UNIVERSITY OF TORONTO
DEPARTMENT OF ELECTRICAL & COMPUTER ENGINEERING

ECE 416F
Communication Systems I
Final Exam

December 11, 1997
Time: 2:00 - 4:30 PM

Examiner: Prof. Elvino S. Sousa

Instructions:

1. Type A exam: No aids allowed except for a non-programmable calculator.
2. Answer all of the 6 questions.
3. All 6 questions are of equal value.
4. Begin the answer to each question on a new page.
5. Answer questions 1,2,3 on one book and questions 4,5,6 on a second book.

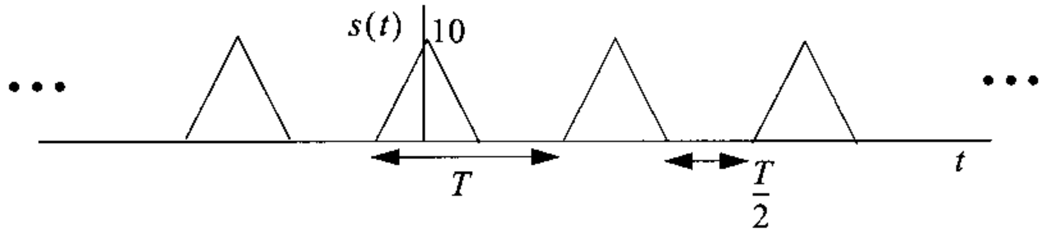
1)

a) The Fourier transform of the signal $x(t)$ is $X(f)$. Determine the Fourier transform of the signal $x(t-t_0)$ in terms of $X(f)$ (t_0 is a constant).

b) Find the Fourier transform of the signal $m(t) = 1$ for $0 < t < 10$ and zero elsewhere.

c) If the above signal $m(t)$ is input to a DSB modulator, give the Fourier transform of the signal at the output of the demodulator assuming that the carrier has an amplitude of 4.

d) Determine the average power of the following signal.



e) If the signal $s(t)$ is the input to a DSB modulator where the carrier input has amplitude 2, determine the average power of the modulator output.

2) An FDM system is to multiplex a number of modulated message signals. Four of the message signals have a bandwidth of 3 KHz and are to be modulated using lower sideband SSB, two of the modulated signals have a bandwidth of 10 KHz and are to be modulated using DSB, and one of the signals has a bandwidth of 20 KHz and is to be modulated using FM with a peak frequency deviation of 40 KHz. The composite FDM signal is to be transmitted through a band-pass channel which has a low-frequency cut-off at 1000 KHz. The signals are to be multiplexed with the SSB signals at the low end of the spectrum, followed by the DSB signals and then the FM signal. A guard band of 1 KHz is required between the spectrum of the various signals. Determine an appropriate set of carrier frequencies for the multiplexed signals. Draw a typical spectrum for the composite FDM signal. What is the bandwidth of the composite FDM signal.

3) A white Gaussian noise process ($n(t)$) with power spectral density $S_n(f) = \frac{N_0}{2}$ is input to an ideal band-pass filter with transfer function $H(f) = \frac{1}{2}$ for $f_0 - \frac{B}{2} \le |f| \le f_0 + \frac{B}{2}$, and $H(f) = 0$, otherwise. (Note that f_0 and B are both positive and $f_0 \gg B$). The filter output is a random process $y(t)$.

a) Determine the autocorrelation function for the noise process $n(t)$.
b) Determine the power spectral density for the noise process $y(t)$ and plot it.
c) Determine the autocorrelation function for the process $y(t)$.
d) The process $y(t)$ is input to a DSB demodulator with carrier $\cos(2\pi(f_0 - B)t)$. Give the power spectral density and the average power of the output of the demodulator.

4) A given low-pass channel has a bandwidth of 100 KHz. This channel is used to transmit a PCM signal that is based on the use of an 8-bit A/D converter and the use of one synchronization bit per coded sample.

a) Determine the maximum bandwidth of a low-pass signal that can be transmitted using the above PCM scheme if the transmitter uses a pulse-shaping filter with 25% excess bandwidth.

b) For the case of a transmitted pulse with zero excess bandwidth and a bit rate of 100 KHz give the basic signalling pulse for transmitting the bits, i.e. $h(t)$, including all the parameters. Give the Fourier transform of this pulse.

5) In a cellular system the total allocated frequency spectrum is 20 MHz for base transmit and 20 MHz for mobile transmit. A frequency re-use cluster size of 7 is used and the modulation scheme used is FM with a bandwidth of 25 KHz. Each 25 KHz channel can carry one voice signal.

a) Determine the capacity of the system in terms of the number of voice calls that can be supported per cell assuming that each cell uses the equivalent of two voice circuits for system control.

b) In the above each of the 25 KHz channels is converted into a digital channel. This channel uses QAM modulation which carries a binary sequence with sinc shaped pulses (in the time domain) in each of the in-phase and quadrature carriers. Determine the maximum bit rate for this channel (i.e. the rate of the two combined sequences). If 20 Kb/s speech coders are used determine the number of users that can be supported in each 25 KHz channel assuming digital transmission.

6) In the computer exercises you generated a low-pass noise process using the computer. Describe the various steps that were carried out in the exercise to generate a low-pass Gaussian process; include a description of the the random variables generated at each step as follows:

a) What was the probability density function for the first random numbers generated? Give a typical plot of the histogram.

b) What was the probability density function for the second set of random numbers generated (i.e. transformed from the 1st set). Give a typical plot of the histogram.

c) What was the probability density function for the 3rd set of random numbers generated (i.e. transformed from the 2nd set). Give a typical plot of the histogram.

d) A low-pass Gaussian random process was created by using an array of Gaussian random variables and then using linear interpolation to create a continuous time random process where the Gaussian random variables are samples. If the samples are spaced T seconds apart and the Gaussian samples have zero mean and variance equal to σ^2, give an expression for the power spectral density of the resulting Gaussian noise process.