SURNAME: ____________ GIVEN NAME: ____________ STUDENT NO.: ______

UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING
DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

Final Examination, December 12, 2001
Fourth Year — Program 5 (Option 5CE) and Program 9

ECE 418F — Data Communications

Examination Type: D
Examiner: F. R. Kschischang

Instructions:

- This examination paper consists of eleven [11] pages (including this one). Please make sure that you have a complete paper.
- Write your name and student number in the space provided at the top of each page.
- Answer each question directly on the examination paper, using the back of each page if necessary. Indicate clearly where your work can be found.
- Show all steps and present all results clearly. **State any assumptions that you make.**
- Answer **all** five [5] questions. Each question is worth 20 marks. A total of 100 marks is available.
- This is a type D examination. **The only aids permitted are class notes and problem set solutions, the textbook by Haykin, and a calculator.**
- Time: $2\frac{1}{2}$ hours.

EXAMINER'S REPORT

1.		/20
2.		/20
3.		/20
4.		/20
5.		/20
Total:		/100

Design Table

$\mathrm{erfc}(\sqrt{x})$	x	$\mathrm{erfc}(\sqrt{x})$	x	$\mathrm{erfc}(\sqrt{x})$	x
9×10^{-1}	0.007895	9×10^{-3}	3.411413	9×10^{-5}	7.667845
8×10^{-1}	0.032092	8×10^{-3}	3.516737	8×10^{-5}	7.779146
7×10^{-1}	0.074236	7×10^{-3}	3.636484	7×10^{-5}	7.905424
6×10^{-1}	0.137498	6×10^{-3}	3.775151	6×10^{-5}	8.051325
5×10^{-1}	0.227468	5×10^{-3}	3.939719	5×10^{-5}	8.224055
4×10^{-1}	0.354163	4×10^{-3}	4.141907	4×10^{-5}	8.435695
3×10^{-1}	0.537097	3×10^{-3}	4.403734	3×10^{-5}	8.708911
2×10^{-1}	0.821187	2×10^{-3}	4.774768	2×10^{-5}	9.094647
1×10^{-1}	1.352772	1×10^{-3}	5.413783	1×10^{-5}	9.755710
9×10^{-2}	1.437187	9×10^{-4}	5.511380	9×10^{-6}	9.856363
8×10^{-2}	1.532451	8×10^{-4}	5.620616	8×10^{-6}	9.968934
7×10^{-2}	1.641510	7×10^{-4}	5.744623	7×10^{-6}	10.096618
6×10^{-2}	1.768692	6×10^{-4}	5.887989	6×10^{-6}	10.244100
5×10^{-2}	1.920729	5×10^{-4}	6.057833	5×10^{-6}	10.418644
4×10^{-2}	2.108942	4×10^{-4}	6.266097	4×10^{-6}	10.632424
3×10^{-2}	2.354646	3×10^{-4}	6.535197	3×10^{-6}	10.908279
2×10^{-2}	2.705947	2×10^{-4}	6.915542	2×10^{-6}	11.297521
1×10^{-2}	3.317448	1×10^{-4}	7.568353	1×10^{-6}	11.964063

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1. **Source Coding**—A binary memoryless source emits symbols from the alphabet $\{a, b\}$ at a rate of 1000 symbols per second. On average, 1/4th of the output symbols are 'a' and the rest are 'b'.

3 marks

(a) Determine the minimum possible channel capacity (bits/s) needed to convey the output of this source, assuming a source code that achieves the source entropy.

(b) Three different coding schemes are proposed for this source. Each of them involves parsing the output of the source into strings from a particular set, and then applying Huffman coding to the strings.

5 marks

i. Suppose the source is parsed into fixed-length strings of length 3, i.e., the intermediate alphabet is

$$\{\mathsf{aaa}, \mathsf{aab}, \mathsf{aba}, \mathsf{abb}, \mathsf{baa}, \mathsf{bab}, \mathsf{bba}, \mathsf{bbb}\}.$$

Find a Huffman code for this alphabet, and determine the resulting compressed bit rate.

8 marks

5 marks

ii. Suppose the source is parsed according to "runs" of the symbol b of up to length 7, i.e., the intermediate alphabet is

$$\{\mathsf{a}, \mathsf{ba}, \mathsf{bba}, \mathsf{bbba}, \mathsf{bbbba}, \mathsf{bbbbba}, \mathsf{bbbbbba}, \mathsf{bbbbbbb}\}.$$

Find a Huffman code for this alphabet, and determine the resulting compressed bit rate.

5 marks

iii. Suppose the source is parsed into the words of the following alphabet:

$$\{\mathsf{aa}, \mathsf{ab}, \mathsf{ba}, \mathsf{bba}, \mathsf{bbba}, \mathsf{bbbba}, \mathsf{bbbbba}, \mathsf{bbbbbb}\}.$$

Find a Huffman code for this alphabet, and determine the resulting compressed bit rate.

2 marks

(c) Which of the three schemes yields the smallest bit rate?

12 marks

2. **QAM Modem Design**—You are given an additive white Gaussian noise channel of bandwidth 5MHz in the range 2400MHz to 2405MHz in which the noise has a two-sided power spectral density of $N_0/2$. Over this channel, you may transmit a signal with enough power so that the received signal-to-noise ratio E_s/N_0 is 33.3 dB.

2 marks (a) Assuming QAM modulation using pulses with $33\frac{1}{3}\%$ excess bandwidth, give the carrier frequency and symbol rate that you would use to make best use of the given channel.

5 marks (b) Estimate the maximum transmission rate (bits per second) that you could achieve using the system of part (a) for some reasonable symbol error probability ($< 10^{-5}$). [*Hint:* you may assume a required $\mathrm{SNR}_{\mathrm{norm}}$ of 9 dB.]

3 marks (c) Describe the QAM signal constellation that you would use.

10 marks

5 marks

(d) Find the maximum theoretically achievable transmission rate for the given channel, i.e., the channel capacity, assuming ideal 0% excess bandwidth pulses.

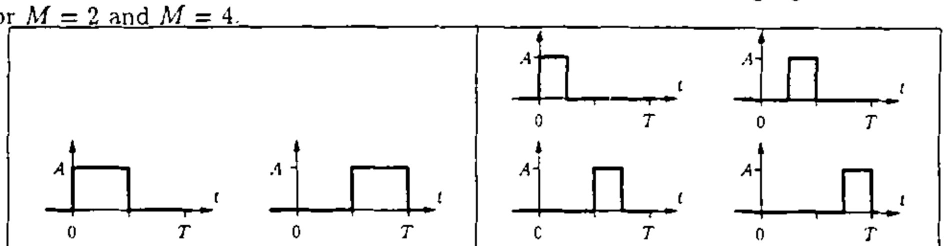
5 marks

(e) You are given an analog source bandlimited to 1.5 MHz, which is sampled at the Nyquist rate. It is observed that the samples are uniformly distributed in the interval $[-A, A]$. Design a 2^m-level uniform quantizer that results in maximum possible signal-to-quantization noise ratio (SQNR) while still permitting transmission of the resulting bit stream using your QAM modem of part (b). (Describe your quantizer explicitly, giving the value of m, the locations of the boundaries between quantization cells, and also the reconstruction value associated with each cell.) What is the resulting SQNR? (Express your answer in dB).

10 marks

3. **Pulse Position Modulation**—A widely used transmission scheme, particularly in wireless optical communication, is M-ary pulse position modulation (M-PPM). The signalling interval $[0, T)$ is divided into M disjoint sub-intervals of equal duration. Depending on the bits to be transmitted, the transmitter sends a nonzero pulse during exactly one of the M sub-intervals (and zero in the remaining sub-intervals).

For the purposes of this question, you may assume that the transmitter sends a signal of constant level A during the "ON" sub-interval, and zero at other times. The following figure illustrates the signal sets for $M = 2$ and $M = 4$.



2-PPM signal set | 4-ppm signal set

In this question you will design a modem for the 2-PPM signal set, assuming a transmission channel that is an additive white Gaussian noise channel with two-sided noise power spectral density $N_0/2$.

2 marks (a) Find an orthonormal basis $\{\phi_1(t), \phi_2(t)\}$ for the signal set.

2 marks (b) Find the coordinates of each signal with respect to the basis found in part (a), and sketch the resulting signal constellation.

2 marks (c) Determine the average transmitted energy and the squared Euclidean distance between the signal constellation points.

2 marks (d) Explain how a pair of matched filters can be used to project the received signal $r(t)$ into the signal space spanned by $\{\phi_1(t), \phi_2(t)\}$. Give the impulse response of each filter explicitly. Precisely specify the time at which the output of each filter is to be sampled.

8 marks

2 marks (e) Sketch the response of each of the matched filters to each of the possible transmitted signals.

2 marks (f) Specify the maximum-likelihood decision regions, and design a decision device that processes the output of the matched filters to make the maximum-likelihood decision.

2 marks (g) Write an expression for the probability of error.

2 marks (h) Find the smallest value of A^2/N_0 needed to achieve an error probability of 10^{-4}.

4 marks (i) Design an optimum detector that uses only one matched filter. [*Hint:* consider the rotated basis $\{(\phi_1(t) + \phi_2(t))/\sqrt{2}, (\phi_1(t) - \phi_2(t))/\sqrt{2}\}$.]

12 marks

4. **M-PPM** Consider now case of M-PPM, where $M \geq 2$.

2 marks (a) Specify an orthonormal basis for the signal set. How many basis functions are needed?

2 marks (b) Find the coordinates of each signal with respect to the basis found in part (a).

2 marks (c) Determine the average transmitted energy and the squared Euclidean distance between each pair of signal constellation points.

5 marks (d) Find an upper bound on the probability of error for the optimum signal detector.

11 marks

5 marks

(e) Under the "maximum correlation" decision rule, the receiver computes the inner product between the received waveform $r(t)$ and each of the possible transmitted waveforms, and chooses the signal that has maximum inner product. Show that the "maximum correlation" decision rule is equivalent to the maximum-likelihood rule for the M-PPM signal set.

4 marks

(f) Find a simple modification of the M-PPM set that minimizes the average transmitted signal energy, without changing the the distance between any pair of signal constellation points. Sketch the modified signals in the case $M = 2$ and $M = 4$.

9 marks

5. **Channel Coding**—Let C be the binary linear polynomial code of length 6, with generator polynomial $G(D) = 1 + D^2 + D^3$. Codewords are transmitted over a binary symmetric channel with crossover probability p.

2 marks (a) Show that

$$G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

is a generator matrix for this code.

2 marks (b) What is the dimension of this code? What is the rate of this code?

2 marks (c) Find a parity-check matrix for this code.

2 marks (d) Is the code self-dual?

2 marks (e) Find the weight enumerator for this code.

2 marks (f) What is the minimum distance of this code?

10 marks

4 marks (g) For each possible syndrome value associated with this code, determine what action a maximum-likelihood decoder for this code would take, assuming a binary symmetric channel with crossover probability $p < 1/2$. (Assume a complete decoder, i.e., a decoder for which decoding failures are not permitted.)

4 marks (h) For your decoder, what set of error patterns result in correct decoding? Determine the *exact* probability of decoding error for your decoder as a function of p.

2 marks (i) This code is modified by adding an overall parity-check bit. Determine the weight enumerator for the modified code.

10 marks