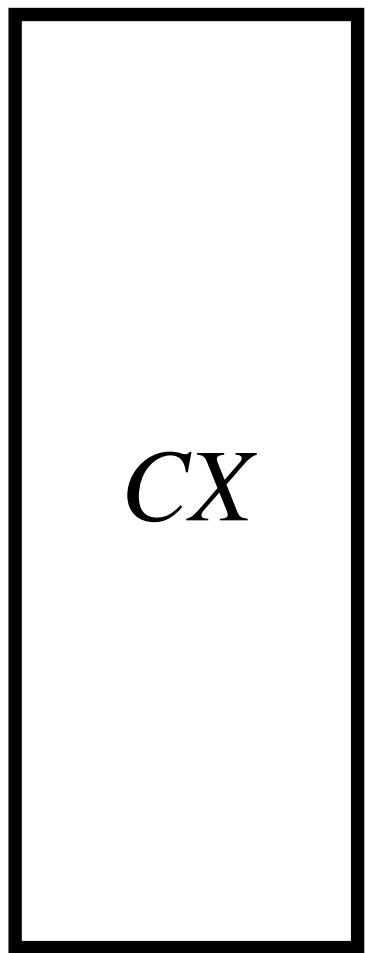
$$U_I := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



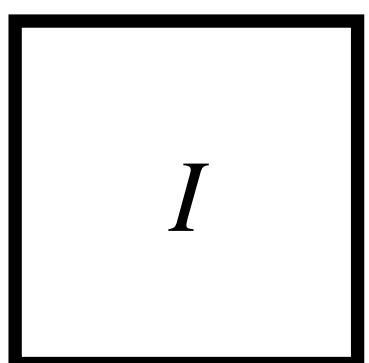
# Circuit to Unitary



$$U_{R_z^\theta} := \begin{bmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{bmatrix}$$



$$U_{CX} := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

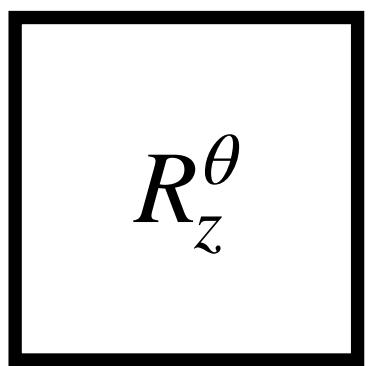


$$U_I := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

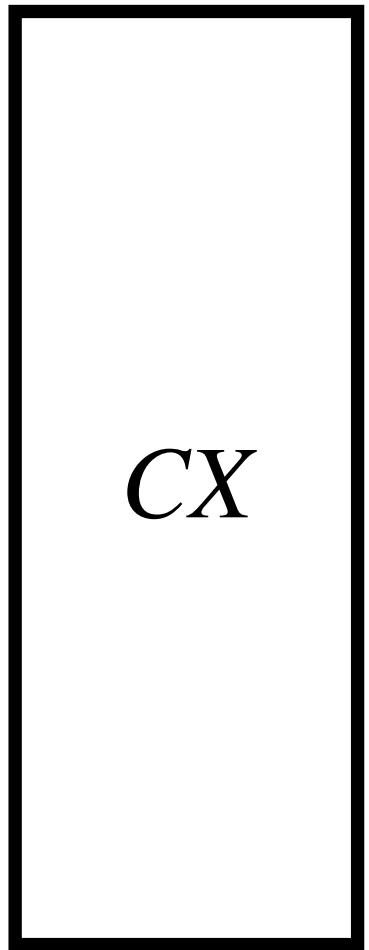
$$C := \begin{array}{c} q_0 \xrightarrow{R_z^\pi} \bullet \\ \text{---} \quad | \\ q_1 \xrightarrow{\oplus} R_z^\pi \end{array}$$

$U_C = U_{R_z^\pi}$

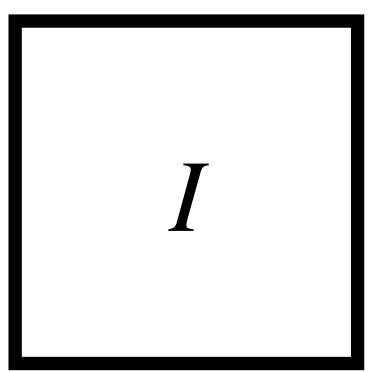
# Circuit to Unitary



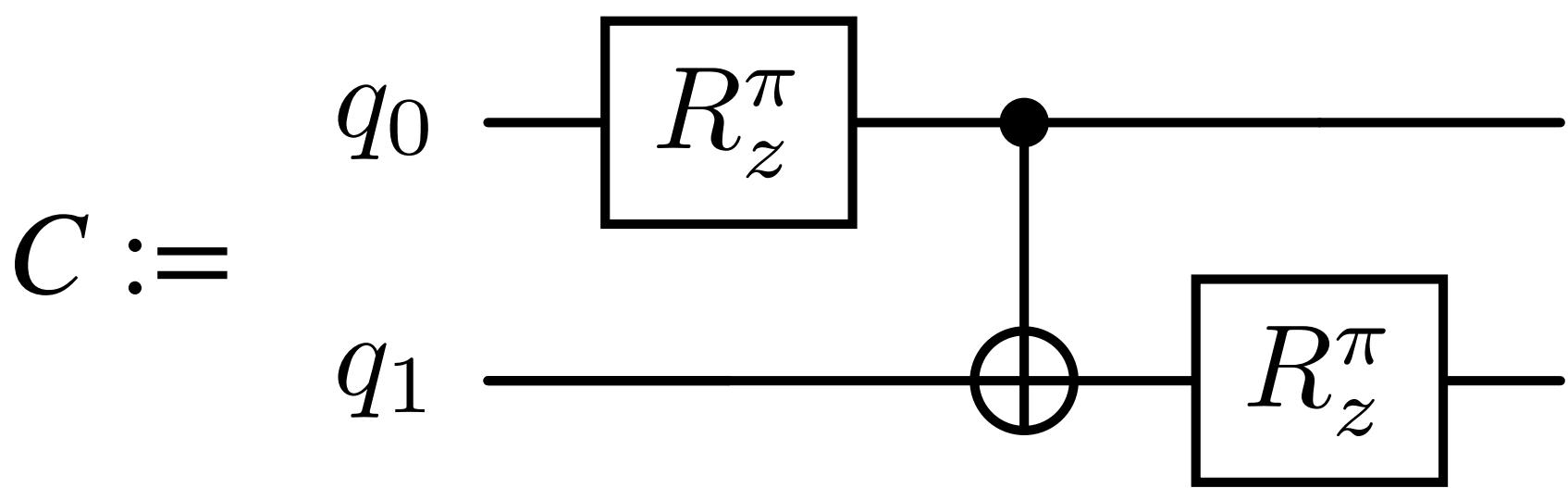
$$U_{R_z^\theta} := \begin{bmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{bmatrix}$$



$$U_{CX} := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



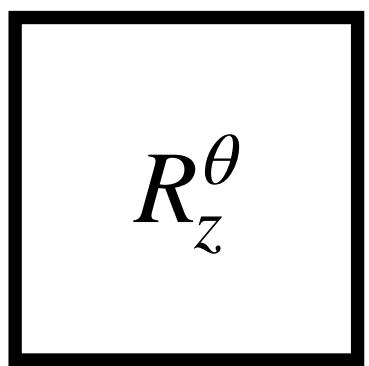
$$U_I := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



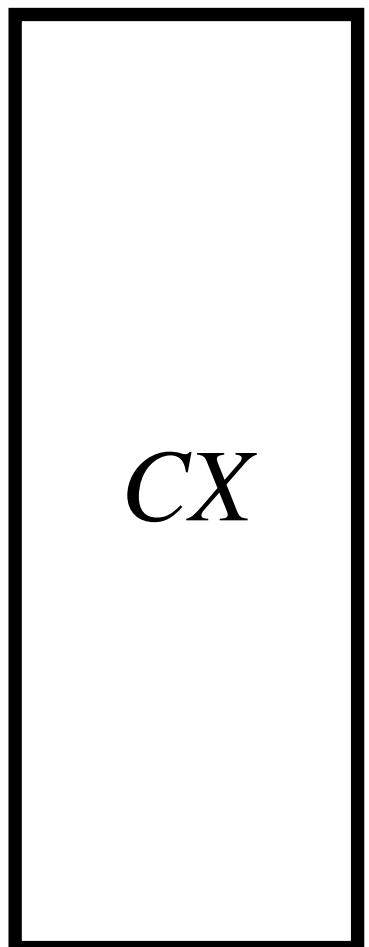
$$U_C =$$
  
$$2^2 \times 2^2$$

$$U_{R_z^\pi}$$

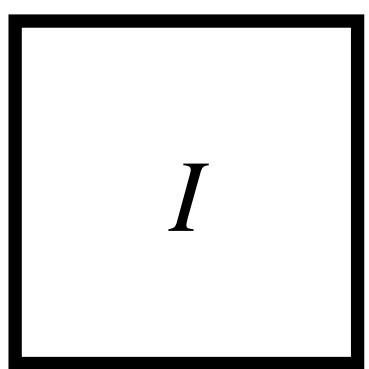
# Circuit to Unitary



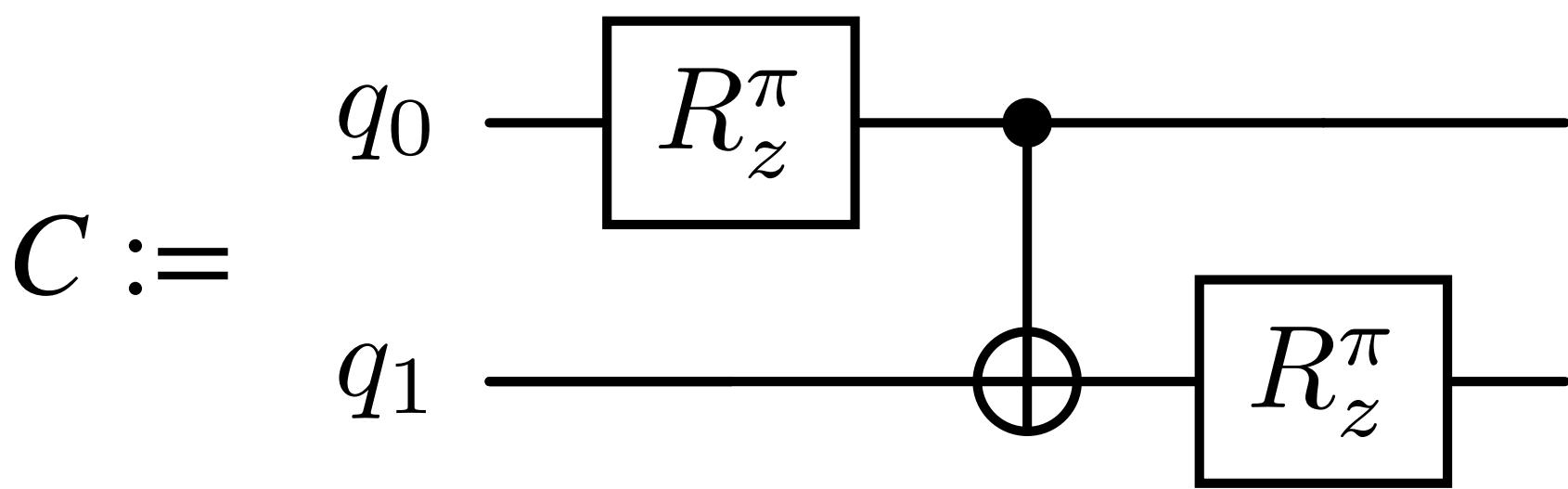
$$U_{R_z^\theta} := \begin{bmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{bmatrix}$$



$$U_{CX} := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



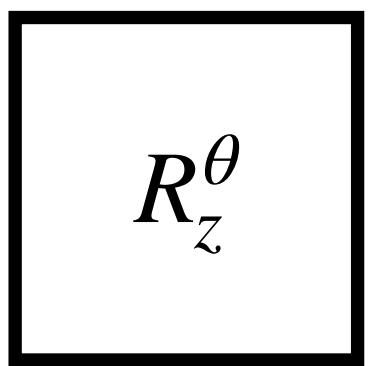
$$U_I := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



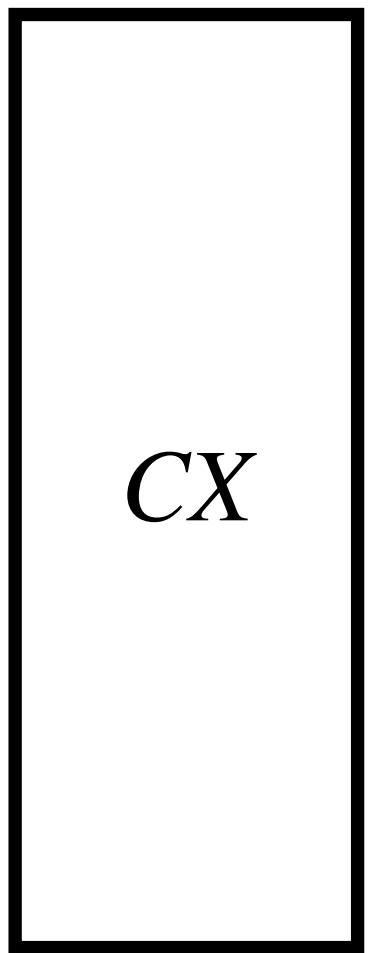
$$U_C =$$
  
$$2^2 \times 2^2$$

$$U_{R_z^\pi}$$
  
$$2 \times 2$$

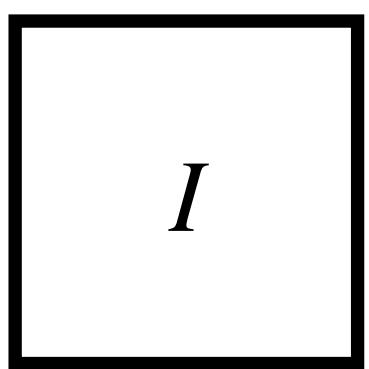
# Circuit to Unitary



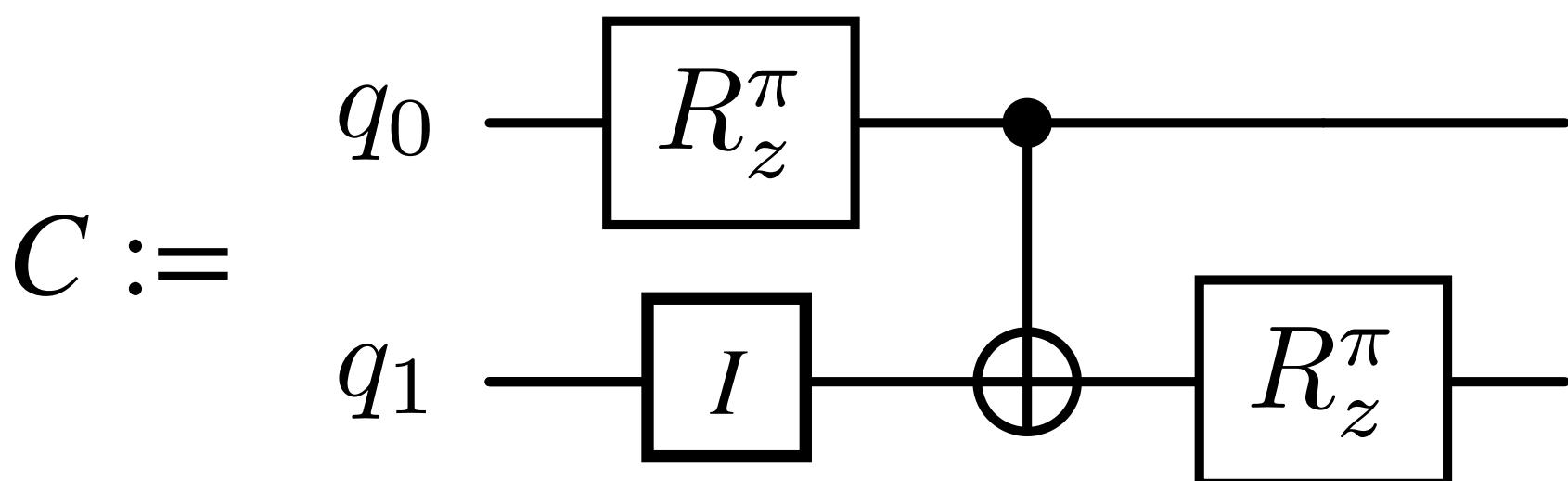
$$U_{R_z^\theta} := \begin{bmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{bmatrix}$$



$$U_{CX} := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



$$U_I := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



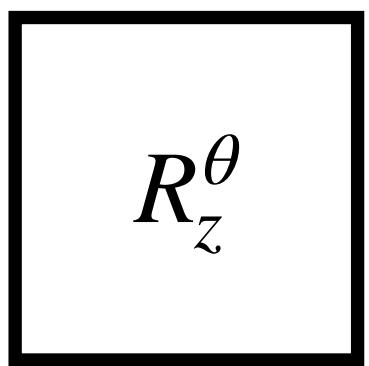
$$U_C =$$

$2^2 \times 2^2$

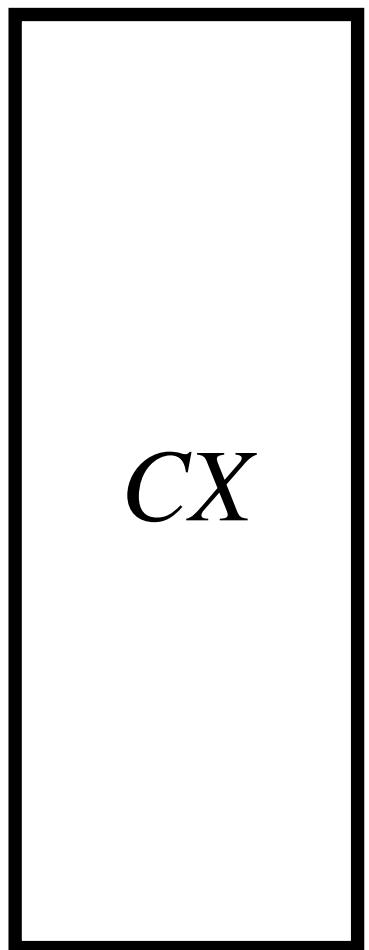
$$U_{R_z^\pi}$$

$2 \times 2$

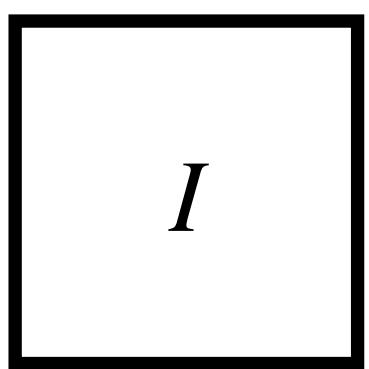
# Circuit to Unitary



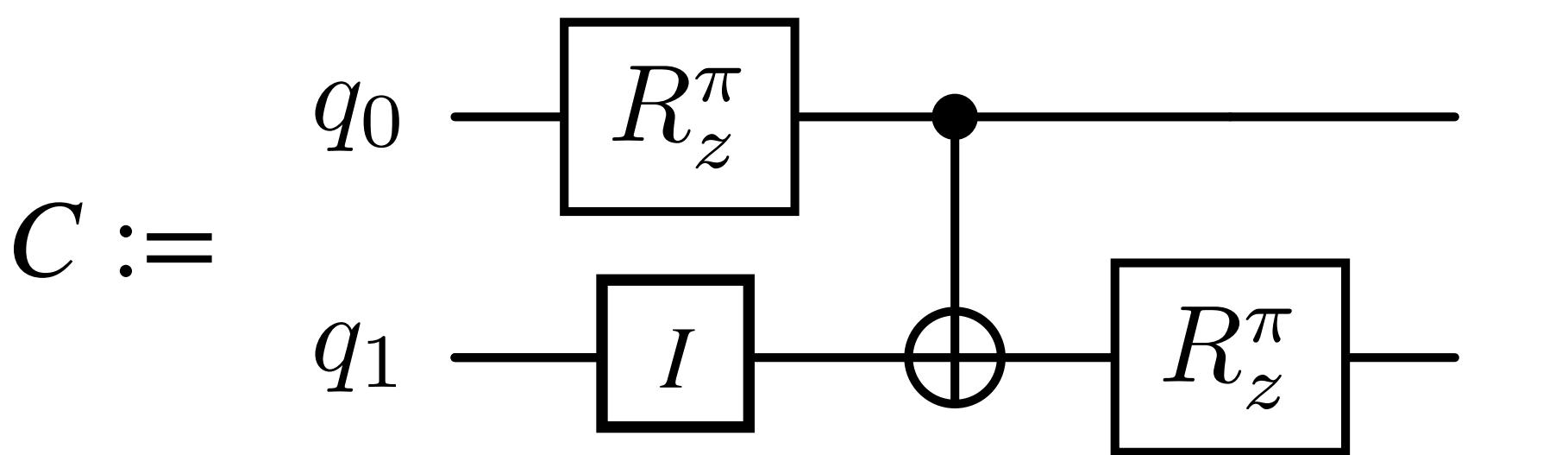
$$U_{R_z^\theta} := \begin{bmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{bmatrix}$$



$$U_{CX} := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

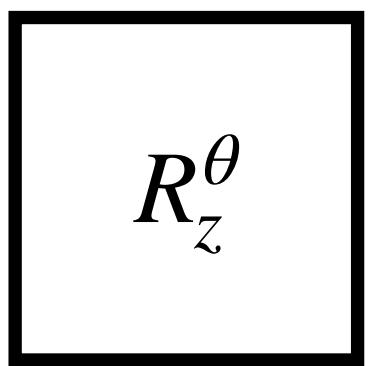


$$U_I := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

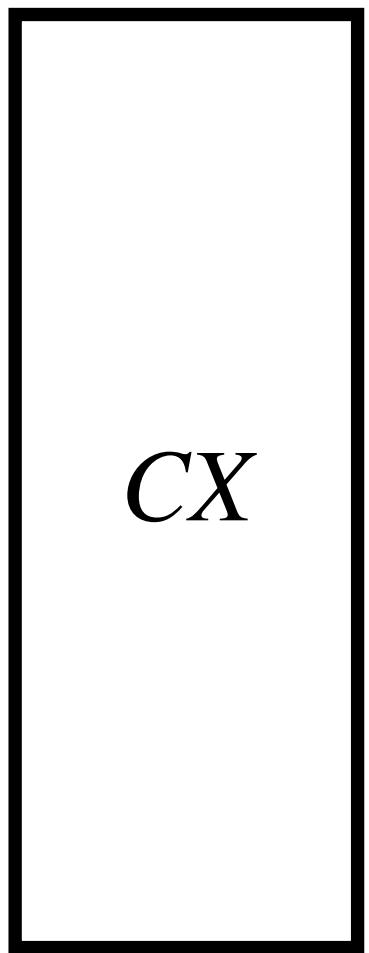


$$U_C = \frac{(U_{R_z^\pi} \otimes U_I)}{2 \times 2}$$

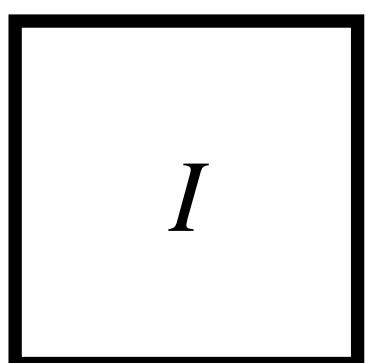
# Circuit to Unitary



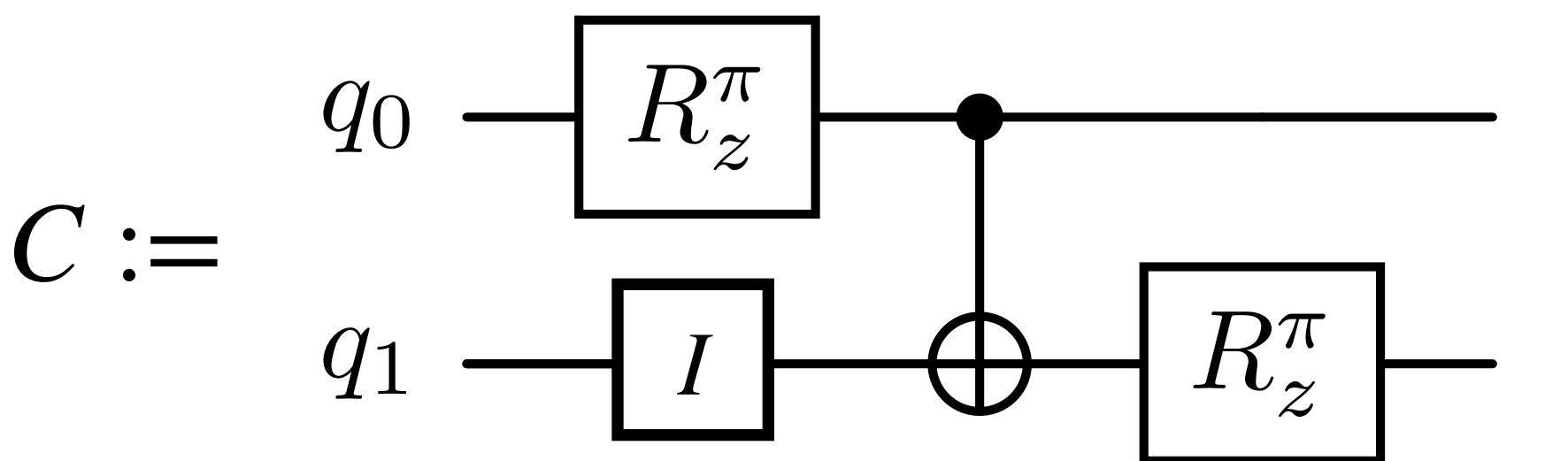
$$U_{R_z^\theta} := \begin{bmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{bmatrix}$$



$$U_{CX} := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



$$U_I := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

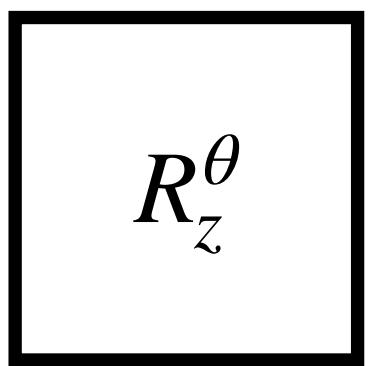


$$U_C =$$

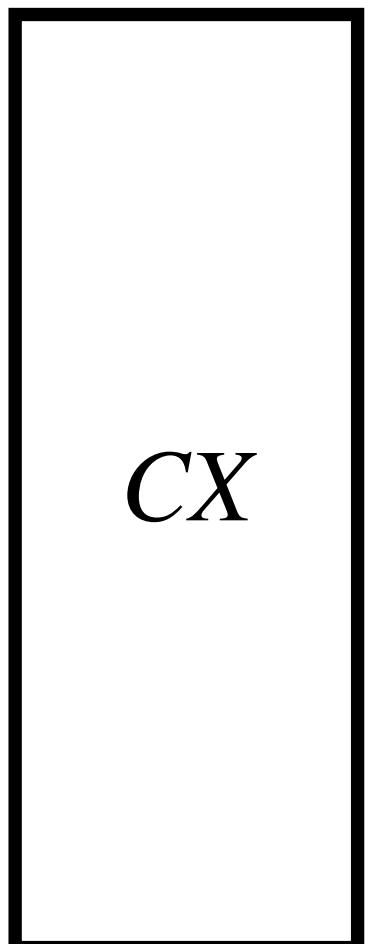
$2^2 \times 2^2$

$$\frac{(U_{R_z^\pi} \otimes U_I)}{2 \times 2 \quad 2 \times 2}$$

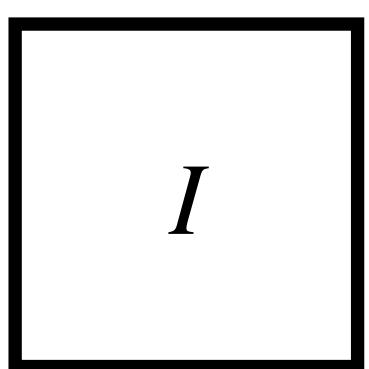
# Circuit to Unitary



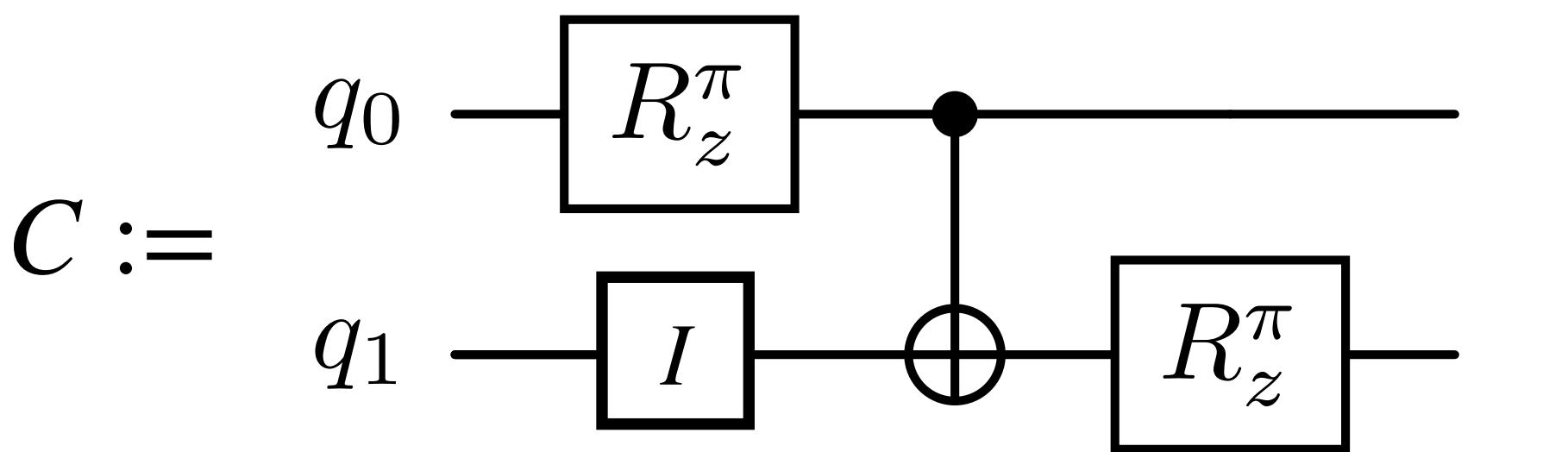
$$U_{R_z^\theta} := \begin{bmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{bmatrix}$$



$$U_{CX} := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



$$U_I := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



$$U_C =$$

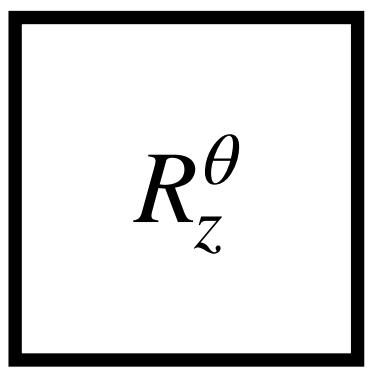
$2^2 \times 2^2$

$$\frac{(U_{R_z^\pi} \otimes U_I)}{2 \times 2 \quad 2 \times 2}$$

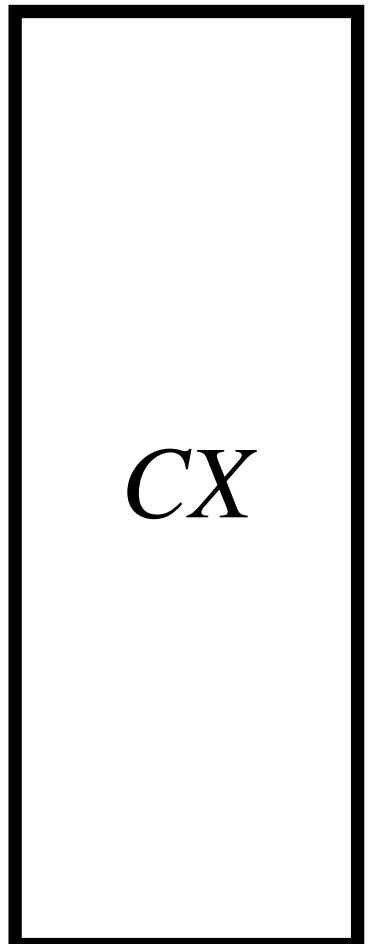
$\frac{2^2 \times 2^2}{2^2 \times 2^2}$

$$(U_{R_z^\pi} \otimes U_I) = \begin{bmatrix} e^{-i\frac{\pi}{2}} \cdot U_I & 0 \cdot U_I \\ 0 \cdot U_I & e^{i\frac{\pi}{2}} \cdot U_I \end{bmatrix}$$

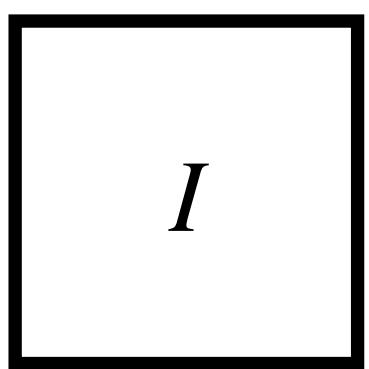
# Circuit to Unitary



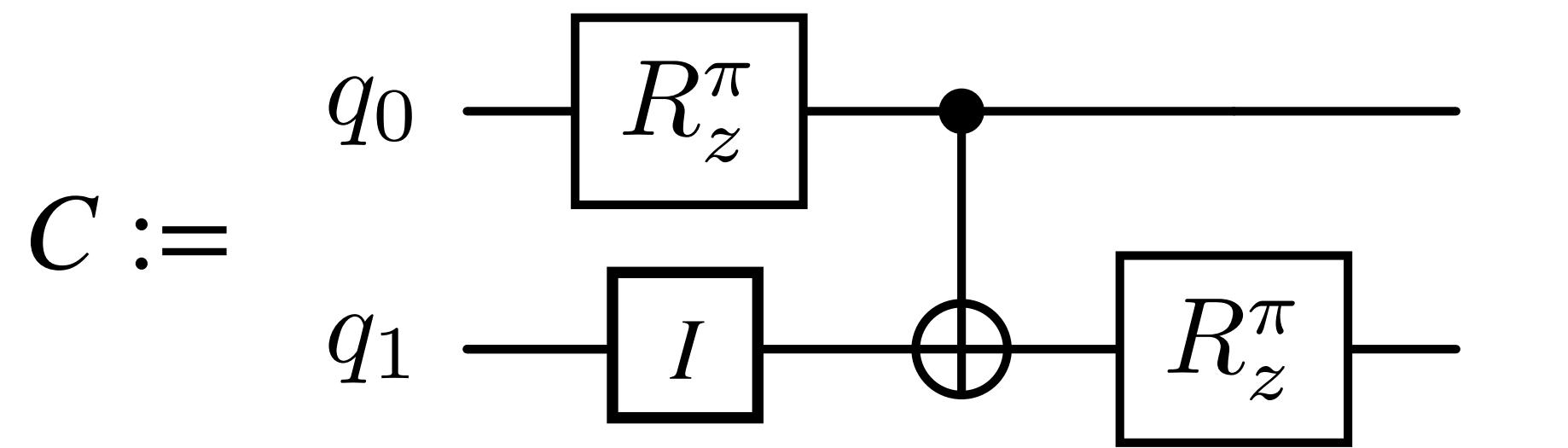
$$U_{R_z^\theta} := \begin{bmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{bmatrix}$$



$$U_{CX} := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



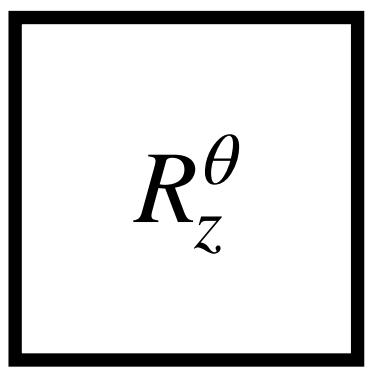
$$U_I := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



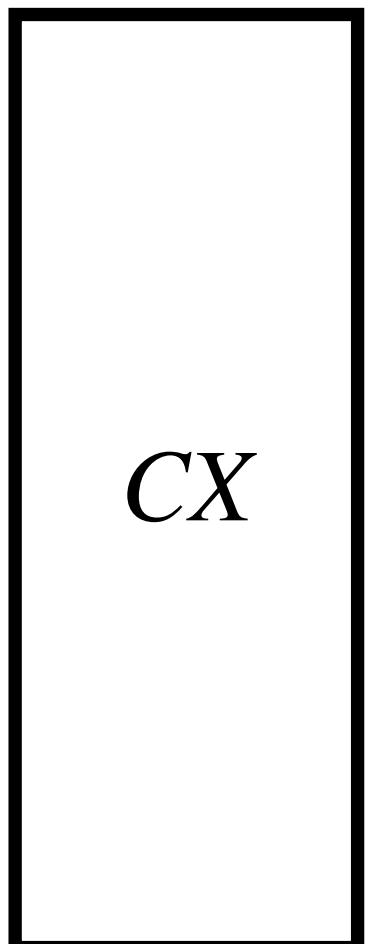
$$U_C = \frac{(U_{R_z^\pi} \otimes U_I)}{2^2 \times 2^2}$$

$$(U_{R_z^\pi} \otimes U_I) = \begin{bmatrix} e^{-i\frac{\pi}{2}} \cdot U_I & 0 \cdot U_I \\ 0 \cdot U_I & e^{i\frac{\pi}{2}} \cdot U_I \end{bmatrix}_{2 \times 2}$$

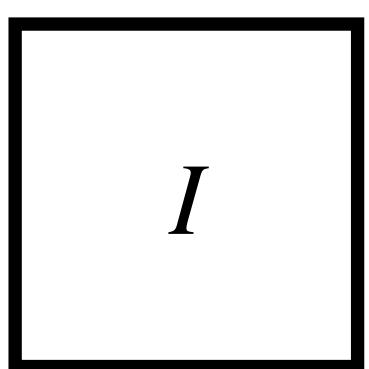
# Circuit to Unitary



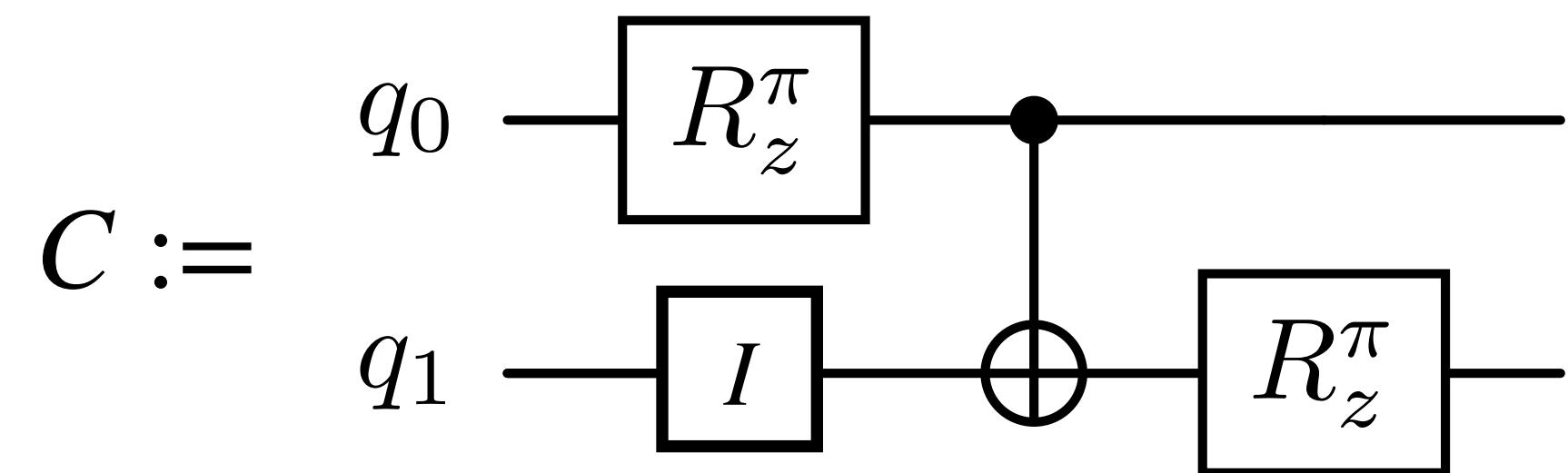
$$U_{R_z^\theta} := \begin{bmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{bmatrix}$$



$$U_{CX} := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



$$U_I := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



$$U_C =$$

$2^2 \times 2^2$

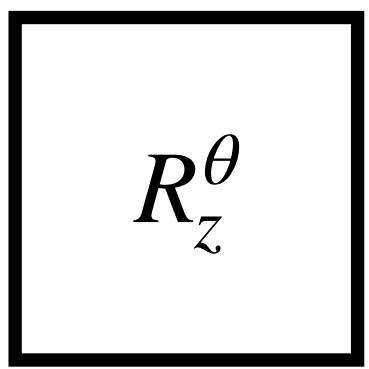
$$U_{CX} \cdot (U_{R_z^\pi} \otimes U_I)$$

$\frac{2 \times 2 \quad 2 \times 2}{2^2 \times 2^2}$

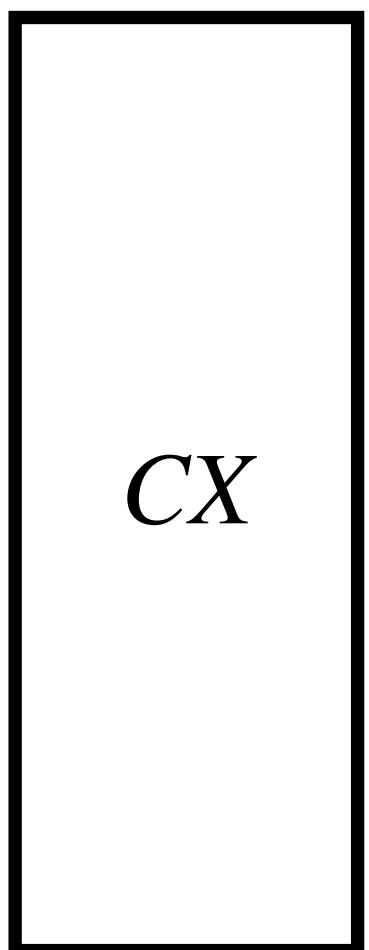
$$(U_{R_z^\pi} \otimes U_I) = \begin{bmatrix} e^{-i\frac{\pi}{2}} \cdot U_I & 0 \cdot U_I \\ 0 \cdot U_I & e^{i\frac{\pi}{2}} \cdot U_I \end{bmatrix}$$

$\frac{2 \times 2 \quad 2 \times 2}{2 \times 2 \quad 2 \times 2}$

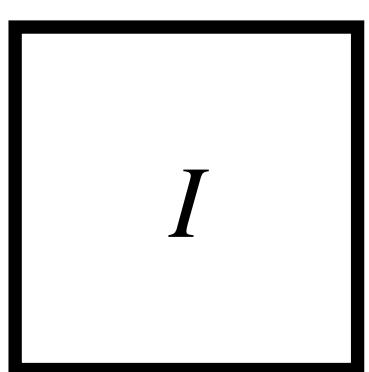
# Circuit to Unitary



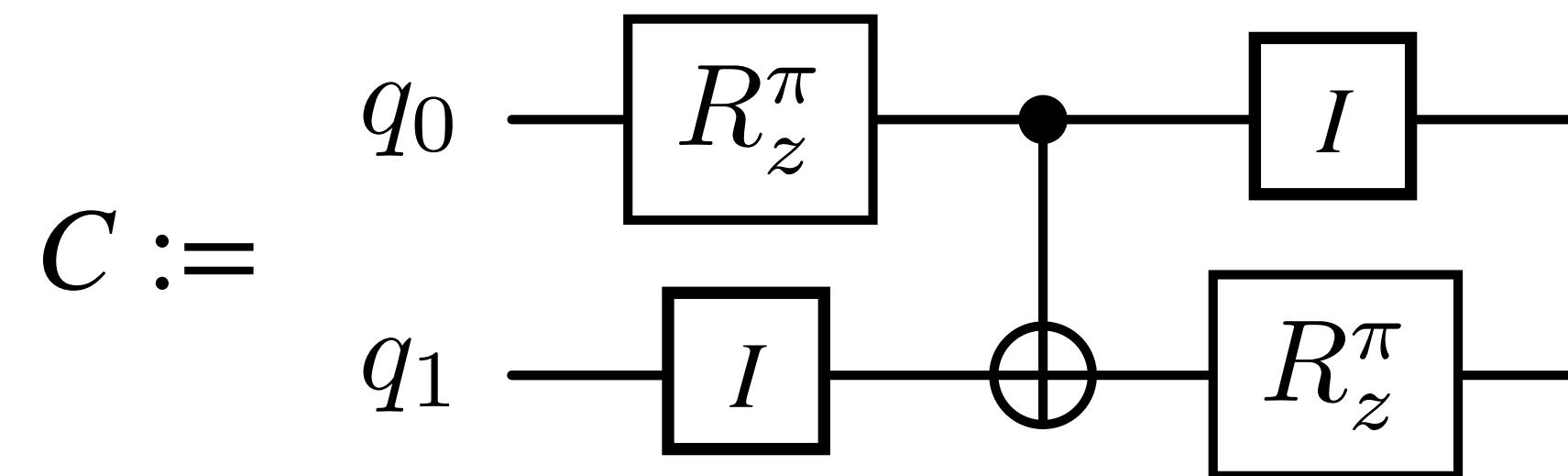
$$U_{R_z^\theta} := \begin{bmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{bmatrix}$$



$$U_{CX} := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



$$U_I := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



$$U_C = (U_I \otimes U_{R_z^\pi}) \cdot U_{CX} \cdot (U_{R_z^\pi} \otimes U_I)$$

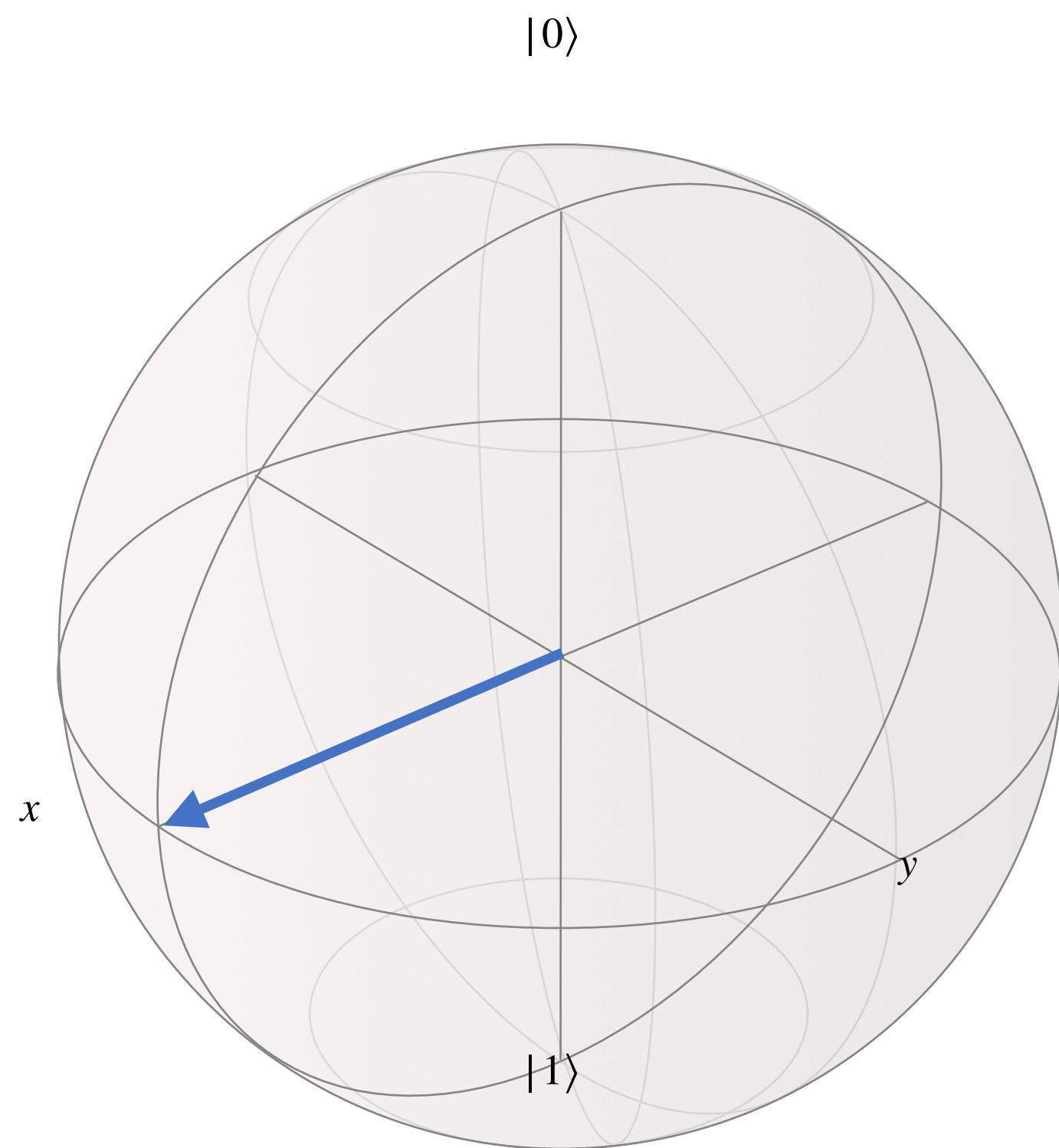
$$\frac{2^2 \times 2^2}{2^2 \times 2^2}$$

$$\frac{2 \times 2}{2^2 \times 2^2}$$

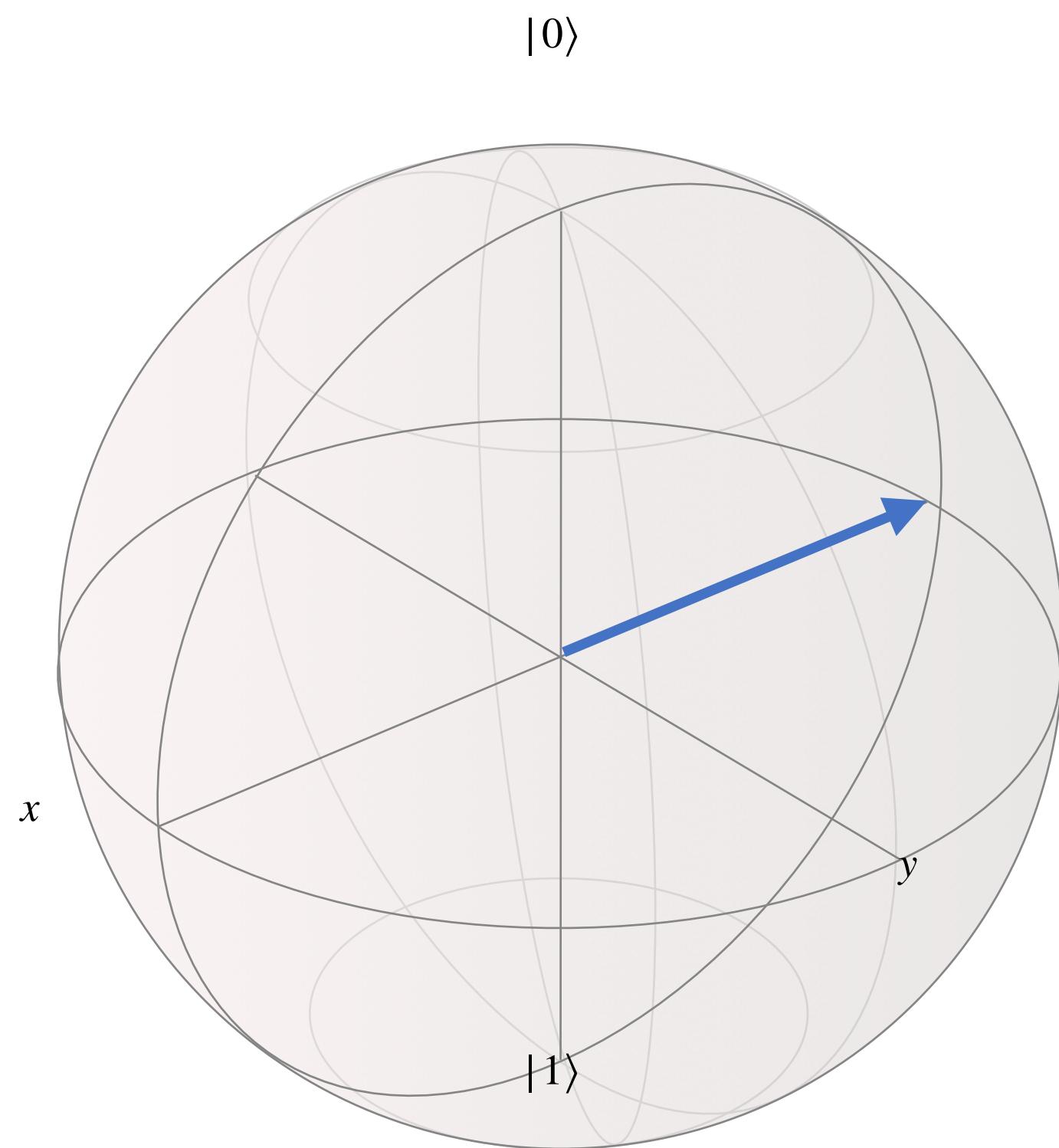
$$(U_{R_z^\pi} \otimes U_I) = \begin{bmatrix} e^{-i\frac{\pi}{2}} \cdot U_I & 0 \cdot U_I \\ 0 \cdot U_I & e^{i\frac{\pi}{2}} \cdot U_I \end{bmatrix}$$

$$\frac{2 \times 2}{2 \times 2}$$

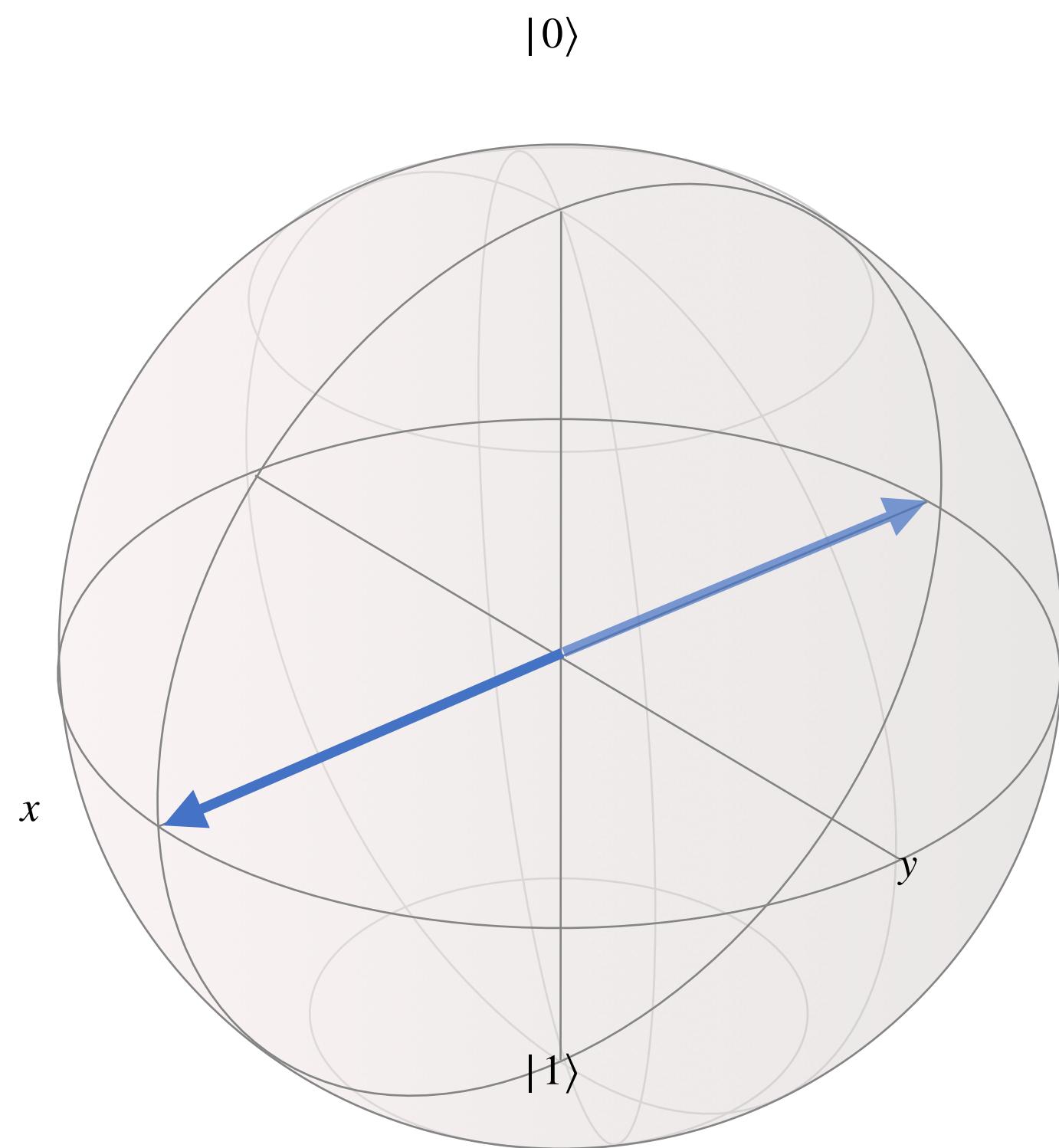
# Approximate Circuit Equivalence



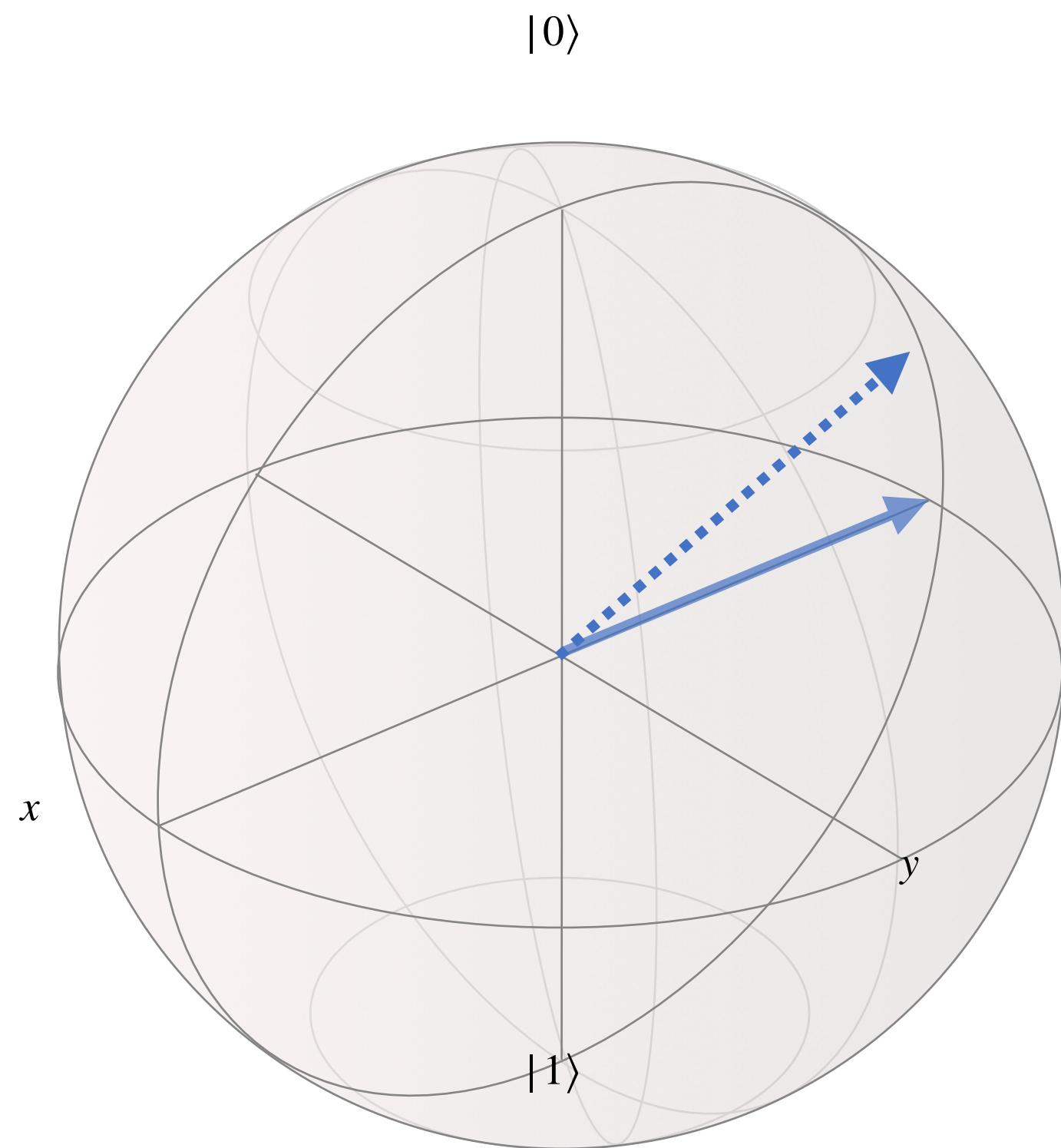
# Approximate Circuit Equivalence



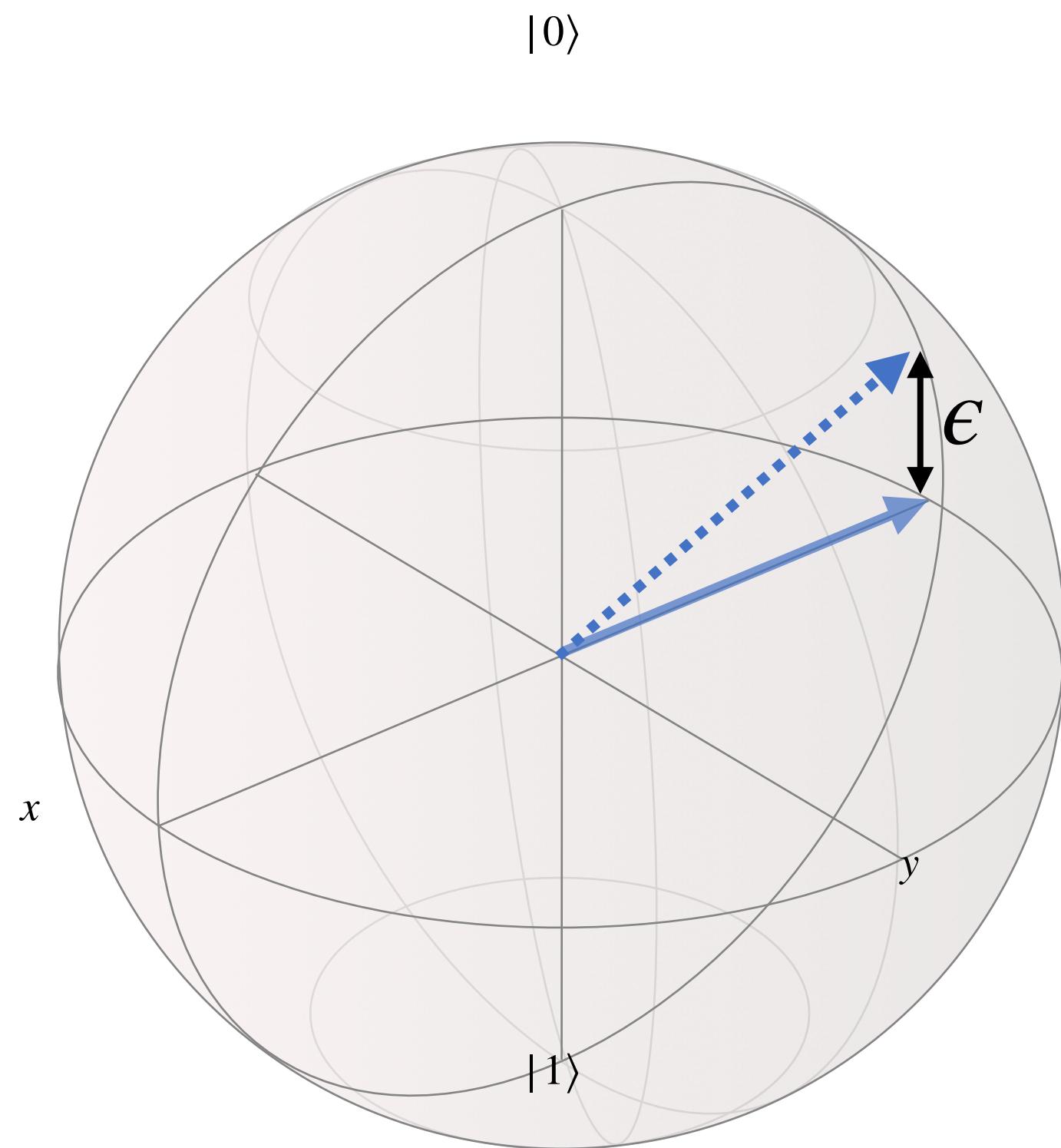
# Approximate Circuit Equivalence



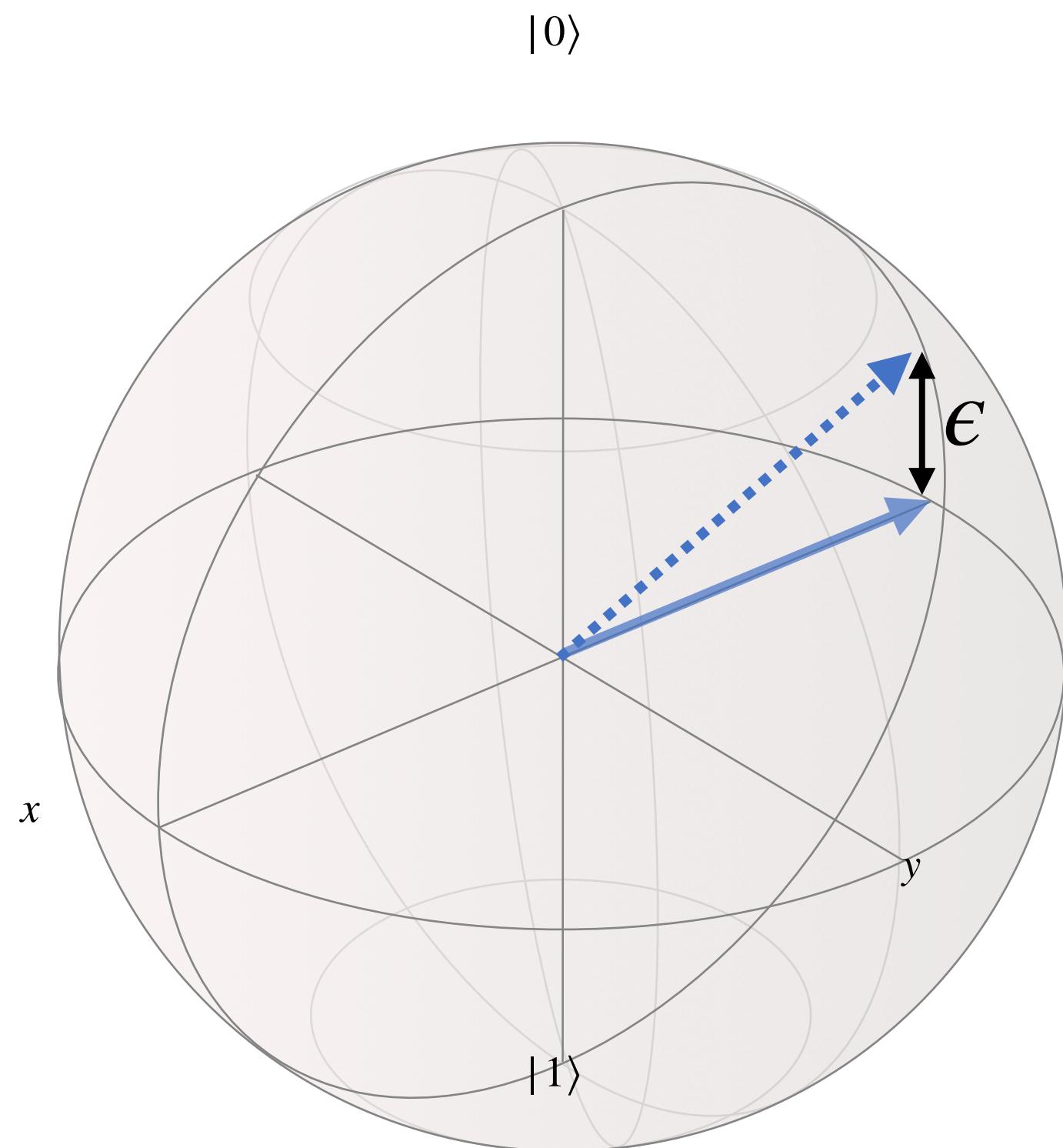
# Approximate Circuit Equivalence



# Approximate Circuit Equivalence



# Approximate Circuit Equivalence

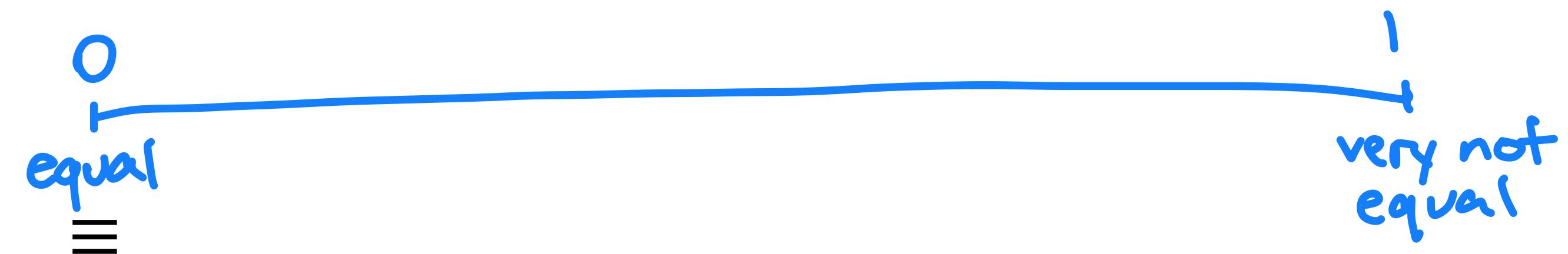


$$\Delta(U_C, U_{C'}) \leq \epsilon \iff C \equiv_{\epsilon} C'$$

# Hilbert–Schmidt Distance

"easy" to compute

$$\Delta_{HS}(U, U') := \sqrt{1 - \frac{|Tr(U^\dagger U')|^2}{2^{2n}}}$$



Upper bound on total variation distance

# Why Approximate Circuits?

FTQC **requires**  
approximations for  
arbitrary angle rotations

Ideal

Circuit  $C$

$$+ \text{ No noise} = U_C$$

Reality

Circuit  $C$

$$+ \text{ Noise} = U_{C_{noisy}}$$

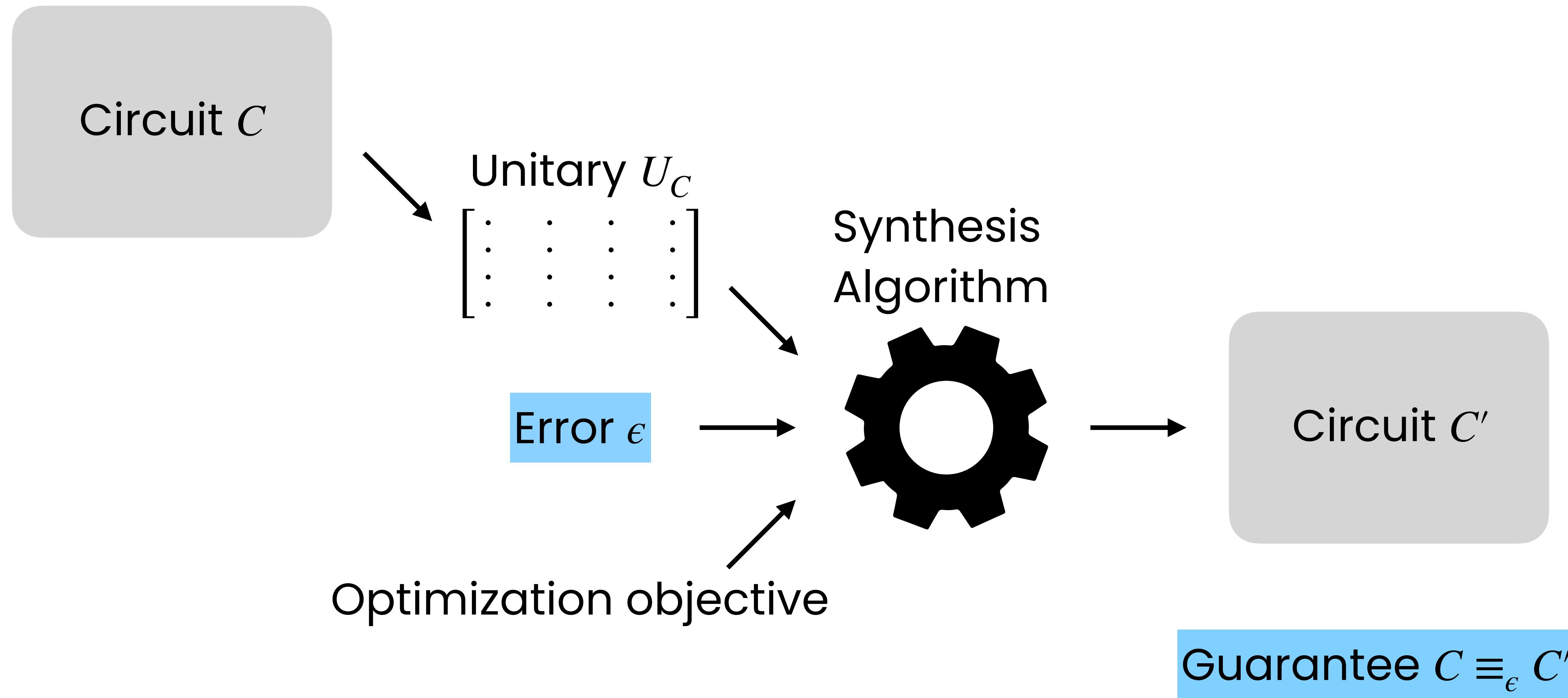
Circuit  
 $C' \equiv_\epsilon C$

$k$  fewer gates than  $C$

The hope:  $U_{C'_{noisy}}$  closer to  $U_C$  than  $U_{C_{noisy}}$

Noise from  $k$  gates  $> \epsilon$

# Unitary Synthesis



# Rich Research Area

## Synthesis of Quantum-Logic Circuits

Vivek V. Shende, Stephen S. Bullock, and Igor L. Markov, *Senior Member, IEEE*

### Towards Optimal Topology Aware Quantum Circuit Synthesis

Marc G. Davis, Ethan Smith, Ana Tudor, Koushik Sen, Irfan Siddiqi

{marc.davis, ethanhs, koushik.sen, irfan.siddiqi}@berkeley.edu



Public

Berkeley Quantum Synthesis Toolkit

● OpenQASM ⭐ 132 📂 38

Costin Iancu  
Lawrence Berkeley National Laboratory

### LEAP: Scaling Numerical Optimization Based Synthesis Using an Incremental Approach

ETHAN SMITH and MARC GRAU DAVIS, University of California, Berkeley  
JEFFREY LARSON, Argonne National Laboratory

RIJSEN, and COSTIN IANCU,

### Exact synthesis of multiqubit Clifford+ $T$ circuits

Brett Giles

Department of Computer Science  
University of Calgary

Peter Selinger

Department of Mathematics

### Modular Component-Based Quantum Circuit Synthesis

CHAN GU KANG, Korea University, Republic of Korea

Public of Korea

### SyntheTiq: Fast and Versatile Quantum Circuit Synthesis

ANOUK PARADIS\*, ETH Zurich, Switzerland  
JASPER DEKONINCK\*, ETH Zurich, Switzerland  
BENJAMIN BICHSEL, ETH Zurich, Switzerland  
MARTIN VECHEV, ETH Zurich, Switzerland



Public

● OpenQASM ⭐ 3 📂 MIT

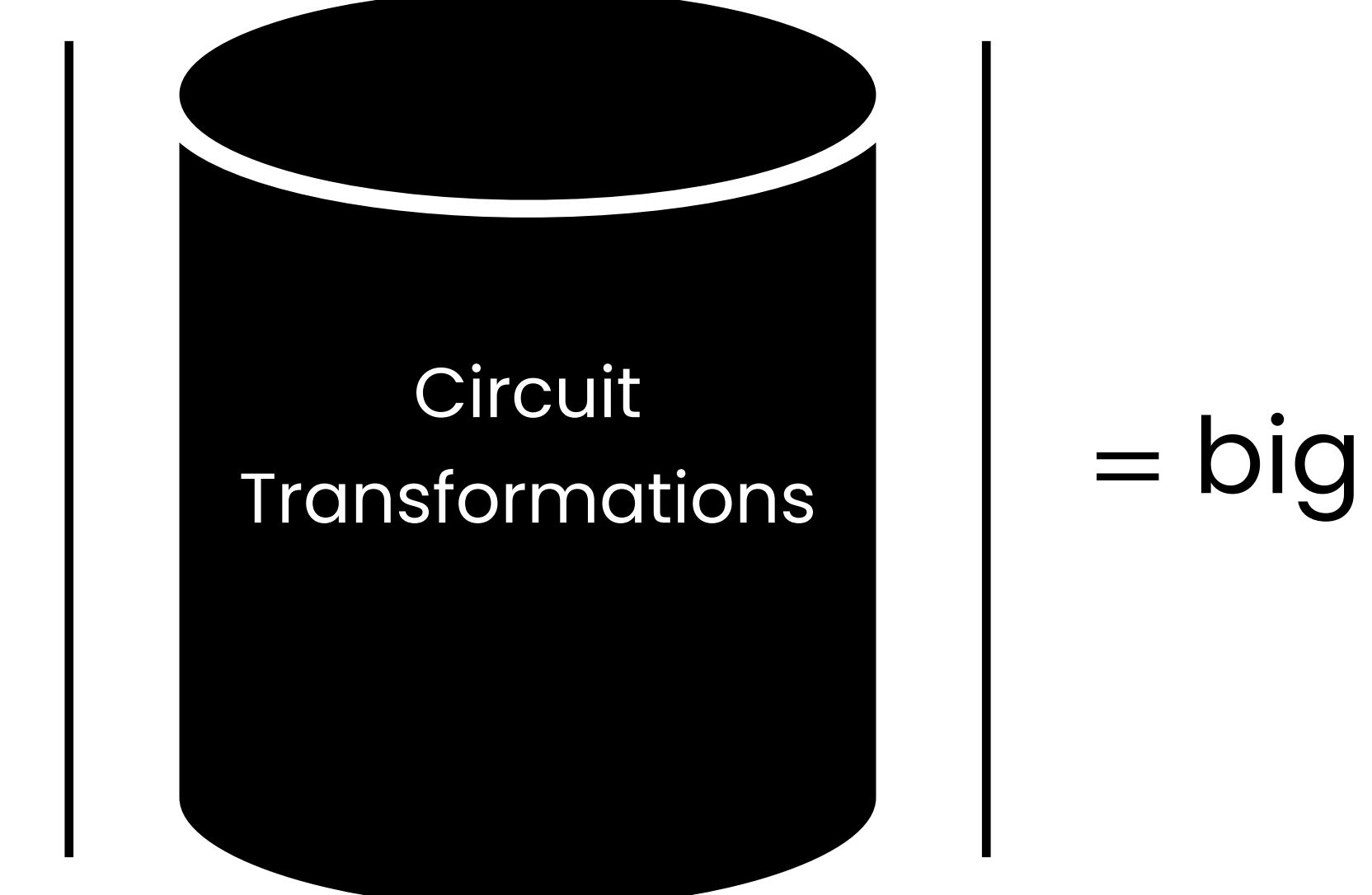
## Techniques:

- analytical
- numerical optimization
- explicit search

# Scheduling Transformations

# Scheduling is hard

In what order to apply???

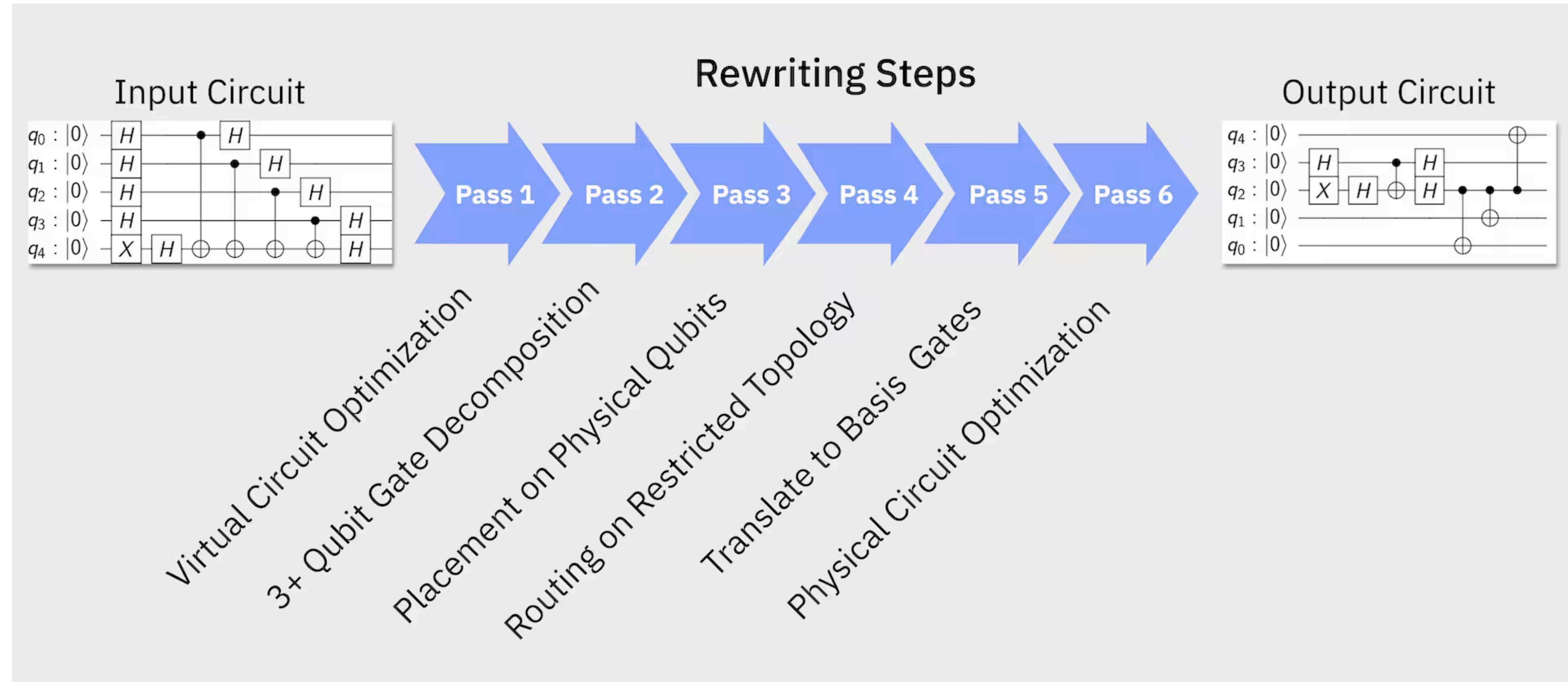


"phase-ordering problem"

# Qiskit

<https://docs.quantum.ibm.com/api/qiskit/transpiler#optimization-stage>

[https://github.com/Qiskit/qiskit/blob/stable/1.4/qiskit/transpiler/preset\\_passmanagers/level3.py](https://github.com/Qiskit/qiskit/blob/stable/1.4/qiskit/transpiler/preset_passmanagers/level3.py)



<https://docs.quantum.ibm.com/api/qiskit/transpiler#optimization-stage>

# Qiskit

[https://github.com/Qiskit/qiskit/blob/stable/1.4/qiskit/transpiler/preset\\_passmanagers/level3.py](https://github.com/Qiskit/qiskit/blob/stable/1.4/qiskit/transpiler/preset_passmanagers/level3.py)

qiskit / qiskit / transpiler / preset\_passmanagers / level3.py ↑ Top

Code Blame 119 lines (104 loc) · 4.8 KB · ⚡

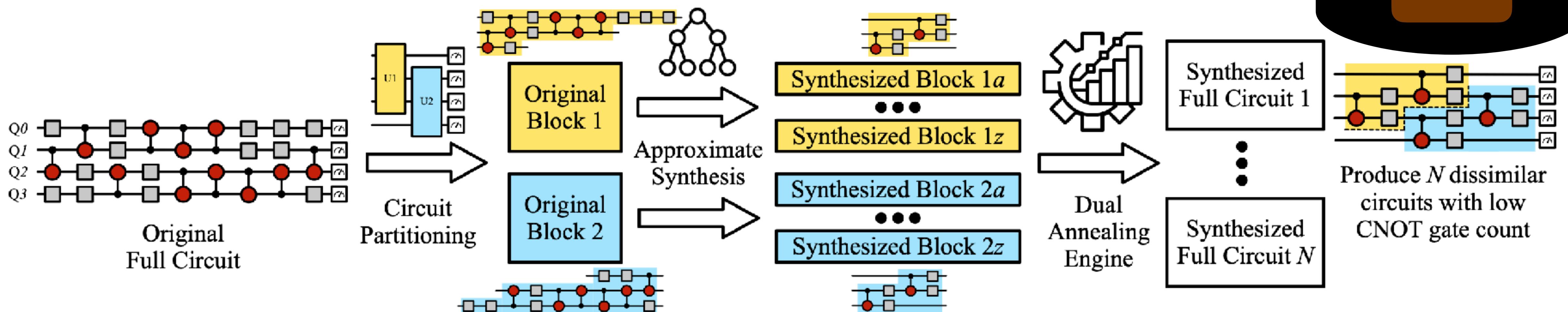
```
26
27 def level_3_pass_manager(pass_manager_config: PassManagerConfig) -> StagedPassManager:
28     """Level 3 pass manager: heavy optimization by noise adaptive qubit mapping and
29     gate cancellation using commutativity rules and unitary synthesis.
30
31     This pass manager applies the user-given initial layout. If none is given, a search
32     for a perfect layout (i.e. one that satisfies all 2-qubit interactions) is conducted.
33     If no such layout is found, and device calibration information is available, the
34     circuit is mapped to the qubits with best readouts and to CX gates with highest fidelit
35
36     The pass manager then tra... the coupling constraints.
37     It is then unrolled to th... rections are fixed.
38
39     Finally, optimizations in the form of commutative gate cancellation, resynthesis
40     of two-qubit unitary blocks, and redundant reset removal are performed.
```

Circuit Transformations

Rewrite rules Resynthesis

**coarse fixed passes**

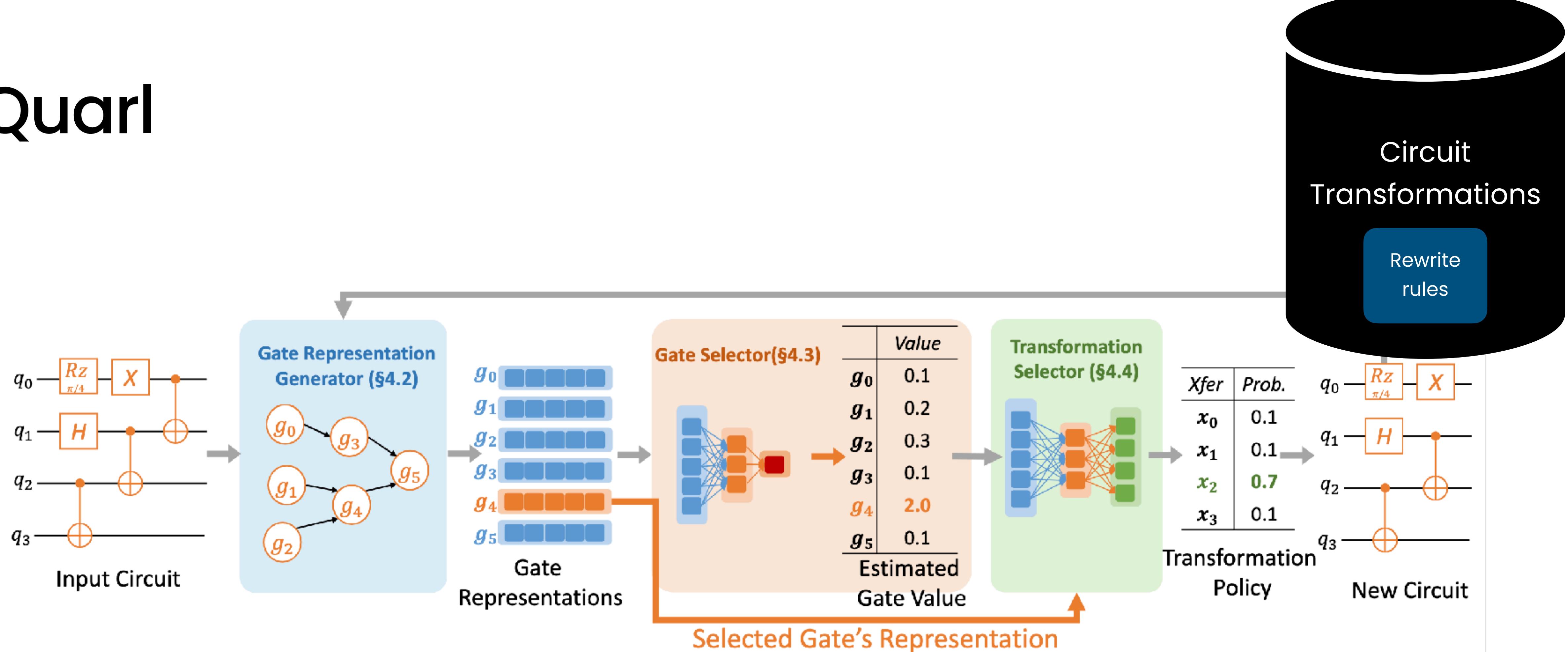
# BQSKit



QUEST: systematically approximating quantum circuits for higher output fidelity [Patel et al. 2022]

<https://github.com/BQSKit/bqskit>

# Quarl



\* requires an NVIDIA A100 GPU (\$\$\$)

# GUOQ

## Optimizing Quantum Circuits, Fast and Slow

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axu44@wisc.edu

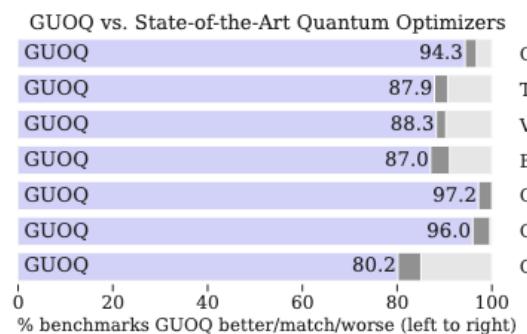
Abtin Molavi  
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Swamit Tannu  
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Aws Albarghouthi  
University of Wisconsin-Madison  
Madison, WI, USA  
aws@cs.wisc.edu

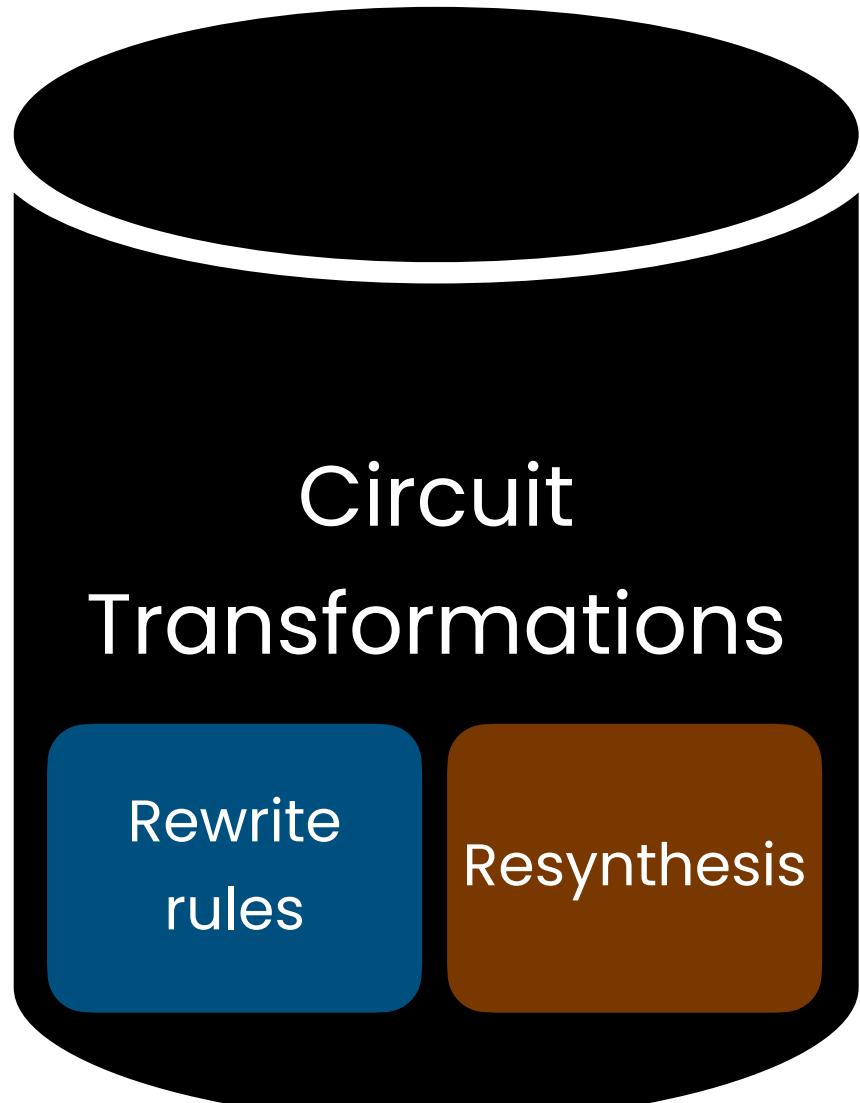
### Abstract

Optimizing quantum circuits is critical: the number of quantum operations needs to be minimized for a successful evaluation of a circuit on a quantum processor. In this paper we unify two disparate ideas for optimizing quantum circuits, *rewrite rules*, which are fast standard optimizer passes, and *unitary synthesis*, which is slow, requiring a search through the space of circuits. We present a clean, unifying framework for thinking of rewriting and resynthesis as abstract circuit transformations. We then present a radically simple algorithm, `guoq`, for optimizing quantum circuits that exploits the synergies of rewriting and resynthesis. Our extensive evaluation demonstrates the ability of `guoq` to strongly outperform existing optimizers on a wide range of benchmarks.



**Figure 1.** Summary of `guoq` compared to state-of-the-art on 2-qubit-gate reduction for the IBMQ20 gate set. `guoq` and BQSKit are allowed to approximate the circuit up to  $\epsilon = 10^{-8}$ .  
\*Quar requires an NVIDIA A100 (40GB) GPU to run.

## ASPLOS talk on April 1st!



1. Randomly pick a transformation.
2. Randomly pick a subcircuit to transform.
3. Accept the result if better than current.  
Else, reject with high probability.

# GUOQ

## Optimizing Quantum Circuits, Fast and Slow

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Swarnit Tammu  
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swarnit@cs.wisc.edu

### Abstract

Optimizing quantum circuits is critical: the number of quantum operations needs to be minimized for a successful evaluation of a circuit on a quantum processor. In this paper we unify two disparate ideas for optimizing quantum circuits, *rewrite rules*, which are fast standard optimizer passes, and *anytime synthesis*, which is slow, requiring a search through the space of circuits. We present a clean, unifying framework for thinking of rewriting and resynthesis as abstract circuit transformations. We then present a radically simple algorithm, *crog*, for optimizing quantum circuits that exploits the synergies of rewriting and resynthesis. Our extensive evaluation demonstrates the ability of *crog* to strongly outperform existing optimizers on a wide range of benchmarks.

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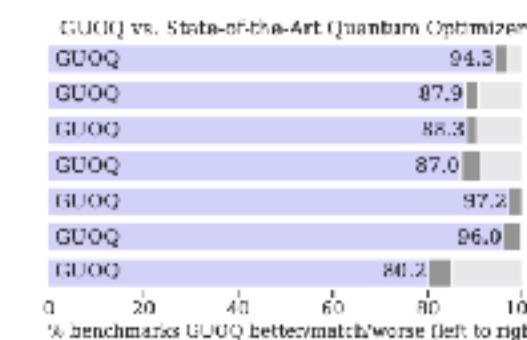
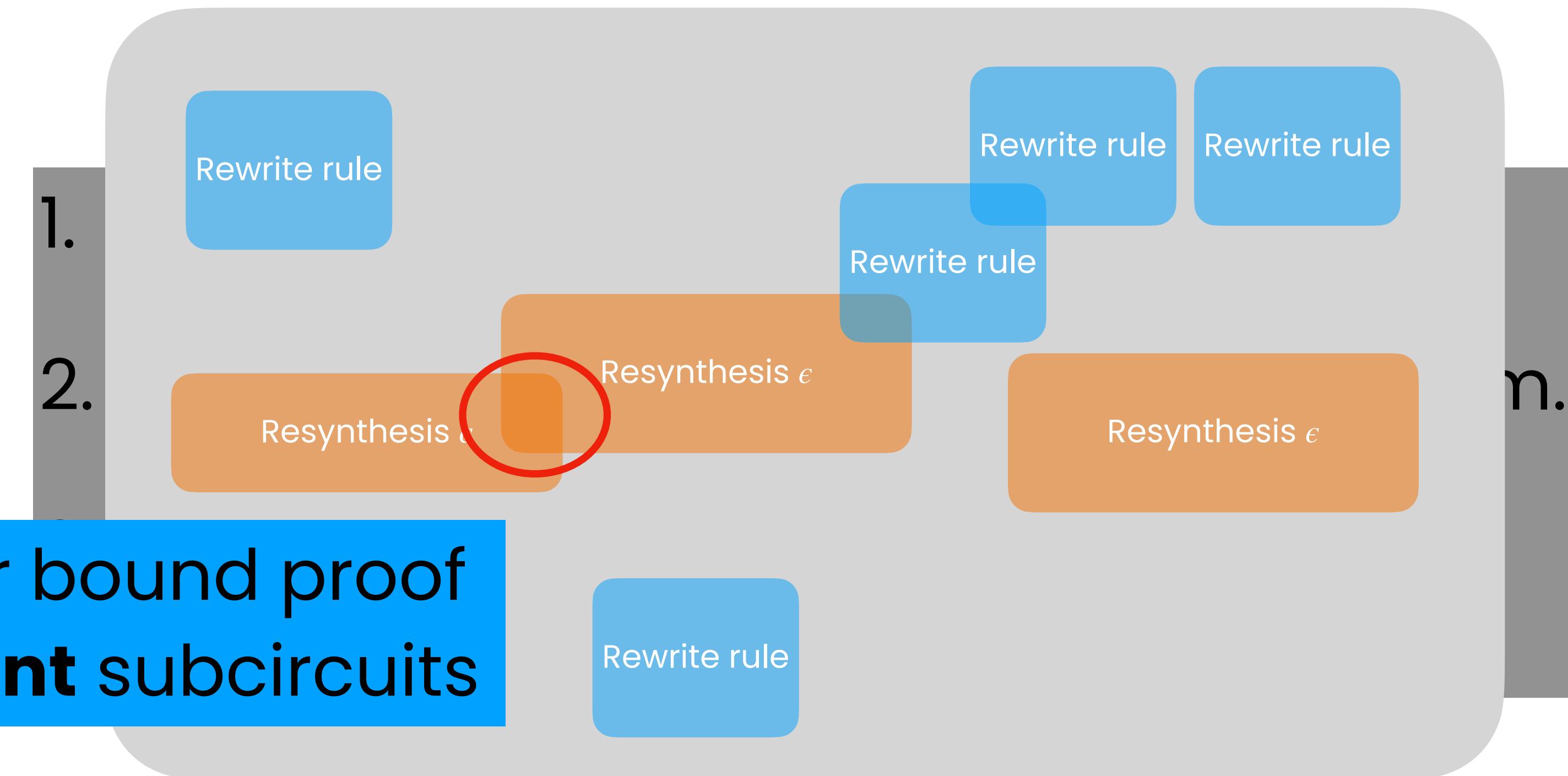
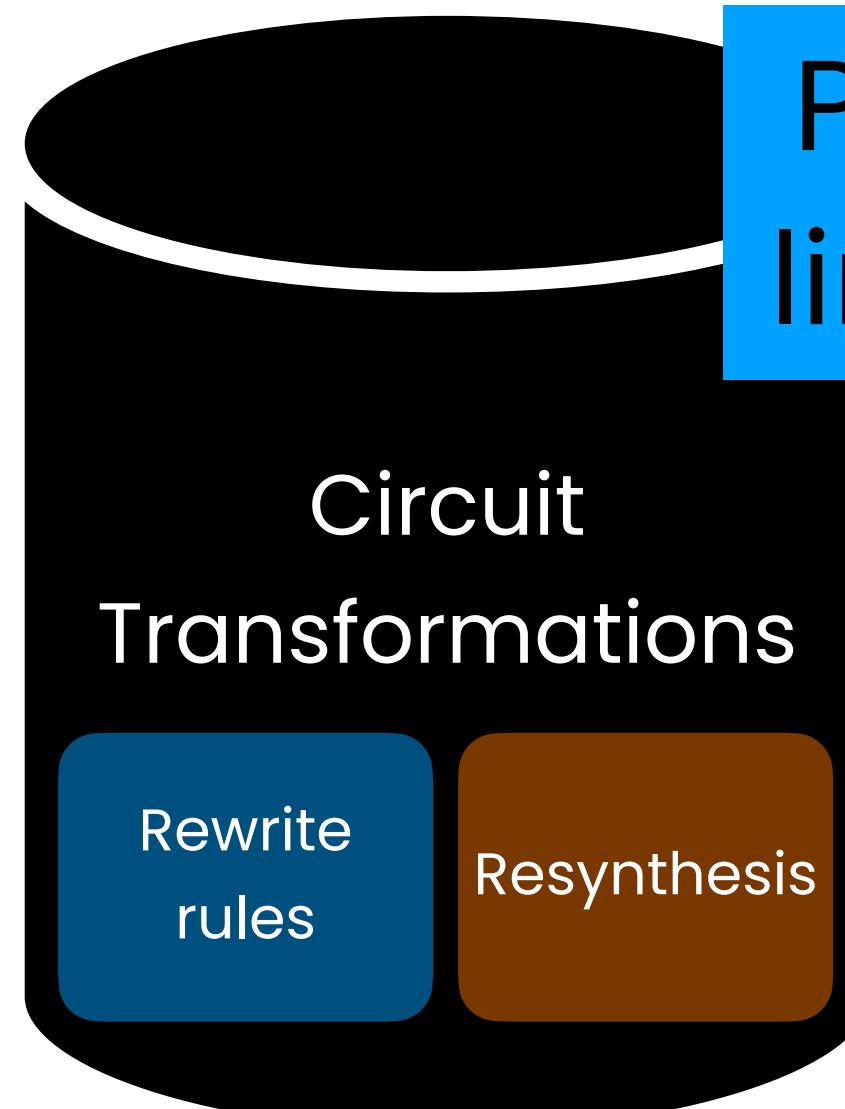


Figure 1. Summary of *crog* compared to state-of-the-art on 2-qubit gate reduction for the nqsim gate set. *crog* and ROSQT are allowed to approximate the circuit up to  $\epsilon = 10^{-6}$ . \*Quart requires an NVIDIA A100 (40GB) GPU to run.

ASPLOS talk on April 1st!



$$\epsilon_{total} \leq \epsilon + \epsilon + \epsilon$$

We extended to handle arbitrary subcircuits

# GUOQ Motivation: Two Disparate Techniques

## Rewrite Rules

$$\begin{array}{c} \text{---} \\ | \quad | \quad | \\ R_z Q(k) \quad H \quad \oplus \quad H \end{array} \equiv \begin{array}{c} \text{---} \\ | \quad | \\ H \quad \oplus \quad H \quad R_z Q(k) \end{array}$$

$$\begin{array}{c} \text{---} \\ | \quad | \quad | \quad | \\ R_z Q(k) \quad \oplus \quad R_z Q(k') \quad \oplus \quad | \\ | \quad | \quad | \quad | \end{array} \equiv \begin{array}{c} \text{---} \\ | \quad | \quad | \quad | \\ \oplus \quad R_z Q(k') \quad \oplus \quad R_z Q(k) \end{array}$$

$$\begin{array}{c} \text{---} \\ | \quad | \\ R_z Q(k) \quad | \\ | \quad | \end{array} \equiv \begin{array}{c} \text{---} \\ | \quad | \\ R_z Q(k) \end{array}$$

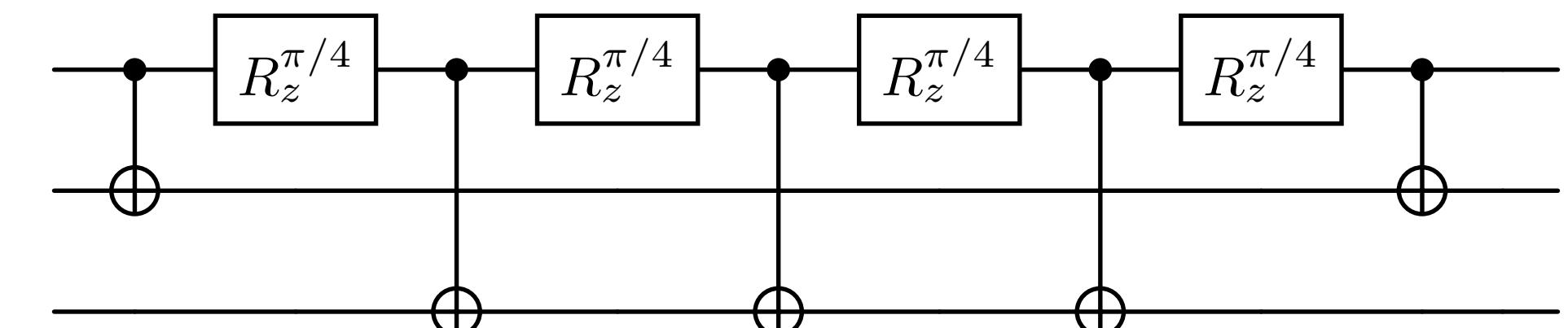
$$\begin{array}{c} \text{---} \\ | \quad | \quad | \\ | \quad | \quad | \\ \oplus \quad \oplus \end{array} \equiv \begin{array}{c} \text{---} \\ | \quad | \quad | \\ | \quad | \quad | \\ \oplus \quad \oplus \end{array}$$

$$\begin{array}{c} \text{---} \\ | \quad | \quad | \\ | \quad | \quad | \\ \bullet \quad \bullet \quad \bullet \end{array} \equiv \begin{array}{c} \text{---} \\ | \quad | \quad | \\ | \quad | \quad | \\ \oplus \quad \oplus \end{array}$$

$$\begin{array}{c} \text{---} \\ | \quad | \quad | \\ | \quad | \quad | \\ \oplus \quad H \quad \bullet \quad H \end{array} \equiv \begin{array}{c} \text{---} \\ | \quad | \quad | \\ | \quad | \quad | \\ H \quad \bullet \quad H \quad \oplus \end{array}$$

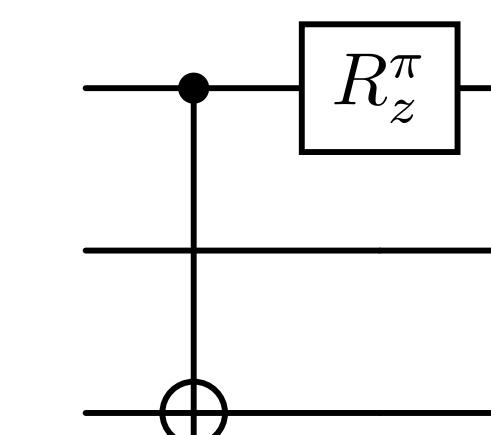
• • •

## Circuit Resynthesis



specification

synthesis



# GUOQ Motivation: Two Disparate Techniques

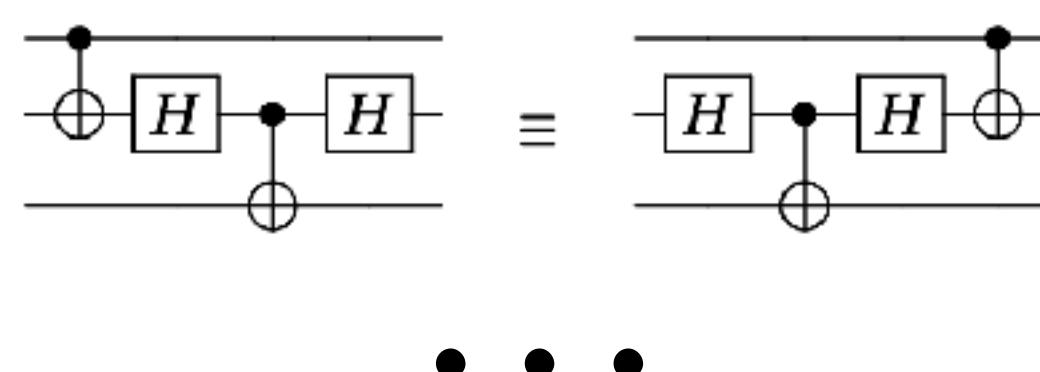
## Rewrite Rules



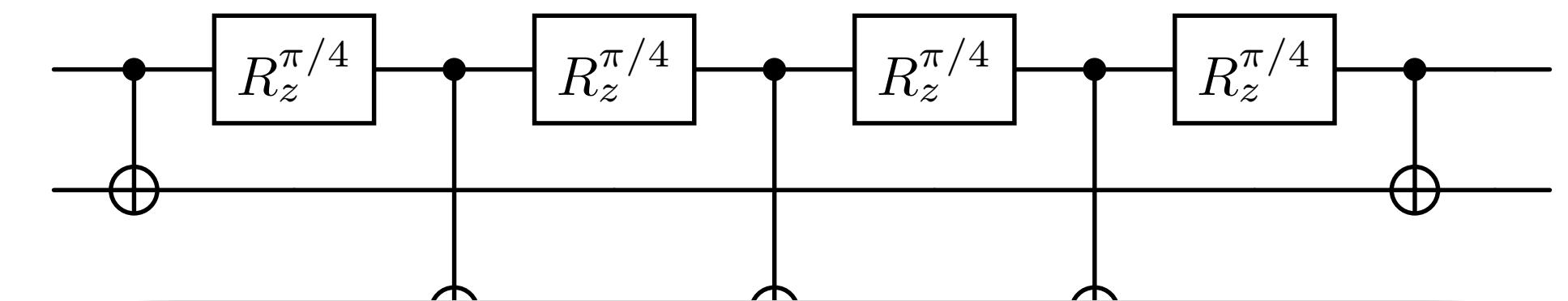
Fast

Limited by # gates

Exact



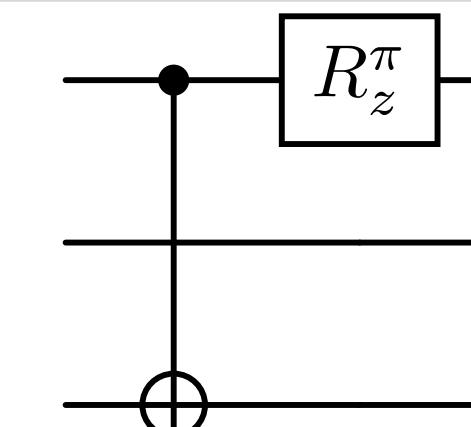
## Circuit Resynthesis



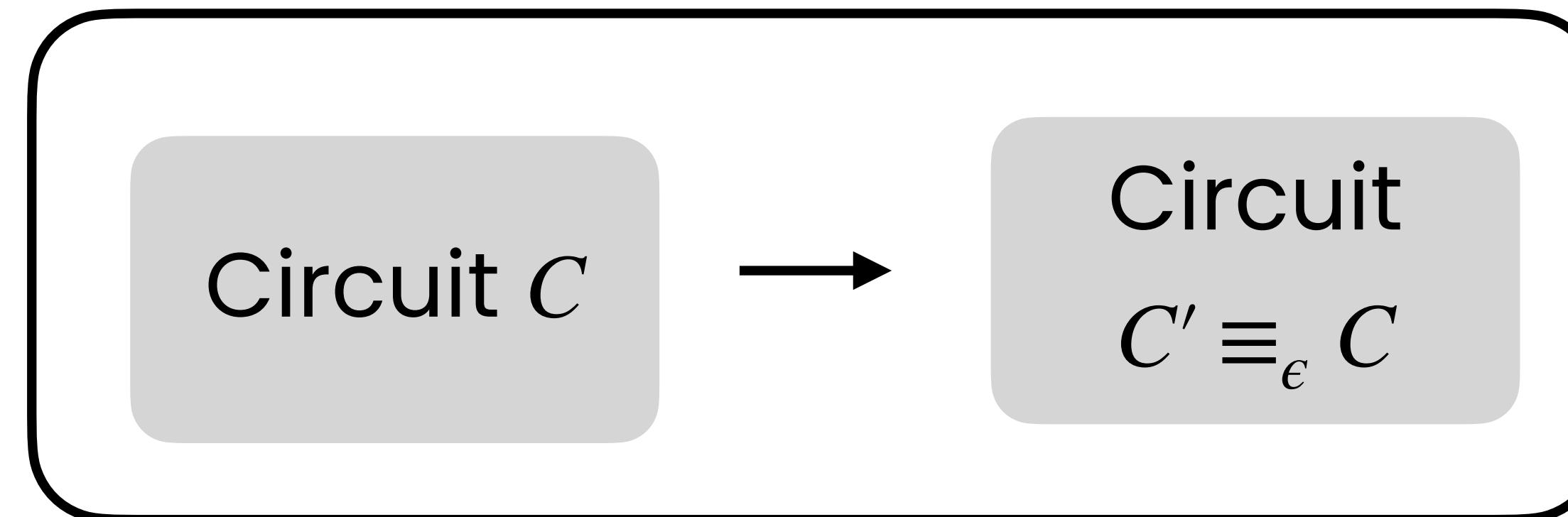
Slow

Limited by # qubits

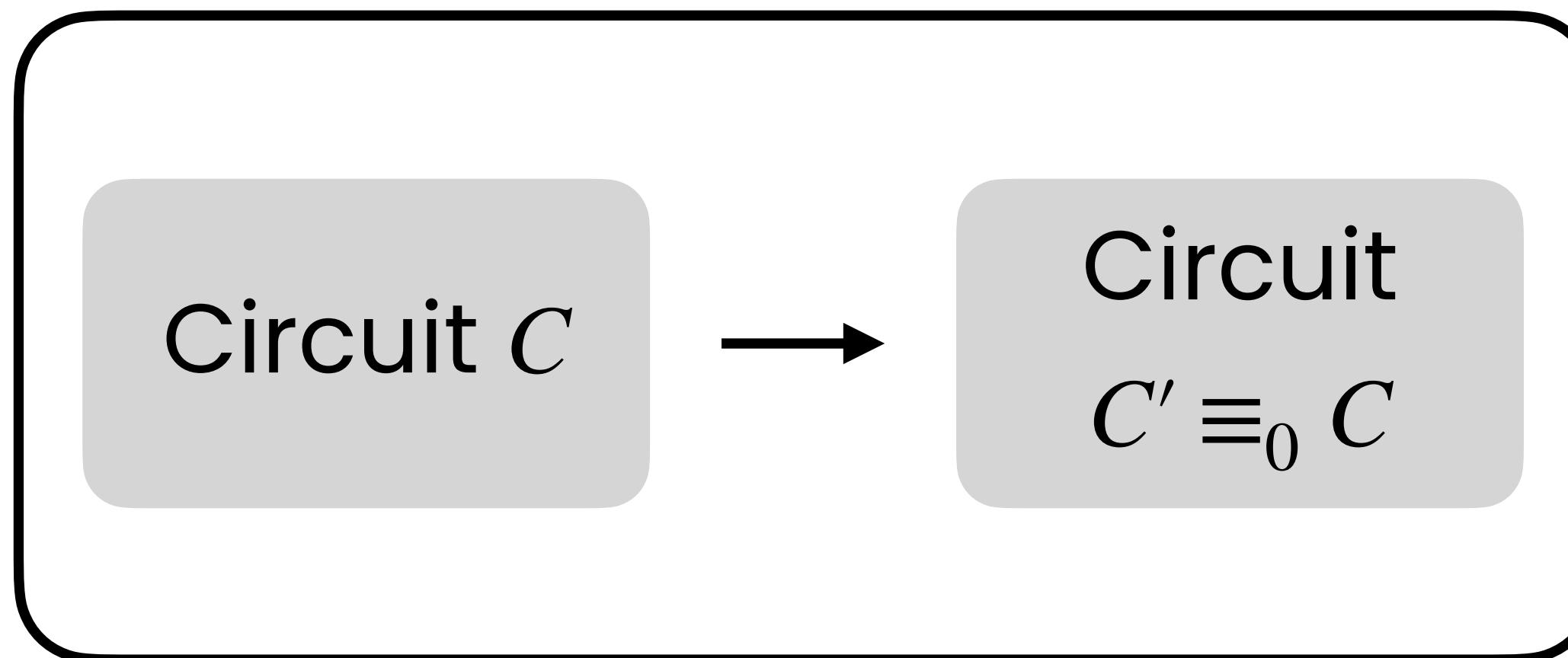
Approximate



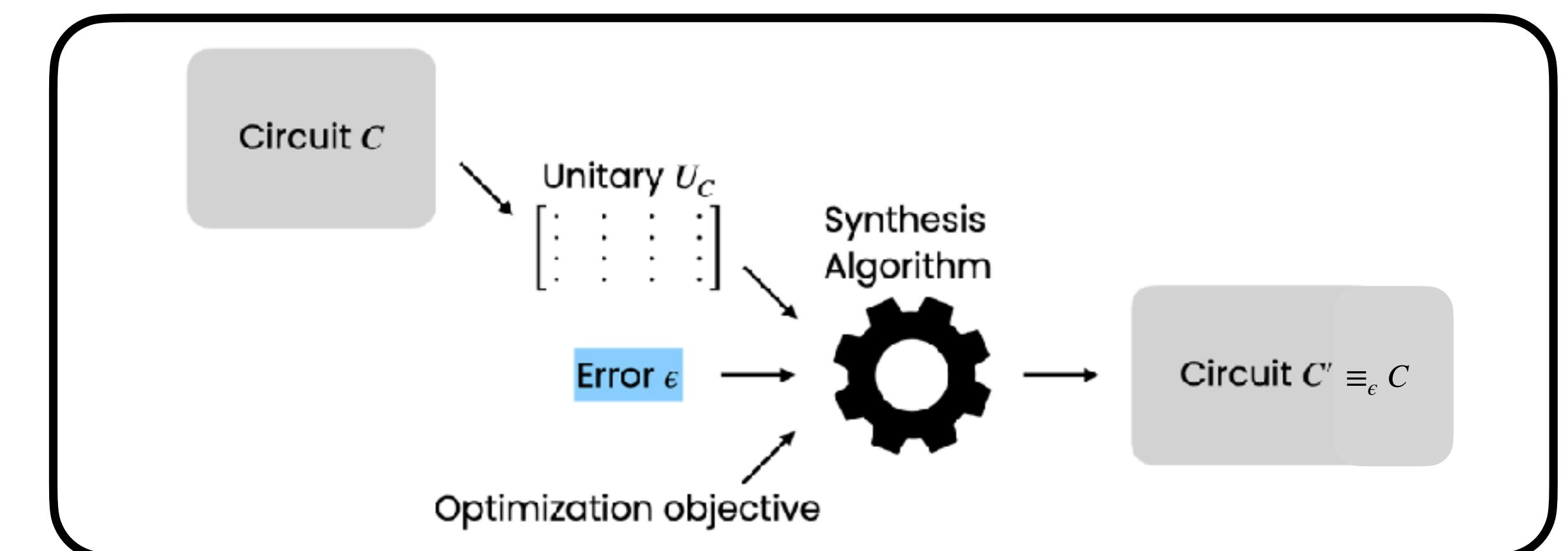
# Unifying Abstract Circuit Transformations



transformation  $\tau_{\epsilon} : \mathcal{C} \rightarrow \mathcal{C}$

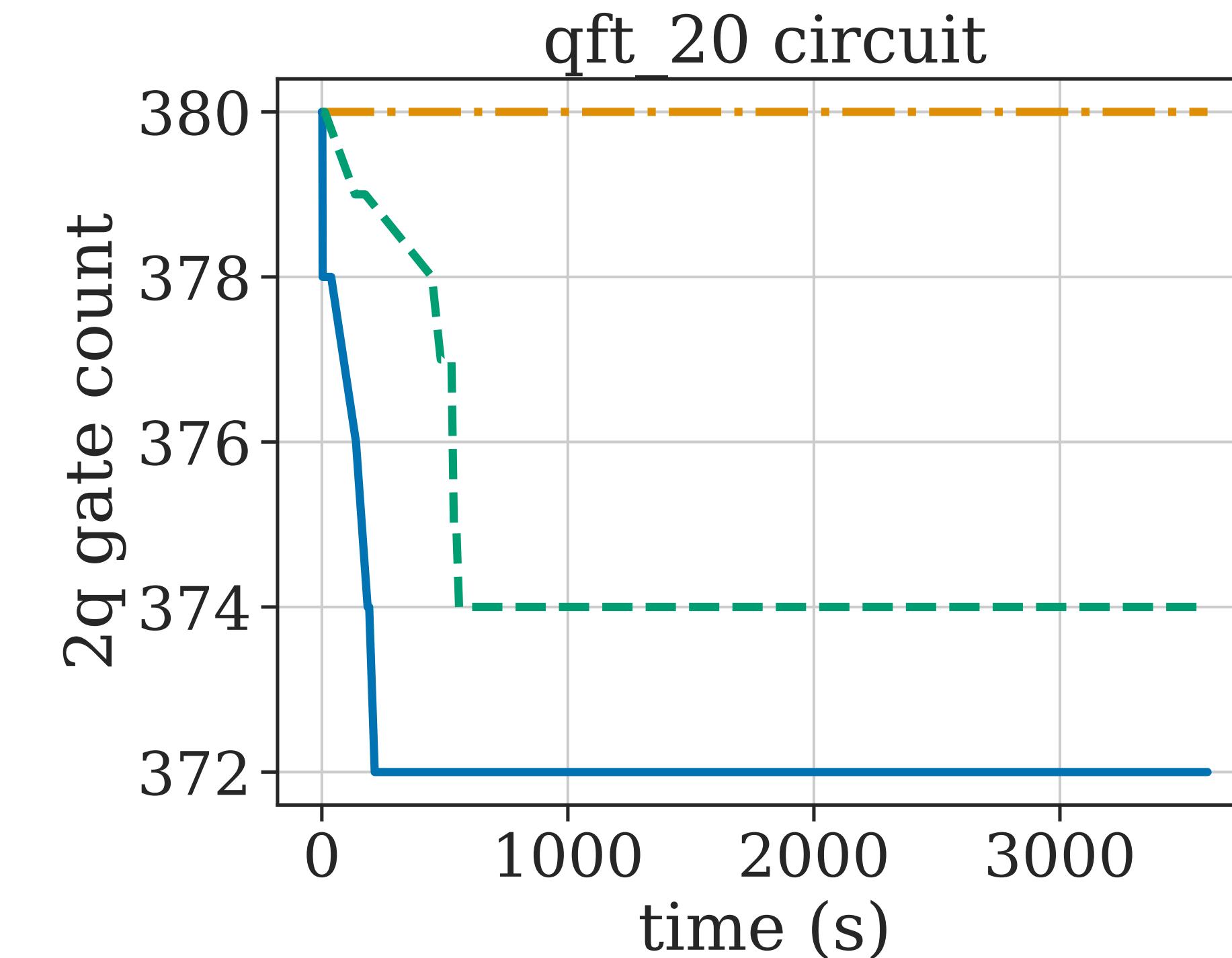
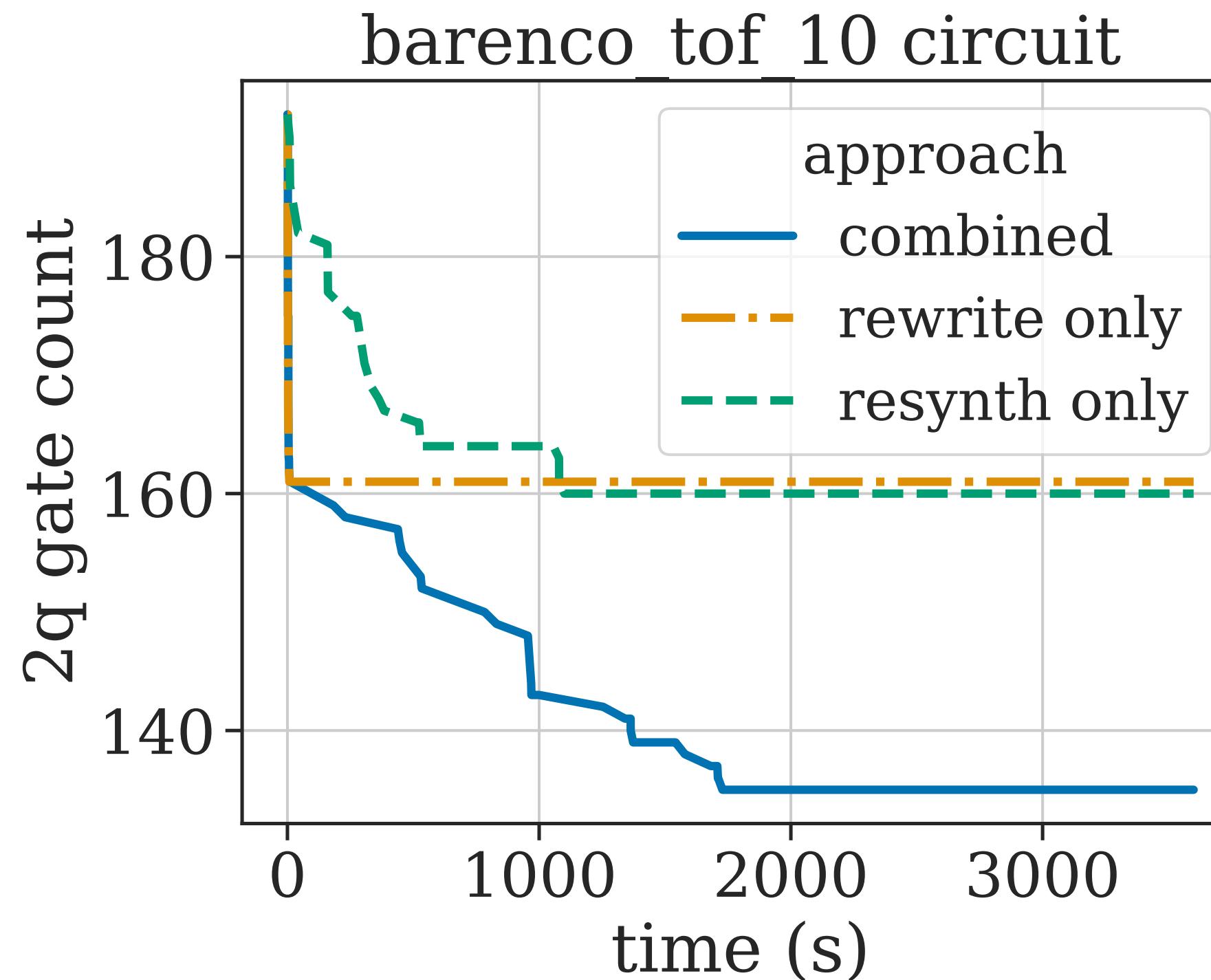


rewrite rule



resynthesis

# GUOQ Search Exploits Synergy



Also significantly outperforms state-of-the-art!

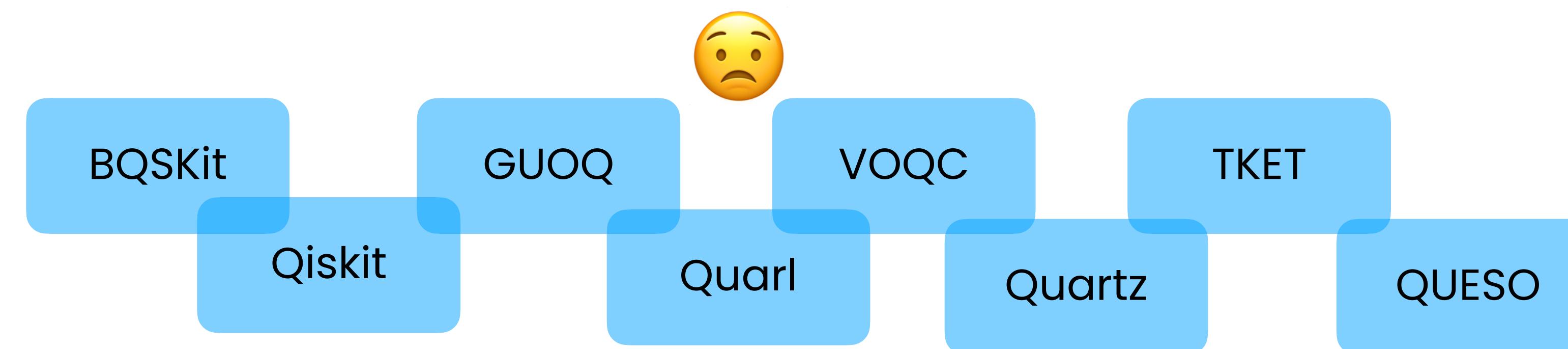
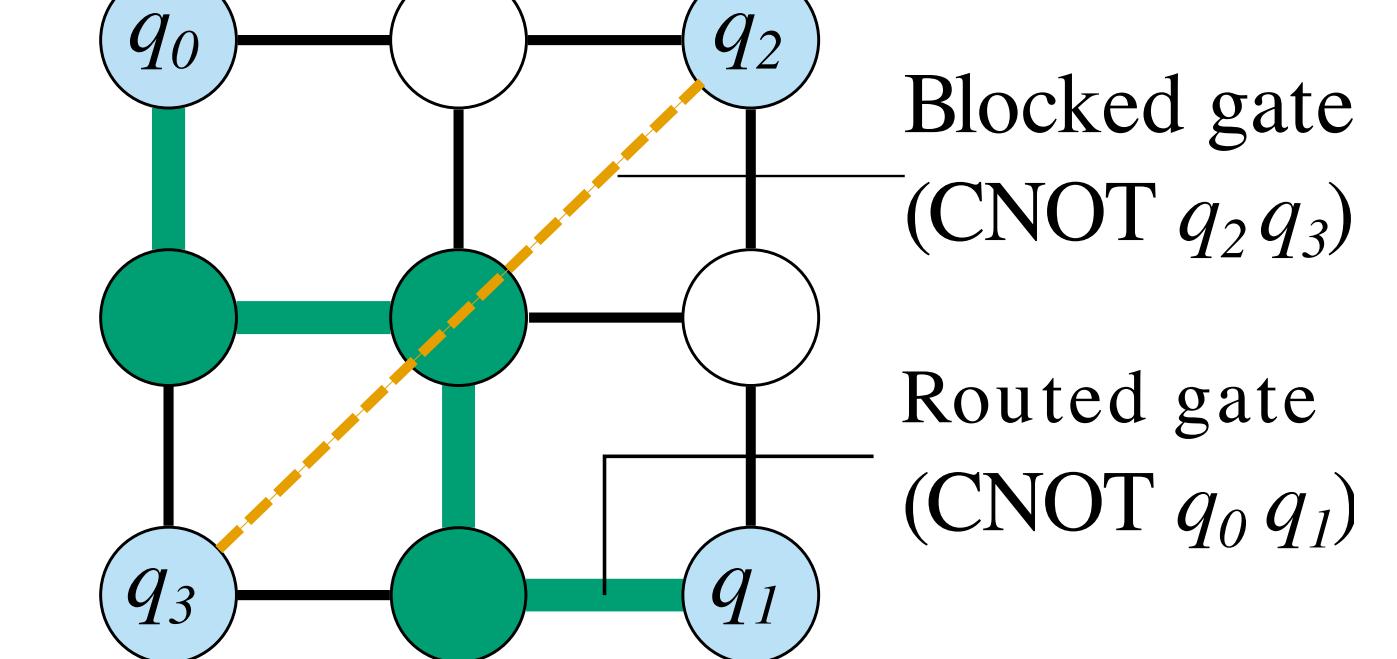
**Come to the ASPLOS talk on April 1st!**

# Optimizing for FTQC

# Optimizing for FTQC

Clifford + T gate set:  $\{T, T^\dagger, S, S^\dagger, H, CX\}$  *discrete*  
"magic state distillation"

Eastin–Knill theorem: no transversal universal set



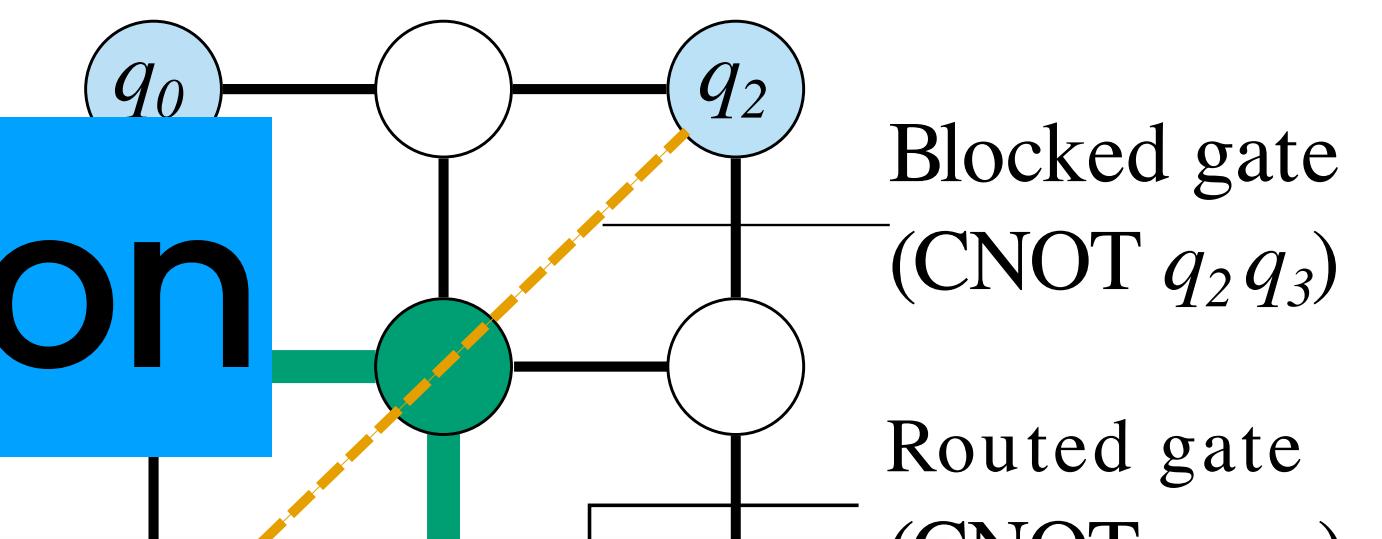
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Clifford + T gate set:  $\{T, T^\dagger, S, S^\dagger, H, CX\}$

"magic state distillation"

Eastin-Knill th

## Open Research Question



Reducing T-count with the ZX-calculus

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January 20, 2020



Linear and Non-linear Relational Analyses for Quantum Program Optimization

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