

**UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING**

FINAL EXAMINATION, APRIL 2001

Third Year – Program: Industrial (Systems) Engineering

**MIE364S – Methods of Quality Control and Improvement
Exam Type: C**

EXAMINER: V. MAKIS

One double-sided 8 ½" x 11" aid sheet and a non-programmable calculator are permitted. Relevant statistical tables are attached

Marks

1. An industrial engineer studied the effect of tool speed (A), depth (B), and feed rate (C), on the life of a cutting tool. The experiment was repeated 3 times using tools of the same brand from 3 different batches.

The data are in the table below.

Speed (A) (rpm)	Depth (B) (in.)	Feed (c) (ips)	Tool Life (min)		
			Y1	Y2	Y3
1000	0.02	2	200	180	210
1000	0.08	2	28	72	52
1000	0.02	10	150	220	160
1000	0.08	10	17	56	40
2000	0.02	2	225	240	170
2000	0.08	2	35	58	38
2000	0.02	10	154	192	208
2000	0.08	10	21	42	65

- 11 a) Formulate a model for this experiment and find the estimates of the model parameters.
- 9 b) Find significant effects ($\alpha = 0.1$). Is the batch effect significant in this experiment? Find the estimates of the standard errors of significant model coefficients.
- 3 c) What combination of factor levels would you recommend?
- 7 d) Are the mean tool lives different for the treatment combinations $(-, -, -)$ and $(+, -, +)$? Formulate and test the appropriate hypothesis at significance level $\alpha = 0.05$. Estimate the p-value.
- 6 e) Find the predicted tool life for the speed of 1,500 rpm, depth of 0.06 in. and feed of 5 ips.
2. Consider the data in Problem 1 for replicate 1 only (the data in the column Y1).
 - 7 a) Using the attached table of random numbers (start with the first row), determine the order of the experimentation. What assumptions are you making? Explain how the experiment is conducted.
 - 8 b) Find the significant effects ($\alpha = 0.10$). Find 90% confidence intervals for the main effects.

Marks

- 3 c) Find the residuals for the treatment combinations $(-, -, -)$ and $(+, -, +)$.

3. A consulting firm wishes to evaluate the performance of four software packages as measured by the computational time. Three problem types were considered and each software package was used once on each problem type. The data in the table below are the computational times in milliseconds.

Problem Type	Software Package			
	1	2	3	4
1	37	14	23	24
2	40	27	18	19
3	36	17	13	20

- 7 a) What kind of an experiment is this? Formulate a model for this experiment and find the estimates of the model parameters.
- 6 b) Formulate and test the appropriate hypotheses at $\alpha = 0.05$ level of significance.
- 6 c) Compare the average computational times with a scaled t distribution. Comment on the plot.
- 3 d) Which software packages appear to be the best? Use $\alpha = 0.05$.
- 5 e) Find a 95% confidence interval for the difference between the mean computational times for packages 3 and 4.
4. To perform a capability study, a sample size $N = 50$ was taken from an in-control process (sample average = 70.5 and the sample variance = 125). The process is controlled by \bar{X} and R charts with probability limits ($\alpha = 0.002$ for both charts) and a constant sample size $n = 4$. The following information is available. \bar{X} chart: LCL = 52, UCL = 86.
- 3 a) Find the in-control process parameters. Determine the lower and the upper control limit for the R chart.
- 6 b) The lower specification limit $LSL = 30$ and the $USL = 110$. Calculate the estimates of the capability indexes C_p , C_{pk} and C_{pm} . Comment. Estimate the proportion of defectives below LSL and above USL . Find the natural tolerance limits for the process, $\alpha = 0.05$ and the confidence level $\gamma = 0.9$. Explain the meaning of these limits.

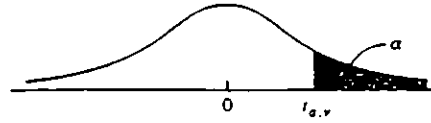
Marks

- 10 c) To control future production, it is required to design an \bar{X} chart with probability limits such that when the process is in control, the probability that the run length is at least 200 should be equal to 0.8 and a shift in the process mean from μ_0 (the in-control value) to $\mu_1 = 80$ should be detected on the first or second sample following the shift with the probability at least 0.6. Determine the sample size, find the lower and the upper control limit for this chart and calculate the ARL for μ_0 and for $\mu_1 = 80$.

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Note: $F_{010, \sigma, \sigma} = 1/F_{010, \sigma, \sigma}$.

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Appendix IV Percentage Points of the t Distribution*

v	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0025	0.001	0.0005
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657	127.32	318.31	636.62
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925	14.089	23.326	31.598
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.213	12.924
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.265	0.727	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.263	0.711	1.415	1.895	2.365	2.998	3.49	4.019	4.785	5.408
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.260	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.258	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.992
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.256	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
24	0.256	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.256	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.256	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.256	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.256	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.256	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.256	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.255	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	0.254	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	0.254	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.253	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

 v = degrees of freedom.* Adapted with permission from *Biometrika Tables for Statisticians*, Vol. I, 3rd ed., by E. S. Pearson and H. O. Hartley, Cambridge University Press, Cambridge, 1966

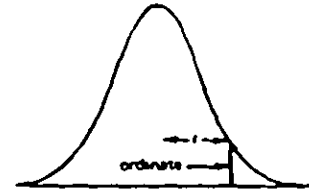
Appendix IX Random Numbers

10480	15011	01536	02011	81647	91646	69179	14194	62590
22368	46573	25595	85393	30995	89198	27982	53302	93965
24130	48360	22527	97265	76393	64809	15179	24830	49340
42167	93093	06243	61680	07856	16376	39440	53537	71341
37570	39975	81837	16656	06121	91782	60468	81305	49684
77921	06907	11008	42751	27756	53498	18602	70659	90655
99562	72905	56420	69994	98872	31016	71194	18738	44013
96301	91977	05463	07972	18876	20922	94595	56869	69014
89579	14342	63661	10281	17453	18103	57740	84378	25331
85475	36857	53342	53988	53060	59533	38867	62300	08158
28918	69578	88231	33276	70997	79936	56865	05859	90106
63553	40961	48235	03427	49626	69445	18663	72695	52180
09429	93969	52636	92737	88974	33488	36320	17617	30015
10365	61129	87529	85689	48237	52267	67689	93394	01511
07119	97336	71048	08178	77233	13916	47564	81056	97735
51085	12765	51821	51259	77452	16308	60756	92144	49442
02368	21382	52404	60268	89368	19885	55322	44819	01188
01011	54092	33362	94904	31273	04146	18594	29852	71585
52162	53916	46369	58586	23216	14513	83149	98736	23495
07056	97628	33787	09998	42698	06691	76988	13602	51851
48663	91245	85828	14346	09172	30168	90229	04734	59193
54164	58492	22421	74103	47070	25306	76468	26384	58151
32639	32363	05597	24200	13363	38005	94342	28728	35806
29334	27001	87637	87308	58731	00256	45834	15398	46557
02488	33062	28834	07351	19731	92420	60952	61280	50001
81525	72295	04839	96423	24878	82651	66566	14778	76797
29676	20591	68086	26432	46901	20849	89768	81536	86645
00742	57392	39064	66432	84673	40027	32832	61362	98947
05366	04213	25669	26422	44407	44048	37937	63904	45766
91921	26418	64117	94305	26766	25940	39972	22209	71500
00582	04711	87917	77341	42206	35126	74087	99547	81817
00725	69884	62797	56170	86324	88072	76222	36086	84637
69011	65795	95876	55293	18988	27354	26575	08625	40801
25976	57948	29888	88604	67917	48708	18912	82271	65424
09763	83473	73577	12908	30883	18317	28290	35797	05998
91567	42595	27958	30134	04024	86385	29880	99730	55536
17955	56349	90999	49127	20044	59931	06115	20542	18059
46503	18584	18845	49618	02304	51038	20655	58727	28168
92157	89634	94824	78171	84610	82834	09922	25417	44137
14577	62765	35605	81263	39667	47358	56873	56307	61607
98427	07523	33362	64270	01638	92477	66969	98420	04880
34914	63976	88720	82765	34476	17032	87589	40836	32427
70060	28277	39475	46473	23219	53416	94970	25832	69975
53976	54914	06990	67245	68350	82948	11398	42878	80287
76072	29515	40980	07391	58745	25774	22987	80059	39911
90725	52210	83974	29992	65831	38857	50490	83765	55657
64364	67412	33339	31926	14883	24413	59744	92351	97473
08962	00358	31662	25388	61642	34072	81249	35648	56891
95012	68379	93526	70765	10592	04542	76463	54328	02349
15664	10493	20492	38391	91132	21999	59516	81652	27195

Appendix V (Continued)

ν_1	ν_2	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
Degrees of freedom for the denominator (ν_2)																				
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	249.5	250.1	251.1	252.2	253.3	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.48	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.50	4.46	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	2.00	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.46	1.39	1.32	1.22	1.00

Note: $F_{\alpha, \nu_1, \nu_2} = t/F_{\alpha, \nu_1, \nu_2}$

TABLE B2. Ordinates of t distribution with v degrees of freedom

v	value of t												
	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00
1	0.318	0.300	0.255	0.204	0.159	0.124	0.098	0.078	0.064	0.053	0.044	0.037	0.032
2	0.354	0.338	0.296	0.244	0.193	0.149	0.114	0.088	0.068	0.053	0.042	0.034	0.027
3	0.368	0.353	0.313	0.261	0.207	0.159	0.120	0.090	0.068	0.051	0.039	0.030	0.023
4	0.375	0.361	0.322	0.270	0.215	0.164	0.123	0.091	0.066	0.049	0.036	0.026	0.020
5	0.380	0.366	0.328	0.276	0.220	0.168	0.125	0.091	0.065	0.047	0.033	0.024	0.017
6	0.383	0.369	0.332	0.280	0.223	0.170	0.126	0.090	0.064	0.045	0.032	0.022	0.016
7	0.385	0.372	0.335	0.283	0.226	0.172	0.126	0.090	0.063	0.044	0.030	0.021	0.014
8	0.387	0.373	0.337	0.285	0.228	0.173	0.127	0.090	0.062	0.043	0.029	0.019	0.013
9	0.388	0.375	0.338	0.287	0.229	0.174	0.127	0.090	0.062	0.042	0.028	0.018	0.012
10	0.389	0.376	0.340	0.288	0.230	0.175	0.127	0.090	0.061	0.041	0.027	0.018	0.011
11	0.390	0.377	0.341	0.289	0.231	0.176	0.128	0.089	0.061	0.040	0.026	0.017	0.011
12	0.391	0.378	0.342	0.290	0.232	0.176	0.128	0.089	0.060	0.040	0.026	0.016	0.010
13	0.391	0.378	0.343	0.291	0.233	0.177	0.128	0.089	0.060	0.039	0.025	0.016	0.010
14	0.392	0.379	0.343	0.292	0.234	0.177	0.128	0.089	0.060	0.039	0.025	0.015	0.010
15	0.392	0.380	0.344	0.292	0.234	0.177	0.128	0.089	0.059	0.038	0.024	0.015	0.009
16	0.393	0.380	0.344	0.293	0.235	0.178	0.128	0.089	0.059	0.038	0.024	0.015	0.009
17	0.393	0.380	0.345	0.293	0.235	0.178	0.128	0.089	0.059	0.038	0.024	0.014	0.009
18	0.393	0.381	0.345	0.294	0.235	0.178	0.129	0.088	0.059	0.037	0.023	0.014	0.008
19	0.394	0.381	0.346	0.294	0.236	0.179	0.129	0.088	0.058	0.037	0.023	0.014	0.008
20	0.394	0.381	0.346	0.294	0.236	0.179	0.129	0.088	0.058	0.037	0.023	0.014	0.008
22	0.394	0.382	0.346	0.295	0.237	0.179	0.129	0.088	0.058	0.036	0.022	0.013	0.008
24	0.395	0.382	0.347	0.296	0.237	0.179	0.129	0.088	0.057	0.036	0.022	0.013	0.007
26	0.395	0.383	0.347	0.296	0.237	0.180	0.129	0.088	0.057	0.036	0.022	0.013	0.007
28	0.395	0.383	0.348	0.296	0.238	0.180	0.129	0.088	0.057	0.036	0.021	0.012	0.007
30	0.396	0.383	0.348	0.297	0.238	0.180	0.129	0.088	0.057	0.035	0.021	0.012	0.007
35	0.396	0.384	0.348	0.297	0.239	0.180	0.129	0.088	0.056	0.035	0.021	0.012	0.006
40	0.396	0.384	0.349	0.298	0.239	0.181	0.129	0.087	0.056	0.035	0.020	0.011	0.006
45	0.397	0.384	0.349	0.298	0.239	0.181	0.129	0.087	0.056	0.034	0.020	0.011	0.006
50	0.397	0.385	0.350	0.298	0.240	0.181	0.129	0.087	0.056	0.034	0.020	0.011	0.006
∞	0.399	0.387	0.352	0.301	0.242	0.183	0.130	0.086	0.054	0.032	0.018	0.009	0.004

Table VI Multiple comparisons with the best and Dunnett tests;
 $d_{\alpha, k, \nu}$ for $P(d \geq d_{\alpha, k, \nu}) = \alpha$

ν	$k = 2$	$\alpha = 0.05$ (one-sided)											
		3	4	5	6	7	8	9	10	11	12	15	20
5	2.44	2.68	2.85	2.98	3.08	3.16	3.24	3.30	3.36	3.41	3.45	3.57	3.72
6	2.34	2.56	2.71	2.83	2.92	3.00	3.07	3.12	3.17	3.22	3.26	3.37	3.50
7	2.27	2.48	2.62	2.73	2.82	2.89	2.95	3.01	3.05	3.10	3.13	3.23	3.36
8	2.22	2.42	2.55	2.66	2.74	2.81	2.87	2.92	2.96	3.01	3.04	3.14	3.25
9	2.18	2.37	2.50	2.60	2.68	2.75	2.81	2.86	2.90	2.94	2.97	3.06	3.18
10	2.15	2.34	2.47	2.56	2.64	2.70	2.76	2.81	2.85	2.89	2.92	3.01	3.12
11	2.13	2.31	2.44	2.53	2.60	2.67	2.72	2.77	2.81	2.85	2.88	2.96	3.07
12	2.11	2.29	2.41	2.50	2.58	2.64	2.69	2.74	2.78	2.81	2.84	2.93	3.03
13	2.09	2.27	2.39	2.48	2.55	2.61	2.66	2.71	2.75	2.78	2.82	2.90	3.00
14	2.08	2.25	2.37	2.46	2.53	2.59	2.64	2.69	2.72	2.76	2.79	2.87	2.97
15	2.07	2.24	2.36	2.44	2.51	2.57	2.62	2.67	2.70	2.74	2.77	2.85	2.95
16	2.06	2.23	2.34	2.43	2.50	2.56	2.61	2.65	2.69	2.72	2.75	2.83	2.93
17	2.05	2.22	2.33	2.42	2.49	2.54	2.59	2.64	2.67	2.71	2.74	2.81	2.91
18	2.04	2.21	2.32	2.41	2.48	2.53	2.58	2.62	2.66	2.69	2.72	2.80	2.89
19	2.03	2.20	2.31	2.40	2.47	2.52	2.57	2.61	2.65	2.68	2.71	2.79	2.88
20	2.03	2.19	2.30	2.39	2.46	2.51	2.56	2.60	2.64	2.67	2.70	2.77	2.87
24	2.01	2.17	2.28	2.36	2.43	2.48	2.53	2.57	2.60	2.64	2.66	2.74	2.83
30	1.99	2.15	2.25	2.33	2.40	2.45	2.50	2.54	2.57	2.60	2.63	2.70	2.79
40	1.97	2.13	2.23	2.31	2.37	2.42	2.47	2.51	2.54	2.57	2.60	2.67	2.75
60	1.95	2.10	2.21	2.28	2.35	2.39	2.44	2.48	2.51	2.54	2.56	2.63	2.72
120	1.93	2.08	2.18	2.26	2.32	2.37	2.41	2.45	2.48	2.51	2.53	2.60	2.68
∞	1.92	2.06	2.16	2.23	2.29	2.34	2.38	2.42	2.45	2.48	2.50	2.56	2.64

From C. W. Dunnett (1955) "A Multiple Comparison Procedure for Comparing Several Treatments with a Control." *Journal of the American Statistical Association* 50, 1112-1118. Reprinted with permission from *Journal of the American Statistical Association*. Copyright 1955 by the American Statistical Association. All rights reserved. C. W. Dunnett (1964) "New Tables for Multiple Comparisons with a Control." *Biometrics* 20, 482-491; and additional tables produced by C. W. Dunnett in 1980.

TABLE 1

Percentage points of the range for samples of n from $N(\mu, 1)$
(Values for which the cumulative probability is P)

$P \backslash n$	2	3	4	5	6	7
.0001	0.000177	0.019046	0.092394	0.205489	0.334168	0.464515
.0005	0.000886	0.042594	0.158155	0.308222	0.463700	0.612589
.0010	0.001772	0.080245	0.199446	0.367392	0.534736	0.691347
.0050	0.008862	0.138447	0.342702	0.554904	0.748983	0.921825
.0100	0.017725	0.193945	0.433676	0.665015	0.869515	1.048144
.0250	0.044319	0.303071	0.594643	0.849672	1.065951	1.250500
.0500	0.088681	0.431402	0.757533	1.029940	1.252885	1.440141
.1000	0.177712	0.618352	0.979366	1.261398	1.488195	1.676051
.2000	0.358287	0.900092	1.285672	1.573441	1.799995	1.985445
.3000	0.544925	1.138259	1.531485	1.818447	2.042028	2.223993
.4000	0.741614	1.362597	1.756529	2.040097	2.259641	2.437706
.5000	0.953873	1.587788	1.978320	2.256882	2.471652	2.645452
.6000	1.190232	1.826320	2.210281	2.482427	2.691658	2.860733
.7000	1.465738	2.074590	2.468799	2.732888	2.935559	3.099199
.8000	1.812388	2.423529	2.783758	3.037317	3.231739	3.388684
.9000	2.326174	2.902380	3.240446	3.478281	3.660721	3.808098
.9500	2.771808	3.314493	3.633160	3.857656	4.030092	4.169554
.9750	3.169822	3.682268	3.984015	4.197026	4.360906	4.493624
.9900	3.642773	4.120303	4.402801	4.602821	4.757047	4.882166
.9950	3.969745	4.424235	4.694087	4.885585	5.033479	5.153613
.9990	4.653508	5.063453	5.308804	5.483754	5.619333	5.729754
.9995	4.922533	5.216400	5.552855	5.721773	5.852849	5.959710
.9999	5.502128	5.864137	6.082864	6.239691	6.361710	6.461392

$P \backslash n$	8	9	10	11	12	13
.0001	0.590186	0.708709	0.819433	0.922514	1.018443	1.107820
.0005	0.751013	0.878357	0.995220	1.102585	1.201493	1.292918
.0010	0.834826	0.965508	1.084583	1.193404	1.293250	1.385252
.0050	1.075281	1.212115	1.334927	1.445920	1.546898	1.639327
.0100	1.204819	1.343385	1.467033	1.578303	1.679205	1.771331
.0250	1.410019	1.549720	1.673517	1.784355	1.884474	1.975611
.0500	1.600414	1.739853	1.862843	1.972582	2.071455	2.161277
.1000	1.835449	1.973327	2.094446	2.202195	2.299037	2.386902
.2000	2.141656	2.276121	2.393844	2.498317	2.592064	2.676969
.3000	2.376728	2.507898	2.622556	2.724195	2.815329	2.897818
.4000	2.586852	2.714772	2.826491	2.925467	3.014177	3.094450
.5000	2.790841	2.915438	3.024202	3.120531	3.206853	3.284960
.6000	3.002059	3.123122	3.228778	3.322347	3.406194	3.482065
.7000	3.235931	3.353046	3.455258	3.545785	3.626919	3.700346
.8000	3.519834	3.632192	3.730280	3.817183	3.895093	3.965627
.9000	3.931349	4.037023	4.129346	4.211200	4.284635	4.351158
.9500	4.286309	4.386509	4.474124	4.551864	4.621655	4.684920
.9750	4.604857	4.700411	4.784033	4.858286	4.924993	4.985497
.9900	4.987183	5.077506	5.156635	5.226963	5.290196	5.347592
.9950	5.254550	5.341439	5.417616	5.485364	5.546312	5.601663
.9990	5.822728	5.902906	5.971307	6.036000	6.097468	6.153802
.9995	6.049760	6.127468	6.195739	6.256568	6.311378	6.361227
.9999	6.545530	6.618237	6.682189	6.739227	6.790668	6.837491

Appendix VII Factors for Two-Sided Normal Tolerance Limits

n	90% Confidence That Percentage of Population Between Limits Is			95% Confidence That Percentage of Population Between Limits Is			99% Confidence That Percentage of Population Between Limits Is		
	90%	95%	99%	90%	95%	99%	90%	95%	99%
2	15.98	18.80	24.17	32.02	37.67	48.43	160.2	188.5	242.3
3	5.847	6.919	8.974	8.380	9.916	12.86	18.93	22.40	29.06
4	4.166	4.943	6.440	5.369	6.370	8.299	9.398	11.15	14.53
5	3.494	4.152	5.423	4.275	5.079	6.634	6.612	7.855	10.26
6	3.131	3.723	4.870	3.712	4.414	5.775	5.337	6.345	8.301
7	2.902	3.452	4.521	3.369	4.007	5.248	4.613	5.448	7.187
8	2.743	3.264	4.278	3.136	3.732	4.891	4.147	4.936	6.468
9	2.626	3.125	4.098	2.967	3.532	4.631	3.822	4.550	5.966
10	2.535	3.018	3.959	2.829	3.379	4.433	3.582	4.265	5.594
11	2.463	2.933	3.849	2.737	3.259	4.277	3.397	4.045	5.308
12	2.404	2.863	3.758	2.655	3.162	4.150	3.250	3.870	5.079
13	2.355	2.805	3.682	2.587	3.081	4.044	3.130	3.727	4.893
14	2.314	2.756	3.618	2.529	3.012	3.955	3.029	3.608	4.737
15	2.278	2.713	3.562	2.480	2.954	3.878	2.945	3.507	4.605
16	2.246	2.676	3.514	2.437	2.903	3.812	2.872	3.421	4.492
17	2.219	2.643	3.471	2.400	2.858	3.754	2.808	3.345	4.393
18	2.194	2.614	3.433	2.366	2.819	3.702	2.753	3.279	4.307
19	2.172	2.588	3.399	2.337	2.784	3.656	2.703	3.221	4.230
20	2.152	2.564	3.368	2.310	2.752	3.615	2.659	3.168	4.161
21	2.135	2.543	3.340	2.286	2.723	3.577	2.620	3.121	4.100
22	2.118	2.524	3.315	2.264	2.697	3.543	2.584	3.078	4.044
23	2.103	2.506	3.292	2.244	2.673	3.512	2.551	3.040	3.993
24	2.089	2.489	3.270	2.225	2.651	3.483	2.522	3.004	3.947
25	2.077	2.474	3.251	2.208	2.631	3.457	2.494	2.972	3.904
26	2.065	2.460	3.232	2.193	2.612	3.432	2.469	2.941	3.865
27	2.054	2.447	3.215	2.178	2.595	3.409	2.446	2.914	3.828
28	2.044	2.435	3.199	2.164	2.579	3.388	2.424	2.888	3.794
29	2.034	2.424	3.184	2.152	2.554	3.368	2.404	2.864	3.763
30	2.025	2.413	3.170	2.140	2.549	3.350	2.385	2.841	3.733
35	1.988	2.368	3.112	2.090	2.490	3.272	2.306	2.748	3.611
40	1.959	2.334	3.066	2.052	2.445	3.213	2.247	2.677	3.518
50	1.916	2.284	3.001	1.996	2.379	3.126	2.162	2.576	3.385
60	1.887	2.248	2.955	1.958	2.333	3.066	2.103	2.506	3.293
80	1.848	2.202	2.894	1.907	2.272	2.985	2.026	2.414	3.173
100	1.822	2.172	2.854	1.874	2.233	2.934	1.977	2.355	3.096
200	1.764	2.102	2.762	1.798	2.143	2.816	1.865	2.222	2.921
500	1.717	2.046	2.689	1.737	2.070	2.721	1.777	2.117	2.783
1000	1.695	2.019	2.654	1.709	2.036	2.676	1.736	2.068	2.718
∞	1.645	1.960	2.576	1.645	1.960	2.576	1.645	1.960	2.576

Appendix VI Factors for Constructing Variables Control Charts

Observations in Sample, <i>n</i>	Chart for Averages					Chart for Standard Deviations					Chart for Ranges				
	Factors for Control Limits					Factors for Center Line					Factors for Center Line				
	<i>A</i>	<i>A</i> ₂	<i>A</i> ₁	<i>c</i> ₄	<i>1/c</i> ₄	<i>B</i> ₃	<i>B</i> ₄	<i>B</i> ₅	<i>B</i> ₆	<i>d</i> ₃	<i>1/d</i> ₃	<i>d</i> ₃	<i>D</i> ₁	<i>D</i> ₂	<i>D</i> ₄
2	2.121	1.880	2.659	0.7979	1.2533	0	3.267	0	2.606	1.128	0.8865	0.853	0	3.686	0
3	1.732	1.023	1.954	0.8862	1.1284	0	2.568	0	2.276	1.693	0.5907	0.888	0	4.358	0
4	1.500	0.729	1.628	0.9213	1.0854	0	2.266	0	2.088	2.059	0.4857	0.880	0	4.698	0
5	1.342	0.577	1.427	0.9400	1.0638	0	2.089	0	1.964	2.326	0.4299	0.864	0	4.918	0
6	1.225	0.483	1.287	0.9515	1.0510	0.030	1.970	0.029	1.874	2.534	0.3946	0.848	0	5.078	0
7	1.134	0.419	1.182	0.9594	1.0423	0.118	1.882	0.113	1.806	2.704	0.3698	0.833	0.204	5.204	0.076
8	1.061	0.373	1.099	0.9650	1.0363	0.185	1.815	0.179	1.751	2.847	0.3512	0.820	0.388	5.306	0.136
9	1.000	0.337	1.032	0.9693	1.0317	0.239	1.761	0.232	1.707	2.970	0.3367	0.808	0.547	5.393	0.184
10	0.949	0.308	0.975	0.9727	1.0281	0.284	1.716	0.276	1.669	3.078	0.3249	0.797	0.687	5.469	0.223
11	0.905	0.285	0.927	0.9754	1.0252	0.321	1.679	0.313	1.637	3.173	0.3152	0.787	0.811	5.535	0.256
12	0.866	0.266	0.886	0.9776	1.0229	0.354	1.646	0.346	1.610	3.258	0.3069	0.778	0.922	5.594	0.283
13	0.832	0.249	0.850	0.9794	1.0210	0.382	1.618	0.374	1.585	3.336	0.2998	0.770	1.025	5.647	0.307
14	0.802	0.235	0.817	0.9810	1.0194	0.406	1.594	0.399	1.563	3.407	0.2935	0.763	1.118	5.696	0.328
15	0.775	0.223	0.789	0.9823	1.0180	0.428	1.572	0.421	1.544	3.472	0.2880	0.756	1.203	5.741	0.347
16	0.750	0.212	0.763	0.9835	1.0168	0.448	1.552	0.440	1.526	3.532	0.2831	0.750	1.282	5.782	0.363
17	0.728	0.203	0.739	0.9845	1.0157	0.466	1.534	0.458	1.511	3.588	0.2787	0.744	1.356	5.820	0.378
18	0.707	0.194	0.718	0.9854	1.0148	0.482	1.518	0.475	1.496	3.640	0.2747	0.739	1.424	5.856	0.391
19	0.688	0.187	0.698	0.9862	1.0140	0.497	1.503	0.490	1.483	3.689	0.2711	0.734	1.487	5.891	0.403
20	0.671	0.180	0.680	0.9869	1.0133	0.510	1.490	0.504	1.470	3.735	0.2677	0.729	1.549	5.921	0.415
21	0.655	0.173	0.663	0.9876	1.0126	0.523	1.477	0.516	1.459	3.778	0.2647	0.724	1.605	5.951	0.425
22	0.640	0.167	0.647	0.9882	1.0119	0.534	1.466	0.528	1.448	3.819	0.2618	0.720	1.659	5.979	0.434
23	0.626	0.162	0.633	0.9887	1.0114	0.545	1.455	0.539	1.438	3.858	0.2592	0.716	1.710	6.006	0.443
24	0.612	0.157	0.619	0.9892	1.0109	0.555	1.445	0.549	1.429	3.895	0.2567	0.712	1.759	6.031	0.451
25	0.600	0.153	0.606	0.9896	1.0105	0.565	1.435	0.559	1.420	3.931	0.2544	0.708	1.806	6.056	0.459

For *n* > 25

$$A = \frac{3}{\sqrt{n}}, \quad A_2 = \frac{3}{c_4 \sqrt{n}}, \quad c_4 = \frac{4(n-1)}{4n-3}$$

$$B_3 = 1 - \frac{3}{c_4 \sqrt{2(n-1)}}, \quad B_4 = 1 + \frac{3}{c_4 \sqrt{2(n-1)}}$$

$$B_5 = c_4 - \frac{3}{\sqrt{2(n-1)}}, \quad B_6 = c_4 + \frac{3}{\sqrt{2(n-1)}}$$

Appendix II Cumulative Standard Normal Distribution

$$\Phi(z) = \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du$$



z	0.00	0.01	0.02	0.03	0.04	z
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.0
0.1	0.53983	0.54379	0.54776	0.55172	0.55567	0.1
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.2
0.3	0.61791	0.62172	0.62551	0.62930	0.63307	0.3
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.4
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.5
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.6
0.7	0.75803	0.76115	0.76424	0.76730	0.77035	0.7
0.8	0.78814	0.79103	0.79389	0.79673	0.79954	0.8
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.9
1.0	0.84134	0.84375	0.84613	0.84849	0.85083	1.0
1.1	0.86433	0.86650	0.86864	0.87076	0.87285	1.1
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	1.2
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	1.3
1.4	0.91924	0.92073	0.92219	0.92364	0.92506	1.4
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	1.5
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	1.6
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	1.7
1.8	0.96407	0.96485	0.96562	0.96637	0.96711	1.8
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	1.9
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	2.0
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	2.1
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	2.2
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	2.3
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	2.4
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	2.5
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	2.6
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	2.7
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	2.8
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	2.9
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	3.0
3.1	0.99903	0.99906	0.99910	0.99913	0.99916	3.1
3.2	0.99931	0.99934	0.99936	0.99938	0.99940	3.2
3.3	0.99952	0.99953	0.99955	0.99957	0.99958	3.3
3.4	0.99966	0.99968	0.99969	0.99970	0.99971	3.4
3.5	0.99977	0.99978	0.99978	0.99979	0.99980	3.5
3.6	0.99984	0.99985	0.99985	0.99986	0.99986	3.6
3.7	0.99989	0.99990	0.99990	0.99990	0.99991	3.7
3.8	0.99993	0.99993	0.99993	0.99994	0.99994	3.8
3.9	0.99995	0.99995	0.99996	0.99996	0.99996	3.9

Appendix II (Continued)

$$\Phi(z) = \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du$$

z	0.05	0.06	0.07	0.08	0.09	z
0.0	0.51994	0.52392	0.52790	0.53188	0.53586	0.0
0.1	0.55962	0.56356	0.56749	0.57142	0.57534	0.1
0.2	0.59871	0.60257	0.60642	0.61026	0.61409	0.2
0.3	0.63683	0.64058	0.64431	0.64803	0.65173	0.3
0.4	0.67364	0.67724	0.68082	0.68438	0.68793	0.4
0.5	0.70884	0.71226	0.71566	0.71904	0.72240	0.5
0.6	0.74215	0.74537	0.74857	0.75175	0.75490	0.6
0.7	0.77337	0.77637	0.77935	0.78230	0.78523	0.7
0.8	0.80234	0.80510	0.80785	0.81057	0.81327	0.8
0.9	0.82894	0.83147	0.83397	0.83646	0.83891	0.9
1.0	0.85314	0.85543	0.85769	0.85993	0.86214	1.0
1.1	0.87493	0.87697	0.87900	0.88100	0.88297	1.1
1.2	0.89435	0.89616	0.89796	0.89973	0.90147	1.2
1.3	0.91149	0.91308	0.91465	0.91621	0.91773	1.3
1.4	0.92647	0.92785	0.92922	0.93056	0.93189	1.4
1.5	0.93943	0.94062	0.94179	0.94295	0.94408	1.5
1.6	0.95053	0.95154	0.95254	0.95352	0.95448	1.6
1.7	0.95994	0.96080	0.96164	0.96246	0.96327	1.7
1.8	0.96784	0.96856	0.96926	0.96995	0.97062	1.8
1.9	0.97441	0.97500	0.97558	0.97615	0.97670	1.9
2.0	0.97982	0.98030	0.98077	0.98124	0.98169	2.0
2.1	0.98422	0.98461	0.98500	0.98537	0.98574	2.1
2.2	0.98778	0.98809	0.98840	0.98870	0.98899	2.2
2.3	0.99061	0.99086	0.99111	0.99134	0.99158	2.3
2.4	0.99286	0.99305	0.99324	0.99343	0.99361	2.4
2.5	0.99461	0.99477	0.99492	0.99506	0.99520	2.5
2.6	0.99598	0.99609	0.99621	0.99632	0.99643	2.6
2.7	0.99702	0.99711	0.99720	0.99728	0.99736	2.7
2.8	0.99781	0.99788	0.99795	0.99801	0.99807	2.8
2.9	0.99841	0.99846	0.99851	0.99856	0.99861	2.9
3.0	0.99886	0.99889	0.99893	0.99897	0.99900	3.0
3.1	0.99918	0.99921	0.99924	0.99926	0.99929	3.1
3.2	0.99942	0.99944	0.99945	0.99948	0.99950	3.2
3.3	0.99960	0.99961	0.99962	0.99964	0.99965	3.3
3.4	0.99972	0.99973	0.99974	0.99975	0.99976	3.4
3.5	0.99981	0.99981	0.99982	0.99983	0.99983	3.5
3.6	0.99987	0.99987	0.99988	0.99988	0.99989	3.6
3.7	0.99991	0.99992	0.99992	0.99992	0.99992	3.7
3.8	0.99994	0.99994	0.99995	0.99995	0.99995	3.8
3.9	0.99996	0.99996	0.99996	0.99997	0.99997	3.9

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