

FACULTY OF APPLIED SCIENCE AND ENGINEERING  
UNIVERSITY OF TORONTO

MIE 444F      Final Exam  
December 10, 2001  
Examiner: Prof. R. Ben Mrad

General Comments:

1. You have 2.5 hours to complete the exam.
2. The maximum number of points you can get on the exam is 100 points.
3. Write your name and student number on the front page to ensure proper identification.
4. Calculators are allowed. You are allowed to use an 8.5"x11" formula sheet. No additional material is allowed.
5. The exam contains 10 pages.

Name: ---

Student Number: ---

Signature: ---

You may use the following table:

| <u>Laplace Transform</u> | <u>Discrete Signal</u> | <u>Z-Transform</u>   |
|--------------------------|------------------------|----------------------|
| $\frac{1}{s}$            | $u(kT)$                | $\frac{z}{z-1}$      |
| $\frac{1}{s^2}$          | $kT$                   | $\frac{Tz}{(z-1)^2}$ |

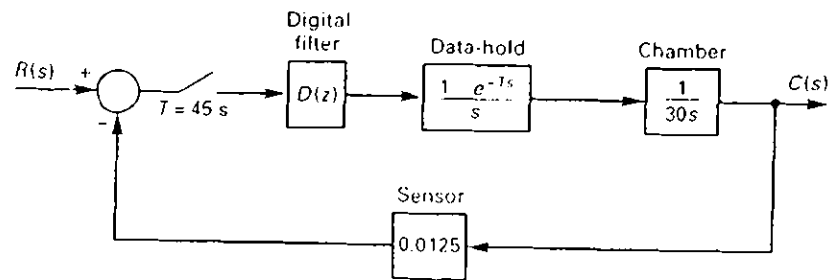
### Problem 1 (20%):

Shown below is the block diagram of a carbon dioxide control system of an environmental plant chamber. The digital filter has the form:

$$D(z) = K_p + K_i T \frac{z}{z - 1}$$

which represents a discrete PI controller.

1. Determine the closed-loop transfer function of the sampled-data system.
2. Determine the difference equation relating the input to the output.
3. Is the closed-loop system stable for  $K_p = 53.33$  and  $K_i = 1$ ? (explain your answer)
4. What are the time constants of the closed-loop system ?





Problem 2 (30%):

**Part A:** The continuous-time signal:

$$x(t) = \sin(20\pi t) + \cos(40\pi t)$$

is sampled with a sampling period  $T$  to obtain the discrete-time signal:

$$x[n] = \sin\left(\frac{\pi n}{5}\right) + \cos\left(\frac{2\pi n}{5}\right)$$

1. Determine a choice for  $T$  consistent with this information.
2. Find a Fourier Series representation of the periodic discrete-time signal  $x[n]$ .



**Part B:** Consider a sequence  $x[n]$  for which the z-transform is:

$$X(z) = \frac{\frac{1}{3}}{1 - 0.5z^{-1}} + \frac{\frac{1}{4}}{1 - 0.2z^{-1}}$$

Determine the value of the sequence  $x[n]$  as  $n$  goes to infinity.

**Part C:**

1. Comment on the difference between the pole-zero matching method and Tustin's method.
2. Determine a digital approximation of the following filter using the pole-zero matching method and then using Tustin's Method:

$$D(s) = \frac{s + 2}{(s + 1)(s + 4)}$$

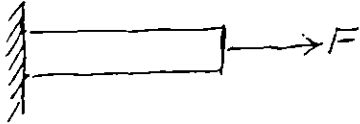
Problem 3 (20%): A digital filter is described by the difference equation:

$$y[n] = \frac{1}{2}(x[n] + x[n - 1])$$

1. Obtain an expression for the frequency response function of the filter.
2. Sketch roughly the magnitude of the frequency response function as a function of frequency.



Problem 4 (15%): A steel bar with an elastic modulus  $E = 205 \cdot 10^6 \text{ kN/m}^2$  and a cross section area  $A = 6.5 \text{ cm}^2$  is subject to an axial force  $F$ . For measuring this force, a strain gauge is cemented on the bar. The nominal resistance of the strain gauge is  $R = 100\Omega$ . The strain gauge is connected in a branch of Wheatstone bridge with all other branches with resistances equal to  $R = 100\Omega$ . Wheatstone bridge voltage output is conditioned using a difference amplifier with gain equal to 1. The strain gauge factor is  $G = 2.1$  and the supply voltage  $V_s = 8.5V$ . Calculate the force  $F$  given a measured voltage  $V_o = 0.5V$ .



Problem 5 (15%):

1. Using an R-2R resistor ladder network and a supply voltage of 8V, determine the analog equivalent to a 3-bit binary number 101 (Please show all work).
2. It is desired to obtain a positive voltage as output of the R-2R ladder network. How would you achieve that ?