UNIVERSITY OF TORONTO

FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION, DECEMBER 2001

ECE350F – PHYSICAL ELECTRONICS

Exam Type: A

Examiner – Prof. E. H. Sargent

Answer any 5 questions. All questions are of equal value.

Good luck!

1. Key parameters of an n-channel silicon MOSFET with aluminum gate are given in Table 1.

(a) **(2 marks)** Determine the work function difference between the metal and semiconductor. Use the approximation that the intrinsic Fermi energy of silicon lies exactly in the middle of the bandgap. Draw band diagrams for the metal and semiconductor both before and after being brought together.

(b) **(2 marks)** Determine the threshold voltage increase/decrease due to charge trapped at the dielectric-semiconductor interface. With the help of a labelled diagram, explain the increase/decrease in threshold voltage.

(c) **(2 marks)** Determine the threshold voltage of the device. Draw a carefully-labelled energy vs. position diagram cutting through the gate, oxide, and semiconductor, for an applied gate voltage equal to the threshold voltage.

(d) **(2 marks)** The voltage on the gate is now increased to exceed the threshold voltage by 3 V. A drain-source voltage of 2 V is applied. What current flows from drain to source?

(e) **(2 marks)** The drain-source voltage is doubled from its value in (d), while all other parameters are held fixed. Does the current double as well? Explain the mechanism underlying your answer with the help of a labelled diagram.

Table 1 – Properties of NMOS transistor considered in Question 1

Work function of aluminum used as gate metal	4.28 eV
Electron affinity of silicon	4.01 eV
Bandgap of silicon	1.1 eV
Acceptor doping concentration in the channel	10^{14} cm^{-3}
Intrinsic carrier concentration of silicon at room temperature	1.5×10^{10} cm^{-3}
Distance of Fermi level below the intrinsic Fermi level in 10^{14} cm^{-3} p-doped silicon	0.23 eV
Effective interface charge per unit area trapped in SiO_2 dielectric	1×10^{-8} C/cm^2
Oxide capacitance per unit area	1×10^{-7} F/cm^2
Relative dielectric constant of silicon	11.8
Width of the depletion region at the onset of strong inversion	2.5 μm
Magnitude of depletion charge per unit area at the onset of strong inversion	4×10^{-9} C/cm^2
Effective channel mobility	1000 cm^2/Vs
Depth of the channel (conventional symbol = Z)	10 μm
Length of the channel (conventional symbol = L)	10 μm

2. (a) **(4 marks).** Consider an <u>**n-p-n**</u> bipolar junction transistor. Draw a carefully-labelled spatial band diagram for the device in normal active mode. Indicate and explain:

- depletion regions and their relative widths
- how the band diagram reflects the imposition of suitable boundary conditions
- how the band diagram shows regions of strong nonequilibrium versus regions which are essentially at equilibium
- how the emitter-base and collector-base voltages may be measured from the diagram.

(b) **(3 marks)** Referring back to your band diagram and defining appropriate symbols in your band diagram, obtain an expression for the emitter injection efficiency.

(c) **(3 marks).** The emitter or base material is changed in order to provide a heterojunction and improve thereby the emitter injection efficiency. A large-bandgap material is available with conduction band offset 0.2 eV and valence band offset 0.1 eV relative to the original material for a total bandgap difference of 0.3 eV.

(i) Would you replace the base or the emitter with this new larger-bandgap material?

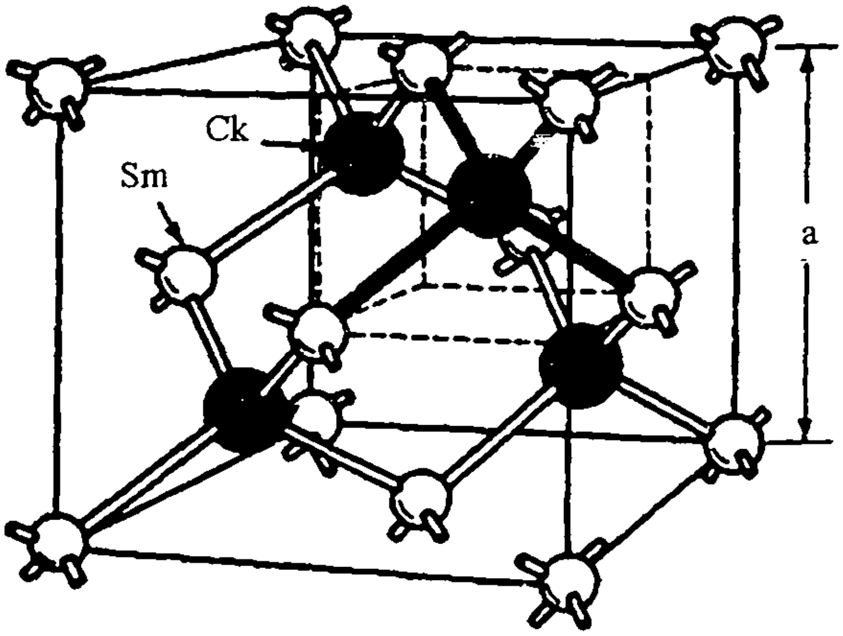
(ii) Estimate the new emitter injection efficiency (expressed as a function of your answer in (b)) at room temperature.

3. A rectangular piece of silicon is of length 1 cm and has cross-sectional dimensions 1 mm x 1 mm. It is doped n-type 10^{15} cm^{-3}. The electron mobility is 1000 cm^2/Vs and the hole mobility 100 cm^2/Vs. The minority carrier recombination time for holes is 1 μs.

(a) **(4 marks).** Draw a diagram showing how you would carry out the Hall experiment using this sample. Pick a suitable current and magnetic field to apply, and calculate the Hall voltage which you would measure. Indicate the polarity of the Hall voltage given the directions of current and magnetic field. Discuss which electronic properties of the material are revealed through the Hall experiment.

(b) **(6 marks).** Draw a diagram showing how you would carry out the Haynes-Shockley experiment using this sample. Pick a suitable longitudinal potential drop to apply. Draw a labelled, quantitative diagram (show quantitative time delays, quantitative pulse spreads) of what you would observe on an oscilloscope comparing the input and output pulses. With the sample given have you been able to ensure through your choice of excitation and measurement conditions that the standard approximations using the Haynes-Shockley approximations are satisfied?

4. **(10 marks).** Consider performing the Debye-Scherrer (powder diffraction, monochromatic illumination) experiment on smellium crackenide (SmCk). A SmCk crystal is of the zincblende type: it consists of two interpenetrating face-centered cubic lattices, one of Sm, the other of Ck shifted ¼ along the diagonal of the nonprimitive cubic cell, as illustrated in the figure.



The cubic lattice period (nonprimitive lattice) depicted in the diagram as a is 5 Å. The atomic form factor for smellium is twice that of crackenic.

Choose a suitable wavelength of x-ray which will reveal a maximum of information about the crystal in the Debye-Scherrer experiment.

Depict a suitable measurement geometry (e.g. beam, powder, x-ray film placement and angles) and quantify any angular and length relationships that you have chosen. Draw what you would observe on the x-ray film for these conditions. In particular, quantify peak heights, indicate any missing peaks (zero structure factor), and quantify the position of the peaks on the film and show quantitatively how they reveal the crystal's lattice constant.

5.

(a) **(5 marks)** Draw a fully quantitatively labelled spatial band diagram (diagrams of band edges and fermi level(s) across the spatial extent of a device) for an unbiased p-n junction made of silicon in which the p-material is doped $1.5\text{x}10^{14}$ cm^{-3} and the n-type material is doped $1.5\text{x}10^{15}$ cm^{-3}. Quantify and indicate explicitly the spatial extent of depletion region on each side of the junction; quantitatively the portion of built-in voltage falling on each side of the junction; and quantitatively the energetic positions of the Fermi level relative to the band edges on each side.

$$\ln(10) = 2.3 \approx 2$$

(b) **(5 marks)** Draw a fairly accurate but non-quantitative spatial band diagram for:

(i) **(1 mark)** An unbiased, uniformly-illuminated (photon energy exceeding bandgap energy) p-n junction. Label the directions of flow of the four components of current and discuss their relative magnitudes.

(ii) **(1 mark)** A reverse-biased, uniformly-illuminated (photon energy exceeding bandgap energy) p-n junction. Label the directions of flow of the four components of current and discuss their relative magnitudes.

(iii) **(1 mark)** An unbiased pnpn device:

p	n	p	n

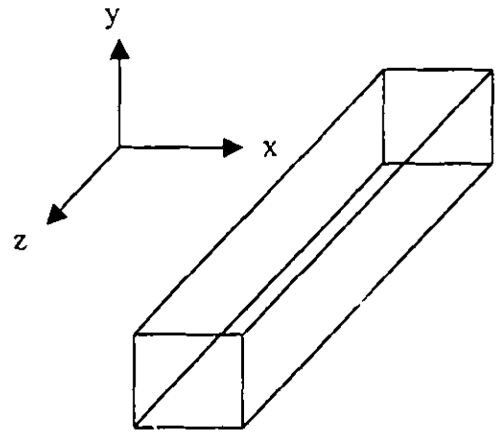
(iv) **(1 mark)** A pnpn device with a positive potential of 1V applied on the p-side relative to the n-side. Be clear about where most of the voltage falls and where the Fermi level/levels bends/bend. In relative terms, how much current do you expect to flow compared to the 1-V-forward-biased pn junction?

(v) **(1 mark)** The 1-V-forward-biased pnpn device is now illuminated. Draw the spatial band diagram and comment on current.

6. **(10 marks).** The wavefunction of an electron at the Brillouin zone boundary is approximated by $\psi_+ = c_1 \cos(n \pi x / a)$ (higher potential energy solution) and $\psi_- = c_1 \sin(n \pi x / a)$ (lower potential energy solution) where n is a positive integer which may be used to label sequentially the bandgaps. The quantity a is the spatial period of the potential. The quantity c_1 is a suitably-chosen normalization constant. Normalization may be carried out over a single unit cell.

Suppose that the periodic potential is given by a 50%-duty-cycle square wave of amplitude 1 eV. The atoms making up the semiconductor provide four valence electrons per primitive unit cell. What range of energies of light (photon energies, in eV) will be strongly absorbed by this semiconductor? Draw the absorption spectrum over the pertinent range.

7. A long wire has rectangular cross-section 5 nm by 5 nm, as illustrated in the figure.



$\{x,y\}$ constitutes the transverse plane (confinement direction) while z constitutes the propagation direction. The wire is made out of a semiconductor with known conduction and valence band energies E_c, E_v which are separated in energy by 1 eV. Electrons and holes have effective mass $m^* = 1\ m_0$ in this material.

(a) **(3 marks).** Treat the potential outside the wire as infinite and the potential inside as 0 V (arbitrary reference). Calculate the eigenenergies of the lowest three confined modes assuming, for the moment, that there is no propagation along the z axis. Draw the wavefunctions associated with these purely transverse modes.

(b) **(2 marks).** Now consider propagating modes, i.e. solutions whose wavefunctions are of the form $\psi(x,y,z) = \psi_{transverse}(x,y)\ \exp(-j\ k_z\ z)$. Depict the dispersion relation E vs. k_z for conduction band electrons which are confined to the wire. Label energies measured relative to the conduction band edge quantitatively.

(c) **(1 mark).** Confined holes may be thought of analogously to confined electrons, except that for holes energy increases in the downward direction on an electron dispersion relation. If the potential confining the holes may also be thought of as infinite (in the downward direction), then the confined-hole valence band dispersion relation looks looks like your answer of (b), but mirrored over the horizontal line at the half-bandgap. Draw a single dispersion relation E vs. k_z which depicts both electrons and holes. What is the lowest-energy photon that would be absorbed by this quantum wire?

(d) **(4 marks).** Now consider the wire to be doped p-type for $z < 0$ and n-type for $z > 0$. A 1-D quantum p-n junction is thus formed. Develop an ideal diode equation for this device. Compare its threshold voltage and the character of its temperature dependence to that of a bulk p-n junction. You may simplify your analysis by focusing your attention only the first quantum-confined levels in each band.

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appendix II

PHYSICAL CONSTANTS AND CONVERSION FACTORS†

Avogadro's number	$N_A = 6.02 \times 10^{23}$ molecules/mole
Boltzmann's constant	$k = 1.38 \times 10^{-23}$ J/K
	$= 8.62 \times 10^{-5}$ eV/K
Electronic charge (magnitude)	$q = 1.60 \times 10^{-19}$ C
Electronic rest mass	$m_0 = 9.11 \times 10^{-31}$ kg
Permittivity of free space	$\epsilon_0 = 8.85 \times 10^{-14}$ F/cm
	$= 8.85 \times 10^{-12}$ F/m
Planck's constant	$h = 6.63 \times 10^{-34}$ J-s
	$= 4.14 \times 10^{-15}$ eV-s
Room temperature value of kT	$kT = 0.0259$ eV
Speed of light	$c = 2.998 \times 10^{10}$ cm/s
	Prefixes:
1 Å (angstrom) = 10^{-8} cm	milli-, m- = 10^{-3}
1 μm (micron) = 10^{-4} cm	micro-, μ- = 10^{-6}
1 mil = 10^{-3} in.	nano-, n- = 10^{-9}
2.54 cm = 1 in	pico-, p- = 10^{-12}
1 eV = 1.6×10^{-19} J	kilo-, k- = 10^{3}
	mega-, M- = 10^{6}
	giga-, G- = 10^{9}

A wavelength λ of 1 μm corresponds to a photon energy of 1.24 eV

† Since cm is used as the unit of length for many semiconductor quantities, caution must be exercised to avoid unit errors in calculations. When using quantities involving length in formulas which contain quantities measured in MKS units, it is usually best to use all MKS quantities. Conversion to standard semiconductor usage involving cm can be accomplished as a last step. Similar caution is recommended in using J and eV as energy units.

appendix III

PROPERTIES OF SEMICONDUCTOR MATERIALS

	E_g (eV)	μ_n (cm²/V-s)	μ_p (cm²/V-s)	ρ (Ω-cm)	Transition	Doping	Lattice	a (Å)	ϵ_r	Density (g/cm³)	Melting point (°C)
Si	1.11	1350	480	2.5×10^{5}*	*i*	n, p	*D*	5.43	11.8	2.33	1415
Ge	0.67	3900	1900	43	*i*	n, p	*D*	5.66	16	5.32	936
SiC(α)	2.86	500		10^{10}	*i*	n, p	*W*	3.08	10.2	3.21	2830
AlP	2.45	80		10^{-1}	*i*	n, p	*Z*	5.46	9.8	2.40	2000
AlAs	2.16	180		0.1	*i*	n, p	*Z*	5.66	10.9	3.60	1740
AlSb	1.6	200	300	5	*i*	n, p	*Z*	6.14	11	4.26	1080
GaP	2.26	300	150	1	*i*	n, p	*Z*	5.45	11.1	4.13	1467
GaAs	1.43	8500	400	4×10^{8}*	*d*	n, p	*Z*	5.65	13.2	5.31	1238
GaSb	0.7	5000	1000	0.04	*d*	n, p	*Z*	6.09	15.7	5.61	712
InP	1.35	4000	100	8×10^{-3}	*d*	n, p	*Z*	5.87	12.4	4.79	1070
InAs	0.36	22600	200	0.03	*d*	n, p	*Z*	6.06	14.6	5.67	943
InSb	0.18	10^{5}	1700	0.06	*d*	n, p	*Z*	6.48	17.7	5.78	525
ZnS	3.6	110		10^{10}	*d*	n	*Z, W*	5.409	8.9	4.09	1650†
ZnSe	2.7	600		10^{4}	*d*	n	*Z*	5.671	9.2	5.65	1100†
ZnTe	2.25		100	100	*d*	p	*Z*	6.101	10.4	5.51	1238†
CdS	2.42	250	15		*d*	n	*W, Z*	4.137	8.9	4.82	1475
CdSe	1.73	650		10^{3}	*d*	n	*W*	4.30	10.2	5.81	1258
CdTe	1.58	1050	100	10^{9}	*d*	n, p	*Z*	6.482	10.2	6.20	1098
PbS	0.37	575	200	5×10^{-1}	*i*	n, p	*H*	5.936	161	7.6	1119
PbSe	0.27	1000	1000	10^{-1}	*i*	n, p	*H*	6.147	280	8.73	1081
PbTe	0.29	1600	700	10^{-2}	*i*	n, p	*H*	6.452	360	8.16	925

All values at 300 K   *intrinsic resistivity.   †vaporizes.

Definitions of symbols: ρ is resistivity of high-purity material; *i* is indirect; *d* is direct; *D* is diamond; *Z* is zincblende; *W* is wurtzite; *H* is halite (NaCl). Values of mobility and resistivity are for material of available purity; these values are considered approximate (exception: Si and GaAs resistivities are extrapolated to intrinsic material). Most of the values in this table were taken from publications of the Electronic Properties Information Center (EPIC), Hughes Aircraft Co., Culver City, California; also, M. Neuberger, "III-V Semiconducting Compounds—Data Tables," published with permission from Plenum Publishing Corporation, copyright 1970.

Crystals in the wurtzite structure are not described completely by the single lattice constant given here, since the unit cell is not cubic. Several II-VI compounds can be grown in either the zincblende or wurtzite structures.

Many values quoted here are approximate or uncertain, particularly for the II-VI and IV-VI compounds.

Semiconductor Terms

Variable Subscripts

n - looking at negative charges (electrons)
p - " " positive " (holes)
c - conduction
V - valence
i - intrinsic
f - fermi
qf - quasi fermi

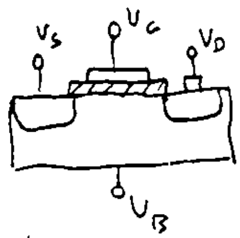
Important Quantities

J - Current Density
I - Current
A - Area (of cross-section
μ - Mobility
L - Diffusion length
D - Diffusion Coefficient
τ - minority carrier lifetime
$V_T \equiv \frac{KT}{q} \overset{stp}{\Longrightarrow} 25.9\,mV$ at room temperature
$\mathcal{E}$ - electric field
V - voltage
$N_{a,d}$ - acceptor/Donor Concentration
K - Boltzman Constant
q - electronic charge 1.60×10^{-19}
σ - conductivity
ρ - resistivity

Terms in P-n Junction

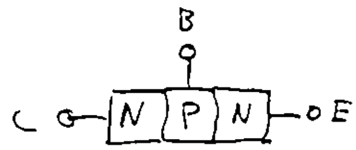
V_o - built in potential
W - depletion region width
ϵ - permitivity $\epsilon = \epsilon_r \epsilon_0$
ϵ_0 - permitivity of free space 8.85×10^{-12} F/m
n - ideality factor

Terms in MosFET



C_i - the insulator capacitance
Φ_{ms} - metal-semiconductor work function.
ϕ_s - Surface Potential
V_G - Gate Voltage; V_B - Body or Substrate Voltage $V_{GS} = V_G - V_S$
V_D - Drain Voltage; V_S - source Voltage $V_{DS} = V_D - V_S$

Terms in BJT



i_c - collector current
i_e - emitter current
i_B - base current
α - ~0.99 β - ~100

MODULE ONE: CRYSTALS

LATTICE: defined by 3 fundamental translation vectors $\vec{a}_1, \vec{a}_2, \vec{a}_3$ such that the appearance from $\vec{r}$ is the same as from $\vec{r}+\vec{R}$, where $\vec{R} = \sum_{i=1}^{3} u_i \vec{a}_i$. Crystal is just lattice w/ basis attached to each point.

UNIT CELL: defined by unit traslation in $\vec{a}_1, \vec{a}_2, \vec{a}_3$. PRIMITIVE ⇒ smallest.

MILLER INDICES: $(hkl) \leftrightarrow (\frac{1}{a_1}\,\frac{1}{a_2}\,\frac{1}{a_3})$ where a_1, a_2, a_3 are the intercepts of the (hkl) plane with $\vec{a}_1, \vec{a}_2, \vec{a}_3$.

COORDINATION NUMBER: # of nearest neighbours.

PACKING FRACTION $\triangleq \frac{\text{volume of closely packed hard spheres in 1 unit cell}}{\text{volume of 1 unit cell}}$

BCC: $\vec{a}_1 = \frac{1}{2}a(\hat{x}+\hat{y}-\hat{z})$; $\vec{a}_2 = \frac{1}{2}a(-\hat{x}+\hat{y}+\hat{z})$; $\vec{a}_3 = \frac{1}{2}a(\hat{x}-\hat{y}+\hat{z})$

FCC: $\vec{a}_1 = \frac{1}{2}a(\hat{x}+\hat{y})$; $\vec{a}_2 = \frac{1}{2}a(\hat{y}+\hat{z})$; $\vec{a}_3 = \frac{1}{2}a(\hat{x}+\hat{z})$ a = SC length

DIAMOND: FCC with 2 atom basis, one at (000), one at $(\frac{1}{4}\frac{1}{4}\frac{1}{4})$.

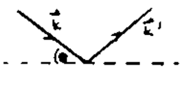
RECIPROCAL LATTICE: denoted $\vec{G} = \sum n_i \vec{b}_i$ exists in k-space. To satisfy translational invariance, require $\vec{G}\cdot\vec{R} = 2\pi m$

Or $\vec{a}_i \cdot \vec{b}_j = 2\pi\delta_{ij}$. In 1D, $b = \frac{2\pi}{a}$.

In 3D: $\vec{b}_1 = \frac{\vec{a}_2\times\vec{a}_3}{\vec{a}_1\cdot(\vec{a}_2\times\vec{a}_3)}\cdot 2\pi$, $\vec{b}_2 = \frac{\vec{a}_3\times\vec{a}_1}{\vec{a}_1\cdot(\vec{a}_2\times\vec{a}_3)}\cdot 2\pi$, $\vec{b}_3 = \frac{\vec{a}_1\times\vec{a}_2}{\vec{a}_1\cdot(\vec{a}_2\times\vec{a}_3)}\cdot 2\pi$

NOTE: $\vec{G} = h\vec{b}_1 + k\vec{b}_2 + l\vec{b}_3$ is $\perp$ to (hkl) plane.

DIFFRACTION CONDITIONS: strong scattering occurs when $\Delta\vec{k} = \vec{G}$



in an elastic scattering: $|G|^2 = 2\vec{k}\cdot\vec{G}$

note: $\vec{k} = \frac{2\pi}{\lambda}\hat{k}$, where λ is the wavelength of the incident beam. Also d_{hkl} (distance between hkl planes) $= \frac{2\pi}{|G|}$

distance between crystal planes

BRAGG'S LAW: $2d\sin\theta = n\lambda$

STRUCTURE FACTOR: $S_G = \sum_k f_k e^{-j\vec{G}\cdot\vec{r}_k}$ where k runs over all atoms of the basis. f_k is the atomic factor of the k^{th} atom and $\vec{r}_k$ its position.

MODULE TWO: WAVES

QUANTUM REVIEW: $p = \hbar k$ (momentum; k = wave #), $E = \hbar\omega$

For a travelling wave, $\Psi = e^{j(\vec{k}\cdot\vec{r}-\omega t)}$, $V_{group} = \frac{\partial\omega}{\partial k}$, $E = \frac{\hbar^2 k^2}{2m}$

Schrödinger's Equation: $[-\frac{\hbar^2}{2m}\nabla^2 + V]\Psi = E\Psi$ or $i\hbar\frac{\partial\Psi}{\partial t}$. (SE)

For 1D-infinite well of length L, $\Psi(x,t) = \sqrt{\frac{2}{L}}\sin(\frac{n\pi x}{L})e^{-i\frac{E_n}{\hbar}t}$

$E_n = \frac{h^2 n^2}{8mL^2}$.

Normalization Condition: $\int_{\text{all } x} |\Psi(x,0)|^2 dx = 1$.

Expectation Value: $\langle f(x)\rangle = \int_{\text{all } x} f(x)|\Psi(x,t)|^2 dx$

PERIODIC MEDIA: Given a periodic potential where $V(\vec{r}) = V(\vec{r}+\vec{R})$, then $\Psi(\vec{r}) = u(\vec{r})e^{j\vec{k}\cdot\vec{r}}$ where $u(\vec{r}) = u(\vec{r}+\vec{R})$.

By examining the Kronig-Penney Model, it was found that there exists energy gaps where $k = \frac{n\pi}{a}$.

periodicity of 1D potential

EFFECTIVE MASS: $m^* = \frac{\hbar^2}{\partial^2 E/\partial k^2}$, $F_{ext} = m^* a$.

HOLES: act as positive charged particles $k_h = -k_e$, $m_h^* = -m_e^*$.

MODULE THREE: EQUILIBRIUM

NUMBER OF STATES PER BAND $= \frac{V_{k\text{-space-per-band}}}{V_{k\text{-space-per-state}}} = \frac{V_{real\text{-space-total}}}{V_{real\text{-space-unit}}}$

= # of unit cells in a crystal

In a band, one electronic state per unit cell, 2 with spin degeneracy.

DENSITY OF STATES: # of states present in the energy range dE.

For bulk material, assuming PARABOLIC bands:

$g(E) = \frac{1}{V_{real}}\frac{dN}{dE} = \frac{1}{4\pi^2}\left(\frac{2m^*}{\hbar^2}\right)^{3/2}\sqrt{E-E_{c,bot}}$ for $E > E_{c,bot}$ (conduction band energy)

For 2D: $g(E) = \frac{m^*}{\pi\hbar^2 L_z}\sum_n u(E-E_n)$ where E_n are the bound states

For 1D: $g(E) = \frac{1}{\pi L_x L_y}\sqrt{\frac{2m^*}{\hbar^2}}\sum_{n_x}\sum_{n_y}\frac{u(E-E_{nx}-E_{ny})}{\sqrt{E-E_{nx}-E_{ny}}}$

For 0D: $g(E) = \sum_{n_x}\sum_{n_y}\sum_{n_z}\frac{2}{L_x L_y L_z}\delta(E-E_{nx}-E_{ny}-E_{nz})$

FERMI-DIRAC: probability of occupation of existing energy state E is $f(E) = \frac{1}{1+e^{(E-E_f)/kT}}$ (T: temp in Kelvin; k: Boltzmann constant) E_f = Fermi energy.

When $E = E_f$, $f(E) = 1/2$. If E is many kTs above E_f, then $f(E)$ is well approximated by $e^{-(E-E_f)/kT}$ (MB stats).

CARRIER CONCENTRATION: $n = \int_{E_c(bot)}^{E_c(top)} g(E)f(E)\,dE$

using MB stats, $n = C_n e^{-(E_c-E_f)/kT}$, $C_n = 2\left(\frac{2\pi m^* kT}{h^2}\right)^{3/2}$

$p = C_p e^{-(E_f-E_v)/kT}$ (E_v: valence band edge),

$C_n = C_p$ if $m^* = m_0$ for both. @ 300K, $C_n = 2.5\times10^{19}\,cm^{-3}$

LAW OF MASS ACTION: $np = C_n C_p e^{-E_g/kT}$ for Si @ 300K

INTRINSIC CARRIER DENSITY: $n_i^2 = C_n C_p e^{-E_g/kT}$, $n_i = 1.5\times10^{10}\,cm^{-3}$

DOPING: Group V donates electrons (N_d)

Group III donates holes (N_a)

$N_d^+ + p = N_a^- + n$

$\therefore n = \frac{(N_d^+ - N_a^-)}{2} + \sqrt{(N_d^+ - N_a^-)^2/4 + n_i^2}$, $p = n_i^2/n$

MODULE FOUR: TRANSPORT

DRIFT: MOBILITY $\mu = \frac{V_{drift}}{\mathcal{E}}$, $\mu_n = -\frac{V_n}{\mathcal{E}}$, $\mu_p = \frac{V_p}{\mathcal{E}}$.

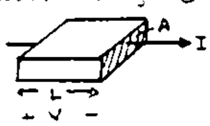
Let τ_n and τ_p be the mean free time between scattering, then $\mu = \frac{q\tau}{m^*}$. $\tau_{phonon} \propto T^{-3/2}$, $\tau_{impurities} \propto T^{3/2}$ (scattering processes)

Combined effect: $\frac{1}{\tau} = \frac{1}{\tau_{phonon}} + \frac{1}{\tau_{impurities}}$

CURRENT DENSITY: $J_{drift} = q(n\mu_n + p\mu_p)\mathcal{E}$

CONDUCTIVITY: $\sigma = \frac{J_{drift}}{\mathcal{E}} = (n\mu_n + p\mu_p)q$

RESISTIVITY: $\rho = \frac{1}{\sigma}$



$J = \frac{I}{A}$, $V = L\mathcal{E}$

$R = \rho\frac{L}{A}$

DIFFUSION: $\frac{m V_{thermal}^2}{2} = \frac{1}{2} k_B T$

$\bar{l}$ ≡ mean free path ≡ dist. btw. collisions = $V_{thermal} \times \tau$

Electron flux: $\phi_n(x) = -\frac{\bar{l}^2}{2\tau}\frac{dn}{dx} = -D_n \frac{dn}{dx}$ ← Diffusion coefficient.

$J_{ndiff} = |q| D_n \frac{dn}{dx}$, $J_{pdiff} = -|q| D_p \frac{dp}{dx}$

TOTAL CURRENT: $J = J_{drift} + J_{diff}$

EINSTEIN'S RELATION: $\frac{D}{\mu} = \frac{kT}{q}$

, Derived assuming equilibrium w/ no potential applied.

MODULE 5: NON-EQUILIBRIUM SEMI-CONDUCTORS

Let G = rate of generation ; R = rate of recombination.

At equilibrium, for an intrinsic S/C, G = R.

Beyond equilibrium: $n = n_0 + \delta n$, $p = p_0 + \delta p$ ← excess carriers ($\delta n = \delta p$).

Let τ_{no}, τ_{po} be excess carrier recombination lifetime.

Then for low-level injection ($n_0 + p_0 \gg \delta n$) :

$$D_n \frac{\partial^2(\delta n)}{\partial x^2} + \mu_n E \frac{\partial(\delta n)}{\partial x} + g' - \frac{\delta n}{\tau_{no}} = \frac{\partial(\delta n)}{\partial t}$$

$$D_p \frac{\partial^2(\delta p)}{\partial x^2} - \mu_p E \frac{\partial(\delta p)}{\partial x} + g' - \frac{\delta p}{\tau_{po}} = \frac{\partial(\delta p)}{\partial t}$$

Steady-state: $\frac{\partial(\delta n)}{\partial t} = 0$

Uniform distribution of excess carriers: $D_n \frac{\partial^2(\delta n)}{\partial x^2} = 0$

Zero electric field: $E \frac{\partial(\delta n)}{\partial x} = 0$

No excess carrier generation: $g' = 0$

No excess carrier recombination: $\frac{\delta n}{\tau_{no}} = 0$

Let $L_n^2 = D_n \tau_n$ (minority carrier diffusion length). If there ∃ little drift, then $\delta n(x) = \delta n(0) e^{-x/L_n}$.

HAYNES-SHOCKLEY: Given a sample of length L, generate at t=0 a pulse of minority carriers (ΔP_{peak}), apply V_A to cause drift

If t_d is the time it takes for the peak to reach L

For $t_d \ll \tau_p$, then $\delta p(x,t) = \frac{\Delta P_{peak}}{2\sqrt{\pi D_p t}} e^{-x^2/4D_p t}$

Δx = ½ width of the pulse = $4\sqrt{D_p t_d}$

$\Delta t = \frac{\Delta x}{V_{drift}}$ (at x=L) $= \frac{4\sqrt{D_p t_d^3}}{L}$ ⇒ $D_p = \frac{L^2 \Delta t^2}{16 t_d^3}$

QUASI-FERMI LEVELS:

Define E_{qfn} = e^- quasi-fermi-level by

$n = C_n e^{-(E_c - E_{qfn})/kT} = n_i e^{-(E_{qfn} - E_{fi})/kT}$ ← intrinsic Fermi level

Similarly: $p = C_p e^{-(E_{qfp} - E_v)/kT} = n_i e^{-(E_{fi} - E_{qfp})/kT}$

Then: $J_n = \frac{\sigma_n}{q}\frac{\partial(E_{qfn})}{\partial x}$; $J_p = \frac{\sigma_p}{q}\frac{\partial(E_{qfp})}{\partial x}$

COMMON TAYLOR SERIES EXPANSIONS:

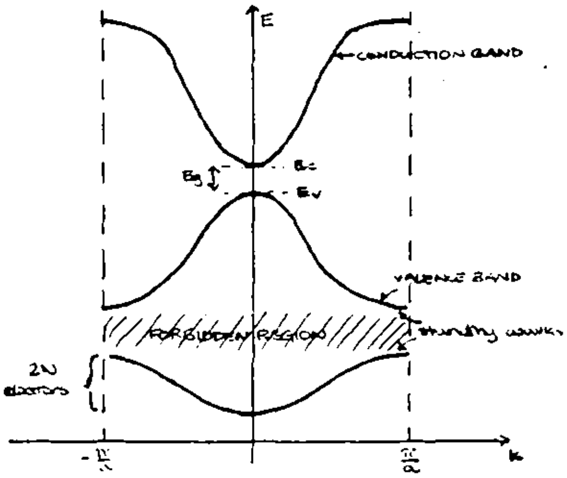
$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$; $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + \cdots$

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$; $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$

$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$; $\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$

$(1+x)^{1/2} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \cdots$; $(1+x)^{p/q} = 1 + \frac{p}{q}x + \frac{p(p-q)}{q \cdot 2q}x^2 + \cdots$

GENERIC BAND DIAGRAM FOR SEMI-CONDUCTOR:



REDUCED K-SPACE REPRESENTATION

$$J_n(x) = q\mu_n n(x)\mathscr{E}(x) + qD_n\frac{dn(x)}{dx}$$

Conduction Current: drift diffusion (4–23)

$$J_p(x) = q\mu_p p(x)\mathscr{E}(x) - qD_p\frac{dp(x)}{dx}$$

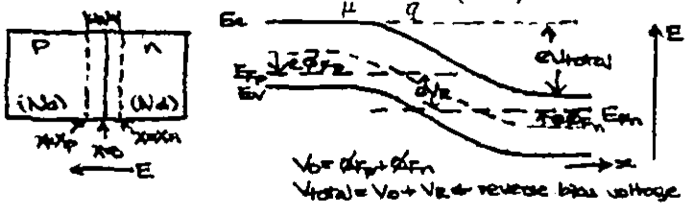
$$J_{total} = J_{conduction} + J_{displacement} = J_n + J_p + C\frac{dV}{dt}$$

Continuity: $\frac{\partial p(x,t)}{\partial t} = \frac{\partial \delta p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} - \frac{\delta p}{\tau_p}$ $\quad \frac{\partial \delta n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} - \frac{\delta n}{\tau_n}$ (4–31)

For steady state diffusion: $\frac{d^2\delta n}{dx^2} = \frac{\delta n}{D_n\tau_n} \equiv \frac{\delta n}{L_n^2}$ $\quad \frac{d^2\delta p}{dx^2} = \frac{\delta p}{L_p^2}$ (4–34)

Diffusion length: $L \equiv \sqrt{D\tau}$ Einstein relation: $\frac{D}{\mu} = \frac{kT}{q}$ (4–29)

p-n JUNCTIONS



Equilibrium: $V_0 = \frac{kT}{q}\ln\frac{p_p}{p_n} = \frac{kT}{q}\ln\frac{N_a}{n_i^2/N_d} = \frac{kT}{q}\ln\frac{N_aN_d}{n_i^2}$ (5–8)

applied forward voltage

$\frac{p_p}{p_n} = \frac{n_n}{n_p} = e^{qV_0/kT}$ (5–10) $\quad W = \left[\frac{2\epsilon(V_0 - V)}{q}\left(\frac{N_a + N_d}{N_aN_d}\right)\right]^{1/2}$ (5–57)

One-sided abrupt p^+-n: $x_{n0} = \frac{WN_a}{N_a + N_d} \simeq W$ (5–23) $\quad V_0 = \frac{qN_dW^2}{2\epsilon}$

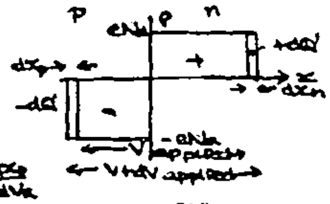
$\Delta p_n = p(x_{n0}) - p_n = p_n(e^{qV/kT} - 1)$ (5–29) where $p_n = \frac{n_i^2}{N_d}$; $n_p = \frac{n_i^2}{N_a}$

$\delta p(x_n) = \Delta p_n e^{-x_n/L_p} = p_n(e^{qV/kT} - 1)e^{-x_n/L_p}$ (5–31b) $\quad I = I_p + I_n$; $I_0 = I_s$

Ideal diode: $I = qA\left(\frac{D_p}{L_p}p_n + \frac{D_n}{L_n}n_p\right)(e^{qV/kT} - 1) = I_0(e^{qV/kT} - 1)$ (5–36)

Non-ideal: $I = I_0'(e^{qV/\mathbf{n}kT} - 1)$ ($\mathbf{n}$ = 1 to 2) (5–74)

With light: $I_{op} = qAg_{op}(L_p + L_n + W)$ (8–1)

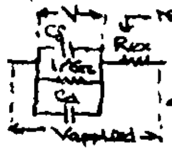


Capacitance: $C = \left|\frac{dQ}{dV}\right|$ (5–55)

$= eN_a \frac{dx_{p0}}{dV_R} = eN_d \frac{dx_{n0}}{dV_R}$

Junction Depletion: $C_j = \epsilon A \left[\frac{q}{2\epsilon(V_0 - V)} \frac{N_d N_a}{N_d + N_a}\right]^{1/2} = \frac{\epsilon A}{W}$ (5–62)

Stored charge exp. hole dist.: $Q_p = qA\int_0^\infty \delta p(x_n)dx_n = qA\Delta p_n \int_0^\infty e^{-x_n/L_p}dx_n = qAL_p\Delta p_n$ (5–39)



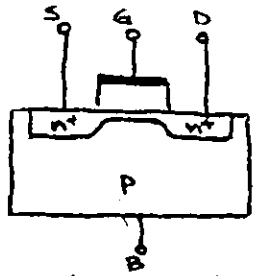
$I_p(x_n = 0) = \frac{Q_p}{\tau_p} = qA\frac{L_p}{\tau_p}\Delta p_n = qA\frac{D_p}{L_p}p_n(e^{qV/kT} - 1)$ (5–40)

small signal response

$G_s = \frac{dI}{dV} = \frac{qAL_p p_n}{\tau_p}\frac{d}{dV}(e^{qV/kT}) = \frac{q}{kT}I$ (5–67c)

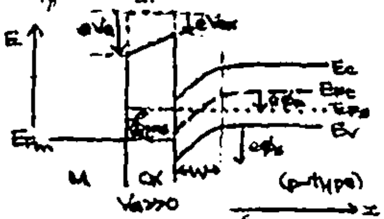
diffusion capacitance (small-signal)

$C_d = \left(\frac{1}{2V_T}\right)\left(I_p(x_n=0)\tau_p + I_n(x_p=0)\tau_n\right)$



Long p⁺-n: $i(t) = \frac{Q_p(t)}{\tau_p} + \frac{dQ_p(t)}{dt}$ (5–47)

MOS-n CHANNEL



Oxide: $C_i = \frac{\epsilon_i}{d}$ Depletion: $C_d = \frac{\epsilon_s}{W}$ MOS: $C = \frac{C_i C_d}{C_i + C_d}$ (6–36)

$\epsilon_i = \epsilon_r\epsilon_0$ (oxide)
$\epsilon_s = \epsilon_r\epsilon_0$ (S/C)

Effective positive charge at interface btw ox. and S/C
Depletion charge in the S/C

Threshold: $V_T = \underbrace{\Phi_{ms} - \frac{Q_i}{C_i}}_{\text{Flat band}} - \frac{Q_d}{C_i} + 2\phi_F$ (6–38)

Inversion: $\phi_s(\text{inv.}) = 2\phi_F = 2\frac{kT}{q}\ln\frac{N_a}{n_i}$ (6–15) $W = \left[\frac{2\epsilon_s\phi_s}{qN_a}\right]^{1/2}$ (6–30)

$Q_d = -qN_aW_m = -2(\epsilon_s qN_a\phi_F)^{1/2}$ (6–32) At V_{FB}: $C_{FB} = \frac{C_i C_{debye}}{C_i + C_{debye}}$

Debye screening length: $L_D = \sqrt{\frac{\epsilon_s kT}{q^2 p_0}}$ (6–25) $C_{debye} = \frac{\sqrt{2}\,\epsilon_s}{L_D}$ (6–40)

Substrate bias: $\Delta V_T = \frac{\sqrt{2\epsilon_s qN_a}}{C_i}(-V_B)^{1/2}$ (n channel) (6–63)

MOSFET OPERATION for $V_{GS} \geq V_{t}$, $V_{DS} \leq V_{GS} - V_{t}$ (n-type)

$$I_D = \frac{\bar{\mu}_n Z C_i}{L}\left\{\left(V_G - V_{FB} - 2\phi_F - \tfrac{1}{2}V_D\right)V_D - \frac{2}{3}\,\frac{\sqrt{2\epsilon_s q N_a}}{C_i}\left[(V_D + 2\phi_F)^{3/2} - (2\phi_F)^{3/2}\right]\right\} \quad (6\text{–}50)$$

$$I_D \simeq \frac{\bar{\mu}_n Z C_i}{L}\left[(V_G - V_T)V_D - \tfrac{1}{2}V_D^2\right] \quad (6\text{–}49)$$

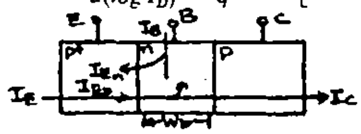
Saturation: $I_D(\text{sat.}) \simeq \frac{1}{2}\bar{\mu}_n C_i \frac{Z}{L}(V_G - V_T)^2 = \frac{Z}{2L}\bar{\mu}_n C_i V_D^2(\text{sat.})$ (6–53) $V_{DS} \geq V_{GS} - V_{t}$ (n-type)

$$g_m = \frac{\partial I_D}{\partial V_G} \;;\; g_m(\text{sat.}) = \frac{\partial I_D(\text{sat.})}{\partial V_G} \simeq \frac{Z}{L}\bar{\mu}_n C_i (V_G - V_T) \quad (6\text{–}54)$$

For short L: $I_D \simeq Z C_i (V_G - V_T) v_s$ (6–60)

Subthreshold slope: $S = \frac{dV_G}{d(\log I_D)} = \frac{kT}{q}\ln 10\left[1 + \frac{C_d + C_{it}}{C_i}\right]$ (6–66)

BJT-p-n-p



$$I_{Ep} = qA\frac{D_p}{L_p}\left(\Delta p_E \operatorname{ctnh}\frac{W_b}{L_p} - \Delta p_C \operatorname{csch}\frac{W_b}{L_p}\right) \quad (7\text{–}18) \qquad \begin{aligned}\Delta p_E &= p_n(e^{qV_{EB}/kT} - 1)\\ \Delta p_C &= p_n(e^{qV_{CB}/kT} - 1)\end{aligned} \quad (7\text{–}8)$$

$$I_C = qA\frac{D_p}{L_p}\left(\Delta p_E \operatorname{csch}\frac{W_b}{L_p} - \Delta p_C \operatorname{ctnh}\frac{W_b}{L_p}\right)$$

$$I_B = qA\frac{D_p}{L_p}\left[(\Delta p_E + \Delta p_C)\tanh\frac{W_b}{2L_p}\right] \quad (7\text{–}19)$$

$$B = \frac{I_C}{I_{Ep}} = \frac{\operatorname{csch} W_b/L_p}{\operatorname{ctnh} W_b/L_p} = \operatorname{sech}\frac{W_b}{L_p} \simeq 1 - \left(\frac{W_b^2}{2L_p^2}\right) \quad (7\text{–}26)$$

(Base transport factor)

In n-region, In p-region

$$\gamma = \frac{I_{Ep}}{I_{En} + I_{Ep}} = \left[1 + \frac{L_p^n n_n \mu_n^p}{L_n^p p_p \mu_p^n}\tanh\frac{W_b}{L_p^n}\right]^{-1} \simeq \left[1 + \frac{W_b n_n \mu_n^p}{L_n^p p_p \mu_p^n}\right]^{-1} \quad (7\text{–}25)$$

of e^-, of holes

(Emitter injection efficiency)

$\frac{i_C}{i_E} = B\gamma \equiv \alpha$ (7–3)

(Common base gain)

$\frac{i_C}{i_B} = \frac{B\gamma}{1 - B\gamma} = \frac{\alpha}{1 - \alpha} \equiv \beta$ (7–6)

(Common emitter gain)

$\frac{i_C}{i_B} = \beta = \frac{\tau_p}{\tau_t}$ (7–7)

(For $\gamma = 1$)