UNIVERSITY OF TORONTO

FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION, FRIDAY DECEMBER 15, 2000

Third Year – Engineering Science (Biomedical, Computer, Electrical)
Fourth Year – Engineering Science (Manufacturing)

ECE355F - System and Signal Analysis I

Exam Type: A

Examiner: D. Kundur

- No aids allowed (including calculators).
- Three important tables from the text book have been provided on the last pages of the exam.
- If a particular question seems unclear, please explicitly state any reasonable assumptions and proceed with the problem.
- "Sketch" questions require that all axes and significant points on the graph be labeled.
- All questions are of equal weight.

1. **(10 marks)**

(a) True or False: the series interconnection of two linear time-invariant systems is also linear time-invariant.

(b) Consider the series interconnection of three subsystems shown in Figure 1.

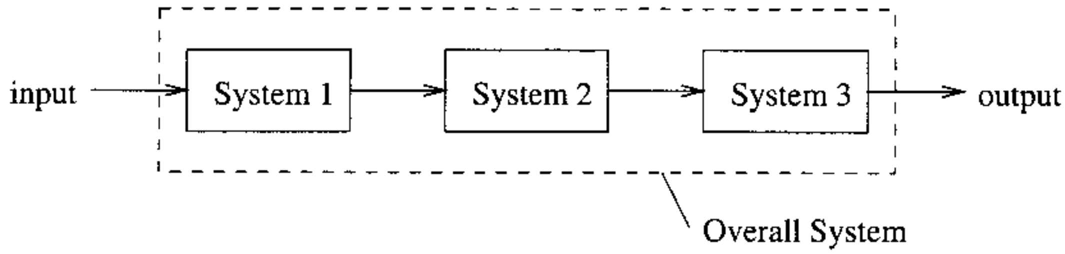


Figure 1: Series interconnection of three subsystems.

The input-output relationships for each subsystem are:

$$\begin{aligned}
\text{System 1} &: \quad y[n] = \begin{cases} x[n/2] & \text{for } n \text{ even} \\ 0 & \text{for } n \text{ odd} \end{cases} \\
\text{System 2} &: \quad y[n] = x[n] + \frac{1}{2}x[n-1] + \frac{1}{4}x[n-2] \\
\text{System 3} &: \quad y[n] = x[2n]
\end{aligned}$$

What is the input-output relationship for the overall interconnected system? Show your analysis.

(c) True or False: the series interconnection of two time variant systems is necessarily time variant. Justify your answer (i.e., if your answer is true, then provide reasoning, if it is false, provide a counter-example for the statement.)

(d) True or False: the series interconnection of two systems which are not linear is necessarily not linear. Justify your answer (i.e., if your answer is true, then provide reasoning, if it is false, provide a counter-example for the statement.)

2. **(10 marks)**

The step response $s[n]$ of a discrete-time system is defined as the output of the given system when the input is the unit step function $u[n]$. The scenario is shown in Figure 2.

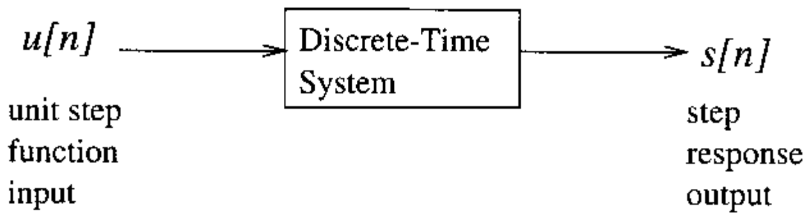


Figure 2: The unit step response of a system.

Assume that the discrete-time system is LTI.

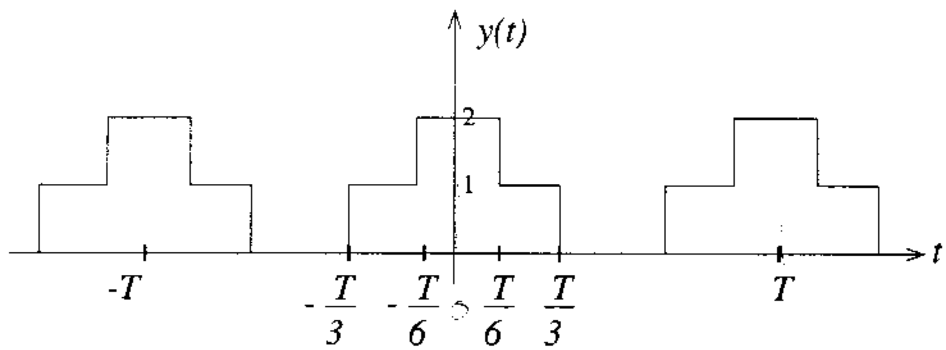
(a) Find $s[n]$ in terms of the system impulse response $h[n]$.

(b) Determine $h[n]$ in terms of $s[n]$. *Hint:* $\delta[n] = u[n] - u[n-1]$ *where* $\delta[n]$ *is the discrete-time unit impulse function.*

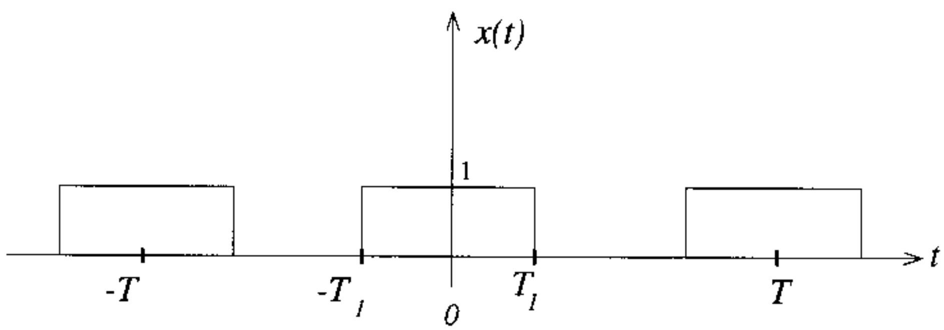
(c) Compute the unit step response of the system when $h[n] = 3^n u[-n]$.

3. **(10 marks)**

(a) Find the Fourier Series coefficients b_k of the following periodic signal:



Hint: The Fourier Series coefficients of the following periodic function



are

$$a_k = \frac{\sin(k\omega_0 T_1)}{k\pi} \tag{1}$$

where $\omega_0 = \frac{2\pi}{T}$.

(b) Determine the Fourier Transform of $y(t)$ in part (a) of this question.

4. **(10 marks)**

Consider a LTI system with frequency response

$$H(j\omega) = \frac{j\omega + 4}{6 - \omega^2 + 5j\omega}$$

(a) Determine the differential equation relating the input $x(t)$ and the output $y(t)$ of the overall system. What are the auxiliary conditions of the LTI system?

(b) Compute the impulse response $h(t)$ of the system.

(c) Let the input to the system be $x(t) = e^{-4t}u(t) - te^{-4t}u(t)$. Find the spectrum of $x(t)$.

(d) If the input to the system is as given in part (c), then determine the corresponding output $y(t)$ of the LTI system.

5. **(10 marks)**

A real signal $x(t)$ with Fourier Transform $X(j\omega)$ undergoes impulse-train sampling to generate

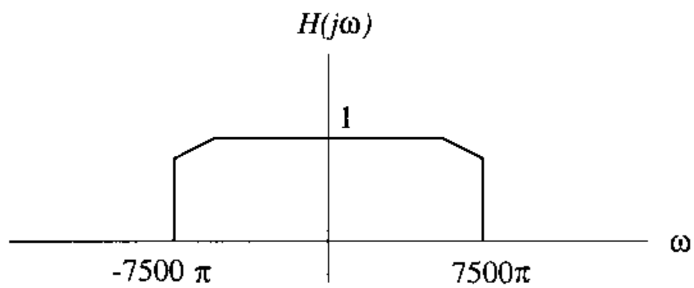
$$x_p(t) = \sum_{n=-\infty}^{\infty} x(nT)\delta(t - nT)$$

where $T = 10^{-4}$. For each of the following sets of constraints on $x(t)$ or $X(j\omega)$, does the sampling theorem guarantee that $x(t)$ can be recovered exactly from $x_p(t)$.

(a) $X(j\omega) = 0$ for $|\omega| > 5000\pi$

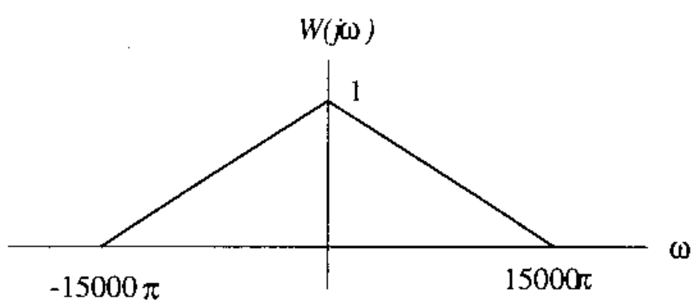
(b) $X(j\omega) = 0$ for $|\omega| > 15000\pi$

(c) $x(t)$ is the output of a filter with frequency response



(d) $x(t)$ is the output of a filter with frequency response $H(j\omega) = \left(\frac{7}{10}\right)^{-\omega^2}$

(e) The signal $w(t) = x^2(t)$ has frequency spectrum $W(j\omega)$



6. **(10 marks)**

Consider a real signal $x(t)$ with Fourier Transform $X(j\omega)$ which undergoes impulse-train sampling as follows

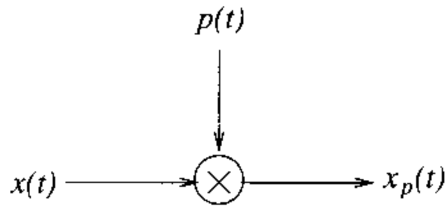


Figure 3: Impulse-train sampling.

where $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$.

The spectrum of $x(t)$ is given by

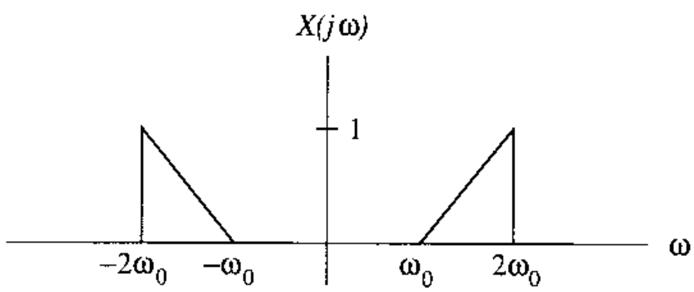


Figure 4: Spectrum $x(t)$.

Let $T = \frac{\pi}{\omega_0}$.

(a) Sketch the spectrum of $x_p(t)$. Label all axes. Are we sampling above the Nyquist rate?

(b) In a standard recovery system, we filter $x_p(t)$ using a lowpass filter to produce $x_r(t)$. Will a lowpass filter be able to recover $x(t)$ from its samples (Yes or No)?

(c) To recover $x(t)$ from $x_p(t)$, we can use the scenario shown in Figure 5 in which $H(j\omega)$ is the frequency response of any arbitrary filter. Sketch the frequency response of the filter $H(j\omega)$ which will provide perfect recovery.

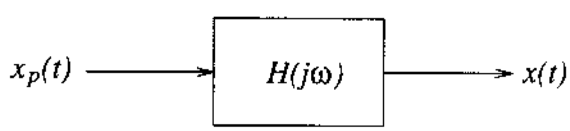


Figure 5: Proposed recovery system that produces $x(t)$ from $x_p(t)$.

7. **(10 marks)**

Consider three information bearing signals $x_a(t)$, $x_b(t)$, and $x_c(t)$ with spectra $X_a(j\omega)$, $X_b(j\omega)$, and $X_c(j\omega)$, respectively, shown in Figure 6.

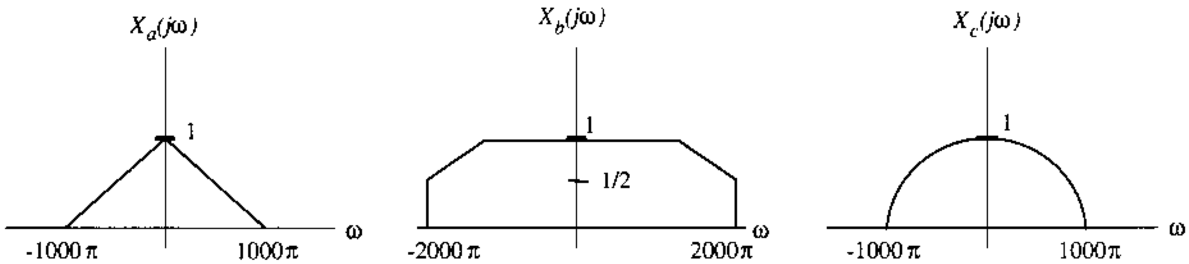


Figure 6: Frequency spectra of $x_a(t)$, $x_b(t)$, and $x_c(t)$.

We would like to design a frequency division multiplexing (FDM) scheme whereby $x_a(t)$ is modulated with carrier frequency ω_a, $x_b(t)$ is modulated with carrier frequency ω_b, and $x_c(t)$ is modulated with carrier frequency ω_c.

The following FDM system details are specified:

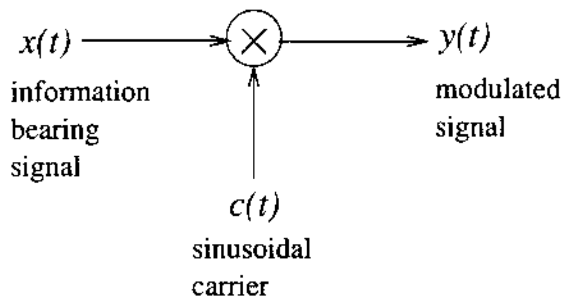
- $\omega_a < \omega_b < \omega_c$
- $\omega_a = 4000\pi$
- lower single sideband amplitude modulation is used in the transmission of each individual information bearing signal.

(a) What are the minimum values of ω_b and ω_c, such that ideal and accurate demultiplexing and demodulation is possible for any of the information bearing signals.

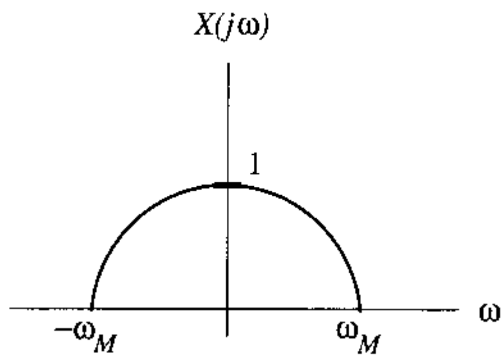
(b) Sketch the spectrum of the FDM system output.

8. **(10 marks)**

Consider the following modulation system:



where $c(t) = \sin(\omega_c t)$, and the spectrum of $x(t)$ is given by



such that $\omega_M \ll \omega_c$.

(a) Sketch the real and imaginary components (on separate plots) of $Y(j\omega)$, the spectrum of $y(t)$. Show all steps in obtaining $Y(j\omega)$ and provide reasoning where necessary.

(b) Is $y(t)$ even, odd or neither? Explain.

(c) Design a synchronous demodulation scheme which uses a multiplier and a filter with frequency response $H(j\omega)$ to recover $x(t)$ exactly. Completely specify all signals in the system as well as $H(j\omega)$.

Sketch the frequency response of the signals right after the multiplier and the filter. Again, draw the real and imaginary parts separately.

(d) Is the recovered signal after demodulation even, odd or neither? Explain.