UNIVERSITY OF TORONTO

FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION, December 2001

CHE393 – TRANSPORT PHENOMENA

Examiner: C. R. Ethier

STUDENT NAME: ______________________________

STUDENT ID NUMBER: __________________________

Open book (Type X)
All calculator types allowed
Time allotted: 2.5 hours
Circled quantities in margins are marks for each subquestion

Reminder: for questions requiring numericai answers, units are required and worth 50%

Question	Maximum Mark	Actual Mark
1	10	
2	18	
3	24	
4	24	
5	24	
Total	**100**	

Question 1.

A cyclist rides her bike for a distance d due north, then turns around and rides due south to return to her starting point. Neglect rolling resistance and assume that the cycling route is on flat, level ground.

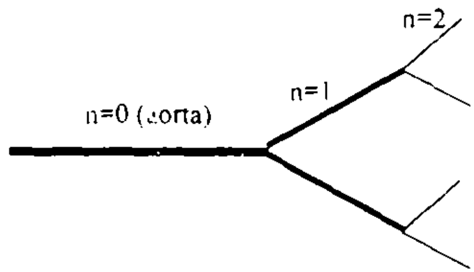
3 (a) If she rides at constant speed V for the entire trip, compute her total energy expenditure and average power output as a function of her drag coefficient, C_D, frontal area, A, and other relevant parameters. Recall that power is work done per unit time.

7 (b) The next day she does the same ride at the same constant speed V, even though a steady wind blows due north at speed U for the entire trip. Is her average power output on the windy day the same, less than, or greater than her average power output on the still day? What is the difference between her power outputs on the two days, if any?

Question 2.

The circulatory system can be modeled as a branching network in which each junction contains a parent tube and two equal diameter daughters. This model can be used to give a rough estimate of pressure drop as a function of position within the arterial tree. Properties of the pipes in generation n are designated by the subscript n. The aorta is considered to be generation zero. The bifurcations are characterized by the area ratio α, defined as the total cross-sectional area of the daughter tubes divided by the cross-sectional area of the parent tube.

You may assume that α is the same for each generation, and also that the length to diameter ratio L_n/D_n is the same for all generations.



5 (a) Show that $D_n = (\alpha/2)^{n/2} D_o$.

13 (b) Show that the total pressure drop from the root of the aorta to the end of the nth generation pipe, Δp, is given by

$$\Delta p = \Delta p_o \frac{(2/\alpha^3)^{(n+1)/2} - 1}{\sqrt{(2/\alpha^3)} - 1}$$

where Δp_o is the pressure drop along the aorta. Assume laminar, Newtonian, steady, fully developed flow in each generation. **Hint:** in the above you will need to use the formula for summation of a geometric series: $\sum_{i=0}^{n} r^i = \frac{1 - r^{n+1}}{1 - r}$

Question 3.

A reactor is cooled by a circular pipe of length $L = 20$ m and diameter $D = 8$ cm. The heat flux through the pipe wall is known to vary with axial position x according to $q/A = B\,(1 - x/L)$, where $B = 9500\ W/m^2$. Water flows in the pipe with an entry temperature of 20 °C at a flow rate of 40 litres/min.

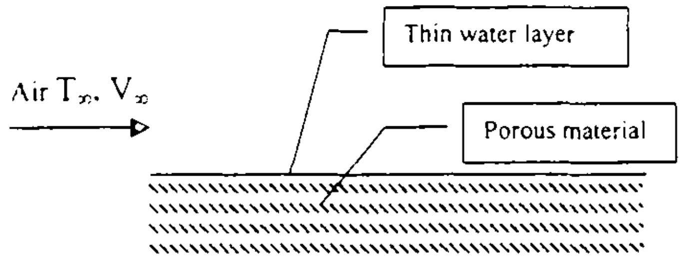
11 (a) What is the water exit temperature?

13 (b) Where does the maximum pipe wall temperature occur? <u>You may neglect entrance effects.</u> Note that this problem can be solved analytically, and does not require graphing or tedious numerical computations. You do not actually have to solve for the maximum pipe wall temperature, just its location.

Property values: use property values for water at 293 K. You do not need to take into account the variations in property values with temperature.

Question 4.

Completely dry air flows over a flat piece of porous material. Water at 7 °C is pumped into the bottom of the material and seeps upwards through the pores so that the surface of the material is covered with a very thin film of water. The air velocity is $V_\infty = 5$ m/s and the air is at 1 atmosphere pressure.



13 (a) What is the rate of water transfer (evaporation) to the air from the plate at a distance of 0.1 m from the leading edge of the material? The vapour pressure of water at 7 °C is 0.02 atm.

11 (b) Assume the porous material is perfectly insulating (transfers no heat). What free-stream temperature, T_∞, is needed so as to supply enough heat to evaporate the water at x = 0.1 m? Note that it requires 4.464 x 10^4 J to vaporize 1 gmol of water at 7 °C.

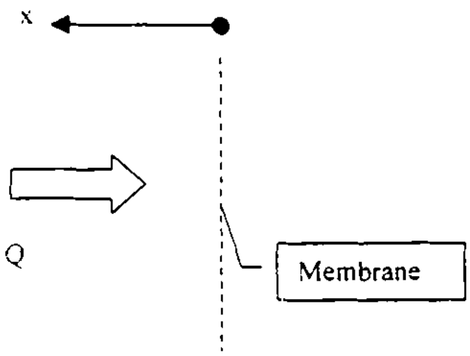
Note: for this question, evaluate all air properties at 280 K, and neglect property variations with temperature. For diffusion coefficient(s), read them from Appendix J and neglect variations with temperature.

Question 5.

A protein solution is being concentrated by filtration. You may assume that the solution consists of a single protein in water, that the membrane freely passes the water, and that the membrane prevents most of the protein from passing through. Specifically, the amount of protein retained by the filter is quantified by the filter rejection coefficient, R, defined by

$$1 - R = \frac{\text{protein mass flux passing through filter}}{|v|\ \rho_1^{o}}$$

where $|v|$ is the magnitude of the mass-average velocity, $\rho_1(x)$ is the protein mass concentration, and ρ_1^{o} is the value of ρ_1 at x=0, i.e. at the filter surface. Note that as R approaches 1, less and less protein passes through the filter.



5 (a) Draw a suitable control volume and show by a mass balance that at steady state, the protein mass flux leaking through the membrane must equal the product $|v|\ \rho_1^{\infty}$, where ρ_1^{∞} is the protein density in the feed solution far from the membrane.

3 (b) Using the same control volume and the definition of R, show that $\rho_1^{o} = \rho_1^{\infty}/(1-R)$.

16 (c) If the protein's diffusion coefficient in water is $D = 6 \times 10^{-6}\ cm^2/s$, $\rho_1^{\infty} = 10\ \mu g/ml$, and R = 98%, determine the steady state protein mass concentration, $\rho_1(x)$, at x = 0.001 cm. The solution is pumped at a steady flow rate of Q = 0.1 ml/s through a membrane with surface area 5 cm^2. This part of the question requires you to compute $\rho_1(x)$.

You may assume that $\rho_{protein} << \rho_{water}$, which implies that the density of the solution is constant and that the mass average velocity does not depend on x.