UNIVERSITY OF TORONTO

FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION

ECE 424F Microwave Circuits

December 20, 2001

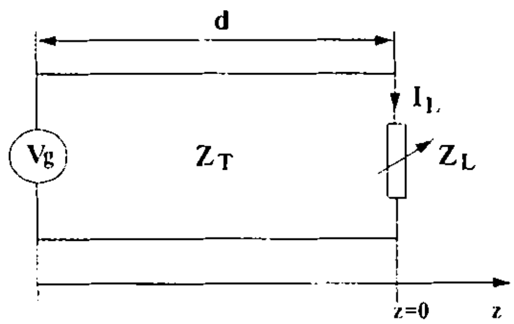
9:30-12:00 a.m.
Exam Type D

Examiner: George V. Eleftheriades

Total Points: 100

Problem 1. *(25 points)*

A load Z_L is fed by a voltage source $V_g = j25\,V$ through an ideal transmission line of character-



istic impedance Z_T. The transmission line is air-filled and the operating frequency is 3 GHz.

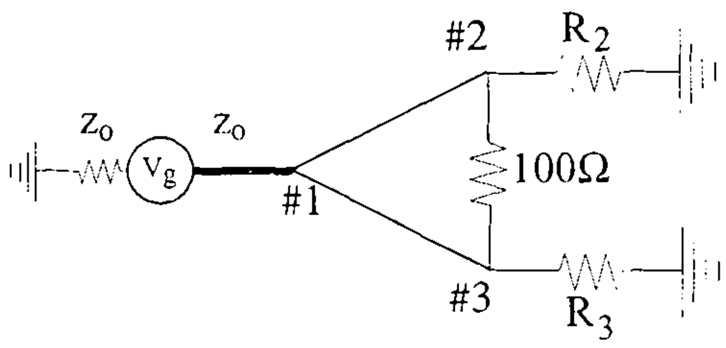
(1) Calculate the phase velocity V_ϕ and propagation constant β along the transmission line.

(2) Write general equations in terms of z *(distance)*, Z_T, β and Z_L describing the voltage $V(z)$ and current $I(z)$ along the transmission line $-d \le z \le 0$. Leave your expressions in symbolic form.

(3) Determine the length d and the characteristic impedance Z_T so that the load current becomes $I_L = 0.25A$ and is INDEPENDENT of the load Z_L (which may be variable). For your convenience, choose (but justify) your answer among the following electrical lengths: $\lambda/8$, $\lambda/4$, $\lambda/2$ and λ .

Problem 2. *(25 points)*

You are given a Wilkinson power divider which is excited from port #1 by a voltage source $V_g = 10V$ and is terminated with two resistive loads $R_2 = 150\Omega$ and $R_3 = 350\Omega$ as shown below. The system characteristic impedance is $Z_o = 50\Omega$.

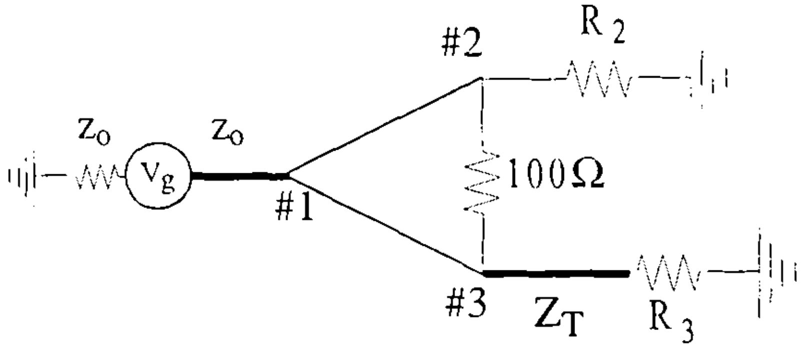


(1) Find the power available from the source.

(2) Find the power delivered to each one of the load resistors R_2 and R_3.

(3) How much power is it dissipated on the 100Ω resistor?

A piece of a transmission line is now inserted at port #3 as shown below.

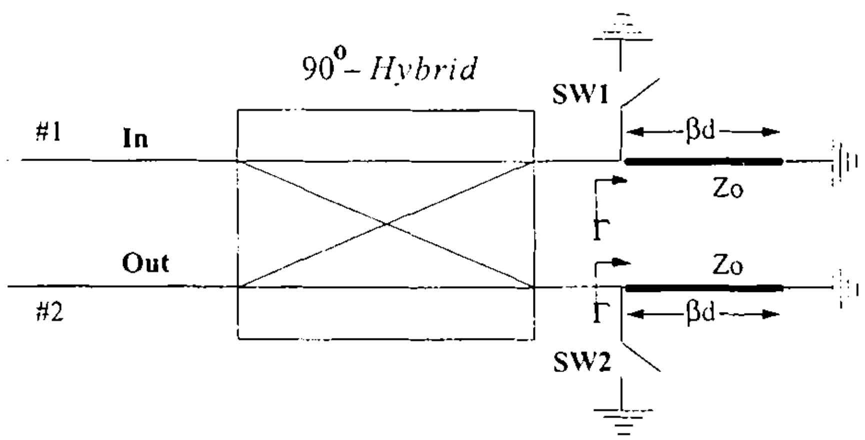


(4) Determine the characteristic impedance Z_T and length (in wavelengths) of the inserted line at port #3 so that NO power is dissipated by the 100Ω resistor.

Problem 3.

(25 points)

Consider the 2-port device shown below. The 90^o-hybrid is assumed ideal (i.e. infinite directivity). As well, the switches SW1 and SW2 are assumed identical and ideal (i.e. exact shorts in the ON state and exact opens in the OFF state). The system characteristic impedance is $Zo = 50\Omega$ and is equal to the characteristic impedance of the terminating lines βd.



(1) Calculate the transmission and reflection coefficients S_{21} and S_{11} when both switches are ON.

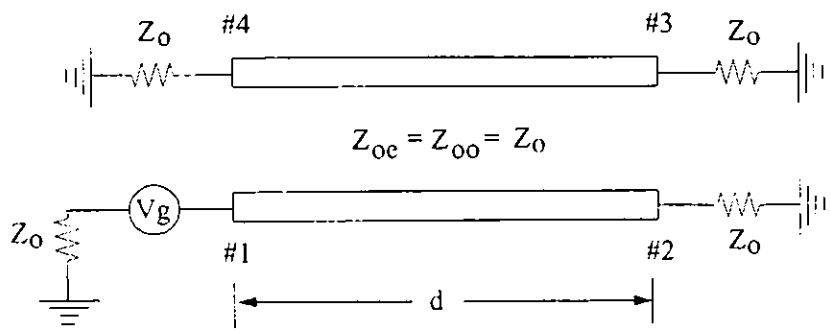
(2) Calculate the transmission and reflection coefficients $\bar{S}_{21}$ and $\bar{S}_{11}$ when both switches are OFF. Express your result in terms of the reflection coefficient Γ at the input of each terminating line βd.

(3) Calculate the transmission line length d (in wavelengths) so that the phase difference between the OFF and ON transmission coefficients is 225^o.

(4) Suggest a possible application for this device.

Problem 4. *(25 points)*

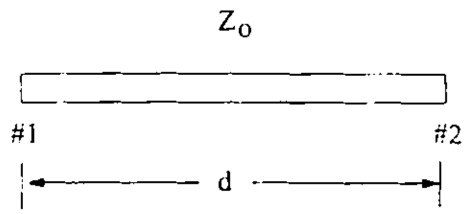
Consider the coupled line arrangement shown below. The lines are apart far enough so that the even and odd characteristic impedances are EQUAL to the isolated line impedance Z_o i.e. $Z_{oe} = Z_{oo} = Z_o = 50\Omega$. However, the even propagation constant β_e is DIFFERENT from the odd propagation constant β_o. As shown, port #1 is excited by a voltage source $V_g = 10V$.



(1) Calculate the available power from the voltage source V_g.

(2) Calculate the power delivered to port #1.

(3) Calculate the reflection and transmission coefficients S_{11} and S_{21} for a line of characteristic impedance Z_o , propagation constant β and length d, as shown below. The system impedance is also Z_o. Note: This is an auxiliary calculation for completing step #5 below.



(4) Decompose the original coupled line problem into even and odd mode excitations.

(5) Using even/odd mode analysis, calculate,

(a) The power delivered to port #4.

(b) The length of the line d so that maximum power is transferred to Port #3 and zero power to Port #2. Leave your answer in terms of β_c and β_o. In this case, how much power is delivered to Port#3?

USEFUL INFORMATION

Impedance transformation $$Z_{in} = Z_o \frac{Z_L + jZ_o \tan(\beta d)}{Z_o + jZ_L \tan(\beta d)}$$

Generalized Reflection Coefficient $$\Gamma = \Gamma_L e^{-2j\beta d}$$

$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}$$

Condition for a lossless network: $$[S]^T [S]^* = [U]$$

Speed of light in vacuum: $c = 3 \times 10^8 m/s$

Scattering Matrix of a Wilkinson Power Divider:

$$[S] = -j/\sqrt{2}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Available power from a matched generator: $P_{av} = |V_g|^2/(8Z_o) = |a_1|^2/2$

Make sure you answer ALL questions!

GOOD LUCK

The usefulness of the $ABCD$ matrix representation lies in the fact that a library of $ABCD$ matrices for elementary two-port networks can be built up, and applied in building-block fashion to more complicated microwave networks that consist of cascades of these simpler two-ports. Table 4.1 lists a number of useful two-port networks and their $ABCD$ matrices.

EXAMPLE 4.6 Evaluation of $ABCD$ Parameters

Find the $ABCD$ parameters of a two-port network consisting of a series impedance Z between ports 1 and 2 (the first entry in Table 4.1).

TABLE 4.1 The $ABCD$ Parameters of Some Useful Two-Port Circuits

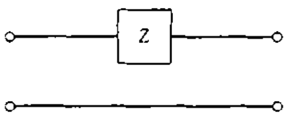
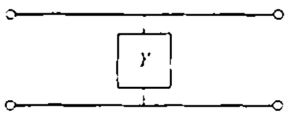
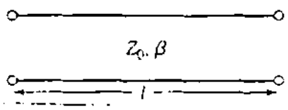
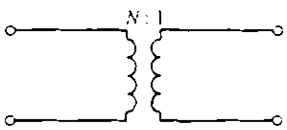
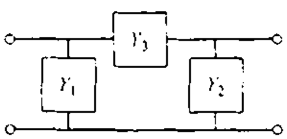
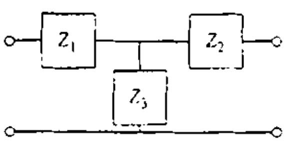
Circuit	$ABCD$ Parameters	
	$A = 1$ $C = 0$	$B = Z$ $D = 1$
	$A = 1$ $C = Y$	$B = 0$ $D = 1$
	$A = \cos \beta l$ $C = jY_0 \sin \beta l$	$B = jZ_0 \sin \beta l$ $D = \cos \beta l$
	$A = N$ $C = 0$	$B = 0$ $D = \frac{1}{N}$
	$A = 1 + \frac{Y_2}{Y_3}$ $C = Y_1 + Y_2 + \frac{Y_1 Y_2}{Y_3}$	$B = \frac{1}{Y_3}$ $D = 1 + \frac{Y_1}{Y_3}$
	$A = 1 + \frac{Z_1}{Z_3}$ $C = \frac{1}{Z_3}$	$B = Z_1 + Z_2 + \frac{Z_1 Z_2}{Z_3}$ $D = 1 + \frac{Z_2}{Z_3}$

TABLE 4.2 Conversions Between Two-Port Network Parameters

	S	Z	Y	$ABCD$
S_{11}	S_{11}	$\dfrac{(Z_{11}-Z_0)(Z_{22}+Z_0)-Z_{12}Z_{21}}{\Delta Z}$	$\dfrac{(Y_0-Y_{11})(Y_0+Y_{22})+Y_{12}Y_{21}}{\Delta Y}$	$\dfrac{A+B/Z_0-CZ_0-D}{A+B/Z_0+CZ_0+D}$
S_{12}	S_{12}	$\dfrac{2Z_{12}Z_0}{\Delta Z}$	$\dfrac{-2Y_{12}Y_0}{\Delta Y}$	$\dfrac{2(AD-BC)}{A+B/Z_0+CZ_0+D}$
S_{21}	S_{21}	$\dfrac{2Z_{21}Z_0}{\Delta Z}$	$\dfrac{-2Y_{21}Y_0}{\Delta Y}$	$\dfrac{2}{A+B/Z_0+CZ_0+D}$
S_{22}	S_{22}	$\dfrac{(Z_{11}+Z_0)(Z_{22}-Z_0)-Z_{12}Z_{21}}{\Delta Z}$	$\dfrac{(Y_0+Y_{11})(Y_0-Y_{22})+Y_{12}Y_{21}}{\Delta Y}$	$\dfrac{-A+B/Z_0-CZ_0+D}{A+B/Z_0+CZ_0+D}$
Z_{11}	$Z_0\dfrac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	Z_{11}	$\dfrac{Y_{22}}{\lvert Y\rvert}$	$\dfrac{A}{C}$
Z_{12}	$Z_0\dfrac{2S_{12}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	Z_{12}	$\dfrac{-Y_{12}}{\lvert Y\rvert}$	$\dfrac{AD-BC}{C}$
Z_{21}	$Z_0\dfrac{2S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	Z_{21}	$\dfrac{-Y_{21}}{\lvert Y\rvert}$	$\dfrac{1}{C}$
Z_{22}	$Z_0\dfrac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	Z_{22}	$\dfrac{Y_{11}}{\lvert Y\rvert}$	$\dfrac{D}{C}$
Y_{11}	$Y_0\dfrac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\dfrac{Z_{22}}{\lvert Z\rvert}$	Y_{11}	$\dfrac{D}{B}$
Y_{12}	$Y_0\dfrac{-2S_{12}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\dfrac{-Z_{12}}{\lvert Z\rvert}$	Y_{12}	$\dfrac{BC-AD}{B}$
Y_{21}	$Y_0\dfrac{-2S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\dfrac{-Z_{21}}{\lvert Z\rvert}$	Y_{21}	$\dfrac{-1}{B}$
Y_{22}	$Y_0\dfrac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\dfrac{Z_{11}}{\lvert Z\rvert}$	Y_{22}	$\dfrac{A}{B}$
A	$\dfrac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{2S_{21}}$	$\dfrac{Z_{11}}{Z_{21}}$	$\dfrac{-Y_{22}}{Y_{21}}$	A
B	$Z_0\dfrac{(1+S_{11})(1+S_{22})-S_{12}S_{21}}{2S_{21}}$	$\dfrac{\lvert Z\rvert}{Z_{21}}$	$\dfrac{-1}{Y_{21}}$	B
C	$\dfrac{1}{Z_0}\dfrac{(1-S_{11})(1-S_{22})-S_{12}S_{21}}{2S_{21}}$	$\dfrac{1}{Z_{21}}$	$\dfrac{-\lvert Y\rvert}{Y_{21}}$	C
D	$\dfrac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{2S_{21}}$	$\dfrac{Z_{22}}{Z_{21}}$	$\dfrac{-Y_{11}}{Y_{21}}$	D

$|Z| = Z_{11}Z_{22} - Z_{12}Z_{21}$; $|Y| = Y_{11}Y_{22} - Y_{12}Y_{21}$; $\Delta Y = (Y_{11}+Y_0)(Y_{22}+Y_0) - Y_{12}Y_{21}$; $\Delta Z = (Z_{11}+Z_0)(Z_{22}+Z_0) - Z_{12}Z_{21}$; $Y_0 = 1/Z_0$