

UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING
FINAL EXAMINATIONS, APRIL 2001

CHE 507S - PROCESS MODELLING AND SIMULATION
Examiner - R. Luus

Answer all questions. All questions are of equal value.

Name _____

Student Number _____

Marks report

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Total number of pages = 7.
Please check that all pages are in your exam booklet.
Do not unstaple the pages.

1.(a) Show how the following set of 4 equations in 4 unknowns can be arranged into two groups, one consisting of 3 simple equations and the other group consisting of a single difficult equation.

$$x_1 + x_2 + 2x_3 + 5x_4 = 2$$

$$\cos x_2 + x_3 + e^{x_4} + \sin x_1 = 2$$

$$x_2^2 + 2x_4 - x_1^3 = 5$$

$$x_2 + x_1^2 - \sin x_1 = 1.$$

(b) Give an efficient procedure for solving this set of equations. Be as explicit as possible, giving an algorithm that can be converted readily into a computer program. Include the choice of the starting conditions.

2. (a) Parameter estimation is very important in chemical engineering because we frequently know the structure of the model and wish to obtain values for the parameters through experimental measurements. Discuss parameter estimation, pointing out some advances made in recent years.
- (b) Discuss model reduction as quantitatively as possible, using equations where necessary.

3. (a) Solve $(4D^2 - 4D + 1)y = \sin 3t$

(b) By using Laplace transform, solve, with the initial condition $y(0) = 1$, $y'(0) = 2$, the differential equation $y'' - 6y' + 13y = 26$.

4. (a) By using standard notation, show that under suitable assumptions the mass balance for a component flowing down a bed packed with adsorbent is

$$\frac{\partial c}{\partial x} + \frac{\partial n}{\partial y} = 0 \text{ where } y = \bar{v}t - \alpha x.$$

(b) In predicting the isotherm from the output, we define $\tau = y/x$ and assume equilibrium $n = f(c)$. Show that then the equation in (a) becomes

$$[f'(c) - \tau] \frac{\partial c}{\partial \tau} = 0$$

and show how the isotherm may be obtained.

5. (a) Find the transition matrix for the system

$$\frac{d\mathbf{x}}{dt} = \mathbf{A}\mathbf{x} \quad \text{where} \quad \mathbf{A} = \begin{pmatrix} -1 & 2 \\ -2 & 4 \end{pmatrix}$$

(b) By using the transition matrix in (a), solve the equation

$$\frac{d\mathbf{x}}{dt} = \mathbf{A}\mathbf{x} + \mathbf{b} \quad \text{where} \quad \mathbf{b} = \begin{pmatrix} 54te^{3t} \\ 9e^{3t} \end{pmatrix}$$

with the initial condition $\mathbf{x}(0) = \begin{bmatrix} 1 & 3 \end{bmatrix}^T$.

6. By feeding a diet of goat milk, carrots and viagra to specially cloned rabbits, a researcher found that the breeding rate exceeded the breeding rate of rabbits predicted by the Fibonacci series. She found that the number of these rabbits gave the series:

1, 2, 5, 12, 29, 70, ...

(a) What is the next term in this series?

(b) Obtain an expression for the n th term y_n , giving the number of rabbits after n years.

(c) Suppose the lab can hold only 10,000 rabbits; how many years will it take to reach this maximum number of rabbits?

(d) Find the limit as $n \rightarrow \infty$ of the ratio y_{n+1} / y_n .