Name: ______________________ Student #: ______________________

UNIVERSITY OF TORONTO, Department of Civil Engineering

CIV 368F Engineering Economics and Decision Making

Examiner: Professor H. MacLean

Final Examination, December 12, 2001

Exam Type A: Closed book exam. Calculators are permitted. A formula sheet and tables are attached.

There are 6 questions. Do all questions. Read each question carefully. Please work each question on its page. If you continue your answer on the back of the page, clearly indicate this and show the question number.

Show all of your work, including formulae used and all steps in calculations. Clearly state and justify any assumptions that you make.

The length of the exam is 2 ½ hours.

Question	Possible Marks	Earned Marks
1	12	
2	16	
3	20	
4	16	
5	20	
6	16	
Total	100	

Question 1 **12 Marks**

A city is considering whether to purchase or lease a truck. Both would provide the same benefits, but differ in costs to the city as shown below. The vehicle would be used approximately 50,000 km per year for 3 years. Gasoline costs are $0.50 per litre. Ignore taxes.

	Purchase Truck 1	**Lease Truck 2**
Initial Cost	$30,000	-----
Annual End of Year (EOY) Lease Cost	-----	$ 8000
Annual EOY Maintenance Cost	$ 2000	$ 2000
Fuel Efficiency	20 km/litre	25 km/litre

If truck 1 is purchased, it would be sold for one tenth of its purchase price after 3 years. There is no salvage value for the leased vehicle.

Compare the purchase and lease alternatives on the basis of <u>annual worth</u> over 3 years using a discount rate of 8%. What should the city do?

Question 2 **16 Marks**

A city on a river is concerned about flooding each spring. A consultant has estimated that there is a probability of 0.50 that the river will flood the town in each year. If there is a flood, she estimated the following damages and their probabilities of occurrence:

Severity	Probability	Cost of Flooding ($ million)
Medium	0.60	0.5
High	0.40	0.8

The town is considering the construction of a dyke at a cost of $2.5 million. The dyke, which would last in perpetuity, would reduce the annual probability of a flood from 0.50 to 0.25, but would not reduce the severity or cost of flooding if flooding occurs. The city's discount rate is 8%.

10 marks a) Analyze this problem for the city using a decision tree and expected value criterion with annualized costs. Clearly label all parts of the tree. Should they build the dyke?

6 Marks b) Which alternative is best on the basis of the minimax regret criterion? Show your reasoning including a regret matrix.

Question 3 **20 Marks**

A power corporation is proposing to build a hydroelectric plant on a river in order to provide additional electricity to the people of the province. Electricity rates are currently \$0.059 per kilowatt-hour (kwh) of electricity, and consumption is 10^9 kwh per year. The proposed hydroelectric plant would provide an additional 10^8 kwh per year. The dam would also create a reservoir around which the government could build a park.

1. Initial Cost of Hydroelectric Plant = \$50.0 million.
2. Annual Operating and Maintenance Cost of Hydroelectric Plant = \$1.0 million per year.
3. If the new plant is built, electricity rates would continue to be \$0.059 per kwh.
4. Assume all of the electricity produced can be sold in the province.
5. The government uses a discount rate of 10%.
6. The province would own and operate any park at the reservoir. It would cost the province \$1.0 million to build the park plus \$50,000 per year to operate it. On average, there would be 75,000 potential users of the park each year; 30,000 of these would perceive \$10/day in benefits from the use of the park, 20,000 would perceive \$6/day in benefits and 25,000 would perceive \$2/day in benefits. An entrance fee of \$3 per day would be charged to users.
7. The power plant and park proposals would have lives of at least 60 years.

15 Marks a) Evaluate the power plant and park proposals using a cost benefit analysis, using the (B – C) measure (use PW) from the point of view of the people in the province as a whole. What do you recommend?

5 Marks b) Draw a graph, labelling axes, curve(s), etc. that clearly illustrates the concept of 'consumer surplus' for the park. Describe the concepts of your graph in a maximum of 2 sentences.

Question 4 16 Marks

Recalling your project on bulk water exports to the U.S., answer the following questions. Provide concisely written answers using complete sentences. No calculations are necessary. Be specific in your answers.

4 Marks

a) Assume that the interest rate on loans for Watertech for the base case that you analyzed in your project was 8%. How would your analysis of the base/initial case change if the interest rate on loans dropped to 4%? In other words, how would this affect the relative attractiveness of your alternatives?

4 Marks

b) Compared to your base case, how would your analysis and recommendations change if the population of California doubled and limits on carbon dioxide (CO_2) emissions prevented California from using desalination or treatment/reuse of wastewater?

4 Marks

c) The Canadian government is considering a public sector project of bulk water exports. After completing your analysis for Watertech, your consulting firm is hired by the federal government. The government asks you to analyze for them the same options as Watertech was interested in. What, if anything, would change in your analysis? Be specific.

4 Marks

d) In your analysis in part c) for the government, discuss the implications of using:

- a very low discount rate
- a very high discount rate.

Question 5   20 Marks

A private company that tests water samples for contaminants is considering whether to build a new laboratory in the Toronto area. If the lab is built, it must do certain basic tests, termed 'Basic Operation'. It estimates the following costs and revenues for Basic Operation:

Initial (Set up) Costs	= \$ 2.0 million
Annual Fixed Costs	= \$ 300,000
Variable Cost	= \$ 100 per sample tested
Revenue	= \$ 300 per sample tested
Salvage Value	= \$ 750,000 at end of life
Life	= 8 years

With the Basic Operation, the company could have additional equipment to perform additional tests on each water sample that is brought in. The marginal costs and revenues for this equipment are:

	Equipment A	Equipment B
Initial (Set up) Costs	\$ 500,000	\$ 400,000
Variable Cost	\$ 50 per test	\$ 40 per test
Revenue	\$ 75 per test	\$ 70 per test
Salvage Value	\$ 0	\$ 0
Life	8 years	4 years

Equipment B cannot be used without equipment A, but the reverse is not true.

Each water sample that is brought for testing would undergo all of the tests available. However, the company is uncertain how many samples per year would be brought in for testing. It estimates the number to be between 3500 and 4000 per year. The MARR of the company is 10%.

14 Marks a) Analyze the various alternatives, including "do nothing", using the annual worth measure. Note: Breakeven analysis is required.

Question 5 (cont'd)

6 Marks b) Draw a breakeven graph and interpret the results. For which numbers of samples are the various alternatives preferred?

Question 6 **16 Marks**

Metro Toronto Council is considering whether to continue repairing the eastern section of the Gardiner Expressway or tear it down and replace it with a new ramp and road system. Last year, \$700,000 was spent repairing this section of the expressway. To maintain the road, it would cost another \$1,000,000 initially plus \$400,000 per year for at least 50 years for ongoing repairs. Tearing it down would cost \$2,000,000, and the new ramp and road system to replace it would cost \$4,500,000 plus \$100,000 per year for maintenance. In addition to these costs, the new ramp and road would reduce travel time by an estimated 10,000 hours per year. Metro Council has specified a monetary value for travel time savings of \$5.00 per hour. All costs above are in constant worth dollars. Metro uses an interest rate of 10% per year (which includes an inflation component of 3% per year) when evaluating projects.

12 Marks a) Analyze this problem for Metro Council using the B/C ratio. What option would you recommend to the Council?

4 Marks b) There is uncertainty in the value of travel time savings. What is the breakeven value (\$/hour)?

Table 3.2 Summary of Discrete Compounding Interest Factors

To Find	Given	Factor	Symbol	
P	F	$(1+i)^{-n}$	$(P\|F\ i,n)$	
F	P	$(1+i)^{n}$	$(F\|P\ i,n)$	
P	A	$\frac{(1+i)^n-1}{i(1+i)^n}$	$(P\|A\ i,n)$	
A	P	$\frac{i(1+i)^n}{(1+i)^n-1}$	$(A\|P\ i,n)$	
F	A	$\frac{(1+i)^n-1}{i}$	$(F\|A\ i,n)$	
A	F	$\frac{i}{(1+i)^n-1}$	$(A\|F\ i,n)$	
P	G	$\frac{1-(1+ni)(1+i)^{-n}}{i^2}$	$(P\|G\ i,n)$	
A	G	$\frac{(1+i)^n-(1+ni)}{i\left[(1+i)^n-1\right]}$	$(A\|G\ i,n)$	
P	$A_{1,j}$	$\frac{1-(1+j)^n(1+i)^{-n}}{i-j}$	$(P\|A_1\ i,j,n)^*$	$i=j \quad P=\frac{nA_1}{1+i}$
F	$A_{1,j}$	$\frac{(1+i)^n-(1+j)^n}{i-j}$	$(F\|A_1\ i,j,n)^*$	$i=j \quad F=nA_1(1+i)^{n-1}$

$^*\,i \neq j$

Continuous Compounding

$i_{eff} = e^r - 1$

$P = Fe^{-rn}$

$F = Pe^{rn}$

$P = \frac{A}{i}$

$d = i + j + ij$

$T_K = C_K(1+j)^K$

$$PW_j(i) = \sum_{t=0}^{n} A_{jt}(1+i)^{-t}$$

with

$PW_j(i)$ = present worth of Alternative j using *MARR* of i%
n = planning period
A_{jt} = cash flow for Alternative j at the end of period t
i = *MARR*

$$AW_j(i) = \left[\sum_{t=0}^{n} A_{jt}(P|F\ i,t)\right](A|P\ i,n)$$

$$FW_j(i) = \sum_{t=0}^{n} A_{jt}(1+i)^{n-t}$$

$$\sum_{t=1}^{m_j} R_{jt} \geq C_{0j}$$

$$0 = \sum_{t=0}^{n} A_{jt}(1+i_j^*)^{n-t}$$

$$SIR_j(i) = \frac{\sum_{t=1}^{n} R_{jt}(1+i)^{-t}}{\sum_{t=0}^{n} C_{jt}(1+i)^{-t}}$$

Depreciation
$D_t = (P-F)/n$
$P - [(P-F)/n]t$
$D_t = pB_{t-1}$
$B_t = P(1-p)^t$
$D_t = pP(1-p)^{t-1}$

Capital Recovery
CR = P(A/P, i, n) – F(A/F, i, n)

Loan Payment
E_k = A(P/F, i, n – k + 1)
I_k = A - E_k

$$B/C_j(i) = \frac{\sum_{t=1}^{n} B_{jt}(1+i)^{-t}}{\sum_{t=0}^{n} C_{jt}(1+i)^{-t}}$$

$$(B-C)_j(i) = \sum_{t=0}^{n} \left(B_{jt} - C_{jt}\right)(1+i)^{-t}$$

B_{jt} = benefits associated with project j during year t, t = 1, 2, ...n
C_{jt} = costs associated with project j during year t, t = 1, 2, ...n
i = social discount rate

Expected Value
$E(x) = \sum_{\text{all } x} x\, p(x)$

TABLE A.6 Discrete Compounding: i = 6%

	Single payment		Uniform series				Gradient series	
n	Compound amount factor	Present worth factor	Compound amount factor	Sinking fund factor	Present worth factor	Capital recovery factor	Uniform series factor	Present worth factor
	To find F given P $F\|P\ i,n$	To find P given F $P\|F\ i,n$	To find F given A $F\|A\ i,n$	To find A given F $A\|F\ i,n$	To find P given A $P\|A\ i,n$	To find A given P $A\|P\ i,n$	To find A given G $A\|G\ i,n$	To find P given G $P\|G\ i,n$
1	1.0600	0.9434	1.0000	1.0000	0.9434	1.0600	0.0000	0.0000
2	1.1236	0.8900	2.0600	0.4854	1.8334	0.5454	0.4854	0.8900
3	1.1910	0.8396	3.1836	0.3141	2.6730	0.3741	0.9612	2.5692
4	1.2625	0.7921	4.3746	0.2286	3.4651	0.2886	1.4272	4.9455
5	1.3382	0.7473	5.6371	0.1774	4.2124	0.2374	1.8836	7.9345
6	1.4185	0.7050	6.9753	0.1434	4.9173	0.2034	2.3304	11.4594
7	1.5036	0.6651	8.3938	0.1191	5.5824	0.1791	2.7676	15.4497
8	1.5938	0.6274	9.8975	0.1010	6.2098	0.1610	3.1952	19.8416
9	1.6895	0.5919	11.4913	0.0870	6.8017	0.1470	3.6133	24.5768
10	1.7908	0.5584	13.1808	0.0759	7.3601	0.1359	4.0220	29.6023
11	1.8983	0.5268	14.9716	0.0668	7.8869	0.1268	4.4213	34.8702
12	2.0122	0.4970	16.8699	0.0593	8.3838	0.1193	4.8113	40.3369
13	2.1329	0.4688	18.8821	0.0530	8.8527	0.1130	5.1920	45.9629
14	2.2609	0.4423	21.0151	0.0476	9.2950	0.1076	5.5635	51.7128
15	2.3966	0.4173	23.2760	0.0430	9.7122	0.1030	5.9260	57.5546
16	2.5404	0.3936	25.6725	0.0390	10.1059	0.0990	6.2794	63.4592
17	2.6928	0.3714	28.2129	0.0354	10.4773	0.0954	6.6240	69.4011
18	2.8543	0.3503	30.9057	0.0324	10.8276	0.0924	6.9597	75.3569
19	3.0256	0.3305	33.7600	0.0296	11.1581	0.0896	7.2867	81.3062
20	3.2071	0.3118	36.7856	0.0272	11.4699	0.0872	7.6051	87.2304
21	3.3996	0.2942	39.9927	0.0250	11.7641	0.0850	7.9151	93.1136
22	3.6035	0.2775	43.3923	0.0230	12.0416	0.0830	8.2166	98.9412
23	3.8197	0.2618	46.9958	0.0213	12.3034	0.0813	8.5099	104.7007
24	4.0489	0.2470	50.8156	0.0197	12.5504	0.0797	8.7951	110.3812
25	4.2919	0.2330	43.8645	0.0182	12.7834	0.0782	9.0722	115.9732
26	4.5494	0.2198	59.1564	0.0169	13.0032	0.0769	9.3414	121.4684
27	4.8223	0.2074	63.7058	0.0157	13.2105	0.0757	9.6029	126.8600
28	5.1117	0.1956	68.5281	0.0146	13.4062	0.0746	9.8568	132.1420
29	5.4184	0.1846	73.6398	0.0136	13.5907	0.0736	10.1032	137.3096
30	5.7435	0.1741	79.0582	0.0126	13.7648	0.0726	10.3422	142.3588
31	6.0881	0.1643	84.8017	0.0118	13.9291	0.0718	10.5740	147.2864
32	6.4534	0.1550	90.8898	0.0110	14.0840	0.0710	10.7988	152.0901
33	6.8406	0.1462	97.3432	0.0103	14.2302	0.0703	11.0166	156.7681
34	7.2510	0.1379	104.1838	0.0096	14.3681	0.0696	11.2276	161.3192
35	7.6861	0.1301	111.4348	0.0090	14.4982	0.0690	11.4319	165.7427
40	10.2857	0.0972	154.7620	0.0065	15.0463	0.0665	12.3590	185.9568
45	13.7646	0.0727	212.7435	0.0047	15.4558	0.0647	13.1413	203.1096
50	18.4202	0.0543	290.3359	0.0034	15.7619	0.0634	13.7964	217.4574
55	24.6503	0.0406	394.1720	0.0025	15.9905	0.0625	14.3411	229.3222
60	32.9877	0.0303	533.1282	0.0019	16.1614	0.0619	14.7909	239.0428
65	44.1450	0.0227	719.0829	0.0014	16.2891	0.0614	15.1601	246.9450
70	59.0759	0.0169	967.9322	0.0010	16.3845	0.0610	15.4613	253.3271
75	79.0569	0.0126	1300.9487	0.0008	16.4558	0.0608	15.7058	258.4527
80	105.7960	0.0095	1746.5999	0.0006	16.5091	0.0606	15.9033	262.5493
85	141.5789	0.0071	2342.9817	0.0004	16.5489	0.0604	16.0620	265.8096
90	189.4645	0.0053	3141.0752	0.0003	16.5787	0.0603	16.1891	268.3946
95	253.5463	0.0039	4209.1042	0.0002	16.6009	0.0602	16.2905	270.4375
100	339.3021	0.0029	5638.3681	0.0002	16.6175	0.0602	16.3711	272.0471

TABLE A.7 Discrete Compounding: $i = 7\%$

	Single payment		Uniform series				Gradient series	
n	Compound amount factor	Present worth factor	Compound amount factor	Sinking fund factor	Present worth factor	Capital recovery factor	Uniform series factor	Present worth factor
	To find F given P $F\|P\ i,n$	To find P given F $P\|F\ i,n$	To find F given A $F\|A\ i,n$	To find A given F $A\|F\ i,n$	To find P given A $P\|A\ i,n$	To find A given P $A\|P\ i,n$	To find A given G $A\|G\ i,n$	To find P given G $P\|G\ i,n$
1	1.0700	0.9346	1.0000	1.0000	0.9346	1.0700	0.0000	0.0000
2	1.1449	0.8734	2.0700	0.4831	1.8080	0.5531	0.4831	0.8734
3	1.2250	0.8163	3.2149	0.3111	2.6243	0.3811	0.9549	2.5060
4	1.3108	0.7629	4.4399	0.2252	3.3872	0.2952	1.4155	4.7947
5	1.4026	0.7130	5.7507	0.1739	4.1002	0.2439	1.8650	7.6467
6	1.5007	0.6663	7.1533	0.1398	4.7665	0.2098	2.3032	10.9784
7	1.6058	0.6227	8.6540	0.1156	5.3893	0.1856	2.7304	14.7149
8	1.7182	0.5820	10.2598	0.0975	5.9713	0.1675	3.1465	18.7889
9	1.8385	0.5439	11.9780	0.0835	6.5152	0.1535	3.5517	23.1404
10	1.9672	0.5083	13.8164	0.0724	7.0236	0.1424	3.9461	27.7156
11	2.1049	0.4751	15.7836	0.0634	7.4987	0.1334	4.3296	32.4665
12	2.2522	0.4440	17.8885	0.0559	7.9427	0.1259	4.7025	37.3506
13	2.4098	0.4150	20.1406	0.0497	8.3577	0.1197	5.0648	42.3302
14	2.5785	0.3878	22.5505	0.0443	8.7455	0.1143	5.4167	47.3718
15	2.7590	0.3624	25.1290	0.0398	9.1079	0.1098	5.7583	52.4461
16	2.9522	0.3387	27.8881	0.0359	9.4466	0.1059	6.0897	57.5271
17	3.1588	0.3166	30.8402	0.0324	9.7632	0.1024	6.4110	62.5923
18	3.3799	0.2959	33.9990	0.0294	10.0591	0.0994	6.7225	67.6219
19	3.6165	0.2765	37.3790	0.0268	10.3356	0.0968	7.0242	72.5991
20	3.8697	0.2584	40.9955	0.0244	10.5940	0.0944	7.3163	77.5091
21	4.1406	0.2415	44.8652	0.0223	10.8355	0.0923	7.5990	82.3393
22	4.4304	0.2257	49.0057	0.0204	11.0612	0.0904	7.8725	87.0793
23	4.7405	0.2109	53.4361	0.0187	11.2722	0.0887	8.1369	91.7201
24	5.0724	0.1971	58.1767	0.0172	11.4693	0.0872	8.3923	96.2545
25	5.4274	0.1842	63.2490	0.0158	11.6536	0.0858	8.6391	100.6765
26	5.8074	0.1722	68.6765	0.0146	11.8258	0.0846	8.8773	104.9814
27	6.2139	0.1609	74.4838	0.0134	11.9867	0.0834	9.1072	109.1656
28	6.6488	0.1504	80.6977	0.0124	12.1371	0.0824	9.3289	113.2264
29	7.1143	0.1406	87.3465	0.0114	12.2777	0.0814	9.5427	117.1622
30	7.6123	0.1314	94.4608	0.0106	12.4090	0.0806	9.7487	120.9718
31	8.1451	0.1228	102.0730	0.0098	12.5318	0.0798	9.9471	124.6550
32	8.7153	0.1147	110.2182	0.0091	12.6466	0.0791	10.1381	128.2120
33	9.3253	0.1072	118.9334	0.0084	12.7538	0.0784	10.3219	131.6435
34	9.9781	0.1002	128.2588	0.0078	12.8540	0.0778	10.4987	134.9507
35	10.6766	0.0937	138.2369	0.0072	12.9477	0.0772	10.6687	138.1353
40	14.9745	0.0668	199.6351	0.0050	13.3317	0.0750	11.4233	152.2928
45	21.0025	0.0476	285.7493	0.0035	13.6055	0.0735	12.0360	163.7559
50	29.4570	0.0339	406.5289	0.0025	13.8007	0.0725	12.5287	172.9051
55	41.3150	0.0242	575.9286	0.0017	13.9399	0.0717	12.9215	180.1243
60	57.9464	0.0173	813.5240	0.0012	14.0392	0.0712	13.2321	185.7677
65	81.2729	0.0123	1146.7552	0.0009	14.1099	0.0709	13.4760	190.1452
70	113.9894	0.0088	1614.1342	0.0006	14.1604	0.0706	13.6662	193.5185
75	159.8760	0.0063	2269.6574	0.0004	14.1964	0.0704	13.8136	196.1035
80	224.2344	0.0045	3189.0627	0.0003	14.2220	0.0703	13.9273	198.0748
85	314.5003	0.0032	4478.5761	0.0002	14.2403	0.0702	14.0146	199.5717
90	441.1030	0.0023	6287.1854	0.0002	14.2533	0.0702	14.0812	200.7042
95	618.6697	0.0016	8823.8535	0.0001	14.2626	0.0701	14.1319	201.5581
100	867.7163	0.0012	12381.6618	0.0001	14.2693	0.0701	14.1703	202.2001

TABLE A.8 Discrete Compounding: i = 8%

	Single payment		Uniform series				Gradient series	
n	Compound amount factor	Present worth factor	Compound amount factor	Sinking fund factor	Present worth factor	Capital recovery factor	Uniform series factor	Present worth factor
	To find F given P $F\|P\ i,n$	To find P given F $P\|F\ i,n$	To find F given A $F\|A\ i,n$	To find A given F $A\|F\ i,n$	To find P given A $P\|A\ i,n$	To find A given P $A\|P\ i,n$	To find A given G $A\|G\ i,n$	To find P given G $P\|G\ i,n$
1	1.0800	0.9259	1.0000	1.0000	0.9259	1.0800	0.0000	0.0000
2	1.1664	0.8573	2.0800	0.4808	1.7833	0.5608	0.4808	0.8573
3	1.2597	0.7938	3.2464	0.3080	2.5771	0.3880	0.9487	2.4450
4	1.3605	0.7350	4.5061	0.2219	3.3121	0.3019	1.4040	4.6501
5	1.4693	0.6806	5.8666	0.1705	3.9927	0.2505	1.8465	7.3724
6	1.5869	0.6302	7.3359	0.1363	4.6229	0.2163	2.2763	10.5233
7	1.7138	0.5835	8.9228	0.1121	5.2064	0.1921	2.6937	14.0242
8	1.8509	0.5403	10.6366	0.0940	5.7466	0.1740	3.0985	17.8061
9	1.9990	0.5002	12.4876	0.0801	6.2469	0.1601	3.4910	21.8081
10	2.1589	0.4632	14.4866	0.0690	6.7101	0.1490	3.8713	25.9768
11	2.3316	0.4289	16.6455	0.0601	7.1390	0.1401	4.2395	30.2657
12	2.5182	0.3971	18.9771	0.0527	7.5361	0.1327	4.5957	34.6339
13	2.7196	0.3677	21.4953	0.0465	7.9038	0.1265	4.9402	39.0463
14	2.9372	0.3405	24.2149	0.0413	8.2442	0.1213	5.2731	43.4723
15	3.1722	0.3152	27.1521	0.0368	8.5595	0.1168	5.5945	47.8857
16	3.4259	0.2919	30.3243	0.0330	8.8514	0.1130	5.9046	52.2640
17	3.7000	0.2703	33.7502	0.0296	9.1216	0.1096	6.2037	56.5883
18	3.9960	0.2502	37.4502	0.0267	9.3719	0.1067	6.4920	60.8426
19	4.3157	0.2317	41.4463	0.0241	9.6036	0.1041	6.7697	65.0134
20	4.6610	0.2145	45.7620	0.0219	9.8181	0.1019	7.0369	69.0898
21	5.0338	0.1987	50.4229	0.0198	10.0168	0.0998	7.2940	73.0629
22	5.4365	0.1839	55.4568	0.0180	10.2007	0.0980	7.5412	76.9257
23	5.8715	0.1703	60.8933	0.0164	10.3711	0.0964	7.7786	80.6726
24	6.3412	0.1577	66.7648	0.0150	10.5288	0.0950	8.0066	84.2997
25	6.8485	0.1460	73.1059	0.0137	10.6748	0.0937	8.2254	87.8041
26	7.3964	0.1352	79.9544	0.0125	10.8100	0.0925	8.4352	91.1842
27	7.9881	0.1252	87.3508	0.0114	10.9353	0.0914	8.6363	94.4390
28	8.6271	0.1159	95.3388	0.0105	11.0511	0.0905	8.8289	97.5687
29	9.3173	0.1073	103.9659	0.0096	11.1584	0.0896	9.0133	100.5738
30	10.0627	0.0994	113.2832	0.0088	11.2578	0.0888	9.1897	103.4558
31	10.8677	0.0920	123.3459	0.0081	11.3498	0.0881	9.3584	106.2163
32	11.7371	0.0852	134.2135	0.0075	11.4350	0.0875	9.5197	108.8575
33	12.6760	0.0789	145.9506	0.0069	11.5139	0.0869	9.6737	111.3819
34	13.6901	0.0730	158.6267	0.0063	11.5869	0.0863	9.8208	113.7924
35	14.7853	0.0676	172.3168	0.0058	11.6546	0.0858	9.9611	116.0920
40	21.7245	0.0460	259.0565	0.0039	11.9246	0.0839	10.5699	126.0422
45	31.9204	0.0313	386.5056	0.0026	12.1084	0.0826	11.0447	133.7331
50	46.9016	0.0213	573.7702	0.0017	12.2335	0.0817	11.4107	139.5928
55	68.9139	0.0145	848.9232	0.0012	12.3186	0.0812	11.6902	144.0065
60	101.2571	0.0099	1253.2133	0.0008	12.3766	0.0808	11.9015	147.3000
65	148.7798	0.0067	1847.2481	0.0005	12.4160	0.0805	12.0602	149.7387
70	218.6064	0.0046	2720.0801	0.0004	12.4428	0.0804	12.1783	151.5326
75	321.2045	0.0031	4002.5566	0.0002	12.4611	0.0802	12.2658	152.8448
80	471.9548	0.0021	5886.9354	0.0002	12.4735	0.0802	12.3301	153.8001
85	693.4565	0.0014	8655.7061	0.0001	12.4820	0.0801	12.3772	154.4925
90	1018.9151	0.0010	12723.9386	0.0001	12.4877	0.0801	12.4116	154.9925
95	1497.1205	0.0007	18701.5069	0.0001	12.4971	0.0801	12.4365	155.3524
100	2199.7613	0.0005	27484.5157	0.0000	12.4943	0.0800	12.4545	155.6107

TABLE A.10 Discrete Compounding: $i = 10\%$

	Single payment		Uniform series				Gradient series	
n	Compound amount factor	Present worth factor	Compound amount factor	Sinking fund factor	Present worth factor	Capital recovery factor	Uniform series factor	Present worth factor
	To find F given P $F\|P\ i,n$	To find P given F $P\|F\ i,n$	To find F given A $F\|A\ i,n$	To find A given F $A\|F\ i,n$	To find P given A $P\|A\ i,n$	To find A given P $A\|P\ i,n$	To find A given G $A\|G\ i,n$	To find P given G $P\|G\ i,n$
1	1.1000	0.9091	1.0000	1.0000	0.9091	1.1000	0.0000	0.0000
2	1.2100	0.8264	2.1000	0.4762	1.7355	0.5762	0.4762	0.8264
3	1.3310	0.7513	3.3100	0.3021	2.4869	0.4021	0.9366	2.3291
4	1.4641	0.6830	4.6410	0.2155	3.1699	0.3155	1.3812	4.3781
5	1.6105	0.6209	6.1051	0.1638	3.7908	0.2638	1.8101	6.8618
6	1.7716	0.5645	7.7156	0.1296	4.3553	0.2296	2.2236	9.6842
7	1.9487	0.5132	9.4872	0.1054	4.8684	0.2054	2.6216	12.7631
8	2.1436	0.4665	11.4359	0.0874	5.3349	0.1874	3.0045	16.0287
9	2.3579	0.4241	13.5795	0.0736	5.7590	0.1736	3.3724	19.4215
10	2.5937	0.3855	15.9347	0.0627	6.1446	0.1627	3.7255	22.8913
11	2.8531	0.3505	18.5312	0.0540	6.4951	0.1540	4.0641	26.3963
12	3.1384	0.3186	21.3843	0.0468	6.8137	0.1468	4.3884	29.9012
13	3.4523	0.2897	24.5227	0.0408	7.1034	0.1408	4.6988	33.3772
14	3.7975	0.2633	27.9750	0.0357	7.3667	0.1357	4.9955	36.8005
15	4.1772	0.2394	31.7725	0.0315	7.6061	0.1315	5.2789	40.1520
16	4.5950	0.2176	35.9497	0.0278	7.8237	0.1278	5.5493	43.4164
17	5.0545	0.1978	40.5447	0.0247	8.0216	0.1247	5.8071	46.5819
18	5.5599	0.1799	45.5992	0.0219	8.2014	0.1219	6.0526	49.6395
19	6.1159	0.1635	51.1591	0.0195	8.3649	0.1195	6.2861	52.5827
20	6.7275	0.1486	57.2750	0.0175	8.5136	0.1175	6.5081	55.4069
21	7.4002	0.1351	64.0025	0.0156	8.6487	0.1156	6.7189	58.1095
22	8.1403	0.1228	71.4027	0.0140	8.7715	0.1140	6.9189	60.6893
23	8.9543	0.1117	79.5430	0.0126	8.8832	0.1126	7.1085	63.1462
24	9.8497	0.1015	88.4973	0.0113	8.9847	0.1113	7.2881	65.4813
25	10.8347	0.0923	98.3471	0.0102	9.0770	0.1102	7.4580	67.6964
26	11.9182	0.0839	109.1818	0.0092	9.1609	0.1092	7.6186	69.7940
27	13.1100	0.0763	121.0999	0.0083	9.2372	0.1083	7.7704	71.7773
28	14.4210	0.0693	134.2099	0.0075	9.3066	0.1075	7.9137	73.6495
29	14.8631	0.0630	148.6309	0.0067	9.3696	0.1067	8.0489	75.4146
30	17.4494	0.0573	164.4940	0.0061	9.4269	0.1061	8.1762	77.0766
31	19.1943	0.0521	181.9434	0.0055	9.4790	0.1055	8.2962	78.6395
32	21.1138	0.0474	201.1378	0.0050	9.5264	0.1050	8.4091	80.1078
33	23.2252	0.0431	222.2515	0.0045	9.5694	0.1045	8.5152	81.4856
34	25.5477	0.0391	245.4767	0.0041	9.6086	0.1041	8.6149	82.7773
35	28.1024	0.0356	271.0244	0.0037	9.6442	0.1037	8.7086	83.9872
40	45.2593	0.0221	442.5926	0.0023	9.7791	0.1023	9.0962	88.9525
45	72.8905	0.0137	718.9048	0.0014	9.8628	0.1014	9.3740	92.4544
50	117.3909	0.0085	1163.9085	0.0009	9.9148	0.1009	9.5704	94.8889
55	189.0591	0.0053	1880.5914	0.0005	9.9471	0.1005	9.7075	96.5619
60	304.4816	0.0033	3034.8164	0.0003	9.9672	0.1003	9.8023	97.7010
65	490.3707	0.0020	4893.7073	0.0002	9.9796	0.1002	9.8672	98.4705
70	789.7470	0.0013	7887.4696	0.0001	9.9873	0.1001	9.9113	98.9870
75	1271.8954	0.0008	12708.9537	0.0001	9.9921	0.1001	9.9410	99.3317
80	2048.4002	0.0005	20474.0021	0.0000	9.9951	0.1000	9.9609	99.5606
85	3298.9690	0.0003	32979.6903	0.0000	9.9970	0.1000	9.9742	99.7120
90	5313.0226	0.0002	53120.2261	0.0000	9.9981	0.1000	9.9831	99.8118
95	8556.6760	0.0001	85556.7605	0.0000	9.9988	0.1000	9.9889	99.8773
100	13780.6123	0.0001	137796.1234	0.0000	9.9993	0.1000	9.9927	99.9202