University of Toronto
FACULTY OF APPLIED SCIENCE AND ENGINEERING
First Year - Program 5

# FINAL EXAMINATIONS, APRIL 1995

## MAT 195S
### Calculus II

Examiners: Prof. J. D. de Laurier
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Duration – $2\frac{1}{2}$ hours

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1. (a) Integrate

[5] (i) $\int xe^{-x}\,dx$ [6] (ii) $\int \frac{x^3\,dx}{(x^2-1)^{\frac{1}{2}}}$

[8] 1. (b) Find the arc length of the curve $y = \frac{1+2x^6}{8x^2}$ from the point $(1, \frac{3}{8})$ to the point $(2, \frac{129}{32})$.

[6] 2. A Japanese samurai sword, a *Takana*, is made by hammering out a long red-hot steel blank, sprinkling carbon on the surface, then folding it over. This process is repeated 22 times.

(a) What is the sequence that describes the number of layers as a function of the number of folds?

(b) How many layers does the finished sword have?

(3.) Calculate $\lim_{h\to 0} \frac{f(x+h)+f(x-h)-2f(x)}{h^2}$ where $f$ is a twice differentiable function. [Hint: Use L'Hôpital's rule.]

[5] 4. (a) Find the Taylor series for $x^{-4}$ in powers of $(x+1)$.

[8] (b) Find the Taylor series for $\int_0^t e^{-u^3}\,du$, and hence calculate $\int_0^1 e^{-u^3}\,du$ to two decimal places.

5. (a) For what values of $x$ are the following series convergent:

[7] (i) $\sum_{k=1}^{\infty} \frac{\tan^k x}{k}$ [7] (ii) $\sum_{k=1}^{\infty} \frac{1}{x^k + x^{-k}}$

[7] 5. (b) Sum the series $\sum_{k=1}^{\infty} \frac{k^2+1}{k!}$

[10] 6. Give a rough sketch of the hyperbola $r^{-1} = 1 - 2\cos\theta$, and find the equations of the asymptotes to this hyperbola (in cartesian or polar coordinates).

[10] 7. Given that $f(x,y) = x^{\frac{1}{3}} F\left(\frac{y}{x^{\frac{2}{3}}}\right)$, show that $f$ satisfies the equation

$$f_{yyy} = f_y f_{xy} - f_x f_{yy} \text{ when } F''' + \tfrac{1}{3}\{FF'' + (F')^2\} = 0.$$

[10] 8. Find the equation of the tangent plane and the normal line to the ellipsoid $x^2 + 2y^2 + 3z^2 = 6$ at the point $(1,1,1)$.

[10] 9. An instrument gives three readings: $R_1$, $R_2$, and $R_3$, by which one may measure a physical process described by

$$T = 24.7R_1 + 1.83R_2^3 - R_3^5/47.8:$$

the readings and their errors are given by

$$R_1 = 1.82 \pm 0.5: \quad R_2 = 22.1 \pm 0.5: \quad R_3 = 4.2 \pm 0.5.$$

(a) What is the maximum error that $T$ can have?

(b) What is the percentage of maximum error compared with the nominal reading?

[12] 10. Given the plane curve $C$ defined by $\mathbf{r}(t)$, let $Q$ be the centre of curvature of $C$ at $P$. As $P$ traces out $C$, then $Q$ traces out another curve which is called the evolute of $C$. Find the evolute when $C$ is the ellipse $\mathbf{r} = a\cos t\,\hat{i} + b\sin t\,\hat{j}$.