NAME (PRINT):____________________

STUDENT NUMBER:________________

UNIVERSITY OF TORONTO
DEPARTMENT OF CIVIL ENGINEERING
FINAL EXAMINATION
10 DECEMBER 2001

CIV 362 ENGINEERING MATHEMATICS II

Exam Type A (Closed Book, No Exam Aids, Approved Calculators)

Examiner: Andrew Colombo

Time: 2.5 hours

There are four questions. Answer all questions.

Show all of your calculations.

You are expected to present your work neatly and, when unclear as to the interpretation of a question, to state your assumptions clearly. This will assist in awarding partial credit if necessary.

Question	Value	Grade
1	20	
2	10	
3	10	
4	10	
TOTAL	50	
Bonus	3	

$$\int u\,dv = uv - \int v\,du$$

$$f(x) = f(x_0) + (x - x_0)f'(x_0) + \frac{(x - x_0)^2}{2} f''(x_0) + \cdots + \frac{(x - x_0)^n}{n!} f^{(n)}(x_0)$$

Question 1. 20 marks (4 marks each part)
For the following initial value problem

$$y' = te^{3t} - 2y \qquad y(0) = 0 \qquad 0 \le t \le 1$$

solve for $y(t)$ at t=0.5 and t=1 using:

(a) the analytical solution.
(b) the Taylor Series method with a series expansion of degree 2 about the basepoint t=0.
(a) a second order Taylor method with a step size of Δt=0.5.
(b) the classical fourth order Runge-Kutta method with a step size of Δt=0.5.
Enter your results in the table provided.

(e) Briefly comment on the accuracy of each method and how the accuracy of the numerical solutions in general can be improved.

t	y Analytical	y Taylor series	y 2nd-order Taylor method	y 4th- order Runge-Kutta
0.0				
0.5				
1.0				

Question 2. 10 marks

A reservoir is used for runoff control from a small urban catchment. The reservoir has a plan area that is rectangular in shape, with a length of 70 m and a width of 40 m. The sides of the reservoir are vertical and the water level in the reservoir H is initially at 2 m above the reservoir bottom. Furthermore, the reservoir has a maximum working depth of 4 m. After this depth is exceeded overflow occurs to a spillway and emergency drainage canal. When the depth is less than 4 m, drainage from the reservoir occurs through a pipe, with discharge Q (m^3/s) given by

$$Q = 1.5\sqrt{H - 1.0}$$

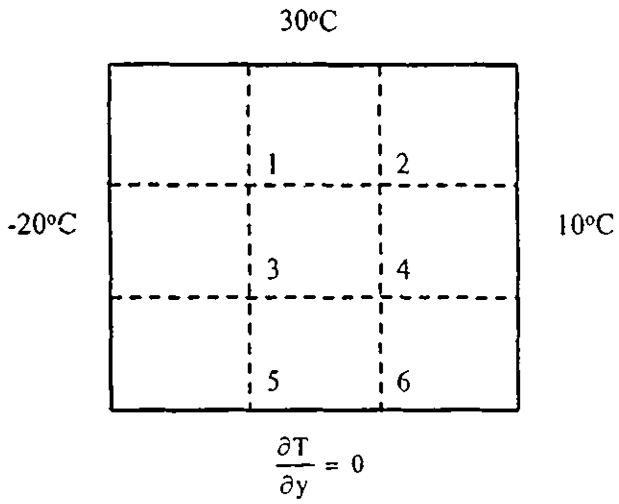
Over the next few hours, the runoff into the reservoir is as follows:

Time (hr)	0	1	2	3
Instantaneous Inflow (m^3/s)	1.5	2.5	3.0	2.5

For the first two hours only, approximate the depth of water in the reservoir using the **Modified Euler method**. Will the storm cause the spillway to be used during the first two hours?

Question 3. 10 marks

The following problem represents the steady state heat flow in a uniform plate. Formulate, **but do not solve**, the finite difference algebraic equations required to solve for the unknown temperatures at the numbered nodes. Assume that $\Delta x = \Delta y = 1$ unit.



Question 4. 10 marks

The *Green Solutions Company* operates a reclamation center that collects four types of solid waste materials and then forms them so that they can be resold. Three different grades of this product can be made, depending upon the mix of the materials used. Quality standards specify a minimum or maximum percentage (by weight) of certain reclaimed materials allowed in a particular product grade. Table 1 shows these specifications along with the formation costs and the selling price for each product grade.

There are restrictions on the weekly availability of each of the four materials. Table 2 gives the quantities available for collection and treatment each week, as well as the cost of treatment, for each type of material.

The problem is to determine how much of each product grade to produce *and* the exact mix of materials to be used for each grade so as to maximize the total weekly profit (total sales income minus the total costs of *both* amalgamation and treatment).

Formulate, **but do not solve**, this problem as a Linear Programming problem.

Table 1. Product data.

Grade	Specification	Formation cost ($) per pound	Selling price ($) per pound
A	Not more than 30% of material 1 Not less than 40% of material 2 Not more than 50% of material 3	3.00	8.50
B	Not more than 50% of material 1 Not less than 10% of material 2	2.50	7.00
C	Not more than 70% of material 1	2.00	5.50

Table 2. Solid waste material data.

Material	Pounds/week available	Treatment cost ($) per pound
1	3000	3
2	2000	6
3	4000	4
4	1000	5

Bonus Question: Ancient Farmer's Problem (3 marks)
Imagine you are a farmer living along the Tigris river in ancient Sumer 6000 years ago. One question you frequently ask is "How much grain should I plant in the spring for an adequate supply of food for the next year, with enough grain left over to plant again for the following year?" You know that your seeds will increase threefold. Hence, 1/3 of a bushel planted in the spring will yield one whole bushel in the fall. You also know that you will need one bushel of grain to feed your family during the winter. How many bushels should you plant this spring in order to ensure a sustainable supply of grain in the future?