UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING
FINAL EXAMINATION, DECEMBER 2001

Third Year – Engineering Science
APM 384 H1F - PARTIAL DIFFERENTIAL EQUATIONS
Exam Type: C

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Questions have equal value. Total marks - 100

1. Solve the initial-boundary value problem:

$$\partial_t^2 u = \partial_x^2 u - 2u \quad \text{for } x \in (0,1) \text{ and } t \geq 0;$$

$$u(0,t) = u(\pi,t) = 0, \quad u(x,0) = x(\pi - x), \quad \partial_t u(x,0) = 0.$$

where ∂_t means $\partial / \partial t$ etc. Check if the solution remains symmetric about $x = \pi/2$ for $t > 0$. Briefly describe a physical situation which leads to such a problem.

2. Solve the heat equation $\partial_t u = k \nabla^2 u$ inside a circle of radius a with zero temperature around its entire boundary, if initially $u(r,\theta,0) = f(r,\theta)$ is given. Identify any special functions required in your solution.

Briefly analyze $\lim\limits_{t \to \infty} u(r,\theta,t)$.

3. Solve Laplace's equation $\nabla^2 u = 0$ inside a square with opposite corners at $(0,0)$ and (π, π) in the xy plane, given that the boundary values are

$f(x) = 2 \sin x + \sin 3x$ on one side and 0 on the on the other three sides.

Evaluate any Fourier coefficients that occur in your solution.

4. Use the method of images to find the Green's function for Poisson's equation $\nabla^2 u = f$ in the unit circle with $u = 0$ on the boundary. Using this Green's function, derive a formula for the solution of Poisson's equation with given boundary values $h(\theta)$, where θ is the polar angle.