

UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING
FINAL EXAMINATION
MMS430S ELECTRONIC MATERIALS

Examiner: Professor Harry E. Ruda
April 27, 2001

**** Answer any FOUR questions only ****

Question 1. (25%)

Please define and explain briefly the following terms:

1. Use the LCAO approach to provide an expression for all possible sp^3 hybrids on a Si atom ($1s^2 2s^2 2p^6 3s^2 3p^2$)
2. Density of electron states as a function of energy for a two-dimensional electron system
3. Form of wavefunction for an electron tunneling into a finite potential barrier
4. Wavevector of an electron having an energy equal to the Fermi energy
5. Electron velocity for free and quasi-free electrons from $k=0$ to 1st Brillouin Zone boundary
6. Hall coefficient for a heavily compensated n-type semiconductor
7. Deep-level electron trap
8. Ion implantation process for providing a sharp shallow doping profile
9. The requirement for an optical absorption transition in a semiconductor, based on energy and momentum conservation
10. The AM1.5 and Black Body Spectra.

Question 2. (25%)

This question is concerned with exploring the origins of band structure from the perspective of atomic interactions. The sp^3 hybrid orbitals discussed in class, can be formed by combining $|s\rangle$, $|p_x\rangle$, $|p_y\rangle$ and $|p_z\rangle$ atomic orbitals.

- (a) How many sp^3 hybrid orbits $|h_i\rangle$ can be formed on a given atom by combining these orbitals together, where i is the index of a possible orbital
- (b) Write down an expression of each of the hybrids $|h_i\rangle$ that can form on a given atom using the LCAO principle
- (c) Given the definition for ϵ_s and ϵ_p (below) in terms of the Hamiltonian H , what is the average hybrid energy ϵ_h on a given atom in terms of ϵ_s and ϵ_p ?
$$\epsilon_s = \langle s | H | s \rangle$$
$$\epsilon_p = \langle p_x | H | p_x \rangle = \langle p_y | H | p_y \rangle = \langle p_z | H | p_z \rangle$$
with all other interaction terms zero; e.g., $\langle p_i | H | p_j \rangle = \langle p_i | H | s \rangle = \langle s | H | p_i \rangle = 0$ where $i, j = x, y, z$ and $i \neq j$.
- (d) Assume that two atoms A and B, each having hybrids $|h^A\rangle$ and $|h^B\rangle$, approach each other and start to interact. Derive an expression for the allowed energies of this system at equilibrium, explaining any assumptions made.
- (e) Based on your answers to the questions above, draw on a single diagram the allowed energies under the following conditions: (i) isolated atoms, (ii) hybrid states formed

on each atom, (iii) interactions occurring between hybrids on each atom and (iv) interactions occurring between a very large number of identical atoms with hybrids on each atom.

Question 3. (25%)

This question is concerned with understanding the behaviour of a pn-junction.

1. Provide two equations describing the contributions of drift and diffusion to the total electron and hole current densities, respectively, in a pn-junction – define and explain all terms used.
2. Using the expressions from part 1, derive expressions for $\frac{dn}{dx}$ and $\frac{dp}{dx}$ at thermal equilibrium.
3. Provide expressions for the spatial variation in electron and hole concentrations, $n(x)$ and $p(x)$, respectively. Express your equations in terms of the Fermi Energy, the Intrinsic Fermi Energy, and other parameters as appropriate.
4. Given that the electric field \mathcal{E} can be written in terms of the Intrinsic Fermi Energy E_i as $\mathcal{E}(x) = \frac{1}{e} \frac{dE_i}{dx}$ derive expressions for D_n/μ_n and D_p/μ_p , respectively, in terms of kT .
5. Describe with the aid of diagrams (as appropriate) how a built-in potential arises in a pn-junction, and what are the controlling factors
6. Derive an expression for the dependence of the reverse saturation current I_0 of a pn-junction on a semiconductors bandgap E_g . Assume a constant of proportionality C in your answer.
7. Using the result from part 6., and given a Ge ($E_g=0.6$ eV) pn-junction with $N_a=10^{16}$ cm^{-3} and $N_d=10^{18}$ cm^{-3} at 300K ($kT=0.026$ eV), with a built-in potential of 0.4V and having $I_0=1 \times 10^{-10}$ A, determine the constant C (of part 6)

Question 4. (25%)

Based on your knowledge of state of the art quantum cascade lasers, and extensive crystal growth expertise, you have been requested by your boss at Brightlight Photonics Inc. to design and grow a quantum cascade laser on a GaAs substrate. Refer to data provided on the last pages of this exam to answer this question.

1. If the active structure of the laser is to be made from either AlGaAs or GaAsP. How would you select the bandgap for the barrier and well material, respectively, and what material parameters are required to satisfy this, in both cases.
2. What role does the thickness of the barrier layer play in controlling a physical process in the structure.
3. How does varying the thickness of the well material affect the output wavelength of emitted light.
4. What physical constraints are there on the compositions of the layers you can grow.
5. Describe briefly the bulk growth technique you would use to prepare the GaAs substrate.
6. Describe the technique that you would chose to fabricate the cascade laser structure on the substrate. Discuss in detail, how doping, layer thickness and composition would be controlled.

Question 5 (25%)

This question is concerned with the properties and fabrication of pn Junction based solar cells.

(1). Draw I-V curves for a pn junction in the dark and under illumination, all on the same figure, identifying the following:

- a) forward current
- b) reverse saturation current
- c) short circuit current
- d) open circuit voltage
- e) power quadrant
- f) illumination current
- g) fill factor

(2). Given that the solar power arriving on the surface of a solar cell follows a "Black Body" curve, $P(\lambda) = [H/\lambda^5] \cdot \exp(-a/\lambda)$ for the power P in a wavelength interval $d\lambda$. (i) Find the total power that is arriving on the solar cell, P_{total} in $\text{W} \cdot \text{m}^{-2}$, given that $a = 2.5 \times 10^{-6} \text{ m}$ and $H = 3.33 \times 10^{-21} \text{ W} \cdot \text{m}^2$. Please take note of the data provided at the end of this question. (ii) Derive an expression for J_{sc} assuming no reflection losses and unity quantum efficiency. Provide your answer in terms of a constant times by a function of E_g . Evaluate the constant and provide units.

(3). How does absorption behaviour influence a solar cell's spectral response and what impact does this have on solar cell design strategy?

(4). What are (a) "Back Surface Field", (b) "Vertical Multi-Junction" and (c) "Tandem" solar cells and for which materials properties would each approach be most useful?

(5). Discuss the utility of concentrator solar cells. Why are they important and how do cell operating parameters vary with concentration?

(6). How can texturing of solar cells improve their performance?

DATA:

$$m_0 = 9.11 \times 10^{-31} \text{ kg}$$

$$h = 6.626 \times 10^{-34} \text{ Js}$$

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$$

$$1 \text{ nm} = 1 \times 10^{-9} \text{ m}$$

$$c = 3 \times 10^8 \text{ m} \cdot \text{s}^{-1}$$

$$e = 1.602 \times 10^{-19} \text{ C}$$

$$\lambda(\mu\text{m}) = 1.24/E(\text{eV})$$

$$\int x^m e^{ax} dx = \frac{x^m e^{ax}}{a} - \frac{m}{a} \int x^{m-1} e^{ax}$$

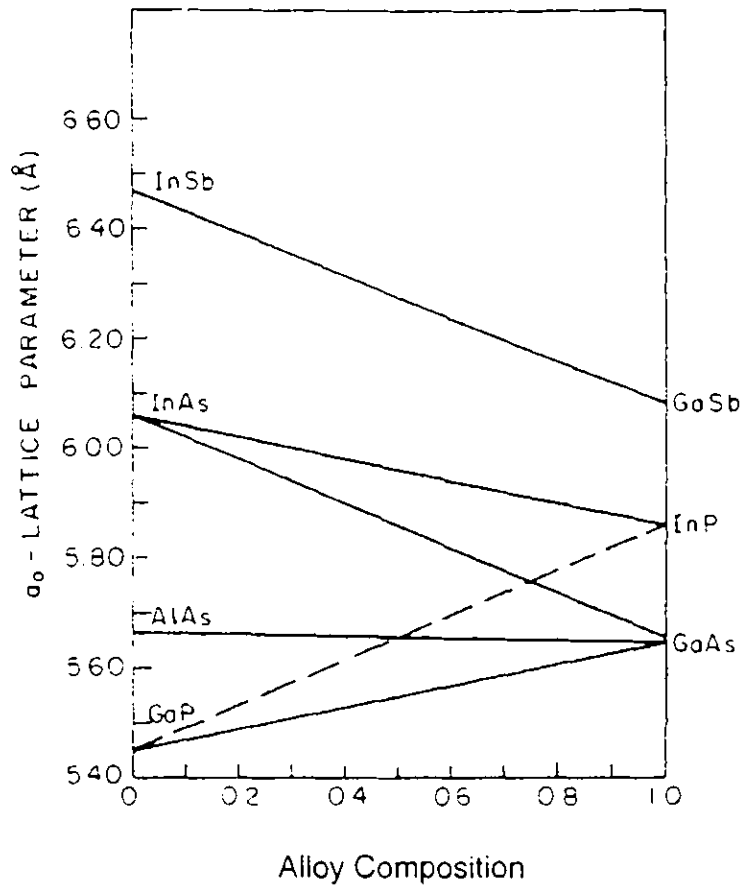
$$\int x e^{ax} dx = \frac{e^{ax}}{a^2} (ax - 1)$$

$$\int_0^{\infty} x^3 e^{-x} dx = 6$$

$$\int x^2 e^{-x} dx = -e^{-x} (1 + 2x + x^2)$$

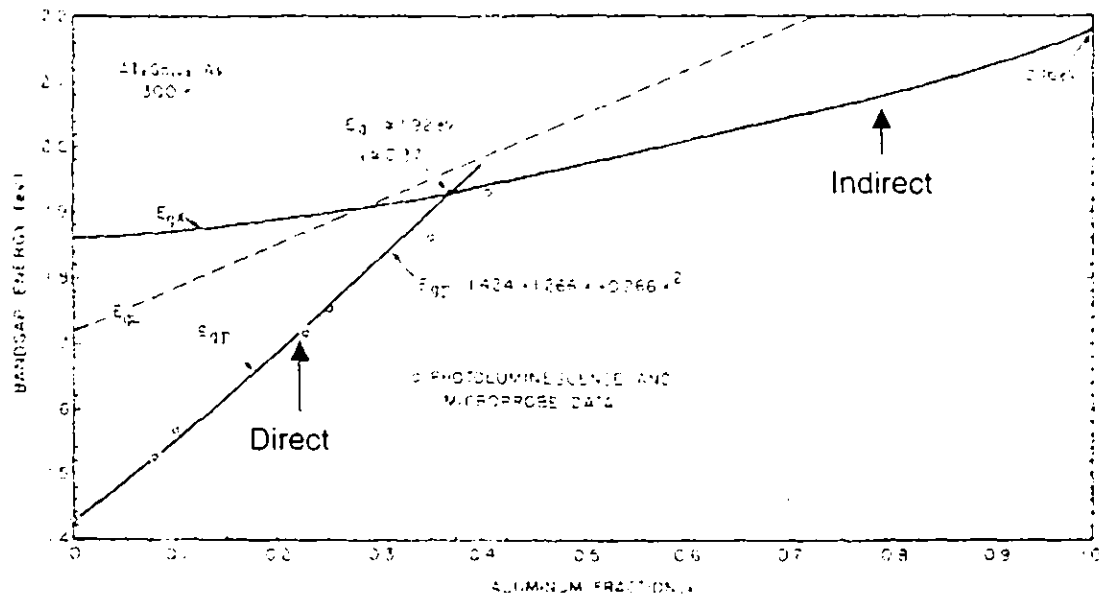
DATA ON DIFFERENT SEMICONDUCTOR ALLOYS

Lattice Parameter Versus Alloy Composition for Different Semiconductor Alloys



Alloys included are: GaAsP, AlGaAs, InGaP, GaInAs, InAsP, and GaInSb

Bandgap versus composition for (a) AlGaAs Alloys, and



(b) GaAsP Alloys:

