FACULTY OF APPLIED SCIENCE AND ENGINEERING
UNIVERSITY OF TORONTO

MIE 404F Final Exam
December 14, 2001
Examiner: Prof. R. Ben Mrad

General Comments:
1. You have 2.5 hours to complete the exam.
2. The maximum number of points you can get on the exam is 100 points.
3. Write your name and student number on the front page to ensure proper identification.
4. Calculators are allowed. The exam is open textbook, and open lecture and tutorial notes and handouts. No additional material is allowed.
5. The exam contains 9 pages.
6. Note that a quadratic equation of the form $s^2 + bs + c = 0$ admits, if $b^2 - 4c > 0$, two solutions of the form: $s_{1,2} = \frac{-b \pm \sqrt{b^2 - 4c}}{2}$

Name: ____________________

Student Number: ____________________

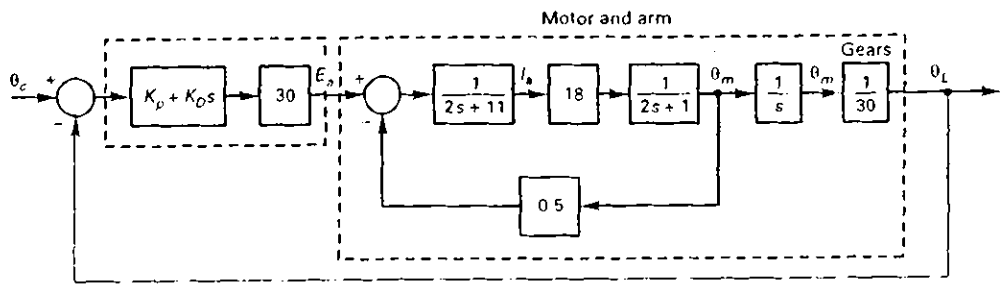
Signature: ____________________

Problem 1 (25%): Shown below is the control system for one joint of a robot arm. The controller is a PD controller.

(a) Determine the plant transfer function $\frac{\Theta_L(s)}{E_a(s)}$.

(b) Find conditions on the controller gains K_p and K_D, with these gains positive, such that the closed-loop system is stable.

(c) Let $K_D = 1$. Fing K_p such that the system will have a steady-state oscillation once a step input is applied to it, and find the period of such an oscillation.

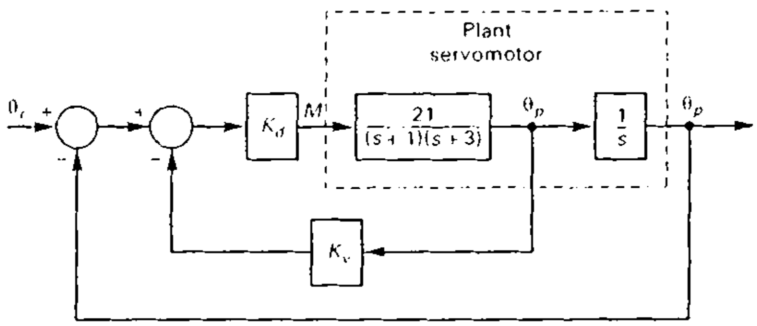


Problem 2 (25%): The block diagram below represents the servo control system for one of the axes of a digital plotter.

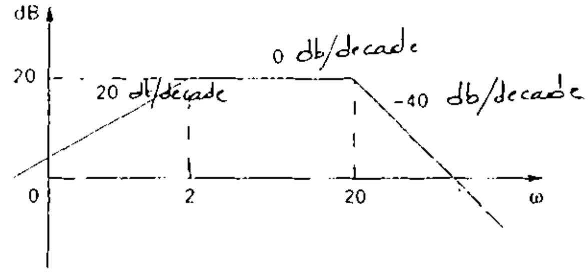
(a) Let $K_v = 0$. Sketch the root locus for this system as the positive gain K_d varies from zero to infinity. In drawing the root locus make sure you determine:

1. arrival and departure angles if there are any.
2. angles of the asymptotes if there are any.
3. imaginary axis crossing points if there are any.
4. break-away/break-in points if there are any.

(b) Remove the rate feedback path ($K_v = 0$) and replace K_d with a PD controller, with the transfer function $G_c(s) = K_p + K_D s$. Calculate the controller gains K_p and K_D to give a characteristic equation of the overall closed loop PD controlled system to have roots at $s = -1 \pm j$.

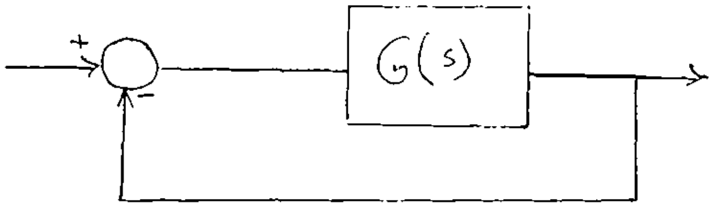


Problem 3 (15%): Consider the magnitude plot of the straight line Bode diagram for the transfer function $G(s)$. Find $G(s)$.



Problem 4 (25%): The unity feedback system shown below has a plant transfer function:

$$G(s) = \frac{Ks}{(0.1s+1)(s+2)}$$



1. Draw the Bode plot of the function $G(s)$ with $K = 30$.
2. What are the approximate values of the gain and phase crossover frequencies ?
3. What are the approximate values of the gain and phase margins of the system ? Is the closed-loop system stable ?
4. Is the closed-loop system stable for the value of $K = 100$? (Explain)

Problem 5 (10%): Shown below is the block diagram of the lateral control system of an aircraft landing system. The output $Y(s)$ is the aircraft lateral position, and the input $Y_c(s)$ is the desired aircraft position. The aircraft position is determined by radar, which is modeled as unity gain with an added noise signal $D_r(s)$. This noise signal represents the inaccuracies of the radar and will be neglected in the following analysis ($D_r(s) = 0$). The signal $D_w(s)$ represents the wind disturbance on the aircraft.

The transfer functions $G_p(s)$ and $G_d(s)$ each have two poles at $s = 0$ in the actual aircraft models. Suppose that the wind on the aircraft is constant, such that $d_w(t)$ is modeled as a constant signal. A system design criterion requires that steady-state effects of the wind disturbance alone on the aircraft are zero. Assuming the system is stable, what form should $G_c(s)$ take such that this criterion is satisfied?