UNIVERSITY OF TORONTO

FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATIONS, APRIL 1999

Third Year – Programs 5bm(e), 5a, 5e, 5p

ECE 357 S – ELECTROMAGNETIC FIELDS

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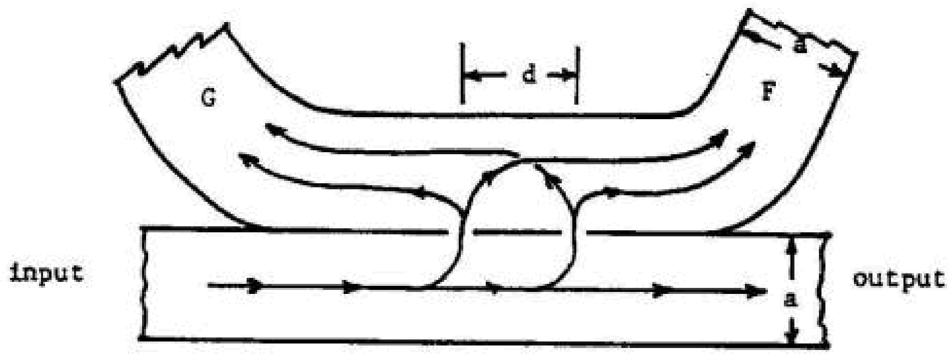
ALL QUESTIONS HAVE EQUAL VALUE. CREDIT WILL BE GIVEN FOR CLARITY.
NO CALCULATORS, BOOKS OR NOTES ARE PERMITTED.

1. A lossless transmission line with $Z_0 = 50\Omega$ is 1 m long and the phase velocity on it is $u = (2/3)c$ where c is the velocity of light in vacuum. At the sending end, the line is driven by a 0-60 V step function generator starting at $t = 0$ and having an internal resistance of 100Ω. At the receiving end, the line is terminated with a 12.5Ω resistor. Find numeric values for (a) the one-way propagation time in ns of a transient on the line, (b) the voltage reflection coefficient at the receiving end, (c) the voltage reflection coefficient at the sending end, (d) the initial current at the sending end, and (e) the final current at the sending end. Finally, (f) sketch a "bounce diagram" (time vs. distance) for the *voltage* on the line from $t = 0$ to $t = 15\text{ns}$, showing clearly the voltage for each region on the diagram.

2. The normalized impedance plane (ζ) maps into the reflection coefficient plane (Γ) through the transformation $\Gamma = \dfrac{\zeta - 1}{\zeta + 1}$ or $\zeta = \dfrac{1 + \Gamma}{1 - \Gamma}$ where $\Gamma = u + jv$ and $\zeta = r + jx$. Consider the line $u = 2v$ in the Γ plane, and show that it maps into a circle in the ζ plane by finding the equation of the circle. Deduce the radius of the circle and state the coordinates of its center. Make appropriate sketches of the ζ and Γ planes, and indicate three corresponding points on the line and circle referred to above.

A waveguide directional coupler can be constructed as shown below, using rectangular waveguides which carry only the TE_{10} mode. Two waveguides having the same transverse dimensions share a common narrow wall, the shared wall having width b and the other wall (the broad one) having width a. Two small holes are drilled in the common wall. For a given hole spacing d the contributions from the two holes will always add at point F. However, it is possible that these two contributions will cancel at point G: this is the directional coupler condition.

Question: for the directional coupler condition, express the distance d in terms of the free-space wavelength λ and the waveguide dimensions, noting that λ is the wavelength of a uniform plane wave measured in its direction of propagation (i.e. its wave-normal direction).



arrows indicate power flow

4. A medium-frequency linear array of vertical antenna towers is required to have a horizontal-plane radiation pattern that is unidirectional in the $\phi = 0$ direction where ϕ is the angle with respect to the array axis. The pattern is to have nulls in the $\phi = 180°$ and $\phi = \pm 120°$ directions, at least.

(a) Under the assumption that a *uniform linear array* is appropriate, use a graphical method to select a minimum number of elements n, a progressive phase shift α radians between adjacent elements, and an inter-element spacing d (expressed in terms of the wavelength λ) that together will satisfy these requirements.

(b) Suppose that the tower farthest from the origin of coordinates blows down in a storm, leaving the excitation unchanged for the remaining towers. Sketch the radiation pattern.

3. Consider a fully-ionized, collisionless plasma in which both the electrons and the ions are mobile. The total particle current density is then the sum of the electron and ion current densities, that is, $\bar{J} = \bar{J}_e + \bar{J}_i$ (in the lectures, we discussed only $\bar{J}_e$). Take the particle properties to be

	electrons	ions
mass	m_e	m_i
charge	$-e$	$+e$
density	N	N
velocity	$\bar{v}_e$	$\bar{v}_i$
plasma frequencies ω_e and ω_i	$\omega_e^2 = \dfrac{Ne^2}{m_e \varepsilon_0}$	$\omega_i^2 = \dfrac{Ne^2}{m_i \varepsilon_0}$

(a) Find an expression for the relative permittivity in terms of ω_e, ω_i and ω.

(b) State the propagation constant for a uniform plane wave in this medium and state its cutoff frequency. For the special case $m_i = 500\, m_e$, indicate by what approximate percentage the cutoff frequency has been changed relative to the case in which ions were assumed to be immobile.