UNIVERSITY OF TORONTO

FACULTY OF APPLIED SCIENCE AND ENGINEERING
FINAL EXAMINATION, DECEMBER 2001

Second Year - Program 6
CHE221F - CALCULUS AND NUMERICAL METHODS
Exam Type : C
Examiner : C.M. Yip

Student First Name: ______________________________

Student Last Name: ______________________________

Student Number: ______________________________

- Attempt all problems
- Calculator Type 2 - All non-programmable calculators are allowed. No programmable calculators are permitted.
- Each candidate may use both sides of a single authorized aid sheet.
- **All work must be done on these sheets. Do not separate the sheets.**
- Use the back of the sheets if you need more space. Be sure to **clearly** indicate if your answer continues elsewhere. **Draw a box around your answer**.
- Read each question carefully. The exam lasts 2 hr and 30 minutes.

Question 1	/15
Question 2	/20
Question 3	/10
Question 4	/10
Question 5	/10
Question 6	/15
Question 7	/10
Question 8	/10

Question 1 (15 points possible)

(a) Identify the maximum and minimum values for the following:

$$f(x,y,z) = x + 2y \text{ subject to the following constraint equations}$$

$$g(x,y,z) = x + y + z - 1$$

$$h(x,y,z) = y^2 + z^2 - 4$$

(b) Let $f(x; y; z) = xy + yz + zx$, and let P be the point $P(1; 2; 3)$.

- Find the direction in which the function is increasing the most rapidly at the point P, and the corresponding rate of increase.
- Find one direction in which the directional derivative of the function at P is 0.
- Find the directional derivative of f in the direction of $\mathbf{b} = 2\mathbf{i}+4\mathbf{j} - 10\mathbf{k}$ at P.
- Find the equations of the tangent plane and normal line to the surface $f(x; y; z) = 11$ at P.
- Estimate the value of $f(1{:}1; 1{:}8; 3{:}2)$.

Question 2. (20 points possible)

Evaluate the following

(a)

$$\int_0^1\int_0^1 \frac{xy}{\sqrt{x^2+y^2+1}}\,dydx$$

(b)

Calculate the line integral of $\mathbf{F}(x,y)=\left(2x\sin y, x^2\cos y-3y^2\right)$

along the straight line from (-1,0) to (5,1)

Question 3. (10 points possible)

Consider the surface parameterized by

$\mathbf{r}(u,v) = \left(u\cos v, u\sin v, u^2\right)$

where $0 \le u \le 1$ and $0 \le v \le 2\pi$

(a) Sketch this surface

(b) Calculate the surface area.

Question 4. (10 points possible)

Prove the Divergence Theorem for the region bounded by the cone

$z = \sqrt{x^2 + y^2}$ and the plane $z = 2$ and $\mathbf{F}(x, y, z) = (x, 3xz, y)$

Question 5. (10 points possible)

(a)

The curve C has parameterization

$\mathbf{r}(t) = t^2\mathbf{i} + \cos(t)\mathbf{j} - \mathbf{k}$

Find the unit tangent vector $\mathbf{T}(t)$

(b)

For $f(x, y) = x^2 y$, evaluate the following :

(1) Gradient of $f(x, y)$

(2) Find the directional derivative of $f(x, y)$ at the point $(2,3)$ in the direction of the vector $\mathbf{i} - \mathbf{j}$

Question 6. (15 points possible)

A rectangular solid has dimensions a, b, c. Its density is 1.

(a) Find its moment of inertia about an edge of length c

(b) Suppose the dimensions of the box are $a = 1, b = 2, c = 3$. About which edge would the moment of inertia be the greatest? About which edge would the moment of inertia be the least?. Predict the answer by physical intuition and then verify it by using the formula you found in part (a)

Question 7. (10 points possible)

Verify Green's Theorem for the disk D defined by $x^2+y^2 \leq R^2$
given the following functions

$$P(x,y) = x+y,$$
$$Q(x,y) = y$$

Question 8. (10 points possible)

(a)

Find the volume under the surface $z = x^2 y$ and above the region in the xy - plane bounded by the curves $y = 0$ and $y = 1 - x^2$

(b)

Find the volume under the graph $z = f(x, y) = \cos x \cos(\cos y)$ and over the triangle in the x - y plane with vertices $(0,0)$, $(0,1)$ and $(1,1)$.