UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION, APRIL 2001

First Year - Program 5

MAT195S - CALCULUS II

Exam Type: A

Time allowed: 2 and 1/2 hours

Examiners: J. W. Davis
E. J. Barbeau

Last Name: ______________________________

Given Name: ______________________________

Student #: ______________________________

- Please circle the name of your instructor above
- There are 10 questions of equal value. In multiple part questions, each part has approximately the same value.
- **No calculators** or other aids are allowed. Do not remove any pages from this examination booklet.
- Please write your name on each page. Write your answers on the page that contains the question. If extra space is needed, use the back of the page.

FOR MARKER USE ONLY	
Question	Mark
1	
2	
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10	
TOTAL	/100

1) Evaluate the following integrals:

a) $\int \frac{x}{x^4+6x^2+5} dx$

b) $\int_0^2 \frac{1}{\sqrt{2x-x^2}} dx$

2) a) Determine whether the following series converge or diverge:

i) $$\sum_{k=1}^{\infty}\left[\sqrt{k^2+k}-k\right]$$

ii) $$\sum_{k=1}^{\infty}\frac{1}{(\ln k)^3}$$

b) Evaluate the following limit:

$$\lim_{x\to 0}\frac{x\tan 2x}{1-\cos 3x}$$

3) Let $\Sigma\, a_n$ be an absolutely convergent series, i.e., $\Sigma\, |a_n|$ converges.

a) Prove that $\Sigma\, a_n$ converges

b) Prove that $\Sigma\, a_n^2$ converges.

4) a) Find the surface area generated by rotating the given curve about the y-axis:

$x(t) = e^t - t$ $\qquad y(t) = 4e^{t/2}$ $\qquad 0 \leq t \leq 1$

b) Find the area inside the circle $r = 2$, but outside the 4-leaf rose $r=2\cos 2\theta$.

5) a) For what values of x does the following series converge absolutely? Conditionally? Give the interval of convergence.

$$\sum_{n=0}^{\infty} (n+1)^2 \left(\frac{x}{x+2}\right)^n$$

b) Determine the Taylor series expansion for $\sin^{-1}x$ about $x = 0$.

6) A spaceship is following the following trajectory:

$$\mathbf{r}(t) = t^2\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$$

and the coordinates of a space station are (5,5,7). The Captain wants to turn off the engines, and allow the spaceship to coast into the space station without power. When should the engines be turned off? Starting at $t = 0$, how far will the space ship travel in getting to the station? What is the curvature of the trajectory at the point where the engines are turned off (or at a general point t)?

7) a) Determine whether the vector function: $(e^x+2xy)\mathbf{i} + (x^2+\sin y)\mathbf{j}$ is the gradient of a function $\nabla f(x,y)$. If so, find such a function f.

b) Show that the function:

$$u(x,y,z,t) = t^{-3/2}e^{-(x^2+y^2+z^2)/4t}$$

satisfies the three-dimensional heat equation:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$$

8) Determine all of the critical values of:

$f(x,y) = (x+1)(x+y-2)(x-y-2)$

for $-2 \leq x \leq 3, \quad -4 \leq y \leq 4$

9) Find a, b and c so that the volume $V = 4\pi abc/3$ of an ellipsoid $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$, passing through the point (1,2,1) is as small as possible.

10) Does the following integral exist?

$$\int_0^\infty e^{-x} \ln x \, dx$$

Justify your answer.