UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING
FINAL EXAMINATION, APRIL 2001
AER 336S - SCIENTIFIC COMPUTING
2 pages

1. Find the two-point Gaussian quadrature approximation to

$$I = \int_1^{1.5} e^{-x^2} dx$$

The exact value to seven figures is 0.1093643. (10 marks)

2. Consider the initial-value problem given by

$$u' = u - t^2 + 1, \quad u(0) = 0.5$$

and the second-order time-marching method given by

$$\begin{aligned} \tilde{u}_{n+1/2} &= u_n + \frac{1}{2} h u_n' \\ u_{n+1} &= u_n + h \tilde{u}_{n+1/2}' \end{aligned}$$

Calculate an estimate of the solution at $t = 1$ by applying the time-marching method for one step with $h = 1$. Calculate another estimate by applying the method for two steps with $h = 1/2$. Combine the two estimates using Richardson extrapolation to obtain an improved estimate. The exact value to eight figures is 2.6408591. (30 marks)

3. Consider the following ODE:

$$\frac{d^2u}{dt^2} = \frac{5000 - 0.1[\frac{du}{dt}]^2}{300 - 10t} - 9.81$$

Write this ODE as a first-order system of ODE's in the form

$$\vec{u}' = \vec{F}(\vec{u}, t)$$

Find the Jacobian matrix and $\partial\vec{F}/\partial t$. Write the delta form resulting from application of the second-order backwards time-marching method given by

$$u_{n+1} = \frac{1}{3}(4u_n - u_{n-1} + 2hu'_{n+1})$$

with local time linearization. (30 marks)

4. Using a Taylor table, find an approximation to a third derivative in the form:

$$(\delta_{xxx}u)_j = \frac{1}{\Delta x^3}(au_{j-2} + bu_{j-1} + cu_j + du_{j+1} + eu_{j+2})$$

What is the leading truncation error term? (30 marks)