UNIVERSITY OF TORONTO

Faculty of Applied Science and Engineering

Final Examination

Third Year — Program 5a

AER301F — Dynamics

Examiner: G M T D'Eleuterio

13 December 2001

Student Name

Last Name First Name

Student Number

Instructions:

1. Attempt *all* questions
2. The value of each question is indicated in the table opposite.
3. Write the final answers *only* in the boxed space provided for each question
4. This is a Type D examination. Lecture and student notes are permitted
5. Type I calculators are permitted
6. There are 13 pages and 8 problems in this examination paper

For Examiner Only		
Problem	**Value**	**Mark**
A		
1	20	
B		
2	5	
3	5	
4	5	
5	5	
C		
6	10	
7	20	
8	30	
Total	100	

A. True or False

Determine if the following statements are true or false and indicate by "T" (for true) and "F" (for false) in the box beside the question. The value of each question is 2 marks.

1(a). The time-rate of change of momentum is the same regardless of reference frame.

1(b). The Earth is an inertial reference frame.

1(c). If a ball were dropped from the Leaning Tower of Pisa and air resistance were not a factor, the ball would fall slightly to the west of the point from which it was dropped.

1(d). A scale measuring the weight of a goldfish and a bowl filled with water will read less if the goldfish is in the bowl than if it is placed on the scale beside the bowl.

1(e). If the rules of the road in Canada were changed to make everyone drive on the other side of the road (*i.e.*, to drive on the left side instead of the right), the rotation rate of the Earth would slightly increase.

1(f). A nonlinear spring with restoring force $f_s = -k_1 x + k_3 x^3$ is conservative.

1(g). Lagrange's equations can be applied in any reference frame.

1(h). Lagrange multipliers can be interpreted as the forces or torques required to maintain the constraints.

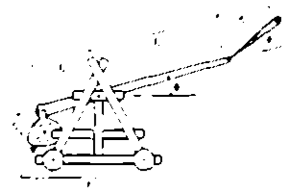
1(i). The minor-axis spin of any body is stable.

1(j). A zero natural frequency signifies the presence of a rigid-body mode.

B. Questions

Provide short answers. Qualitative arguments are acceptable.

2. A trebuchet was a medieval catapult consisting of a lever arm with a sling holding the projectile at one end and a very large counterweight at the other. It was discovered that the range of the projectile was significantly increased if the platform were free to roll as opposed to being fixed. Why?



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3. What will happen to each of the cannonballs after the fired ball strikes the others on the table? Assume the balls are all identical and moreover that all collisions are perfectly elastic and head-on. (The figure is modified from Marcus Marci's *De proportione motus*.)

/5

4. A satellite is in orbit around the Earth with a substantial inclination to the equatorial plane. How will the orbit be affected by the oblateness of the Earth?

/5

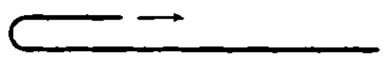
5. When we whirl an object at the end of a tether we instinctively make a small circle with the hand holding the tether. Why?

/5

C. Problems

Not-so-short answers may be required.

6. A 5-m carpet runner (long and narrow for hallways) weighing 100 N is pulled back at a rate of 1 m s^{-1}. Determine the total work required to pull back the runner completely. (Assume $g = 10$ m s^{-2} and that the pulled portion of the runner does not drag on the floor nor on any other portion of the runner. The runner is assumed to have padding on the underside that prevents slipping.)



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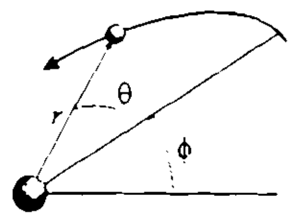
7. Two bodies of mass m_1 and m_2 are mutually attracted by gravity. Derive the equations of motion in three dimensions. Use the coordinates r, θ and ϕ.



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7. . . . cont'd

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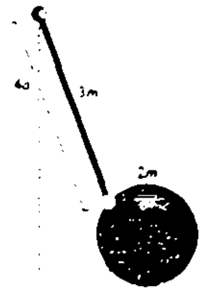
8. A double pendulum consists of a uniform rigid rod of length $4a$ and mass $3m$ and a uniform rigid disk of radius a and mass $2m$.



(*a*) Determine the second moment of mass (inertia) of the rod about an end point and of the disk about its center.

(*b*) Derive an expression for the kinetic energy. (Identify your coordinates and do not linearize!)

(*c*) Derive an expression for the potential energy. (Do not linearize!)

(*d*) Obtain the equations of motion. (Do not linearize!)

(*e*) Determine the linearized equations of motion. (Linearize!)

(*f*) Determine the natural frequencies of vibration.

(*g*) Determine and describe the natural modes of vibration.

8(a). Determine the second moment of mass (inertia) of the rod about an end point and of the disk about its center.

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8(b). Derive an expression for the kinetic energy. (Identify your coordinates and do not linearize!)

/4

8(c). Derive an expression for the potential energy. (Do not linearize!)

/2

8(d). Obtain the equations of motion. (Do not linearize!)

. . . cont'd

8. . . . cont'd

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8(e). Determine the linearized equations of motion. (Linearize!)

/4

8(f). Determine the natural frequencies of vibration.

/4

8(g). Determine and describe the natural modes of vibration.

/6

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