UNIVERSITY OF TORONTO

FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION, DECEMBER 2001

Third Year Industrial Engineering Program

MIE 337H1F - STATISTICS & EXPERIMENTAL DESIGN

Exam Type: B

Aids Allowed: 1. Aid Sheet supplied with this exam
2. One non-programmable calculator

Examiners - D. R. Edwards

DO NOT WRITE YOUR ANSWERS ON THIS QUESTION PAPER; USE ONLY BOOKLETS PROVIDED

VALUE

4 **1.** X and Y are two independent continuous random variables with means μ_x and μ_y and variances σ_x^2 and σ_y^2. Specify the mean and variance of Z where $Z = aX - bY - c$ and a, b and c are constants?

3 **2.** Find the probability that in successive tosses of a fair die a 3 comes up for the first time on the fifth toss. What distribution models this set of events?

4 **3.** Houses in a coastal residential area were found to have dry basements in only 62.4% of cases. How many houses would it be necessary to check to estimate the population proportion of dry basements within .03 with 95% confidence?

5 **4.** Construct the 99% confidence interval for σ^2 from the following sample data: 4, 1, 8, 0, 1, 9, 8, 3, 2, 2. The null hypothesis is that $\sigma^2 = 18$. What decision would you make concerning this null hypothesis?

x	x-xbar	(x-xbar)^2
4	0.2	0.04
1	-2.8	7.84
8	4.2	17.64
0	-3.8	14.44
1	-2.8	7.84
9	5.2	27.04
8	4.2	17.64
3	-0.8	0.64
2	-1.8	3.24
2	-1.8	3.24
Σ		99.60

5. The EPA wants to model the gas consumption ratings, Y, of automobiles as a function of their engine displacement, $_1X$. A quadratic model:

$$Y_i = \beta_0 + \beta_1\,{}_1X_i + \beta_2\,{}_1X_i^{\,2} + \varepsilon_i$$

is proposed. A sample of 50 engines with displacements from 90 to 212 cubic inches with an average of 150 are selected and the fuel consumption in miles per gallon is measured. $_1X_i$ is scaled so that $_1X = 1.00$ when the engine displacement is 100 cubic incles. The least squares model is:

$$\textbf{Y-hat}_i = 51.3 - 10.1\,{}_1X_i + 0.15\,{}_1X_i^2$$

and $\quad$ **MSE** = 4.72 $\quad$ $S_{b_2}^{\;2} = 0.0037$ $\quad$ $R^2 = 0.93$

4 a) sketch the fitted model between $_1X = 1$ and $_1X = 3$ in 0.5 increments.

6 b) We are interested in knowing whether there is evidence that the quadratic term in the model is contributing to the prediction of Y. Conduct a t-test at $\alpha = 0.01$ and appropriate degrees of freedom.

2 c) Assuming that the model is appropriate, use the model to estimate the mean miles per gallon rating for all cars with 195 cubic inch engines (i.e., $_1X_o = 1.95$).

4 d) Suppose the 99% confidence interval for the mean of Y quantity estimated in part (c) is (29.27, 35.09). Interpret this interval. With 99% confidence,what is the largest error, ε, that is likely in estimating $\mu_{Y|x}$ from $\textbf{Yhat}_{|x=1.95}$? What is $\underline{x}_o$ for $_1X_o = 1.95$?

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9 e) Suppose you purchase an automobile with a 350 cubic inch engine (${}_1X_o = 3.50$) and find that the actual miles per gallon rating is 9.7. If you use the fitted model, what fuel consumption is predicted? What is $\mathbf{x_o}$ for ${}_1X_o = 3.50$? What is the 99% prediction interval for mileage rating of this engine when $\mathbf{Y_o}$ is estimated from $\mathbf{Yhat}_{|x=3.50}$? Is the actual miles per gallon rating in the prediction interval? Discuss your result.

11 **6.** For each of the items on the left in the list below there is a corresponding definition, partial definition or instance in the list on in the right. Write the item number from the left list beside the most appropriate item on the right. Please note that the list on the right contains more items than the list on the left.

A	F_{max}
B	multicollinearity
C	randomized blocks design
D	squared Pearson product-moment correlation
E	Mallow's C_p
F	hypergeometric distribution
G	polynomial regression
H	heteroskedasticity
I	regression line through origin
J	Type II ANOVA Model
K	probability of rejecting false null hypothesis
L	probability of rejecting null hypotheisis when it is true
M	Scheffé test
N	a Gauss-Markov condition for regression
O	Poisson distribution
P	multinomial distributed events
Q	random independent sampling of binary events with replacement
R	Normal Distribution
S	Tukey HSD Test
T	eta-squared (η^2)
U	$F_{1,v,\alpha}$
V	dependent events

	model underfit if this statistic greater than number of predictors
	β
	variance of dependent measure non-constant across range of predictor
	random effects
	α
	power
	$(Sxy)^2 / (Sxx\ Syy)$
	Bernoulli trials
	test of $\sigma_1^2 = \sigma_2^2 = ... = \sigma_k^2$ for k groups
	$Y_i = \beta_0 + \beta_{1\cdot1}X_i^2 + \varepsilon_i$
	$t^2_{v,\alpha/2}$
	$P(A \cap B) = P(A)\ P(B\|A)$
	pairwise comparisons of means
	correlated regression predictors
	approximates Binomial for very large n and p not extreme
	treatments assigned randomly to like experimental units
	estimated effect size
	$E(\varepsilon_i) = 0$
	events with more than 2 outcomes each with probability p_i
	a posteriori comparison of 2 or more means
	limiting form of Binomial for very large n, small p and constant mean
	$Y_i = \beta_{1\cdot1}X_i + \beta_{2\cdot2}X_i + \varepsilon_i$
	intercept
	sampling without replacement

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7. A manufacturing firm is interested in boosting the sales of its three product lines by introducing a commission component to supplement the salaries of their sales staff. Managment wishes to examine three commission alternatives as a Treatment Factor in an analysis of variance experiment: 60% of Current Salary + a 10% Commission (t_1), 40% of Current Salary + 20% commission (t_2), and 30% of Current Salary + 30% Commission (t_3). The trial period for the three commission programs (Treatments) is one year, and the dependent measure is the number of sales $10K units surpassed above the sales rep's normal sales quota. Realizing that SALES TERRITORIES, East (a_1), Central (a_2) and West (a_3), as well as the PRODUCT LINES (b_1,b_2,b_3) to which sales persons are dedicated are important determinants of the sales posted by an individual sales rep, TERRITORY and LINE are included as blocking factors in the experiment. It is also assumed that the two Blocking Factors (Territories and Line) and the Treatment Factor (Commission Program) each contribute <u>independently</u> to the prediction of sales. Within each blocking combination, one commission program was assigned to **4** employees with a total of **36** sales reps participating in the experiment..

Table 1: Terriroty (**A**) x Product Line (**B**) x Commission Program (**T**)

Design				**Observed Data**						Σn_{ijk} = 36
	b_1	b_3	b_2		b_1	b_3	b_2	Mean$_{i..}$	$S_{i..}$	Grand Mean
a_2	t_3	t_2	t_1	a_2	6, 8, 12, 7	0, 0, 1, 4	0, 2, 2, 5	2.92	3.80	
a_1	t_2	t_1	t_3	a_1	2, 5, 3, 1	2, 2, 4, 6	9, 10, 12, 12	5.67	4.08	
a_3	t_1	t_3	t_2	a_3	0, 1, 1, 4	2, 1, 1, 5	0, 1, 1, 4	1.75	1.66	
				Mean$_{.j.}$	4.17	2.33	4.83			
				$S_{.j.}$	3.59	1.97	4.67			3.78

Table 2

		Treatments	
	t_1	t_2	t_3
Mean$_{..k}$	2.42	1.83	7.08
$n_{..k}$	12	12	12
$S_{..k}$	1.93	1.75	4.17

Table 3: Factor **B** (Product Line) x **T** (Commission Plan)

Product Line	Commission Plan	Mean Sales Level
1	1	1.50
1	2	2.75
1	3	8.25
2	1	2.25
2	2	1.50
2	3	10.75
3	1	3.50
3	2	1.25
3	3	2.25

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VALUE

2 a) What kind of Design is this?

2 b) Write the model equation representing Sale Level as Y_{ijkl}, Territory as α_i, Product Line as β_j and Commission Plan as τ_k.

3 c) What assumptions (3) about the parameters are necessary to make them estimable?

2 d) State the assumptions about the relation among the blocking factors and the treatment in terms of the parameters $\alpha\beta_{ij}$ ... $\alpha\beta\tau_{ijk}$?

1 e) Both blocking factors include all levels possible (i.e., there are only three terrirtories, and only three product lines sold by the company). The treatment levels were selected from a population of alternatives, but these are the only commission programs upper managment is willing to consider. What type of ANOVA Model Type is appropriate for this experiment?

8 f) Copy the partial ANOVA summary Table below to your answer booklet and complete it.

SOURCE OF VARIATION	SUM OF SQUARES	DF	MEAN SQUARE	F*	YOUR DECISION at $\alpha = .05$
A: Territory	92.39				
B: Prod.Line	40.22				
T: Program	198.72				
Residual	33.39	$(k-1)(k-2)=$			
Within.Cell	99.50	$K^2(n-1)=$			
TOTAL	464.22				

6 g) List **three** assumptions underlying the F-Test of Treatments.

6 h) Plot the Treatment means by Employee Type using Employee Type as the parameter.

4 i) Use the ANOVA results and your plot to discuss whether the design / model is appropriate?

4 j) What is the Treatment factor null hypothesis? Are you in a position to test it? Explain.

4 k) Use Cochran's test to assess the homogeneity of variance within Treatments at $\alpha = .05$.

2 l) Discuss whether it is appropriate to conduct contrasts on the Treatment main effect.

11 m) Making an assumption that contrasts are appropriate, construct and test a contrast at $\alpha = .01$ that assesses whether the 30/30 commission plan (t_3) is significantly more effective than the other two plans in increasing sales. State the null hypothesis for your contrast in terms of the population treatment means and the contrast coefficients.

VALUE	
	n) Suppose that that model has been shown to be inappropriate in i). Management is willing to commit additional funds to expand the experiment with less restrictive assumptions, but has asked you to salvage the data already collected. The company has many hundreds of sales representatives who can be included in the experiment, and it is known that the gross revenues of the company in the coming quarter are typically the same as those of the quarter in which the first phase of the experiment was conducted.
6	i. make a table for our your new design in a format similar to Table 1, indicating how treatments are assigned to sales reps within blocks
1	ii. indicate the number of cases per cell
4	iii. in place of the "Observed Data" in Table 1, copy the data salvaged from phase one into the appropriate cells.
2	iv. what kind of fourth treatment level might serve as a control group for the experiment. Add it to the table you created in 7.n.i.
4	v. discuss why your new design overcomes the shortcomings of the original design.and the benefit of including the control group.
6	vi. without data, set up a partial ANOVA summary table for your new design, including the sources of variation, degrees of freedom, mean squares and composition of the F-ratios.
TOTAL MARKS 130	

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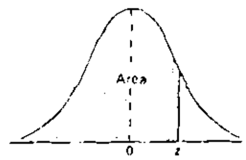

Table A.3 Areas Under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9278	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

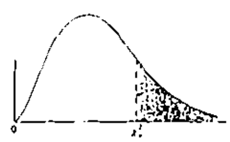

TABLE A.5 Critical Values of the Chi-Squared Distribution

	α										
v	0.995	0.99	0.98	0.975	0.95	0.90	0.80	0.75	0.75	0.50	v
1	0.0^4393	0.0^3157	0.0^3628	0.0^3982	0.00393	0.0158	0.0642	0.102	0.148	0.455	1
2	0.0100	0.0201	0.0404	0.0506	0.103	0.211	0.446	0.575	0.713	1.386	2
3	0.0717	0.115	0.185	0.216	0.352	0.584	1.005	1.213	1.424	2.366	3
4	0.207	0.297	0.429	0.484	0.711	1.064	1.649	1.923	2.195	3.357	4
5	0.412	0.554	0.752	0.831	1.145	1.610	2.343	2.675	3.000	4.351	5
6	0.676	0.872	1.134	1.237	1.635	2.204	3.070	3.455	3.828	5.348	6
7	0.989	1.239	1.564	1.690	2.167	2.833	3.822	4.255	4.671	6.346	7
8	1.344	1.646	2.032	2.180	2.733	3.490	4.594	5.071	5.527	7.344	8
9	1.735	2.088	2.532	2.700	3.325	4.168	5.380	5.899	6.393	8.343	9
10	2.156	2.558	3.059	3.247	3.940	4.865	6.179	6.737	7.267	9.342	10
11	2.603	3.053	3.609	3.816	4.575	5.578	6.989	7.584	8.148	10.341	11
12	3.074	3.571	4.178	4.404	5.226	6.304	7.807	8.438	9.034	11.340	12
13	3.565	4.107	4.765	5.009	5.892	7.042	8.634	9.299	9.926	12.340	13
14	4.075	4.660	5.368	5.629	6.571	7.790	9.467	10.165	10.821	13.339	14
15	4.601	5.229	5.985	6.262	7.261	8.547	10.307	11.036	11.721	14.339	15
16	5.142	5.812	6.614	6.908	7.962	9.312	11.152	11.912	12.624	15.338	16
17	5.697	6.408	7.255	7.564	8.672	10.085	12.002	12.792	13.531	16.338	17
18	6.265	7.015	7.906	8.231	9.390	10.865	12.857	13.675	14.440	17.338	18
19	6.844	7.633	8.567	8.907	10.117	11.651	13.716	14.562	15.352	18.338	19
20	7.434	8.260	9.237	9.591	10.851	12.443	14.578	15.452	16.266	19.337	20
21	8.034	8.897	9.915	10.283	11.591	13.240	15.445	16.344	17.182	20.337	21
22	8.643	9.542	10.600	10.982	12.338	14.041	16.314	17.240	18.101	21.337	22
23	9.260	10.196	11.293	11.688	13.091	14.848	17.187	18.137	19.021	22.337	23
24	9.886	10.856	11.992	12.401	13.848	15.659	18.062	19.037	19.943	23.337	24
25	10.520	11.524	12.697	13.120	14.611	16.473	18.940	19.939	20.867	24.337	25
26	11.160	12.198	13.409	13.844	15.379	17.292	19.820	20.843	21.792	25.336	26
27	11.808	12.879	14.125	14.573	16.151	18.114	20.703	21.749	22.719	26.336	27
28	12.461	13.565	14.847	15.308	16.928	18.939	21.588	22.657	23.647	27.336	28
29	13.121	14.256	15.574	16.047	17.708	19.768	22.475	23.567	24.577	28.336	29
30	13.787	14.953	16.306	16.791	18.493	20.599	23.364	24.478	25.508	29.336	30

TABLE A.5 (continued) Critical Values of the Chi-Squared Distribution

	α									
v	0.30	0.25	0.20	0.10	0.05	0.025	0.02	0.01	0.005	0.001
1	1.074	1.323	1.642	2.706	3.841	5.024	5.412	6.635	7.879	10.827
2	2.408	2.773	3.219	4.605	5.991	7.378	7.824	9.210	10.597	13.815
3	3.665	4.108	4.642	6.251	7.815	9.348	9.837	11.345	12.838	16.268
4	4.878	5.385	5.989	7.779	9.488	11.143	11.668	13.277	14.860	18.465
5	6.064	6.626	7.289	9.236	11.070	12.832	13.388	15.086	16.750	20.517
6	7.231	7.841	8.558	10.645	12.592	14.449	15.033	16.812	18.548	22.457
7	8.383	9.037	9.803	12.017	14.067	16.013	16.622	18.475	20.278	24.322
8	9.524	10.219	11.030	13.362	15.507	17.535	18.168	20.090	21.955	26.125
9	10.656	11.389	12.242	14.684	16.919	19.023	19.679	21.666	23.589	27.877
10	11.781	12.549	13.442	15.987	18.307	20.483	21.161	23.209	25.188	29.588
11	12.899	13.701	14.631	17.275	19.675	21.920	22.618	24.725	26.757	31.264
12	14.011	14.845	15.812	18.549	21.026	23.337	24.054	26.217	28.300	32.909
13	15.119	15.984	16.985	19.812	22.362	24.736	25.472	27.688	29.819	34.528
14	16.222	17.117	18.151	21.064	23.685	26.119	26.873	29.141	31.319	36.123
15	17.322	18.245	19.311	22.307	24.996	27.488	28.259	30.578	32.801	37.697
16	18.418	19.369	20.465	23.542	26.296	28.845	29.633	32.000	34.267	39.252
17	19.511	20.489	21.615	24.769	27.587	30.191	30.995	33.409	35.718	40.790
18	20.601	21.605	22.760	25.989	28.869	31.526	32.346	34.805	37.156	42.312
19	21.689	22.718	23.900	27.204	30.144	32.852	33.687	36.191	38.582	43.820
20	22.775	23.828	25.038	28.412	31.410	34.170	35.020	37.566	39.997	45.315
21	23.858	24.935	26.171	29.615	32.671	35.479	36.343	38.932	41.401	46.797
22	24.939	26.039	27.301	30.813	33.924	36.781	37.659	40.289	42.796	48.268
23	26.018	27.141	28.429	32.007	35.172	38.076	38.968	41.638	44.181	49.728
24	27.096	28.241	29.553	33.196	36.415	39.364	40.270	42.980	45.558	51.179
25	28.172	29.339	30.675	34.382	37.652	40.646	41.566	44.314	46.928	52.620
26	29.246	30.434	31.795	35.563	38.885	41.923	42.856	45.642	48.290	54.052
27	30.319	31.528	32.912	36.741	40.113	43.194	44.140	46.963	49.645	55.476
28	31.391	32.620	34.027	37.916	41.337	44.461	45.419	48.278	50.993	56.893
29	32.461	33.711	35.139	39.087	42.557	45.722	46.693	49.588	52.336	58.302
30	33.530	34.800	36.250	40.256	43.773	46.979	47.962	50.892	53.672	59.703

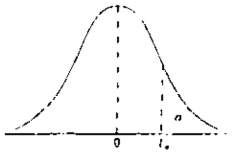

Table A.4 Critical Values of the t-Distribution

v	α: 0.40	0.30	0.20	0.15	0.10	0.05	0.025
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365
8	0.262	0.546	0.589	1.108	1.397	1.860	2.306
9	0.261	0.543	0.683	1.100	1.383	1.833	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179
13	0.259	0.537	0.870	1.079	1.350	1.771	2.160
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052
28	0.256	0.530	0.555	1.056	1.313	1.701	2.048
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042
40	0.255	0.529	0.551	1.050	1.303	1.684	2.021
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960

Table A.4 (continued) Critical Values of the t-Distribution

v	α: 0.02	0.015	0.01	0.0075	0.005	0.0025	0.0005
1	15.895	21.205	31.821	42.434	63.657	127.322	636.590
2	4.849	5.643	6.965	8.073	9.925	14.089	31.598
3	3.482	3.896	4.541	5.047	5.841	7.453	12.924
4	2.999	3.298	3.747	4.088	4.604	5.598	8.610
5	2.757	3.003	3.365	3.634	4.032	4.773	6.869
6	2.612	2.829	3.143	3.372	3.707	4.317	5.959
7	2.517	2.715	2.998	3.203	3.499	4.029	5.408
8	[illegible]	2.634	2.896	3.085	3.355	3.833	5.041
9	2.398	2.574	2.821	2.998	3.250	3.690	4.781
10	2.359	2.527	2.764	2.932	3.169	3.581	4.587
11	2.328	2.491	2.718	2.879	3.106	3.497	4.437
12	2.303	2.461	2.681	2.836	3.055	3.428	4.318
13	2.282	2.436	2.650	2.801	3.012	3.372	4.221
14	2.264	2.415	2.624	2.771	2.977	3.326	4.140
15	2.249	2.397	2.602	2.746	2.947	3.286	4.073
16	2.235	2.382	2.583	2.724	2.921	3.252	4.015
17	2.224	2.368	2.567	2.706	2.898	3.222	3.965
18	2.214	2.356	2.552	2.689	2.878	3.197	3.922
19	2.205	2.346	2.539	2.674	2.861	3.174	3.883
20	2.197	2.336	2.528	2.661	2.845	3.153	3.849
21	2.189	2.328	2.518	2.649	2.831	3.135	3.819
22	2.183	2.320	2.508	2.639	2.819	3.119	3.792
23	2.177	2.313	2.500	2.629	2.807	3.104	3.768
24	2.172	2.307	2.492	2.620	2.797	3.091	3.745
25	2.167	2.301	2.485	2.612	2.787	3.078	3.725
26	2.162	2.296	2.479	2.605	2.779	3.067	3.707
27	2.158	2.291	2.473	2.598	2.771	3.057	3.690
28	2.154	2.286	2.467	2.592	2.763	3.047	3.674
29	2.150	2.282	2.462	2.586	2.756	3.038	3.659
30	2.147	2.278	2.457	2.581	2.750	3.030	3.646
40	2.125	2.250	2.423	2.542	2.704	2.971	3.551
60	2.099	2.223	2.390	2.504	2.660	2.915	3.460
120	2.076	2.196	2.358	2.468	2.617	2.860	3.373
∞	2.054	2.170	2.326	2.432	2.576	2.807	3.291

Table A.11 ***Critical Values for Cochran's Test***

$\alpha = 0.05$

k \ n	2	3	4	5	6	7	8	9	10	11	17	37	145	∞
2	0.9985	0.9750	0.9392	0.9057	0.8772	0.8534	0.8332	0.8159	0.8010	0.7880	0.7341	0.6602	0.5813	0.5000
3	0.9669	0.8709	0.7977	0.7457	0.7071	0.6771	0.6530	0.6333	0.6167	0.6025	0.5466	0.4748	0.4031	0.3333
4	0.9065	0.7679	0.6841	0.6287	0.5895	0.5598	0.5365	0.5175	0.5017	0.4884	0.4366	0.3720	0.3093	0.2500
5	0.8412	0.6838	0.5981	0.5441	0.5065	0.4783	0.4564	0.4387	0.4241	0.4118	0.3645	0.3066	0.2513	0.2000
6	0.7808	0.6161	0.5321	0.4803	0.4447	0.4184	0.3980	0.3817	0.3682	0.3568	0.3135	0.2612	0.2119	0.1667
7	0.7271	0.5612	0.4800	0.4307	0.3974	0.3726	0.3535	0.3384	0.3259	0.3154	0.2756	0.2278	0.1833	0.1429
8	0.6798	0.5157	0.4377	0.3910	0.3595	0.3362	0.3185	0.3043	0.2926	0.2829	0.2462	0.2022	0.1616	0.1250
9	0.6385	0.4775	0.4027	0.3584	0.3286	0.3067	0.2901	0.2768	0.2659	0.2568	0.2226	0.1820	0.1446	0.1111
10	6.6020	0.4450	0.3733	0.3311	0.3029	0.2823	0.2666	0.2541	0.2439	0.2353	0.2032	0.1655	0.1308	0.1000
12	0.5410	0.3924	0.3264	0.2880	0.2624	0.2439	0.2299	0.2187	0.2098	0.2020	0.1737	0.1403	0.1100	0.0833
15	0.4709	0.3346	0.2758	0.2419	0.2195	0.2034	0.1911	0.1815	0.1736	0.1671	0.1429	0.1144	0.0889	0.0667
20	0.3894	0.2705	0.2205	0.1921	0.1735	0.1602	0.1501	0.1422	0.1357	0.1303	0.1108	0.0879	0.0675	0.0500
24	0.3434	0.2354	0.1907	0.1656	0.1493	0.1374	0.1286	0.1216	0.1160	0.1113	0.0942	0.0743	0.0567	0.0417
30	0.2929	0.1980	0.1593	0.1377	0.1237	0.1137	0.1061	0.1002	0.0958	0.0921	0.0771	0.0604	0.0457	0.0333
40	0.2370	0.1576	0.1259	0.1082	0.0968	0.0887	0.0827	0.0780	0.0745	0.0713	0.0595	0.0462	0.0347	0.0250
60	0.1737	0.1131	0.0895	0.0765	0.0682	0.0623	0.0583	0.0552	0.0520	0.0497	0.0411	0.0316	0.0234	0.0167
120	0.0998	0.0632	0.0495	0.0419	0.0371	0.0337	0.0312	0.0292	0.0279	0.0266	0.0218	0.0165	0.0120	0.0083
∞	0	0	0	0	0	0	0	0	0	0	0	0	0	0

TABLE A.6* Critical Values of the *F*-Distribution

$f_{0.05}(v_1, v_2)$

v_2 \ v_1	1	2	3	4	5	6	7	8	9
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88

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TABLE A.6 (continued) Critical Values of the *F*-Distribution

$f_{0.01}(v_1, v_2)$

v_2 \ v_1	1	2	3	4	5	6	7	8	9
1	4052	4999.5	5403	5625	5764	5859	5928	5981	6022
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41