UNIVERSITY OF TORONTO

FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATIONS, APRIL 1998

Third Year – Programs 5e, 5bm, 5a

ECE 357 S – ELECTROMAGNETIC FIELDS

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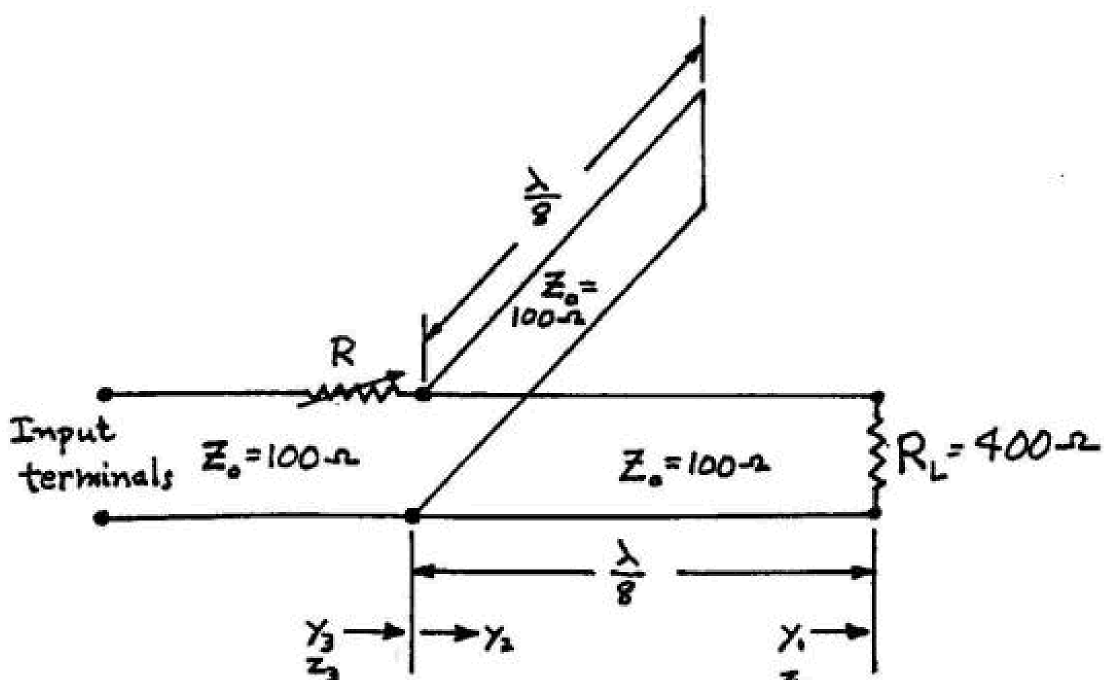
ALL QUESTIONS HAVE EQUAL VALUE. CREDIT WILL BE GIVEN FOR CLARITY. NO CALCULATORS, BOOKS OR NOTES ARE PERMITTED.

1. Solve the following problem graphically, using the Smith chart. For the transmission-line network below, state in your examination booklet (and mark clearly on the Smith chart) the values of the following normalized impedances and admittances:

   z_1, y_1, y_2, y_3, z_3.

   Suppose that R is adjusted for minimum input SWR: state that SWR, along with the value of R in ohms.

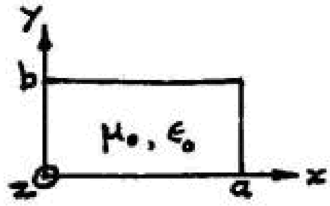


2. In the perfectly-conducting rectangular waveguide with cross-section shown at right, the total electric field strength in the TE_{10} mode is given by

$$\bar{E}(\bar{r}) = \hat{y}\sin\frac{\pi x}{a}\, e^{-jk_z z}$$

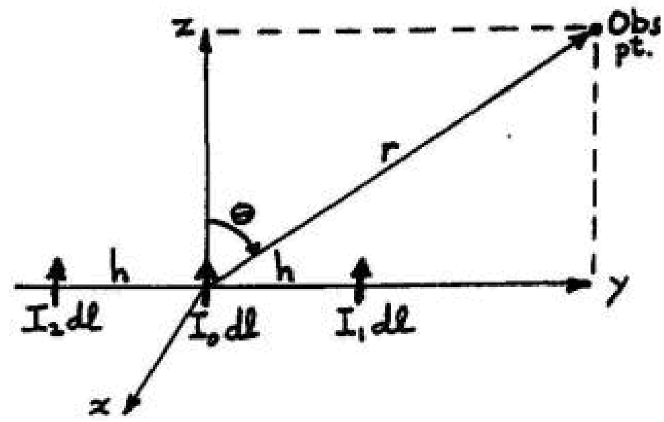
where

$$k_z = \sqrt{\omega^2\mu_0\varepsilon_0 - \left(\frac{\pi}{a}\right)^2}.$$

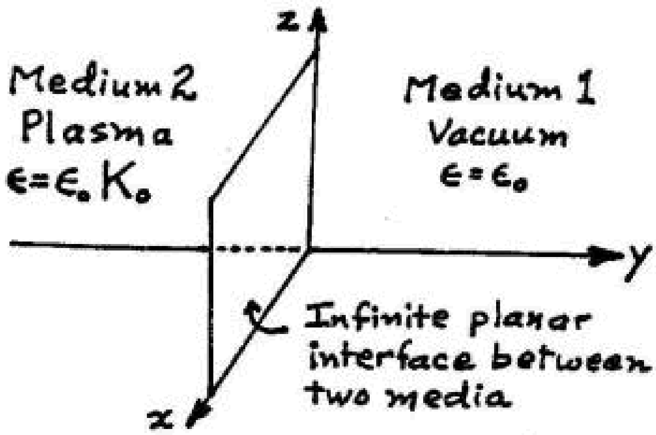


Find expressions (stating units) for the magnetic field strength $\bar{H}(\bar{r})$, and the complex Poynting vector $\bar{P}(\bar{r})$. Note the directions of the real and imaginary parts of $\bar{P}(\bar{r})$ and state why you think your results are reasonable. Then find an expression for the *total* time-average power in *watts* flowing through the waveguide and the direction in which it flows. Note that *total* power means all the power passing through any transverse cross-section of the waveguide.

3. Shown at right are three current elements having currents $I_0 = 1.0A$ and $I_1 = I_2 = 0.5A$. The frequency is ω and the surrounding lossless medium has permittivity ε_0 and permeability μ_0. Find an expression for the E_θ radiation field at a distant observation point in the $y-z$ plane. For the special case with $h = 0.75\lambda$, sketch the array pattern, the element pattern and the total radiation pattern as functions of θ, for the $y-z$ plane. Then sketch the array pattern, the element pattern and the total pattern, for each of the remaining two principal planes.



4. Consider the possible existence of an electromagnetic surface wave that propagates along the infinite planar interface between two lossless, homogeneous media and is bound to the interface, thus decaying exponentially on *both* sides of the interface. The geometry is shown below for the case in which the interface is the $x-z$ plane ($y = 0$). The two media are vacuum with $\varepsilon = \varepsilon_0$ for $y>0$ and isotropic plasma with $\varepsilon = \varepsilon_0 K_0$ for $y<0$. Note that $K_0 = 1-\omega_p^2/\omega^2$ and the plasma frequency ω_p is assumed to be known. Note also that there is no surface charge and no surface current on the interface.



You are given that the surface wave to be considered propagates in the x direction, is independent of z, and is transverse magnetic (TM). Thus the entire magnetic field strength is given by

$$\bar{H}_1(x, y) = \hat{z}H_1(x, y) = \hat{z}H_0 e^{-ay} e^{-jkx} \quad \text{for} \quad y>0$$

$$\bar{H}_2(x, y) = \hat{z}H_2(x, y) = \hat{z}H_0 e^{+by} e^{-jkx} \quad \text{for} \quad y<0$$

where a, b and k are undetermined but they must all be positive real for such a wave to exist. Notice that the given $\bar{H}$ field (with H_0 as a constant) is continuous across the interface, as a result of there being no surface current on the interface at $y = 0$.

You are asked to do the following:

(i) Use one of Maxwell's curl equations to find expressions for $\bar{E}_1(x, y)$ for $y>0$ and $\bar{E}_2(x, y)$ for $y<0$, in terms of H_0, a, b, and k.

(ii) Impose an appropriate boundary condition on the component of $\bar{E}$ tangential to the planar interface and deduce a relationship between b and a.

(iii) Note that the magnetic field strengths in each of the two media must satisfy the source-free Helmholtz equation. Use this fact to derive a relation between k, a and β_1, and a similar relation between k, b and β_2, where $\beta_1 = \omega\sqrt{\mu_0\varepsilon_0} = \beta_0$ and $\beta_2 = \omega\sqrt{\mu_0\varepsilon_0 K_0} = \beta_0\sqrt{K_0}$.

(iv) Eliminate k from the two relations from (iii), and combine this result with the result from (ii) to solve for a and b in terms of K_0 and β_0.

(v) Examine the results of (ii) and (iv) in view of the constraint that a and b must both be positive real, and thereby deduce the frequency range relative to the plasma frequency (ω relative to ω_p) in which propagation of the surface wave should be possible (note that it is not necessary to get an expression for the propagation constant k although you now have all the information needed to do so easily).