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University of Toronto
FACULTY OF APPLIED SCIENCE AND ENGINEERING
Final Examination, December 1998

APM 384F
Year III, Program III — 5a, 5bm, 5env, 5p

Examiner: Prof. R.A. Ross
Duration : $2\frac{1}{2}$ hours

Exam Type C
All questions have EQUAL value

1. Solve $\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$ for $u(r,\theta)$ where $0 < a \le r \le b$, $0 \le \theta \le \frac{\pi}{2}$, with the boundary conditions

$$\frac{\partial u}{\partial \theta}(r,0) = 0 \quad , \quad u\left(r, \frac{\pi}{2}\right) = 0$$
$$u(a,\theta) = 0 \quad , \quad u(b,\theta) = f(\theta) \ .$$

2. Solve $\frac{\partial^2 u}{\partial t^2} - c^2\frac{\partial^2 u}{\partial x^2} + b\frac{\partial u}{\partial t} = 0$ for $u(x,t)$, where $0 \le x \le \ell$, $t \ge 0$, with the boundary conditions

$$u(0,t) = 0 \ , \quad \frac{\partial u}{\partial x}(\ell,t) = 0 \ ,$$

and the initial conditions

$$u(x,0) = f(x) \ , \quad \frac{\partial u}{\partial t}(x,0) = 0 \ .$$

The constants b and c satisfy the condition

$$\cancel{b < \frac{2\pi c}{\ell}}. \qquad b < \frac{\pi c}{\ell}$$

3. The temperature $u(x,y,t)$ in the rectangular plate $0 \le x \le \ell$, $0 \le y \le h$, satisfies the equation $\frac{\partial u}{\partial t} - k\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) = 1$. If the initial temperature of the plate is $u(x,y,0) = 0$ and the boundary conditions on the edges of the plate are

$$u(0,y,t) = 0 \quad , \quad u(\ell,y,t) = 0$$
$$\frac{\partial u}{\partial y}(x,0,t) = x \quad , \quad \frac{\partial u}{\partial y}(x,h,t) = 0 \ ,$$

find $u(x,y,t)$.

4. Solve $\dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial u}{\partial r}\right) + \dfrac{\partial^2 u}{\partial z^2} = 0$ for $u(r, z)$ in the semi-infinite cylinder, $0 \le z < \infty$, $0 \le r \le a$, where $\dfrac{\partial u}{\partial z} = f(r)$ on $z = 0$, $u = 0$ on $r = a$ and $\lim\limits_{z\to\infty} u(r, z) = 0$.

5. Solve $\dfrac{\partial^2 u}{\partial t^2} - c^2\dfrac{\partial^2 u}{\partial x^2} = 0$, $0 \le x < \infty$, $t \ge 0$, for $u(x, t)$, where $u(0, t) = 0$, and the initial conditions are

$$u(x, 0) = 0\ , \quad \frac{\partial u}{\partial t}(x, 0) = \cancel{\delta(x)} \cdot \delta(x - a)\ , \quad a > 0.$$