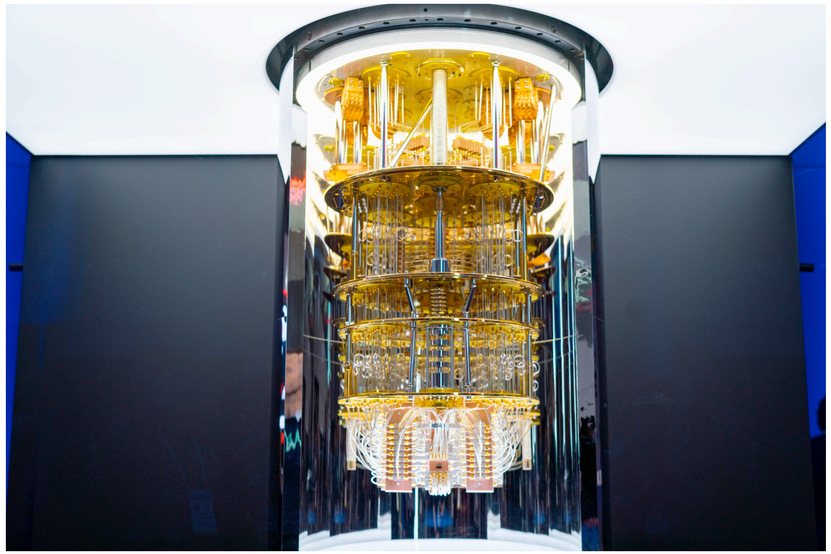
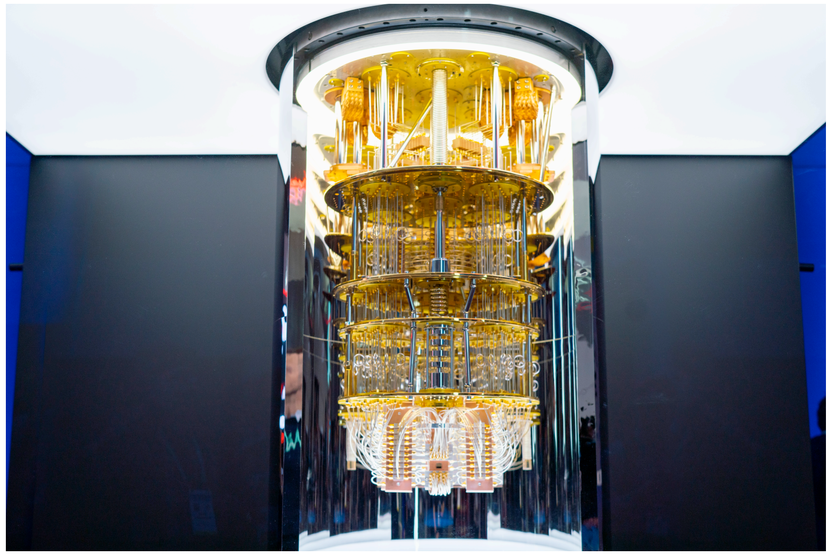
Quantum-Circuit Optimization

Native Basis Gate Set



IBM

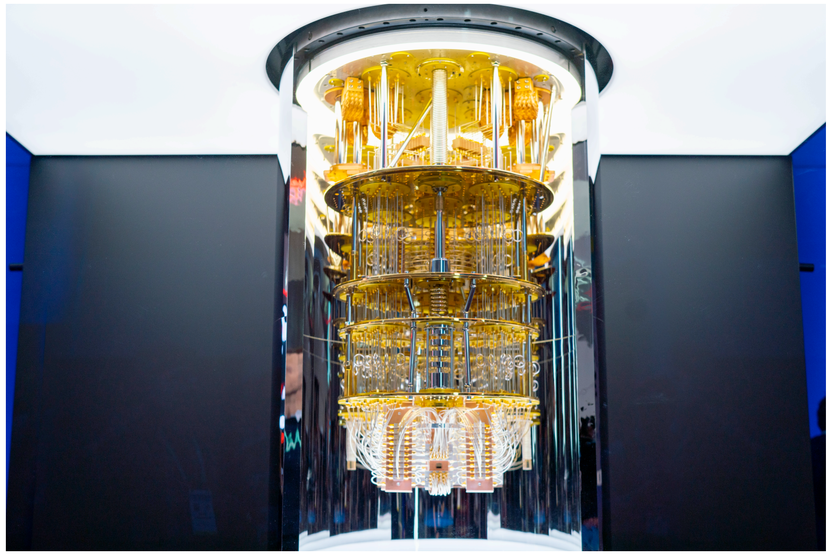
Native Basis Gate Set



IBM

$$R_z^{\theta}$$

Native Basis Gate Set

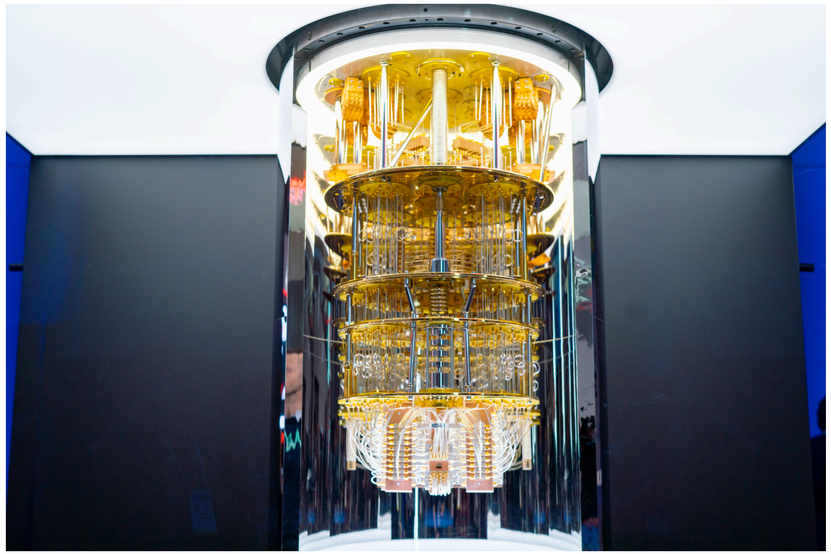


IBM

R_z^θ

SX

Native Basis Gate Set



IBM

R_z^θ SX X

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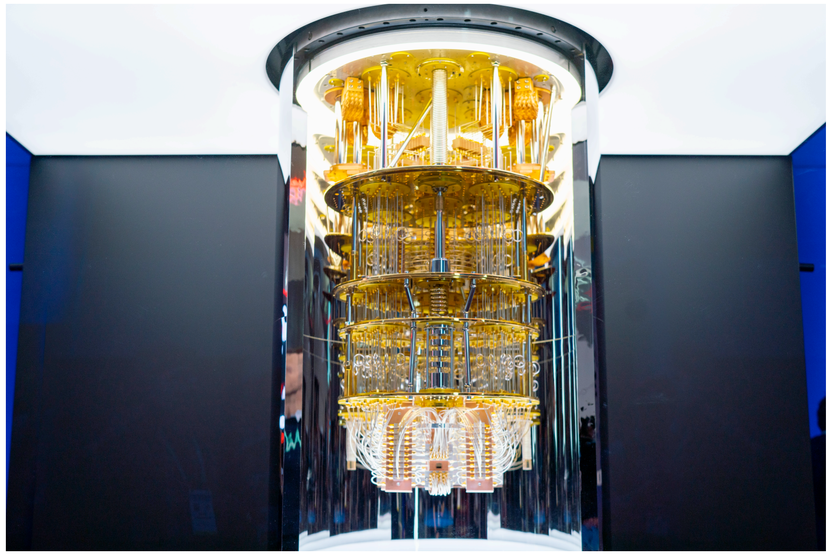

IBM

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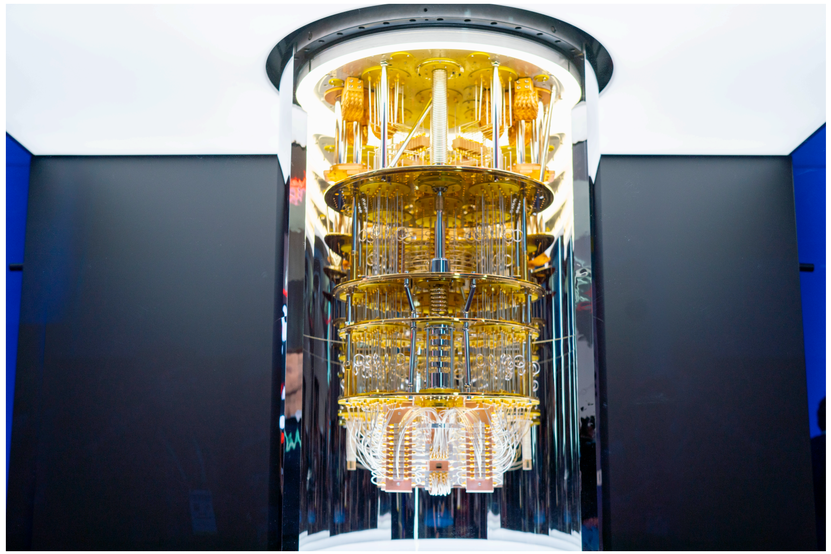
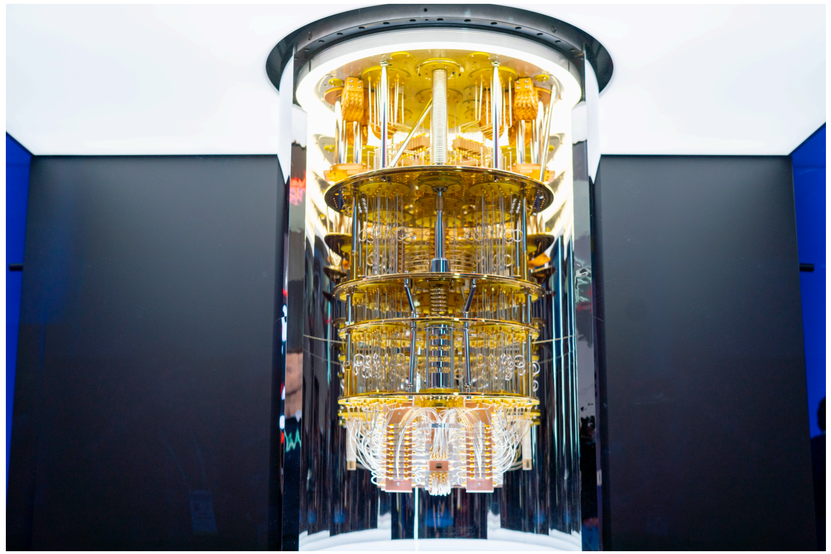


IBM

$\{ R_z^\theta , SX , X , CX \}$

Native Basis Gate Set

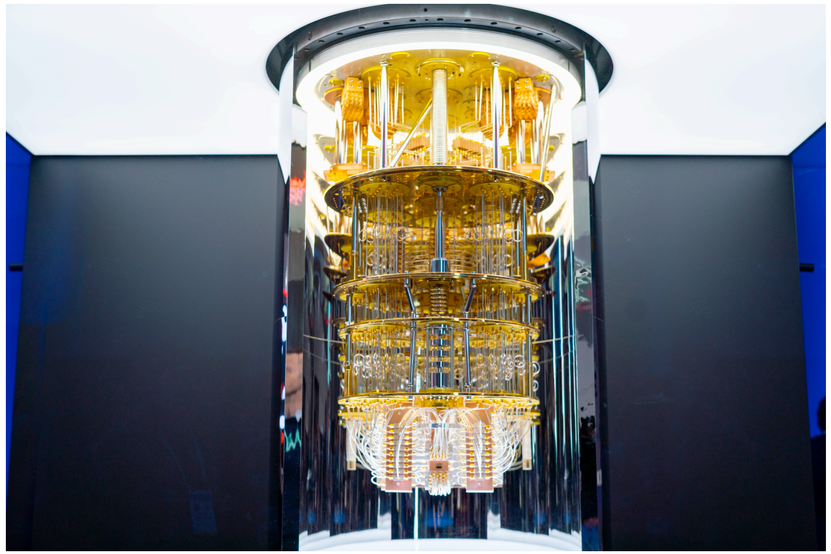


IBM

R_z^θ , SX , X , CX

"universal"

Native Basis Gate Set



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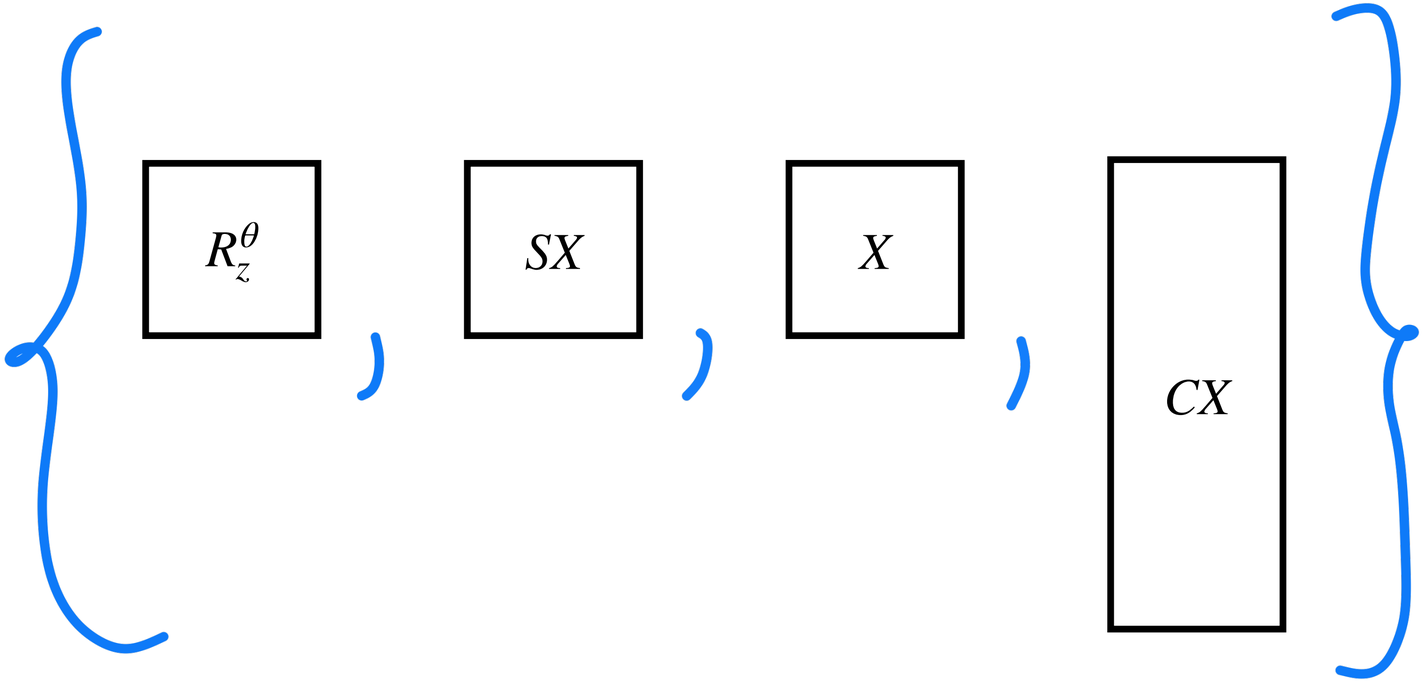
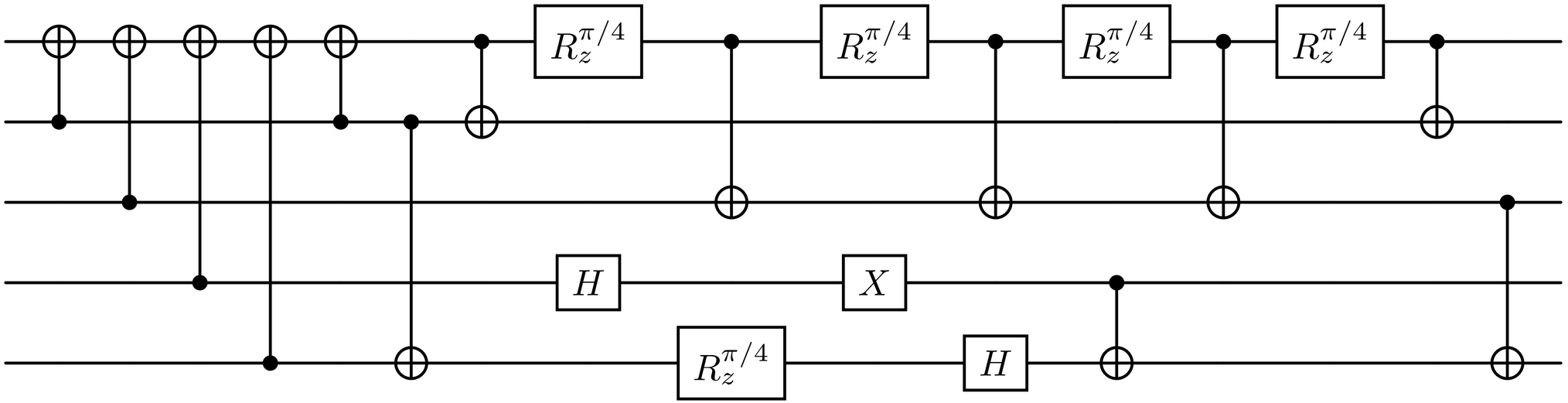


"universal"

i.e. approximate any computation to arbitrary precision

Optimization Objectives



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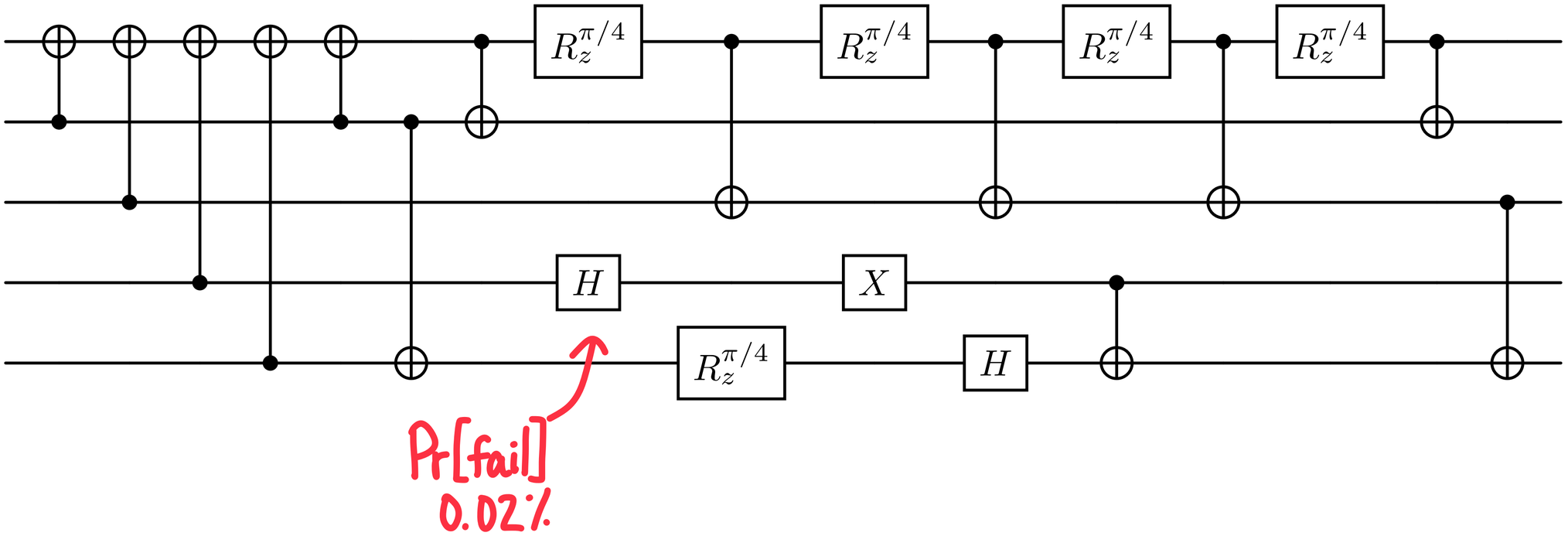

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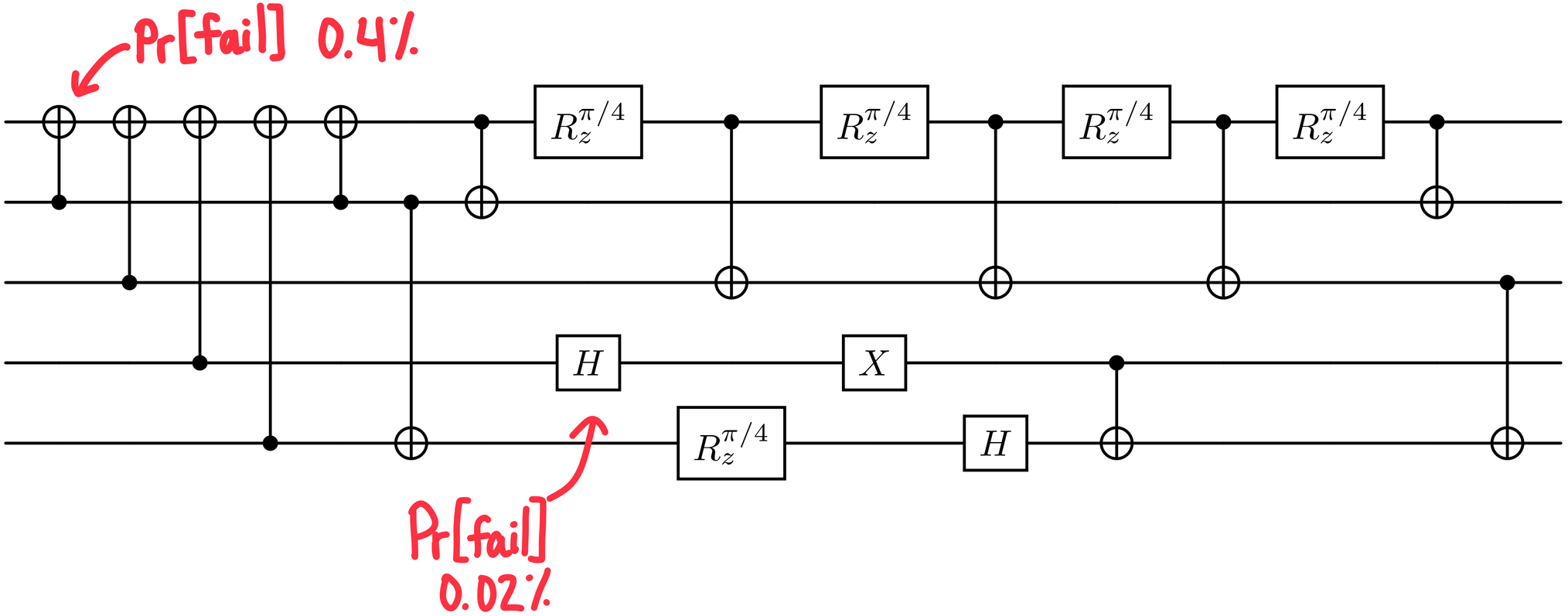

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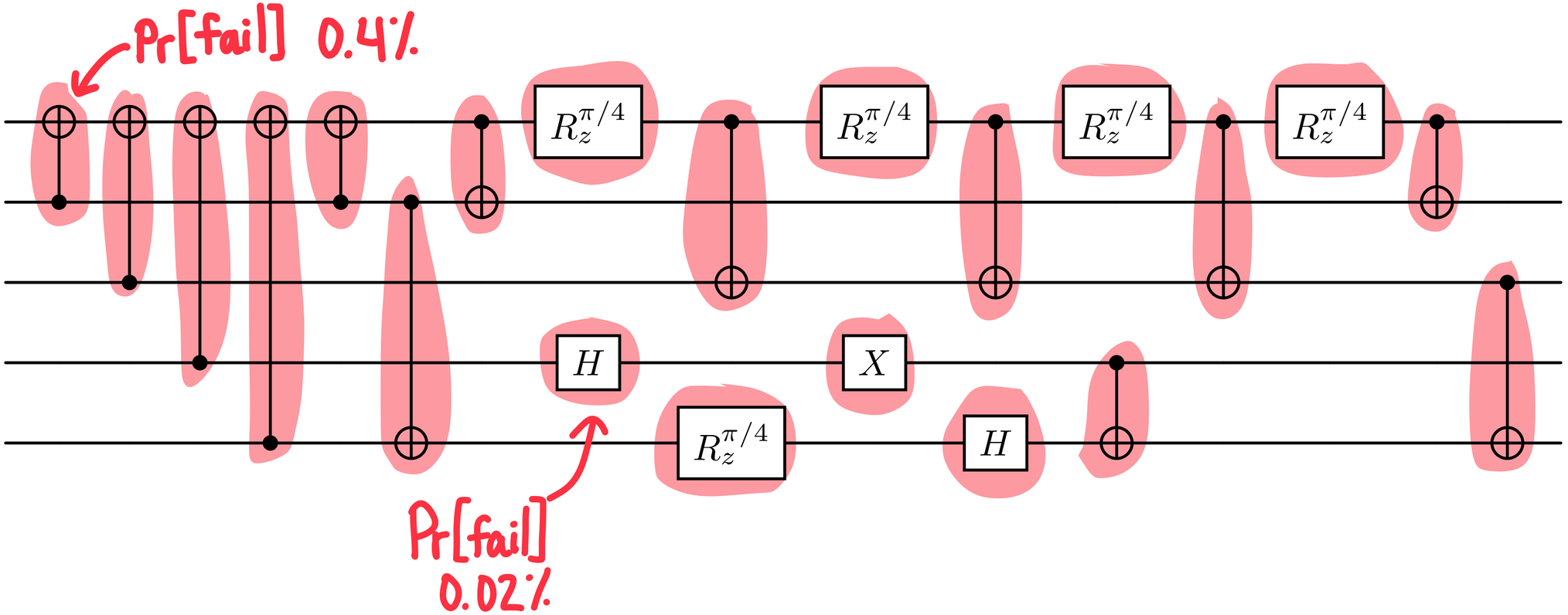
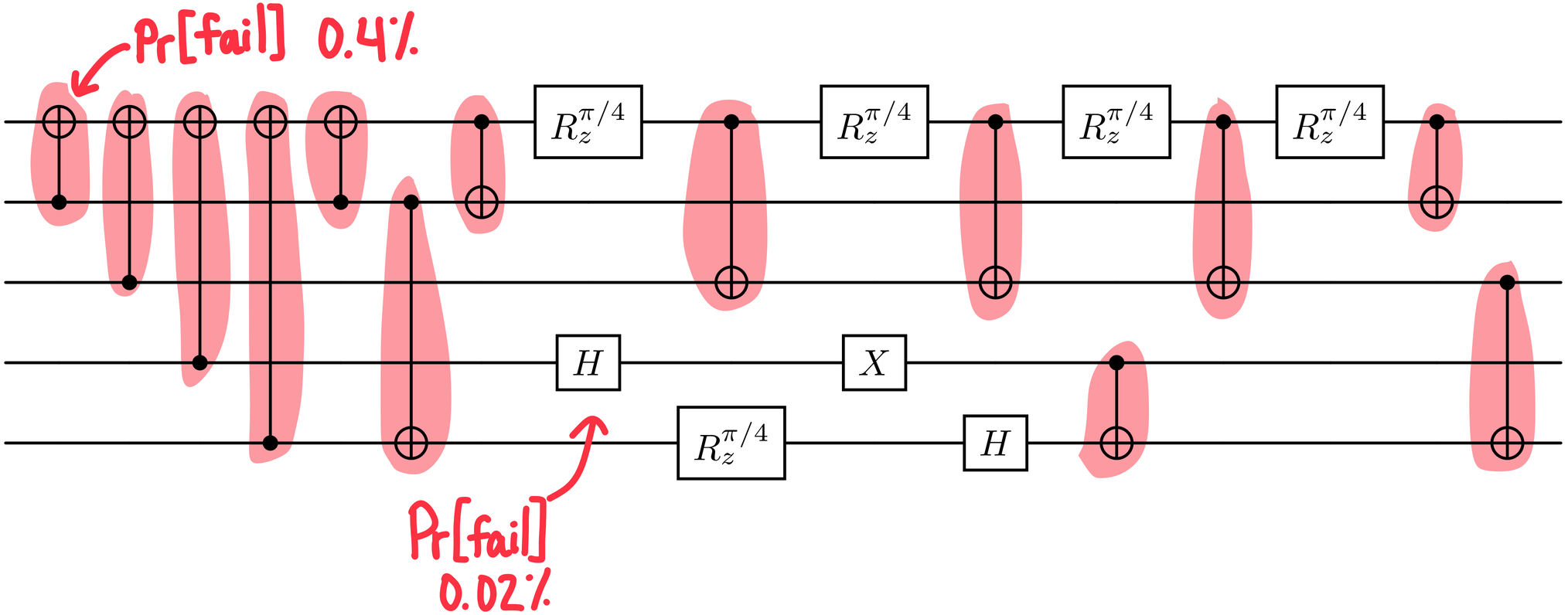


NISQ

- total gate count

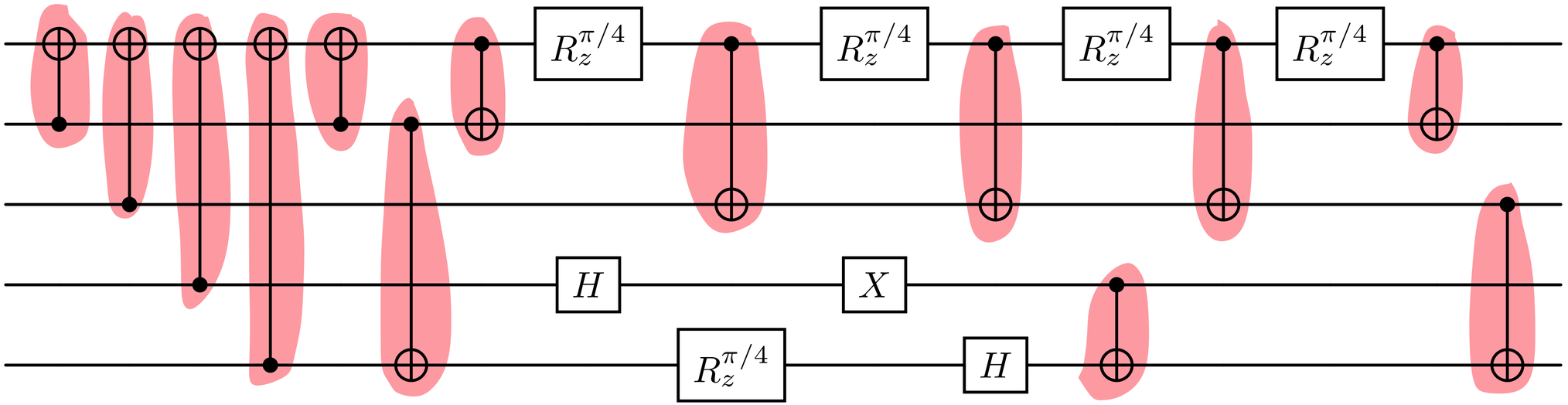
Optimization Objectives



NISQ

- total gate count
- 2q gate count

Optimization Objectives



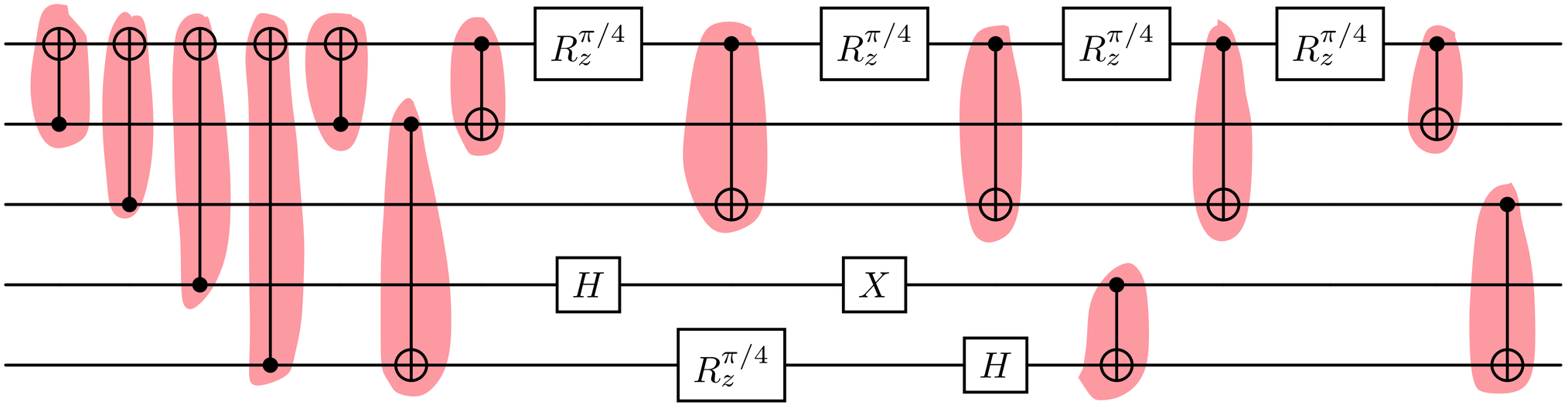
NISQ

- total gate count
- 2q gate count

FTQC

- T count and 2q gate count

Optimization Objectives



$$T = R_z^{\pi/4}$$

NISQ

- total gate count
- 2q gate count

FTQC

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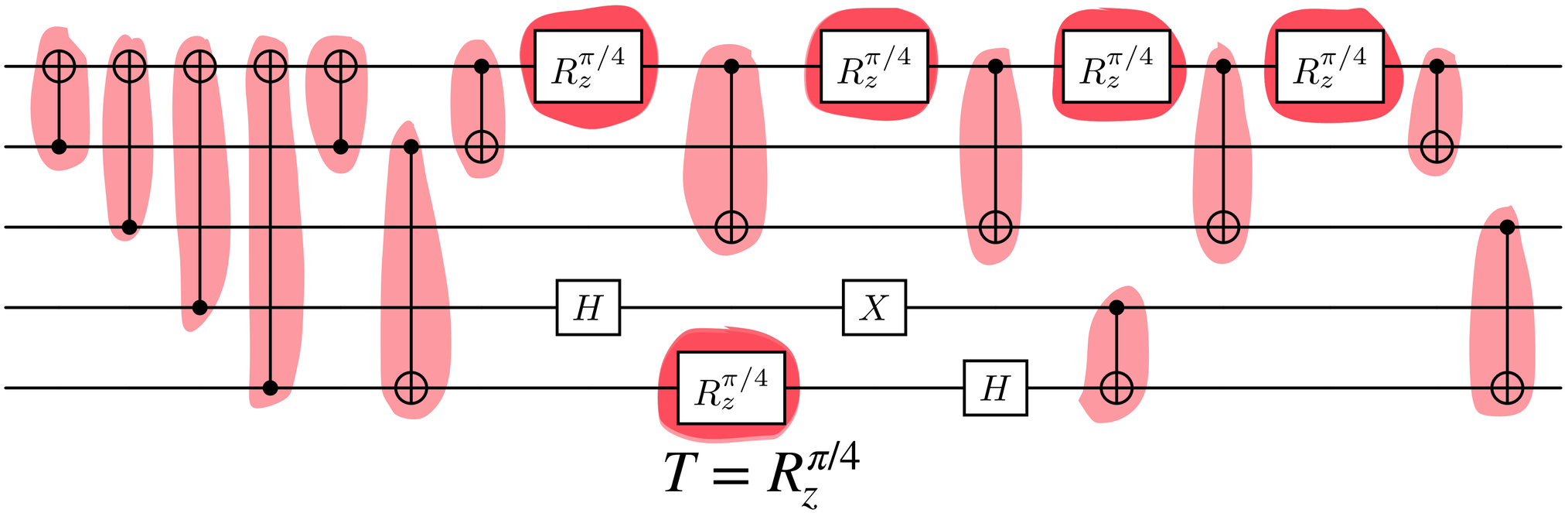
Optimization Objectives



NISQ

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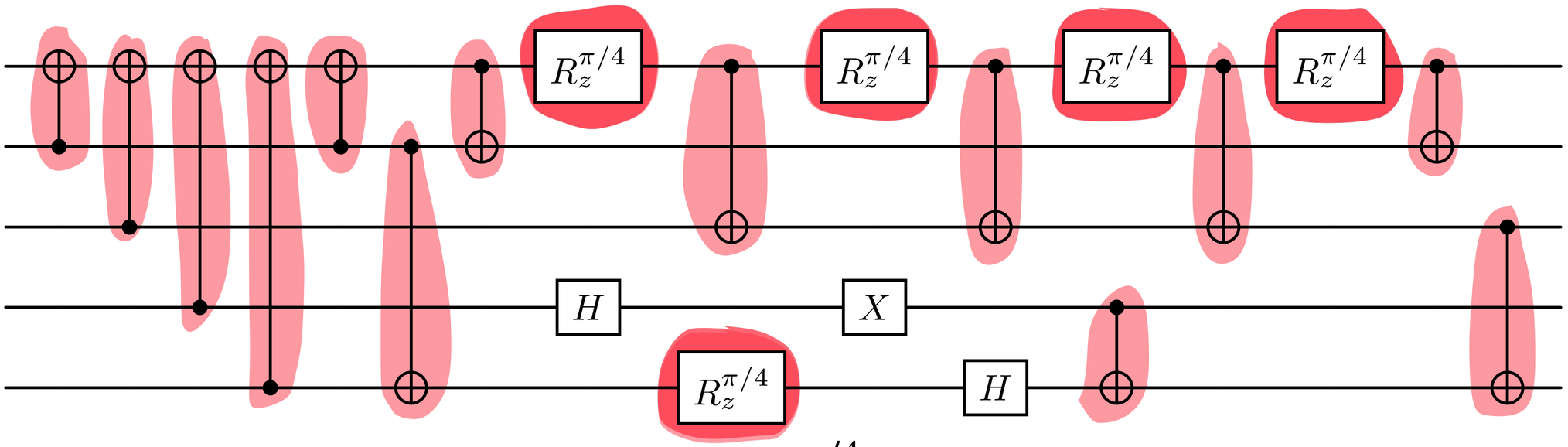

NISQ

- total gate count
- 2q gate count

FTQC

- T count and 2q gate count

and many more...

High-level problem

Given a circuit and optimization objective, output an equivalent circuit that minimizes the optimization objective.

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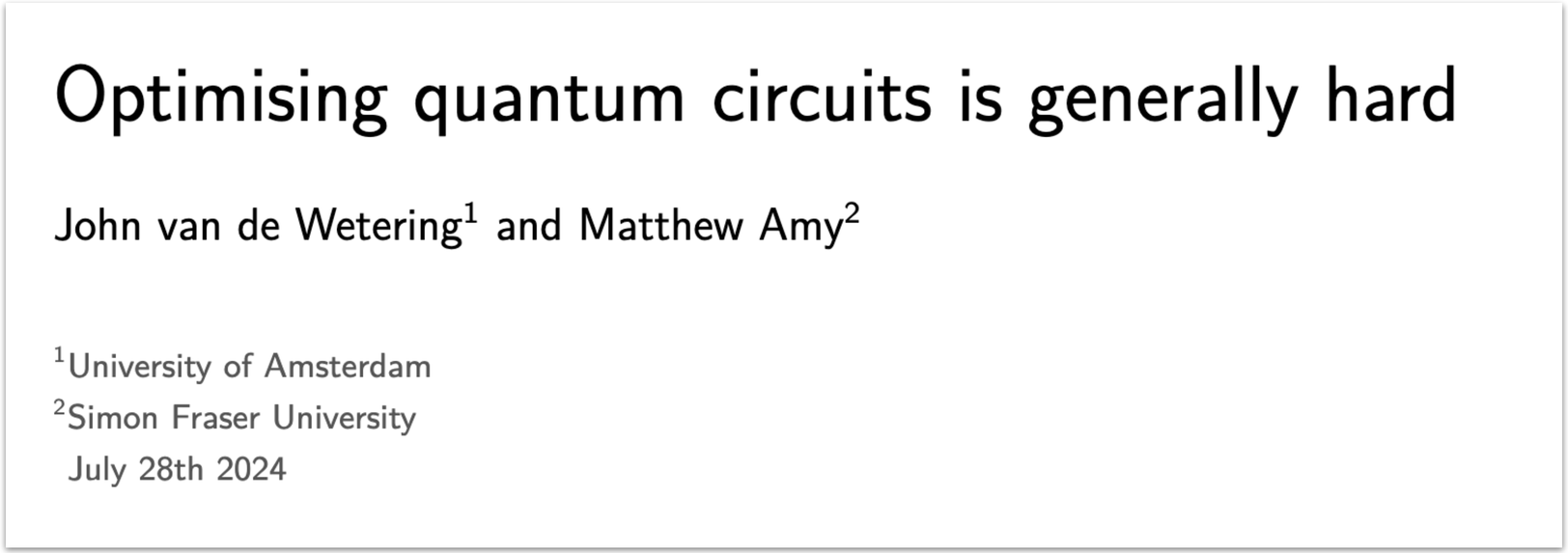
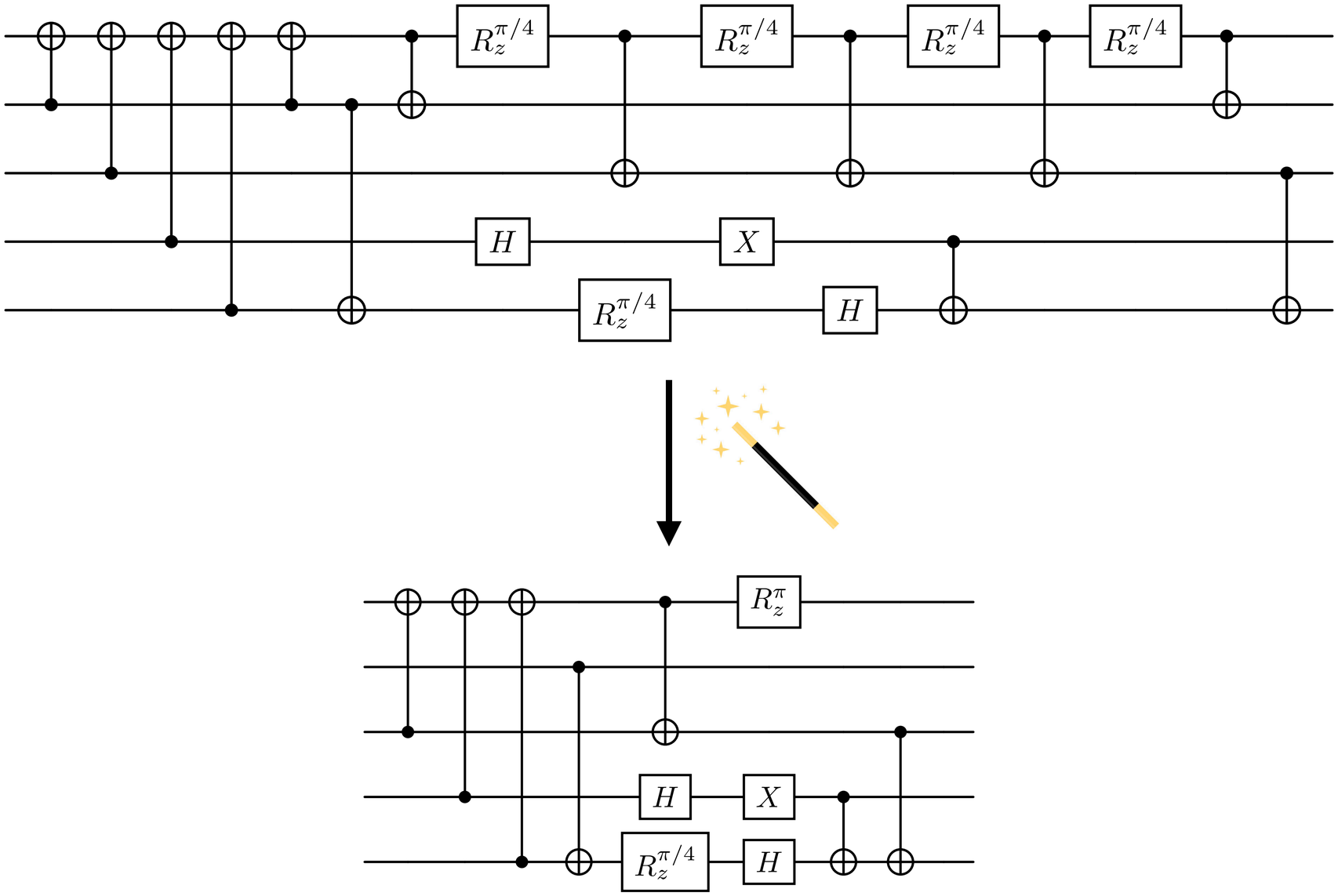


Optimising quantum circuits is generally hard

John van de Wetering[1] and Matthew Amy[2]

[1]University of Amsterdam
[2]Simon Fraser University
July 28th 2024

General Approach



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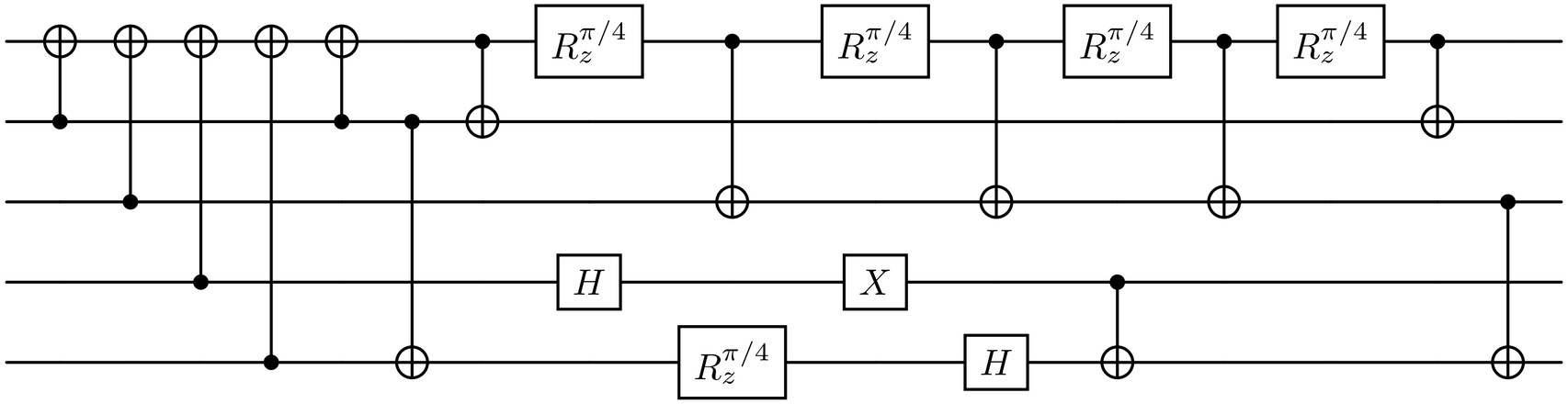

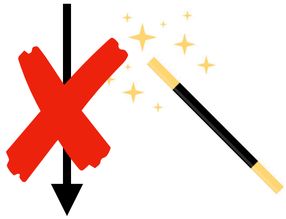

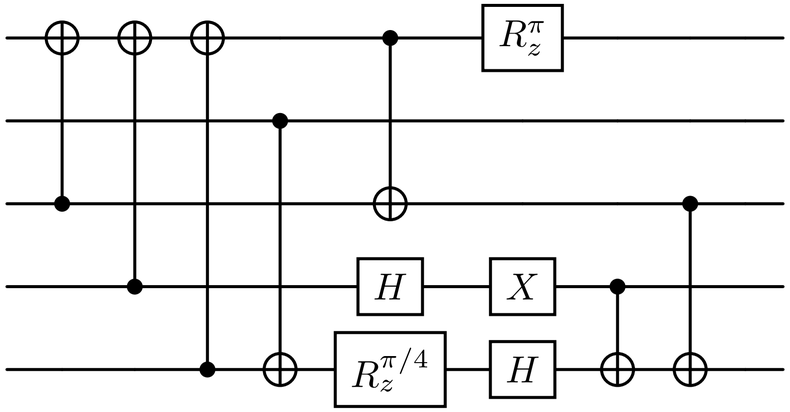

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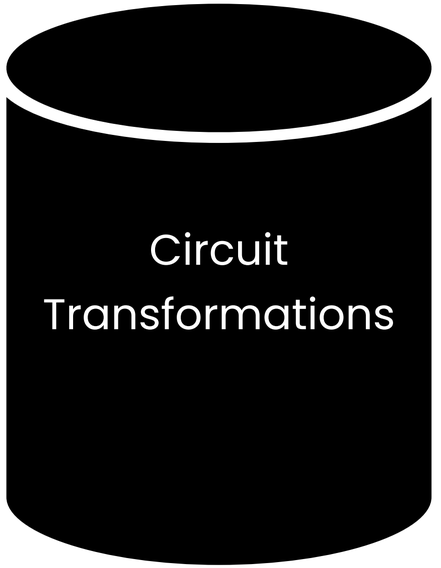

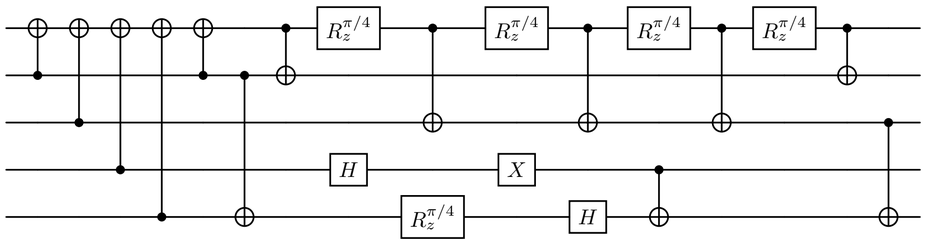

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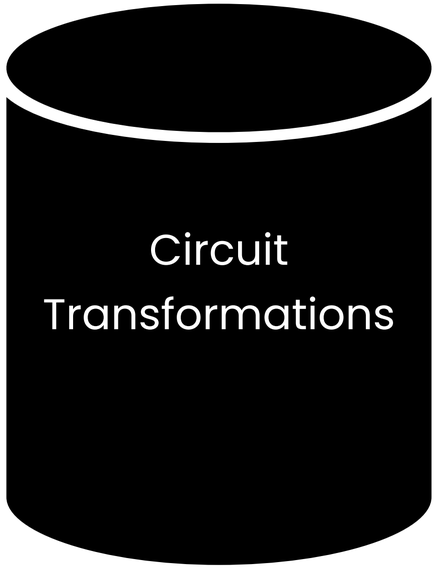

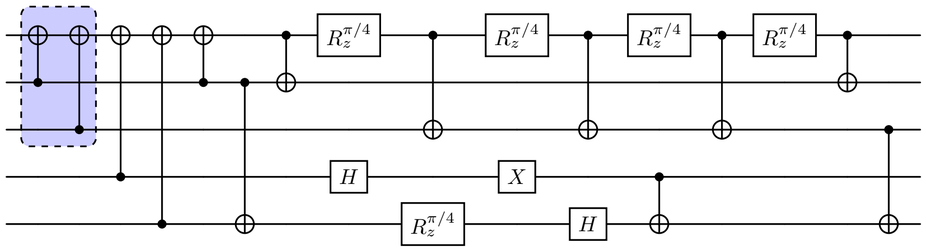

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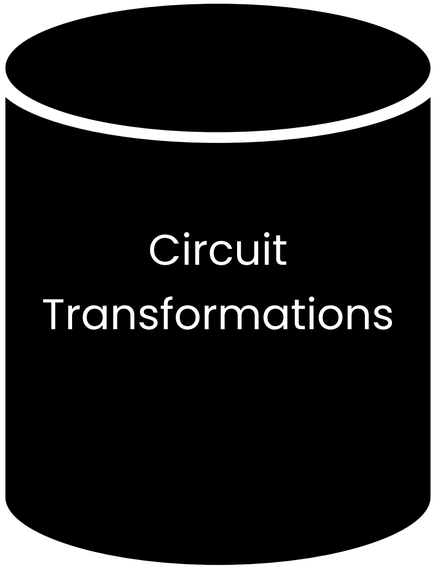
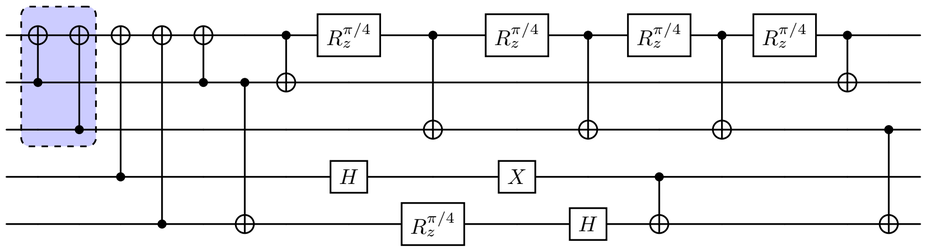


"subcircuit"



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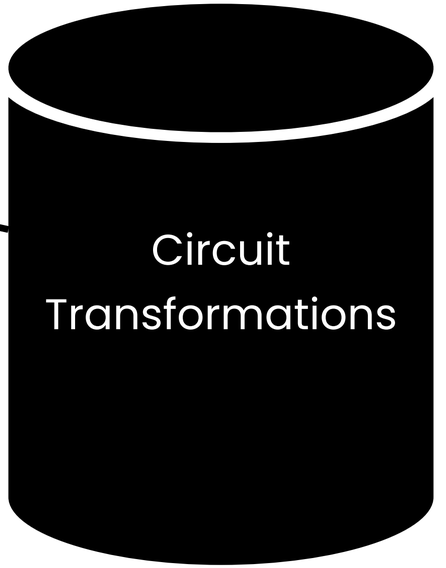

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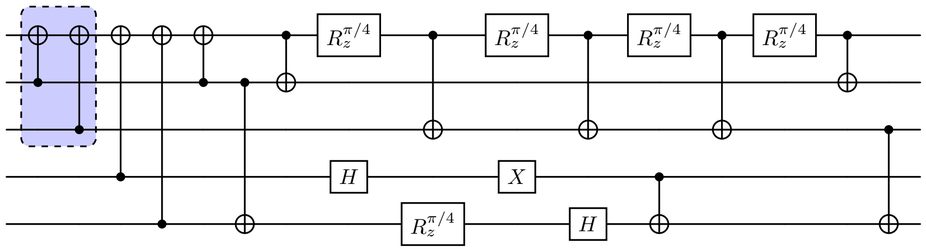

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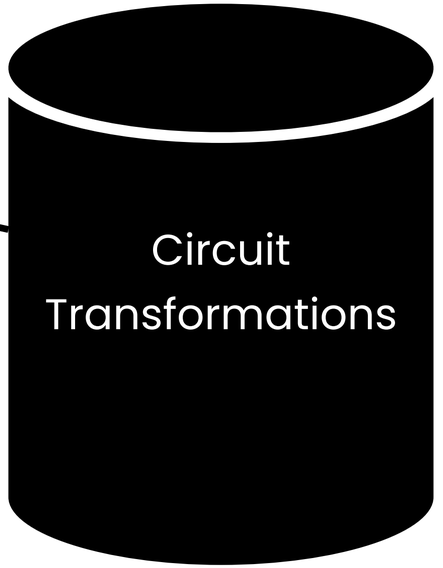

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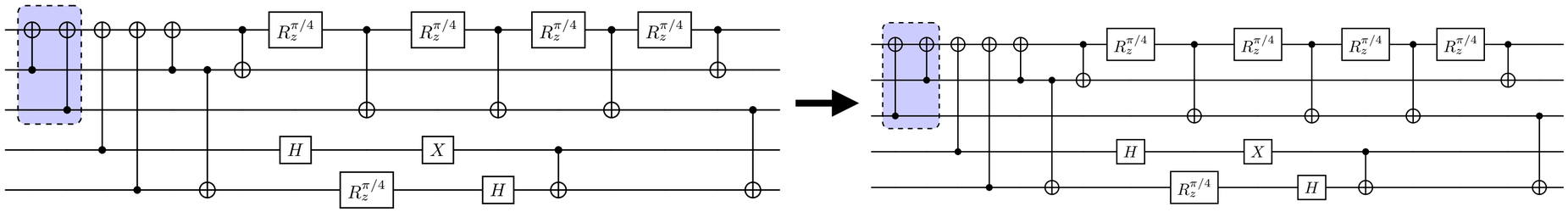

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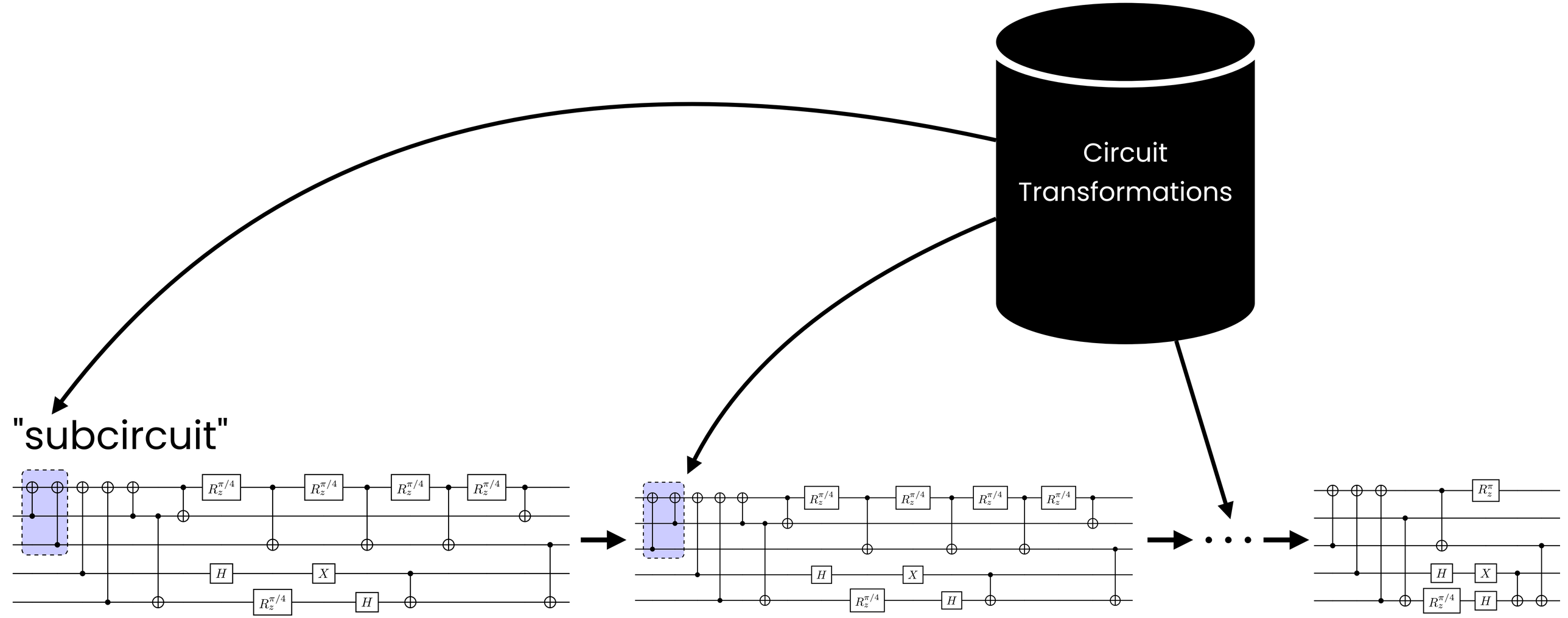

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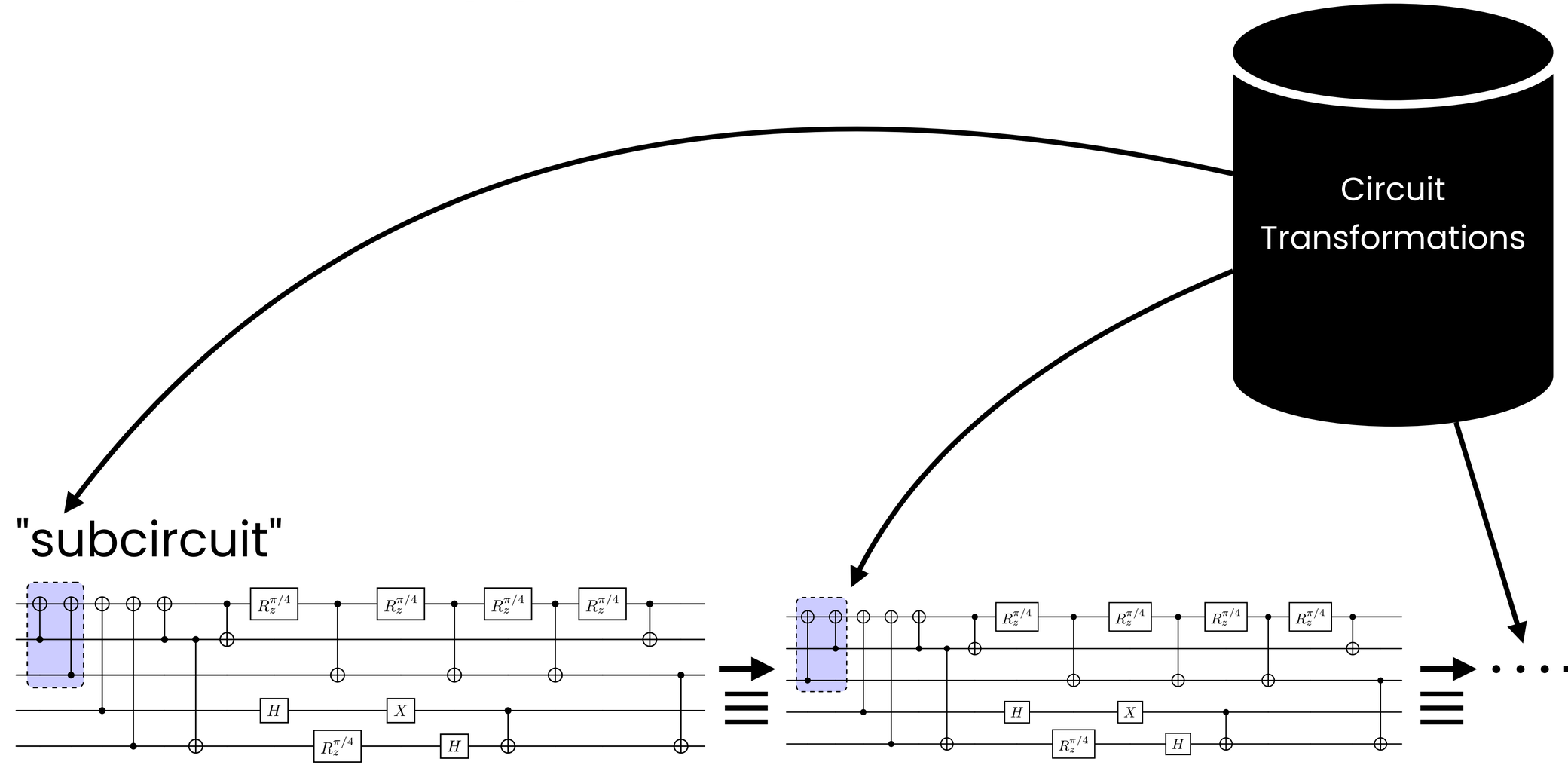

Circuit Equivalence

Circuits are unitary matrices: $U^\dagger U = UU^\dagger = I$

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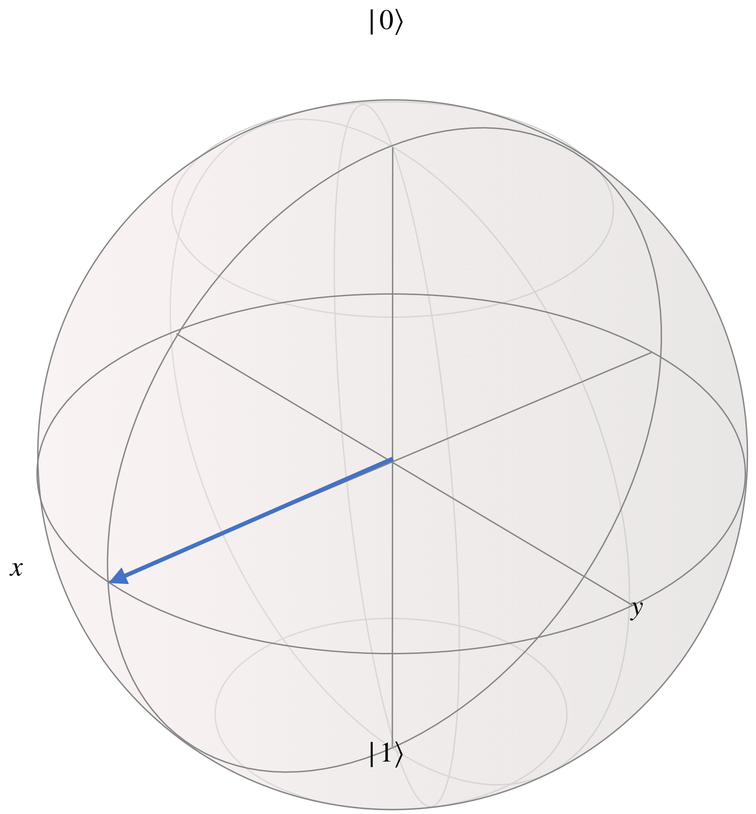

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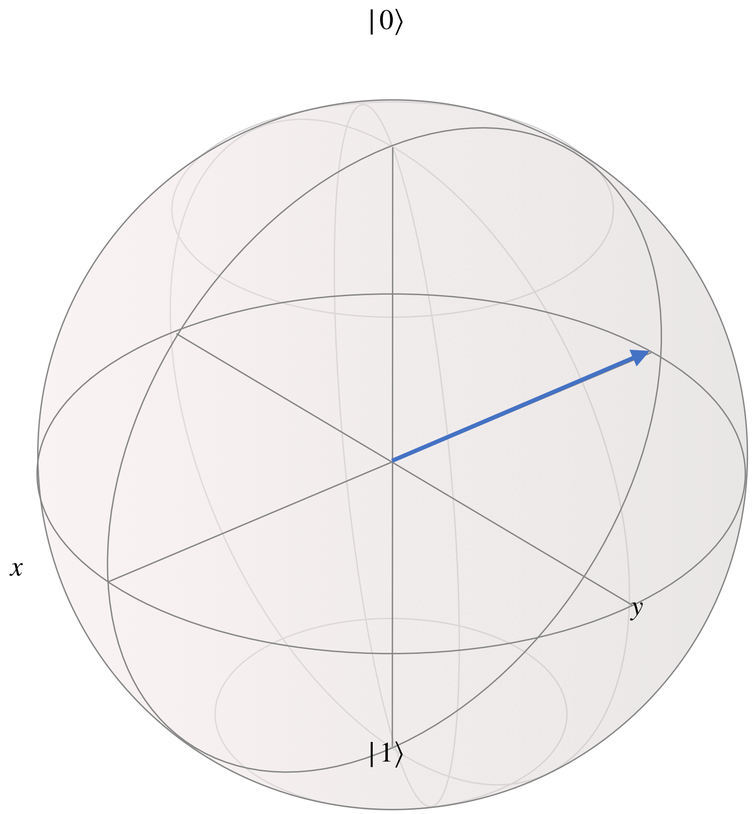

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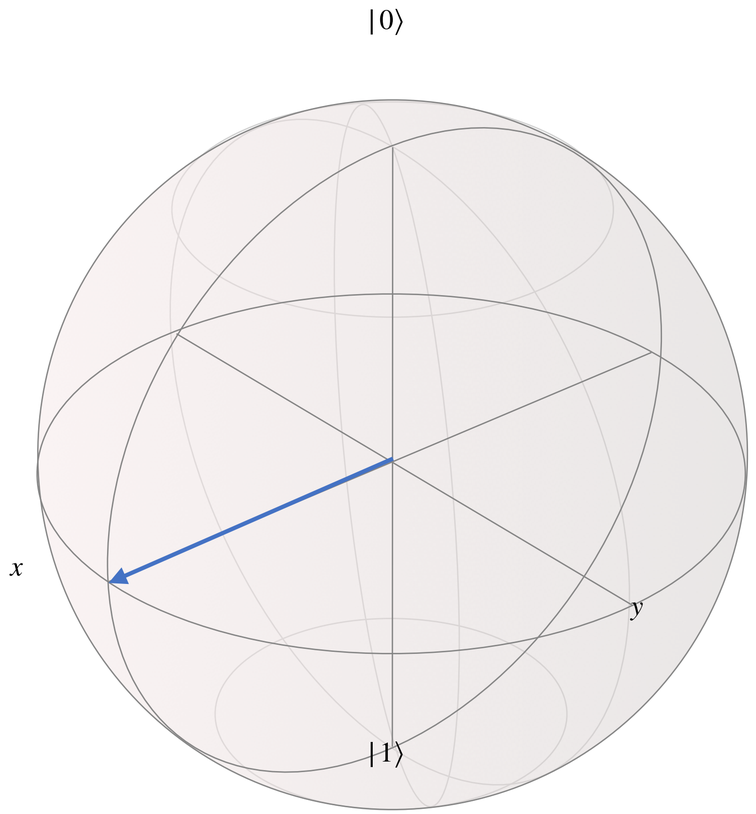

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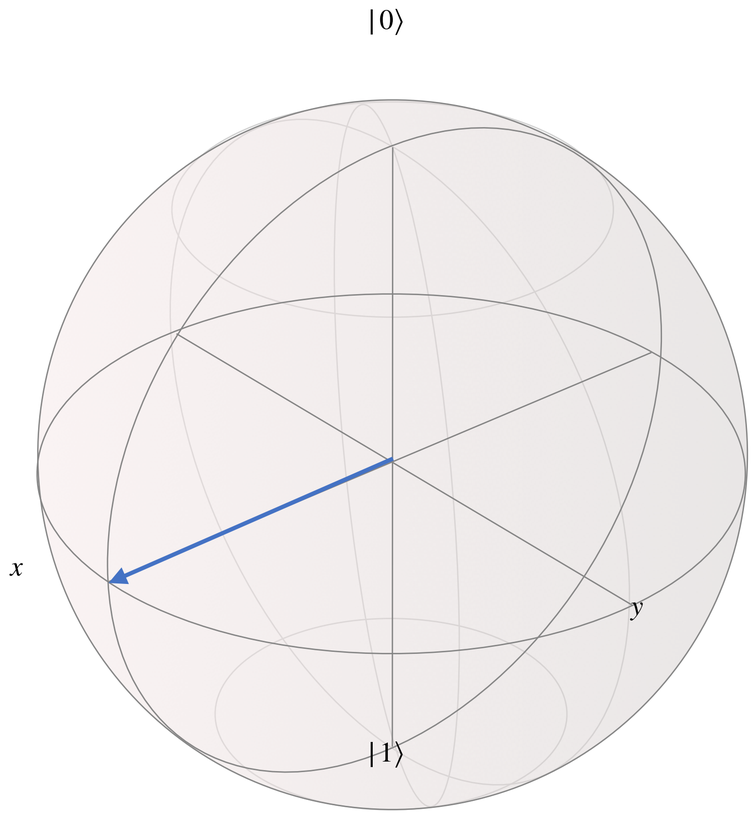

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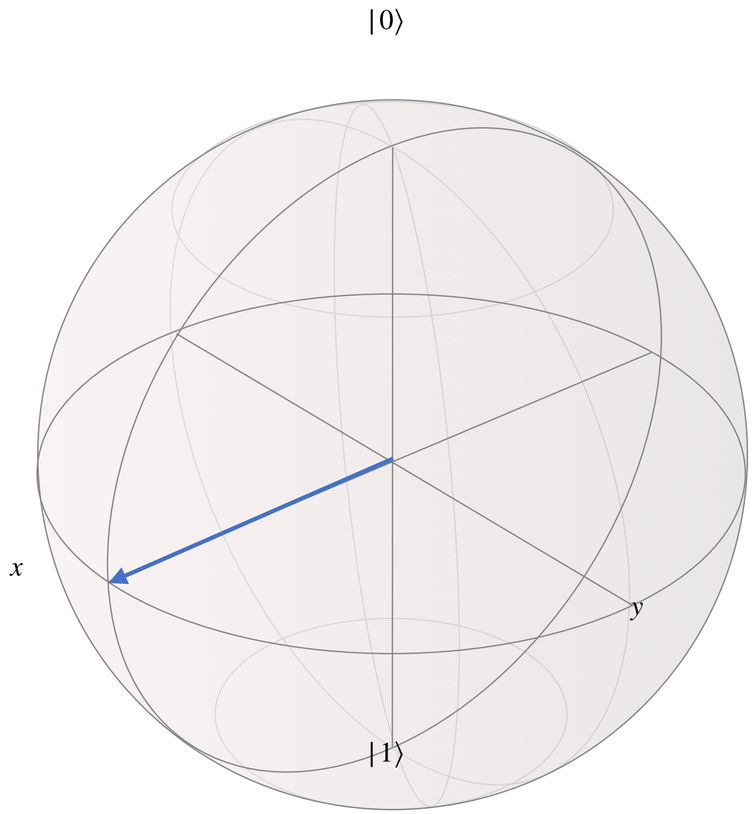
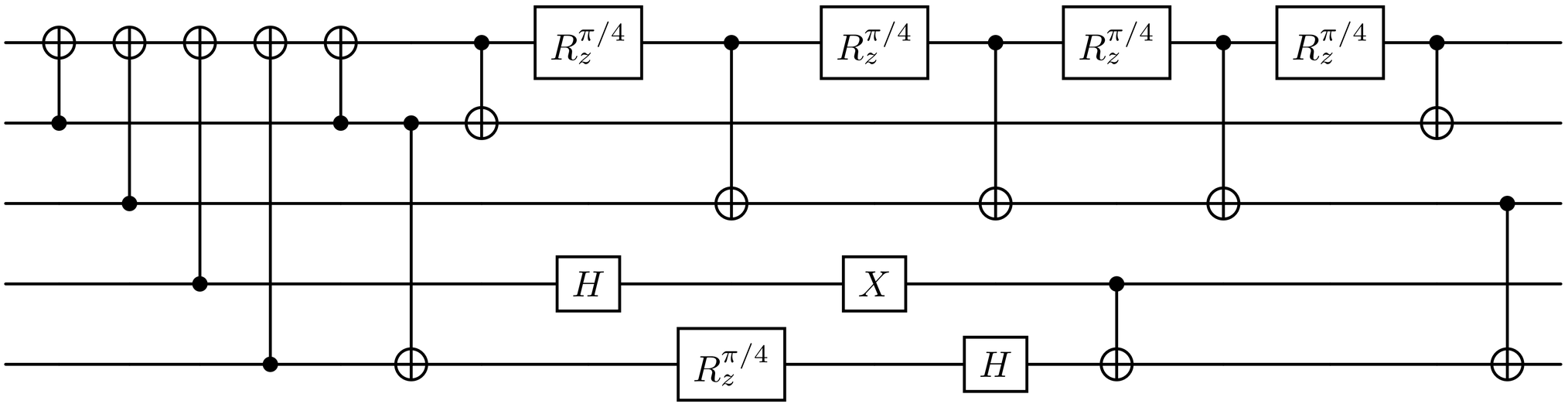
Hard for even a quantum computer to verify

(QMA-Complete)



Subcircuit



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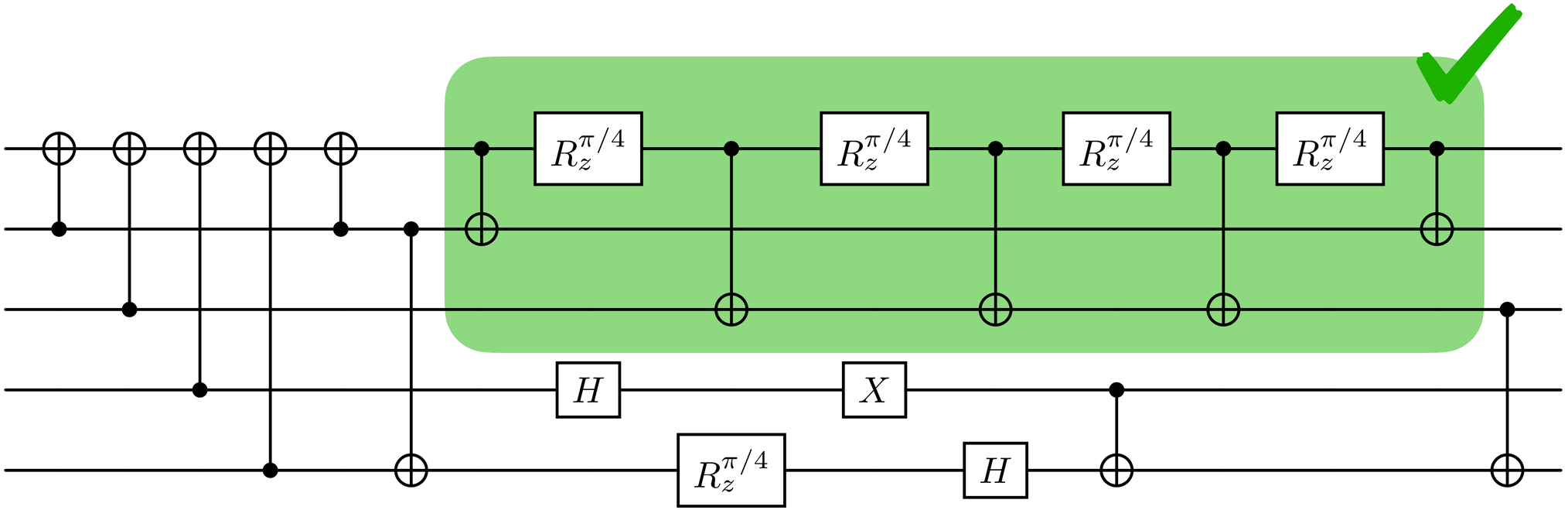

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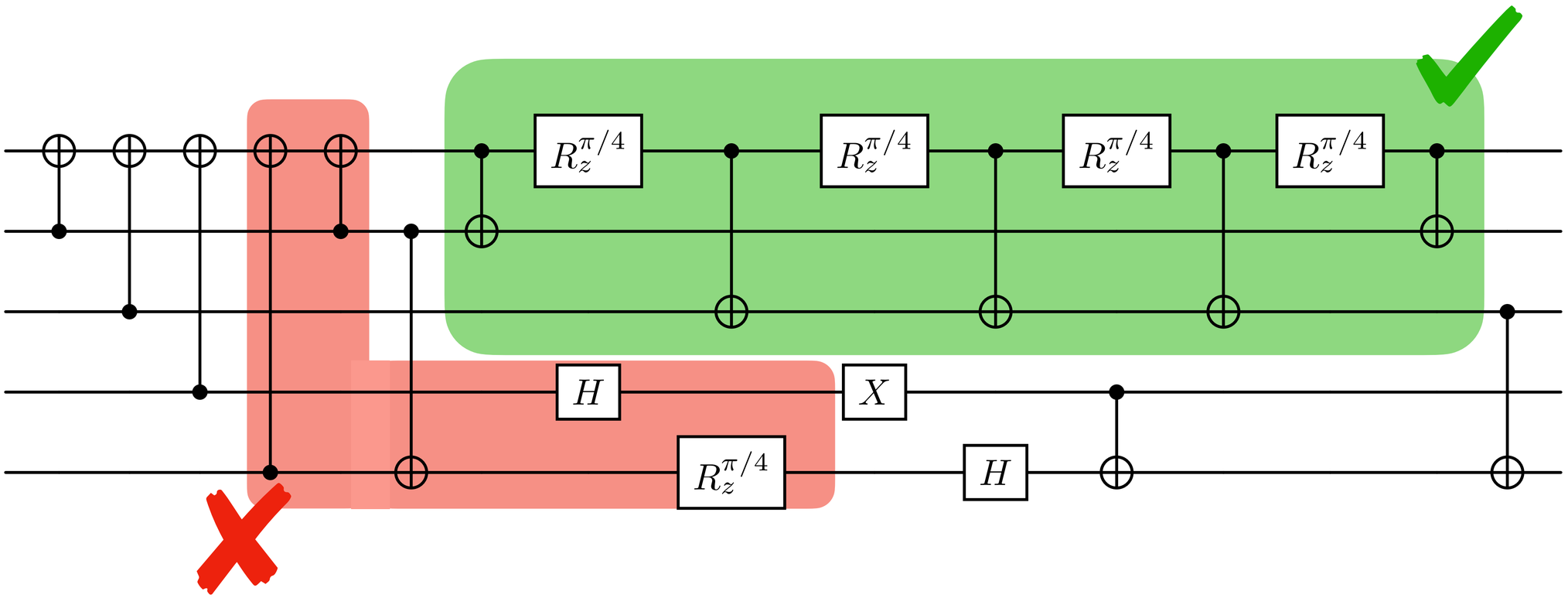

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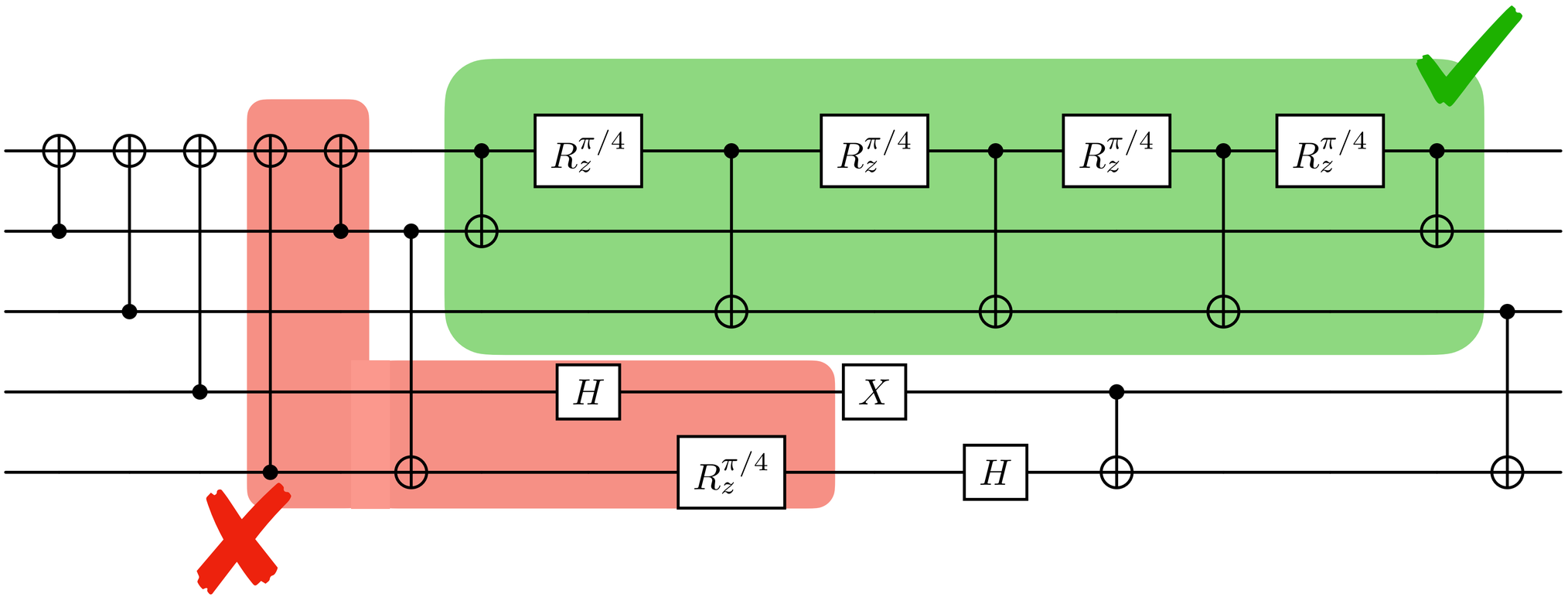
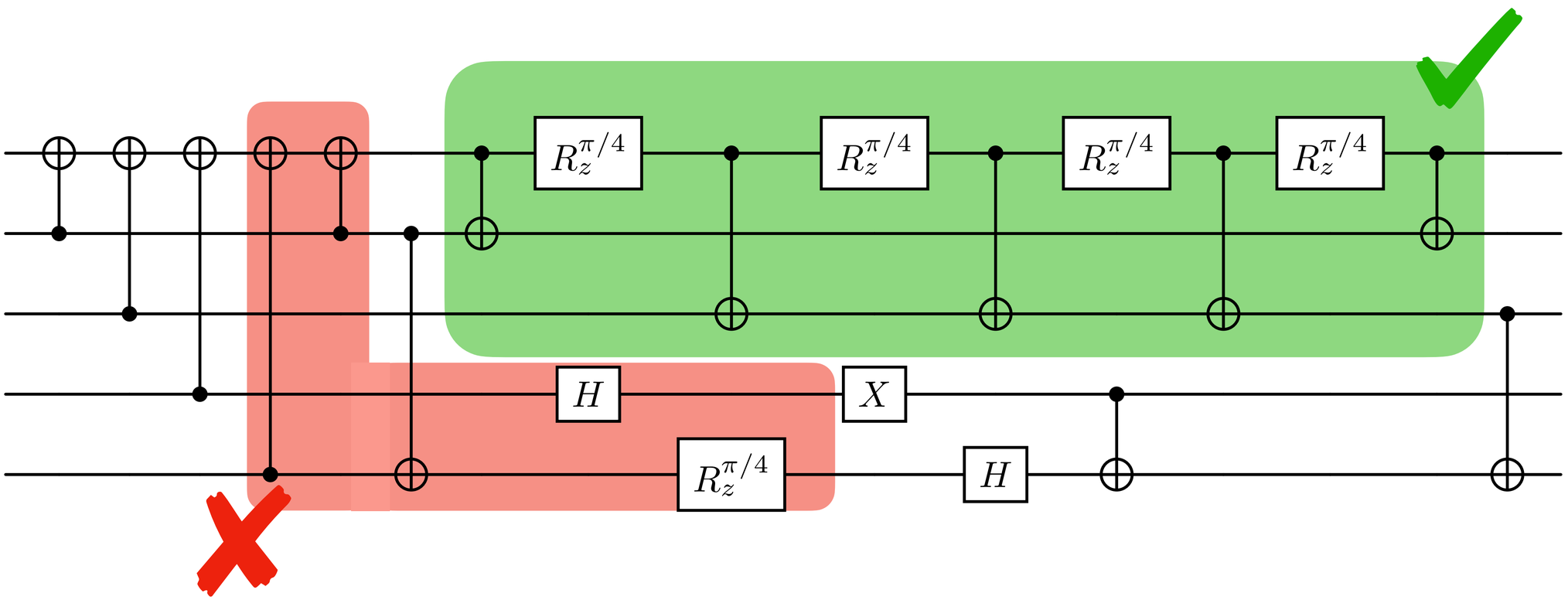


"convex subgraph"

Subcircuit



"convex subgraph"

i.e. any path between two vertices in the DAG also in subgraph

Rewrite Rules

Rewrite Rules

"peephole optimization"

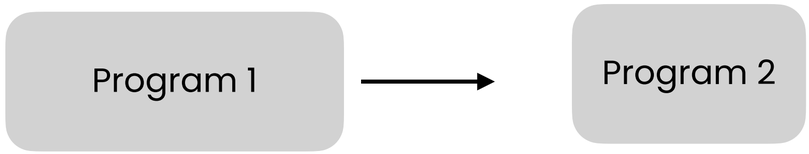
Rewrite Rules

"peephole optimization"

Program 1

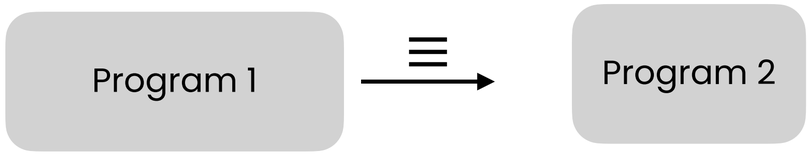
Rewrite Rules

"peephole optimization"



Rewrite Rules

"peephole optimization"



Rewrite Rules

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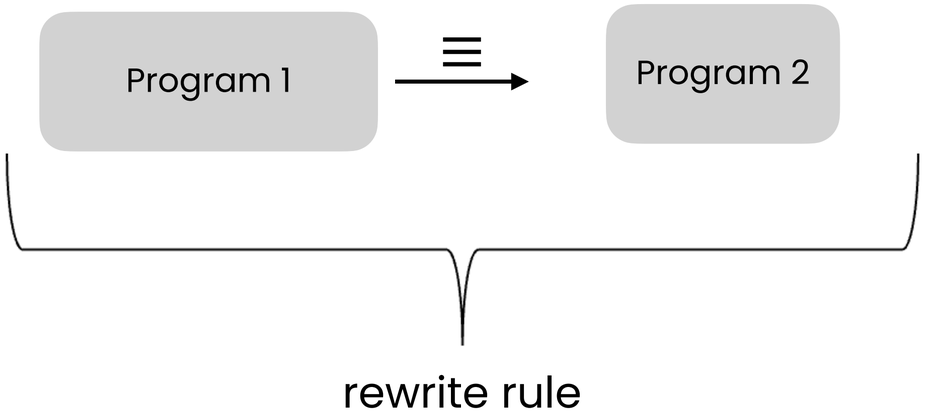
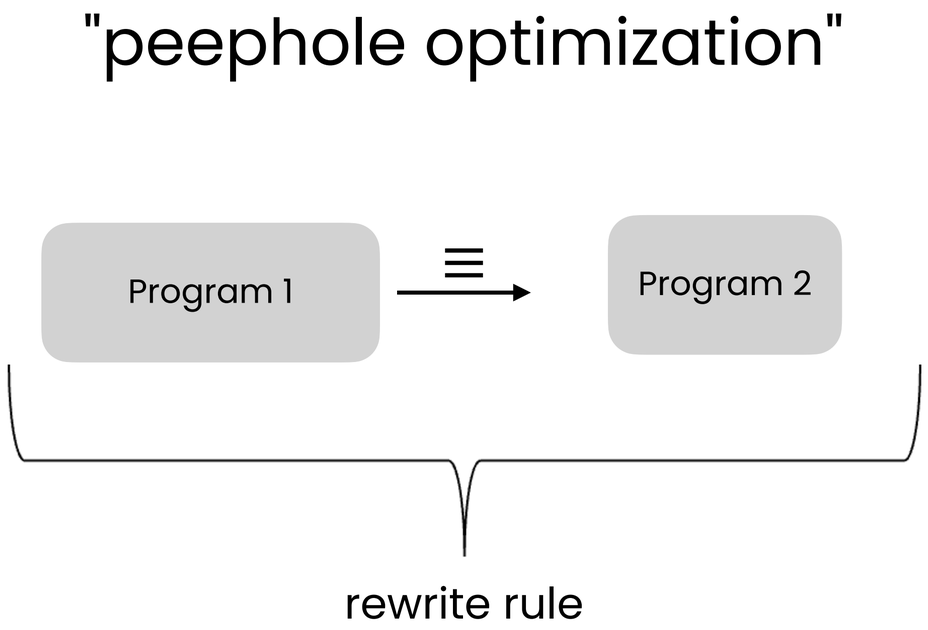
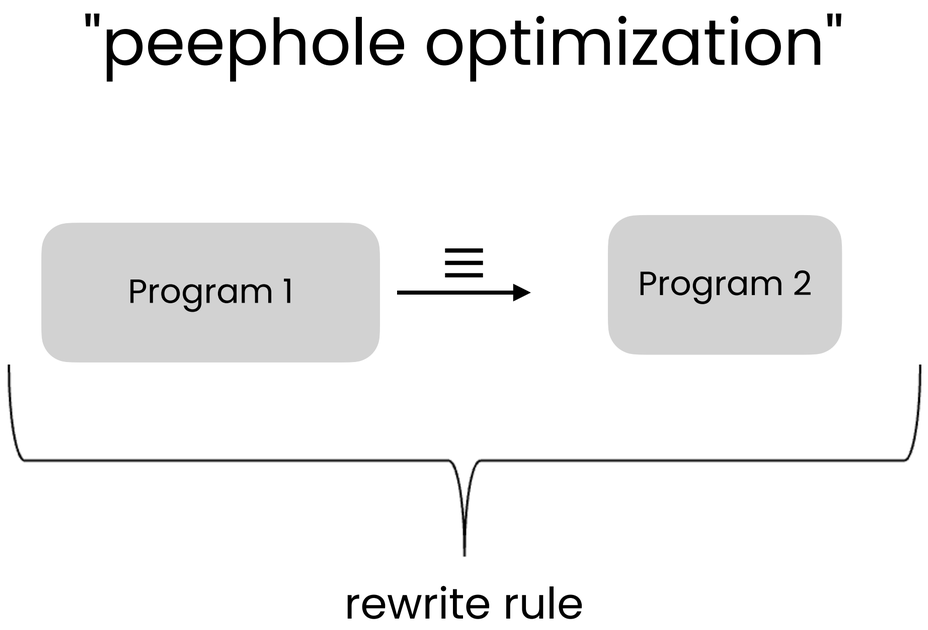


Rewrite Rules



- Optimization: LHS and RHS same gate set

Rewrite Rules



- Optimization: LHS and RHS same gate set
- Decomposition: LHS higher level than RHS

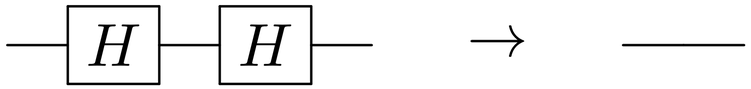
Simple Rewrite Rules

Simple Rewrite Rules

Cancel

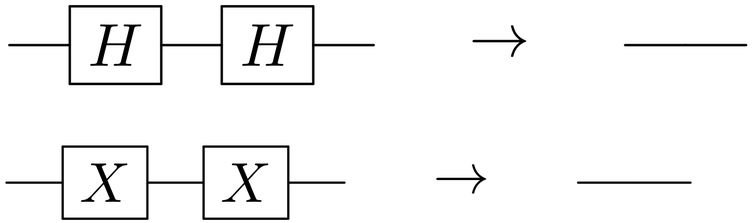
Simple Rewrite Rules

Cancel



Simple Rewrite Rules

Cancel



Simple Rewrite Rules

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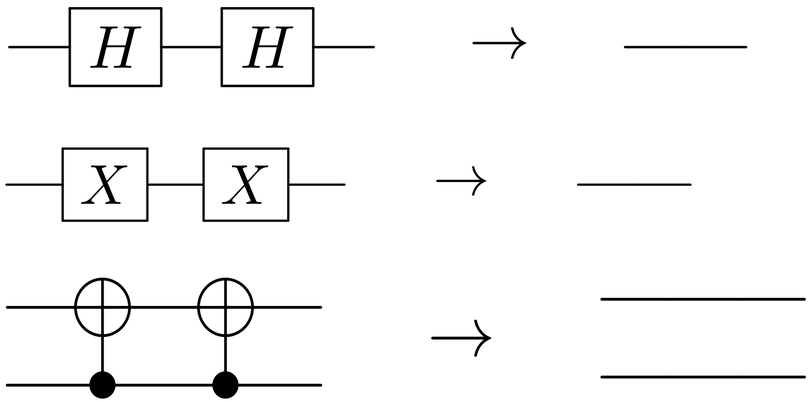
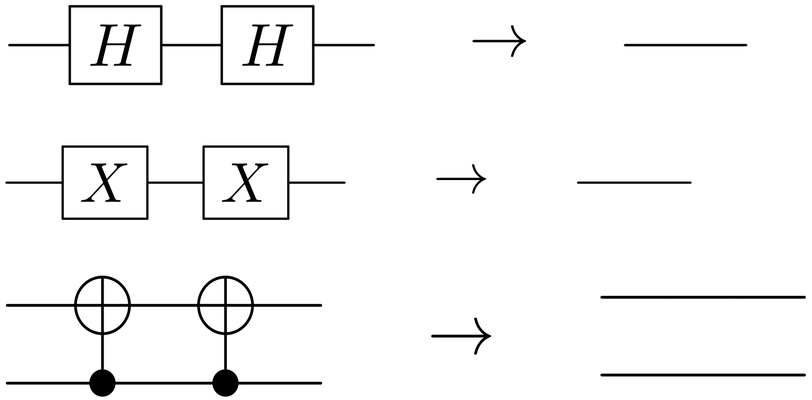


Simple Rewrite Rules

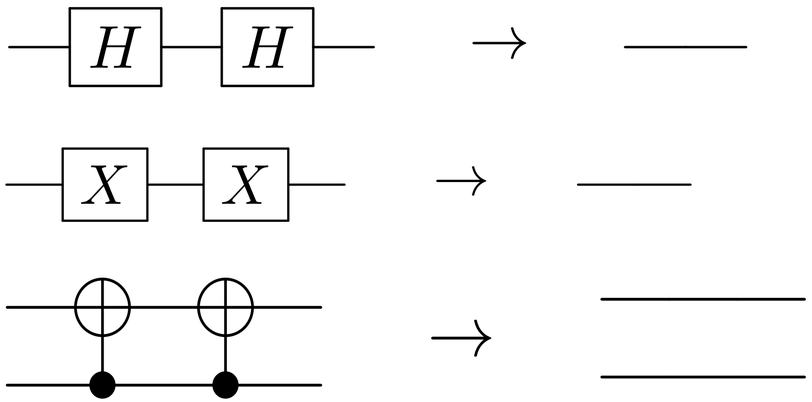
Cancel



Merge

Simple Rewrite Rules

Cancel



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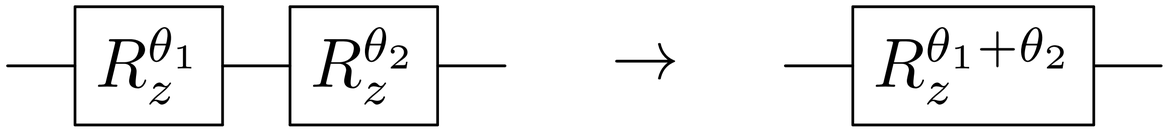
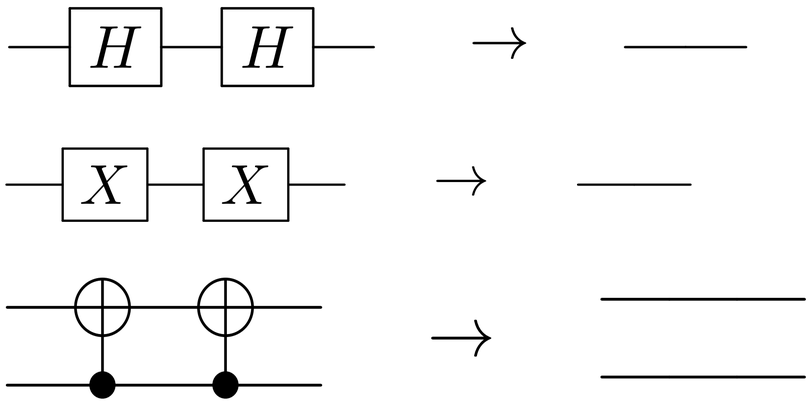


Simple Rewrite Rules

Cancel



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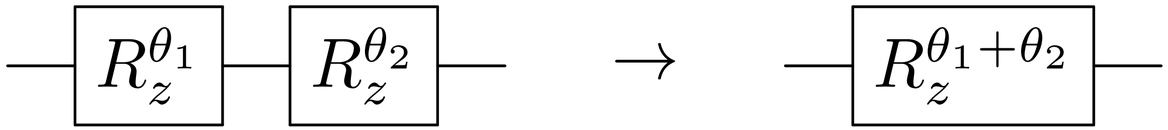
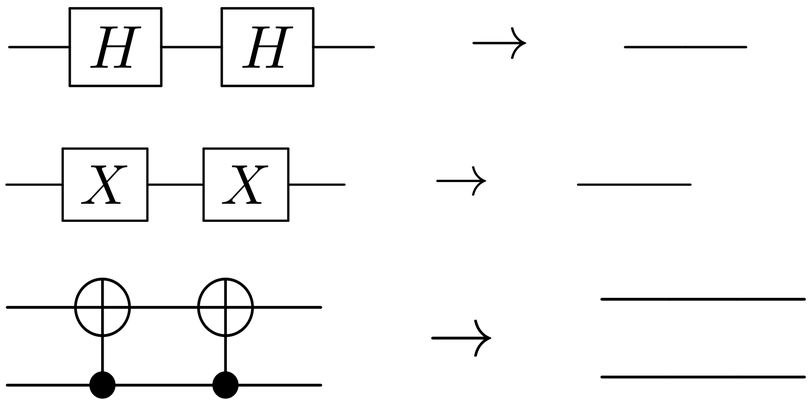


Commute

Simple Rewrite Rules

Cancel



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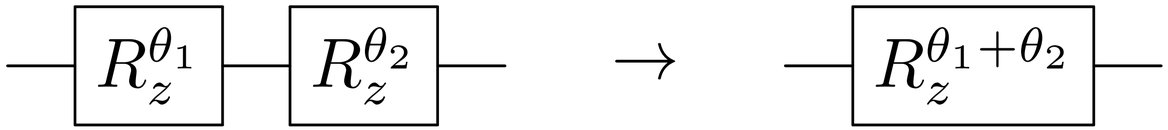

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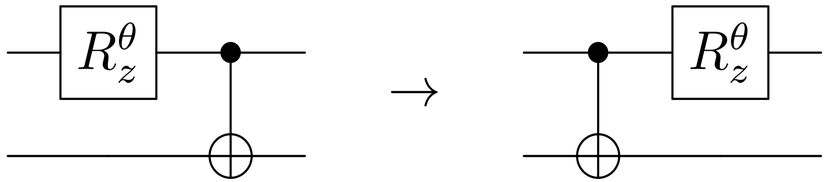
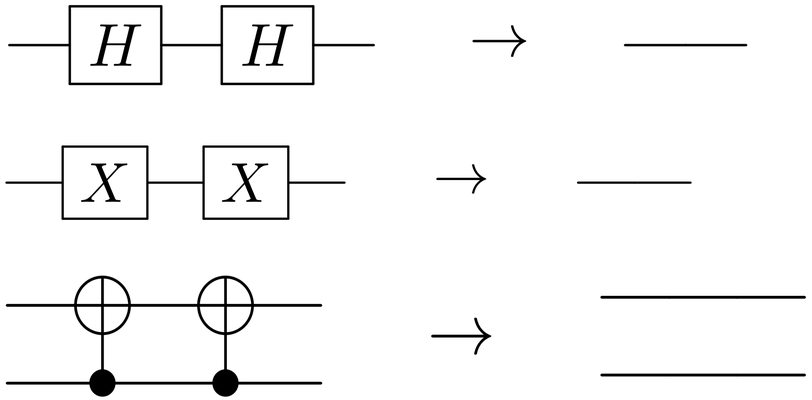
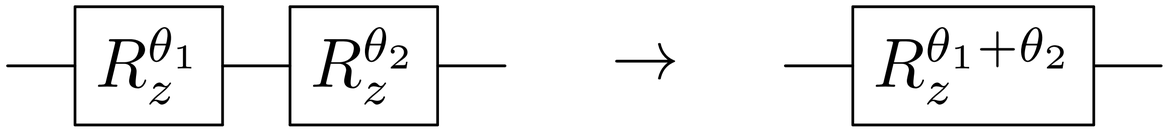


Simple Rewrite Rules

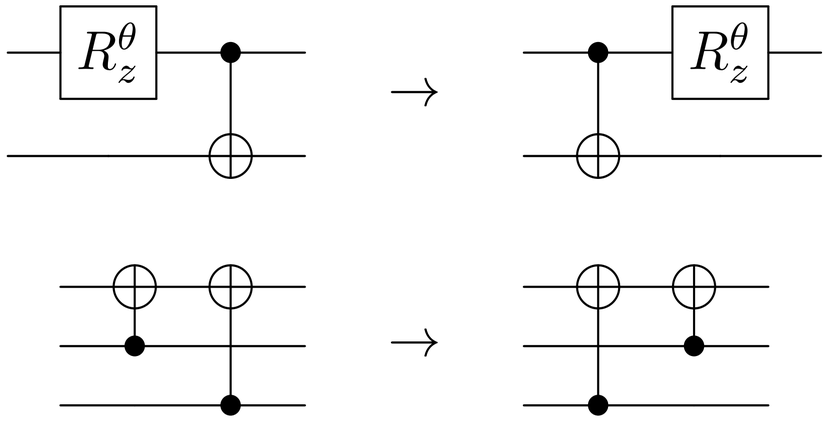
Cancel



Merge

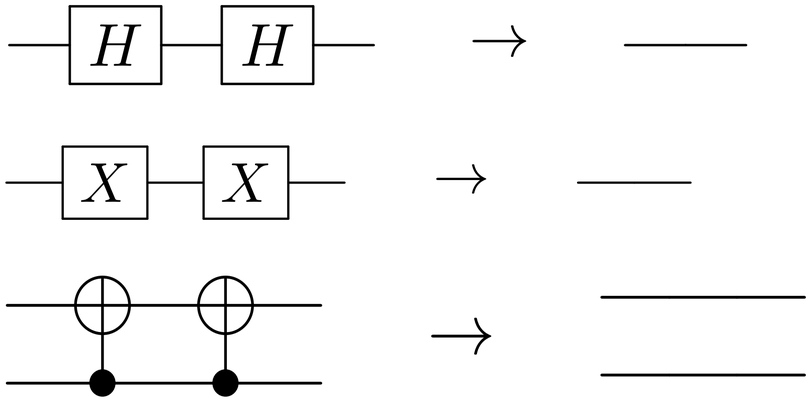


Commute

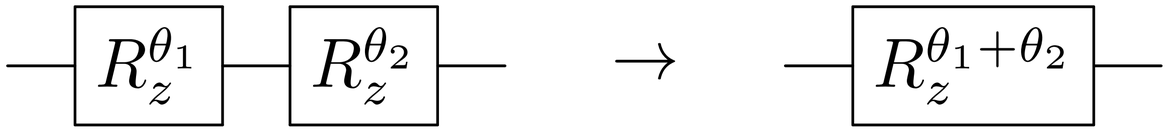


Simple Rewrite Rules

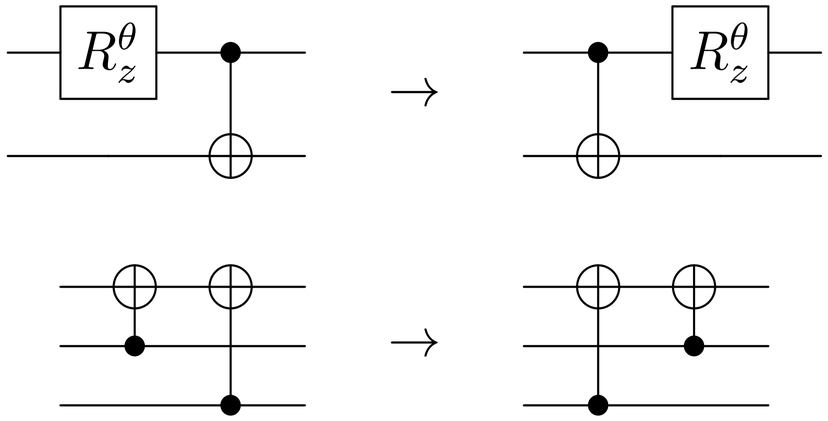
Cancel



Merge



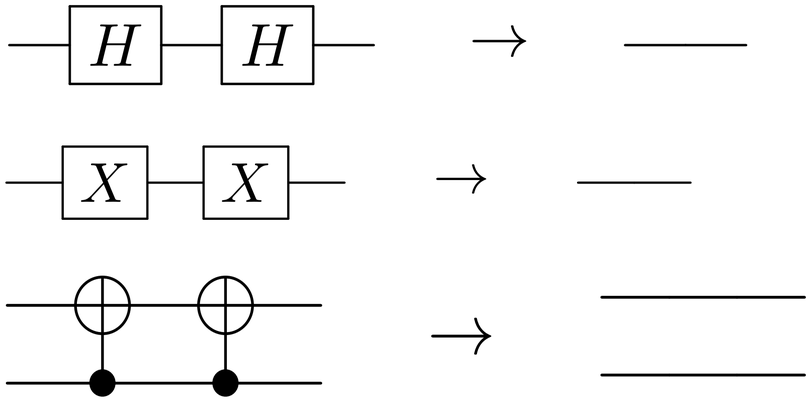
Commute



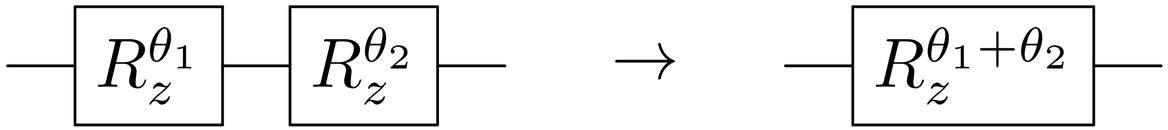
Global Phase

Simple Rewrite Rules

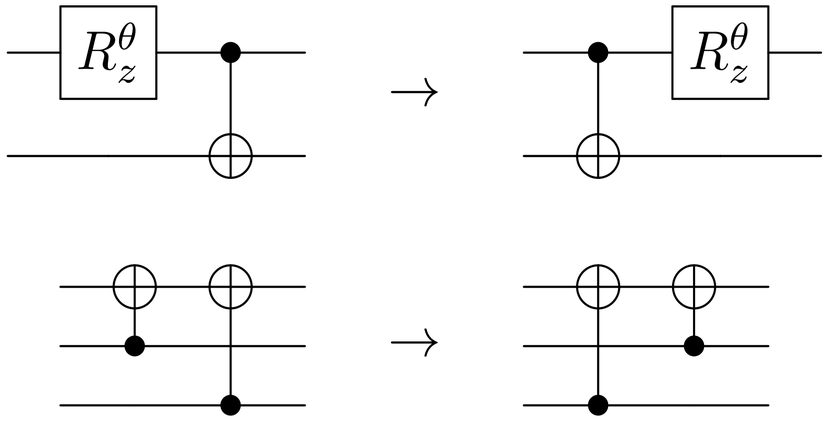
Cancel



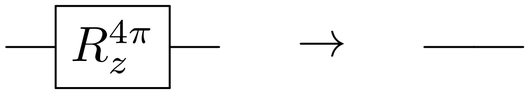
Merge



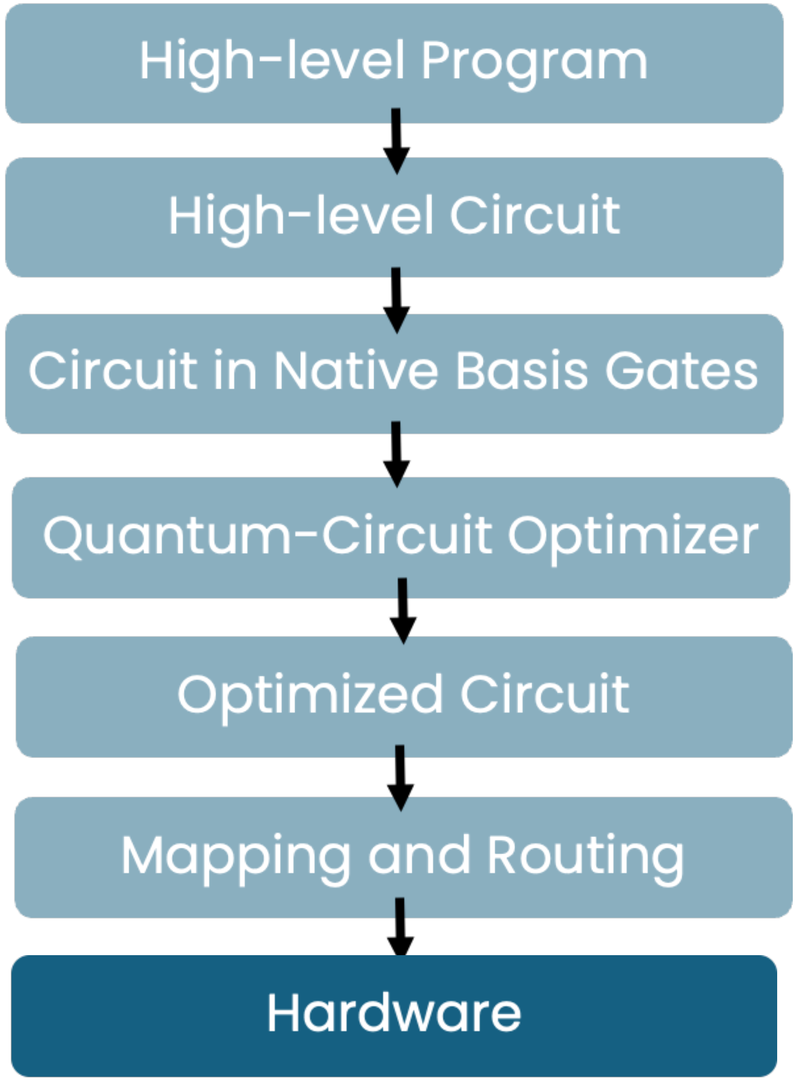
Commute



Global Phase



Diverse Hardware



Diverse Hardware

Hardware

Superconducting

Google
Quantum AI

Diverse Hardware

Hardware

Superconducting

Google Quantum AI

Neutral Atom

Diverse Hardware

Ion Trap

Hardware

Superconducting

Neutral Atom

Diverse Hardware

Ion Trap

IONQ

Neutral Atom

rigetti

OQC

QuEra
COMPUTING INC.

Google Quantum AI

IBM

aws

Diverse Hardware

Ion Trap

Topological

Hardware

Superconducting

Neutral Atom

Diverse Hardware

Ion Trap

Topological

Hardware

Superconducting

Google Quantum AI

U1, U2, U3, CX

Neutral Atom

Diverse Hardware

Ion Trap

Topological

Hardware

Superconducting

Google Quantum AI

Neutral Atom

~~U1, U2, U3, CX~~ → Rz, SX, X, CX

Diverse Hardware

Ion Trap

Topological

Hardware

Superconducting

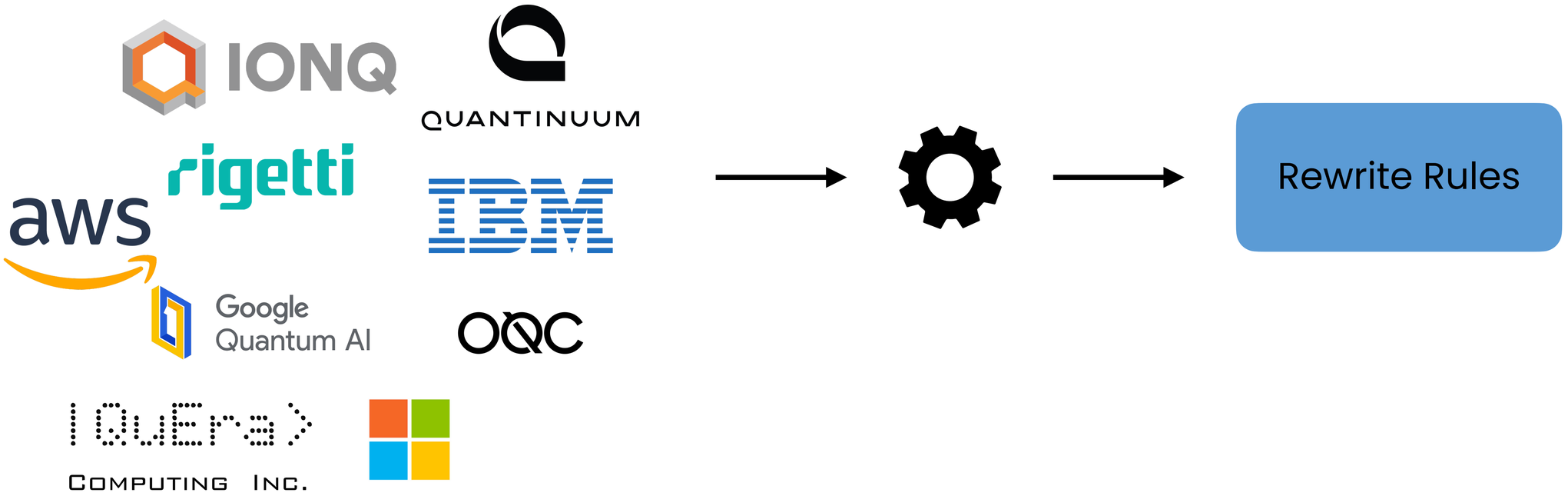
Neutral Atom

~~U1, U2, U3, CX~~ → ~~Rz, SX, X, CX~~

→ Rz, Rx, SX, X, CZ, Rzz

Synthesis!

Arbitrary gate set



Naïve Synthesis

```
rules = []
```

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circuits = enumerate(max_qubits, max_size)
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big

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expensive

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```

big

uh oh...

expensive

QUESO

Synthesizing Quantum-Circuit Optimizers

AMANDA XU, University of Wisconsin-Madison, USA
ABTIN MOLAVI, University of Wisconsin-Madison, USA
LAUREN PICK, University of Wisconsin-Madison, USA
SWAMIT TANNU, University of Wisconsin-Madison, USA
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(Symbolic)
Circuit

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(Symbolic) Circuit → Polynomial

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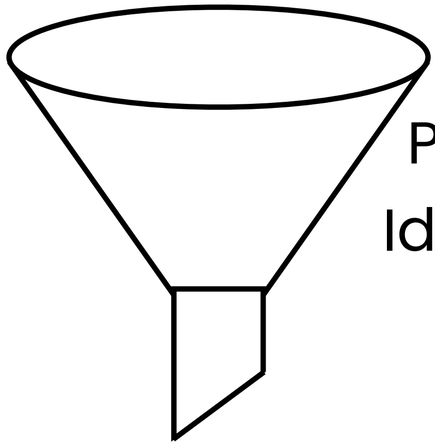
(Symbolic) Circuit → Polynomial →  Polynomial Identity Filter (PIF) → Circuit Equivalence Classes

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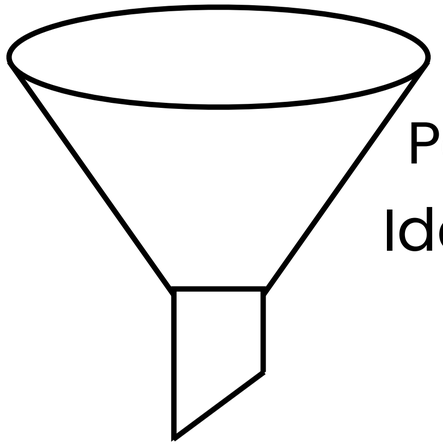
(Symbolic) Circuit



Polynomial Identity Filter (PIF)

Polynomial

Circuit Equivalence Classes

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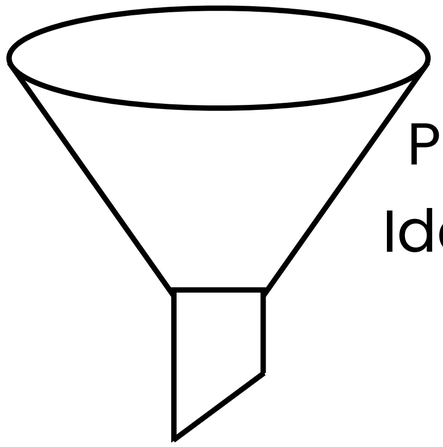
QUESO

Feynman's path integral formulation for quantum mechanics

(Symbolic) Circuit



Polynomial Identity Filter (PIF)

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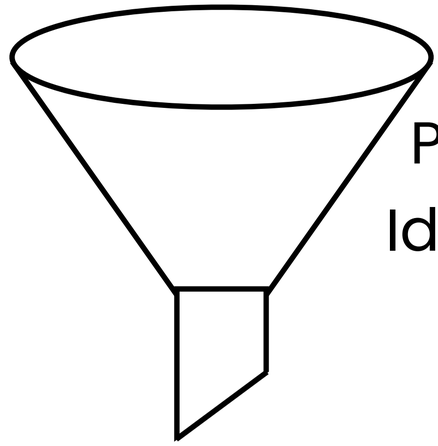
QUESO

Schwartz-Zippel Lemma for polynomial identity testing (PIT)

Feynman's path integral formulation for quantum mechanics

(Symbolic) Circuit



Polynomial Identity Filter (PIF)

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Circuit Equivalence Classes

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QUESO

Feynman's path integral formulation

for quantum mechanics

(Symbolic) Circuit

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Path-sum Semantics

Algebraically represent semantics of quantum gates

$$R_z(\theta) : |x\rangle \to e^{i(2x-1)\theta}|x\rangle$$

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More generally: $|\mathbf{x}\rangle \rightarrow \phi(\mathbf{x}, \rho)\,|f(\mathbf{x})\rangle$

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amplitude state

More generally: $|\mathbf{x}\rangle \rightarrow \phi(\mathbf{x}, \rho)\,|f(\mathbf{x})\rangle$

Simple Circuit as Polynomial

Circuit	Polynomial
`Rz(`θ`) q1;`	$e^{i(2(0)-1)\theta}\|0\rangle$

Simple Circuit as Polynomial

Circuit	Polynomial
`Rz(`θ`) q1;`	$e^{-i\theta}\|0\rangle$

Simple Circuit as Polynomial

Circuit	Polynomial
`Rz(`θ`)  q1;`	$e^{-i\theta}\|0\rangle + e^{i\theta}\|1\rangle$

Simple Circuit as Polynomial

Circuit

`Rz(`θ`)  q1;`

Polynomial

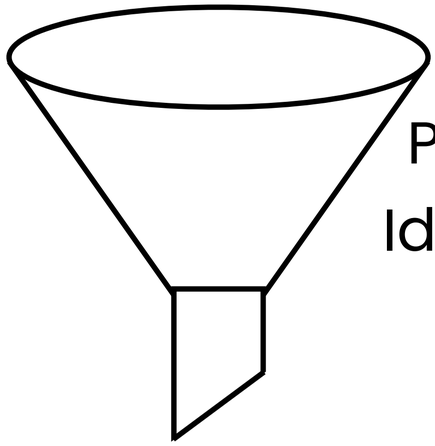
$e^{-i\theta}|0\rangle + e^{i\theta}|1\rangle$

QUESO

Schwartz-Zippel Lemma for polynomial identity testing (PIT)

Feynman's path integral formulation for quantum mechanics

(Symbolic) Circuit → Polynomial →



Polynomial Identity Filter (PIF)

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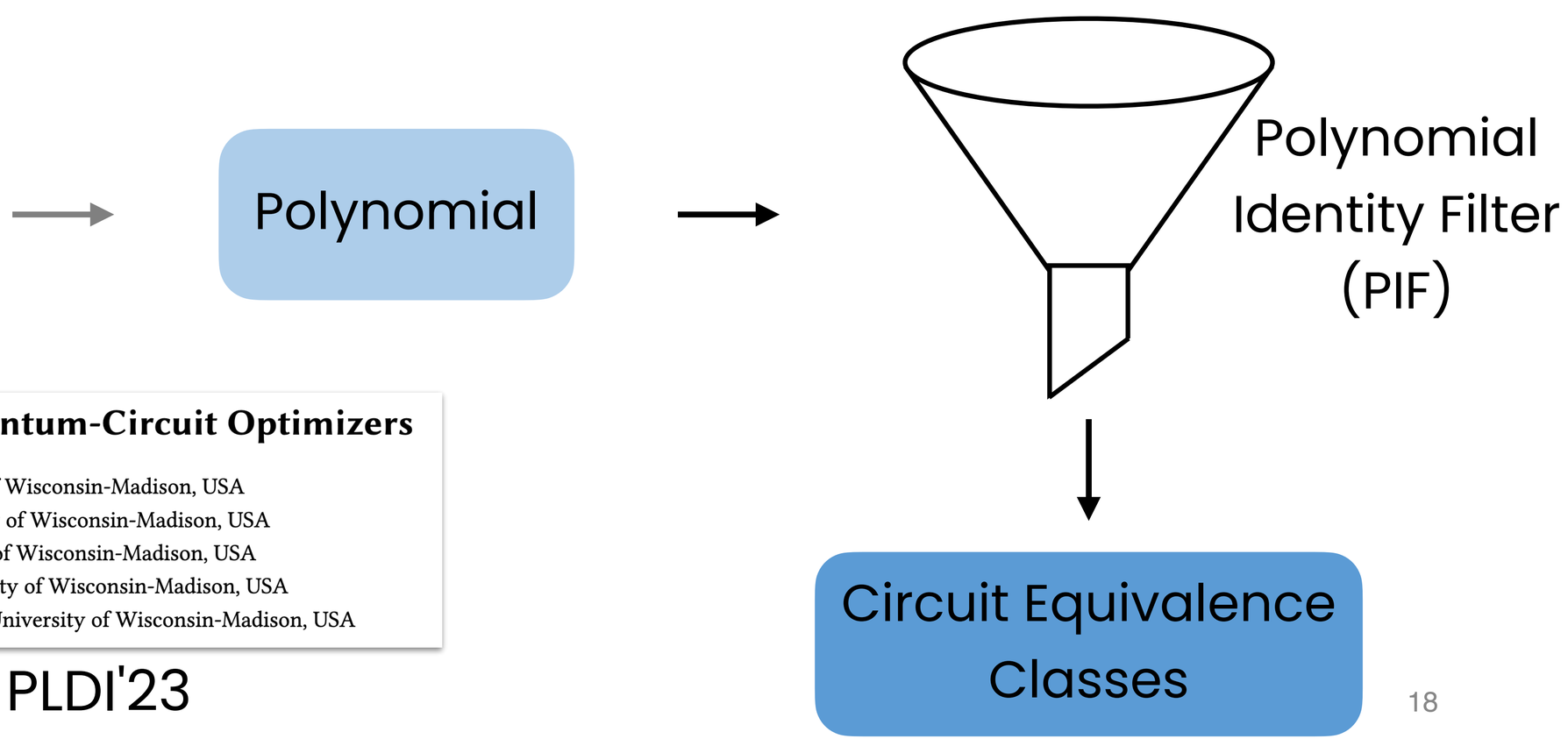
Circuit Equivalence Classes

QUESO

Schwartz-Zippel Lemma for polynomial identity testing (PIT)



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Polynomial Identity Testing (PIT)

$$\mathbf{x} \in \mathbb{C}^3 \qquad p_1(\mathbf{x}) = x_1 x_2$$

Polynomial Identity Testing (PIT)

$$\mathbf{x} \in \mathbb{C}^3 \qquad p_1(\mathbf{x}) = x_1 x_2$$

$$p_2(\mathbf{x}) = x_1 x_2 + x_3 x_2$$

Polynomial Identity Testing (PIT)

$$\mathbf{x} \in \mathbb{C}^3 \qquad p_1(\mathbf{x}) = x_1 x_2$$
$$p_2(\mathbf{x}) = x_1 x_2 + x_3 x_2$$

How can we check $p_1 \equiv p_2$?

Schwartz-Zippel Lemma

$$p_1(\mathbf{x}) = x_1 x_2$$

$$p_2(\mathbf{x}) = x_1 x_2 + x_3 x_2$$

Schwartz-Zippel Lemma

$$p_1(\mathbf{x}) = x_1 x_2$$

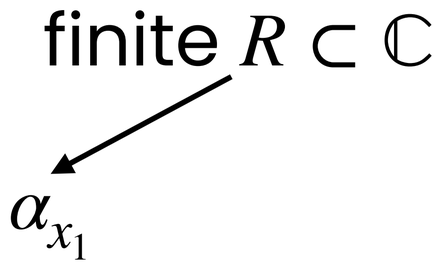
$$p_2(\mathbf{x}) = x_1 x_2 + x_3 x_2$$

finite $R \subset \mathbb{C}$

Schwartz-Zippel Lemma

$$p_1(\mathbf{x}) = x_1 x_2$$

$$p_2(\mathbf{x}) = x_1 x_2 + x_3 x_2$$



Schwartz-Zippel Lemma

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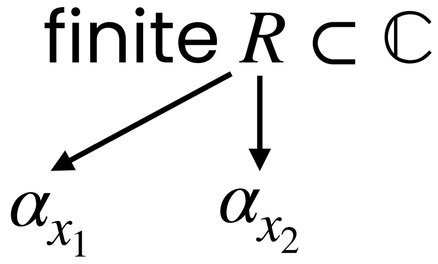
$$p_2(\mathbf{x}) = x_1 x_2 + x_3 x_2$$



Schwartz-Zippel Lemma

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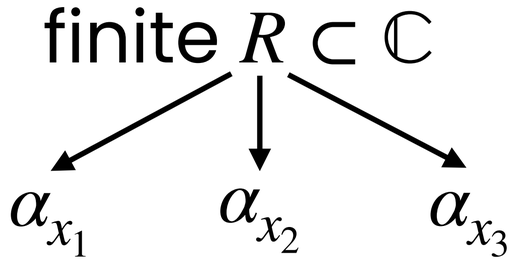

Schwartz-Zippel Lemma

$$p_1(\boldsymbol{\alpha}) = \alpha_{x_1}\alpha_{x_2}$$
$$p_2(\boldsymbol{\alpha}) = \alpha_{x_1}\alpha_{x_2} + \alpha_{x_3}\alpha_{x_2}$$

Schwartz-Zippel Lemma

$$p_1(\boldsymbol{\alpha}) = \alpha_{x_1}\alpha_{x_2}$$
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Check $p_1(\boldsymbol{\alpha}) = p_2(\boldsymbol{\alpha})$

Schwartz-Zippel Lemma

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Check $p_1(\boldsymbol{\alpha}) = p_2(\boldsymbol{\alpha})$

If $p_1 \neq p_2$, then the probability that $p_1(\boldsymbol{\alpha}) = p_2(\boldsymbol{\alpha})$ is at most $\frac{d}{|R|}$ (d (max total degree))

Schwartz-Zippel Lemma

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$$p_2(\boldsymbol{\alpha}) = \alpha_{x_1}\alpha_{x_2} + \alpha_{x_3}\alpha_{x_2}$$

Check $p_1(\boldsymbol{\alpha}) = p_2(\boldsymbol{\alpha})$

If $p_1 \neq p_2$, then the probability that $p_1(\boldsymbol{\alpha}) = p_2(\boldsymbol{\alpha})$ is at most $\frac{d}{|R|}$

small

d (max total degree)

very large

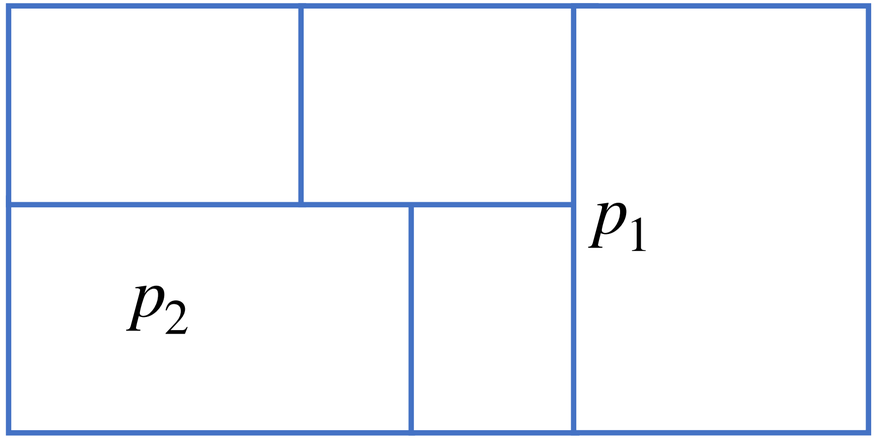
Polynomial Identity Filter (PIF)

Insert p_3

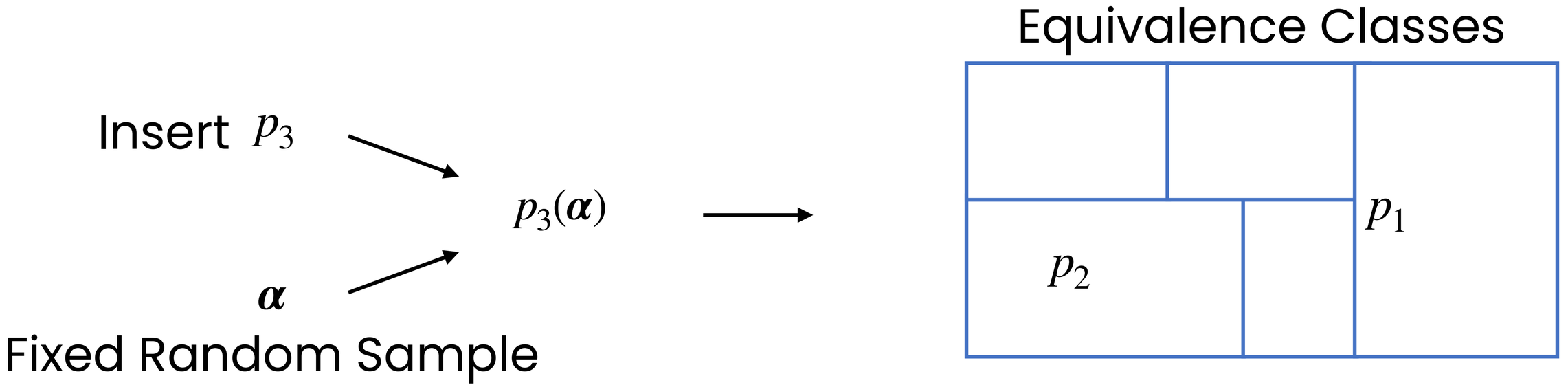
$\boldsymbol{\alpha}$

Fixed Random Sample

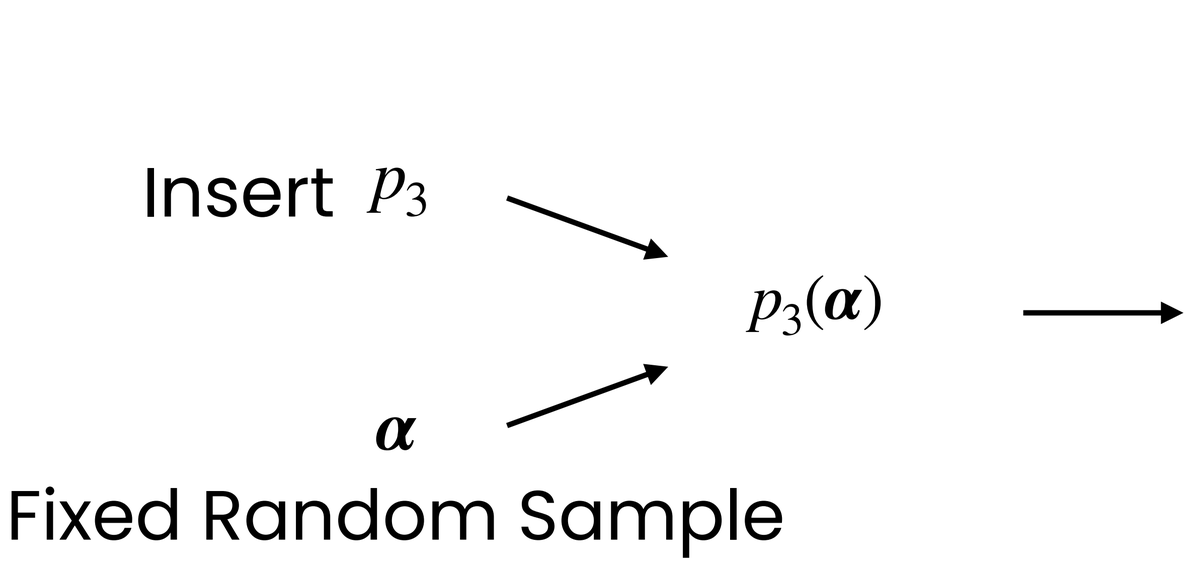
Equivalence Classes

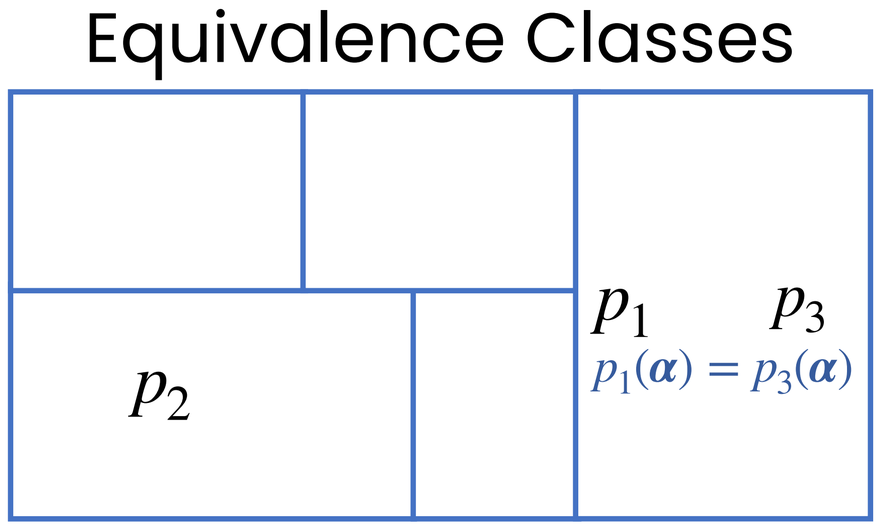


Polynomial Identity Filter (PIF)



Polynomial Identity Filter (PIF)





QUESO's Improvements

```
rules = []
circuits = enumerate(max_qubits, max_size)
for c1 in circuits:
  for c2 in circuits:
    if verify(c1, c2):
      rules.append(c1 → c2)
```

big

uh oh...

expensive

QUESO's Improvements

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rules = []
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uh oh...

expensive

Efficient equivalence check between circuit pairs!

QUESO's Improvements

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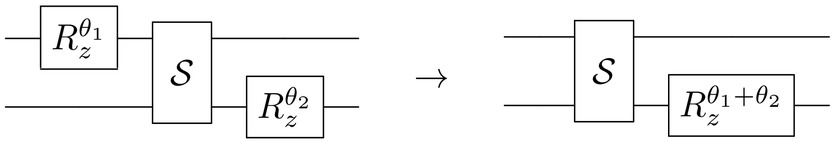
            rules.append(c1 → c2)
```

big

uh oh...

expensive

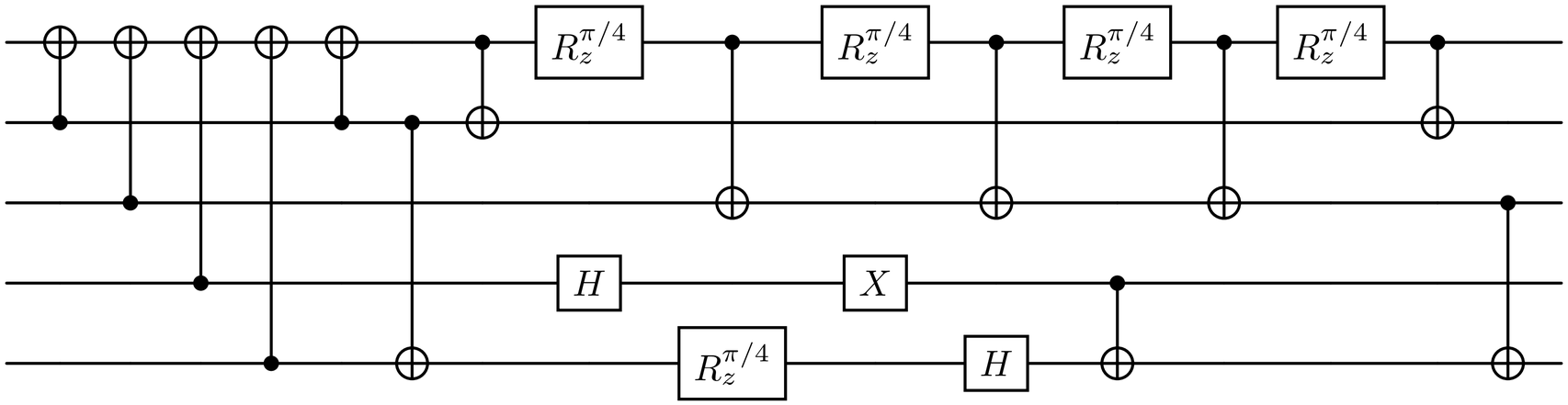
Efficient equivalence check between circuit pairs!

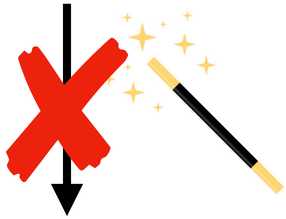
bonus: 

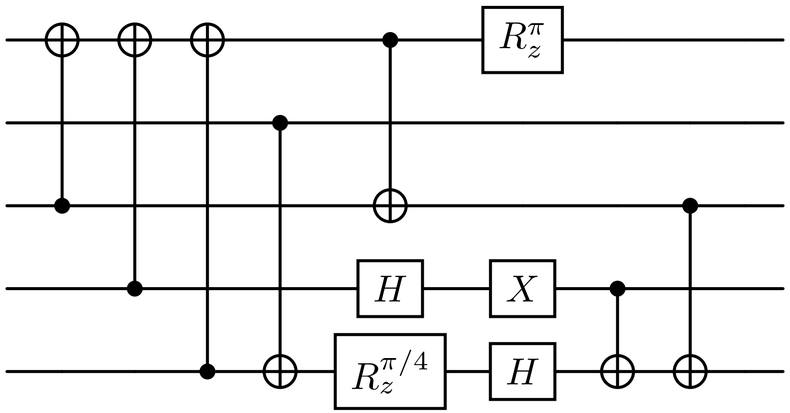
(advanced symbolic rules)

Circuit Resynthesis

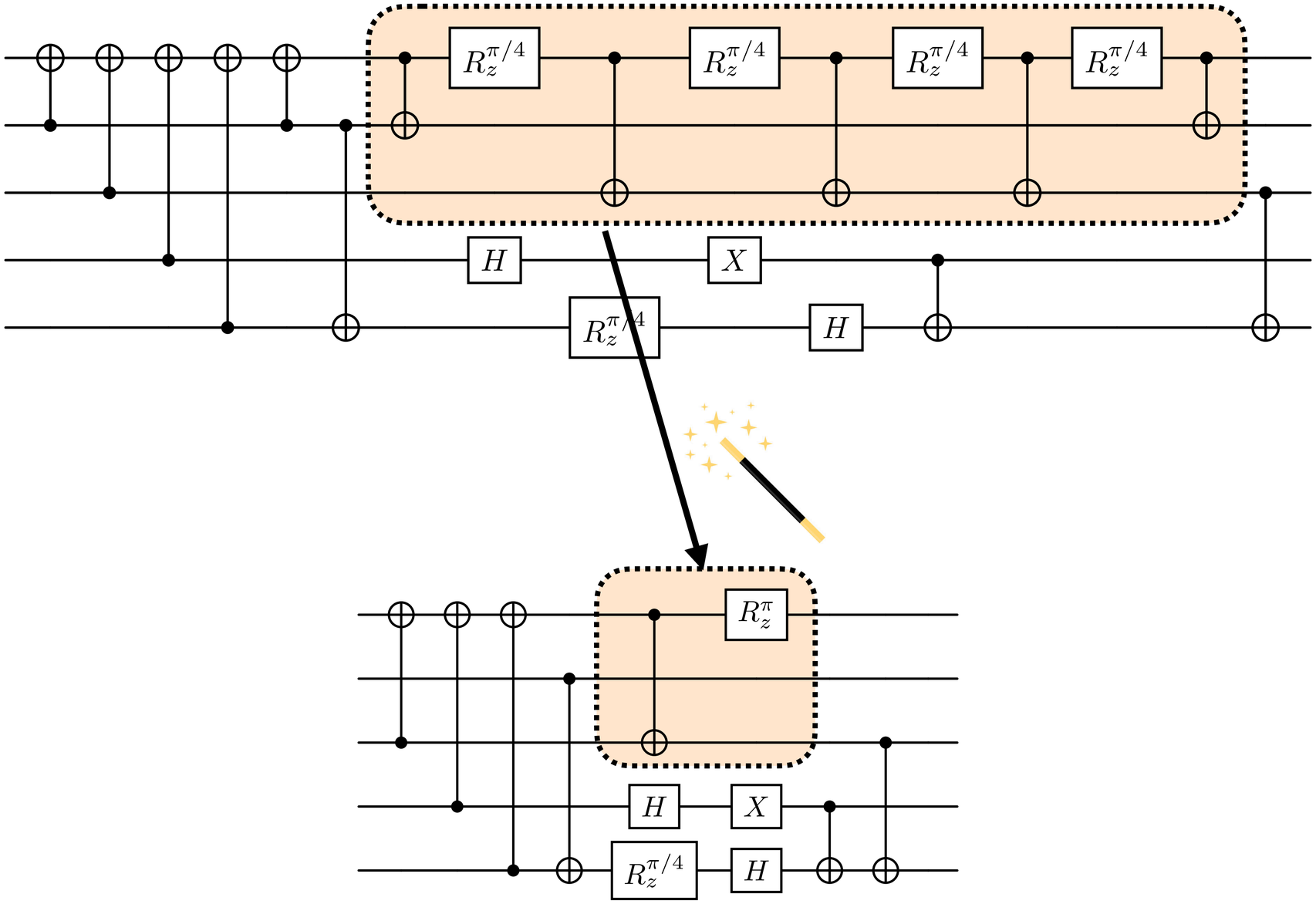
Circuit Resynthesis



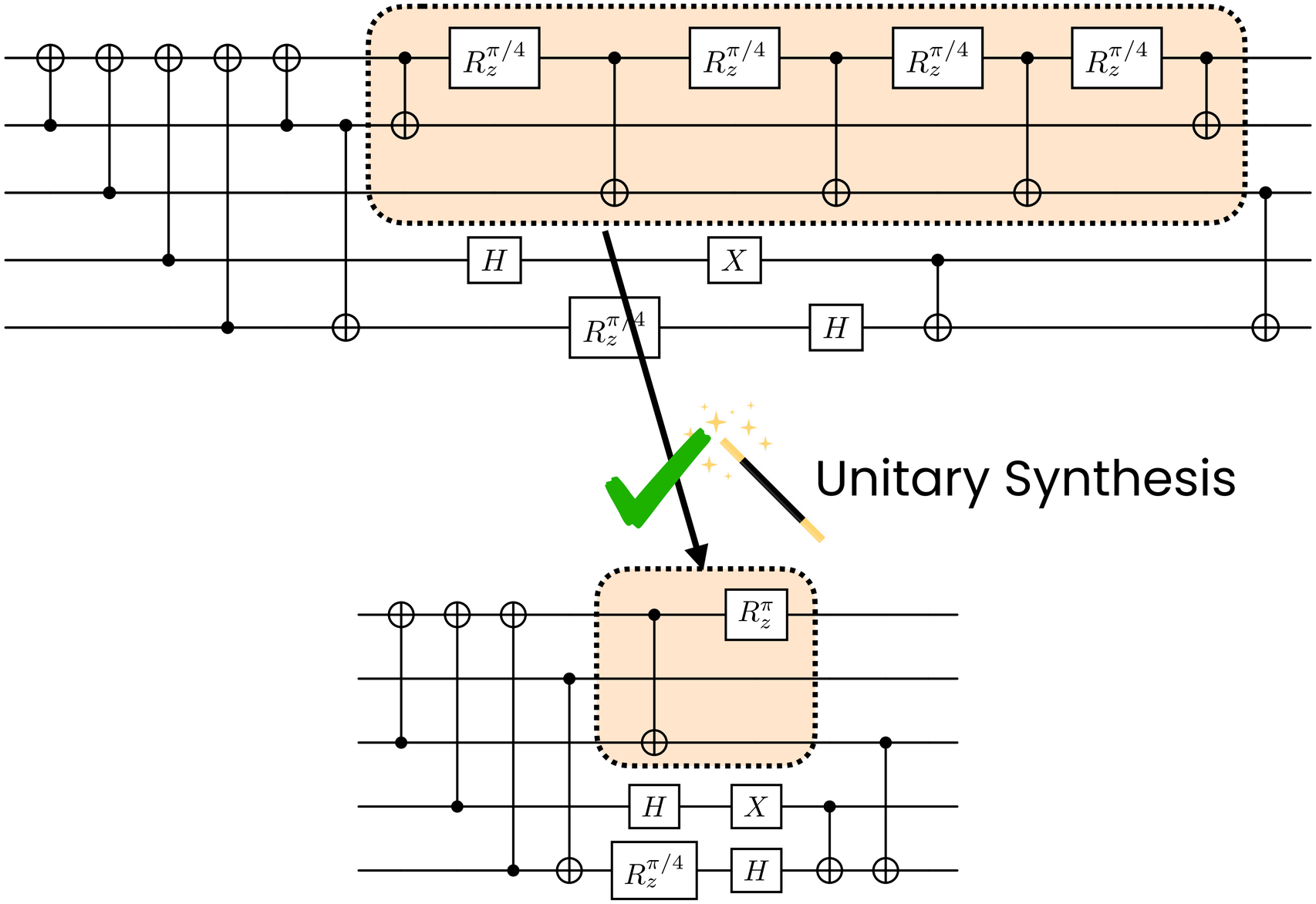




Circuit Resynthesis

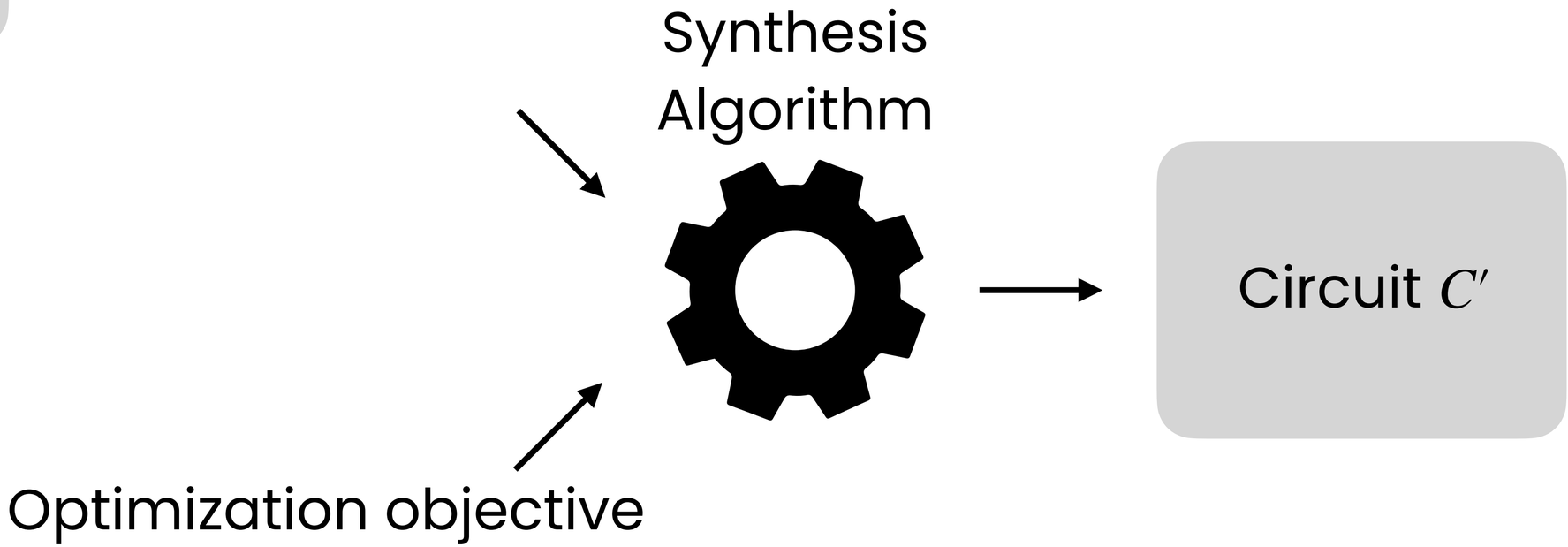


Circuit Resynthesis

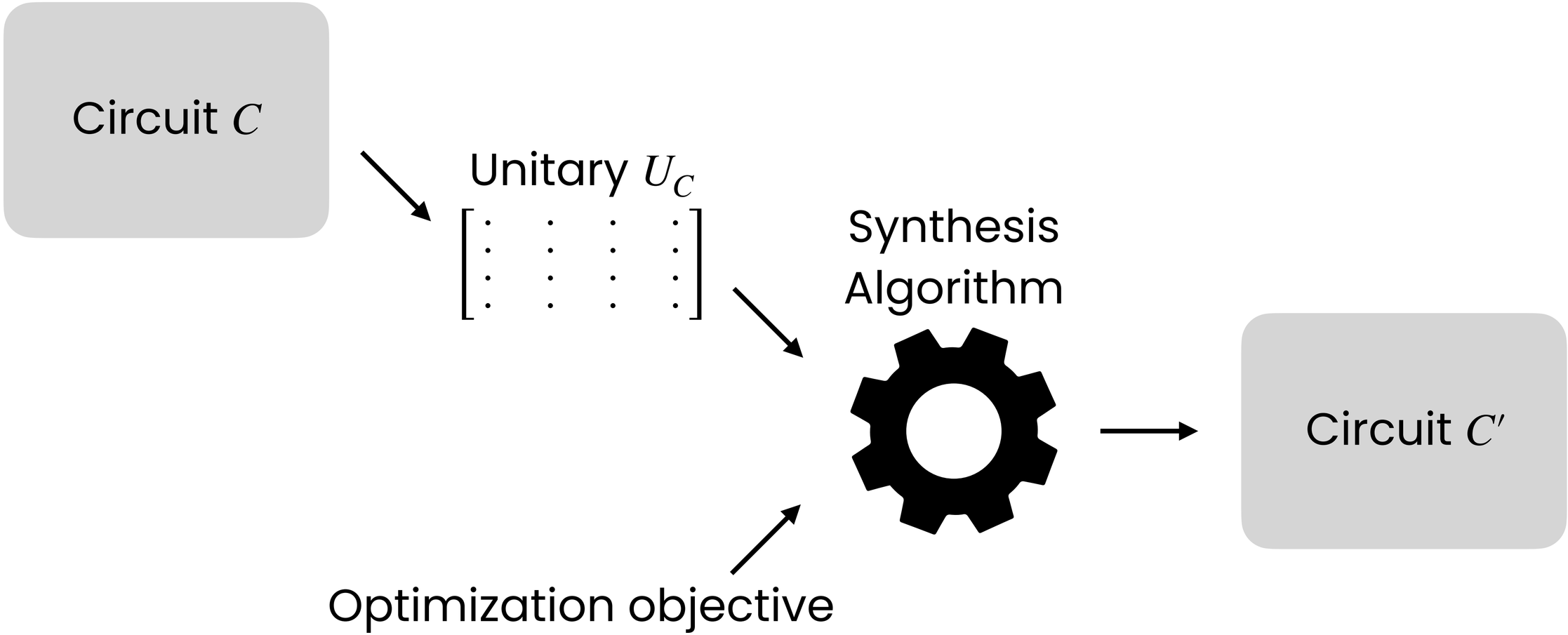


Unitary Synthesis

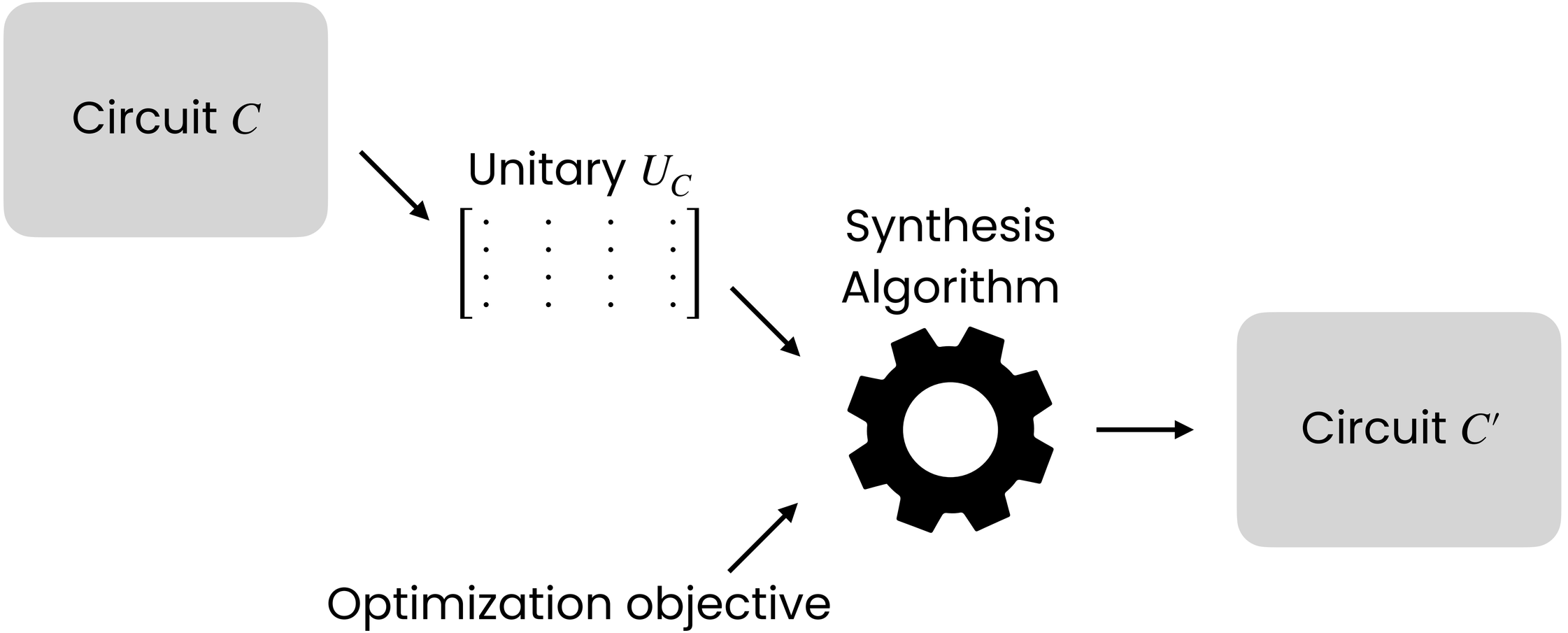
Circuit C



Unitary Synthesis



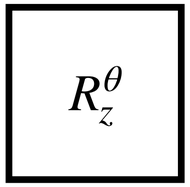
Unitary Synthesis



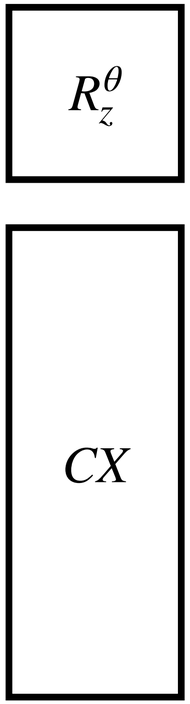
Guarantee $C \equiv C'$

Circuit to Unitary

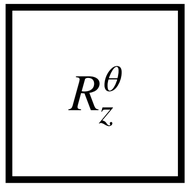
Circuit to Unitary



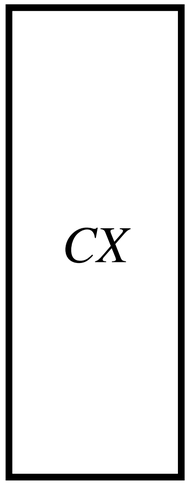
Circuit to Unitary



Circuit to Unitary



$$U_{R_z^\theta} := \begin{bmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{bmatrix}$$



Circuit to Unitary

R_z^θ

$$U_{R_z^\theta} := \begin{bmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{bmatrix}$$

CX

$$U_{CX} := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Circuit to Unitary

R_z^θ

$$U_{R_z^\theta} := \begin{bmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{bmatrix}$$

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$$U_{CX} := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

I

$$U_I := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Circuit to Unitary

R_z^θ

$$U_{R_z^\theta} := \begin{bmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{bmatrix}$$

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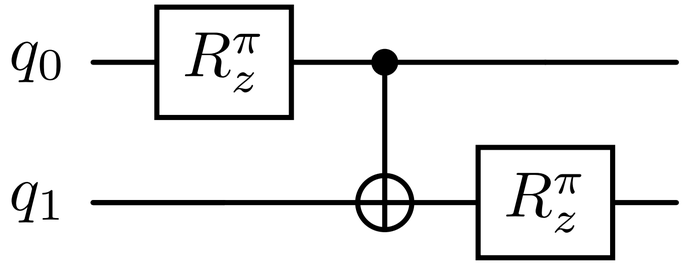
I

$$U_I := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$C :=$



Circuit to Unitary

R_z^θ

$$U_{R_z^\theta} := \begin{bmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{bmatrix}$$

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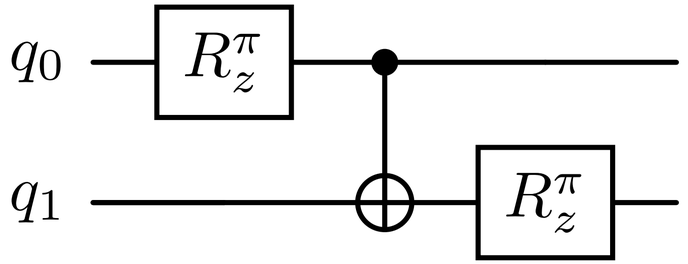
$$U_I := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$C :=$



$U_C =$ $U_{R_z^\pi}$

Circuit to Unitary

R_z^θ

$$U_{R_z^\theta} := \begin{bmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{bmatrix}$$

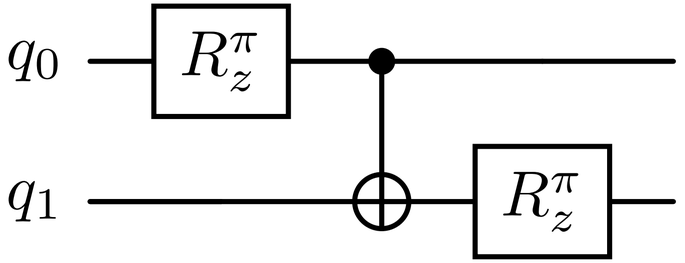
CX

$$U_{CX} := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

I

$$U_I := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$C := \quad q_0 \quad q_1$$



$$U_C = \quad U_{R_z^\pi}$$

$2^2 \times 2^2$

Circuit to Unitary

R_z^θ

$$U_{R_z^\theta} := \begin{bmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{bmatrix}$$

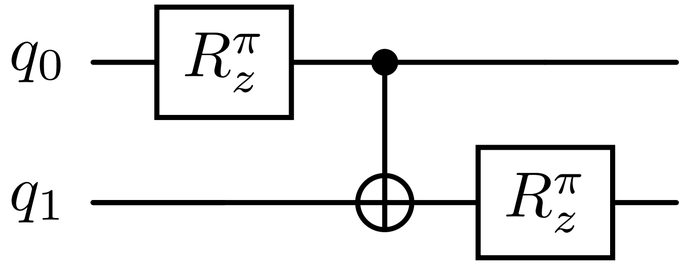
CX

$$U_{CX} := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

I

$$U_I := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$C :=$



$U_C =$
$2^2 \times 2^2$

$U_{R_z^\pi}$
2×2

Circuit to Unitary

R_z^θ

$$U_{R_z^\theta} := \begin{bmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{bmatrix}$$

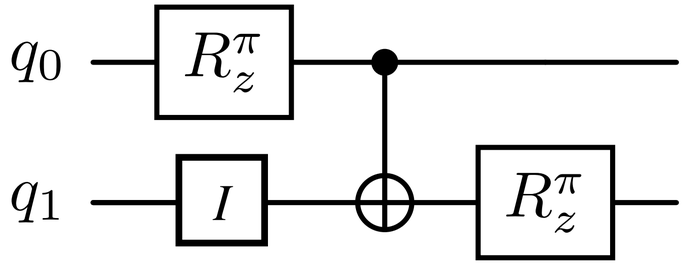
CX

$$U_{CX} := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

I

$$U_I := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$C :=$



$U_C =$
$2^2 \times 2^2$

$U_{R_z^\pi}$
2×2

Circuit to Unitary

R_z^θ

$$U_{R_z^\theta} := \begin{bmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{bmatrix}$$

CX

$$U_{CX} := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

I

$$U_I := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$C := \quad q_0 \text{ — } R_z^\pi \text{ — } \bullet \text{ — }$$
$$\qquad\quad q_1 \text{ — } I \text{ — } \oplus \text{ — } R_z^\pi \text{ — }$$

$$U_C = (U_{R_z^\pi} \otimes U_I)$$
$$2^2 \times 2^2 \qquad 2 \times 2$$

Circuit to Unitary

R_z^θ

$$U_{R_z^\theta} := \begin{bmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{bmatrix}$$

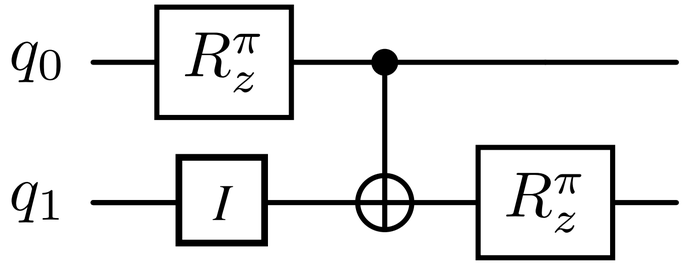
CX

$$U_{CX} := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

I

$$U_I := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$C :=$$



$$U_C = \underbrace{(U_{R_z^\pi} \otimes U_I)}_{2^2 \times 2^2}$$

$$\underbrace{2 \times 2 \quad 2 \times 2}_{2^2 \times 2^2}$$

Circuit to Unitary

R_z^θ

$$U_{R_z^\theta} := \begin{bmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{bmatrix}$$

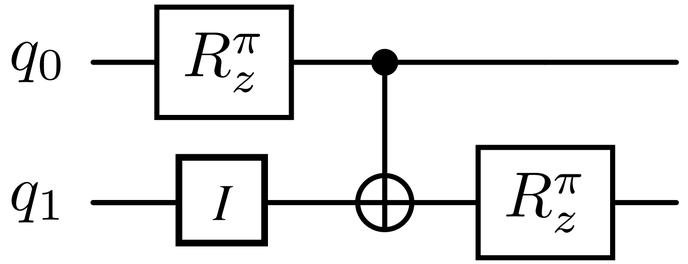
CX

$$U_{CX} := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

I

$$U_I := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$C :=$$



$$U_C = \underbrace{(U_{R_z^\pi} \otimes U_I)}_{2^2 \times 2^2}$$
$$\underbrace{2 \times 2 \quad 2 \times 2}$$

$$(U_{R_z^\pi} \otimes U_I) = \begin{bmatrix} e^{-i\frac{\pi}{2}} \cdot U_I & 0 \cdot U_I \\ 0 \cdot U_I & e^{i\frac{\pi}{2}} \cdot U_I \end{bmatrix}$$

Circuit to Unitary

R_z^θ

$$U_{R_z^\theta} := \begin{bmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{bmatrix}$$

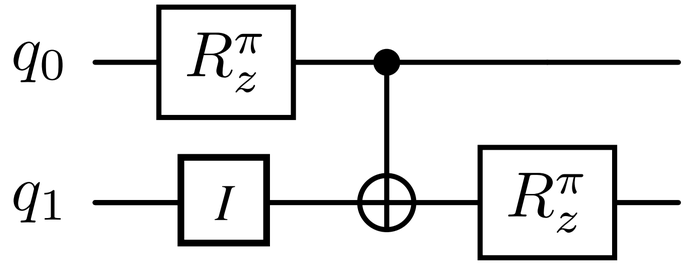
CX

$$U_{CX} := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

I

$$U_I := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$C := \begin{array}{l} q_0 \\ q_1 \end{array} \quad \Qquad$$



$$U_C = \underset{2^2 \times 2^2}{(U_{R_z^\pi} \otimes U_I)}$$

$$\underbrace{\overset{2\times 2}{\quad} \quad \overset{2\times 2}{\quad}}_{2^2 \times 2^2}$$

$$(U_{R_z^\pi} \otimes U_I) = \begin{bmatrix} \overset{2\times 2}{e^{-i\frac{\pi}{2}} \cdot U_I} & \overset{2\times 2}{0 \cdot U_I} \\ \underset{2\times 2}{0 \cdot U_I} & \underset{2\times 2}{e^{i\frac{\pi}{2}} \cdot U_I} \end{bmatrix}$$

Circuit to Unitary

R_z^θ

$$U_{R_z^\theta} := \begin{bmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{bmatrix}$$

CX

$$U_{CX} := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

I

$$U_I := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$C := \begin{array}{l} q_0 \\ q_1 \end{array} \quad \begin{array}{c} R_z^\pi \\ I \end{array} \quad \bullet \quad \oplus \quad R_z^\pi$$

$$U_C = U_{CX} \cdot (U_{R_z^\pi} \otimes U_I)$$

$$2^2 \times 2^2 \qquad \underbrace{2 \times 2 \quad 2 \times 2}_{2^2 \times 2^2}$$

$$(U_{R_z^\pi} \otimes U_I) = \begin{bmatrix} \overset{2\times 2}{e^{-i\frac{\pi}{2}} \cdot U_I} & \overset{2\times 2}{0 \cdot U_I} \\ \underset{2\times 2}{0 \cdot U_I} & \underset{2\times 2}{e^{i\frac{\pi}{2}} \cdot U_I} \end{bmatrix}$$

Circuit to Unitary

R_z^θ

$$U_{R_z^\theta} := \begin{bmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{bmatrix}$$

CX

$$U_{CX} := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

I

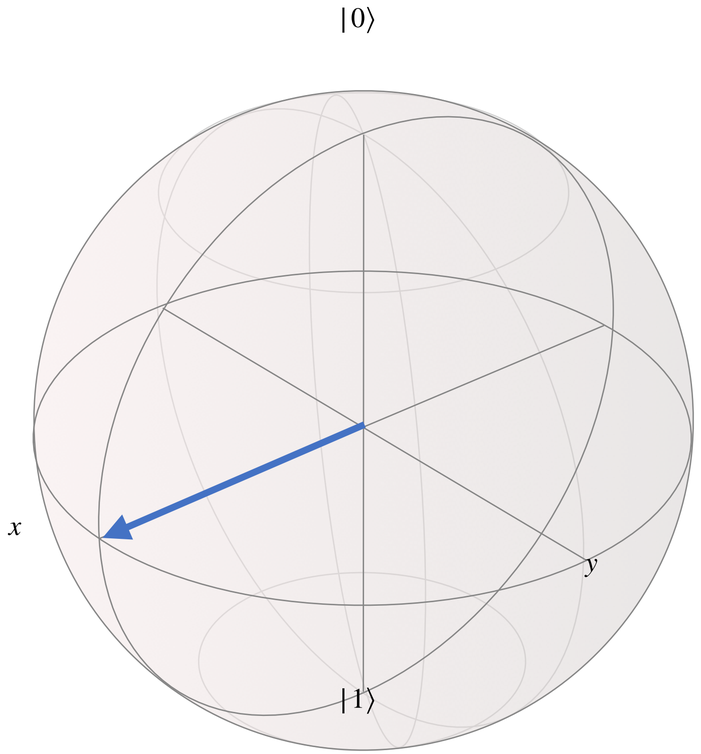
$$U_I := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$C := \begin{array}{l} q_0 \\ q_1 \end{array} \quad \Qcircuit @C=1em @R=1em { & \gate{R_z^\pi} & \ctrl{1} & \gate{I} \\ & \gate{I} & \targ & \gate{R_z^\pi} }$$

$$\underset{2^2 \times 2^2}{U_C} = \underbrace{(U_I \otimes U_{R_z^\pi}) \cdot U_{CX} \cdot (U_{R_z^\pi} \otimes U_I)}_{2^2 \times 2^2}$$

$$(U_{R_z^\pi} \otimes U_I) = \begin{bmatrix} \overset{2\times 2}{e^{-i\frac{\pi}{2}} \cdot U_I} & \overset{2\times 2}{0 \cdot U_I} \\ \underset{2\times 2}{0 \cdot U_I} & \underset{2\times 2}{e^{i\frac{\pi}{2}} \cdot U_I} \end{bmatrix}$$

Approximate Circuit Equivalence



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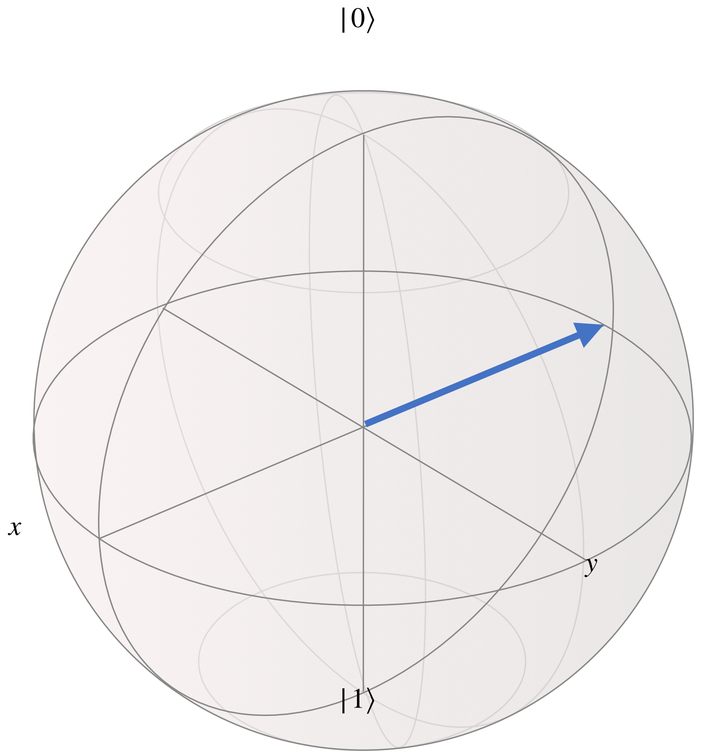

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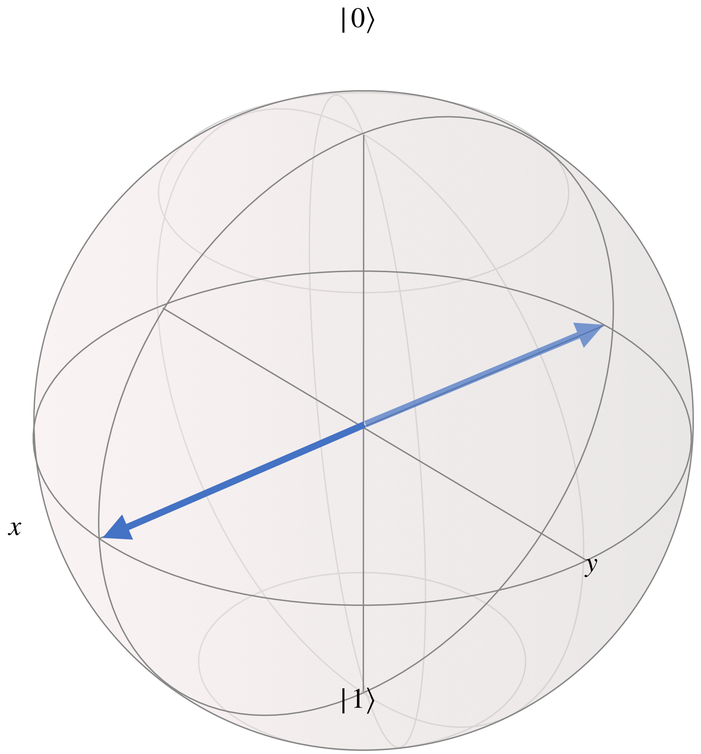

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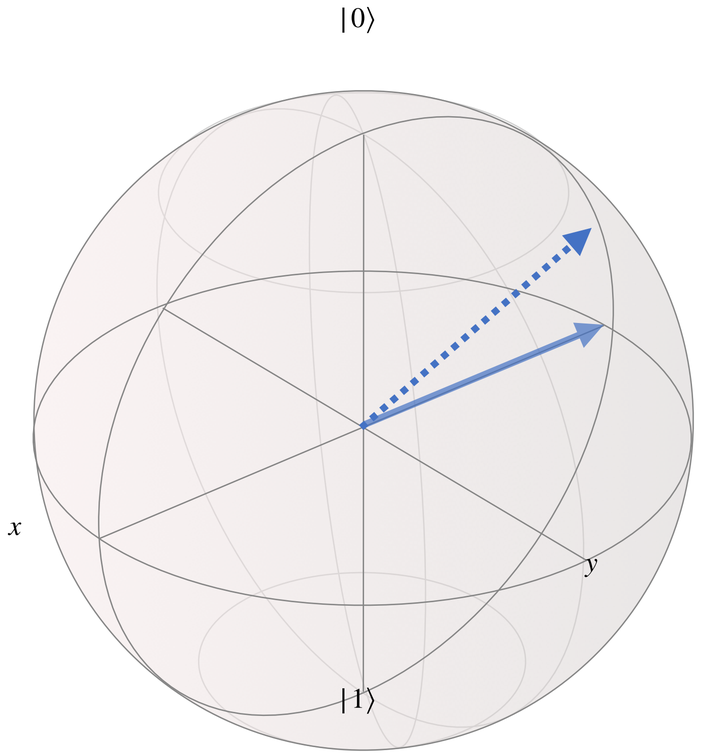

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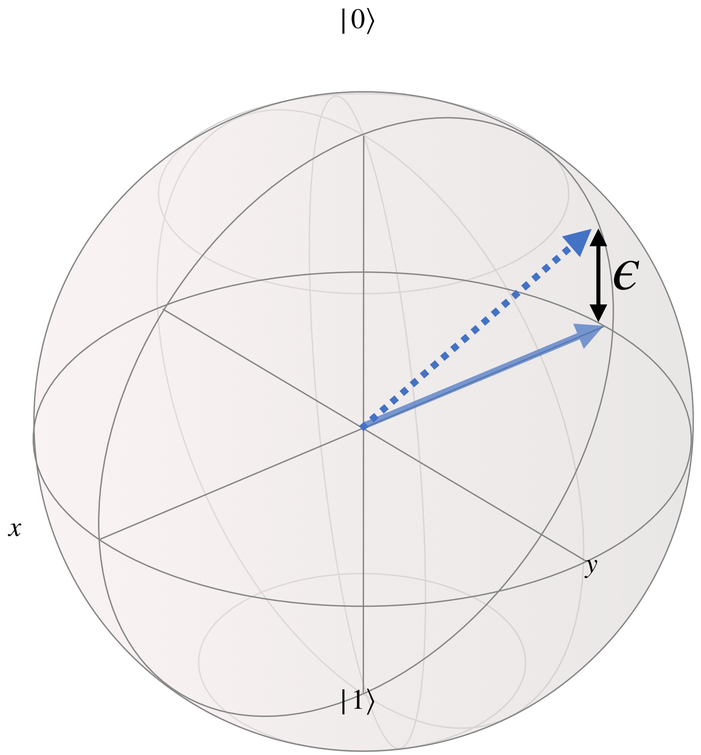

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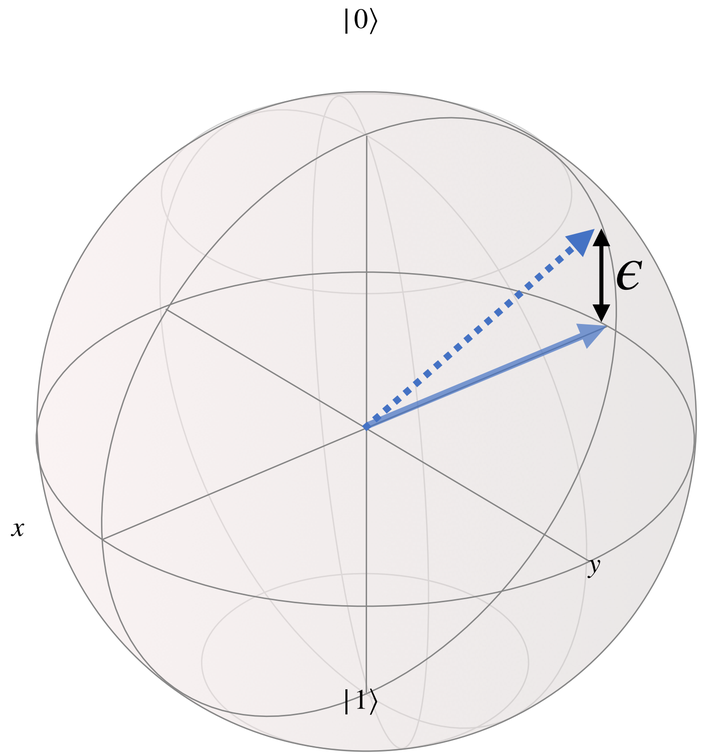

$$\Delta(U_C, U_{C'}) \leq \epsilon \iff C \equiv_\epsilon C'$$

Hilbert-Schmidt Distance

"easy" to compute

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$$\Delta_{HS}(U, U') := \sqrt{1 - \frac{|\,Tr(U^\dagger U')\,|^2}{2^{2n}}}$$

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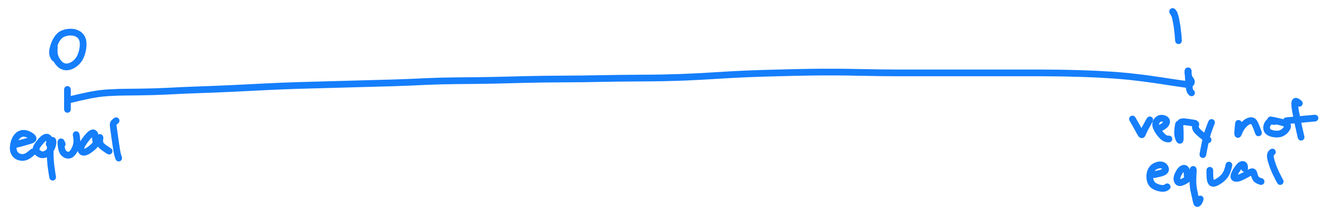

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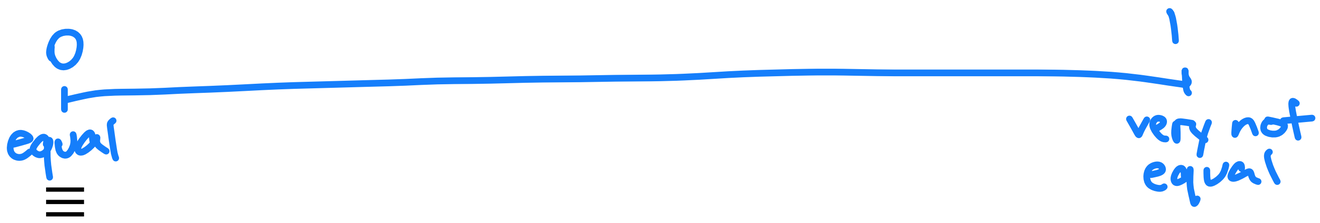

≡

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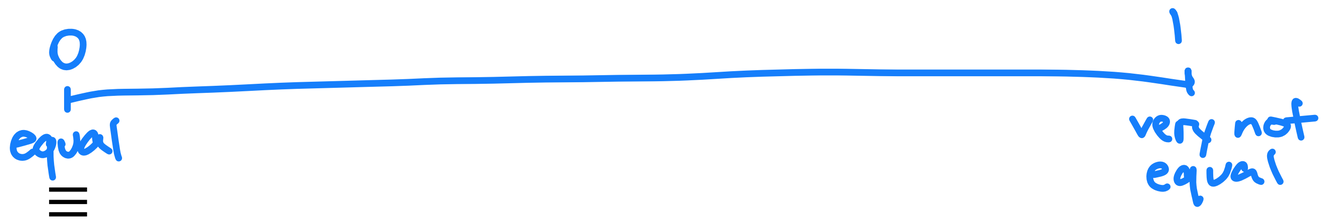

Upper bound on total variation distance

Why Approximate Circuits?

Ideal

Circuit C

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Ideal

Circuit C + No noise

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Circuit C + No noise = U_C

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Circuit C + Noise = $U_{C_{noisy}}$

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Noise from k gates $> \epsilon$

Why Approximate Circuits?

FTQC **requires** approximations for arbitrary angle rotations

Ideal

Circuit C + No noise = U_C

Reality

Circuit C + Noise = $U_{C_{noisy}}$

Circuit $C' \equiv_\epsilon C$

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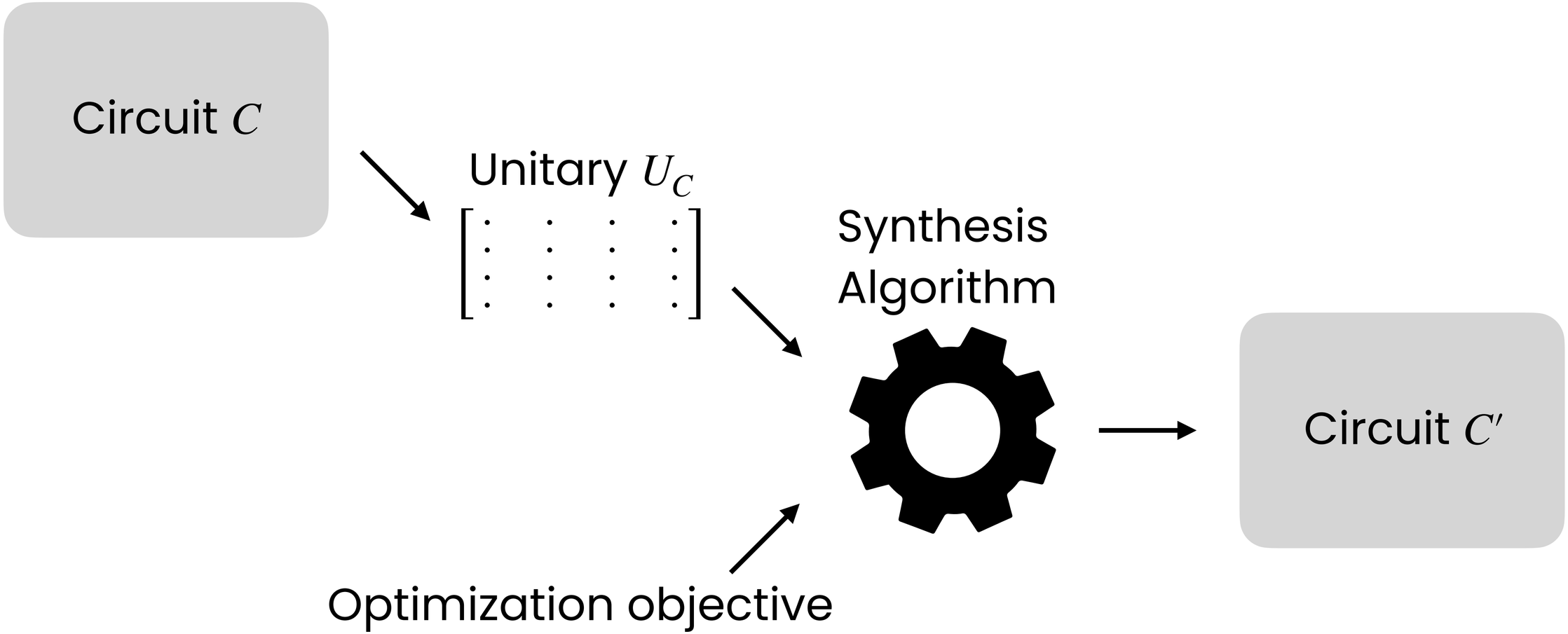
Unitary Synthesis



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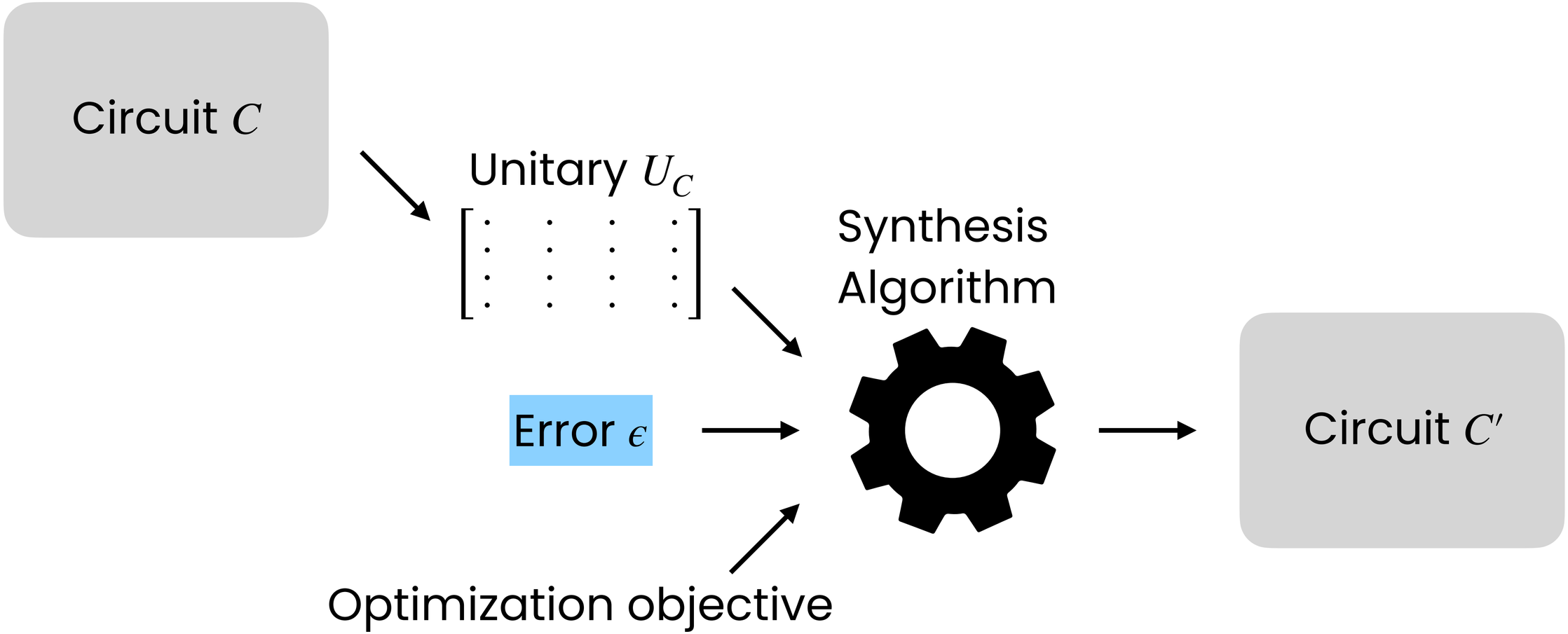

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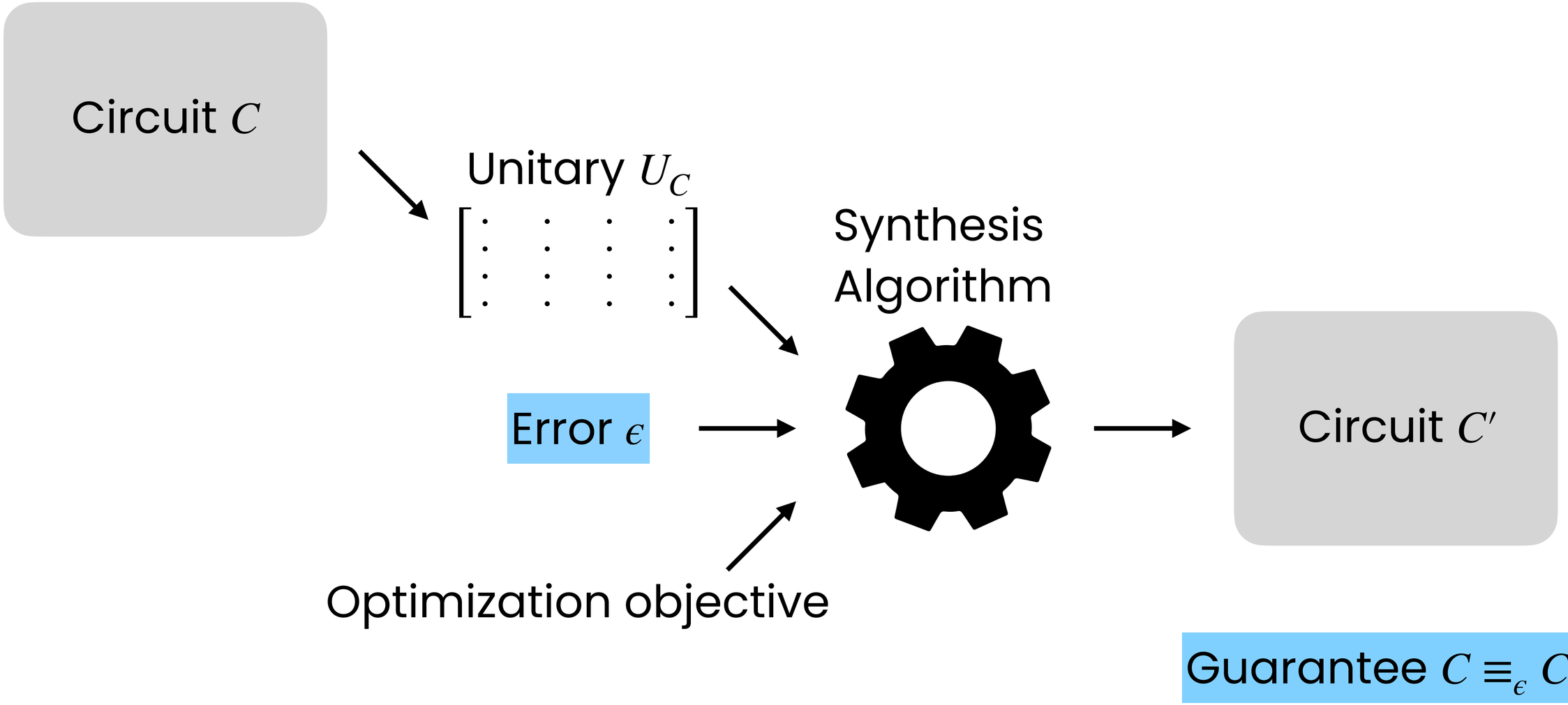

Rich Research Area

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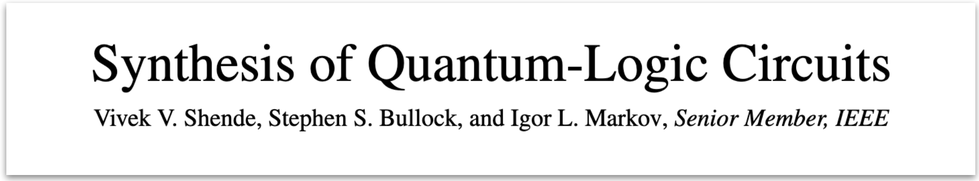
Synthesis of Quantum-Logic Circuits

Vivek V. Shende, Stephen S. Bullock, and Igor L. Markov, *Senior Member, IEEE*

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Towards Optimal Topology Aware Quantum Circuit Synthesis

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Exact synthesis of multiqubit Clifford+T circuits

Brett Giles
Department of Computer Science
University of Calgary

Peter Selinger
Department of Mathematics and Statistics
Dalhousie University

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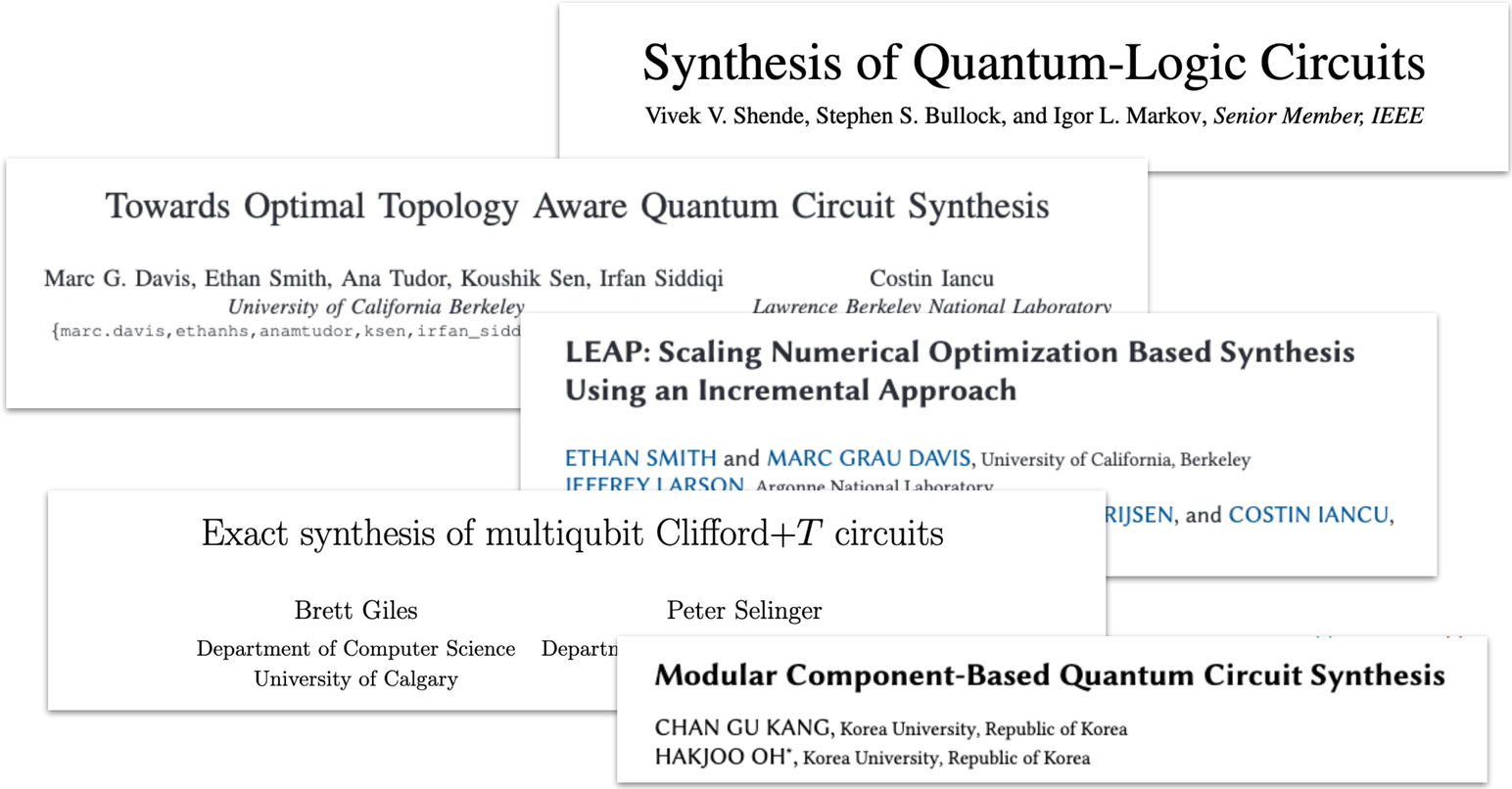

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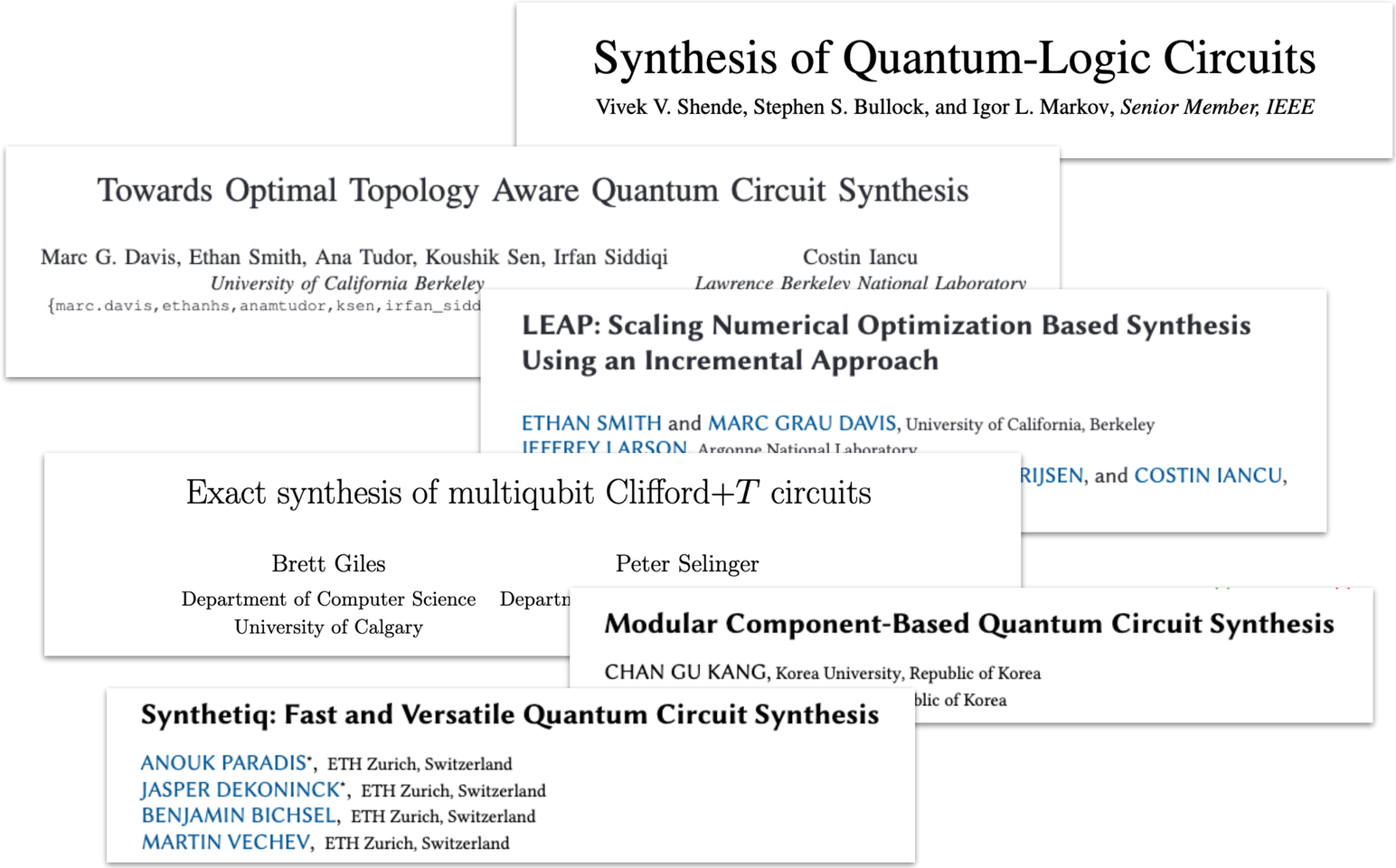

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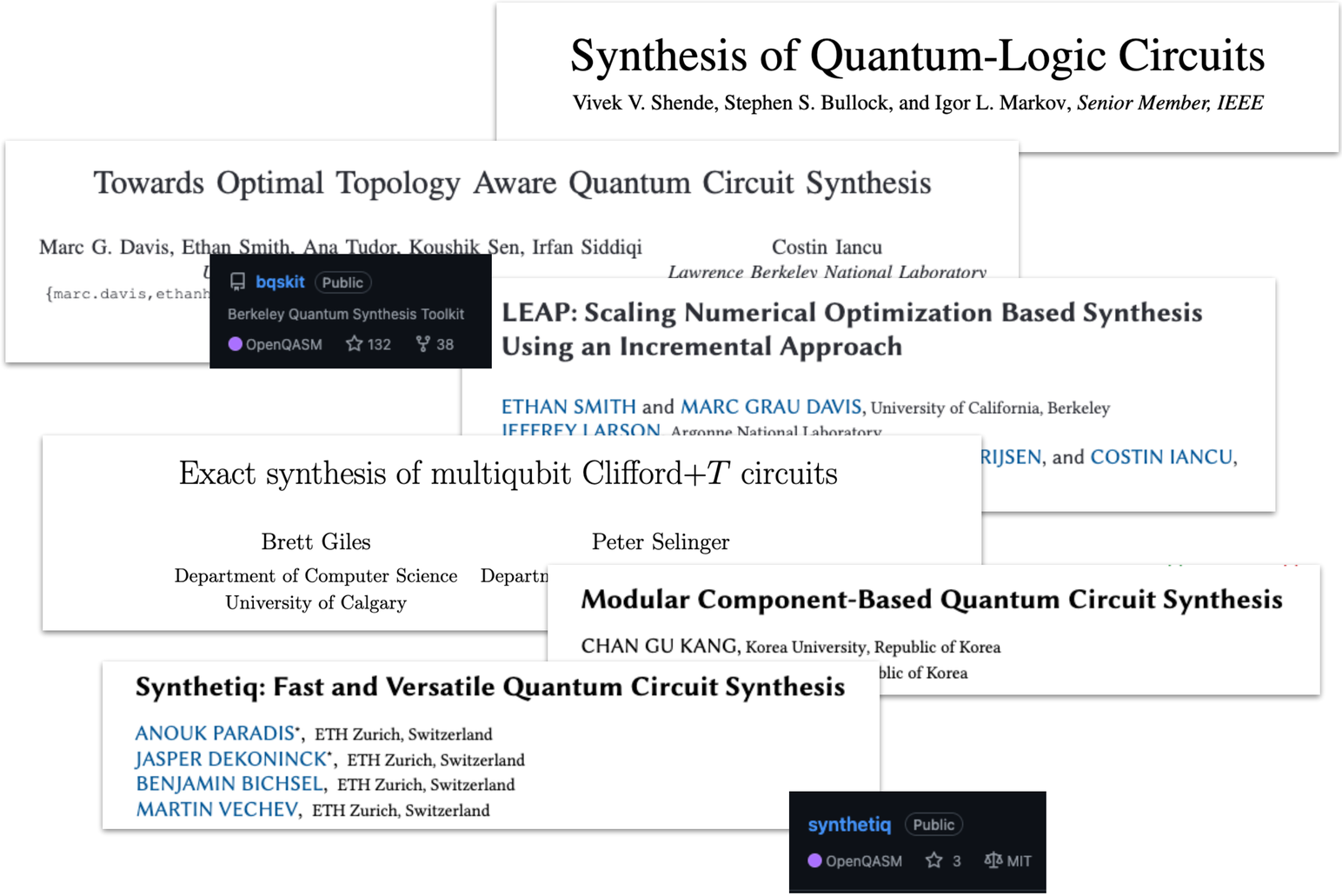

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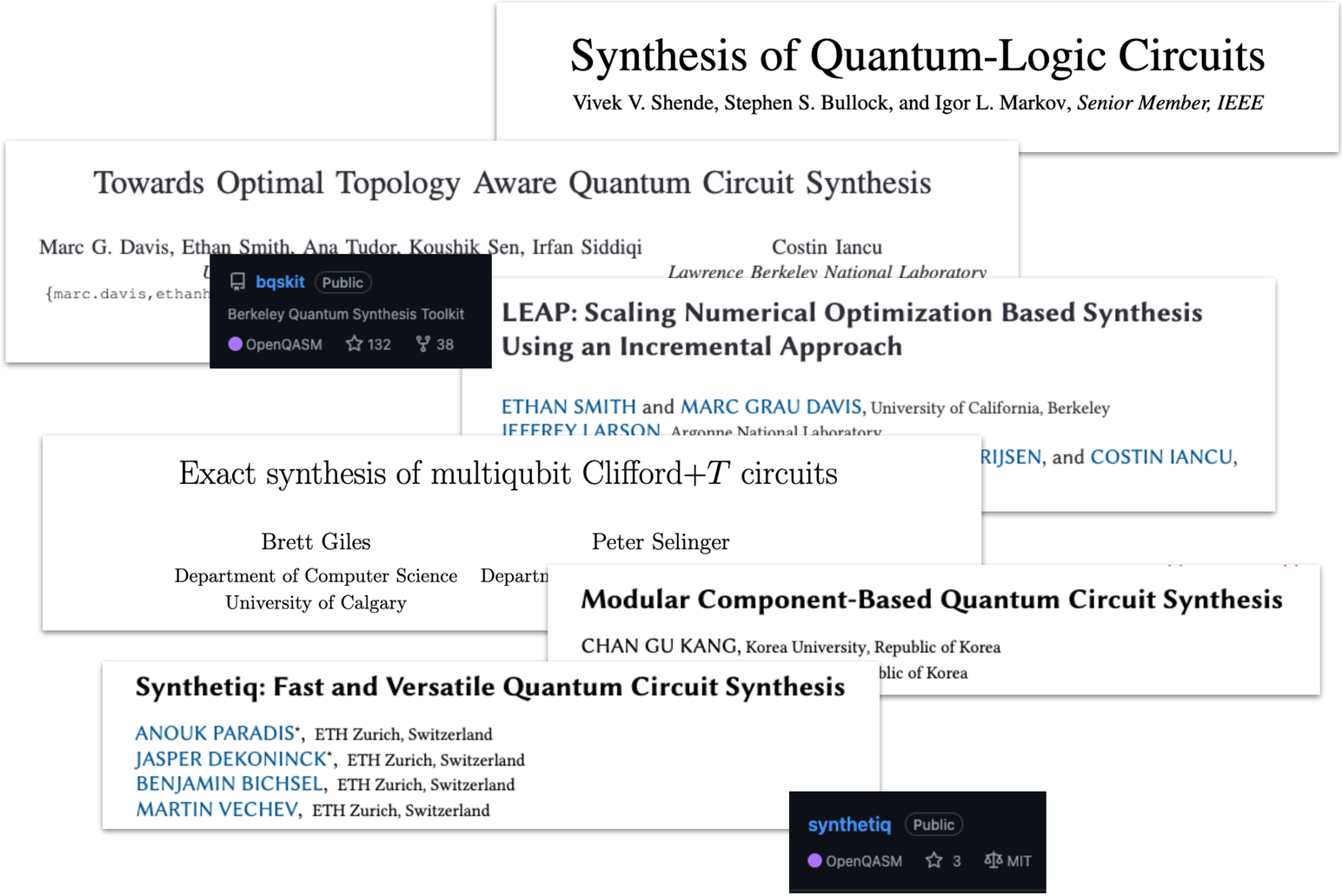
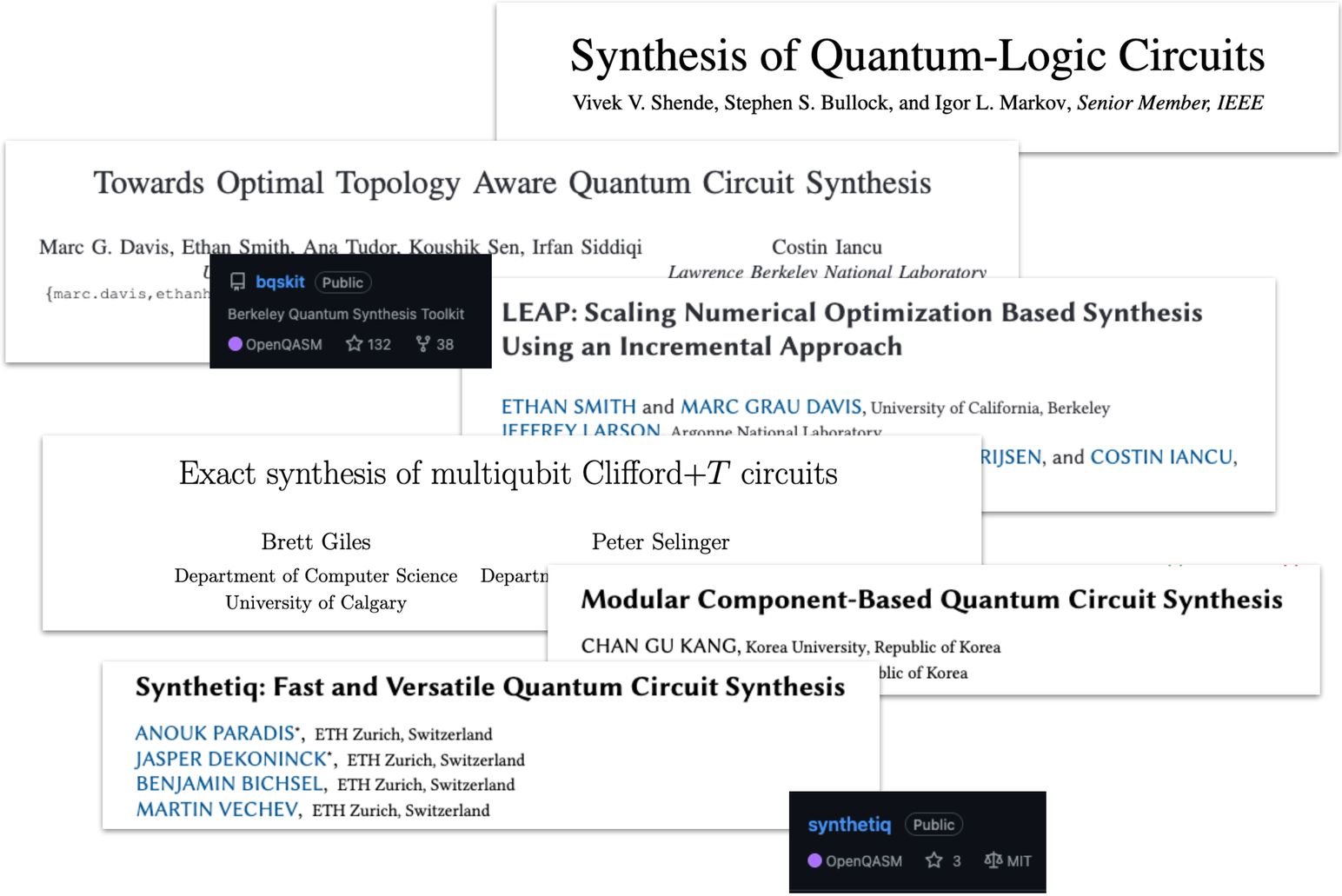


Techniques:

- analytical

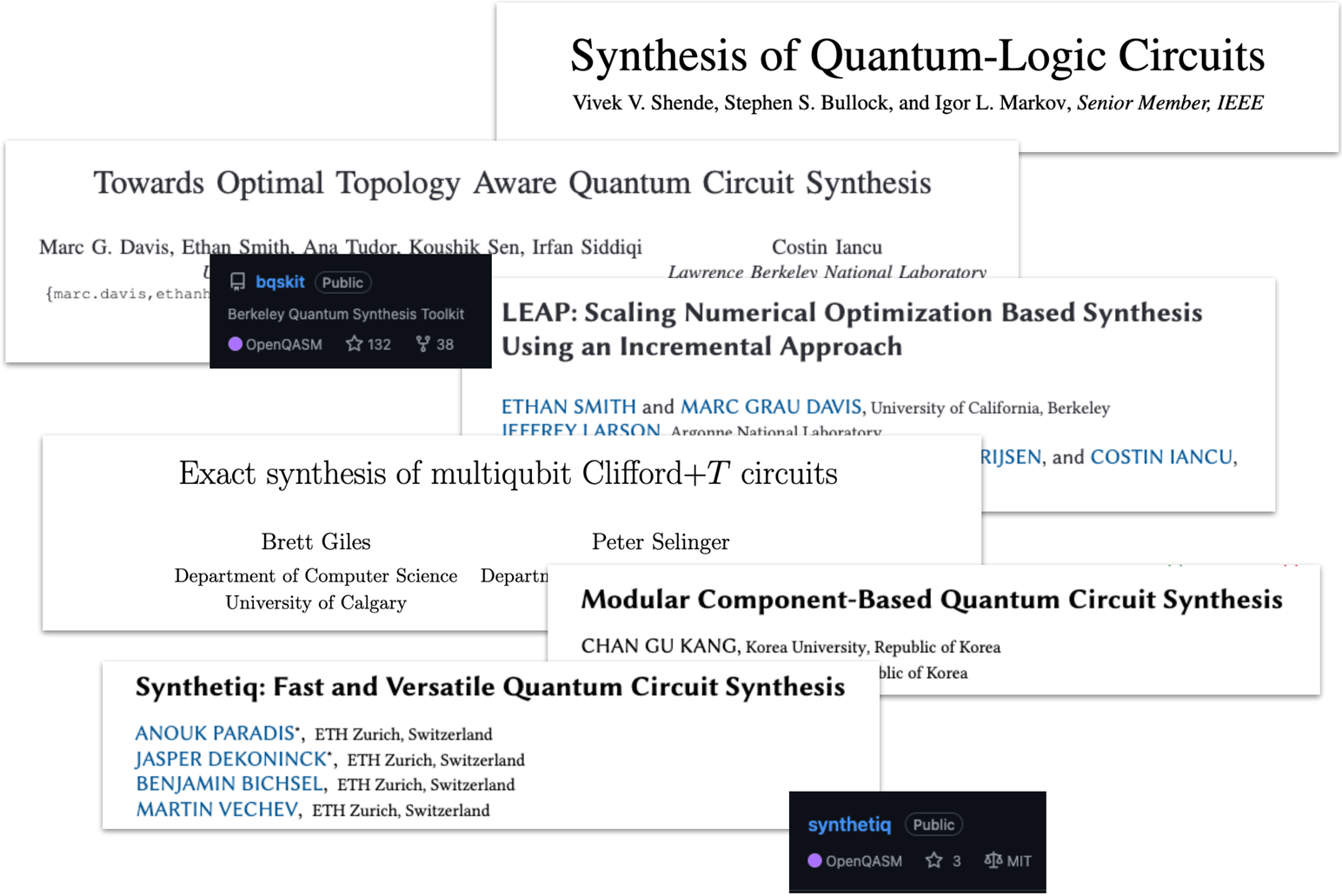
Rich Research Area



Techniques:

- analytical
- numerical optimization

Rich Research Area

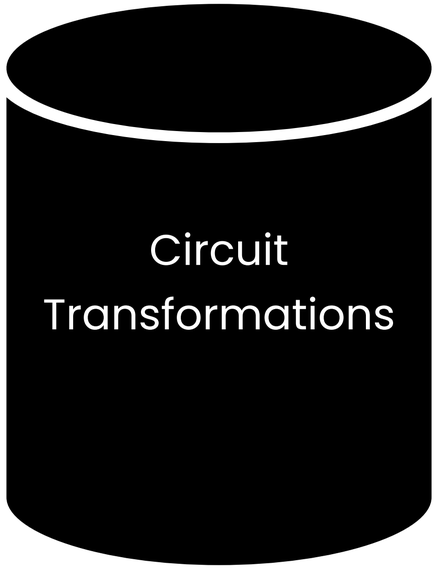


Techniques:

- analytical
- numerical optimization
- explicit search

Scheduling Transformations

Scheduling is hard



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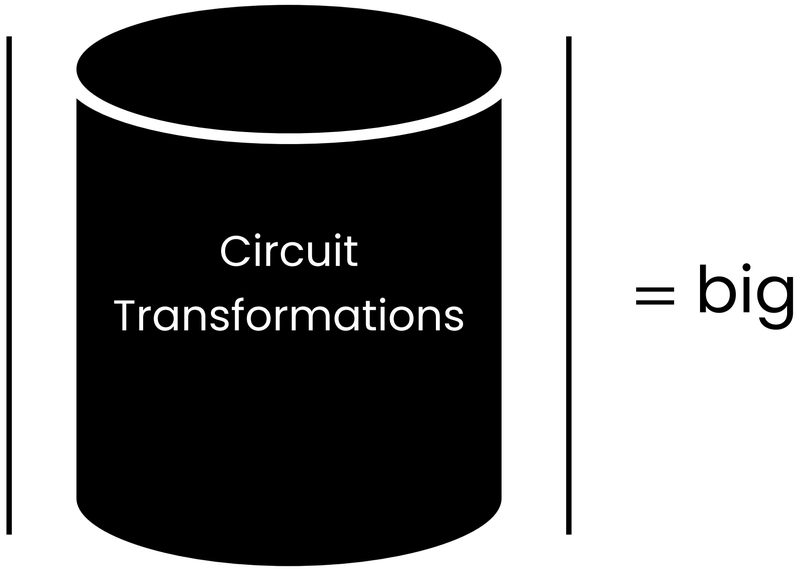
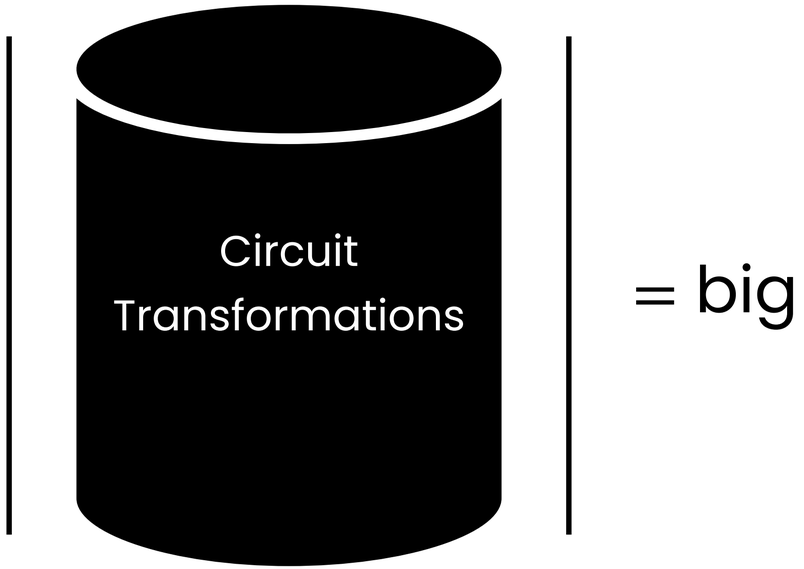


Scheduling is hard

In what order to apply???



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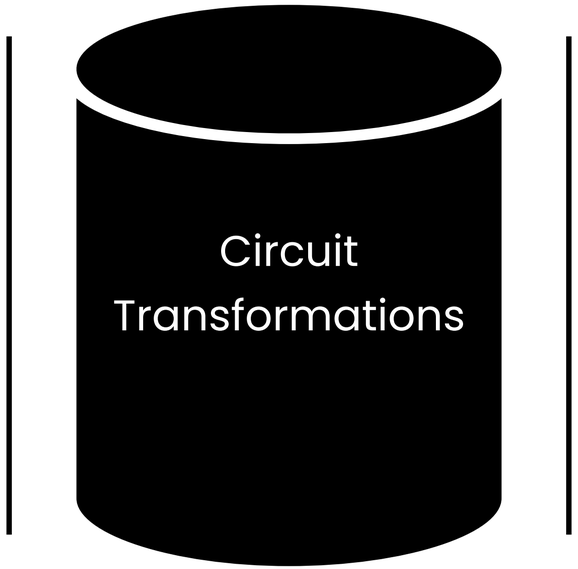

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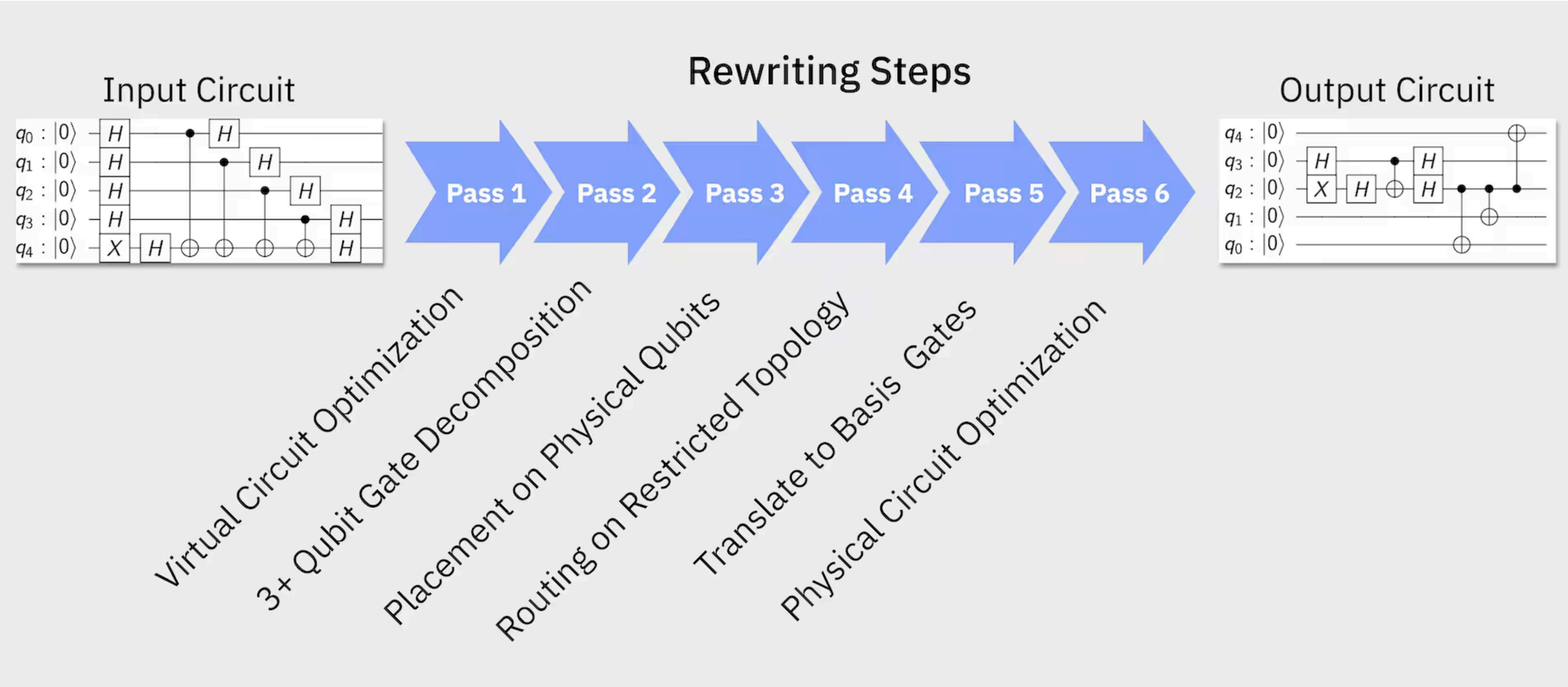
"phase-ordering problem"

Qiskit

https://docs.quantum.ibm.com/api/qiskit/transpiler#optimization-stage

https://github.com/Qiskit/qiskit/blob/stable/1.4/qiskit/transpiler/preset_passmanagers/level3.py



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qiskit / qiskit / transpiler / preset_passmanagers / **level3.py** ↑ Top

Code | Blame | 119 lines (104 loc) · 4.8 KB · | Raw

```
26
27  def level_3_pass_manager(pass_manager_config: PassManagerConfig) -> StagedPassManager:
28      """Level 3 pass manager: heavy optimization by noise adaptive qubit mapping and
29      gate cancellation using commutativity rules and unitary synthesis.
30
31      This pass manager applies the user-given initial layout. If none is given, a search
32      for a perfect layout (i.e. one that satisfies all 2-qubit interactions) is conducted.
33      If no such layout is found, and device calibration information is available, the
34      circuit is mapped to the qubits with best readouts and to CX gates with highest fidelit
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36      The pass manager then transforms the circuit to match the coupling constraints.
37      It is then unrolled to the basis, and any flipped cx directions are fixed.
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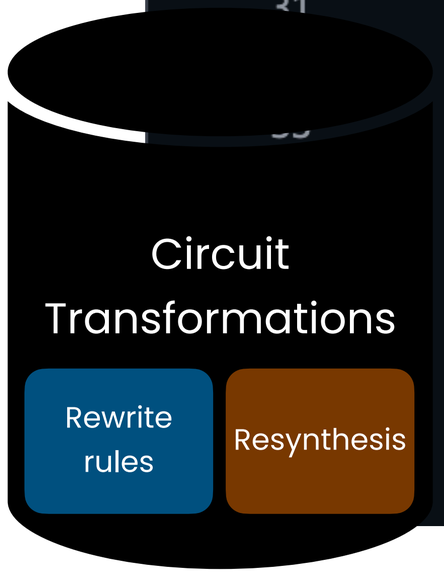

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qiskit / qiskit / transpiler / preset_passmanagers / **level3.py** ↑ Top

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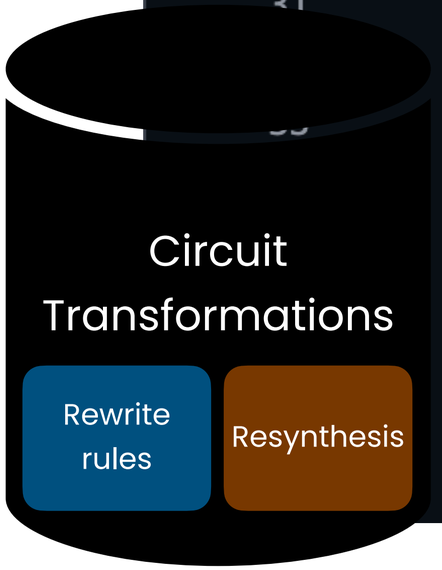
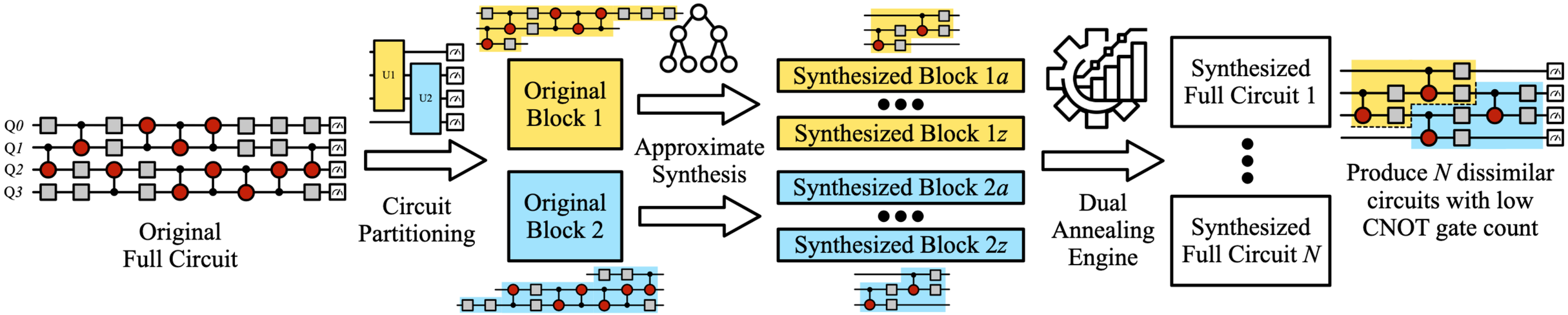
coarse fixed passes



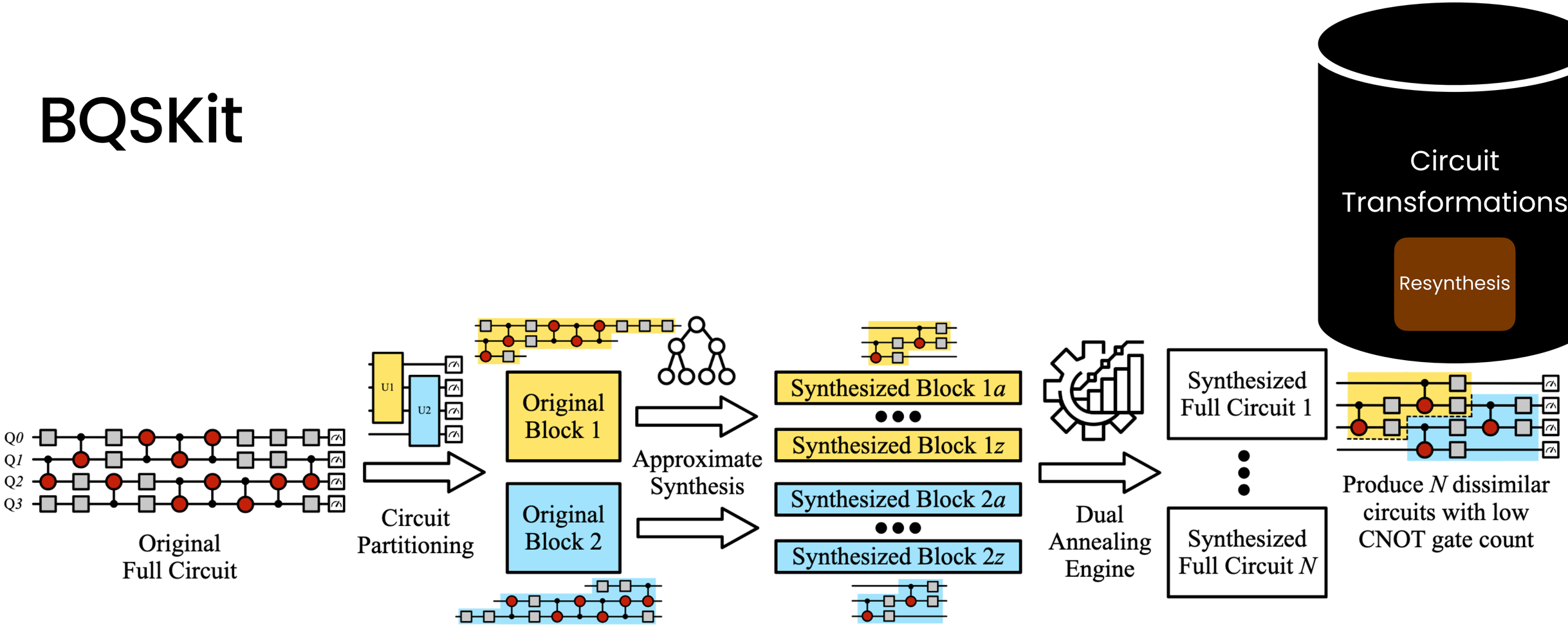
BQSKit



QUEST: systematically approximating quantum circuits for higher output fidelity [Patel et al. 2022]

https://github.com/BQSKit/bqskit

BQSKit



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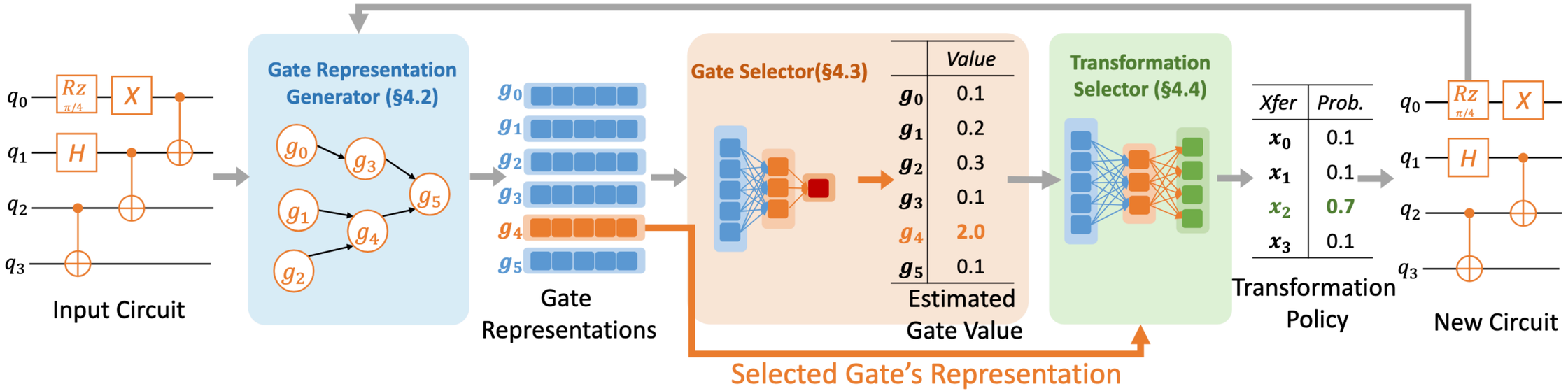
https://github.com/BQSKit/bqskit

Quarl



Quarl: A Learning-Based Quantum Circuit Optimizer [Li et al. 2024]

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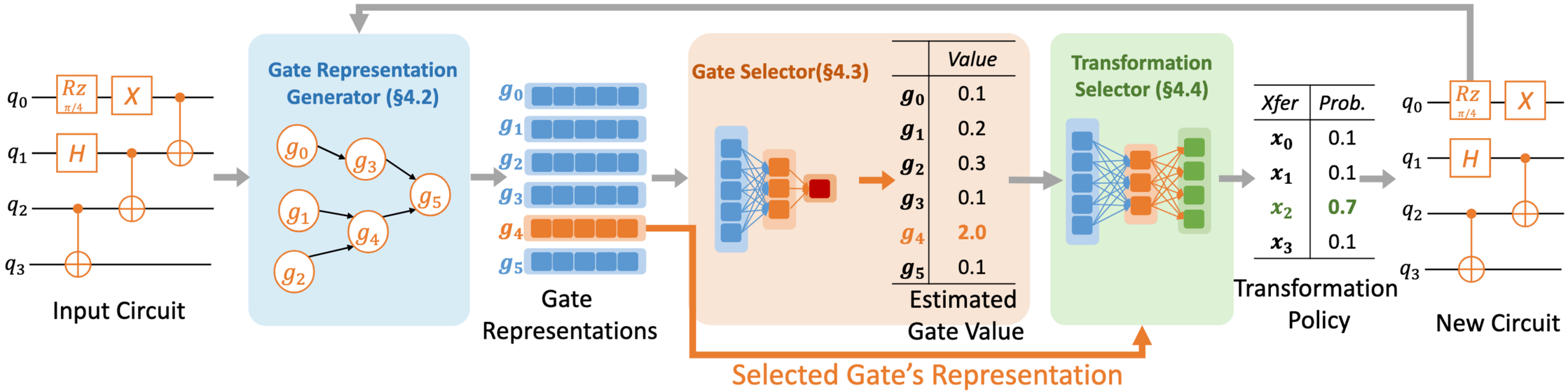

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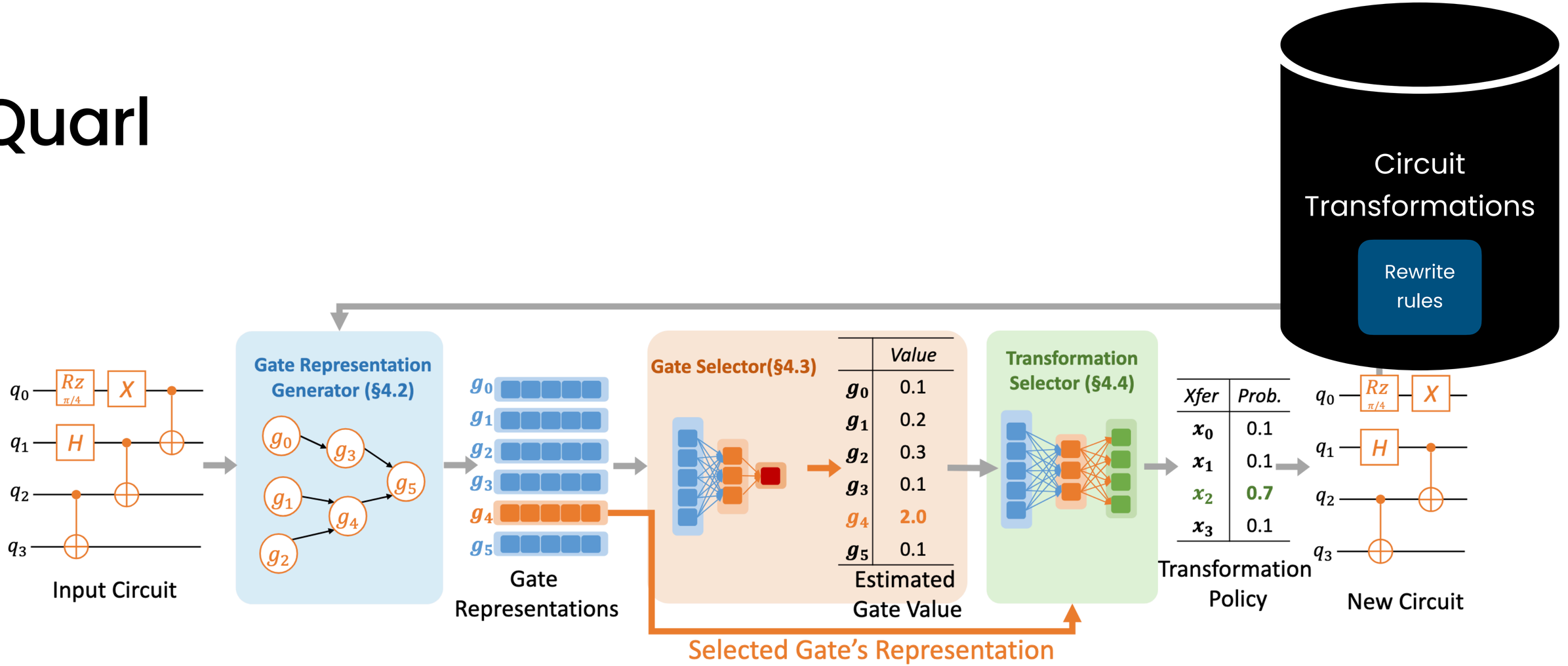

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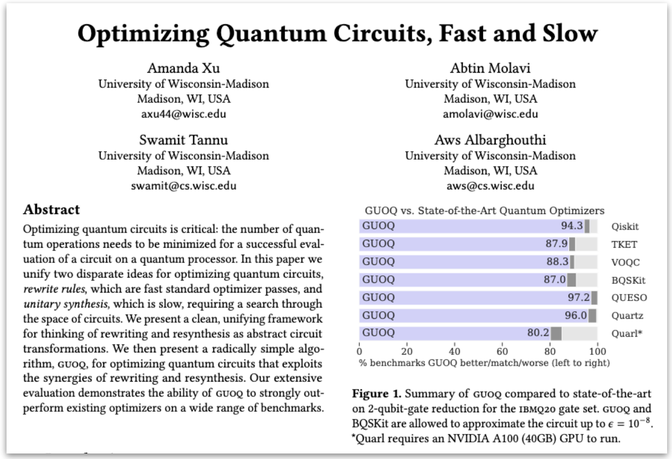
GUOQ



Optimizing Quantum Circuits, Fast and Slow

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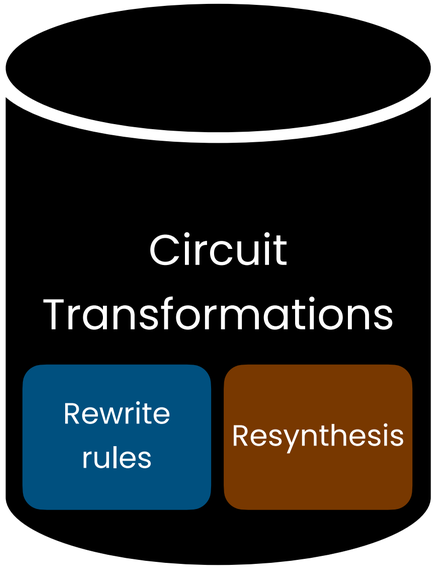

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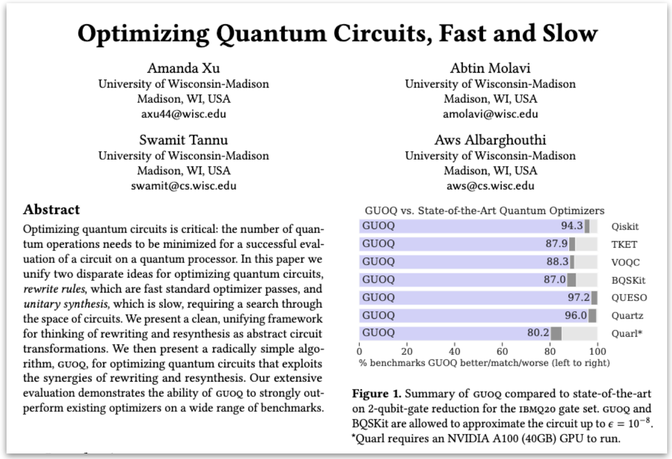

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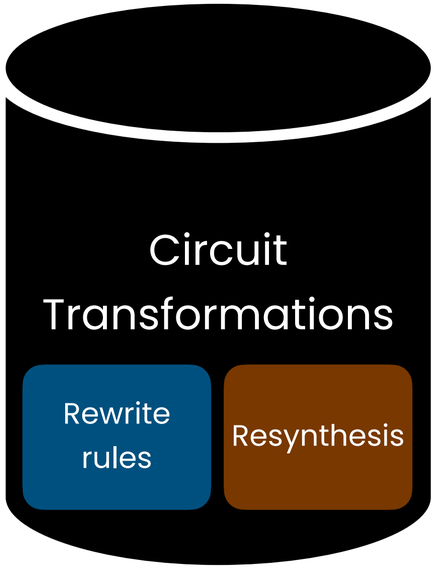

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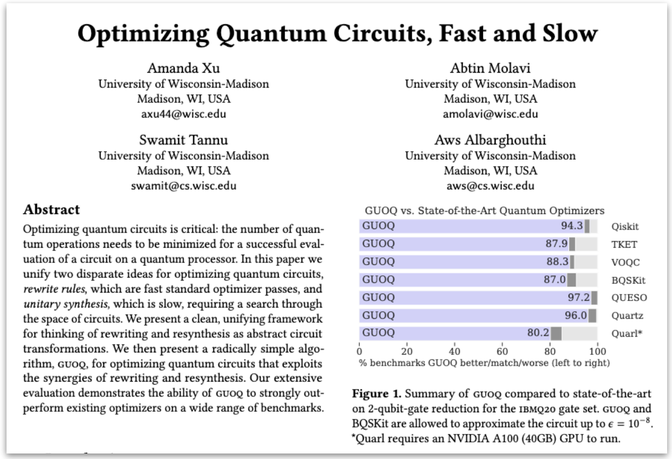
GUOQ



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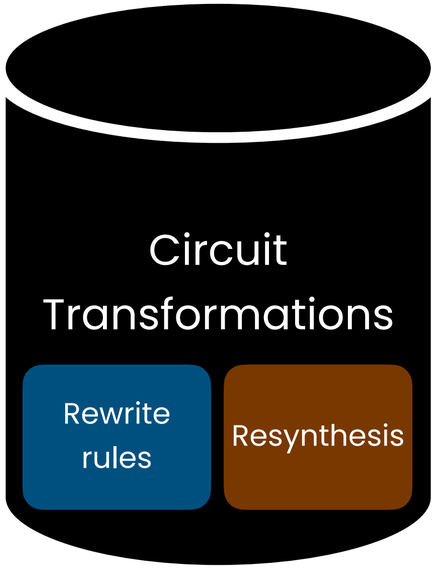

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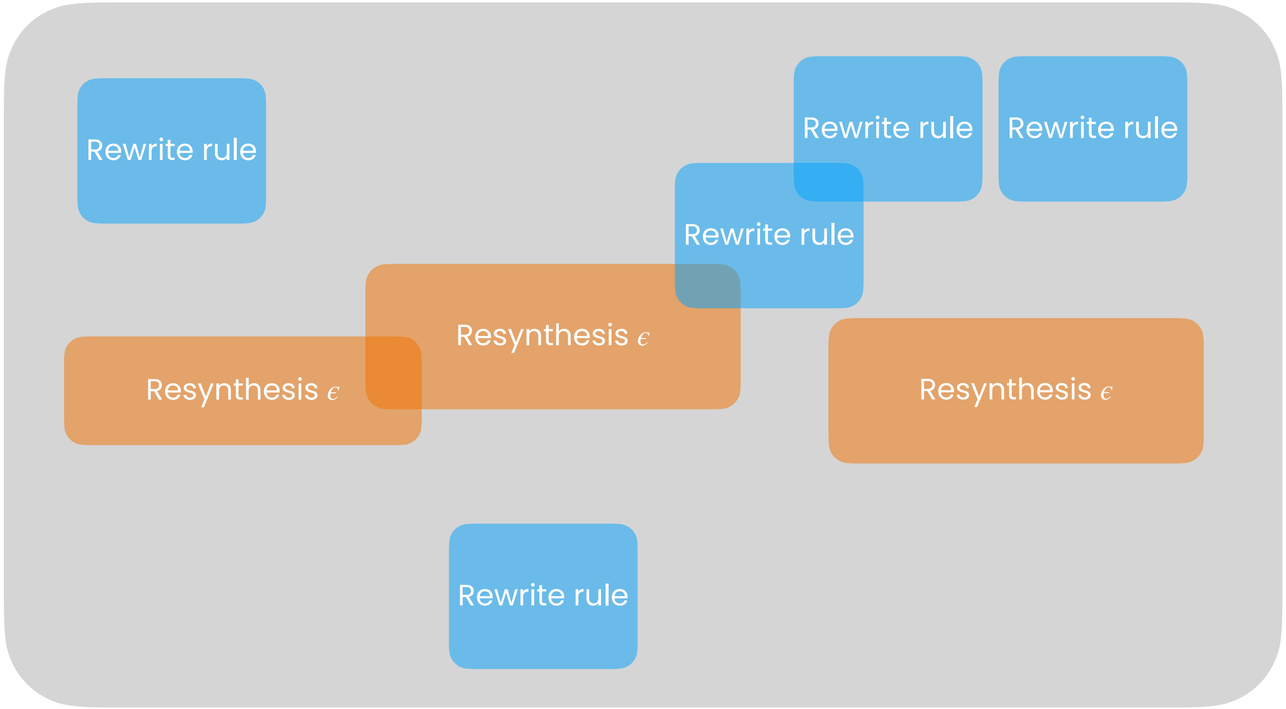

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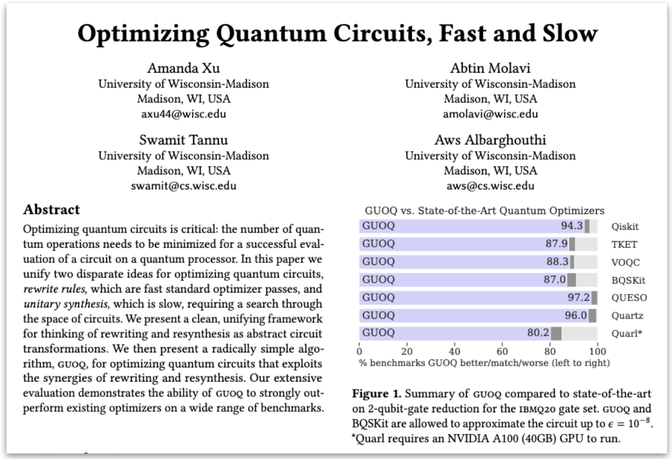

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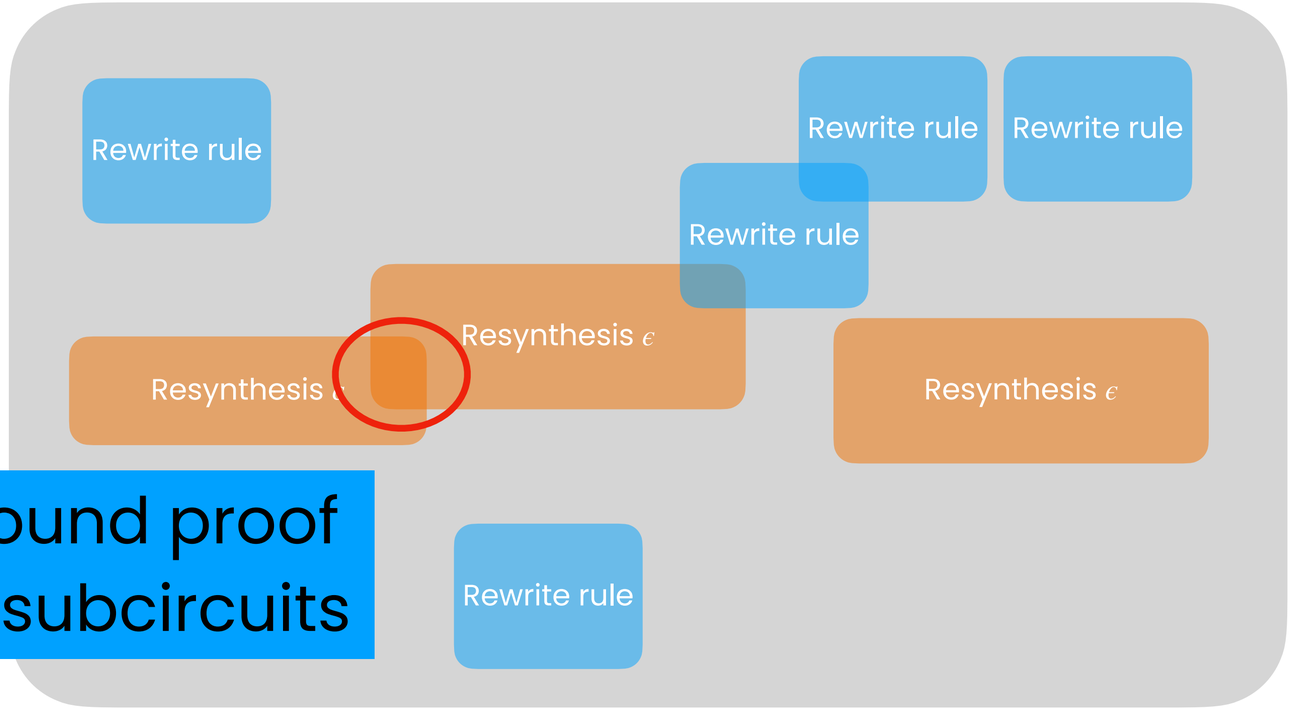
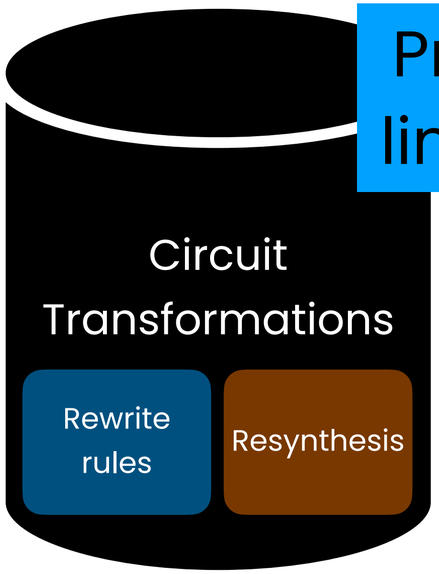
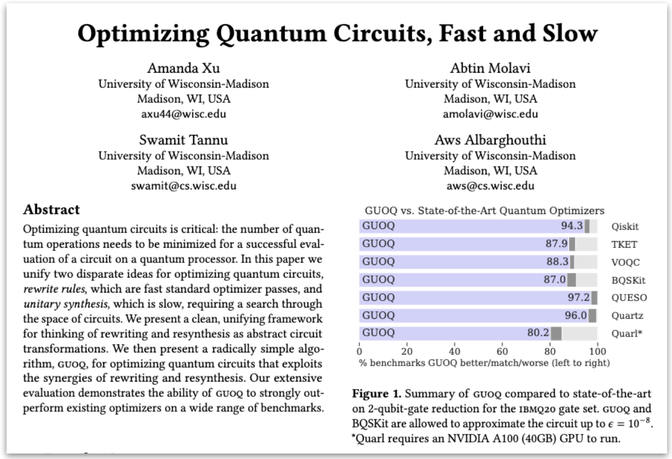




Prior work: error bound proof limited to **disjoint** subcircuits

GUOQ



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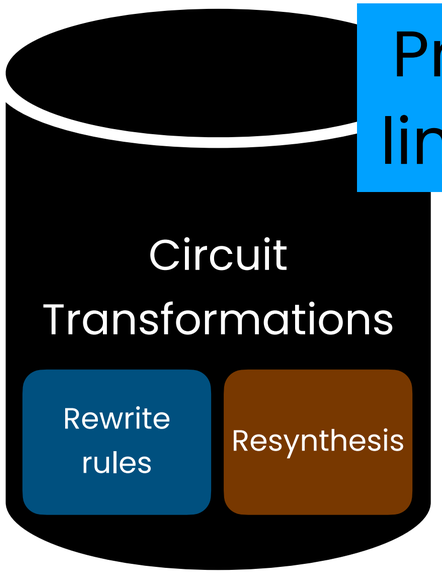

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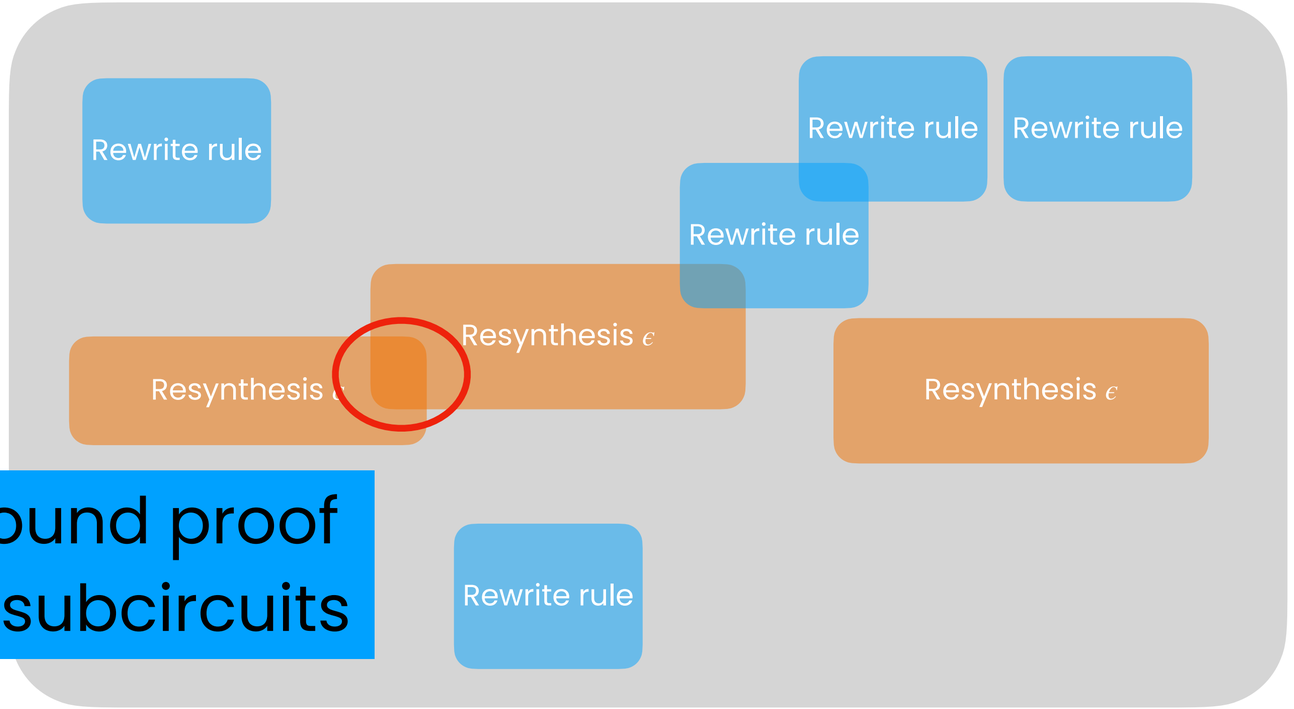

$$\epsilon_{total} \leq \epsilon + \epsilon + \epsilon$$

We extended to handle arbitrary subcircuits

GUOQ Motivation: Two Disparate Techniques

Rewrite Rules

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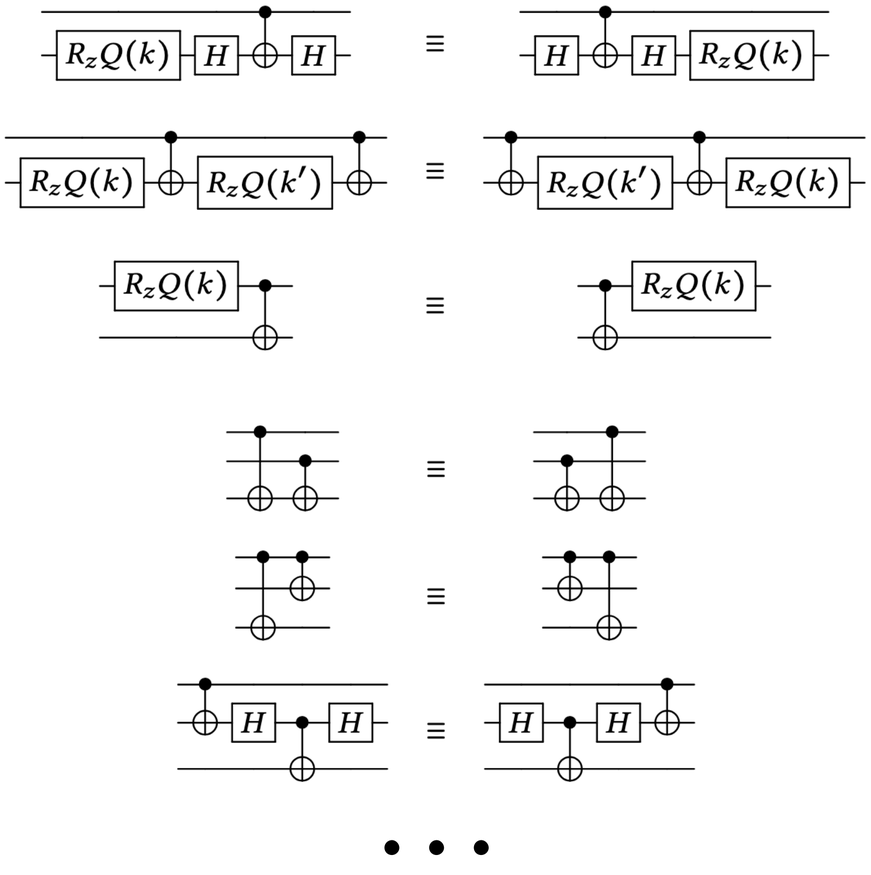

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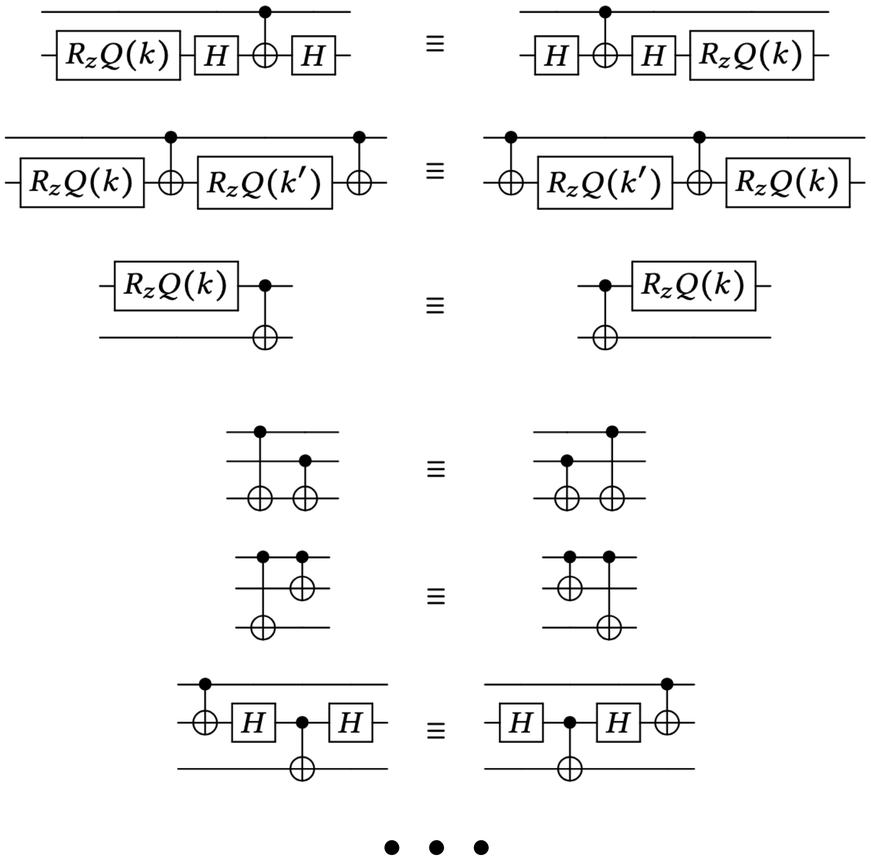

Circuit Resynthesis

GUOQ Motivation: Two Disparate Techniques

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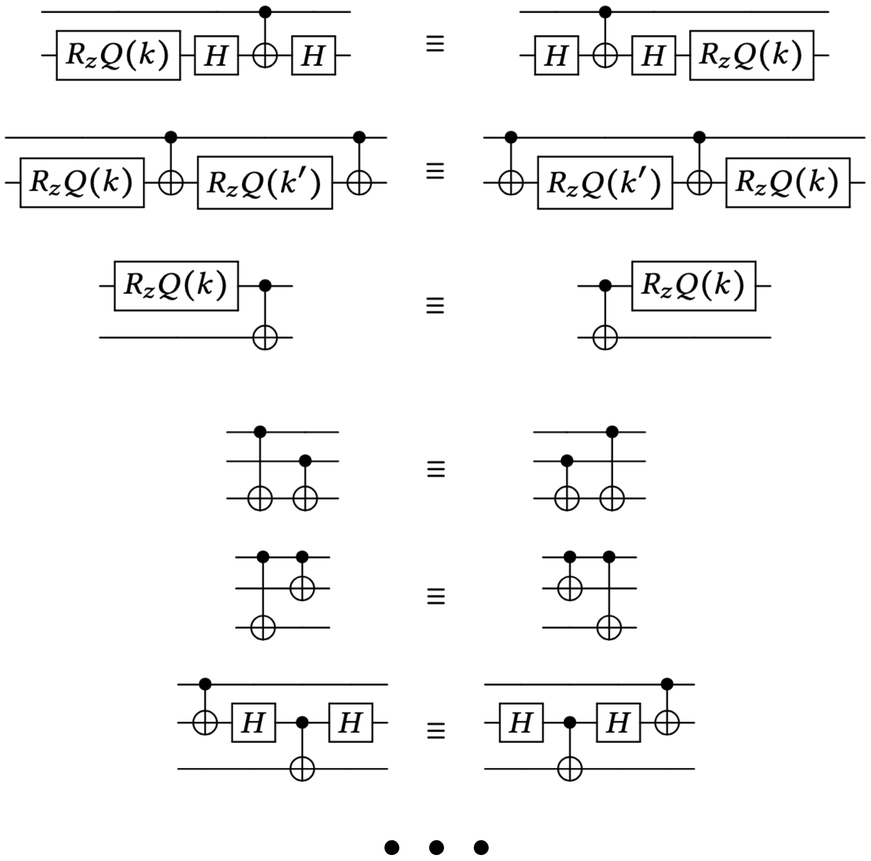

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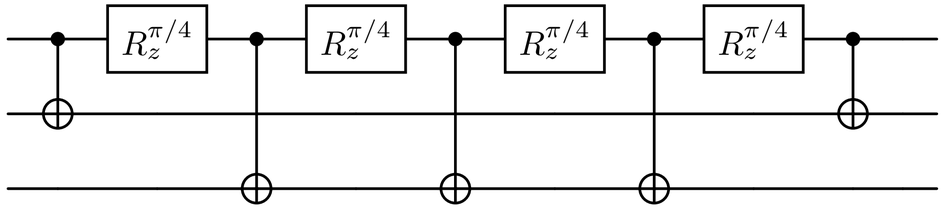

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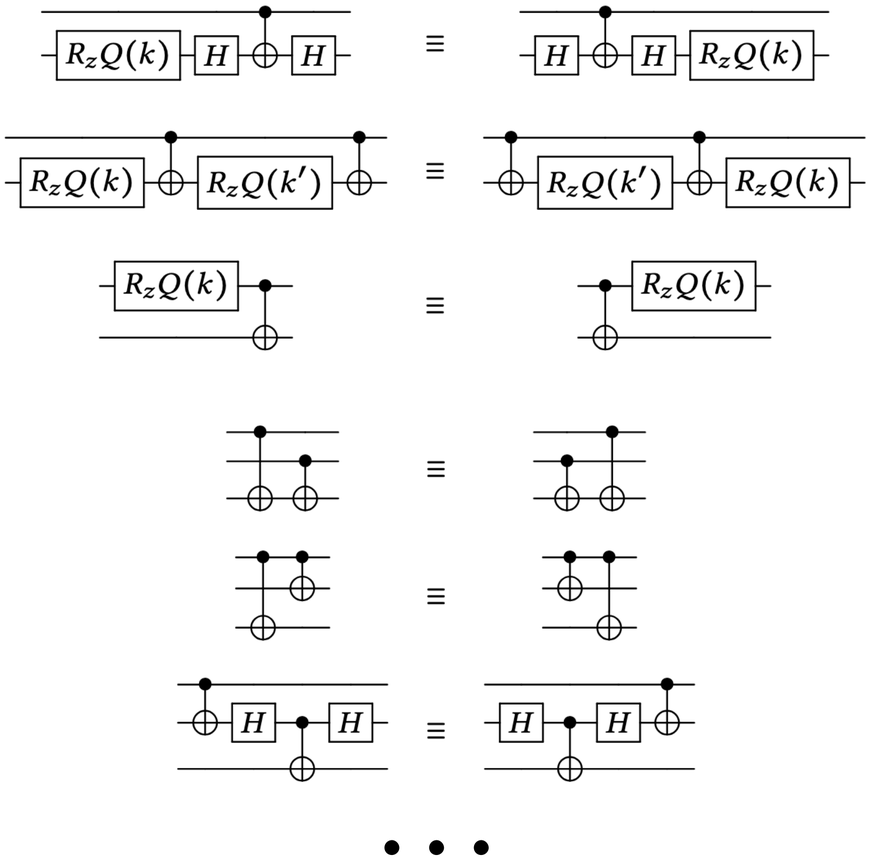

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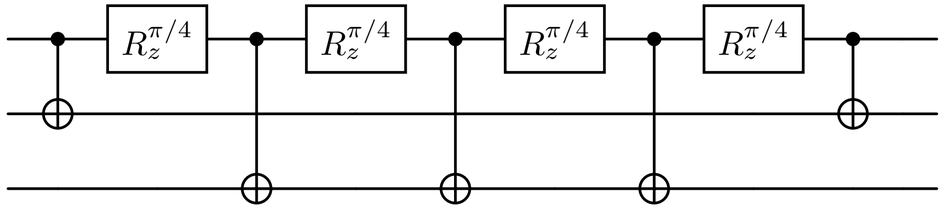

specification

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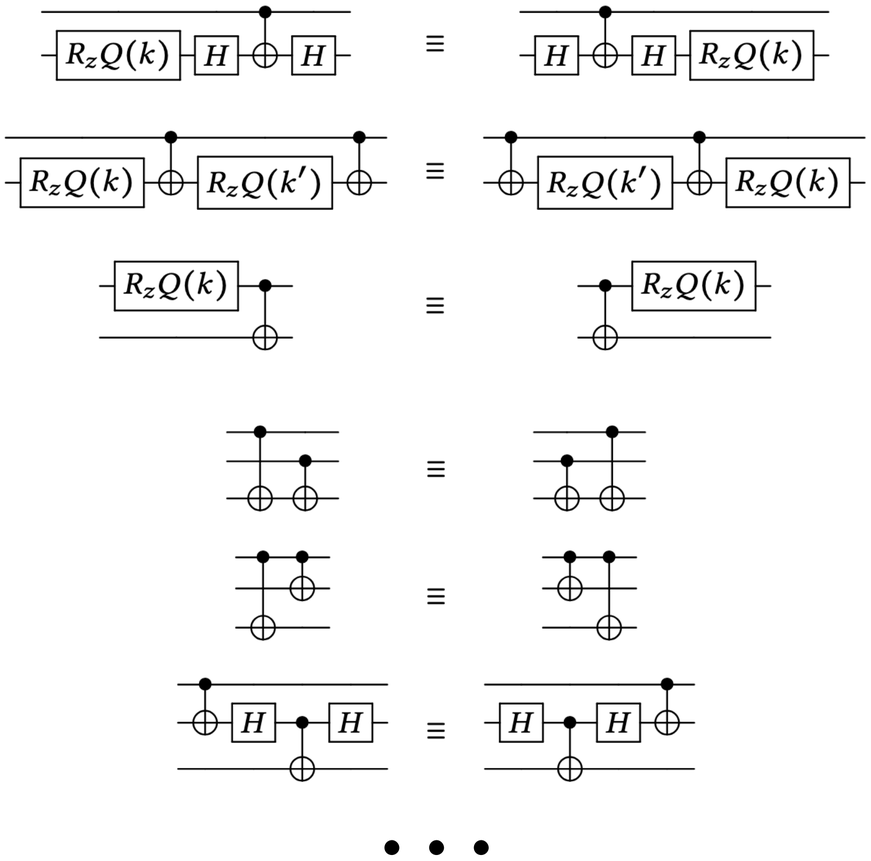

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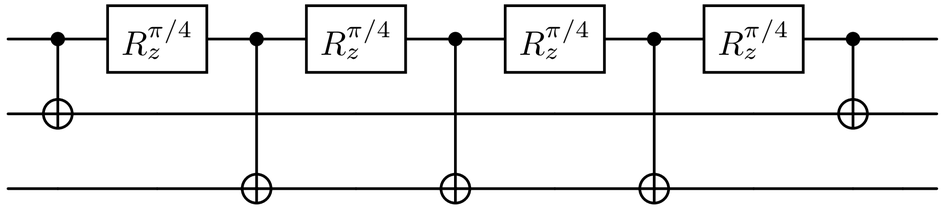

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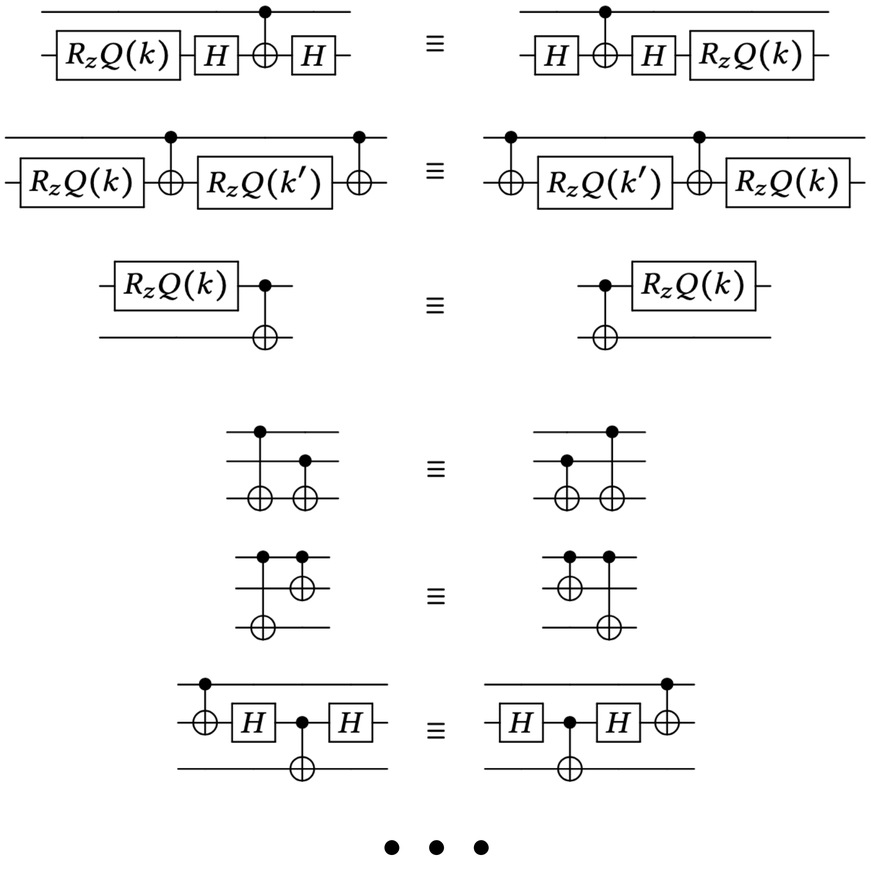

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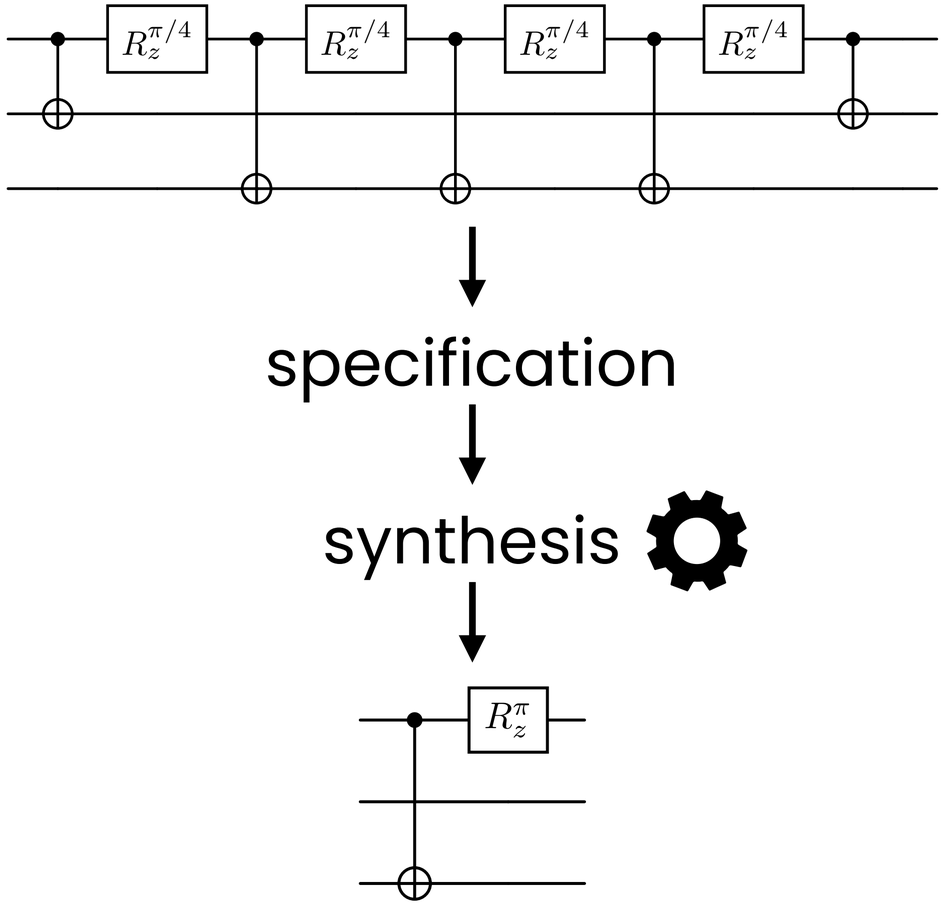

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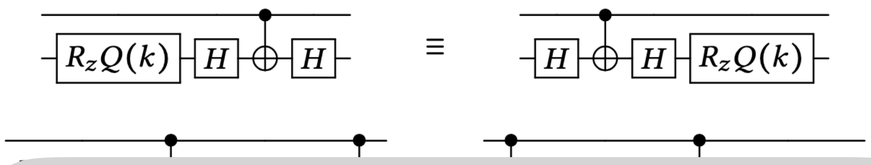

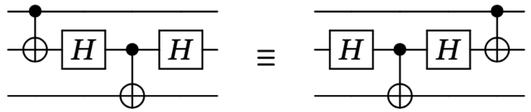

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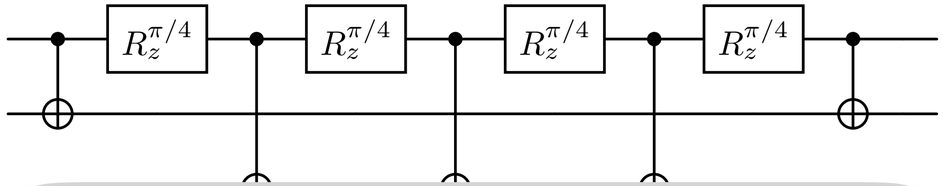

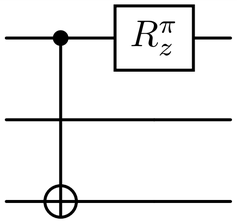

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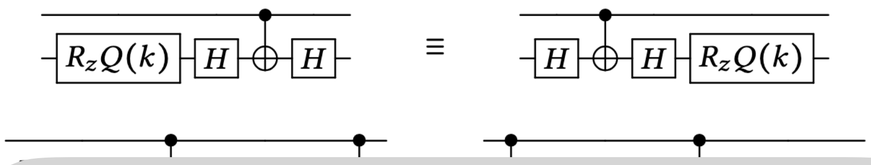

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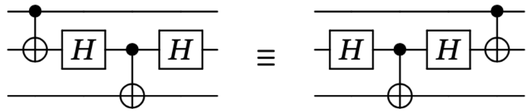

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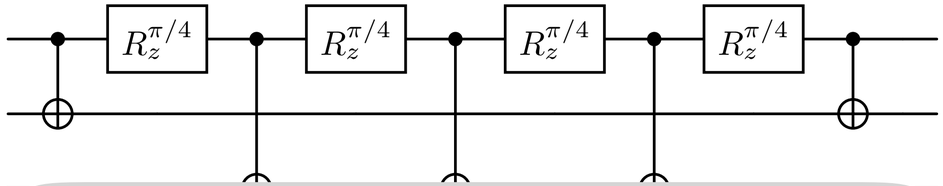

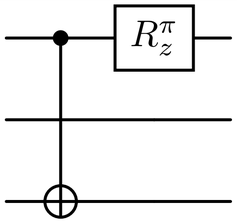

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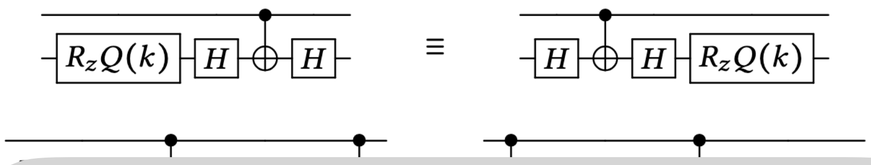

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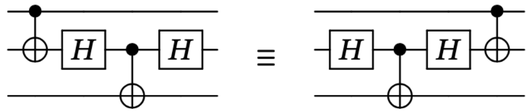

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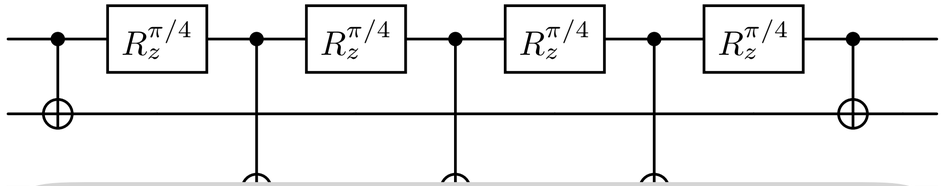
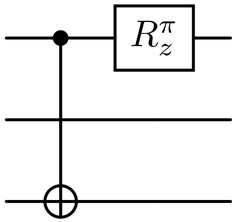
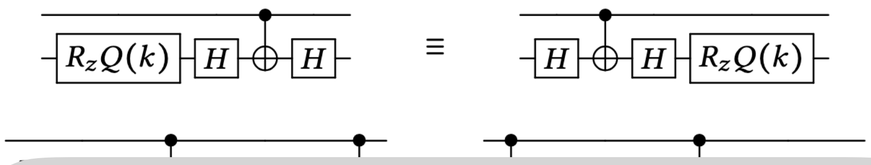


Slow



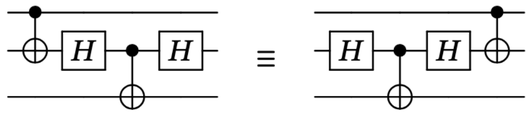
GUOQ Motivation: Two Disparate Techniques

Rewrite Rules



Fast

Limited by # gates



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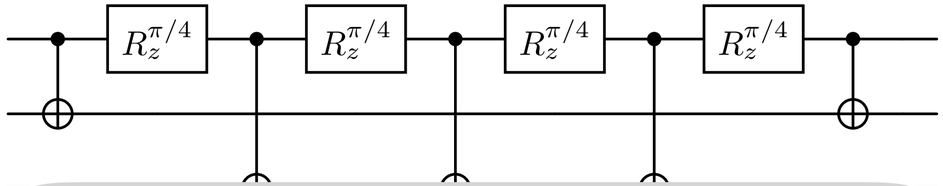
Circuit Resynthesis



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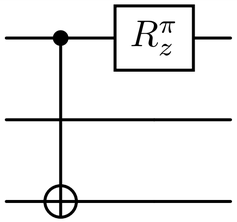
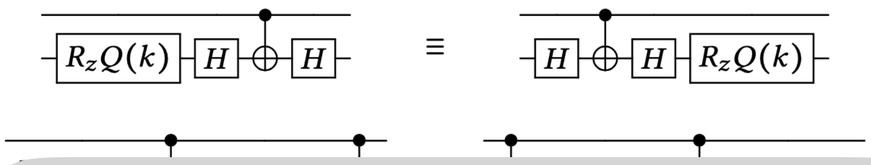


GUOQ Motivation: Two Disparate Techniques

Rewrite Rules



Fast

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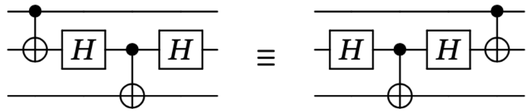

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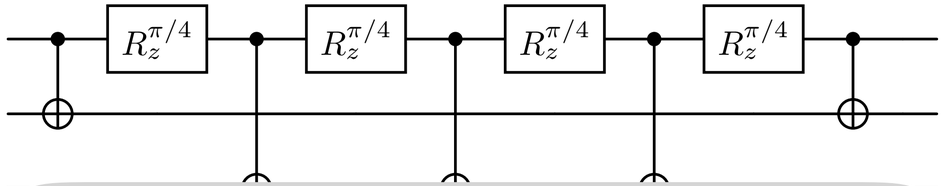
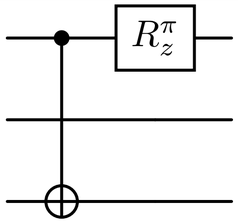
Circuit Resynthesis



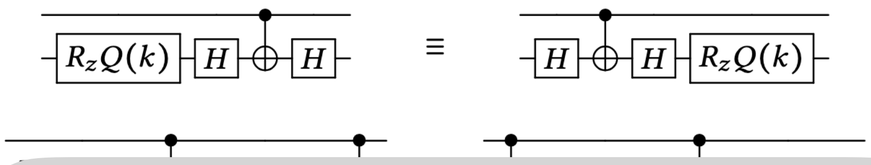
Slow

Limited by # qubits



GUOQ Motivation: Two Disparate Techniques

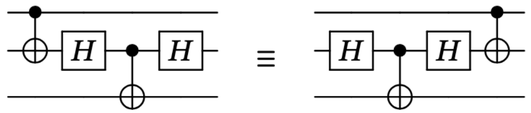
Rewrite Rules



Fast

Limited by # gates

Exact



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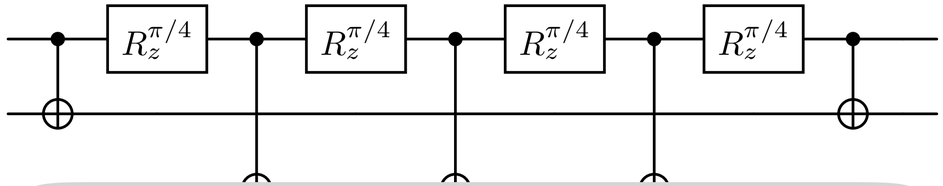
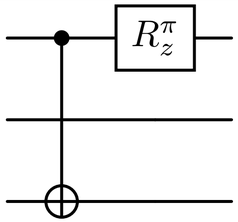
Circuit Resynthesis



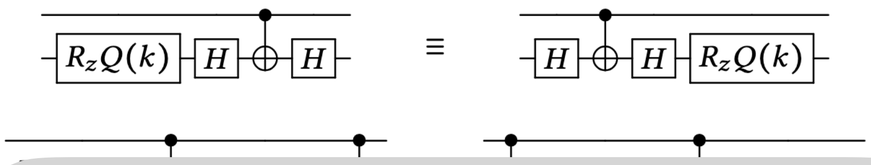
Slow

Limited by # qubits



GUOQ Motivation: Two Disparate Techniques

Rewrite Rules



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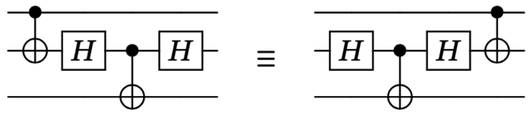

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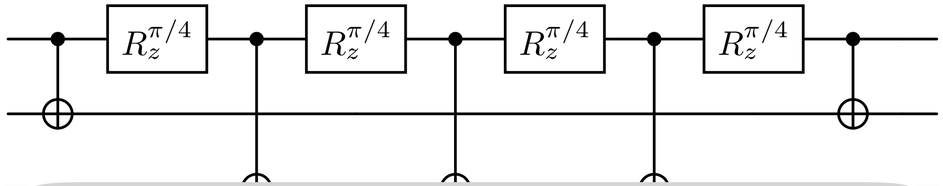
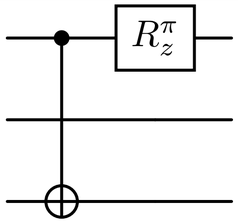
Circuit Resynthesis



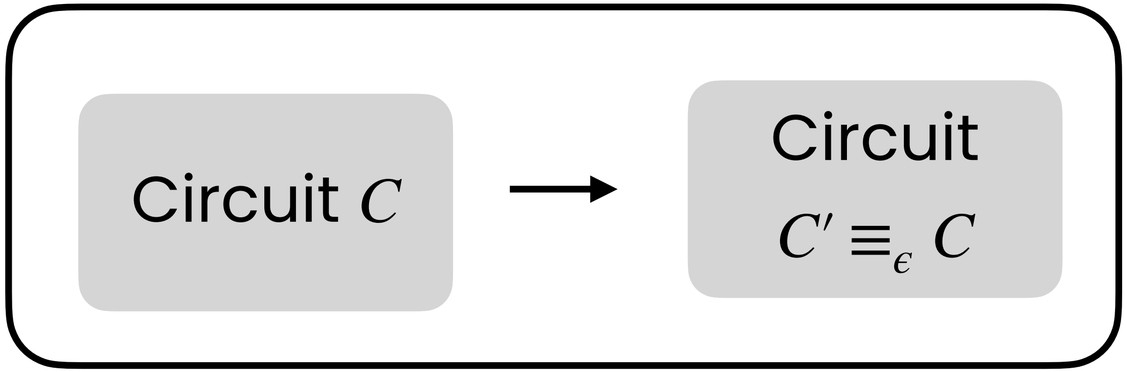
Slow

Limited by # qubits

Approximate

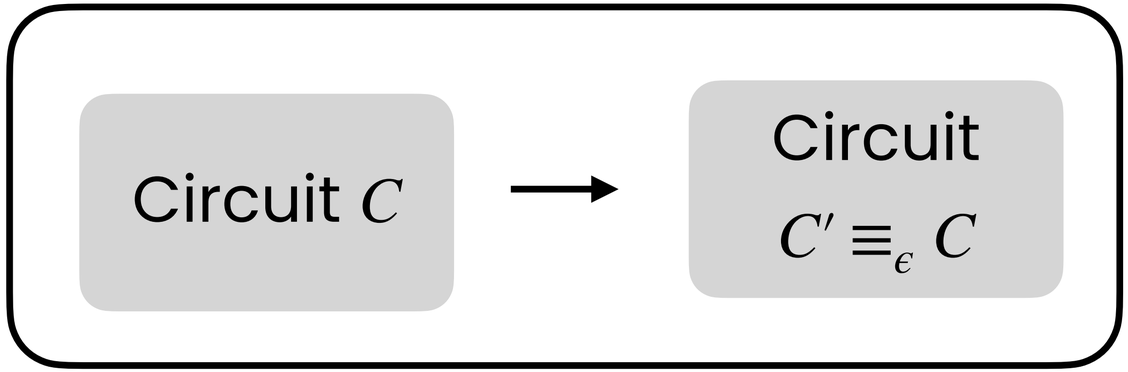


Unifying Abstract Circuit Transformations

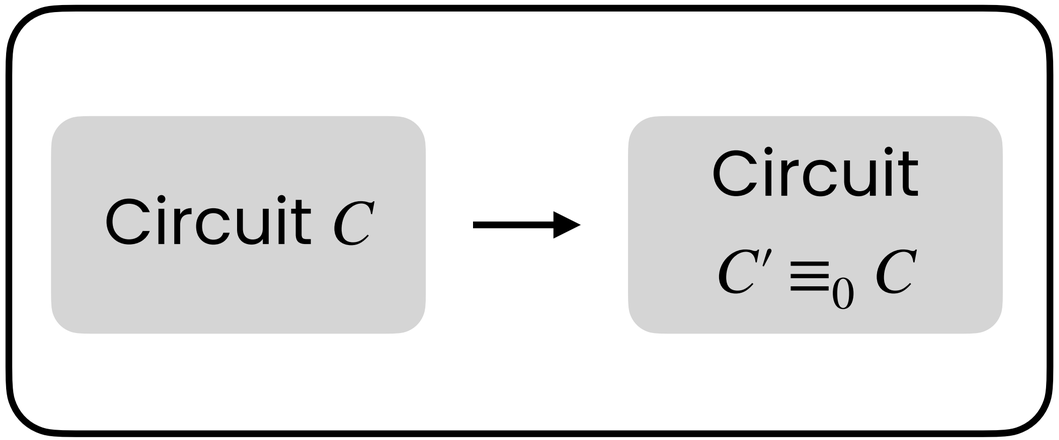


transformation $\tau_\epsilon : \mathscr{C} \to \mathscr{C}$

Unifying Abstract Circuit Transformations

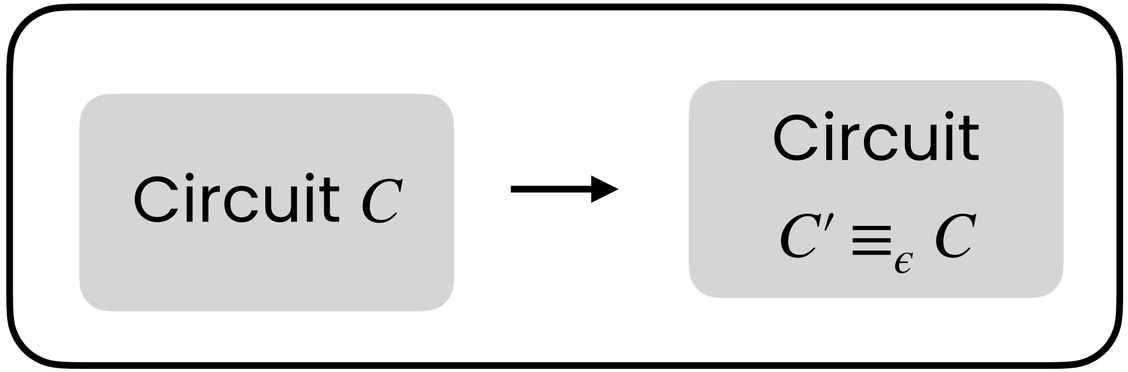


transformation $\tau_\epsilon : \mathscr{C} \to \mathscr{C}$



rewrite rule

Unifying Abstract Circuit Transformations



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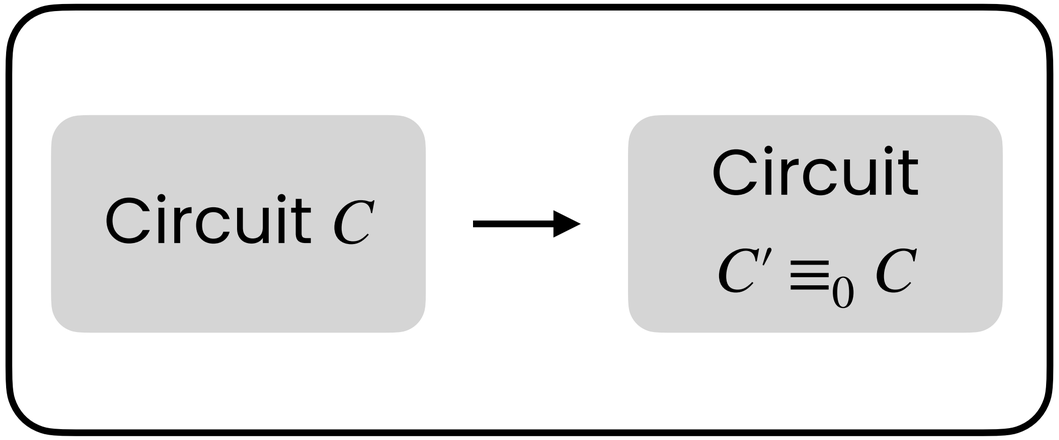
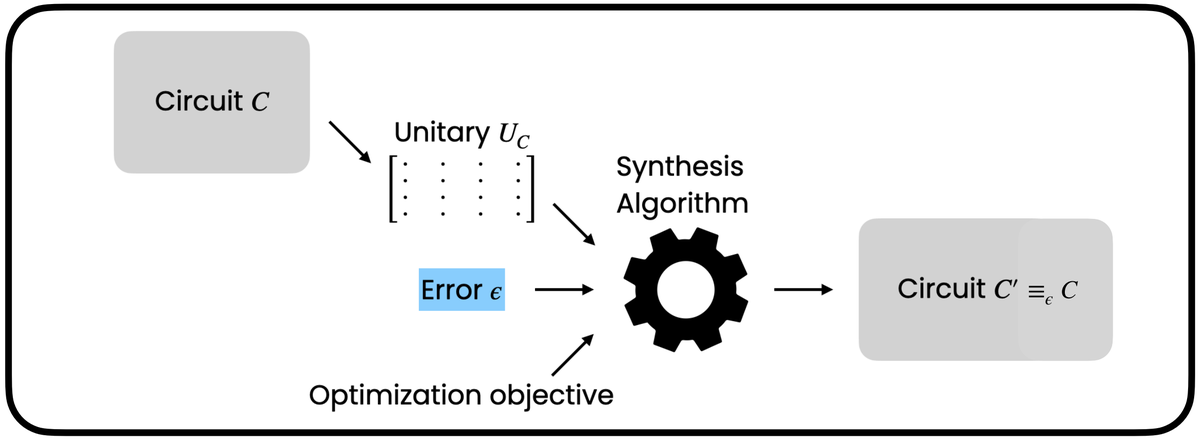
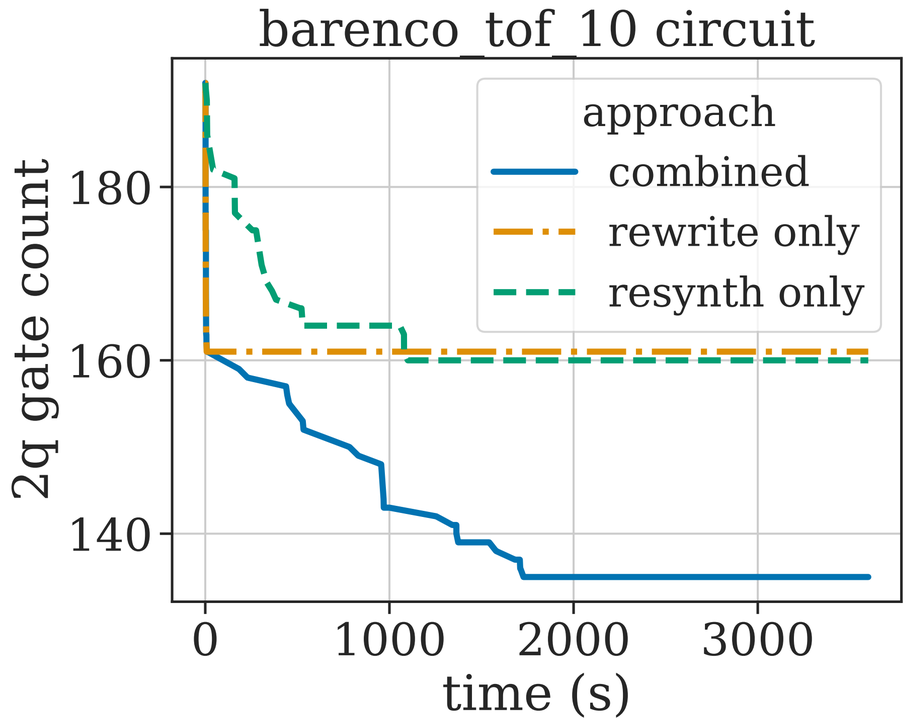


rewrite rule

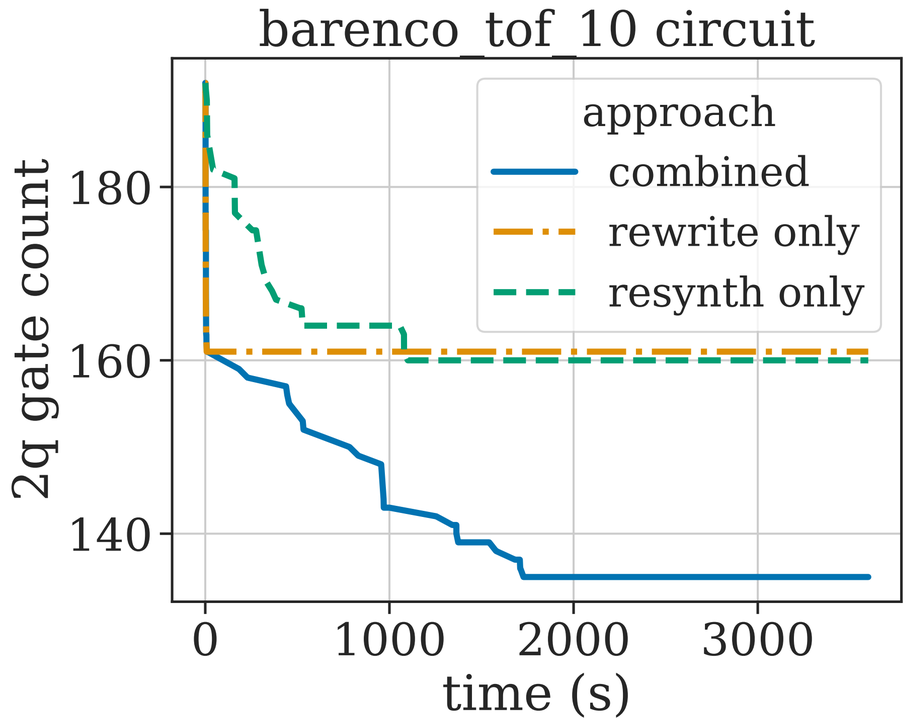


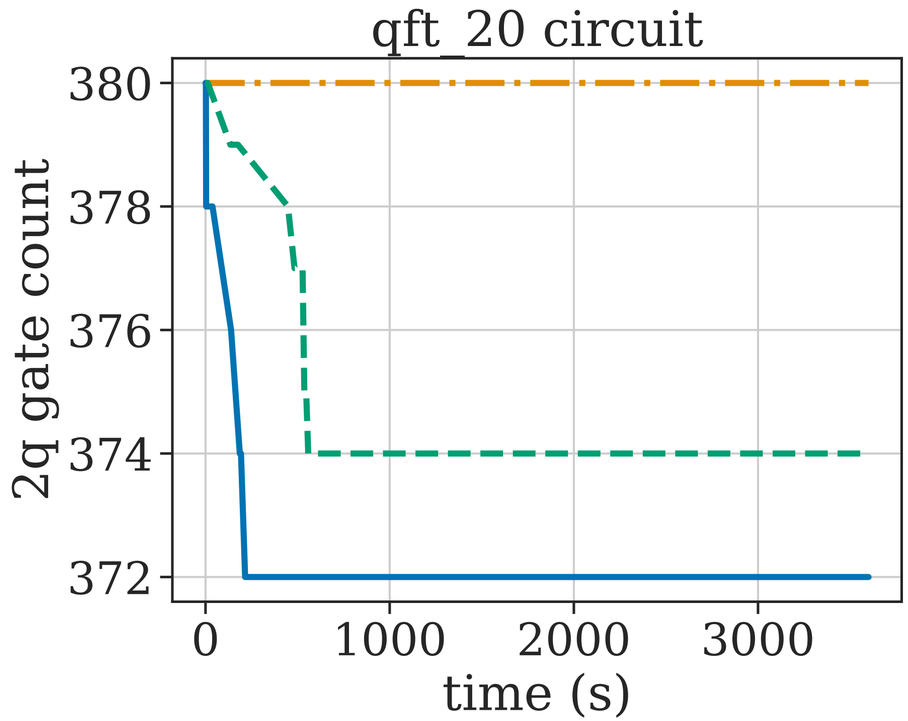
resynthesis

GUOQ Search Exploits Synergy

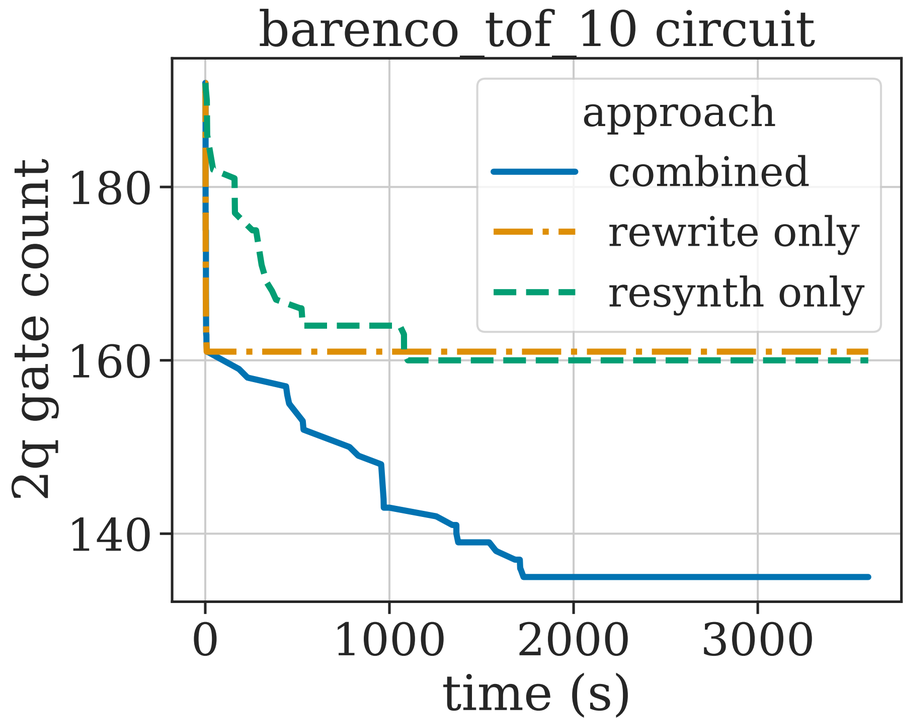


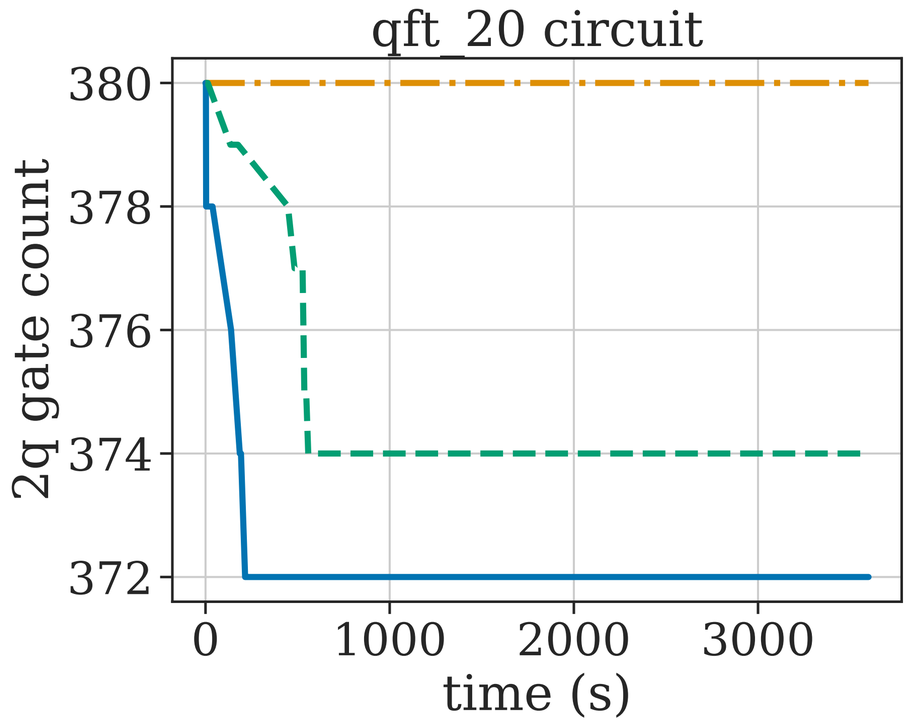
GUOQ Search Exploits Synergy





GUOQ Search Exploits Synergy





Also significantly outperforms state-of-the-art!

Come to the ASPLOS talk on April 1st!

Optimizing for FTQC

Optimizing for FTQC

Clifford + T gate set: $\{T, T^\dagger, S, S^\dagger, H, CX\}$

Optimizing for FTQC

discrete

Clifford + T gate set: $\{T, T^\dagger, S, S^\dagger, H, CX\}$

Optimizing for FTQC

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discrete

"magic state distillation"

Optimizing for FTQC

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"magic state distillation"

Eastin-Knill theorem: no transversal universal set

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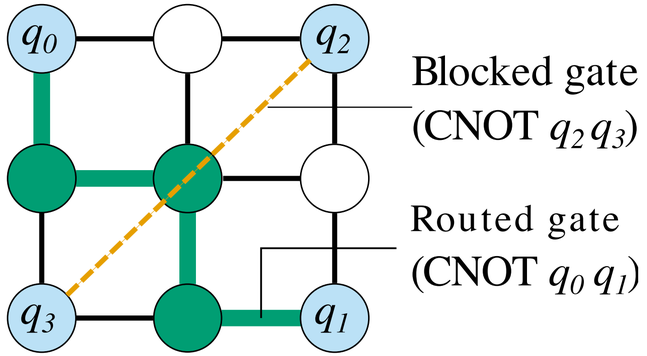
Eastin-Knill theorem: no transversal universal set



Optimizing for FTQC

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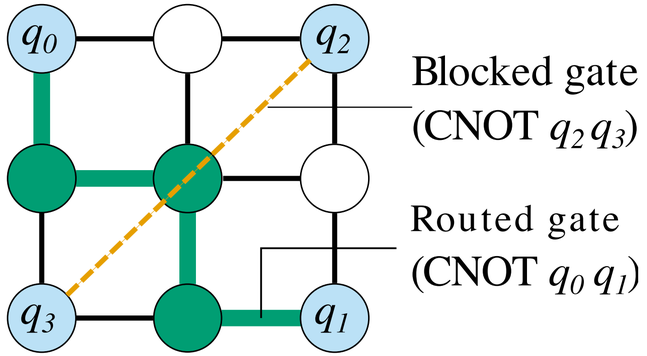

😟

BQSKit
GUOQ
VOQC
TKET
Qiskit
Quarl
Quartz
QUESO

Optimizing for FTQC

Open Research Question

Optimizing for FTQC

Open Research Question

Reducing T-count with the ZX-calculus

Aleks Kissinger and John van de Wetering

Radboud University Nijmegen
January 20, 2020

Linear and Non-linear Relational Analyses for Quantum Program Optimization

MATTHEW AMY, Simon Fraser University, Canada
JOSEPH LUNDERVILLE, Simon Fraser University, Canada