UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING
FINAL EXAMINATION, APRIL 1999
AER 336S - SCIENTIFIC COMPUTING
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1. Assume that f is a given function for which the following values are known: $f(1) = 2$, $f(2) = 3$, $f(3) = 5$, $f(4) = 3$. For these data, find the quadratic spline function that satisfies the condition $q'(1) = 0$. (25 marks)

2. Consider the following function of two variables:

$$f(x, y) = 100(y - x^2)^2 + (1 - x)^2$$

Find the gradient vector and the Hessian matrix. Then apply two steps of Newton's method to "zero the gradient," i.e., to find the minimum of the above function. Use $x = 0$, $y = 0$ as the initial guess. (25 marks)

3. Find a second-order approximation to a third derivative. Give the truncation error term. (25 marks)

4. Consider the following partial differential equation:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

with specified initial conditions and periodic boundary conditions. Use Fourier stability analysis to determine whether the explicit Euler time marching method is stable when second-order centered differencing is used for the spatial derivative. Repeat for the leapfrog time-marching method. If either of these time-marching methods produces conditional stability, find the limit on the time step h. (25 marks)