

**UNIVERSITY OF TORONTO  
FACULTY OF APPLIED SCIENCE AND ENGINEERING**

**FINAL EXAMINATION, DECEMBER 2001  
Fourth Year – Program 5a**

**AER 402F – Atmospheric Flight**

**Examiner – Professor L.D. Reid**

Candidates may use any aids required. Candidates must answer all questions. You may freely use equations from notes or other aids *without derivation*, but you should indicate the source and discuss the limitations in your application. When expedient make use of reasonable simplifying assumptions.

All 3 questions are of equal value.

1. (a) What are trim tabs and how are they employed by the pilot?
- (b) What conditions hold when  $\delta_e$  is equal to (i)  $\delta_{e_{free}}$  and (ii)  $\delta_{e_{trim}}$ ?
- (c) What tends to keep an aircraft flying with  $\beta = 0$ ?
- (d) What is the aileron reversal speed?
- (e) What property does the inertia matrix exhibit when principal axes are employed?
- (f) What undesirable aircraft properties occur when the CG is (i) too far forward or (ii) too far aft?
- (g) What is weathercock stability?
- (h) How can eigenvalues be used to predict aircraft stability?
- (i) How can an aircraft be operated successfully with an unstable spiral mode?
- (j) Following a step elevator input, what is a typical aircraft angle of attack response (i) in the first few seconds, (ii) in the following 10 minutes?

2. Consider an aircraft in level unaccelerated flight with  $L = W$  and  $C_m = 0$ . Assume that propulsive effects can be ignored.

- (i) What is the most aft CG location for which  $C_{m_\alpha} < 0$  ?
- (ii) What is the minimum tail angle  $i_t$  for which  $C_{m_0} > 0$  ?
- (iii) Find  $i_t$  as a function of CG location  $h$  when  $\delta_e = 0$ .
- (iv) Plot the results of (iii) showing the limits imposed by (i) and (ii).
- (v) If elevator deflection (for a fixed  $i_t$ ) is used to compensate for shifts in the CG location, show how the required change of  $\delta_e$  with CG location  $h$ ,

$$\left( \frac{\partial \delta_e}{\partial h} \right), \text{ depends on } C_L.$$

Given:

$S = 86.875 \text{ m}^2$	$V = 123 \text{ m/s}$
$S_t = 21.375 \text{ m}^2$	$m = 22,680 \text{ kg}$
$a = 0.088/\text{deg.}$	$\rho = 1.225 \text{ kg/m}^3$
$a_t = 0.064/\text{deg.}$	$h_{nwb} = 0.25$
$\epsilon_0 = 0.72 \text{ deg.}$	$\bar{V}_H = 0.6104$
$\frac{\partial \epsilon}{\partial \alpha} = 0.30$	$C_{m_{acwb}} = -0.018$

3. Assume that an approximation to the perturbation equations of an aircraft is given by:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$$

$$\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Express the following in terms of ( a, b, c, d ):

- (i) Find the characteristic equation for the system.
- (ii) Find the eigenvalues for the system.
- (iii) What condition must apply for there to be oscillatory modes?
- (iv) What condition must apply for there to be non-oscillatory modes?
- (v) For a given A matrix, can there be one oscillatory mode and one non-oscillatory mode?
- (vi) What condition must apply for there to be a stable oscillatory mode?
- (vii) What condition must apply for there to be a stable non-oscillatory mode?
- (viii) Prove that Routh's criterion for static stability (  $E > 0$  ) applies to this second order system.
- (ix) Find the system eigenvectors.
- (x) Show that the two columns of  $\text{adj}(\lambda \mathbf{I} - \mathbf{A})$  giving the eigenvectors are multiples of one another.