FINAL EXAMINATION

FACULTY OF APPLIED SCIENCE & ENGINEERING

UNIVERSITY OF TORONTO

APRIL 2001

COURSE ECE 311S - "DYNAMIC SYSTEMS & CONTROL"

TYPE "C"

III - AEELEBASC, AECPEBASC

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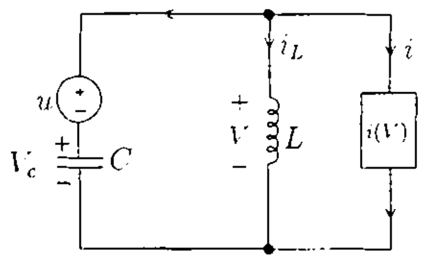
Remarks:
- Please answer all questions.
- A one-page aid sheet is allowed.
- Only non-programmable calculators allowed.
- Duration of exam is $2\frac{1}{2}$ hr.
- All questions carry equal weight.

FAMILY NAME: ______________________________

GIVEN NAME: ______________________________

STUDENT #: ______________________________

1. Consider the nonlinear circuit in Figure 1, where $i(V)$ denotes a voltage-controlled current source with characteristic $i = -V + \frac{1}{3}V^3$, and the voltage u is the control input.



$$i(V) = -V + \frac{1}{3}V^3$$

Figure 1.

(a) Derive a mathematical model of the circuit.

(b) Let $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $x_1 = i_L$, $x_2 = V_c$, let the output $y = V_c$, and write the state equation in the form $\dot{x} = f(x, u)$, $y = g(x, u)$. Verify that $x = 0, u = 0$ is an equilibrium condition of the circuit.

(c) Linearise the state equation around the equilibrium condition $x = 0, u = 0$.

(d) Find the transfer function of the linearized system.

2. Consider the system in Figure 2.

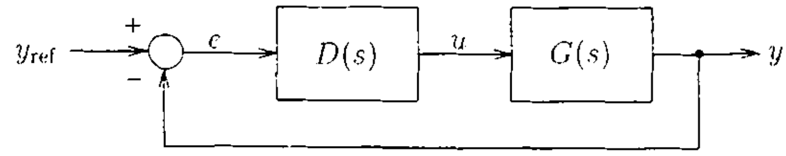


Figure 2.

where $G(s) = \dfrac{s+1}{\left(s+\frac{3}{2}\right)\left[\left(s+\frac{1}{2}\right)^2+1\right]}$

(a) Set $D(s) = k$ and draw the root locus with respect to $k > 0$. Label important points on the locus.

(b) Using the root locus, design a compensator $D(s)$ such that the closed-loop system is stable and all its poles are included in the sector depicted in Figure 2a.

(c) What is the compensator gain in this case?

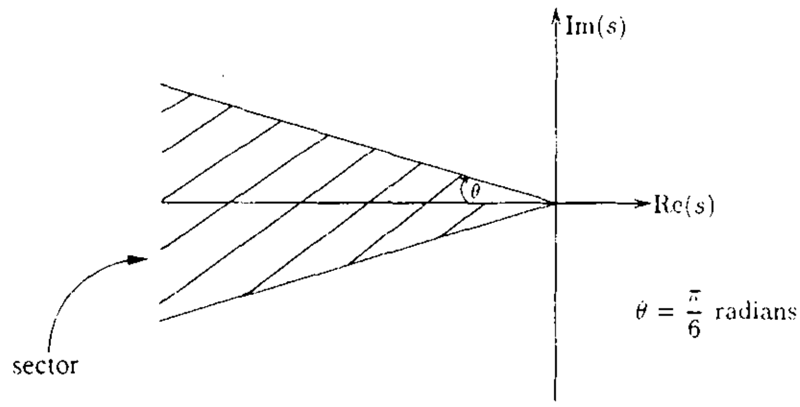
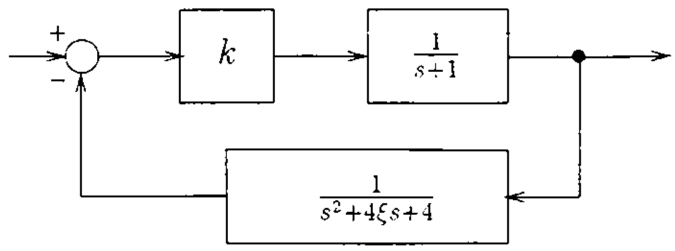


Figure 2a.

3. The figure below shows a control system with proportional gain k, and a sensor with damping factor ξ.

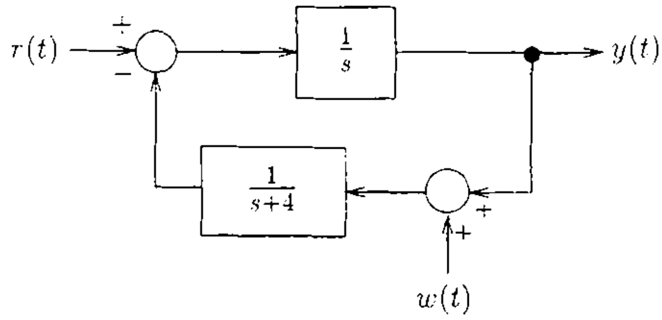


(a) Find the conditions k and ξ must satisfy for the system to be stable.

(b) On the k, ξ plane, **neatly** plot the domain of k and ξ satisfying the conditions found in part (a).

(c) If $\xi = 1/2$ for what values of k (if any) would the roots of the closed loop characteristic equation lie on the $j\omega$-axis ?

4. Consider the block diagram shown below with reference input, $r(t)$, and disturbance input, $w(t)$.

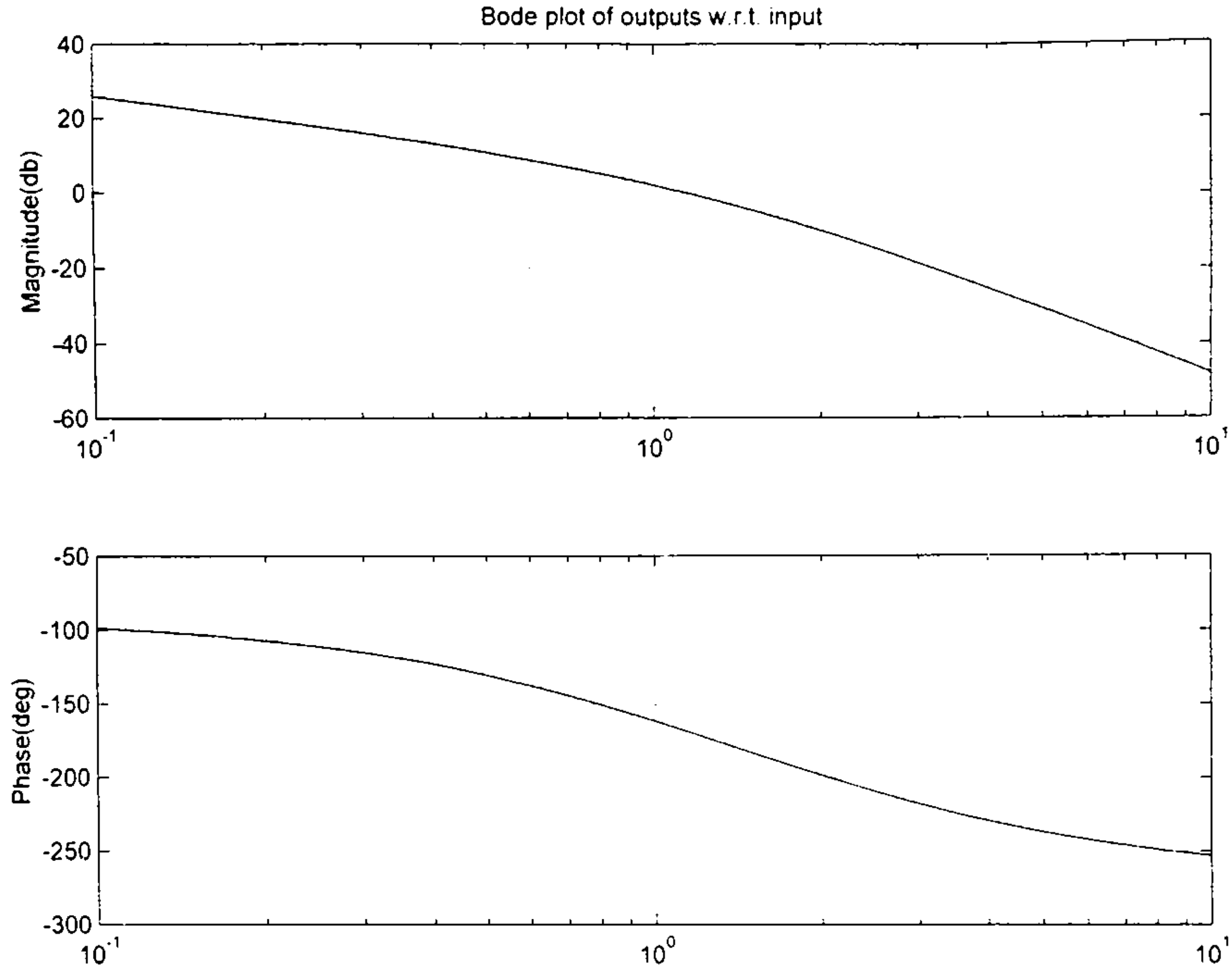


(a) Find $y(s)$ as a function of the reference, $r(s)$, if $w(s) = 0$.

(b) Find $y(s)$ as a function of the disturbance, $w(s)$, if $r(s) = 0$.

(c) The tracking error is defined as $e(s) = r(s) - y(s)$. Find the steady state error, e_{ss}, if $r(t)$ is the unit step, and $w(t)$ is a unit step.

5. Given a plant $y(s) = g(s)\,u(s), g(s) = \dfrac{4}{s(s+1)(s+2)}$, where y is the output and u is the input. the Bode plot of $g(s)$ is given as follows:



Suppose the following controller $u = k(y_{\text{ref}} - y)$, where y_{ref} is the tracking signal, is used to control the plant.

(a) Let $k = 1$. Determine if the resultant closed loop system is stable by using Bode plot analysis.

(b) For what value of k (if any) does the gain margin (GM) of the resultant system = 16 db? What is the resultant phase margin (PM) in this case?

(c) It is desired to find a new controller so that the GM of the resultant system has $GM \geq 1000$ and $PM \geq 45°$, and such that $\lim_{t\to\infty} (y_{\text{ref}} - y) = 0$, for all constant y_{ref}. Find a controller which achieves this.