UNIVERSITY OF TORONTO

FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION — 15 December 1997

FOURTH YEAR GASDYNAMICS — AER–410 F

Examiner: Professor J. J. Gottlieb

Instructions: *(a)* open book test (D) — notes and textbook permitted,
(b) answer or attempt all questions if you can, and
(c) relative mark values for questions are indicated.

20 % *1.* Consider the steady flow of a compressible gas from location (1) to another location (2) farther along a duct between which there can occur a change in area, heat transfer, viscous losses and shaft work. Derive the following expression

$$\frac{p_{02}}{p_{01}}\frac{T_{01}^{1/2}}{T_{02}^{1/2}}\frac{A_2}{A_1} = \frac{M_1}{M_2}\left(\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

for the conservation of mass in terms of the stagnation pressure (p_0) and temperature (T_0), flow Mach number (M) and area A at locations (1) and (2).

20 % *2.* A conventional shock tube with air in the driver (a_4 = 288 m/s) and air in the channel (p_1 = 102 kPa, a_1 = 288 m/s) is used to produce a flow of speed u_2 = 200 m/s between the shock and contact surface.

a) Determine the shock Mach number M_s and shock pressure p_2.

b) Determine the sound speed a_3 in region 3 and driver pressure p_4.

c) Determine the test time between the shock and contact surface for a test section located 5 m downstream of the diaphragm.

20 % *3.* A picture of a supersonic flow field shows that the flow is deflected or turned by a $\delta = 20$ degree wedge and thereby produces an oblique shock wave with an angle $\beta = 20$ degrees from the wedge surface (that is a wave angle $\varepsilon = 40$ degrees). Determine the flow Mach numbers M_1 and M_2 of the flows on both sides of the oblique shock wave.

20 % *4.* Air flows from a reservoir (T_0 = 300 K) through a short convergent nozzle, then along a constant-area duct, and finally exits at sonic speed (M_e = 1) into the surrounding atmosphere (p_{atm} = 1 atm). Heat addition to the flow in the duct increases the stagnation temperature by 400 K.

a) Determine the Mach number M_1 of the flow in the duct before heat addition.

b) Determine the stagnation pressure p_0 in the reservoir.

c) Draw a temperature-entropy diagram to illustrate relevant flow states including the reservoir and atmosphere.

20 % 5. The blow-down wind tunnel shown in the diagram is designed with a reservoir volume $V = 500\ \mathrm{m}^3$, test section area $A_{\mathrm{ts}} = 1\ \mathrm{m}^3$, flow Mach number in the test section $M_{\mathrm{ts}} = 3$, and air flow discharge from the tunnel exit via a free jet into the atmosphere ($p_{\mathrm{atm}} = 1$ atm, $T_{\mathrm{atm}} = 300$ K). The initial reservoir pressure $p_{0i} = 15$ atm and initial sound speed $a_{0i} = 300$ m/s. Heat transfer directly to the reservoir air maintains an essentially constant reservoir temperature during the wind-tunnel run.

a) What are the flow Mach number M_t and area A_t at the throat (that is the minimum flow area)?

b) What type of free jet exits at the start of the wind-tunnel run (i.e., subsonic, supersonic, type of shock or rarefaction wave pattern)? Justify your answer.

c) Describe how wind-tunnel shutdown occurs (i.e., loss of supersonic flow in the test section). Determine the test section pressure p_{ts} and reservoir pressure p_0 when shutdown occurs.

d) Start with the integral equation for the conservation of mass and derive expressions for the change of reservoir density and pressure as a function of time. Note that the reservoir temperature can be assumed constant because of heat transfer and the rapid expansion to the throat can be assumed isentropic.

e) Calculate the wind-tunnel test time (i.e., the time for the reservoir pressure to drop from the initial pressure to the shutdown pressure).

f) Start with the integral equation for the conservation of energy and derive an expression for the amount of heat transfer to maintain a constant reservoir temperature during the wind-tunnel run.