University of Toronto
FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATIONS, DECEMBER 2000

First Year - Programs 1,2,3,4,6,7,8,9

MAT 188H1F
Linear Algebra

Examiners

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SURNAME ______________________

GIVEN NAME ______________________

STUDENT NO. ______________________

SIGNATURE ______________________

INSTRUCTIONS:

Non-programmable calculators permitted.

Answer all questions.

Present your solutions in the space provided; use the back of the **preceding** page if more space is required.

TOTAL MARKS: 100

The value for each question is shown in parentheses after the question number.

MARKER'S REPORT	
Q1	
Q2	
Q3	
Q4	
Q5	
Q6	
Q7	
Q8	
TOTAL	

1. [15 marks: 5 marks for each part] Find the following:

(a) parametric equations of the line of intersection of the two planes with equations $x + y - z = 6$ and $3x - y + 3z = 4$.

(b) $\det \begin{pmatrix} 1 & 1 & 2 & -5 \\ -1 & 0 & 1 & 0 \\ 5 & -1 & 0 & -2 \\ 0 & 1 & -1 & 4 \end{pmatrix}$

(c) the adjoint of $\begin{pmatrix} 1 & 2 & 4 \\ 0 & 1 & -1 \\ 1 & 0 & 2 \end{pmatrix}$

2.(a) [5 marks] Find the LU decomposition of the matrix $A = \begin{pmatrix} 1 & 2 & 7 \\ 1 & 5 & -1 \\ 1 & 0 & 5 \end{pmatrix}$. (Do not use any row interchanges.)

2.(b) [5 marks] Let A and B be 3×3 matrices such that $\det A = -2$ and $\det B = 4$. Find the value of $\det\left(B^2 A^T B^{-3} A^2\right)$.

3. [10 marks] Let $\mathbf{S}$ be the subspace of $\mathbf{R}^4$ consisting of all vectors of the form $(a-c, a-b, c, 2a+b+c)$, where a, b, and c are in $\mathbf{R}$. Find an *orthogonal* basis of $\mathbf{S}$, relative to the usual dot product in $\mathbf{R}^4$.

4. [20 marks: 2 marks for each part] Indicate whether each of the following statements is true (T) or false (F), and give a brief justification for your choice:

(a) If E and F are any 3×3 elementary matrices, then $EF = FE$.

(b) If 3 is an eigenvalue of the square matrix A, then 27 is an eigenvalue of the matrix A^3.

(c) If the 6×6 matrix B is obtained from the 6×6 matrix A by replacing the third column of A with the sum of the second and fourth columns of A, then $\det B = \det A$.

(d) If λ is an eigenvalue of the $n \times n$ matrix A, then $\lambda^2 + 1$ is an eigenvalue of $A^2 + I$, where I is the $n \times n$ identity matrix.

(e) The set of all $n \times n$ symmetric matrices is a subspace of $\mathbf{M}^{n,n}$.

(f) Suppose $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_m\}$ is an orthogonal basis of $\mathbf{R}^m$, with respect to the usual dot product, and A is the $m \times m$ matrix with $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_m$ as its columns. Then the rows of $(14A)^T$ form an orthogonal basis of $\mathbf{R}^m$.

(g) If A and B are two 2×3 matrices such that $A\mathbf{x} = \mathbf{0}$ and $B\mathbf{x} = \mathbf{0}$ have the same solution spaces, then $A = B$.

(h) $(3, 2, -4)$ is the coordinate vector of $-3 + x - 4x^2$, relative to the ordered basis $\{x + 1, x - 1, 1 + x + x^2\}$ of P_2.

(i) The value of y in the solution of the system of equations

$$\begin{array}{rcrcrcr} 2x & + & 4y & + & z & = & 6 \\ x & - & y & + & 3z & = & 3 \\ -x & + & y & - & 4z & = & -3 \end{array}$$

is $y = 0$.

(j) $\dim P_n = n + 1$

5. [10 marks] For which values of k does the following system of equations

$$\begin{array}{rcrcrcl} 2x & + & ky & - & z & = & 2 \\ & & y & + & kz & = & 2 \\ kx & + & y & & & = & 2 \end{array}$$

have

(a) no solutions?

(b) a unique solution?

(c) infinitely many solutions?

6. [15 marks] Let $A = \begin{pmatrix} 2 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 3 & 0 \end{pmatrix}$. Find an invertible matrix P and a diagonal matrix D such that $D = P^{-1}AP$.

7. [10 marks] For this question let the inner product on $\mathbf{R}^3$ be defined by

$$\langle \mathbf{u}, \mathbf{v} \rangle = 2u_1v_1 + 4u_2v_2 + u_3v_3.$$

Let $\mathbf{S}$ be the subset of vectors in $\mathbf{R}^3$ which are orthogonal to $(1, -1, 2)$, with respect to the above inner product.

(a) Show that $\mathbf{S}$ is a subspace of $\mathbf{R}^3$.

(b) Find an *orthonormal* basis of $\mathbf{S}$.

8. [10 marks] Let $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$.

(a) Show that for any value of θ the matrix A is orthogonal.

(b) Show that for any vector $\mathbf{v}$ in $\mathbf{R}^2$, $||A\mathbf{v}|| = ||\mathbf{v}||$, with respect to the usual dot product of $\mathbf{R}^2$.

(c) Let P be any 2×2 orthogonal matrix with determinant equal to 1. Show that $P = A$, for some value of θ.