UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING
FINAL EXAMINATION, 9:30 AM, APRIL 23, 1996

First Year - Program 5

ECE150S - ELECTRICITY AND MAGNETISM

Examiner: A. Konrad

Question 1.

(a) State the integral form of Gauss's law for electric fields define <u>all</u> the quantities involved, showing their SI units.

(b) A spherical, metallic shell of 10 cm inner radius and 11 outer radius has a 12 nC positive charge located at center. There are no other charges within the 10 cm in radius. The electric field at a radius of 25 cm is everywh 800 V/m and is directed <u>towards the center</u> of the shell. Draw a sketch of the problem geometry; (ii) Find the charge, Q_{net}, of the metallic shell; (iii) Find the surf charge density, σ_{in}, on the inner surface of the metal shell; (iv) Find the surface charge density, σ_{out}, on the outer surface of the metallic shell; (v) Find the elect field at a radius of 5 cm; (vi) Find the electric field a radius of 10.5 cm.

Question 2.

Consider a parallel plate capacitor with negligible fringing. plate areas are 0.2 m², and the plates are 0.01 m apart. capacitor is connected to a battery and charged to a poten difference of V_b = 3000 volts. Then it is disconnected from battery, and a sheet of insulating plastic material is inse between the plates, completely filling the space between them. voltmeter indicates that the potential difference decreases V_a = 1000 volts.

(a) Compute the capacitance, C, and the magnitude of charge, q, each plate <u>before</u> and <u>after</u> the insertion of the plas material between the plates. Find the <u>relative</u> permittivity the plastic material.

(b) Compute the electric fields E_b and E_a, and the ene densities u_b and u_a, <u>before</u> and <u>after</u> the insertion the plastic material, respectively.

(c) Compute the forces F_b and F_a, with which the two paral plates attract each other <u>before</u> and <u>after</u> the insertion the plastic material, respectively.

Question 3.

(a) State the law of Biot and Savart and define all the quantiti... involved together with their SI units.

(b) State the relationship between magnetic flux density $\overline{B}$ a... magnetic field intensity $\overline{H}$ in free space.

(c) A wire carries a current i and consists of a circular arc c... radius R and central angle $\pi/2$ radians, and two very lo... straight sections whose extensions intersect the center C c... the arc. (i) Draw a sketch of the problem geometry; (ii) I... $R = 17$ cm, find the current i which produces a magnetic fie... intensity $H = 0.017$ A/m at the center C of the arc.

Question 4.

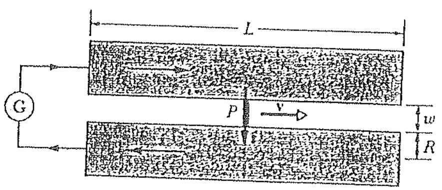
Consider a simple rail gun consisting of two straight, rigid parallel, conducting rails of circular cross-section, each havin... radius R and length L as shown below. The distance between th... two rails is w. A conducting projectile P of length w make... electrical contact with the circular rails. At the left end th... rails are connected to a source which sends a current i throug... the rails and the projectile.

(a) Assuming that $L >> R$, $w < R$, and the positive x-coordinat... direction is to the right, show that the force on the projec... tile is approximately given by

$$\overline{F} = \frac{\mu_o i^2}{2} \ln\left(1 + \frac{w}{R}\right) \overline{a}_x$$

(Hint: Treat the rails as infinitely long, thin straight wire... separated by a distance of $2R + w$).

(b) Given that $L = 6$ m, $R = 10$ cm, and $w = 1$ cm, compute the current i required to accelerate a projectile of mass $m = 12$ g, from rest at $x = 0$ (left end of the rails) to a speed of 3.... km/s at $x = L$ (right end of the rails). (Hint: The work done by $\overline{F}$ equals the change in kinetic energy of the projectile.)



WRITE YOUR FINAL ANSWER TO QUESTION 4(b) IN THE BOX!

i =	A

Question 5.

A tightly-wound, short-circuited, flat, square coil of si... length 0.05 m is lying in the xy-plane at a distance of 0.025 both from the x-axis and from the y-axis. The zy-plane is ... infinite current sheet with a time-dependent surface curre... density of $K(t) = 2000t\ \overline{a}_y$ amperes per meter for time $t > 0$; K = ... for $t < 0$.

(a) Let the positive y-axis point towards the top of the page. With the coil located to the right of the current shee... draw a sketch of the current sheet and coil arrangeme... showing all dimensions.

(b) Find the magnetic flux density $\overline{B}(t)$, due to the current shee... Name the law used to find $\overline{B}(t)$.

(c) Find the magnetic flux $\Phi_B(t)$, due to the current shee... passing through the coil.

(d) Find the induced emf $\mathcal{E}$ in the coil, if the coil consists ... $N = 15$ turns. Name the law used to find $\mathcal{E}$.

(e) What is the differential equation that governs the behavio... of the induced current $I(t)$ in the coil? Find $I(t)$ if th... resistance and self-inductance of the coil are 1.65 ohm a... 0.05 millihenry, respectively. Is the current $I(t)$ in the co... clockwise or counter-clockwise? Name the law used to arriv... at the answer.

Question 6.

Two thin, long parallel wires, each of radius a, whose centers a... a distance d apart carry equal currents in opposite direction...

(a) Show that, neglecting the flux within the wires themselve... the inductance of a length l of such a pair of wires ... given by

$$L = \frac{\mu_o l}{\pi} \ln\left(\frac{d - a}{a}\right)$$

(b) Compute L for $l = 1.5$ m, $d = 10$ cm, and $a = 1$ mm.

(c) Find the radius r of the ideal solenoid with 1000 turns/mete... whose inductance per length l is the same as the inductance ... the parallel wires computed in (b).

WRITE YOUR FINAL ANSWER TO QUESTIONS 6(b) AND 6(c) IN THE BOXES!

(b)	L = mH	(c)	r = m