University of Toronto
FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATIONS, APRIL 2001
Year 1 - Programs 1, 2, 3, 4, 6, 7, 8, 9

MAT 186H1S
CALCULUS I
Examiner: D. Burbulla

INSTRUCTIONS

Non-programmable calculator permitted; no other aids allowed.

Present your solutions to all of the following questions in the exam booklets supplied. The marks for each question are indicated in parantheses.

TOTAL MARKS: 100.

1. [15 marks; each part is worth 5 marks] Find the following:

   (a) $\lim_{x \to 0} \dfrac{1 - x^2 - \cos x}{x^2}$

   (b) $\lim_{x \to 0^+} \left(\dfrac{1}{\sqrt{x}} + \ln x \right)$

   (c) $\lim_{x \to \infty} \left(e^{2x} + x \right)^{1/x}$

2. [15 marks; each part is worth 5 marks] Find the following:

   (a) the average value of $f(x) = x^3 + x$ on the interval $1 \leq x \leq 3$

   (b) the area of the region between the graphs of $f(x) = x^3$ and $g(x) = x$ for $0 \leq x \leq 2$

   (c) $F'(-1)$ if $F(x) = \displaystyle\int_x^{x^2} \frac{1}{\sqrt{1 + t^2}}\, dt$.

3. [15 marks; 5 marks for each part] Find the following:

   (a) $\int \frac{x^2+3x+7}{x+1}\,dx.$

   (b) $\int x^3\sqrt{1-x^2}\,dx.$

   (c) $\int \frac{1}{1+\sin x}\,dx.$

4. [15 marks] Let $f(x) = x^3 + x - 1$.

   (a) [6 marks] How many real solutions are there to the equation $f(x) = 0$? (Make sure you justify your answer.)

   (b) [9 marks] Approximate any one solution to the equation $f(x) = 0$ correct to 4 decimal places by using Newton's method.

5. [10 marks; 5 marks for each part] Consider the function $f(x) = x^2$ on the interval $0 \le x \le 1$. Find the following:

   (a) the volume of the solid obtained by revolving the function $f(x)$ around the line $y = -1$, for $0 \le x \le 1$

   (b) the area of the surface of revolution obtained by revolving the function $f(x)$ around the y-axis, for $0 \le x \le 1$

6. [10 marks] A rain gutter is to be constructed from a metal sheet of width 30 cm by bending up one-third of the sheet on each side through an angle θ. How should θ be chosen so that the gutter will carry the maximum amount of water?

7. [10 marks] Torricelli's Law states that

$$A(y)\frac{dy}{dt} = -a\sqrt{2gy},$$

   where y is the depth of a fluid in a tank at time t, $A(y)$ is the cross-sectional area of the tank at height y above the exit hole, a is the cross-sectional area of the exit hole, and $g = 9.8$ m/sec^2 is the acceleration due to gravity.

   A container in the form of an inverted right circular cone of radius 1 m and height 3 m is full of water. A circular plug of radius 1 cm is pulled open at the bottom of the container. How long will it take for the container to become completely empty of all water?

8. [10 marks] A tank filled with water of density $\rho = 1000$ kg/m^3 has the shape of a sphere of radius 3 m. Find the work done in pumping all of the water out of the tank and up to a horizontal pipe 1 m above the top of the tank. (Use acceleration due to gravity $g = 9.8$ m/sec^2.)