# UNIVERSITY OF TORONTO FACULTY OF APPLIED SCIENCE AND ENGINEERING

#### FINAL EXAMINATION, APRIL 2001

Second Year - Program 6

#### CHE 212S - HEAT TRANFER

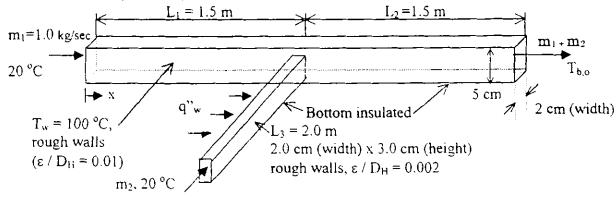
#### Examiner - M. Kawaji

- Lecture notes, a textbook by Kreith (or equivalent) and a calculator of any type are permitted.
- Answer all three questions stating any assumptions you are making in your answer.
- If an iterative solution method is used, show the first step and briefly indicate how subsequent iteration will be performed.
- Tables of water and air properties from Kreith are provided on page 4.

### 1) Forced Convection (35 Marks)

Consider turbulent convection of water in a heated duct with a 2 cm x 5 cm rectangular cross section and 3.0 m length as shown below. The duct has **rough** inner walls with a relative roughness of ( $\epsilon$  /  $D_H$  = 0.01). The duct walls are kept at  $T_w$  = 100 °C everywhere, except for the bottom floor, which is **insulated**. Water enters the duct at 20 °C flowing at a rate of  $m_1$  = 1.0 kg/sec. A second stream of water flows through a smaller rectangular duct (2 cm x 3 cm cross section and 2.0 m length) also with **rough** walls ( $\epsilon$  /  $D_H$  = 0.002) and an **insulated bottom wall**. The second stream joins the main flow in the duct at x = 1.5 m from the main duct entrance.

- a) Calculate the bulk temperature of water,  $T_{b,o}$  at the outlet of the main duct when there is no flow in the second duct, i.e.,  $m_2 = 0$ . Neglect any entrance effects. (20 marks)
- b) The second water stream is now turned on at a certain flow rate,  $m_2$ , less than 1.0 kg/s. This stream is heated at a constant wall heat flux of  $q_w$ " = 50,000 W/m², so that the bulk temperature increases from 20 °C to  $T_{b2}$  at the junction with the main duct. The mass flow rate in the first half of the main duct ( $L_1$ ) is still  $m_1 = 1.0$  kg/sec, but after mixing with the second stream, the total mass flow rate increases to  $m_1 + m_2$ . Determine the mass flow rate,  $m_2$ , such that the bulk temperature of the mixture would reach  $T_{b,0} = 65$  °C at the outlet of the main duct. Hint: For heat transfer rate calculation in the second half ( $L_2$ ) of the main duct, assume the <u>average</u> bulk temperature of the mixture,  $T_b$ , to be 55 ~ 60 °C. (15 marks)



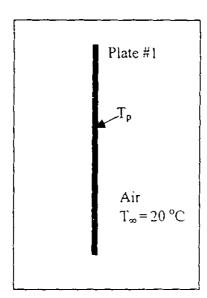
## 2) Natural Convection and Radiation Heat Transfer (25 Marks)

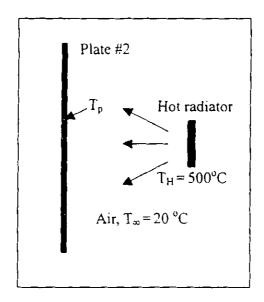
Thick vertical plates (1.0 m wide, 2.0 m high and 10 mm thick) are cooled by natural convection to ambient air at 20 °C as shown below. Both plates are made of steel having a thermal conductivity of  $k = 50 \text{ W/m}^{\circ}\text{C}$ . The first plate generates heat internally at a volumetric heat generation rate of  $Q_{gen}$ " = 100,000 W/m<sup>3</sup>. The second plate does not generate heat internally but receives heat by radiation from a hot radiator ( $T_H = 500 \, ^{\circ}\text{C}$ ) placed 2.0 m away.

- a) Determine the average steady-state temperature of the first plate, T<sub>p</sub>, assuming that the temperature is uniform throughout the plate. In reality, the temperature of the first plate would not be uniform. By roughly estimating the temperature variations in the plate, sketch the temperature distributions across the plate thickness and along the height. (15 marks)
- b) Calculate the average temperature of the second plate if the plate and the hot radiator surface are both assumed to be black bodies and the rate of radiation heat transfer to the second plate is given by,

$$Q_{rad}(W) = \sigma F_{1-2} (T_H^4 - T_p^4) A_p$$

where  $\sigma$  is a Stefan-Boltzmann constant (= 5.669x10<sup>-8</sup> W/m<sup>2</sup>K<sup>4</sup>),  $F_{1-2}$  is a view factor having a value of 0.07,  $T_p$  is the average plate temperature, and  $A_p$  is the surface area of the plate facing the hot radiator. (10 marks)



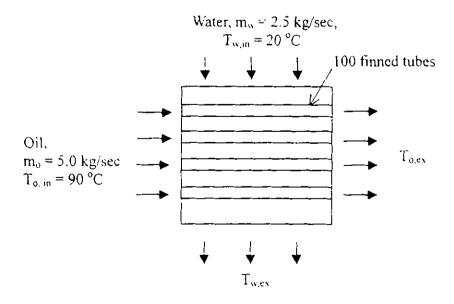


## 3) Heat Exchanger (40 Marks)

A hot stream of oil is to be cooled using cold water in a special cross-flow heat exchanger with both fluids <u>unmixed</u>. The oil flows through 100 finned tubes at a total mass flow rate of  $m_o = 5.0$  kg/sec. Cold water enters the heat exchanger at  $T_{w,in} = 20$  °C, and flows across the tube bundle at a total mass flow rate of  $m_w = 2.5$  kg/sec. The tubes are equipped with fins such that the overall heat transfer coefficient is increased to a value of U = 1,000 W/m<sup>2</sup> °C. Answer the following questions using the following values of specific heat capacities.

Oil: Co = 2,100 J/ kg 
$$^{\circ}$$
C Water: C<sub>w</sub> = 4,200 J/ kg  $^{\circ}$ C

- a) The heat exchanger is operated to obtain equal outlet oil and water temperatures, i.e.,  $T_{o,ex} = T_{w,ex}$ . For this case, draw a temperature diagram for an equivalent double-pipe counterflow heat exchanger, showing all the temperatures at the inlet and outlet. (5 marks)
- b) Using both the LMTD and effectiveness-NTU methods, determine the heat transfer area, A<sub>H</sub> (m<sup>2</sup>), which would provide equal outlet temperatures of oil and water. What is the total heat transfer rate, Q (W)? (15 marks)
- c) The oil flow rate is now reduced to 3.0 kg/sec, while the water flow rate and all other parameters remain unchanged. For this new operating condition, determine the total heat transfer rate, Q (W), and the outlet temperatures of both fluids. If you are not sure of the heat transfer area obtained in part (b), use a value of A<sub>H</sub> = 10 m<sup>2</sup>. (10 marks)
- d) If the exit temperatures of oil and water are to be made equal again for the reduced oil flow rate,  $m_0 = 3.0 \text{ kg/sec}$ , either the water flow rate and/or the inlet water temperature,  $T_{w,in}$ , may be changed. At what new inlet water temperature,  $T_{w,in}$ , would the two fluids have the same exit temperatures if the water flow rate is to be kept unchanged at 2.5 kg/sec? Comment on the feasibility of this option. (10 marks)



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TABLE 13 Water at saturation pressure

| Temperature, |      | ure,     | Density,<br>p<br>(kg/m <sup>3</sup> )                  | Coefficient of Thermal Expansion, $\beta \times 10^4$ (1/K) | *   | Thermal Conductivity, k (V/m K) | Thermal Diffusivity, $\alpha \times 10^6$ (m <sup>2</sup> /s) | Absolute<br>Viscosity,<br>$\mu \times 10^6$<br>(N s/m <sup>2</sup> ) | Kinematic Viscosity, $v \times 10^6$ (m <sup>2</sup> /s) | Prandtl<br>Number,<br>Pr | $\frac{g\beta}{\mu^2} \times 10^{-9}$ (1/K m <sup>3</sup> ) |
|--------------|------|----------|--|---|---|---------------------------------|---|--|--|--------------------------|---|
| "li          | K    | <u> </u> | $\times 6.243 \times 10^{-2}$<br>= $(10_{\rm m}/10^3)$ | × 0.5556<br>= (1/R)   | $\times 2.388 \times 10^{-4}$<br>= (B(u/lb <sub>m</sub> °F) | × 0.5777<br>= (Btu/hr ft °F)    | $\times 3.874 \times 10^{4}$<br>= $(6t^{2}/hv)$               | $ \times 0.6720 $ $= (10_{m}/f(s))$                                  | $ \times 3.874 \times 10^4 $ $= (ft^2/hc)$               |                          | $\times 1.573 \times 10^{-2}$<br>= (1/R fc <sup>3</sup> )   |
| 32           | 273  | 0        | 999.9  | -0.7  | 4226  | 0.558                           | 0.131   | 1794   | 1.789  | 13.7                     | _   |
| 41           | 278  | 5        | 1000   | _   | 4206  | 0.568                           | 0.135   | 1.735  | 1.535  | 11.4                     | _   |
| 50           | 283  | 10       | 999.7  | 0.95  | 4195  | 0.577                           | 0.137   | 1296   | 1.300  | 9.5                      | 0.551   |
| 50           | 288  | 15       | 999.1  | _   | 4187  | 0.585                           | 0.141   | 1136   | 1.146  | 8.1                      | _   |
| 68           | 293  | 20       | 998.2  | 2.1   | 4182  | 0.597                           | 0.143   | 993  | F.000  | 7.0                      | 2.035   |
| 77           | 298  | 2.5      | 997.1  | _   | 4178  | 0.606                           | 0.146   | 880.6  | 0.884  | 6.1                      | _   |
| 86           | 303  | 30       | 995.7  | 3.0   | 4176  | 0.615                           | 0.149   | 792.4  | 0.805  | 5.4                      | 4.540   |
| 95           | 109  | 35       | 994.1  | <del>-</del> -  | 4175  | 0.624                           | 0.150   | 719.8  | 0.725  | 4.8                      | _   |
| 104          | 313  | 40       | 992.2  | 3.9   | 4175  | 0.633                           | 0.151   | 658.0  | 0.658  | 4.3                      | 8.\$33  |
| 113          | 316  | 45       | 990.2  | _   | 4176  | 0.640                           | 0.155   | 605.1  | 0.611  | 3.9                      | _   |
| 122          | 323  | 50       | 988.1  | 4.6   | 4178  | 0.647                           | 0.157   | \$55.1   | 0.556  | 3.55                     | 14.59   |
|              | 148  | 75       | 974.9  | _   | 4190  | 0.671                           | 0.164   | 376.6  | 0.366  | 2.23                     | _   |
| 212          | 373  | LOO      | 958.4  | 7.5   | 4211  | 0.682                           | 0.169   | 277.5  | 0.294  | 1.75                     | 85.0 <del>9</del>   |
| 248          | 393  | 120      | 943.5  | 8.5   | 4232  | 0.685                           | 0.171   | 235.4  | 0.244  | 1 43                     | 140.0   |
| 284          | 413  | 140      | 926.3  | 9.7   | 4257  | 0.684                           | 0.172   | 201.0  | 0.212  | 1.23                     | 211.7   |
| 320          | 433  | 160      | 907.6  | 10.8  | 4285  | 0.680                           | 0.173   | 171.6  | 0.191  | 1.10                     | 290.3   |
| 356          | 153  | 180      | 886.6  | 12.1  | 4396  | 0.673                           | 0.172   | 152.0  | 0.173  | 1.01                     | 396.5   |
| 392          | 47,1 | 200      | 862.8  | 13.5  | 4501  | 0.665                           | 0.170   | 139.3  | 0.160  | 0.95                     | 517.2   |
|              | 493  |          | 837.0  | 15.2  | 4605  | 0.652                           | 0.167   | 124.5  | 0.149  | 0.90                     | 671.4   |
| 464          | 513  | 240      | 809.0  | 17.2  | 4731  | 0.634                           | 0.162   | 113.8  | 0.141  | 0.86                     | 848.5   |
|              | 533  |          | 779.0  | 20.0  | 4982  | 0.613                           | 0.156   | 104.9  | 0.135  | 0.86                     | 1076  |
| 536          | 553  | 280      | 750.0  | 23.8  | 5234  | 0.588                           | 0.147   | 98 07  | 0.131  | 0.89                     | 1360  |
|              |      | 300      |  | 29.5  | 5694  | 0.564                           | 0.132   | 92.18  | 0.128  | 0.98                     | 1766  |

TABLE 27 Dry air at atmospheric pressure

| Temperature,<br>7 |      | re,  | Density, $\rho = (\text{kg/m}^3)$                  | Coefficient of Thermal Expansion, $\beta \times 10^{\circ}$ (1/K) | •   | Thermal<br>Conductivity,<br>k<br>(W/m K) | Thermal Diffusivity, $\alpha \times 10^6$ (m <sup>2</sup> /s) | Absolute<br>Viscosity,<br>$\mu \times 10^6$<br>(N s/m <sup>2</sup> ) | Kinematic<br>Viscosity,<br>$\nu \times 10^6$<br>(m <sup>2</sup> /s) | Prandd<br>Number,<br>Pr | $\frac{g\beta}{v^2} \times 10^{-3}$ (1/K m')             |
|-------------------|------|------|--|---|---|--|---|--|---|-------------------------|--|
| *1:               | К    | °C   | $\times 6.243 \times 10^{-2}$<br>= $(16_m/f(t^3))$ | × 0.5556<br>= (1/R)   | $\times 2.388 \times 10^{-4}$<br>= (B(u/lb <sub>m</sub> °F) | × 0.5777<br>= (Btu/hr ft °F)             | $ \times 3.874 \times 10^3 $ $= (ft^2/hr)$                    | $ \times 0.6720 $ $= (10_{\rm m}/{\rm ft/s}) $                       | $\times 3.874 \times 10^4$<br>= (ft <sup>2</sup> /hr)               |                         | $\times 1.573 \times 10^{-2}$<br>= $(1/R \text{ (c}^3))$ |
| 32                | 273  | 0    | 1.252  | 3.66  | 1011  | 0.0237                                   | 19.2  | 17.456   | 13.9  | 0.71                    | 1.85   |
| 68                | 293  | 20   | 1.164  | 3.41  | 1012  | 0.0251                                   | 22.0  | 18.240   | 15.7  | 0.71                    | 1.36   |
| 104               | 313  | 40   | 1.092  | 3.19  | 1014  | 0.0265                                   | 24.8  | 19.123   | 17.6  | 0.71                    | 1.01   |
| 140               | 333  | 60   | 1.025  | 3.00  | 1017  | 0.0279                                   | 27.6  | 19.907   | 19,4  | 0.71                    | 0.782  |
| 176               | 353  | 80   | 0.968  | 2.83  | 1019  | 0.0293                                   | 30.6  | 20.790   | 21.5  | 0.71                    | 0.600  |
| 212               | 373  | 100  | 0.916  | 2.68  | 1022  | 0.0307                                   | 33.6  | 21.673   | 23.6  | 0.71                    | 0.472  |
| 392               | 473  | 200  | 0.723  | 2.11  | 1035  | 0.0370                                   | 49.7  | 25.693   | 35.5  | 0.71                    | 0.164  |
| 572               | 573  | 300  | 0.596  | 1.75  | 1047  | 0.0429                                   | 68.9  | 29.322   | 49.2  | 0.71                    | 0.0709   |
| 752               | 673  | 400  | 0.508  | 1 49  | 1059  | 0.0485                                   | 89.4  | 32.754   | 64.6  | 0.72                    | 0.0350   |
| 932               | 773  | 500  | 0.442  | 1.29  | 1076  | 0.0540                                   | 113.2   | 35.794   | 81.0  | 0.72                    | 0.0193   |
| 1832              | 1273 | 1000 | 0.268  | 0.79  | 1139  | 0.0762                                   | 240   | 48.445   | 181   | 0.74                    | 0.00236  |