University of Toronto
Faculty of Applied Science and Engineering

FINAL EXAMINATION
April 23, 2001

ECE 462 – MULTIMEDIA SYSTEMS

Exam Type: C

Examiner: K.N. Plataniotis

Instructions

- This is a TYPE C examination. The **ONLY** aids permitted are a *NON*-PROGRAMMABLE calculator and a single aid sheet (two-sided, 8.5′ × 11′, handwritten). **Work independently**.
- There are 5 questions in this examination. The total value of all questions is 100 marks. Point values are indicated at the beginning of each question. Answer **all** questions.
- Use only the space provided on these sheets. If you have work on the back of any pages make sure to indicate where it can be found. Do not remove any sheet from the examination book. No additional sheets are permitted.
- **IMPORTANT:** Write your name and student number in the space below. Write your student number on the top of each sheet of this examination book.

Last Name (Print): ____________________ **First Names:** ____________________

Student Number: ____________________

(Do not write below this line)

Q1. ________ /15 Q2. ________ /40 Q3. ________ /15 Q4. ________ /15 Q5. ________ /15

Total: /100

Question 1: (15 marks)

1.1 [5 Marks] What feature(s) cause(s) video compression to be quite different from still image compression?

1.2 Both frame-difference predictive and motion-compensated coding are forms of predictive codings.

(i) [$2\frac{1}{2}$ Marks] What is the main difference between the two?

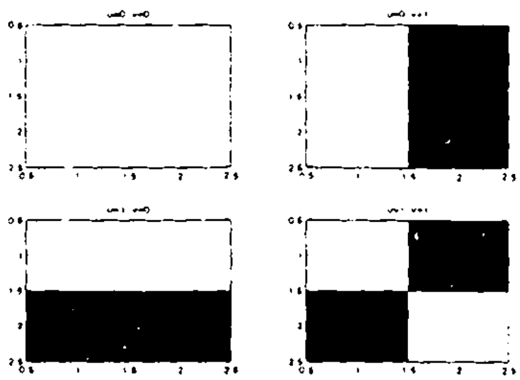
(ii) [5 Marks] Briefly explain why motion-compensated coding is usually more efficient.

(iii) [$2\frac{1}{2}$ Marks] What is the price paid for higher coding efficiency with motion-compensated coding?

Question 1: (Continued)

Question 2: (40 marks)

The basis images for the 2 × 2 DCT are shown in the figure below together with the corresponding matrix entries.



DCT Basis Images for N=2

$$\left[\begin{array}{cc} u=0, v=0 & u=0, v=1 \\ \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix} & \begin{bmatrix} 0.5 & -0.5 \\ 0.5 & -0.5 \end{bmatrix} \\ u=1, v=0 & u=1, v=1 \\ \begin{bmatrix} 0.5 & 0.5 \\ -0.5 & -0.5 \end{bmatrix} & \begin{bmatrix} 0.5 & -0.5 \\ -0.5 & 0.5 \end{bmatrix} \end{array}\right]$$

Recall that any 2 × 2 image $\mathbf{G}$ can be represented in terms of a sum of these four basis images:

$$\mathbf{G} = a \cdot \mathbf{I}_{0,0} + b \cdot \mathbf{I}_{0,1} + c \cdot \mathbf{I}_{1,0} + d \cdot \mathbf{I}_{1,1}$$

where $\mathbf{I}_{u,v}$ is the 2 × 2 matrix at location $[u, v]$ shown above, and $\{a, b, c, d\}$ are scalars.

Consider the following image

$$\mathbf{G}(\epsilon) = \begin{bmatrix} 10+\epsilon & 10 \\ 20 & 20 \end{bmatrix} + \begin{bmatrix} 30 & 40 \\ 30 & 40 \end{bmatrix}$$

where ϵ is some integer such that $0 \leq \epsilon \leq 215$.

(i) [5 Marks] Which basis images will be needed to represent $\mathbf{G}(0)$?

(ii) [10 Marks] Determine the constants $\{a, b, c, d\}$ for the case $\epsilon = 0$.

(iii) [5 Marks] Determine the constants $\{a, b, c, d\}$ for the case $0 \leq \epsilon \leq 215$ (Hint: superposition applies).

(iv) [$2\frac{1}{2}$ Marks] Sketch d^2 vs. ϵ for $0 \leq \epsilon \leq 215$.

(v) [$2\frac{1}{2}$ Marks] Let $\mathbf{T}(\epsilon)$ be the DCT of $\mathbf{G}(\epsilon)$. Determine $\mathbf{T}(\epsilon)$

(vi) [15 Marks] Assume that in the process of quantizing $\mathbf{T}(\epsilon)$, ϵ is discarded (i.e. set to zero). Write $\mathbf{T}(0)$ and compute its inverse DCT, $\mathbf{G}'$. Determine the PSNR between $\mathbf{G}(\epsilon)$ and $\mathbf{G}'$ as a function of ϵ. Sketch your result for $0 \leq \epsilon \leq 215$. Comment on the relationship between this sketch and your previous sketch of d^2.

Question 2: (Continued-1)

Question 2: (Continued-2)

Question 2: (Continued-3)

Question 3: (15 marks)

The optimal rate distortion function of a Gaussian source of arbitrary mean and variance σ^2 with respect to the mean square error (MSE) criterion is:

$$R(D) = \begin{cases} \frac{1}{2}log_2(\frac{\sigma^2}{D}) & 0 \leq D \leq \sigma^2 \\ 0 & D > \sigma^2 \end{cases} \tag{1}$$

(i) [$2\frac{1}{2}$ Marks] Plot the function R(D).

(ii) [5 Marks] What is the maximum distortion D_{max}?

(iii) [$7\frac{1}{2}$ Marks] If a distortion of no more that 75% of the variance is allowed, what is the maximum compression that can be achieved?

Question 3: (Continued)

Question 4: (15 marks)

Consider the Haar wavelet family with mother function

$$\Psi(t) = \begin{cases} 1 & 0 \leq t < 0.5 \\ -1 & 0.5 \leq t < 1 \end{cases} \qquad (2)$$

and scaling function $\Phi(t)$ defined as a unit pulse

$$\Phi(t) = \begin{cases} 1 & 0 \leq t < 1 \\ 0 & \text{otherwise} \end{cases} \qquad (3)$$

(i) [$2\frac{1}{2}$ Marks] Calculate the Haar basis functions $\Psi_{(m,n)}(t)$ for $m = 1, n = 0, 1$.

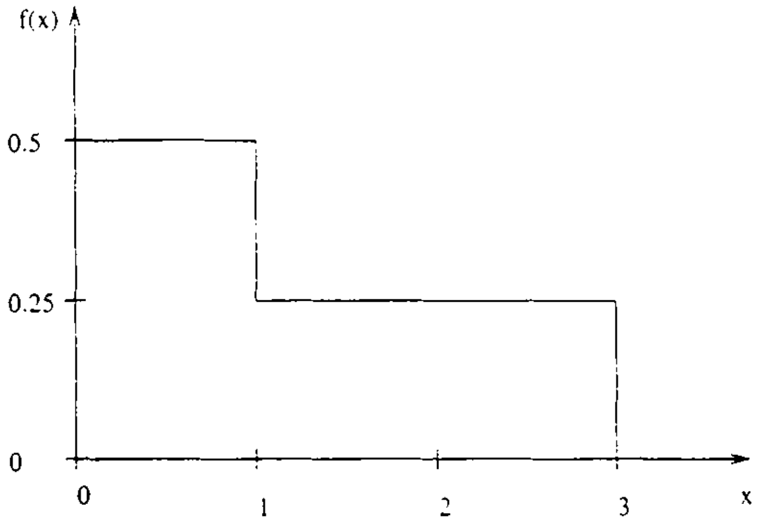
(ii) [$7\frac{1}{2}$ Marks] The wavelet basis functions $\Psi, \Phi, \Psi_{(1,0)}, \Psi_{(1,1)}$ form a linear basis in $\Re^4$. For example, Ψ and Φ can be represented as four-element column vectors: $\Phi = [1, 1, 1, 1]^T$, and $\Psi = [1, 1, -1, -1]^T$, where $\mathbf{x}^T$ is the transpose of vector $\mathbf{x}$. Provide the equivalent four-element column representation for $\Psi_{(1,0)}, \Psi_{(1,1)}, \Psi^*_{(1,0)} = \frac{1}{\sqrt{2}}\Psi_{(1,0)}$, and $\Psi^*_{(1,1)} = \frac{1}{\sqrt{2}}\Psi_{(1,1)}$.

(ii) [5 Marks] Using the column representation for $\Psi, \Phi, \Psi^*_{(1,0)}, \Psi^*_{(1,1)}$ calculate the wavelet coefficients for the input signals $\mathbf{x}_1 = [3, 1, 3, 1]^T$, and $\mathbf{x}_2 = [2, 0, 0, -2]^T$.

Question 4: (Continued)

Question 5: (15 marks)

Consider an image intensity x which can be modeled as a sample whose probability density function is sketched below.



If x is quantized using a two-level, scalar, MID-RISE quantizer with decision levels $t_1 = 0$, $t_2 = 2$, $t_3 = 3$ and reconstruction levels $r_1 = 1$, $r_2 = 2.5$, compute:

(i) [5 Marks] The resulting mean square error (MSE).

(ii) [10 Marks] The entropy of the quantized output.

Question 5: (Continued)

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