# UNIVERSITY OF TORONTO FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION, APRIL 22 1996

Third Year

## ECE357S - ELECTROMAGNETIC FIELDS

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# AIDS PERMITTED:

Double-sided aid sheet and non-programmable calculator







# Electromagnetic Waves

#### Problem 1 (15 points)

A uniform plane wave having the electric field

$$E_i = (\frac{\sqrt{3}}{2}\hat{x} - \frac{1}{2}\hat{z})E_o\cos(6\pi 10^9 t - 10\pi(x + z\sqrt{3})) + aE_o\hat{y}\sin(6\pi 10^9 t - 10\pi(x + z\sqrt{3}))$$

is incident on the interface between free-space and a dielectric medium of  $\varepsilon = 6 \varepsilon_0$  and  $\mu = \mu_0$  Find the value of a for the reflected wave to be right circularly polarized.

#### Problem 2 (15 points)

Consider a distributed current source flowing on an infinite sheet lying in the x-y plane

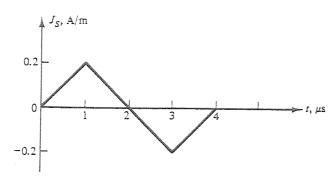
$$J_{S}\big|_{z=0} = -J_{S}(t)\,\hat{x}$$

This current source produces uniform plane waves traveling in the regions z > 0 and z < 0, where

$$E(z,t) = F(t \mp \frac{z}{v_p}) \hat{x} , z \ge 0$$

$$H(z,t) = \pm \frac{1}{\eta_o} F(t \mp \frac{z}{v_p}) \hat{y} , z \ge 0$$

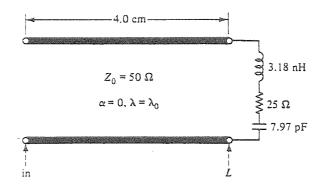
- (i) Using the appropriate boundary conditions at the surface z = 0, show that  $F(t) = \eta_0/2 J_s(t)$ .
- (ii) The medium on either side of the current sheet is now a perfect dielectric of  $\varepsilon = 9\varepsilon_0$  and  $\mu = \mu_0$ . The amplitude of the surface current density  $J_s$  (A/m) is shown below. Find and sketch
  - (a)  $E_x$  versus t for z = 100 m,
  - (b)  $H_y$  versus z for  $t = 5 \mu s$



#### Transmission Lines

#### Problem 3 (15 points)

- (1) For purely reactive networks, the slope of the reactance or susceptance versus frequency curve is always positive. This places a restriction on the possible shape of impedance and admittance versus frequency curves. When plotted on a Smith chart, do the Z and Y curves have a clockwise or counterclockwise trend with increasing frequency? Explain why.
- (2) A frequency sensitive load impedance is shown below.

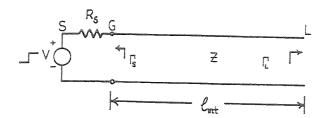


- (i) For the given values of L and C, what is the series resonance frequency  $f_0$ ?
- (ii) Calculate and plot the values of  $Z_{in}$  and  $Z_{L}$  at  $f_{o}$ , and  $f_{o} \stackrel{*}{=} f_{o}/4$  on the Smith chart.
- (iii) Calculate the angular spread of  $Z_{in}$  and  $Z_{L}$  over the given frequency range. Explain the effect of the 4 cm length of lossless line on the shape of the impedance plot.

## Problem 4 (15 points)

When sending high-speed digital signals down transmission lines, it is important to avoid ringing which causes the receiver to switch on and off a few times before the waveform settles. This ringing can be disastrous in a clock or asynchronous line because the glitches can be observed as a transition and can cause the circuit to transit to a wrong state.

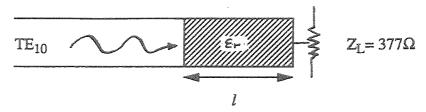
Consider an open-circuited line driven by a source resistance  $R_s$ . Plot  $V_S$ ,  $V_G$  and  $V_L$  as a function of time for  $R_s$ = 10 Z, Z and 0.1 Z. Deduce what condition should be imposed on the source resistance with respect to the characteristic impedance of the line to avoid ringing.



## Waveguides and Antennas

#### Problem 5 (15 points)

A waveguide load with an equivalent  $TE_{10}$  wave impedance of 377 $\Omega$  must be matched to an air-filled X-band rectangular guide (a = 2.286 cm) at 10 GHz. A quarter-wave matching transformer is to be used, and is to consist of a section of guide filled with dielectric. Find the required dielectric constant  $\varepsilon_r$  and physical length l of the matching section.



#### Problem 6 (10 points)

The pattern multiplication technique can be used in reverse to synthesize an array for a specified radiation pattern. Find an arrangement of isotropic elements for the group pattern

$$\frac{\cos^2(6\pi\cos\phi)}{9\cos^2(2\pi\cos\phi)}$$

## Problem 7 (15 points)

A uniform linear array consists of six short dipoles. The spacing between adjacent elements is  $\mathcal{N}4$ , as shown below.

- (i) What should the phase shift  $\xi$  be, in order to point the maximum radiation in the  $\phi = 90^{\circ}$  (i.e. y) direction?
- (ii) Suppose that the E field is due to the first element (the dipole at far left) is given as follows

$$E_{\theta e} = \frac{1000}{r} e^{-jkr} \sin \theta$$

Calculate  $|E_{\theta}|$  of the entire array at point A (0, 1000, 0), point B (1000, 0, 0), point C (0, -1000, 0), and point D (-1000, 0, 0), separately. All positions are given in rectangular coordinates in meters. Use the phase shift found in (i).

