University of Toronto
### FACULTY OF APPLIED SCIENCE AND ENGINEERING
## FINAL EXAMINATION – DECEMBER 2001
### MAT291F – Calculus III        Duration: 2 1/2 hours

Examiners: S. Abou-Ward, I. Soprounov

SURNAME        _____

GIVEN NAME     _____

STUDENT NO.    _____

SIGNATURE      _____


## INSTRUCTIONS:
<u>No Aids of Any Kind.</u>
Answer all questions.
Present your solutions in the space provided; use the back of the **preceding** page if more space is required.
TOTAL MARKS: 100


| For Marker Use Only | |
|---|---|
| Question | Mark |
| 1 | /13 |
| 2 | /19 |
| 3 | /21 |
| 4 | /15 |
| 5 | /17 |
| 6 | /15 |
| TOTAL | /100 |

1. (a) **(5 marks)** Use differentials to approximate $f(2.1, 1.2)$ if $f(x, y) = e^{x-2y}$, using $(2, 1)$ as the starting point.

(b) **(4 marks)** Find $\dfrac{\partial f}{\partial v}$ if $f(x, y) = (x + 3y)^3$, $x = u^2 v$, $y = 2u - v$ when $u = -1$ and $v = 2$.

(c) **(4 marks)** Find $\dfrac{\partial z}{\partial y}$ if $xy \sin(x + z) - e^z = 1$.

2. (a) (**4 marks**) Find and classify the extrema of $f(x,y) = (x^2 + y^2 - 2)^2$.

(b) (**8 marks**) Let $k$ be a positive integer. Find and classify all local extrema of
$f(x,y) = x^k + y^k$.
(P.S. - Consider the two cases: $k = 1$ and $k > 1$).

(c) (**7 marks**) Find the minimum value of the function $f(x, y, z) = \dfrac{1}{x} + \dfrac{1}{y} + \dfrac{1}{z}$ with $x > 0,\ y > 0,\ z > 0$, and $x^2 + y^2 + z^2 = 1$.

3. (a) (**8 marks**) Determine whether the function is a gradient $\nabla \phi(x, y, z)$ and if so. find such a function $\phi$,

$$\left(\frac{y}{z} - e^z\right) i + \left(\frac{x}{z} + 1\right) j - \left(x e^z + \frac{xy}{z^2}\right) k .$$

4. (b) (i) (**4 marks**) Find a parameterization for the hyperboloid $x^2 + y^2 + z^2 = 25$.

(ii) (**4 marks**) Find an expression for a unit normal to this surface.

(iii) (**5 marks**) Find an equation for the plane tangent to the surface at $(x_0, y_0, 0)$. where $x_0^2 + y_0^2 = 25$.

5. (a) **(6 marks)** Evaluate the double integral:

$$\int_0^1 \int_0^{\cos^{-1} y} e^{\sin x} \, dx \, dy \; .$$

(b) **(9 marks)** Let $D$ be the region in the $xy$-plane enclosed by the parallelogram with vertices $(1,2)$, $(2,3)$, $(3,1)$ and $(4,2)$. Use the transformation $x = 2u + v + 1$, $y = -u + v + 2$, to evaluate $\int\int_D (xy^2 - x^2 - xy) \, dx \, dy$.

6. (a) **(5 marks)** Let $F = (x^2 + y - 4)i + 3xyj + (2xz + z^2)k$. Calculate the divergence and curl of $F$.

(b) **(5 marks)** Find the flux of the curl of $F$ across the surface $x^2 + y^2 + z^2 = 16$, $z \geq 0$.

(c) **(7 marks)** Use the divergence theorem to find the flux of $F$ across the surface of the unit cube $[0, 1] \times [0, 1] \times [0, 1]$.

7. **(15 marks)** Let $C$ be the circle $x^2 + y^2 = 1$, $z = 0$, and let

$$F(x, y, z) = (x^2 y^3 + y - \cos(x^3))i + (x^3 y^2 + \sin(y^3) + x)j + zk \; .$$

Use Stoke's theorem to calculate the line integral $\oint_C P dx + Q dy + R dz$.