# MAT 195S
## Calculus

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*Answer all questions.*
*The value of each question appears in the margin.*
*No aids, except for the following formulas. No calculators.*

$$\int e^{ax} \cos bx \, dx \;=\; \frac{e^{ax}}{a^2 + b^2}\,(a\,\cos bx + b\,\sin bx) + C$$

$$\int e^{ax} \sin bx \, dx \;=\; \frac{e^{ax}}{a^2 + b^2}\,(a\,\sin bx - b\,\cos bx) + C$$

1. Solve $y'' + 2y' + 3y = 8\,\cos x$, $y(0) = 3$, $y'(0) = 3$.

2. Find the area enclosed by the $4n$-leafed rose $r = a\,\cos 2n\theta$. What fraction is this of the area of the circumscribing circle?

3. The portion of the curve $\gamma(t) = (e^t\,\cos t, e^t\,\sin t)$ for which $0 \le t \le \dfrac{\pi}{2}$ is revolved about the $x$-axis. Find the surface area generated.

4. (a) Classify the following series as absolutely convergent, conditionally convergent, or divergent. Give reasons.

[3]    (i) $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^{4/3}}$

[3]    (ii) $\displaystyle\sum_{k=1}^{\infty} (-1)^k \frac{\ln k}{k}$

[3]    (iii) $\displaystyle\sum_{k=1}^{\infty} \frac{7k}{\sqrt{k^3 + 3k^2 + 49}}$

[6]    (b) Find the first two nonzero terms in the Taylor series expansion of $\sec x$ in powers of $x$.

[15] 5. (a) Use the relation

$$\frac{1}{1+t} = 1 - t + t^2 - \cdots + (-1)^{n-1}t^{n-1} + (-1)^n \frac{t^n}{1+t}$$

to find the Taylor series expansion of $\ln(1+x)$ in powers of $x$, and to determine precisely for which $x$ this series converges to $\ln(1+x)$.
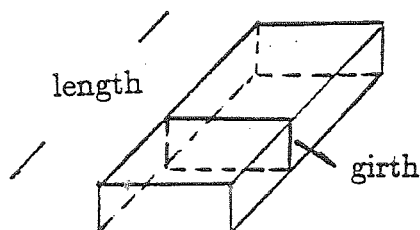
[5]    (b) Explain how the number $\ln 3$ can be calculated using Taylor series.

[5] 6. (a) Find the directional derivative of $f(x,y) = \dfrac{x^2 - y^2}{x^2 + y^2}$ at $(1,3)$ in the direction of $\dfrac{1}{2}\,\vec{i} - \dfrac{\sqrt{3}}{2}\,\vec{j}$.

[10]    (b) Find the point on the hyperbolic paraboloid $z = x^2 - 3y^2$ where the tangent plane is parallel to the plane $8x + 3y - z = 4$.

[15] 7. A package in the shape of a rectangular solid can be mailed by parcel post if the sum of the length and the girth is less than or equal to 108 inches. Find the largest volume $V$ of such a package.

length      girth