Name_____

1. (a) Evaluate the indefinite integral $\displaystyle\int \frac{x^2\,dx}{(1-x^2)^{1/2}}$

(b) Calculate $\displaystyle\int_0^1 x^2 \arctan x\; dx$

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1. (c) Evaluate the indefinite integral $\int \frac{dx}{e^x + 1}$.

(d) Does the improper integral $\int_0^\infty \frac{dx}{e^{x^2} + 1}$ exist?

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2. Calculate the following limits

(a) $\lim_{n\to\infty} \left(1-\frac{1}{n^2}\right)^n$

(b) $\lim_{x\to\infty} x\, ln\left(1-\frac{1}{x}\right)$

(c) $\lim_{x\to\frac{1}{2}\pi} \frac{\sin(x\ \cos x)}{\cos(x\ \sin x)}$

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4. For what values of x are the following series convergent:

(a) $\displaystyle\sum_{k=1}^{\infty} \frac{2^{kx}}{k}$

(b) $\displaystyle\sum_{k=0}^{\infty} \frac{x^k}{k^2+1}$

(c) $\displaystyle\sum_{k=0}^{\infty} \frac{x^k}{1+a^k}$, where $a > 0$ is a constant.

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4. (a) Find the Taylor series for x^{-1} in powers of $(x-2)$.

(b) Use binomial series to calculate $\int_0^{0.8} (1-x^4)^{1/5}\,dx$ to three decimal places.

5. Show that $y(x,t) = t^{1/2}F(xt^{-1/2})$ satisfies the partial differential equation $y_{xx} = y_t$ when $F(z)$ satisfies the ordinary differential equation

$$F'' + \tfrac{1}{2}zF' - \tfrac{1}{2}F = 0 \qquad \text{where} \qquad z = xt^{-1/2} .$$

6. A particle moves along the curve given parametrically by $\underset{\sim}{r} = (t^2 \cos t,\ t^2 \sin t,\ 2t)$, where $t \geq 0$ is the time.

(a) Briefly describe this curve geometrically. spiral

(b) What is the speed of the particle at time t?

(c) At what angle does this curve intersect the plane $z = 2$?

(d) At what angle does this curve intersect the circular cylinder $x^2 + y^2 = 4$?

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7. Find the tangent plane to the surface (in the first octant $x, y, z \geq 0$) $x^{1/2} + y^{1/2} + z^{1/2} = a^{1/2}$ at the point (x_0, y_0, z_0). If this plane intersects the co-ordinate axes at $(x_1, 0, 0)$, $(0, y_1, 0)$, $(0, 0, z_1)$, find $x_1 + y_1 + z_1$.

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8. The temperature distribution in a plate at the point (x, y) is given by $T(x, y) = 200 + 3x^2 - 3y^2$. Find the path a heat seeking particle (which always moves in the direction of greatest increase in temperature) would follow if it starts at the point (X, Y).