UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING
FINAL EXAMINATION, APRIL 1996
Third Year - Programs 05bme, 05ce, 05e
ECE356S - SYSTEM AND SIGNAL ANALYSIS II
Examiner - B.A. Francis

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1. (a) [10 marks] Consider the usual feedback control system with

$$\hat{p}(s) = \frac{s}{(s-1)(s^2+1)}, \quad \hat{c}(s) = \hat{f}(s) = 1.$$

Sketch the Nyquist diagram of $\hat{p}(s)\hat{c}(s)\hat{f}(s)$.

(b) [6 marks] What is the critical point for counting encirclements in this case? How many encirclements must there be for closed-loop stability? How many encirclements are there in this case?

(c) [4 marks] Why does there not exist a stabilizing controller so that the plant output will asymptotically track a step reference input?

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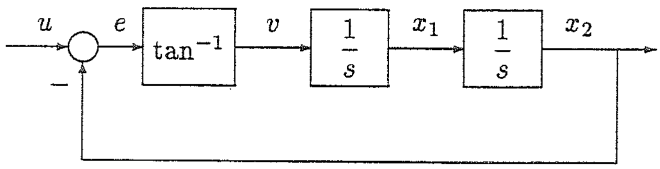
2. (a) [5 marks] Consider the following feedback control system:



The nonlinearity is the saturating function $v = \tan^{-1}(e)$. Taking the state variables as shown, derive the nonlinear state equation

$$\dot{x} = f(x,u).$$

(b) [6 marks] Linearize the system about the equilibrium position where $u = 1$ to find the matrices A and B in the linear equation

$$\dot{\Delta x} = A\Delta x + B\Delta u.$$

You may use the formula $\dfrac{d}{dy}\tan^{-1} y = \cos^2(\tan^{-1} y)$.

(c) [6 marks] Compute the transition matrix e^{tA}. You may use the one-sided Laplace transforms

$$\sin \alpha t \longleftrightarrow \frac{\alpha}{s^2+\alpha^2}, \quad \cos \alpha t \longleftrightarrow \frac{s}{s^2+\alpha^2}.$$

(d) [3 marks] Give an example of a physical system whose state model is linear, time-varying.

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3. (a) [12 marks] Consider the discrete-time system where the input $x(k)$ and output $y(k)$ are related by $\hat{y}(z) = \hat{x}(z^2)$. Find the matrix representation of this system (hint: use the basic definition to construct it column-by-column). Is it time-invariant? Causal? BIBO stable?

   (b) [8 marks] Consider the ideal lowpass discrete-time filter with frequency response

$$\hat{g}(e^{j\omega}) = \begin{cases} 1, & |\omega| \le \pi/4 \\ 0, & \pi/4 < |\omega| \le \pi. \end{cases}$$

   We wish to derive a new filter from this prototype by altering the impulse response $g(k)$. Plot the frequency response $\hat{g}_1(e^{j\omega})$ for the system whose impulse response is $g_1(k) = g(2k)$.

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4. (a) [10 marks] Consider the discrete-time LTI system with transfer function

$$\hat{g}(z) = \frac{z^2}{z+2}.$$

   For the two possible regions of convergence, determine whether the system is causal and/or BIBO stable.

   (b) [10 marks] Take the ROC to be $|z| < 2$ and plot the output $y(k)$ when the input is the unit step starting at time $k = 0$.

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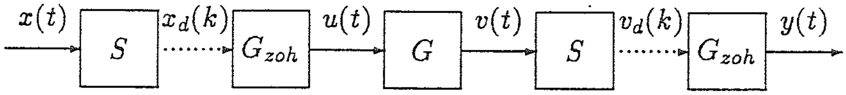
5. (a) [15 marks] In this problem we want to do a digital implementation of the continuous-time system G with transfer function

$$\hat{g}(s) = \frac{1}{s}.$$

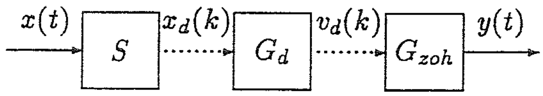
One way to do this is to sample-and-hold the input and output of G as in the following block diagram:



Here S is the ideal periodic sampler with period h and G_{zoh} is the zero-order hold, defined via

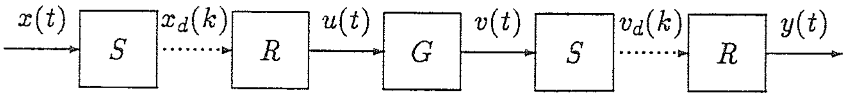
$$u(t) = x_d(k), \quad kh \leq t < (k+1)h.$$

Defining G_d to be the system from $x_d(k)$ to $v_d(k)$, that is, $G_d := SGG_{zoh}$, we have the following system, which can serve as a possible digital implementation of G:



Derive the matrix representation of G_d and conclude it is time-invariant. Find its transfer function, including the ROC.

(b) [5 marks] Consider the preceding development but with G_{zoh} replaced by R, the ideal reconstructor in the sampling theorem; that is,



Now $G_d := SGR$. Is G_d time-invariant? Causal?

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6. (a) [10 marks] The carrier signal $c(t) = \cos(\omega_c t)$ is modulated by the signal

$$x(t) = 2\cos(\omega_1 t) + \cos(\omega_2 t)$$

in the following way:

$$y(t) = [A + x(t)]c(t).$$

Here A is a positive constant. Assume $0 < \omega_1 < \omega_2 < \omega_c$. Sketch the Fourier transform of $y(t)$.

(b) [10 marks] Give a demodulation system to produce $x(t)$ from $y(t)$.
