University of Toronto
Faculty of Applied Science and Engineering
Final Examination, December 14, 2001
MIE 372F - CONTROL SYSTEMS
Examiner: C.B. Park

No aids allowed.

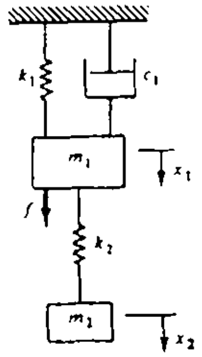
1. The figure shown below describes a dynamic vibration absorber. Assume that the system is in equilibrium when $x_1(t) = x_2(t) = 0$.

a. Draw the block diagram that represents the relationship between the force F(s) (input) and displacement of mass m_1, i.e., $X_1(s)$ (output). You may simplify the diagram, but in your block diagram, show clearly where $X_2(s)$ is (this means that you should not oversimplify the diagram in relating F and X_1). (8 marks)

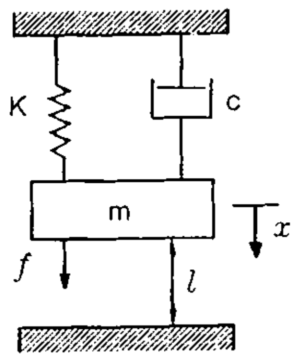
b. Derive the transfer functions X_1/F and X_2/F. Do **not** derive the time responses of $x_1(t)$ and $x_2(t)$. (5 marks)

c. Design a feedback loop system to improve the system responses. Discuss how you can physically implement this feedback loop system. Do not be scared. Simply describe conceptually what you are going to do. Note the force f(t) is the original input and the displacement of mass m_1, i.e., $x_1(t)$ is the desired output. (3 marks)

d. Draw the block diagram of the (negative) feedback loop system. (4 marks)

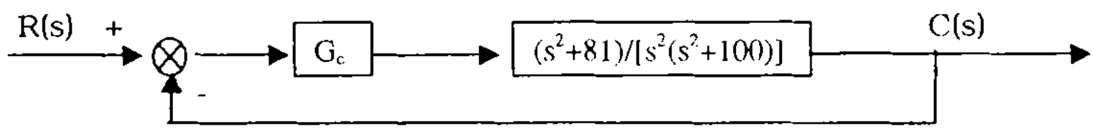


2. Determine the magnitudes of c (linear damping coefficient) and f (step force) that the mass can approach the floor as quick as possible. Derive the time response x(t) for this system. (10 marks)



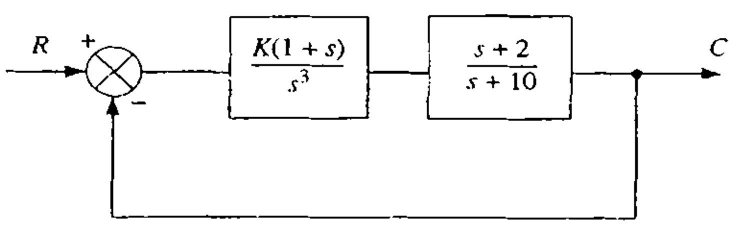
3. Consider an open-loop transfer function $G_c(s)\, G_p(s) = 4K(s+0.5)/[s^2(s+2)(s+3)]$ with unity feedback closed loop.
   a. Sketch a root locus for this system as K varies. (5 marks)
   b. For what values of K is this system stable? (5 marks)
   c. Suppose the input $R(s) = a/s - b/s^2 + c/s^3$ where a, b, and c are real constants. What is the steady-state error e_{ss}? (5 marks)
   d. What is the value of K that results in a pair of closed loop poles at -0.225-j and -0.225+j? What is the corresponding time constant of this system? (7 marks)

4. Consider a control system shown below.



   a. Sketch a root locus of this system for $G_c = K\,(s+1)$ as K varies. Calculate all the departure and arrival angles. But do not calculate the break-in/breakaway points. Discuss the stability of this system. (5 marks)
   b. It is required that the system have a zero steady-state error to parabolic inputs ($1/s^3$). Design a controller that can meet this specification. (5 marks)
   c. Sketch a root locus of your new system. Calculate all the departure and arrival angles. But do not calculate the details such as the break-in/breakaway points and the critical value of K. Discuss the stability of this system. (5 marks)

5. Consider the unity-feedback gain system shown in following figure.
   a. Draw a Bode Diagram of the open-loop transfer function (G_cG_p) when the Bode gain is unity. Use the semi-log graphs attached in next page. Hint: determine K first to make the Bode gain = 1. (10 marks)
   b. With this system, determine the gain margin. If you determine the gain margin by reading directly from the Bode diagram, then you will get only 2 marks, but if you determine it analytically, then you will get 10 marks. (10 marks)
   c. With this system (same value of K), read the phase margin from the Bode diagram (do not attempt to determine it analytically). Discuss the system stability. (3 marks)
   d. Draw the Root Locus and determine the minimum value of K for which the system is stable. (5 marks)
   e. Compare your results obtained from (b) and (c), and discuss them. (5 marks)



Your Name: ____________________

Please note that you are supposed to detach this page and submit it together with your answer booklet(s).