Name__________ ______________________Student Number______________

UNIVERSITY OF TORONTO
Faculty of Applied Science and Engineering

FINAL EXAMINATION, April 14 1999

First Year – Program 7,9

ECE115S – Electricity and Magnetism

Exam Type: A

Examiners – M.L.G. Joy
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Closed book.

Only the following calculators will be allowed: Casio 991; Sharp 520; Texas Instruments 30.

Answer the questions in the spaces provided or on the facing page.

All questions have equal weight.

For numerical answers specify units.

1	2	3	4	5	6	TOTAL

$e = 1.6 \times 10^{-19}\,[C]$ $\quad$ $m_e = 9.11 \times 10^{-31}\,[kg]$

$\mu_0 = 4\pi \times 10^{-7}\left[\frac{T\cdot m}{A}\right]$ $\quad$ $\epsilon_0 = 8.85 \times 10^{-12}\left[\frac{C^2}{N\cdot m^2}\right]$

$\int \frac{x\,dx}{(x^2+a^2)^{3/2}} = \frac{1}{(x^2+a^2)^{1/2}}$ $\quad$ $\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2\,(x^2+a^2)^{1/2}}$

$\int \frac{dx}{\sqrt{x^2+a^2}} = ln\left(x + \sqrt{x^2+a^2}\right)$ $\quad$ $F = \frac{1}{4\pi\epsilon_0}\frac{|q_1||q_2|}{r^2}$

$\tau_E = \mathbf{p} \times \mathbf{E}$ $\quad$ $E = \frac{1}{2\pi\epsilon_0}\frac{p}{z^3}\,(dipole)$

$E = \frac{1}{4\pi\epsilon_0}\frac{qz}{(z^2+R^2)^{3/2}}\,(ring)$ $\quad$ $E = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{z}{\sqrt{z^2-R^2}}\right)\,(disk)$

$\epsilon_0 \oint \mathbf{E}\cdot d\mathbf{A} = q_{enc}$(free space) $\quad$ $E = \frac{\sigma}{\epsilon_0}$ (conducting surface)

$E = \frac{\sigma}{2\epsilon_0}$ (insulating surface) $\quad$ $\Delta V = V_f - V_i = -\frac{W}{q} = -\int_i^f \mathbf{E}\cdot ds$

$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$ $\quad$ $E_x = -\frac{\partial V}{\partial x},\quad E_y = -\frac{\partial V}{\partial y},\quad E_z = -\frac{\partial V}{\partial z}$

$q = CV$ $\quad$ $C = \frac{\epsilon_0 A}{d}$ (plates)

$C = 2\pi\epsilon_0\,L\,/\,ln\,(b/a)$ (cylinder) $\quad$ $C = 4\pi\epsilon_0\,ab\,/\,(b-a)$ (spherical capacitor)

$C = 4\pi\epsilon_0 R$ (sphere) $\quad$ $\epsilon_0 \oint K\mathbf{E}\cdot d\mathbf{A} = q_{enc}$(dielectric)

$I = \frac{dq}{dt}$ $\quad$ $R = \frac{V}{I}$

$\mathbf{E} = \rho\mathbf{J}$ $\quad$ $P = VI$

$EMF = \frac{dW}{dq}$ $\quad$ $W = \frac{1}{2}Vq$

$\tau_B = \boldsymbol{\mu} \times \mathbf{B}$ $\quad$ $\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$

$\mathbf{F}_B = I\,\mathbf{L} \times \mathbf{B}$ $\quad$ $d\mathbf{B} = \frac{\mu_0}{4\pi} I d\mathbf{s} \times \mathbf{r}\,/\,r^3$

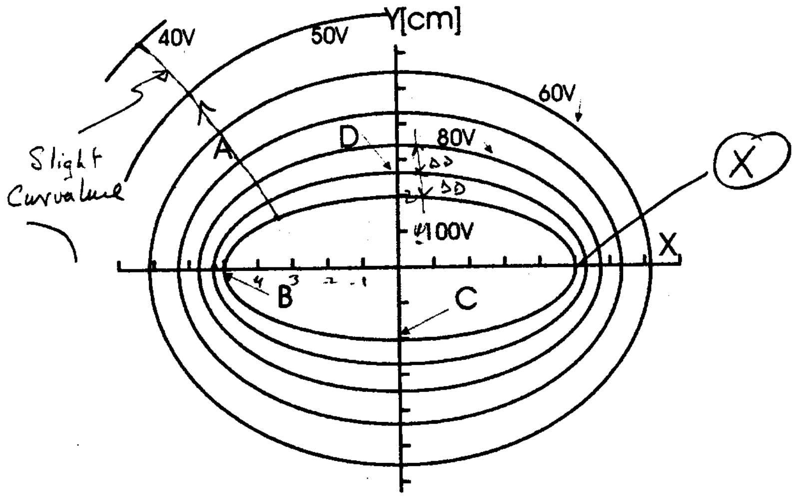
$B = \mu_0 I\,\Phi\,/\,4\pi R$ (arc) $\quad$ $B = \mu_0 I\,n$ (solenoid)

$B = \mu_0 I\,/\,2\pi R$ (wire) $\quad$ $\oint B\cdot ds = \mu_0 I_{enc}$

$EMF = \oint \mathbf{E}\cdot ds = -\frac{d\Phi}{dt}$ $\quad$ $\Phi_B = \int \mathbf{B}\cdot d\mathbf{A}$

Marking scheme.

1. This question concerns a charged "pill shaped" conductive piece of metal isolated from its surroundings. The grey ellipse in the diagram below is a section through the "pill" in the XY plane. A similar section in the XZ plane would be circular. This shape is often referred to as an "ellipsoid of revolution" as it is the gray ellipse rotated about the Y axis. As you can see from scale on the diagram, the major axis of the ellipse is $100mm$ long and the minor axis $40mm$ long. The XY scale is in cm.



The electric potential of the "pill" is $100V$ relative to infinity. The figure shows a plot of the equipotential surfaces surrounding the "pill". The potentials of these surfaces are labeled on the diagram. The potential difference between adjacent surfaces is $10V$.

[6] (a) Draw an electric field line through Point A in the diagram. Continue this line until it goes off the diagram or ends on the "pill". Justify any curvature in the line. Indicate the direction of the field line with an arrow.

(2) Curvature + intersection at $\perp$

(2) Direction outward

(2) Stops at metal surface.

$$-\int \vec{E}\cdot d\vec{s} = \Delta V$$

[6] (b) Estimate the following.

OR. $|E| \cdot \Delta \text{Distance} = \Delta \text{Voltage}.$

1. The electric field, $\mathbf{\vec{E}}$, at point D, $(0, 2.67, 0)$.

(2) $|\vec{E}| \simeq \frac{\Delta V}{\Delta D} = \frac{10}{2.67-2} = \frac{10}{.67} = 15 \text{ V/cm} = 1500 \text{ V/m}$ (2)

(2) Direction: "in +ve Y direction, upwards." $\uparrow$

[6] 2. The charge density (σ_B, σ_C) on the surface of the ~~bar~~ pill near point B, $(-5, 0, 0)$.and C, $(0, -2, 0)$. State the sign of this charge density.

(2) $\sigma = \epsilon_0 E$

(2) $E_C \simeq E_D \simeq 1500 \text{ V/m}$ $\therefore |\sigma_C| = 8.85\times10^{-12} \times 1500 = 1.3\times10^{-8}$

(2) σ_C positive.

$E_B \simeq 2 * E_D$ $\therefore \sigma_B \simeq 2\times\sigma_C$

$E_B = \frac{10}{5.31-5.0} = \frac{10}{.31} = 32 \text{ V/cm}$ Actual

[2] (c) Where is the magnitude of the electric field greatest? Justify your answer.

(1) At surface because 1) equipotential surfaces close together
2) curvature of metal is smallest.

(1) At all points where metal intersects x z plane. especially (X) + A

2. Charge is distributed uniformly in the XY plane with a density of $+8.85 \times 10^{-12} C/m^2$.

[4] (a) Find the Electric Field, $\vec{\mathbf{E}}$ at the following points (distances in metre):

1. $\vec{\mathbf{E}}(1,1,1) = (0 , 0 , \frac{1}{2})$ V/m
2. $\vec{\mathbf{E}}(1,1,-1) = (0 , 0 , -\frac{1}{2})$ V/m
3. $\vec{\mathbf{E}}(-1,1,1) = (0 , 0 , \frac{1}{2})$ V/m

Equal. (2)

$|\vec{E}| = \frac{1}{2}\frac{\Sigma}{\epsilon_0} = \frac{1}{2}\frac{8.85\times10^{-12}}{8.85\times10^{-12}}$

$= 0.5$ V/m or N/C.

[6] (b) What is the electric potential difference between the following *pairs* of points? Which point of each pair is at the higher potential? (Distances are in metre.)

1. Point A $(0,0,-1)$ and point B $(0,0,1)$?

(2) $\Delta V = -\int_i^f \vec{E}\cdot d\vec{s} = V_f - V_i$

(2) $V_{BA} = V_{OA} + V_{BO} = 0.5 - 0.5 = \underline{\text{Zero}}$

Both A + B Same Potential.

2. Point C $(0,3,5)$ and point B $(0,0,1)$?

(2) $V_{CB} = -0.5\times(5-1) = -2$ V

Point B is at the higher Potential

[4] (c) Consider a cubical ($2m \times 2m \times 2m$) Gaussian surface centered on the origin and with every side perpendicular to an axis. What is the electric flux:

1. Out of this Gaussian surface.

(1) Flux $= \oint \vec{E}\cdot d\vec{a} = \frac{Q_{encl}}{\epsilon_0}$ (1) $\quad Q_{Encl.} = 8.85\times10^{-12}\times 2^2 = 35.4\times10^{-12}$ C.

(1) $\therefore$ Flux $= \frac{2^2\times 8.85\times10^{-12}}{8.85\times10^{-12}} = 4\left[\frac{Vm^2}{m}\right] = 4[Vm] = 4\left[\frac{Nm^2}{C}\right]$

2. Out of the face of the cube which intersects the positive Z axis.

By symmetry (1) $\frac{1}{2}$ Flux $= 2$ Vm

(d) Suppose that a point charge of $+1 \times 10^{-9}C$ is placed at point $(0,1,1)$.

Possible misinterp that no sheet charge Now.

[4] 1. Compute the Electric Field, $\vec{\mathbf{E}}$, at point $(0,0,1)$.

$\vec{\mathbf{E}} = (0 , -9 , \frac{1}{2})$ V/m.

superpos 1
magnitude 1
Axis 1
direction 1

$\vec{E} = (0,0,\frac{1}{2}) + \vec{E}_p$ (1) $= (0,0,\frac{1}{2}) + (0,-9,0)$ -superposition

$= (0,-9,\frac{1}{2})$ V/m. (1) (1) sign correct

field point. E_p

$|\vec{E}_p| = \frac{9\times10^9 \times 1\times10^{-9}}{1}$

(1) $= 9$ V/m

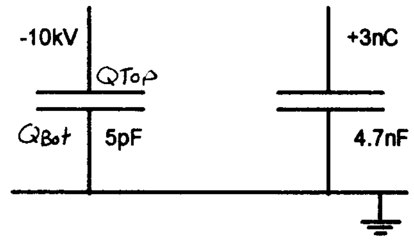
[2] 2. Compute the force, $\vec{\mathbf{F}}$, on the point charge.

(2) $\vec{\mathbf{F}} = (0 , 0 , \frac{1}{2}) \times 1\times10^{-9} = E\cdot q$

$= (0, 0, 0.5\times10^{-9})$ N. (1)

No self force.

3. Two capacitors each have one lead connected to ground and the other lead unconnected. One capacitor has a capacitance of $5 \times 10^{-12} F$ and its unconnected lead is at a potential of $-10 \times 10^3 V$ with respect to ground. The other capacitor has a capacitance of $4.7 \times 10^{-9} F$ and has had a charge of $3 \times 10^{-9} C$ placed on its unconnected lead.



{6}

(a) What are the charges on the two plates of the $5pF$ capacitor?

2 mag $\quad Q_{top} = - 5pF \times 10kV = -50 \times 10^{-9} C$

2 sign $\quad Q_{Bot} = - Q_{top} = 50 \times 10^{-9} C. = 5 \times 10^{-8} C$ //

2 Top Bottom correct.

{2}

(b) What is the potential, V, of the disconnected lead of the $4.7nF$ capacitor?

2 mag + sign c $\quad V = 3nC / 4.7nF = +0.638 V$

(c) The disconnected leads are now connected to each other using a wire with a resistance of $1 \times 10^6 \Omega$ and the charges on the two capacitors are allowed to stabilize. Assuming no charge escapes:

{4}

1. Estimate roughly how long it will take for the charges to stabilize.

① Time constant $\tau = RC. \rightarrow$ 3 TCs. ①

② $R = 10^6 \quad C = C_{eq} =$ 5pF series 4.7nF $\simeq 5pF$

$\therefore RC \doteq 10^6 \times 5 \times 10^{-12} = 5 \times 10^{-6} s. \quad \therefore 15 \mu s$ //

{4}

2. Compute the final electric potential, V_F, (relative to ground) of the previously disconnected lead of the $5pF$ capacitor.

$V = Q_{sum} / C_{eq}$ $\quad$ ② $Q_{sum} = -50 \times 10^{-9} + 3 \times 10^{-9} =$

$= -47 \times 10^{-9} / 4.7 \times 10^{-9}$ $\quad = -47 \times 10^{-9}$

$\simeq -10 V$ $\quad$ ② $C_{eq} = 5 \times 10^{-12} + 4.7 \times 10^{-9} \simeq 4.7 \times 10^{-9}$

{2}

3. Compute the final energy, E_S, stored in the combined capacitors.

2 $\quad E_S = \frac{1}{2} V q = \frac{1}{2}(-10)(-47 \times 10^{-9}) = 2.35 \times 10^{-7} J$

{2}

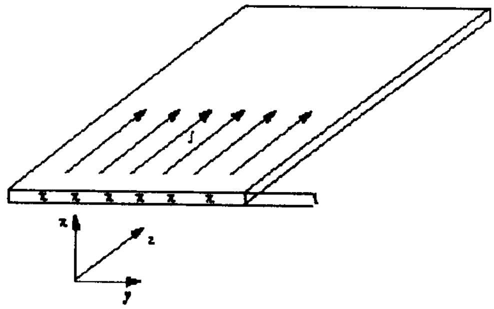
4. Compute the energy, E_D, dissipated as heat in the resistive ($1M\Omega$) wire.

Method: E_D = Initial Energy stored $- E_S$

Initial $= \frac{1}{2} 5 \times 10^{-12} \times (10^3)^2 + \frac{1}{2} \frac{(3 \times 10^{-9})^2}{4.7 \times 10^{-9}} \simeq 2.5 \times 10^{-4} J$

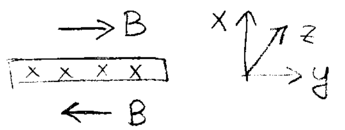
$\therefore E_D \simeq 2.5 \times 10^{-4} J$

4. An **infinite** conducting plate carries a uniform current density $\vec{\mathbf{J}} = 3\hat{\mathbf{z}} \quad A/cm^2$. The plate is assumed to be thin with thickness $t = 2mm$.



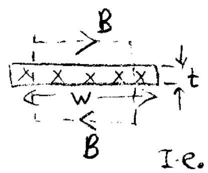
(a) Assume that the magnetic field produced would be Y-directed only and constant. What is the orientation of the magnetic field (i.e. positive or negative in Y) in the space above and below the plate?

From the right-hand rule :



(b) Using Ampere's law, calculate the magnitude of the magnetic field in the space above and below the plate.

Consider a box shaped Amperian loop :



$\oint \bar{B} \cdot d\bar{s} = \mu_o i_{enc} \Rightarrow 2BW = \mu_o i_{enc}$.

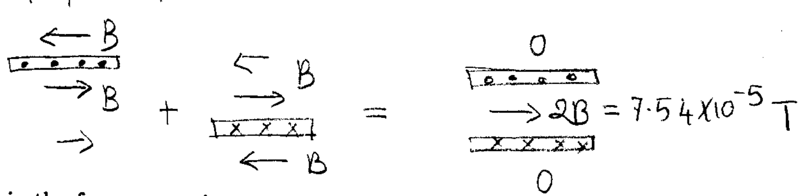
But $i_{enc} = J(Wt)$ therefore $2BW = \mu_o (J W t)$

I.e. $B = \frac{\mu_o}{2}(Jt) = 3.77 \times 10^{-5}$ T

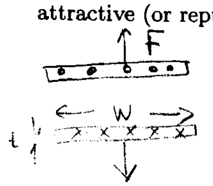
(c) Assume now that a second similar plate but with opposite current density is placed in parallel to the original plate at a distance $d = 5cm$.

1. Compute the magnetic field between and outside the plates.

Use superposition :



2. What is the force per unit area on the second metallic plate? Is this an attractive (or repulsive) force?



Repulsive force $\bar{F} = i\bar{L} \times \bar{B} = iLB$

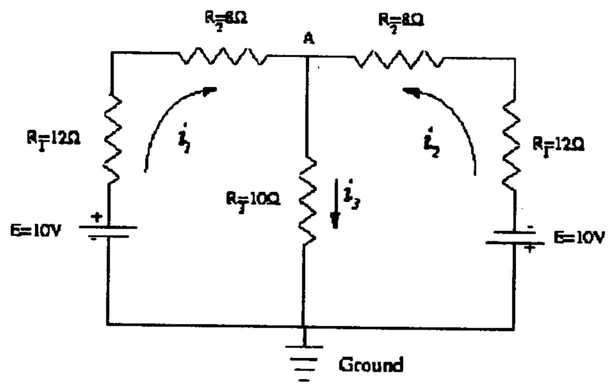
Note that B is only the field produced by the bottom plate.

$F = iLB = (JWt)(L)(B) \Rightarrow$

force-per-area $= F/WL = JBt = \frac{3}{(0.01)^2} Bt = 0.023\ N/m^2$

Hint: Use arguments of symmetry.

5. Consider the circuit below.



(a) By inspecting the given circuit or otherwise, decide which one of the following is the correct value for the potential at point A. Justify your answer.

1. 1.5V
2. 0V
3. $-2V$

Due to the existing antisymmetry
$V_A = 0.$

(b) Using the result of part (a) or otherwise, calculate currents i_1, i_2.

From KVL in the left loop

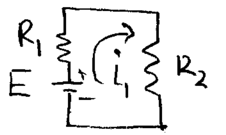
$$-E + i_1 R_1 + i_1 R_2 = 0 \Rightarrow$$

$$i_1 = \frac{E}{R_1+R_2} = \frac{10}{20} = 0.5\,A$$

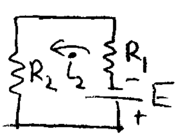
(c) Compute current i_3.

$$V_A = i_3 R_3 = 0 \Rightarrow i_3 = 0.$$

(d) Decompose the given circuit into two independant single loop circuits.



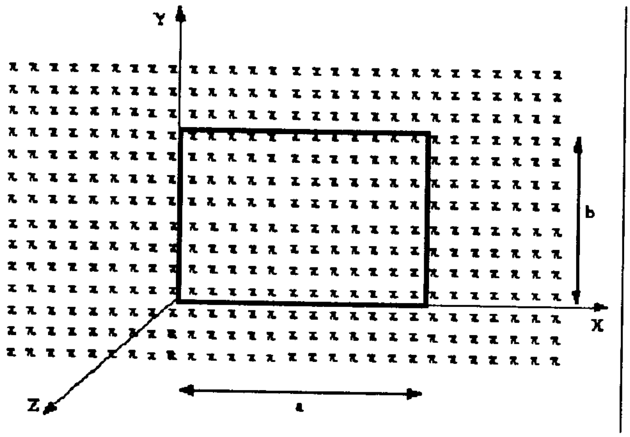
and



(e) Calculate the power that each one of the two generators delivers to the circuit.

$$P = E\,i = (10V)(0.5A) = 5W$$

6. A conducting rectangular loop of dimensions $a \times b = 30cm \times 20cm$ is immersed in a uniform but time varying magnetic field, $\vec{\mathbf{B}} = -2t\hat{\mathbf{z}}\ T$ as shown in the diagram below. The wire making up the loop has a cylindrical cross section of radius, $c = 0.5mm$ and resistivity $\rho = 2 \times 10^{-5}\Omega m$.



(a) Determine the direction of the induced current in the loop. Observe that B is increasing with time. From Lenz's law an opposing magnetic field will be generated if the induced current is counterclockwise

(b) Determine the induced electromotive force in the loop.

$$\mathcal{E} = \frac{d\Phi_B}{dt} = \frac{d}{dt}\int \bar{B}\cdot d\bar{a} = (ab)\frac{dB}{dt} = 0.3 \times 0.2 \times 2 = 0.12\,V$$

(c) Calculate the induced current. What is the value for the induced current density?

$$i = \frac{\mathcal{E}}{R} \quad \text{where} \quad R = 2\left(\frac{\rho a}{A} + \frac{\rho b}{A}\right) = \frac{2\rho}{A}(a+b)$$

I.e. $R = \frac{2(2\times10^{-5})}{\pi(0.0005)^2}(0.3+0.2) \doteq 25\,\Omega$. Hence $i = \frac{0.12V}{25\Omega} = 4.8\,mA$

Also $J = i/A = \frac{4.8mA}{\pi(c)^2} = 6.11 \times 10^4\ A/m^2$

(d) Calculate the magnitude of the electric field in the loop.

Method #1: Ohm's law $J = \frac{E}{\rho} \Rightarrow E = J\rho = 0.12\ V/m$

Method #2: Faraday's law, $\frac{d}{dt}\int \bar{B}\cdot d\bar{A} = \oint \bar{E}\cdot d\bar{s} = \mathcal{E} \Rightarrow$

This assumes that the electric field in the wire is constant. $E\,2(a+b) = \mathcal{E} \Rightarrow E = \frac{\mathcal{E}}{2(a+b)} = \frac{0.12V}{2(0.3+0.2)} = 0.12\,V/m$

(e) How much heat is it dissipated in the loop in 1 second?

$$P = i^2 R = (0.0048)^2(25) = 1.73\,W$$

I.e. In 1 second, 1.73 J of heat is dissipated