UNIVERSITY OF TORONTO
DEPARTMENT OF ELECTRICAL & COMPUTER ENGINEERING
FINAL EXAMINATION - APRIL 2001
Second Year - Programs 7,9
ECE221S - ELECTRIC & MAGNETIC FIELDS
EXAMINERS - R. Adve, F. P. Dawson, A. Konrad, J.D. Lavers

NAME (Please print):		
	Family Name	Given Name
STUDENT NUMBER:		

EXAMINATION TYPE: Type A; Papers for which no data are permitted other than the information printed on the examination paper.

CALCULATORS: Non-programmable scientific type permitted.

DURATION: 2.5 hours.

INSTRUCTIONS:
- **DO NOT UNSTAPLE THIS EXAM BOOK.**
- **Answer all questions.**
- Answer each question neatly and concisely.
- Answers to all questions must be supported by calculations.
- The back side of each adjacent page may also be used for your answer.
- Two pages of possibly useful formulae are found at the end of the examination book on pages 11 and 12.

QUESTION	VALUE	MARKS
A-1 Part (a)	15	
A-1 Part (b)	10	
B-1	7	
B-2	5	
B-3	7	
B-4	6	
C-1	25	
D-1	15	
D-2	10	
TOTAL:	100	

Part A (25 Marks Total) - One Question Consisting of Two Parts

A parallel-plate line shown in Figure 1 is used to connect a driver to two buffers as shown in Fig. 2. Each buffer input is terminated with a resistor R. This type of configuration is referred to as a daisy chain.

The objective is to design the transmission lines so that the reflection coefficient at each buffer input and at the branch point is zero. Also, the signal arriving at each buffer should be delayed by 1 ns with respect to the signal transmitted by the driver.

You are given the following information:

$$\varepsilon_r=4;\ d=2\text{ mm};\ \ l_1=3\text{ cm};\ \ R=75\Omega$$

Note: the transmission line is loss-less and dispersion-less.

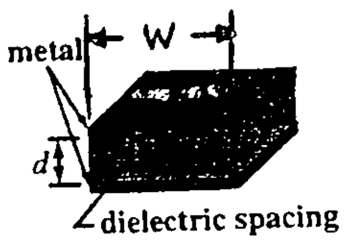


Fig. 1 Parallel-plate line

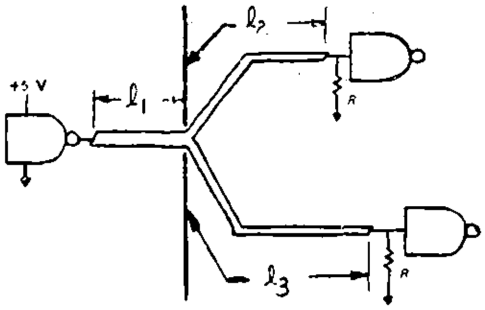


Fig. 2 Daisy chain configuration

Question A-1 - Part (a) (15 marks)

(a) With reference to Figure 2, calculate the length of transmission line 2 (l_2) and transmission line 3 (l_3) and the width of transmission lines 1, 2 and 3. Place your final answers in the box at right.

l_2	
l_3	
w_1	
w_2	
w_3	

Question A-1- Part (b) (10 marks)

(b) Assume that the source impedance (Z_g) is equal to the characteristic impedance of the section represented by the length l_1. Prove that the average power transmitted to each buffer is given by the following expression:

$$P_{buffer} = \frac{3.125}{Z_g}$$

Note: The **source** signal, not the output signal, is a square wave with an amplitude of 5V. The duration of the high state (5V) and low state (0V) are equal and the repetition frequency is 100MHz. **Hint: Voltage and current are in phase. Make use of the expression for P_{ave} obtained from the list of useful equations.**

Part B (25 Marks Total) - Consisting of 4 Questions

Question B-1 (7 Marks)

A spherical capacitor has inner radius *a* and outer radius *b*. The permittivity of the dielectric in between changes with range:

$$\varepsilon(r) = \varepsilon_0 c/r$$

a) If total charge Q is placed on the inner conductor, find the electric field in the dielectric. **(3 Marks)**

b) What is the potential V(r) within the dielectric? **(3 Marks)**

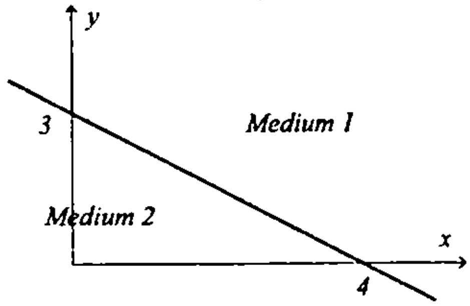
c) Determine the capacitance of this system. **(1 Mark)**

Question B-2 (5 Marks)

What is polarization in a dielectric? How does it relate to the dielectric constant? Explain using words and figures only. **(5 Marks)**

Question B-3 (7 Marks)

As shown in the figure, a plane boundary of infinite extent in the z-direction passes through the points (4,0,0) and (0,3,0). The electric field in medium 1 ($\varepsilon_{r1} = 2.5$) is given by $\mathbf{E} = 25\mathbf{x} + 50\mathbf{y} + 25\mathbf{z}$ V/m. Determine the electric field in medium 2 ($\varepsilon_{r2} = 5$). **(7 Marks)**



Question B-4 (6 Marks)

(a) The potential in a charge free region is given by $V(x,y,z) = ax^2 + by^2 + f(z)$. **(4 Marks)** What is $f(z)$?

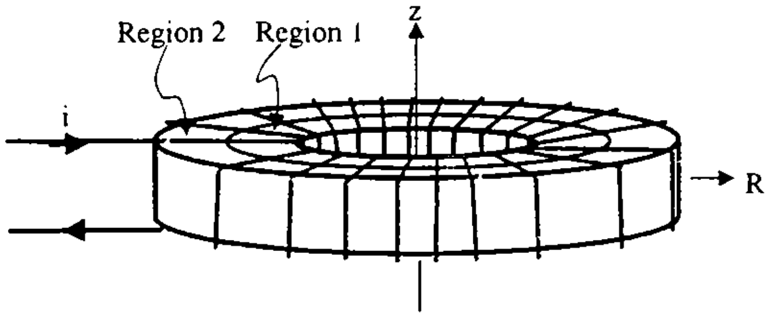
(b) In general the earth is chosen as the reference voltage (V=0). If one were to choose a different reference such that earth were at 10,000 volts, would it be dangerous to walk around? **(2 Marks)**

PAGE FOR CONTINUATION OF ANSWERS TO PART B QUESTIONS

Part C (25 Marks Total) - Consisting of 1 Question

Question C-1:

Consider a toroidal winding as shown below. The core consists of two ferromagnetic materials. Region 1 extends from R_{in} = 10 cm to $R_{interface}$ = 15 cm, has a relative permeability of $\mu_{r,1}$ = 100 and a square cross-section of 5 cm by 5 cm. Region 2 extends from $R_{interface}$ = 15 cm to R_{out} = 20 cm, has a relative permeability of $\mu_{r,2}$ = 150 and a square cross-section of 5 cm by 5 cm. The winding consists of N = 1000 tightly wound turns and carries a DC current of i = 1A as shown.



(a) Make use of Ampere's law to obtain the magnetic flux density $\vec{B}$ as a function of radial distance R for $0 \le R \le 30$ cm.

(b) Plot the magnitude of $\vec{B}$ as a function of R for $0 \le R \le 30$ cm.

(c) Find W_m, the energy stored in the magnetic field of the toroidal winding.

(d) Use the value of W_m obtained in part **(c)** to find the inductance L (in millihenries) of the toroidal winding.

(e) Find ϕ, the total magnetic flux (in webers) inside the toroid.

(f) Find Λ, the magnetic flux linked by the toroidal winding. *Note:* Λ is called *flux linkage*.

(g) Use the value of Λ obtained in part **(f)** to find the self-inductance L (in millihenries) and compare it to the result obtained in part **(d)**.

PAGE FOR CONTINUATION OF ANSWERS TO PART C QUESTION

Part D (25 Marks Total) - Consisting of Two Questions

Question D-1 (15 marks)

Consider a material having $\varepsilon_r = 1$, $\mu_r = 4$ and $\sigma = 2$ S/m. It is known that the magnetic vector potential A in the region is given by:

$$\mathbf{A} = \mathbf{z}\, 10\, r^{3/2}$$

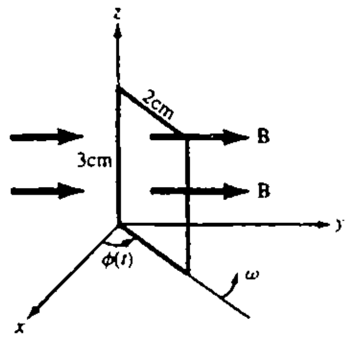
where r and z are the radial coordinate and the axial unit vector, respectively, in a cylindrical coordinate system.

Determine the total current I flowing through a circular surface of radius R = 1 m located at z = 0.5 m and centered about the z-axis.

Question D-2 (15 marks)

The rectangular conducting loop shown in the figure to the right rotates at 3600 rpm in a uniform, time varying magnetic flux density given by:

$$\mathbf{B} = \mathbf{y}\ 1.2 \cos(120\pi t) \quad \text{[Tesla]}$$



a) Determine the current induced in the loop if its internal resistance is 0.5 Ω and it can be assumed that the loop inductance is negligible.

b) Describe how you would verify the assumption that the inductance can be neglected.

ECE221S - ELECTRIC AND MAGNETIC FIELDS
USEFUL FORMULAE AND CONSTANTS

Constants:

μ_0: magnetic permeability of free space — value = $4\pi \times 10^{-7}$ Henry/m

ε_0: electric permittivity of free space — value = 8.85×10^{-12} Farad/m

q_e: charge of one electron — value = -1.60×10^{-19} Coulomb

m_e: mass of one electron — value = 9.11×10^{-31} kg

N_A: Avogadro's constant — value = 6.02×10^{23} mol^{-1}

Transmission Lines:

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')} = \alpha + j\beta \qquad Z_0 = \sqrt{\frac{(R' + j\omega L')}{(G' + j\omega C')}}$$

$$\tilde{V}(z) = V_0^+ e^{-j\beta z} + V_0^- e^{+j\beta z} \qquad \tilde{I}(z) = \frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{+j\beta z} \qquad V_0^- = \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right) V_0^+ = \Gamma V_0^+$$

$$Z_{in}(z) = Z_0 \left[\frac{1 + \Gamma e^{+j2\beta z}}{1 - \Gamma e^{+j2\beta z}}\right] \qquad Z_{in}(-l) = Z_0 \left[\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}\right]$$

$$V_0^+ = \left(\tilde{V}_g \frac{Z_{in}}{Z_{in} + Z_g}\right)\left(\frac{1}{e^{j\beta l} + \Gamma e^{-j\beta l}}\right)$$

$$P_{av} = \frac{\left|V_0^+\right|^2}{2Z_0}\left[1 - |\Gamma|^2\right] \qquad P_{ave} = \frac{1}{T}\int_0^T v \cdot i \cdot dt$$

$$R' = \frac{2R_s}{W} \qquad L' = \frac{\mu d}{W} \qquad G' = \frac{\sigma W}{d} \qquad C' = \frac{\varepsilon W}{d} \qquad R_s = \sqrt{\frac{\pi f \mu_c}{\sigma_c}}$$

Electric Fields:

$$\boldsymbol{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{dq'}{R^2}\hat{R} \text{ where } \boldsymbol{R} = \boldsymbol{r} - \boldsymbol{r}' \qquad R = |\boldsymbol{r} - \boldsymbol{r}'| \qquad \hat{\boldsymbol{R}} = \boldsymbol{R}/R$$

where: the *primed* variable is the variable of integration,
r is the position vector locating the point of observation relative to the origin,
r′ is the position vector locating the point of integration (i.e. source point) relative to the origin,
R is the distance vector from the source point to the point of observation, and

$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq'}{R} \qquad V(b) - V(a) = -\int_a^b \boldsymbol{E} \cdot d\boldsymbol{l} \qquad E_{1t} = E_{2t} \qquad D_{n1} - D_{n2} = \rho_s$$

$$w_E = 0.5\varepsilon E^2 \qquad \boldsymbol{J} = (-\rho_{ve}\mu_e + \rho_{vh}\mu_h)\boldsymbol{E} \qquad I = \int_S \boldsymbol{J} \cdot d\boldsymbol{s}$$

$$\boldsymbol{E} = -\nabla V \qquad \nabla^2 V = -\rho_v/\varepsilon$$

Magnetic Fields:

$$\boldsymbol{B} = \frac{\mu_0}{4\pi}\int\frac{idl' \times \hat{\boldsymbol{a}}_R}{R^2} \qquad \boldsymbol{B} = \nabla \times \boldsymbol{A} \qquad \boldsymbol{F}_m = I\int_C d\boldsymbol{l} \times \boldsymbol{B} \qquad \boldsymbol{T} = \boldsymbol{m} \times \boldsymbol{B}$$

$$H_{2t} - H_{1t} = J_S \qquad B_{n2} = B_{n1} \qquad w_M = 0.5\mu H^2 \qquad \Phi = \int_S \boldsymbol{B} \cdot d\boldsymbol{s}$$

$$V_{emf} = -N\frac{d\phi}{dt} = -N\int_S \frac{\partial \boldsymbol{B}}{\partial t} \cdot d\boldsymbol{s} + N\oint_C (\boldsymbol{u} \times \boldsymbol{B}) \cdot d\boldsymbol{l}$$

Some Integration Formulae:

$$\int\frac{du}{u^2 + a^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} \qquad \int\frac{u\,du}{u^2 + a^2} = \frac{1}{2}\ln\left|u^2 + a^2\right|$$

$$\int\frac{du}{\sqrt{u^2 + a^2}} = \ln\left|u + \sqrt{u^2 + a^2}\right| \qquad \int\frac{du}{\left(u^2 + a^2\right)^{3/2}} = \frac{u}{a^2\sqrt{u^2 + a^2}}$$

Vector Operations in Cylindrical Co-ordinates (r, φ, z)

$$\nabla V = \frac{\partial V}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial V}{\partial \varphi}\hat{\varphi} + \frac{\partial V}{\partial z}\hat{z} \qquad \nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \varphi^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\nabla \cdot \boldsymbol{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\varphi}{\partial \varphi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \boldsymbol{A} = \left(\frac{1}{r}\frac{\partial A_z}{\partial \varphi} - \frac{\partial A_\varphi}{\partial z}\right)\hat{r} + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)\hat{\varphi} + \left(\frac{1}{r}\frac{\partial (rA_\varphi)}{\partial r} - \frac{1}{r}\frac{\partial A_r}{\partial \varphi}\right)\hat{z}$$

Vector Operations in Spherical Co-ordinates (r, θ, φ)

$$\nabla V = \frac{\partial V}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \varphi}\hat{\varphi}$$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial \varphi^2}$$

$$\nabla \cdot \boldsymbol{A} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 A_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta A_\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\varphi}{\partial \varphi}$$

$$\nabla \times \boldsymbol{A} = \frac{1}{r\sin\theta}\left[\frac{\partial(\sin\theta A_\varphi)}{\partial \theta} - \frac{\partial A_\theta}{\partial \varphi}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \varphi} - \frac{\partial (rA_\varphi)}{\partial r}\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial (rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta}\right]\hat{\varphi}$$