UNIVERSITY OF TORONTO

FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION, DECEMBER 2001

Fourth Year – Engineering Science

AER506F - SPACECRAFT DYNAMICS AND CONTROL I

Exam Type: X

Examiner - T D Barfoot

All questions are of equal value. Mark breakdown indicated in left margin.
Your mark will be based on your best 5 (of 6) questions.

1. Short answer

2 (a) Describe qualitatively a sequence of thruster burns that would enable a satellite to get from a circular parking orbit about Earth to a circular parking orbit about Mars. Discuss how all the different intermediate orbits involved would fit together (with a diagram).

3 (b) You would like to alter a satellite's circular orbit such that the radius is larger. Which orbital transfer method is more efficient: the Hohmann transfer or the bielliptic transfer? Answer in terms of both the fuel and time required to carry out the transfer. Discuss.

3 (c) A satellite is intended to be geostationary over longitude 110 degrees West, but at initial orbit insertion it is in fact in a slightly elliptical orbit with perigee 1% too low, apogee 1% too high, and $\lambda_{long} = 70$ degrees West. Suggest a set of maneuvers that will raise the perigee and lower the apogee both to the geostationary radius and drift the satellite to its intended longitude.

2 (d) Two satellites orbit the same planet in coplanar orbits. Their major diameters, a, are equal but their major axes are at an angle of 60 degrees. Their eccentricities are related, $e_1 = 2e_2$. For each intersection point of the two orbits find the ratio of the speed of one satellite to the other at the point. Explain.

5. An axisymmetric spacecraft has the shape of a solid circular cylinder (radius r, length ℓ) and has a uniform mass distribution. It orbits the Earth in a geostationary circular orbit with its axis of symmetry nominally aligned with the pitch axis.

5 (a) If $r = \sqrt{\frac{7}{3}}\ell$, what range(s) of absolute angular velocity, ν, will ensure attitude stability in the presence of the gravity-gradient torque? Does the answer change when internal energy dissipation is considered?

3 (b) What are the frequencies of the roll-yaw modes (nondimensionalized by the orbital frequency, ω_o) when $\hat{\nu} = -1$?

2 (c) How small could the radius be made to still have a stable spin when $\hat{\nu} = -1$ and there is energy dissipation in the cylinder?

6. Consider a gyrostat consisting of a carrier, $\mathcal{R}$ which is nominally spinning about the $\underset{\sim}{b}_2$ axis at a rate $\nu = -\omega_o$, and an axisymmetric rotor, $\mathcal{W}$, which is spinning at a rate of ω_s with respect to the carrier (also about the $\underset{\sim}{b}_2$ axis). The gyrostat is placed in a circular orbit about the Earth with the $\underset{\sim}{b}_2$ axis nominally aligned with the pitch axis. The principal moments of inertia of the combined gyrostat are

$$I_1 = 1000 \text{ kg.m}^2 \qquad I_2 = 1100 \text{ kg.m}^2 \qquad I_3 = 200 \text{ kg.m}^2$$

The moment of inertia of the rotor about its spin axis is $I_s = 50$ kg.m^2.

6 (a) Show that in the presence of the gravity-gradient torque, the conditions for roll-yaw stability become

$$p > 0, \quad q > 0, \quad p^2 - 4q > 0$$

where

$$\begin{aligned} p &\triangleq 1 + 3k_1 + \hat{k}_1\hat{k}_3 & \qquad \hat{k}_1 &\triangleq k_1 + \tfrac{\omega_s}{\nu}\tfrac{I_s}{I_1} & \qquad k_1 &\triangleq \tfrac{I_2 - I_3}{I_1} \\ q &\triangleq \hat{k}_3(\hat{k}_1 + 3k_1) & \qquad \hat{k}_3 &\triangleq k_3 + \tfrac{\omega_s}{\nu}\tfrac{I_s}{I_3} & \qquad k_3 &\triangleq \tfrac{I_2 - I_1}{I_3} \end{aligned}$$

2 (b) Comment on the case that $\frac{\omega_s}{\nu} = 0$. Will this motion be stable (i.e., will the same side of the carrier always face Earth)?

2 (c) Comment on the case when $\frac{\omega_s}{\nu} = -100$. Will this motion be stable (i.e., will the same side of the carrier always face Earth)?