University of Toronto
FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATIONS, DECEMBER 1999

APM 384H1F – Partial Differential Equations

Year III, Program III – 5a, 5bm(c), 5env, 5p

Examiner: Professor R.A. Ross
Duration: $2\frac{1}{2}$ hours

Exam Type C

All questions have EQUAL value.

1. Solve for $u(x,y)$

$$u_{xx} + u_{yy} = 0 \ , \qquad 0 < x < \ell \ , \qquad 0 < y < h$$

where

$$u(0,y) = 0 \ , \qquad u(\ell, y) = y$$
$$u(x,0) = 0 \ , \qquad u(x,h) = x \ .$$

2. In the semicircular plate of radius a, the steady state temperature $u(r,\theta)$ is a solution of

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0 \ , \quad 0 < \theta < \pi \ , \quad 0 < r < a$$

where

$$\frac{\partial u}{\partial \theta}(r,0) = 0 \ , \quad 0 < r < a$$
$$u(r,\pi) = 0 \ , \quad 0 < r < a$$

and

$$u(a,\theta) = f(\theta) \ , \quad 0 < \theta < \pi \ .$$

Find $u(r,\theta)$.

3. (a) Solve for $u(x,t)$

$$\frac{\partial u}{\partial t} - k\frac{\partial^2 u}{\partial x^2} = e^{-t}\,, \qquad 0 < x < \ell\,, \qquad t > 0$$

where $u(x,0) = 0\,,\ u(0,t) = \alpha\,,\ u(\ell,t) = \beta\,.$

(b) Find $\lim_{t\to\infty} u(x,t)\,.$

4. Find the temperature $u(r,t)$ in the annular region

$$0 < a < r < b\,, \qquad -\pi < \theta \le \pi$$

where

$$\frac{\partial u}{\partial t} - \frac{k}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) = 0$$

and

$$u(a,t) = u(b,t) = 0\,, \qquad u(r,0) = f(r)\,.$$

5. Solve for $u(x,t)$

$$\frac{\partial^2 u}{\partial t^2} - c^2\frac{\partial^2 u}{\partial x^2} = \delta(x-a)\sin\omega t\,, \qquad 0 < x < \infty\,, \qquad t > 0$$

$0 < a < \infty$

where

$$u(0,t) = 0\,, \qquad u(x,0) = 0\,, \qquad u_t(x,0) = 0\,.$$