Student Name:____

Student ID No.:____

University of Toronto

The Edward S. Rogers Sr.
Dept. of Electrical and Computer Engineering

Exam - MAT197S - Calculus B

April 2001 Final Examination
Duration: 2 hours 30 minutes

Examiners:

S. Abou-Ward
P. Barker
A. Nabutovsky

For Markers Use Only	
Question	Marks
1	/13
2	/9
3	/8
4	/9
5	/12
6	/12
7	/13
8	/13
9	/11
Total	/100

No Aids Allowed

1. Evaluate the following integral:

[8 marks] (a) $\int \frac{1}{x^3+1}\, dx$.

[5marks] (b) $\int \cos\left(\frac{3x}{2}\right)\ \cos^2\left(\frac{x}{2}\right)\ dx$.

2. Find the following limit (if it exists):

[4 marks] (a) $\lim_{x \to \infty} \dfrac{x - \sin x}{x - \sinh x}$.

[5marks] (b) $\lim_{x \to \infty} \dfrac{f''(x)}{g''(x)}$ if $f(x) = \int_1^x g(t)\left(t + \dfrac{1}{t}\right) dt$, and $g(x) = xe^{x^2}$.

[4 marks] 3. (a) Find the equation(s) of the tangent(s) to the curve $x(t) = (t^3 - t)$, $y(t) = t\sin\left(\frac{\pi t}{2}\right)$ at the point $(1,1)$.

[4 marks] (b) Calculate $\frac{d^2y}{dx^2}$, where $x(t) = \sin^2 t$ *and* $y(t) = \cos t$ *at* $t = \frac{\pi}{4}$.

[5 marks] 4. (a) If $\int \left[\frac{1}{\sqrt{1+2x^2}} - \frac{c}{(x+1)} \right] dx = \ln \left| \frac{\sqrt{1+2x^2} + \sqrt{2}x}{(x+1)^c} \right| + k.$

Find the value c such that

$$\int_0^\infty \left[\frac{1}{\sqrt{1+2x^2}} - \frac{c}{(x+1)} \right] dx \text{ is convergent.}$$

[4 marks] (b) Determine whether $\int_0^\infty \frac{dx}{x^3+1}$ converges or not and if it does determine its value, given that

$$\int \frac{dx}{x^3+1} = \frac{1}{3} \ln \left| \frac{x+1}{\sqrt{x^2+x+1}} \right| + \frac{\sqrt{3}}{3} \tan^{-1}\left(\frac{2x-1}{\sqrt{3}} \right) + c\,.$$

5. Test the following series for (a) absolute Convergence, (b) conditional convergence and/or (c) divergence.

[3 marks] (a) $\sum_{1}^{\infty} \frac{\cos(n\pi)}{n}$

[3 marks] (b) $\sum_{1}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}} \right)$

[3 marks] (c) $\sum_{1}^{\infty} \frac{n^n}{(3^n)^2}$

[3 marks] (d) $\sum_{1}^{\infty} \frac{n^2}{3^n - n^2 + 200}$

[8 marks] 6. (a) Find the Taylor series in powers of $(x-1)$ of the function $f(x) = x^2 \ln x^3$.

[4 marks] (b) What is the interval of convergence of this series?

7. Let $f(x) = xe^x$.

[5 marks] (a) Find a power series representation of f in powers of x.

[4 marks] (b) Integrate the power series in part (a) and show that

$$\sum_{n=1}^{\infty} \frac{1}{n!(n+2)} = \frac{1}{2}.$$

[4 marks] (c) Estimate to within 0.01 the integral $\int_0^1 \tan^{-1} x^2 \, dx$.

8. Let $\vec{r}(t) = (2\sin 3t,\ t,\ 2\cos 3t)$ be a position vector which describes the motion of a particle at some $t \in I$.

[5 marks] (a) Find an equation of the normal plane of this curve at the point $(o, \pi, -2)$.

[5 marks] (b) Find an equation of the osculating plane of this curve at the point $(o, \pi, -2)$.

[3 marks] (c) Find the curvature of this curve at the point $(o, \pi, -2)$. Is it independent of t?

[6 marks] 9. (a) Find the length of the cardioid $r = 1 - \cos\theta$.

[5 marks] (b) Let $f(x,y,z) = \left(x^2 + y^2\right)\cos\left(\dfrac{y+z}{x}\right)$. If $x\dfrac{\partial f}{\partial x} + y\dfrac{\partial f}{\partial y} + z\dfrac{\partial f}{\partial z} = k\, f(x,y,z)$.

Find the value of k.