UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATIONS, APRIL 2001
CIV 263 S – PROBABILITY THEORY FOR CIVIL ENGINEERS
EXAMINER: FELIX J. RECIO

INSTRUCTIONS:

1. ATTEMPT ALL QUESTIONS.
2. SHOW AND EXPLAIN YOUR WORK IN ALL QUESTIONS.
3. GIVE YOUR ANSWERS IN THE SPACE PROVIDED. USE BOTH SIDES OF PAPER, IF NECESSARY.
4. DO NOT TEAR OUT ANY PAGES.
5. USE OF NON-PROGRAMMABLE POCKET CALCULATORS, BUT NO OTHER AIDS ARE PERMITTED.
6. THIS EXAM CONSISTS OF SIX QUESTIONS. THE VALUE OF EACH QUESTION IS INDICATED (IN BRACKETS) BY THE QUESTION NUMBER.
7. THIS EXAM IS WORTH 50% OF YOUR FINAL GRADE.
8. TIME ALLOWED: 2 ½ HOURS.
9. PLEASE WRITE YOUR NAME, YOUR STUDENT NUMBER, AND YOUR SIGNATURE IN THE SPACE PROVIDED AT THE BOTTOM OF THIS PAGE.

PLEASE DO NOT WRITE HERE

QUESTION NUMBER	QUESTION VALUE	GRADE
1	30	
2	30	
3	30	
4	30	
5	30	
6	30	
TOTAL:	180	

NAME: ______________________ , ______________________

(FAMILY NAME. PLEASE PRINT.) (GIVEN NAME.)

STUDENT No.: ______________ SIGNATURE: ______________________

1. The edge roughness of slit paper products increases as knife blades wear. Only 1 % of products slit with new blades have rough edges, 2 % of products slit with blades of average sharpness exhibit roughness, and 5 % of products slit with worn blades exhibit roughness. We known that 30 % of the blades used in the manufacturing process are new, 60 % are of average sharpness, and 10 % are worn. Suppose that one of these slit paper products is randomly selected. (Round all your answers to four decimal places).
   a) (1 mark) What is the probability that the blade used to slit the product was not a blade of average sharpness?
   b) (1 mark) What is the probability that the product exhibits roughness if it was slit with a blade of average sharpness?
   c) (3 marks) What is the probability that the product exhibits roughness and was slit with a blade of average sharpness?
   d) (5 marks) What is the probability that the product exhibits roughness?
   e) (5 marks) What is the probability that the product exhibits roughness or was slit with a blade of average sharpness?
   f) (5 marks) What is the probability that the product was slit with a blade of average sharpness if it exhibits roughness?
   g) (5 marks) Let E denote the event "the product was slit with a blade of average sharpness", and let F denote the event "the product exhibits roughness". Are these two events E and F dependent or independent? Why?
   h) (5 marks) What is the probability that the product does not exhibit roughness if the blade used to slit the product was not a blade of average sharpness?

ANSWERS:

a)	b)	c)	d)
e)	f)	g)	h)

2. The following table represents the joint probability mass function of the discrete random variables X and Y.
a) (1 mark) Compute $P\{X=1, Y=3\}$.
b) (2 marks) Compute $P\{Y=1\}$.
c) (2 marks) Compute $P\{X \geq Y\}$.
d) (5 marks) Find the probability mass function $p(y)$ of the random variable Y.
e) (5 marks) Find the cumulative distribution function $F(x)$ of the random variable X.
f) (5 marks) Compute $E[Y]$.
g) (5 marks) Compute $\mathrm{Var}(X)$.
h) (5 marks) Compute $\mathrm{Cov}(X, Y)$.

		Y: 0	1	2	3
	1	0.10	0.05	0	0.15
X	2	0	0.30	0.25	0.05
	4	0.05	0	0.05	0

ANSWERS:

a) $P\{X=1, Y=3\}=$	b) $P\{Y=1\}=$	c) $P\{X \geq Y\}=$
d) $p(y)=\begin{cases} & \text{if } y=0 \\ & \text{if } y=1 \\ & \text{if } y=2 \\ & \text{if } y=3 \\ & \text{otherwise} \end{cases}$	e) $F(x)=\begin{cases} & \text{if } x<1 \\ & \text{if } 1 \leq x<2 \\ & \text{if } 2 \leq x<4 \\ & \text{if } 4 \leq x \end{cases}$	
f) $E[Y]=$	g) $\mathrm{Var}(X)=$	h) $\mathrm{Cov}(X, Y)=$

3. Round all your answers in each of the following questions to four decimal places.
   a) (5 marks) A shipment of 50 used automobiles contains 20 that are equipped with airbags. If 15 of these cars are randomly assigned to a rental agency, what is the probability that exactly 5 of the assigned cars will be equipped with airbags?
   b) (5 marks) A particular system in a space vehicle must work properly for the spaceship to gain a smooth re-entry into the earth's atmosphere. The system consists of 12 independent components, and it is estimated that each component operates successfully 95 % of the time. To increase the reliability of the system, it has been designed in a way that the system will work properly if at least 10 of the components operate successfully. What is the probability that the system will fail?
   c) (5 marks) Let X be a Poisson random variable such that $P\{X=0\}=0.15$. Compute $P\{X=3\}$.
   d) (5 marks) Let U be a continuous random variable, uniformly distributed over the interval $[-4, 6]$. Find the value of a for which $P\{U^2 > a\} = 0.55$.
   e) (10 marks) Let Y_1, Y_2, and Y_3, be independent, exponential random variables with the common parameter $\lambda = 0.5$, and let $Y = \max(Y_1, Y_2, Y_3)$. Compute $P\{4 \leq Y \leq 6\}$.

ANSWERS:

a)	b)	c) $P\{X=3\}=$	d) $a=$	e) $P\{4 \leq Y \leq 6\}=$

4. Round all your answers in each of the following questions to four decimal places.
   a) (5 marks) A machine makes electrical resistors having a mean resistance of 15.4 ohms and a standard deviation of 0.45 ohms. Assuming normality, what percentage of these resistors will have a resistance between 15 and 16 ohms ?
   b) (5 marks) The average grade for an exam is 71.32 and the variance is 19.58. If the distribution of marks is about normal and only 10 % of the class is to receive an A grade, what is the lowest score required in the exam in order to obtain an A ?
   c) (5 marks) If a 1-litre can of a certain type of paint covers on the average 25 square metre with a standard deviation of 6.5 square metre, what is the probability that a random sample consisting of 70 of these 1-litre cans will be enough to paint an area of 1700 square metre ?
   d) (5 marks) Suppose that a random sample of size n is taken from a normal population with standard deviation $\sigma = 37$. Let $\overline{X}$ denote the sample mean, and let μ denote the population mean. What is the minimum value of n for which $P\{\ |\overline{X} - \mu| > 5\ \} < 0.0750$?
   e) The probability that a person survives a rare blood disease is only 0.43. Medical records indicate that 66 people living in different regions of a large country have contracted the disease.
   e1) (5 marks) What is the probability that exactly 30 of these people will survive ?
   e2) (5 marks) What is the probability that at least 30 of these people will survive ?

ANSWERS:

a)	b)	c)	d)	e1)
				e2)

5. a) A certain type of light bulbs have a length of life that is approximately normally distributed. A random sample of 20 of these light bulbs showed an average life of 785 hours.

   a1) (5 marks) Suppose that it is assumed that the population standard deviation is $\sigma = 52$ hours. Find a 95 % confidence interval for μ, the true mean length of life of that type of light bulbs.

   a2) Suppose now that σ is unknown but the sample standard deviation is found to be $S = 65$ hours.

   a21) (5 marks) Find a 95 % one-sided upper confidence interval for μ.

   a22) (5 marks) Find a 95 % confidence interval for σ.

b) (5 marks) In a random sample of 500 homes in a certain city, it is found that 115 are heated by oil. Find a 96 % one-sided lower confidence interval for p, the true proportion of homes that are heated by oil in that city.

c) (10 marks) An experiment was conducted in which two new types of engines, A and B, were compared. Gas consumption in kilometres per litre was measured. Six experiments were conducted using engine type A and eight using type B. The gasoline used and other conditions were held constant. The following data describes the results obtained in each of the fourteen trials:

Engine A:	12.5	13.4	12.8	12.5	11.9	13.2		
Engine B:	12.6	12.8	13.0	11.4	12.5	11.5	13.2	12.4

Assuming that the populations are normal, with unknown but equal variances, find a 90 % confidence interval for $\mu_A - \mu_B$, the true difference between the mean gas consumptions for engines A and B.

ANSWERS:

a1)	
a21)	a22)
b)	c)

6\. a) (10 marks) Consider the function $f(x)=\begin{cases} a^{-1}x & \text{if } 0 \le x \le 1 \\ a x^{-1/2} & \text{if } 4 \le x \le 9 \\ 0 & \text{otherwise} \end{cases}$. Find the values of the constant a, if any, for which $f(x)$ is the density function of some random variable X.

b) (10 marks) Let $f(x,y)=\begin{cases} 2x^{-2}y^{-3}+3x^{-3}y^{-2} & \text{if } x \ge 1 \text{ and } y \ge 2 \\ 0 & \text{otherwise} \end{cases}$ be the joint density function of the random variables X and Y. Compute $P\{X \ge Y\}$.

c) (10 marks) Let $f(x)=\begin{cases} \theta e^{-\theta^2 x}(\pi x)^{-1/2} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$ (where θ is an unknown positive parameter) be the density function of the random variable X. Suppose that the values $X_1 = 3.8$, $X_2 = 4.1$, $X_3 = 4.3$, $X_4 = 3.7$, and $X_5 = 4.1$ are obtained when a random sample is taken from the distribution of X. Use the maximum likelihood method to estimate the value of the parameter θ.

ANSWERS:

a) $a =$	b) $P\{X \ge Y\} =$	c) $\hat{\theta} =$