

1. This question concerns x-ray imaging (10 marks, time: 30 minutes).

Consider an x-ray imaging system which consists of a nearly ideal x-ray generator, and an amorphous selenium detector. The x-ray generator is a point source of x-rays. It produces a poly-energetic beam for  $50 \mu\text{s}$  consisting of 500 30 keV and 500 60 keV photons per  $\text{cm}^2$  at the object plane. The detector consists of a layer of amorphous selenium, ( $a\text{-Se}$   $\rho=4.26 \text{ g/cm}^3$ ), 1 mm thick evaporated onto an  $1024 \times 1024$  array of pixels, each  $100 \times 100 \mu\text{m}$  in size. The object is located 70 cm from the x-ray generator, and the detector is 100 cm from the generator.

(5 marks):

(2 marks) Assuming there is no blurring in  $a\text{-Se}$ , only the pixel size of the detector causes blurring. What is the modulation transfer function (MTF) of the pixel array?

(2 marks) Using your result for the MTF of the pixel array, and given that the MTF of  $a\text{-Se}$  is  $MTF_{\text{Se}}$ , write the equation for the MTF of the entire system.

iii) (1 mark) Is the blurring due to the pixels pre or post gain blurring?

b) (5 marks):

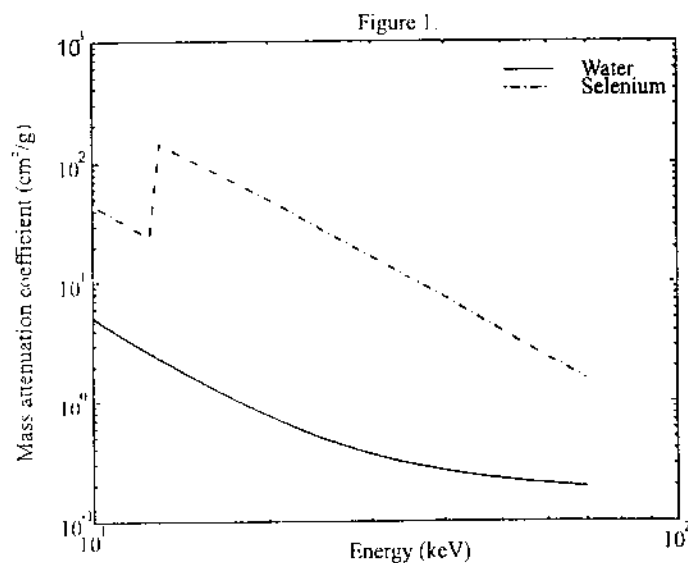
Let the object be a  $7.168 \times 7.168 \times 0.3$  cm "block" of water. The attenuation coefficients for  $a$ -Se and water are shown at the bottom of the page.

iii) (1 mark) Calculate the average energy of the beam at the detector plane after passing through the object.

iv) (1 mark) Using this average energy, calculate the quantum efficiency,  $\eta$ , for the detector.

v) (2 marks) Using the actual beam characteristics, after passing through the object, calculate the quantum efficiency of the detector.

vi) (1 mark) What is the percentage error committed by using the average energy of the beam instead of the actual beam?



2. This question concerns tissue characterization in ultrasonic imaging (10 marks, time: 30 min.)

Figure 1 depicts a system used to characterize an unknown tissue sample. The sample is placed against a perfect reflector within a water bath, and a 10 MHz ultrasound transducer is used to insonate the sample with short pulses. Figure 2 illustrates the signal received at the ultrasound transducer. In this question, you are asked to determine properties of the unknown tissue, including the speed of sound, impedance, and attenuation, based on the provided information. **Ignore the effects of attenuation due to the water bath.** See below for questions.

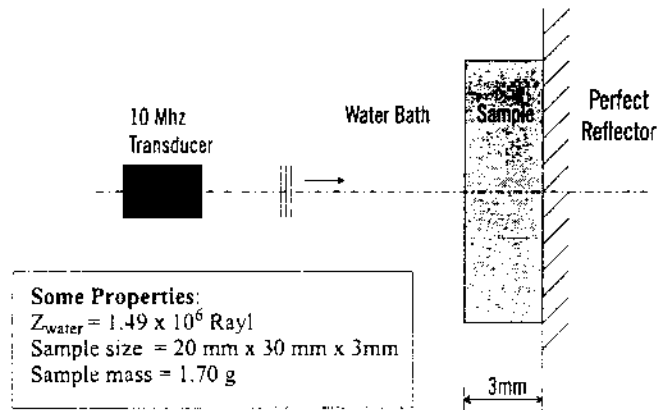


Figure 1. Tissue characterization experimental setup.

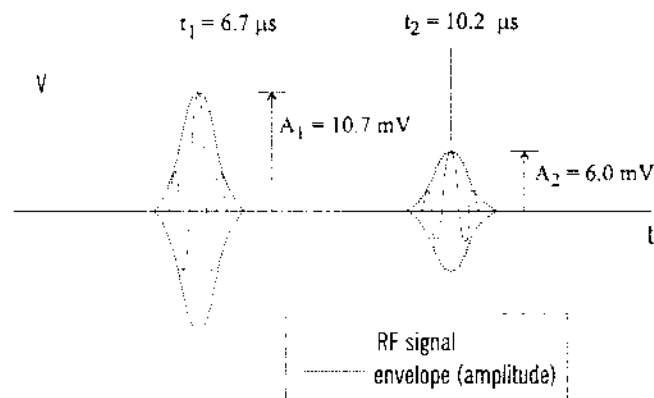


Figure 2 Tissue characterization transducer signal (not to scale!)

Questions:

i) (1 mark) Calculate the speed of sound  $c$  in the sample.

ii) (1 mark) Calculate the acoustic impedance in Rayl for the sample. (Hint: 1 Rayl = 1 kg/(m<sup>2</sup>s))

- iii) (5 marks) Calculate the attenuation coefficient ( $\alpha$  in dB/cm) at this frequency (10 MHz). Remember to ignore attenuation in the water bath. Also, use the following amplitude reflection  $R_a$  and transmission  $T_a$  coefficients:

$$R_a = \frac{Z_2 - Z_1}{Z_2 + Z_1}, \quad T_a = \frac{2Z_2}{Z_2 + Z_1}$$

- iv) (2 marks) Predictions for 5 MHz. Suppose that the frequency dependence for the attenuation coefficient  $\alpha$  is expressed as follows:

$$\alpha = \alpha f^\gamma,$$

If  $\gamma = 1.7$ , determine  $t_1$ ,  $t_2$ ,  $A_1$ , and  $A_2$  for the same experiment using a 5 MHz transducer.

- v) (1 mark) Frequency dependent attenuation (short answer qualitative).

Figure 2 was drawn without consideration of frequency-dependent attenuation. If frequency-dependent attenuation were significant, how would you expect the number of cycles to change in the second pulse, and why?

3. This question concerns doppler ultrasound (10 marks, time: 30 min.)

a) Short answer (5 marks):

i) (1 mark) What is a Doppler "Wall" filter, and why is it used?

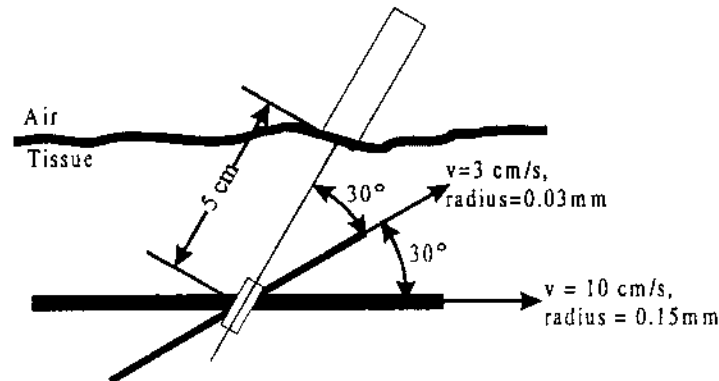
ii) (1.5 marks) List 3 differences between pulsed wave and continuous wave Doppler systems.

iii) (1.5 marks) List three parameters of a colour Doppler imaging system which can affect the frame rate.

iv) (1 mark) Name the two potential sources of ultrasound bioeffects (tissue damage).

b) (5 marks):

A 3 MHz pulsed wave Doppler system operating at a pulse repetition frequency (PRF) of 1 kHz is used to measure blood velocity in two adjacent blood vessels (see diagram below). Both vessels pass through the middle of the Doppler sample volume, which is 1 cm long and is centred at 5 cm distance from the transducer. Assume that the speed of sound in tissue is 1500 cm/s.



i) (1 mark) For each blood vessel, estimate the measured Doppler frequency.

ii) (1.5 marks) If the transducer is a spherically focused circular transducer with a diameter of 2 cm and a focal length of 5 cm, estimate the approximate uncertainty in the measured Doppler frequencies from each vessel resulting from transit-time broadening. State your assumptions.

iii) (1.5 marks) Sketch the resulting Doppler spectrum. What will be the measured mean Doppler frequency?

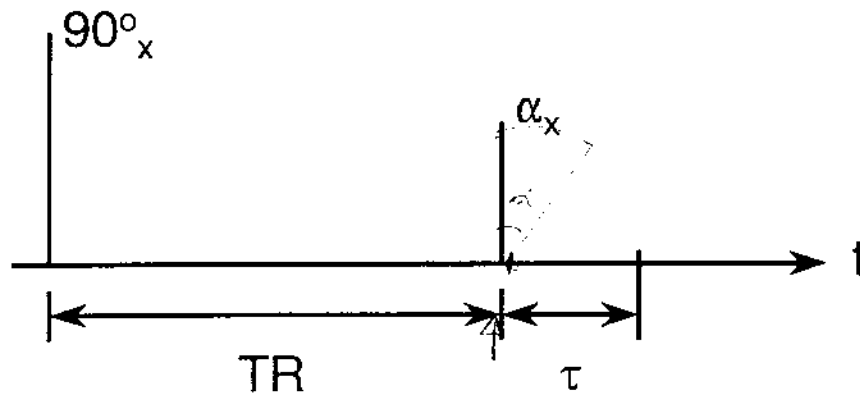
iv) (1 mark) If the PRF is changed to 0.3 kHz, will your answer to part iii) change? Why?

4. This question concerns MRI (10 marks, time: 30 minutes).

a) (5 marks) Describe how the hydrogen nuclei in tissues give rise to bulk magnetization in the presence of a large magnetic field.

b) (5 marks):

Consider a tissue with a certain proton density, and relaxation times  $T_1$  and  $T_2$ , subjected to the two radiofrequency (rf) pulses shown below. The first and second rf pulses tip the net magnetization by  $90$  degrees and  $\alpha$  degrees, respectively, about the x-axis in a frame of reference rotating at the Larmor frequency.



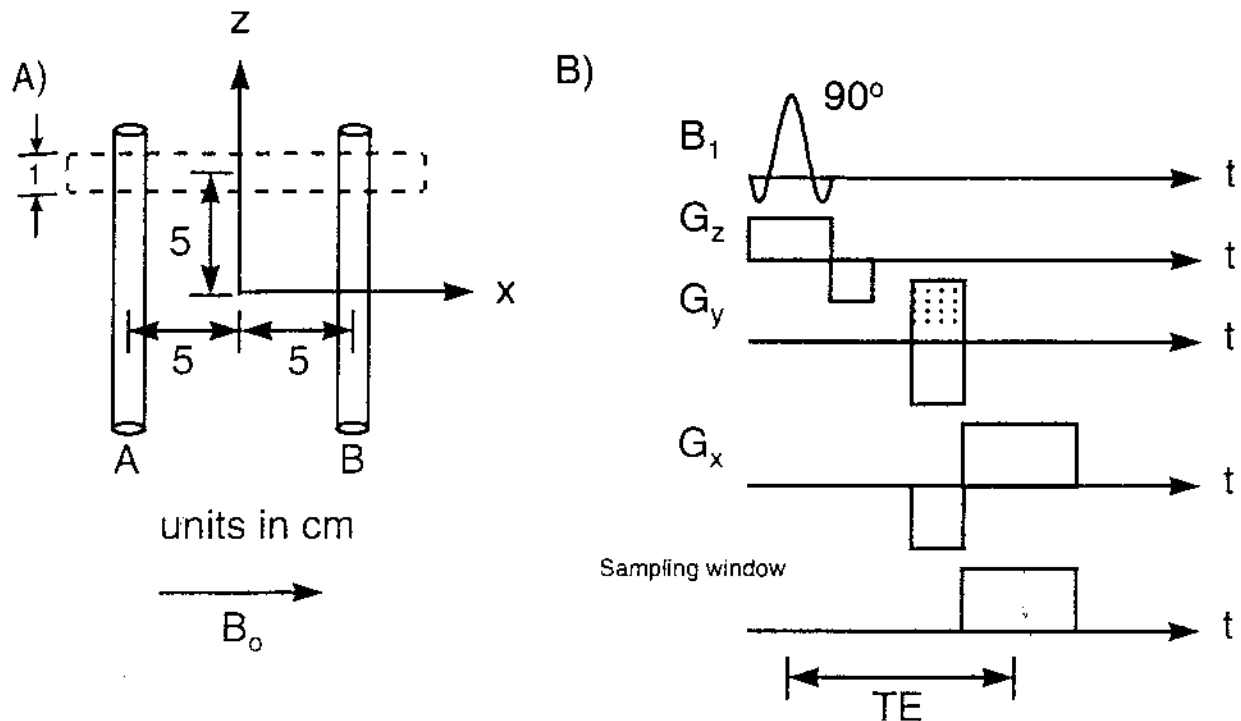
Calculate the amplitude of the transverse magnetization,  $M_y$ , from this tissue at time,  $\tau$ , after the second RF pulse.

The following assumptions simplify the calculation. The frequency of each RF pulse is the same as the Larmor frequency. There is negligible  $T_1$  or  $T_2$  relaxation during each RF pulse. The transverse magnetization decays completely before the second RF pulse. Before the first RF pulse, the longitudinal magnetization,  $M_z$ , is at equilibrium with amplitude,  $M_0$ .



5. This question concerns MRI (10 marks, time: 30 minutes).

Two very thin capillary tubes are placed vertically in a 1.5 T MRI scanner (Fig. A, not to scale) and a 1 cm thick horizontal slice is acquired in the xy plane at spatial location  $z = 5$  cm. Tube A has  $T_2 = 200$  ms, whereas Tube B has  $T_2 = 30$  ms. Images are acquired using the pulse sequence shown in Fig. B. Assume that the  $B_0$  field is perfectly uniform, and that  $T_1$  relaxation can be neglected.



a) (4 marks):

Calculate the ratio of signal intensities (Tube B: Tube A) if:

i) (1 mark) TE is approximately zero.

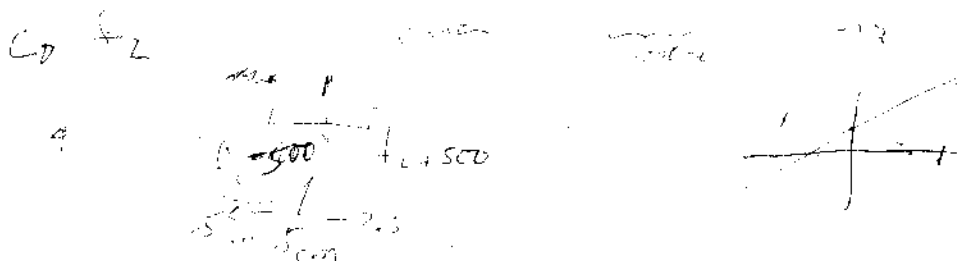
ii) (1 mark) TE = 150 ms.

iii) (2 marks) What does k-space look like in each case? Ignore sampling effects due to frequency and phase encoding.

ii) (1 mark):

The gyromagnetic ratio for protons is  $\gamma/(2\pi) = 42.57 \text{ MHz/T}$ .

- i) (1 mark) If the slice-selective pulse bandwidth is 1000 Hz, what is the associated gradient amplitude in milliTesla per metre?



- ii) (1 mark) What is the frequency of the rf pulse that locates the image slice at  $z=5\text{cm}$ ? State your answer as the difference from the Larmor frequency.

iii) (4 marks):

Assume that TE is very small, the acquisition matrix ( $n_x \times n_y$ ) is  $256 \times 128$ , and that the frequency and phase encoding directions are swapped. The duration of the phase encoding gradient is 4 ms, and the field-of-view is square.

- i) (1 mark) What is the maximum gradient increment  $\Delta G_y$  that ensures the centres of the tubes are not aliased?

- ii) (1 mark) If the field of view is 20 cm and the gradient durations remain unchanged, what are the maximum amplitudes of  $G_x$  and  $G_y$ ?

- iii) (1 mark) For the acquisition described in ii), sketch the reconstructed image, accounting for phase and frequency encoding effects.

- iv) (1 mark) How would the resolution in the frequency encoding direction be altered quantitatively if tube A contained an equal mixture of water and fat?