PRINT

FIRST NAME ____________________ LAST NAME ______________________________

STUDENT NUMBER______________________________

UNIVERSITY OF TORONTO

FINAL EXAMINATION, DECEMBER 1995

SECOND YEAR - PROGRAM 6

CHE 221F - CALCULUS AND NUMERICAL METHODS

EXAMINER - D.C.S. KUHN

1. Attempt all 6 problems; they are of equal value.

2. Calculator Type 2 - All non-programmable calculators are allowed. No programmable calculators are allowed.

3. Paper Type C - Each candidate may use both sides of a single authorized aid sheet.

4. **ALL WORK IS TO BE DONE ON THESE SHEETS!** Use the back of the page if you need more space. Be sure to indicate clearly if your work continues elsewhere. **DO NOT SEPARATE THE SHEETS.**

Useful Integral: $\int \frac{1}{\sin u} du = \ln\left|\tan \frac{u}{2}\right| + C$;

$$\int \cos^2 u du = \frac{u}{2} + \frac{\sin 2u}{4} + C$$

Question	Mark
1	
2	
3	
4	
5	
6	
Total	

Problem 1

(a) Find a particular solution to $\sin x \dfrac{dy}{dx} = x - y\cos x$ where $y(\dfrac{\pi}{2}) = 1$.

Problem 1 [cont'd]

(b) The length of a curve $\bar{f}(t)$ between t_0 and t_1 is:

$$s = \int_{t_0}^{t_1} \left|\bar{f}'(t)\right| dt.$$

Calculate the length of the part of the circle $x^2 + y^2 = 4$ above the x-axis using the above formula. Recall $\left|\bar{f}'(t)\right|$ is the magnitude of the vector $\bar{f}'(t)$.

Problem 2

A lamina lies inside the circle $x^2 + y^2 = 4$, above the line $y=1$, to the left of the line $y = x$ and to the right of the line $y = -x$. Its density is $\rho = (x^2 + y^2)^{-3/2}$. Find the polar moment of inertia, I_O; that is the moment of interia with respect to the z-axis.

Problem 3

Find $\iint_R y \; dA$, where R is the region bounded by 2x+3y =1, 2x +3y = 3, x - 2y =5, and the x-axis, by introducing the change of variables u = 2x + 3y and v = x -2y.

Problem 4

Find the mass of the solid bounded by the cones $x^2 = y^2 + z^2$ and $x^2 = 3y^2 + 3z^2$, and by the planes $x = 1$ and $x = 2$. The density is $\delta = (x^2 + y^2 + z^2)^{1/2}$.

Problem 4 [cont'd]

Problem 5

Health inspectors are interested in the molar flux of CO_2 out through the open section of the Skydome roof during baseball games. The open section may be described by the surface $z = (4 - x^2 - y^2)^{1/2}$ within the cylinder $x^2 + y^2 = 1$. If the CO_2 concentration during a game is the $CO_2 = 12 - x^2 - y^2 - z^2$ [mol/volume] and the velocity field distribution is $\overline{v} = (x, y, z)$; determine the molar flux through the open roof, i.e.

$$\iint_S C_{CO2} \overline{v} \bullet \overline{n} \; dS$$

Problem 5 [cont'd]

<u>Problem 6</u>

Let S be the part of the surface of the paraboloid $z = x^2 + 4y^2$ above the plane $z = 1$ and below the plane $z = 4$ oriented with upward pointing normal vector. Let C be the intersection of these planes and the cones, oriented as in Stokes' Theorem. Let

$$\overline{F} = z\overline{i} + x\overline{j} + y\overline{k}$$

Verify Stokes' Theorem for this case by computing

$$\oint_C \overline{F} \bullet \overline{T} ds \text{ and } \iint_S (\nabla \times \overline{F}) \bullet \overline{n} dS$$

separately and showing that they are equal.

Problem 6 [cont'd]