UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING

Q1	/25
Q2	/25
Q3	/25
Q4	/25
Total	/100

FINAL EXAMINATION

APRIL 2001
EXAM TYPE: A
First Year Programs: CIV, LME, MEC, IND, CHE, MMS

ECE 110S: ELECTRICAL FUNDAMENTALS

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NAME: ______________________________________
Last First

STUDENT NO.: ______________________________________

INSTRUCTIONS:

- This is a Type A examination; no aids are allowed.
- Only non-programmable calculators are allowed.
- Answer all parts of all four questions.
- All four questions are of equal weight.
- The weight of each of the individual parts of each question is stated in the margins.
- All work is to be done on these pages.
- Place your final answers in the provided boxes unless instructed otherwise.
- When answering the questions include all the steps of your work on these pages. For additional space, you may use the back of the preceding page.
- Do not unstaple this exam.

CONSTANTS:

$e = 1.6 \times 10^{-19}\text{C}$ $\quad\quad$ $\varepsilon_0 = 8.85 \times 10^{-12}\text{F/m}$ $\quad\quad$ $\mu_0 = 4\pi \times 10^{-7}\text{H/m}$

Question 1: Electricity and Magnetism

PART A: General

Answer the following questions.

[1] (i) Electric and magnetic phenomena stem from a property of matter known as charge. What is the distinguishing condition of charges that gives rise to a magnetic field?

[1] (ii) The electric field is a *conservative* field. Explain this property of the electric field.

[1] (iii) When we speak of electrical current, the picture that comes to mind is that of electrons drifting down a copper wire. The average velocity or the drift velocity of electrons required to support a current of 1 ampere in a copper wire of 1 mm^2 in cross-section is about (circle the correct answer)

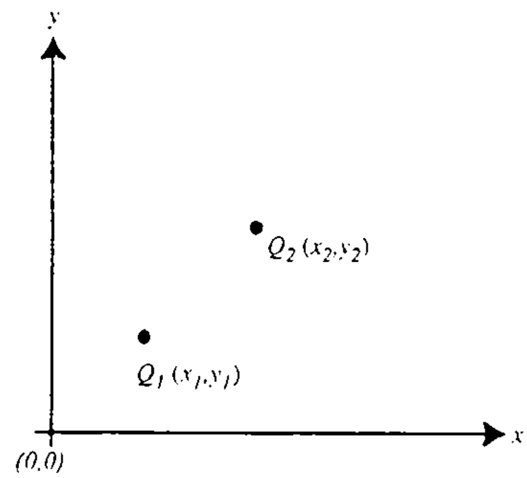
(a) 1000 km/s
(b) 3×10^8 m/s
(c) 1/3 m/h
(d) none of the above.

[1] (iv) The Lorentz force is used, for example, to steer electrons in a cathode ray tube (CRT) or a television tube. Clearly write the Lorentz force expression and define all the terms.

Question 1: Electricity and Magnetism

PART B: Coulomb's Law & Electric Field

Consider the two stationary point charges shown in the figure below. Point charge Q_1 has a charge of 50 μC and is located at $x_1 = y_1 = 0.1$ m, while point charge Q_2 has a charge of 150 μC and is located at $x_2 = y_2 = 0.3$ m



[5] (i) Determine the electric force (magnitude and direction) on charge 2 due to charge 1.

$\mathbf{F}_{21} =$

[1] (ii) Determine the electric force (magnitude and direction) on charge 1 due to charge 2.

$\mathbf{F}_{12} =$

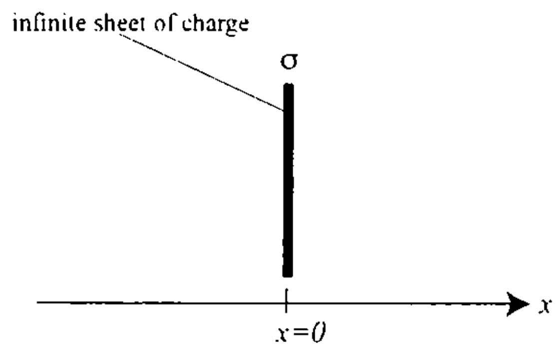
Question 1: Electricity and Magnetism

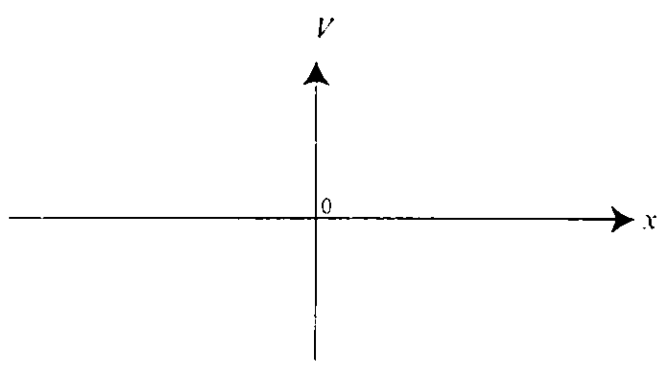
PART C: Electric Field, Potential, and Capacitance

[4] (i) The magnitude of the electric field due to an infinite sheet of charge, of charge density σ, is given by the expression $|E| = \dfrac{\sigma}{2\varepsilon_0}$. For $\sigma = 8.85\times10^{-8}\ \mathrm{C/m^2}$, compute the magnitude of the electric field and the electric potential for all x>0 and all x<0. Assume that the potential at the origin is zero. On the figures below clearly show the direction of the field lines and plot the potential for all x>0 and all x<0.

$|E| =$

$V(x) =$





[3] (ii) Find the capacitance of a parallel plate capacitor (in vacuum) having a spacing of 0.01 m and plate area of 0.01 $\mathrm{m^2}$. Assuming a potential difference of 12 V find the strength of the electric field inside the capacitor.

$C =$

$|E| =$

Question 1: Electricity and Magnetism

PART D: Electromagnetic Induction

A thin copper wire is wound into an essentially flat coil of 100 turns. The coil has a diameter of 0.05 m. The coil is placed in a magnetic field which varies with time, t[s], according to the relation $B(t) = 0.1\cos(120\pi t)$ T. The magnetic field is normal to the surface of the coil. The ends of the coil are connected with heavy gauge copper wire to a true *rms* voltmeter which has a high input impedance.

[3] (i) What is the induced EMF in the coil?

$V(t) =$

[1] (iii) What is the voltage reading displayed on the voltmeter?

$V_{rms} =$

[2] (iv) If the resistance of the coil is 1 Ω, what is the current?

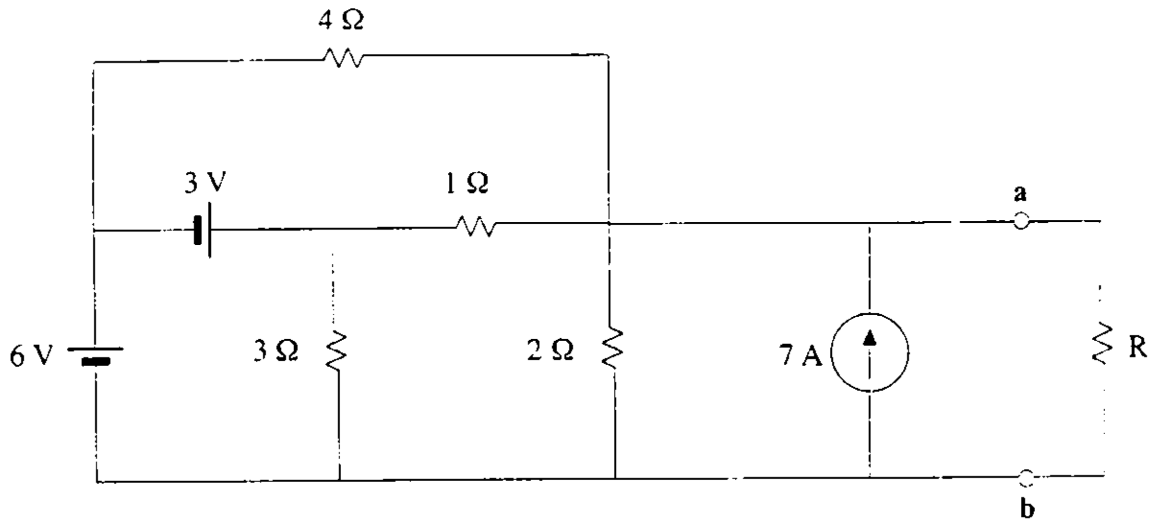
$I(t) =$

[2] (v) What is the power dissipation in the coil, $P(t)$?

$P(t) =$

Question 2: DC Circuits

Consider the circuit shown below.



[7] (i) Using nodal analysis, find the Thévenin's equivalent voltage to the left of terminals **a** and **b**.

V_{Th} =

[3] (ii) Determine the Thévenin's equivalent resistance of this circuit to the left of terminals **a** and **b**.

R_{eq} =

Question 2: DC Circuits

[2] (iii) Draw the Norton equivalent circuit to the left of terminals **a** and **b**.

Norton equivalent circuit:

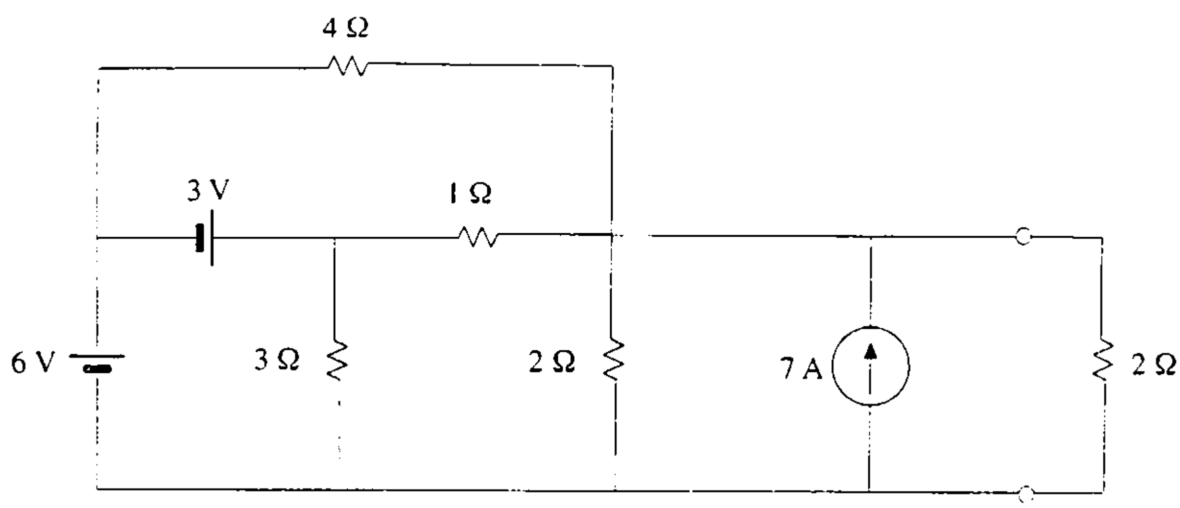
[3] (iv) What is the maximum power that can be transferred to the load, R?

P_{max} =

[3] (v) Assuming R is 2 Ω, what is the actual power delivered to the load?

P_R =

Question 2: DC Circuits



[7] (vi) Use the principle of superposition to determine the current flowing downward in the 3 Ω resistor. (The circuit has been reproduced above for your convenience.)

I_1 (due to 6 V) =

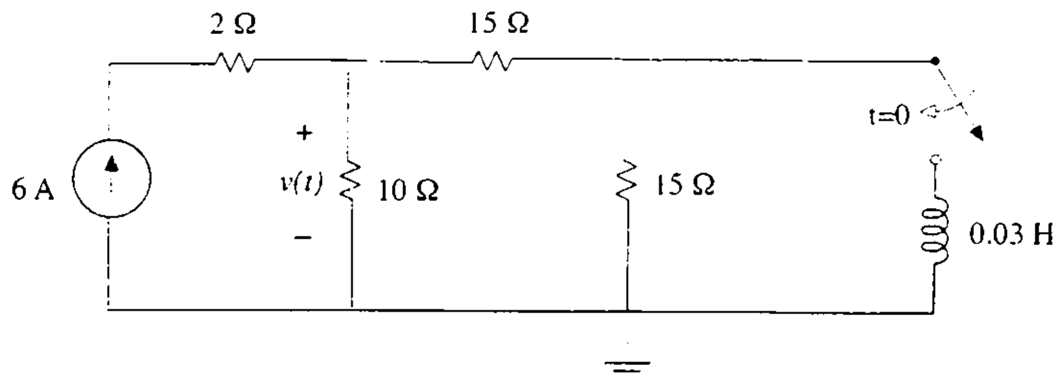
I_2 (due to 3 V) =

I_3 (due to 7 A) =

I =

Question 3: Transient Analysis

A. The switch is closed at time t=0 in the circuit shown below. Assume that the switch was open for a long time prior to time zero.



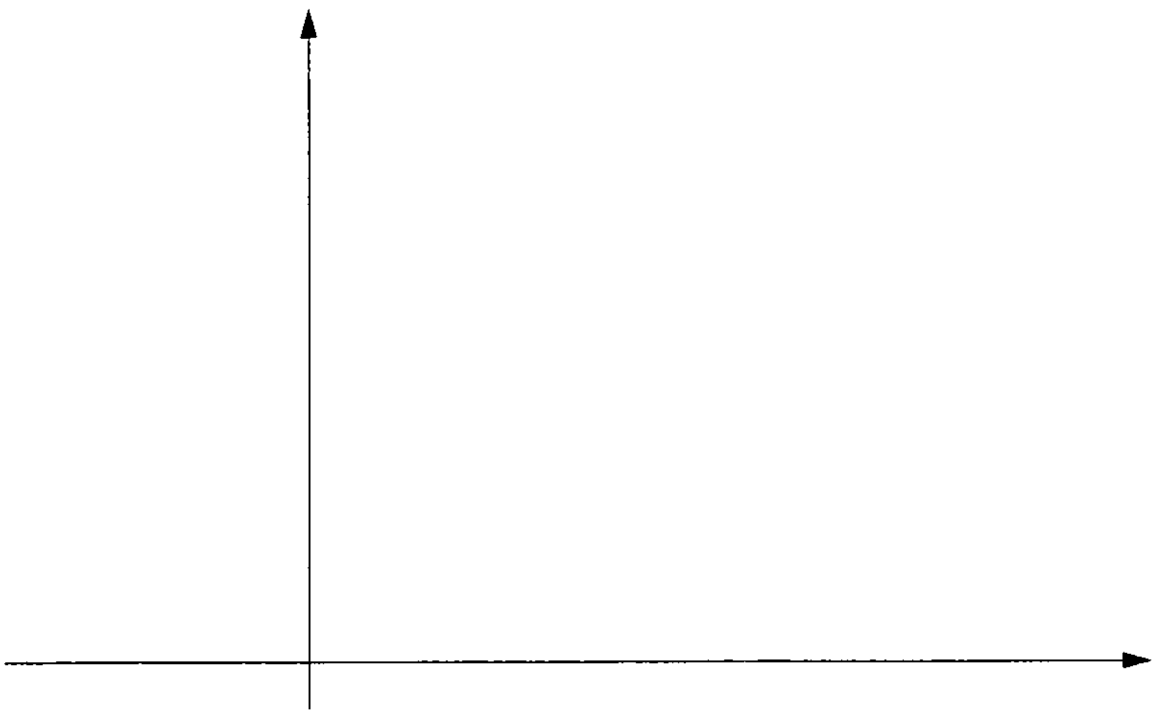
[4] (i) Find $v(t)$ for $t = 0^-$ and $t = 0^+$.

$v(t=0^-)=$
$v(t=0^+)=$

[6] (ii) Find $v(t)$ as a function of time for $t > 0$.

$v(t)$ for $t > 0$:

[5] (iii) Sketch $v(t)$ for all time; clearly label the axes.

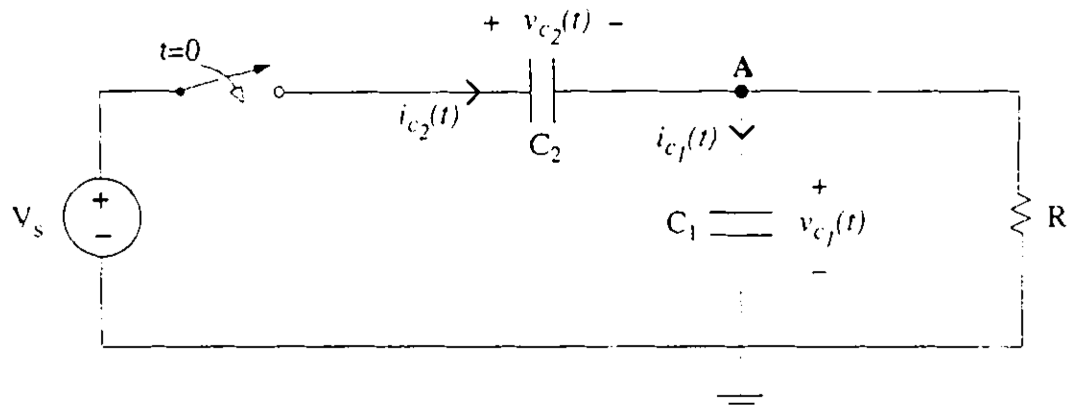


Question 3: Transient Analysis

B. For the circuit shown below, the switch is closed at time t = 0. Prior to time t = 0, you may assume that the switch has been open for a long time.

[8] (i) Derive the differential equation for $v_{c_1}(t)$ for $t>0$.

HINT: Begin by writing the node equation (i.e., applying KCL) at node **A**.



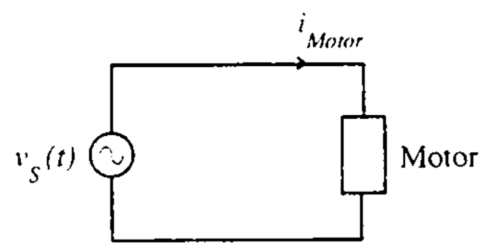
Differential equation for $v_{c_1}(t)$ for $t>0$

[2] (ii) Find the time constant for the above circuit and the value of $v_{c_1}(t)$ a long time after the switching action.

τ =
$v_{c_1}(\infty)$ =

Question 4: AC Circuits

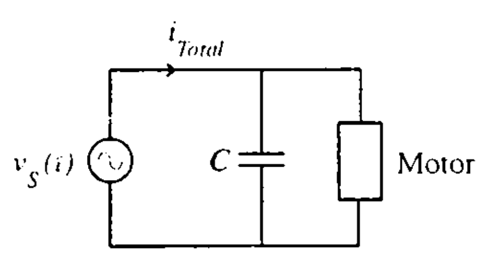
A 1000 watt electric motor is connected to a source of 120 V_{rms} at 60 Hz, which results in a lagging power factor (PF) of 0.8. (Note: lagging power factor means the current in the motor lags the voltage across the motor.)



[5] (i) Calculate the current (*rms* value) drawn from the source.

$i_{Motor(rms)} =$

[5] (ii) The PF is increased to 0.95 lagging by placing a capacitor in parallel with the motor. Calculate the new current (*rms* value) drawn from the source **with** the capacitor connected.



$i_{Total(rms)} =$

Question 4: AC Circuits

[10] (iii) Determine the value of the capacitor required to make the correction.

C =

[5] (iv) The motor can be modeled with a resistor R and an inductor L in series, determine the values of R and L.

R =

L =