Department of Mechanical & Industrial Engineering
University of Toronto

FINAL EXAMINATION, APRIL 2001

Fourth Year
MIE469S – Reliability and Maintainability Engineering

Exam Type: A

Examiner: A.K.S. Jardine

NOTES:

1. Attempt all questions.
2. If doubt exists as to the interpretation of any question, you are urged to submit with your answer a clear statement of any assumption you have made.
3. Show all calculations.

1.a) If $r(t)$ is the instantaneous failure rate of an item then $r(t)\ \delta t$ is the probability of failure of the item in the interval $(t,\ t+\delta t)$ given that it has remained operational until time t. Prove that

$$r(t) = \frac{f(t)}{R(t)}$$

where $f(t)$ is the probability density function of the item's failure times and

$$R(t) = \int_t^{\infty} f(t)dt.$$ **(5 marks)**

1.b) Sketch the 'bath-tub' curve for $r(t)$ and explain the different regions of the curve. Indicate your understanding of how the different regions of the 'bath-tub' may arise in practice. Also indicate any maintenance tactics you would consider employing to ameliorate the effect of the 'bath-tub' curve. **(10 marks)**

2.a) A sample of 10 electronic components were put on test for 75 hours. During that time interval there were 6 failures at the following times in hours:

6 15 26 38 52 70

(i) Use Weibull probability paper to analyze these data. Estimate the parameters of the distribution. **(4 marks)**

(ii) Perform a Kolmogorov-Smirnov goodness-of-fit test to verify the form of the distribution. What is your conclusion? Use $\alpha = 0.05$. **(5 marks)**

(iii) If a preventive replacement costs \$50 and a failure replacement \$500, and the objective is to minimize total cost per unit time, which policy is preferable: age-based preventive replacement, constant interval preventive replacement, or replace-only-on-failure? Why? **(6 marks)**

2.b) The following uncensored grouped data were collected on the failure time of feed-water pumps, in units of 1000 hours.

Interval (in 1000 hours)	Number of failures
0-6	3
6-12	13
12-18	36
18-24	31
24-30	14
30-36	3

It is known that the failure replacement of the pump when it fails takes an average of 5 days at a total cost of $5000, whereas a preventive replacement can be finished in 1 day at accost of $ 1000. Determine the optimal preventive replacement age to minimize total cost per 1000 hours. List all the assumptions made in the calculation.

(4 marks)

2.c) The graphical technique developed by Glasser requires that the optimization goal is the minimization of total cost for both the block and age-replacement policies.

If the goal was the minimization of downtime would it be acceptable to use Glasser's graphs? Clearly indicate the reasoning behind your conclusion.

(6 marks)

3. Due to statutory requirements equipment must be taken off load every 12 months, when a major overhaul occurs. Between these overhauls the efficiency of the equipment deteriorates. This reduction of efficiency can be measured by increasing operating cost. To increase the efficiency minor components of the equipment can be replaced. These replacements cost money in terms of wages, materials and loss of production and a balance is required between the money spent on minor replacements and the resulting increase in efficiency. Construct a mathematical model which could be used to determine the optimal replacement policy (interval between replacements) to minimize the total cost between the annual overhauls.

   Clearly outline your reasoning when constructing the model.

   Give a brief outline of how you might, in practice, expect to obtain values for the cost parameters used in your model. **(10 marks)**

4. The Parks and Recreation Department plans to request funding from Council for replacement of the 4 x 4 Pick-up trucks in their fleet. They review their historical maintenance costs for the fleet, talk to their supplier, talk to finance, and obtain the following data:

   Purchase price of a new truck: $35,000

   Interest rate to be used for discounting 10%

Age of Truck	O & M Cost	Resale Value
1	2,000	31,000
2	4,500	29,000
3	5,000	27,000
4	5,200	25,000
5	5,500	24,000
6	6,000	22,000
7	8,000	21,000

(i) Using the model $C(n) = \dfrac{\sum C_i r^i + (A - S_n) r^n}{1 - r^n}$ establish the EAC associated with the replacement ages 1 to 7 inclusive. **(6 marks)**

(ii) Sketch the shape of the EAC(n) versus n **(4 marks)**

(iii) Draft an executive summary based on your analyses that could be used by the Director of Parks and Recreation to make the case to Council for establishing a replacement policy play based on economic considerations for the ten 4x4 trucks in the fleet. **(10 marks)**

5. Rather than base component hazard rate predictions solely on accumulated utilization it may be possible to use concomitant information to improve predictions. The following model, derived from Cox's proportional hazards model, includes explanatory variables z_1 and z_2 along with cumulative flying hours, t, to predict instantaneous hazard rate, $r(t)$, for wheel motors of a haul truck.

$$r(t, z_i) = \frac{2.891}{23{,}360}\left(\frac{t}{23{,}360}\right)^{1.891} e^{(0.002742 Z_1 + 0.0000539 Z_2)}$$

where

z_1 = Fe concentration in PPM (parts per million) from a SOAP (spectroscopic oil analysis program) analysis

z_2 = Sediment reading from test to measure suspended solids

(i) Given the following inspection data from 3 wheel motors, what do you estimate the failure rate to be for each motor? **(3 marks)**

Wheel Motor Number	Age (hours)	Fe in PPM	Sediment Measurement
1	11770	5	6
2	11660	2	6
3	8460	12	2.4

(ii) You are asked to submit a report to mine maintenance regarding your hazard value for motor number 1. How might you explain the value you have obtained and what maintenance action would you recommend? **(4 marks)**

(iii) What interpretation can you place on the value 2.891 in the PH model? **(3 marks)**

6. In class we focused on four key maintenance decision areas, viz.

- Component replacement
- Capital equipment replacement
- Inspection
- Resource requirements

Outline one maintenance decision problem that might occur in each decision area, within an organization you are familiar with, that could be usefully tackled using a model that was covered in the particular area. Do not repeat any of the decision situations used in the case studies covered in class. **(20 marks)**

$$f(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(t-\mu)^2}{2\sigma^2}}$$

$$= \lambda e^{-\lambda t}$$

$$= 2k^2\lambda e^{-2k\lambda t} + 2\lambda(1-k)^2 e^{-2(1-k)\lambda t}$$

$$= \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} e^{-\left(\frac{t}{\eta}\right)^\beta}$$

$$= \frac{\beta}{\eta}\left(\frac{t-\gamma}{v}\right)^{\beta-1} e^{-\left(\frac{t-\gamma}{\eta}\right)^\beta}$$

$$R(t) = \int_t^\infty f(t)\,dt$$

$$F(t) = \int_{-\infty}^t f(t)\,dt$$

$$r(t) = \frac{f(t)}{1-F(t)}$$

For Weibull: $r(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1}$

$$F(t) = \frac{i}{N+1}$$

$$I = \frac{N + 1 - \text{prev. failure order number}}{1 + \text{no. of items following suspended set}}$$

$$r(t) = \frac{F}{\frac{A + (A - F - C)}{2}} = \frac{F}{A} \quad \text{(if no censorings)}$$

Maintenance and Replacement

Interval Replacement:

$$C(t_p) = \frac{C_p + H(t_p)C_f}{t_p}$$

$$\frac{C_p}{C_f} = t_p\, r(t_p) - H(t_p)$$

$$H(T) = \sum_{i=0}^{T-1} [1 + H(T-1-i)] \int_i^{i+1} f(t)\,dt$$

Age Replacement

$$C(t_p) = \frac{C_p R(t_p) + C_f (1 - R(t_p))}{t_p R(t_p) + M(t_p)[1 - R(t_p)]}$$

$$M(t_p) = \frac{\int_{-\infty}^{t_p} t \cdot f(t)\,dt}{1 - R(t_p)}$$

$$\frac{C_p}{C_f - C_p} = r(t_p)\int_0^{t_p} R(t_p)\,dt - F(t_p)$$

For Glasser's graphs $t_p = \mu + z\sigma$

Economic Life $C(n) = [\sum C_i r^i + (A - S_n) r^n] / [1 - r^n]$

$$C(T) = \sum_{i=1}^{T} C_{p,i}\, r^i + \sum_{j=1}^{n-T} C_{t,j}\, r^{T+j} + Ar^T - (S_{p,T}\, r^T + S_{t,n-T}\, r^n)$$

Inspection $P(n) = V - \frac{V\lambda(n)}{\mu} - \frac{Vn}{i} - R\frac{\lambda(n)}{\mu} - I\frac{n}{i}$

$$D(n) = \frac{\lambda(n)}{\mu} + \frac{n}{i} \qquad A(t_i) = \frac{t_i R(t_i) + \int_{-\infty}^{t_i} t f(t)\,dt}{t_i + T_i + T_r[1 - R(t_i)]}$$

Overhaul and Repair $f_n(I) = \min_{L_J} [C_n(IJ) + f_{n-1}(J)]$

Organizational Structure $C(n) = nC_l + W_s \lambda C_d$ $\qquad C(\mu) = \frac{V\lambda}{\mu - \lambda} - c(\mu)$

$$C(n_m, n_l) = n_m C_m + n_l C_l + W_{s,m} \times \lambda p \times p(n_m, n_l) \times C_d + W_{s,l} \{\lambda(1-p) + \lambda p[1 - p(n_m, n_l)]\} C_d$$

$$C(n) = nC_f + \left(nm\int_{nm}^{\infty} f(r)dr + \int_0^{nm} rf(r)dr\right) C_w + \left(\int_{nm}^{\infty} (r - nm) f(r)dr\right) C_s$$

Sample Size = n

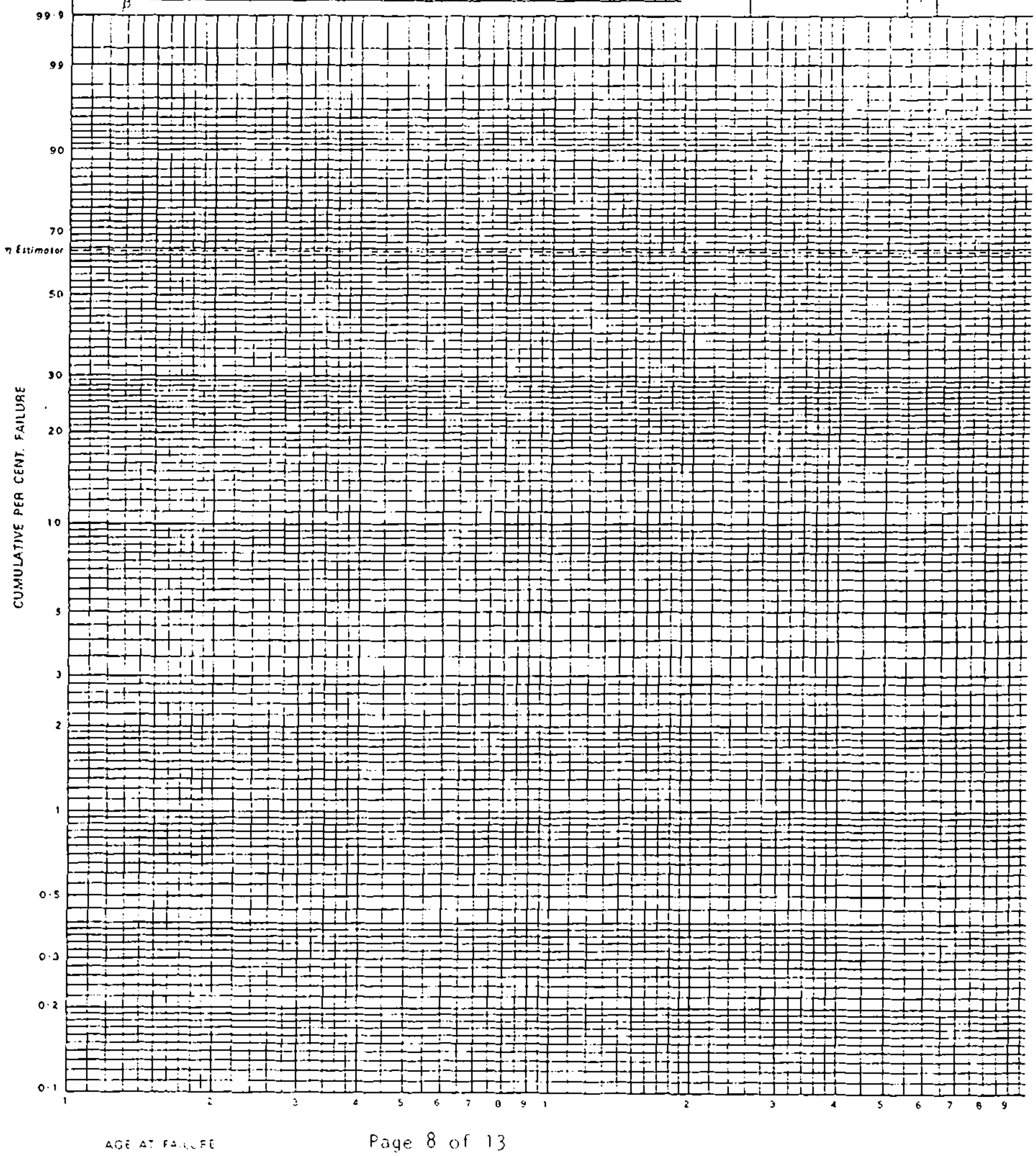
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Rank Order Number
50.0	29.3	20.7	16.0	13.0	11.0	9.5	8.3	7.5	6.8	6.2	5.7	5.2	4.9	4.6	4.3	4.0	3.8	3.6	3.5	1
	70.8	50.0	38.7	31.5	26.6	23.0	20.3	18.1	16.4	14.9	13.7	12.7	11.8	11.1	10.4	9.8	9.3	8.8	8.4	2
		79.4	61.4	50.0	42.2	36.5	32.2	28.8	26.0	23.7	21.8	20.2	18.8	17.6	16.5	15.5	14.7	13.9	13.3	3
			84.1	68.8	57.9	50.0	44.1	39.4	35.6	32.5	29.9	27.6	25.7	24.1	22.6	21.3	20.1	19.1	18.2	4
				87.1	73.5	63.6	56.0	50.0	45.2	41.3	37.9	35.1	32.7	30.6	28.7	27.0	25.6	24.3	23.1	5
					89.1	77.1	67.9	60.7	54.9	50.0	46.0	42.6	39.6	37.0	34.8	32.8	31.0	29.4	28.0	6
						90.6	79.8	71.3	64.5	58.8	54.1	50.0	46.6	43.5	40.9	38.5	36.5	34.6	32.9	7
							91.7	82.0	74.1	67.6	62.2	57.5	53.5	50.0	47.0	44.3	41.9	39.7	37.8	8
								92.6	83.7	76.4	70.2	65.0	60.5	56.5	53.1	50.0	47.3	44.9	42.7	9
									93.3	85.2	78.3	72.4	67.4	63.0	59.2	55.8	52.8	50.0	47.6	10
										93.9	86.4	79.9	74.4	69.5	65.3	61.5	58.2	55.2	52.5	11
											94.4	87.4	81.3	76.0	71.4	67.3	63.6	60.4	57.4	12
												94.9	88.3	82.5	77.5	73.0	69.1	65.6	62.3	13
													95.2	89.0	83.6	78.8	74.5	70.7	67.2	14
														95.5	89.7	84.5	80.0	75.8	72.1	15
															95.8	90.3	85.4	81.0	77.0	16
																96.0	90.8	86.1	81.9	17
																	96.3	91.3	86.8	18
																		96.5	91.7	19
																			96.6	20

Median Ranks (Per Cent)

WEIBULL PROBABILITY CHART

⊙ Estimation Point

Test Number	Article and Source	Sample Size	N
Date	Type of Test	Shape	$\hat{\beta}$
P_{μ}	Mean $\hat{\mu}$	Characteristic Life	$\hat{\eta}$
$\hat{\beta}$		Minimum Life	$\hat{\gamma}$



WEIBULL PROBABILITY CHART

⊙ Estimation Point

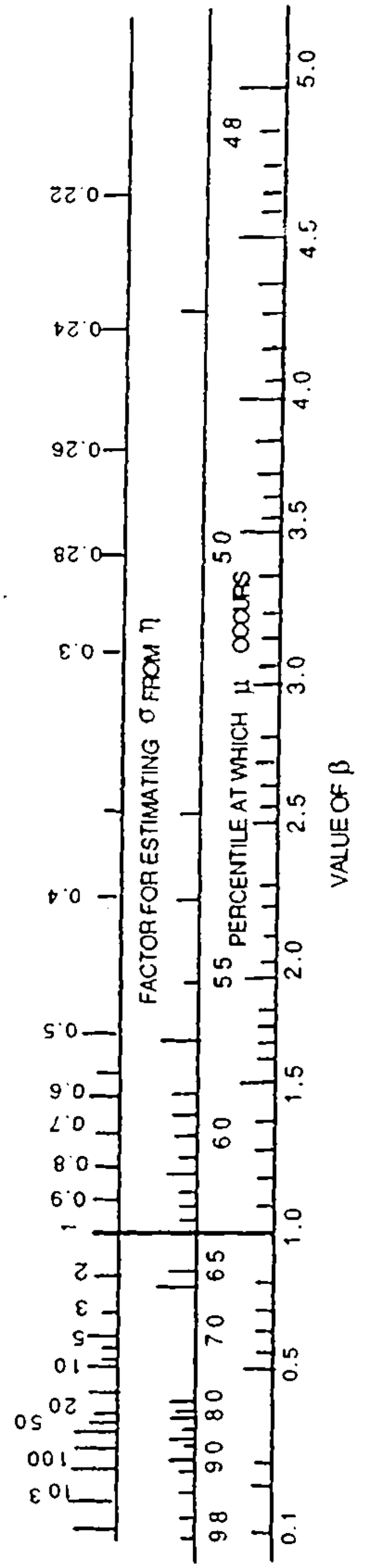
Test Number	Article and Source	Sample Size	N
Date	Type of Test	Shape	$\hat{\beta}$
P_{μ}	Mean $\hat{\mu}$	Characteristic Life	$\hat{\eta}$
$\hat{\beta}$		Minimum Life	$\hat{\gamma}$

CUMULATIVE PER CENT. FAILURE

99·9, 99, 90, 70, η Estimator, 50, 30, 20, 10, 5, 3, 2, 1, 0·5, 0·3, 0·2, 0·1

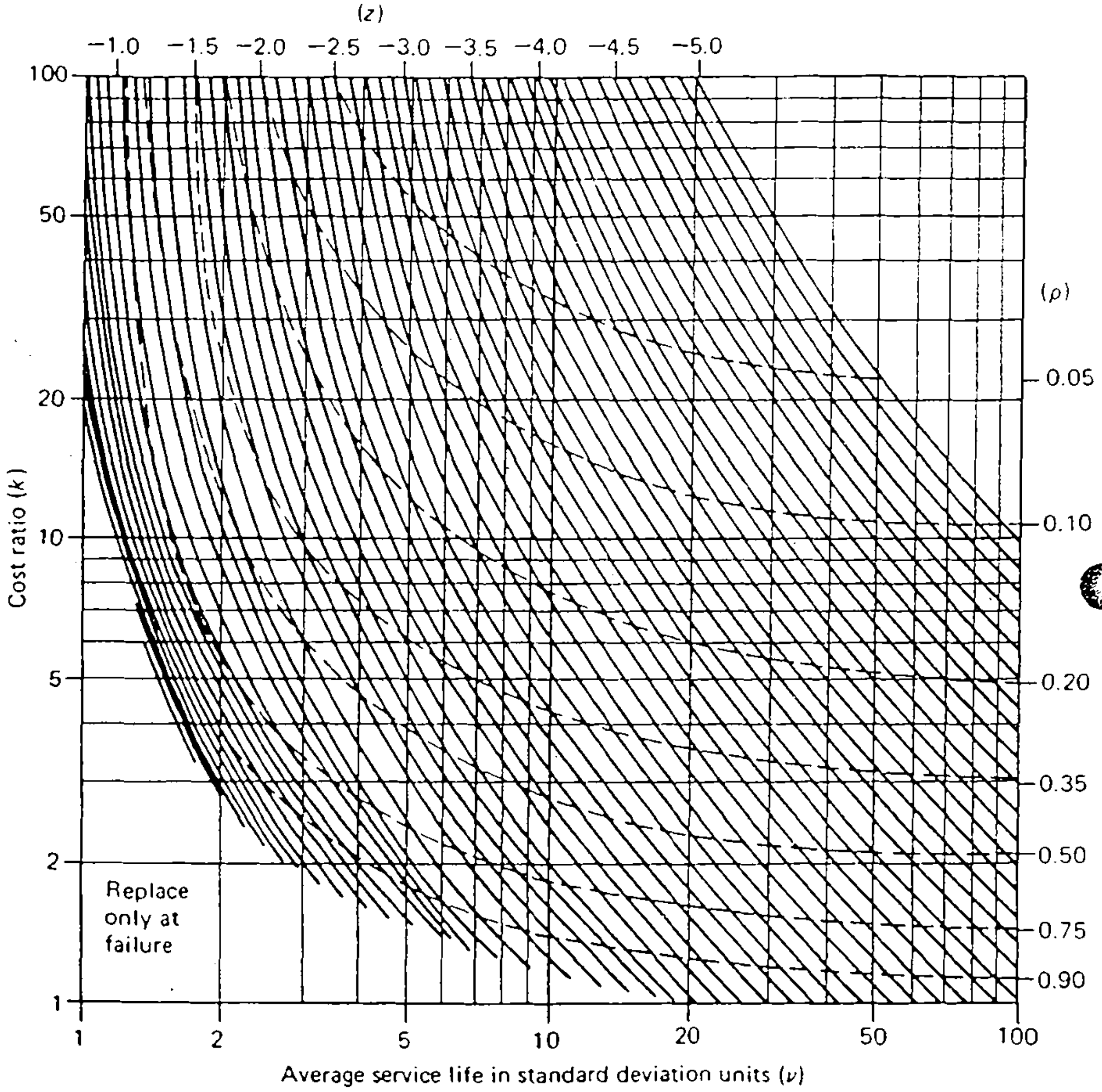
1 2 3 4 5 6 7 8 9 1 2 3 4 5 6 7 8 9

AGE AT FAILURE



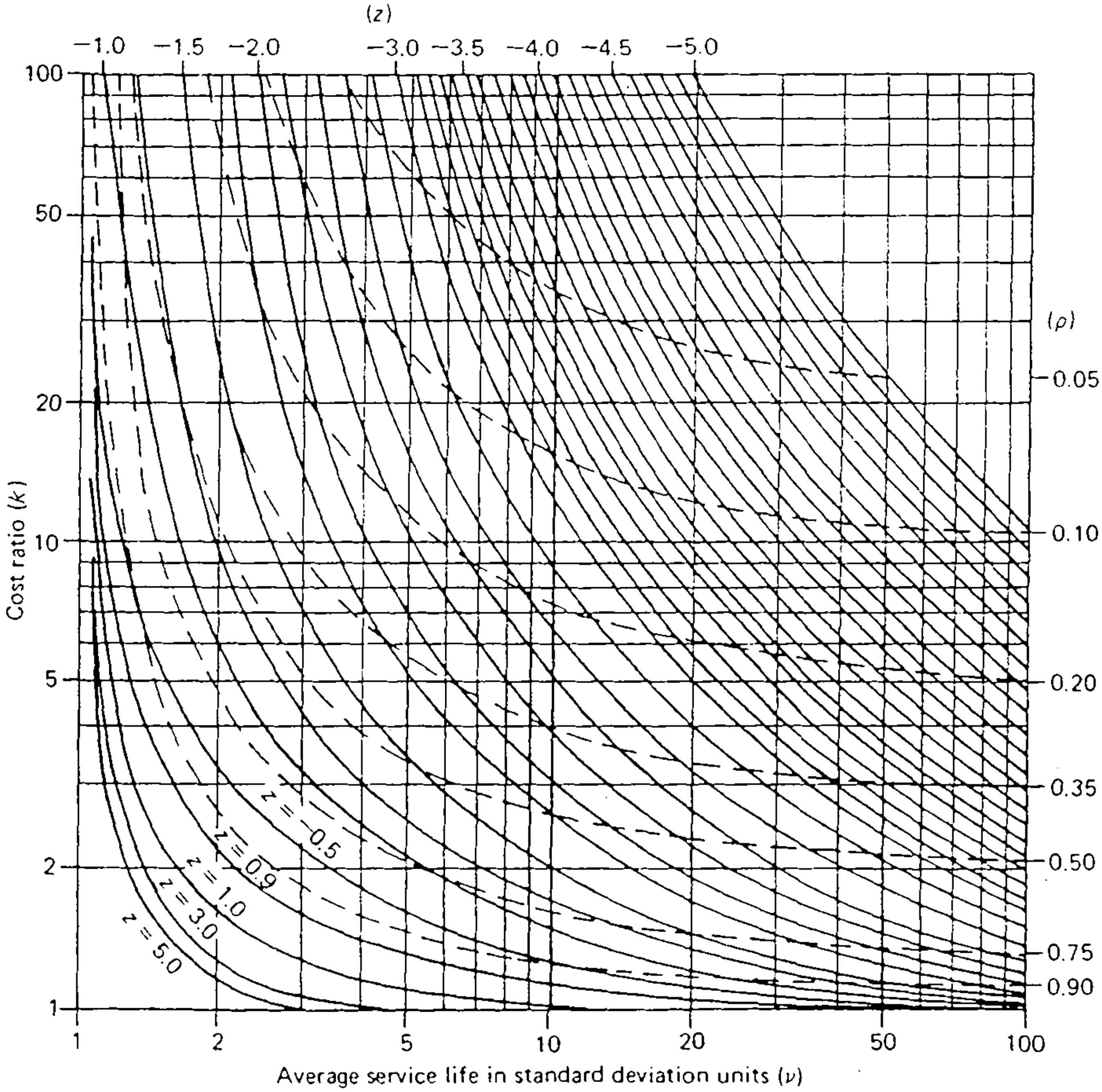
NOMOGRAPH OF THE RELATIONSHIP OF β μ AND σ

SOURCE: TEAM, Vol. 3, No. 1, 1976 - An Introduction to Weibull Distribution Statistics



Optimal policies under block replacement: Weibull distribution.

Figure 1



Optimal policies under age replacement: Weibull distribution.

Figure 2

(VI) Kolmogorov-Smirnov Critical Values (d_α)

Sample Size (n)	Level of Significance(d_α)				
	0.20	0.15	0.10	0.05	0.01
1	0.900	0.925	0.950	0.975	0.995
2	0.684	0.726	0.776	0.842	0.929
3	0.565	0.597	0.642	0.708	0.828
4	0.494	0.525	0.564	0.624	0.783
5	0.446	0.474	0.510	0.565	0.669
6	0.410	0.436	0.470	0.521	0.618
7	0.381	0.405	0.438	0.486	0.577
8	0.358	0.381	0.411	0.457	0.543
9	0.339	0.360	0.388	0.432	0.514
10	0.322	0.342	0.368	0.410	0.490
11	0.307	0.326	0.352	0.391	0.468
12	0.295	0.313	0.338	0.375	0.450
13	0.284	0.302	0.325	0.361	0.433
14	0.274	0.292	0.314	0.349	0.418
15	0.266	0.283	0.304	0.338	0.404
16	0.258	0.274	0.295	0.328	0.392
17	0.250	0.266	0.286	0.318	0.381
18	0.244	0.259	0.278	0.309	0.371
19	0.237	0.252	0.272	0.301	0.363
20	0.231	0.246	0.264	0.294	0.356
25	0.21	0.22	0.24	0.27	0.32
30	0.19	0.20	0.22	0.24	0.29
35	0.18	0.19	0.21	0.23	0.27
Over 35	$\frac{1.07}{\sqrt{n}}$	$\frac{1.14}{\sqrt{n}}$	$\frac{1.22}{\sqrt{n}}$	$\frac{1.36}{\sqrt{n}}$	$\frac{1.63}{\sqrt{n}}$

Reprinted by special permission from the Journal of American Statistical Association (The Kolmogorov-Smirnov Test for Goodness of Fit) by Frank J. Massey, Jr., as published in the "Journal of the American Statistical Association" in March 1951, Volume 46, number 53, page 70.