UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION, DECEMBER 2001
Second Year - Materials and Mining Engineering

MAT294H1F - CALCULUS AND DIFFERENTIAL EQUATIONS
Exam Type: C

Examiner- E. Kerman

A single one-sided aid sheet is allowed.
Total: 80 marks

1. *[10 marks]* KLM airlines requires that the dimensions of a rectangular carry-on bag be such that the length plus the width plus three times the height be less than 81 inches. What is the largest volume that an allowable carry-on bag can have?

2. *[10 marks]* Consider the vector field

$$\bar{F}(x, y, z) = xy^2\hat{\imath} + x^2y\hat{\jmath} + z^3\hat{k}.$$

a. Prove that $\bar{F}$ can be written as $\nabla\phi$ for some function $\phi(x, y, z)$.

b. Find a function ϕ for which $\nabla\phi = \bar{F}$.

c. Use your answer from part b to calculate $\int_C \bar{F} \cdot d\bar{r}$ where C is the curve parameterized by

$$x(t) = \frac{t^2}{(2\pi)^2},\ y(t) = 1 - \cos^2 t,\ z(t) = \frac{t}{2\pi};\ \ 0 \le t \le 2\pi.$$

3. *[10 marks]* Evaluate the triple iterated integral

$$\int_0^1 \int_0^1 \int_0^{\sqrt{1-y^2}} y(y^2 + z^2)^{\frac{1}{2}}\, dz\, dy\, dx.$$

Hint: rewrite it as a triple integral and switch the order of integration to $dy\, dz\, dx$.

4. *[10 marks]* Once you place your money in the Bank of Wishes it increases at a rate proportional to the amount present. If your money doubles in one year, how long will it take for you to have ten times your initial investment?

5. *[10 marks]* Evaluate the line integral $\oint_C \bar{F} \cdot d\bar{r}$ where

$$\bar{F}(x, y, z) = x^2\hat{\imath} + x\hat{\jmath} + xyz\hat{k}$$

and C is the closed curve defined by $x^2 + y^2 = 1$ and $z = 1$.

6. *[10 marks]* Find the volume bounded by the surfaces $z = 0$ and $z = 1 - 2e^{-(x^2+y^2)}$.

7. *[10 marks]* Find a general solution to the differential equation

$$y'' + y = x^2.$$

Use this to find the particular solution of this equation satisfying $y(0) = 0$ and $y'(0) = \pi$.

8. *[10 marks]* The temperature in a room is described by the function

$$T(x, y, z) = x^2 + y^2 + 3xy + z^3 + 3z.$$

a. Find the critical points of this function. For each one decide whether it is a local maximum of the temperature (a hot-spot), a local minimum of the temperature (a cold-spot), or neither.

b. If you are at the origin of the room, $(0, 0, 0)$, and you want to get hotter as quickly as possible, in which direction should you move?