UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION, DECEMBER 2001

Second Year - Program 6

CHE 312 - MASS TRANSFER

Examiner: D.C.S. Kuhn

IMPORTANT NOTES:

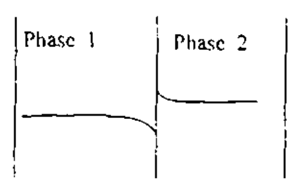
- ***Answer all questions.***
- *State all assumptions clearly.*
- *A Single 8 1/2 inch x 11 inch Aid Sheet is allowed.*
- *Assigned marks are listed following the question number.*
- *All calculators allowed; no computers allowed.*
- *Time allotted: 2 hours and 30 minutes*

Question 1.

a) *(6 marks)* Compound A enters a Continuously Stirred Tank Reactor (CSTR) of volume V at a volumetric flow rate of Q_{in} and a concentration of $c_{A,in}$ where a first order reaction occurs converting A to B with a reaction rate constant k [s^{-1}]. A constant volumetric flow rate of Q_{out} is removed from the tank. Derive an expression for the concentration of A in the tank.

b) *(10 marks)* A 12 m long by 2 m diameter roll of base sheet (a porous type of paper that is coated) is manufactured in Thunder Bay Ontario during a humid summer day. The roll has uniform moisture content of 1 wt % just after manufacture. The roll waits 1 hr before it is coated and converted into a coated writing grade paper. The base sheet must have a moisture content less than 2 wt % or the coating will not properly adhere. If the water content of the paper at the surface of the roll is maintained at 3 wt % due to the humid conditions, to what depth into the roll is the paper unusable after 1 hr? Assume edge effects are negligible and radial effects (i.e. curvature effects) are very small at the time scale considered. The diffusivity coefficient of water through paper is 3.60×10^{-6} m^2/h.

c) (*2 marks*) What direction is the mass transfer in the adjacent figure.



Question 2.

The burning rates of different coals are being evaluated in the third year spring term of the thermodynamics laboratory. Spherical coal pellet are suspended and burnt in a large high temperature (1450 K) pure oxygen reactor at 2 atm. The carbon is consumed at the surface of the pellets in the reaction $C + O_2 \rightarrow CO_2$. The reaction rate is first order, $r = -k\, c_{O2}$ where $k = 0.1$ m/s. The first coal sample to be considered has a binary diffusivity, D_{AB}, of 1.71×10^{-4} m/s at 1450 K and 1 atm. Let A represent O_2.

a) (*8 marks*) Determine the oxygen concentration variation as a function of the radius, r, from the surface of the coal, r_0, to far from the surface of the coal, r_∞. Apply the general boundary conditions $c_{Ar}(r_0) = c_{A0}$ and $c_{Ar}(r_\infty) = c_{A\infty}$.

b) (*6 marks*) Determine c_{A0}, the surface concentration of O_2, in terms of $c_{A\infty}$, k and D_{AB}.

c) (*4 marks*) What is the molar consumption rate of O_2 in kmol/s?

Question 3.

A funky new car air freshener container is a closed permeable 10 cm long cylinder with an outside diameter of 1 cm and wall thickness of 2 mm. It is to be hung from the car mirror. The air freshener fills the container and is a liquid solution initially containing 1 % (molar) scent. The container allows the scent to diffuse through the walls but not the liquid solution containing the scent.

The partition coefficient of the scent between the solution and the wall is $\beta = 5 \times 10^{-3}$ where $c_{wall} = \beta\, c_{solution}$ and the diffusivity coefficient in the permeable fibre wall is 10^{-7} cm^2/s. The diffusivity coefficient of the scent in the liquid solution is three orders of magnitude greater than the diffusivity coefficient of the scent in the permeable fibre wall. A very strong air flow passes over the outside of the air freshener container at all times. Assume the air fresheners reach pseudo-steady state almost immediately after they are removed from their packaging and hung on the mirror. The molecular weight of the solution is 50 kg/kmol and the density of the solution is 500 kg/m^3.

Use the following nomenclature:

- r - radius
- r_i - inner radius of cylinder
- r_o - outer radius of cylinder
- $C_{A,w}(r,t)$ - concentration of scent within the wall
- $C_{A,s}(t)$ – concentration of scent within the solution

a) (*10 marks*) Based on a shell balance derive an equation for the concentration variation through the permeable container wall. Assume pseudo-steady state conditions. State all assumptions and boundary conditions.

b) (*6 marks*) How does the concentration of the scent in the container change with time?

c) (*2 marks*) How many moles of scent have been transferred to the car within the first 2 days?

Question 4.

(18 marks) An old large landfill site was constructed without a bottom lining and sits above an aquifer. The landfill has dimensions 5 m deep and 60 m by 60 m wide. The landfill is filled with 10 cm diameter pellets containing benzene with a bed void fraction of 0.3. Currently the pellets have a uniform mass concentration of benzene of 10^{-5} kmol/m^3. The aquifer is 1.0 m thick with an average water velocity of 0.2 cm/s and runs parallel to the bottom of the landfill in the direction of one of landfill edges. The apparent molecular diffusivity of benzene through the water/pellet bed is 2×10^{-8} m^2/s. The surface area of pellets to landfill volume ratio is 0.4.

When it rains the benzene is leached out of the pellets. During the spring it rains an average of 0.08 mm/hr. Derive an expression for the concentration variation of the benzene in solution through the landfill assuming steady state conditions. What is the flux of benzene into the aquifer? What is the concentration of the benzene in the aquifer after the water flows past the landfill site.

Data and Mass Transfer Correlations:

Molecular weight of benzene is 78 kg/kmol.

Water (@ 293K): $\rho = 998.2$ kg/ m^3, $\mu = 9.93\times10^{-4}$ kg/(m · s), M = 18 kg/(kmol)

Mass transfer of liquids in packed beds

$$j_D = \frac{1.09}{\varepsilon}\mathrm{Re}^{-2/3} \text{ ; for } 0.0016 < \mathrm{Re} < 5,\ 165 < Sc < 70{,}600$$

$$j_D = \frac{0.250}{\varepsilon}\mathrm{Re}^{-0.31} \text{ ; for } 55 < \mathrm{Re} < 1500,\ 165 < Sc < 10{,}690$$

where $\mathrm{Re} = \dfrac{d_p u_{ave}\rho}{\mu}$ and $j_D = \dfrac{Sh}{\mathrm{Re}\, Sc^{1/3}}$

and ε is the void fraction, d_p is the diameter of the pellets, u_{ave} is the superficial mass averaged fluid velocity in the empty bed without spherical pellets, ρ is the mass density of the liquid, and μ is the dynamic viscosity of the liquid.

Error Function

Reproduced from Welty et al., Fundamentals of Momentum, Heat and Mass Transport, 4th edition, Wiley, 2000. Original source J. Crank, The Mathematics of Diffusion, Oxford Press, 1958.

1. $\dfrac{d^2\psi}{d\eta^2} + 2\eta\dfrac{d\psi}{d\eta} = 0$ with boundary conditions $\psi(\eta = 0) = 0$ and $\psi(\eta = \infty) = 1$ has solution $\psi(\eta) = erf(\eta)$.

2. $erf(\eta) = \dfrac{2}{\sqrt{\pi}}\int_0^{\eta} e^{-\phi^2}\,d\phi$

3. $\dfrac{d(erf(\eta))}{d\eta} = \dfrac{2}{\sqrt{\pi}}e^{-\eta^2}$

η	erf(η)	η	erf(η)
0	0.0	0.85	0.7707
0.025	0.0282	0.90	0.7970
0.05	0.0564	0.95	0.8209
0.10	0.1125	1.0	0.8427
0.15	0.1680	1.1	0.8802
0.20	0.2227	1.2	0.9103
0.25	0.2763	1.3	0.9340
0.30	0.3286	1.4	0.9523
0.35	0.3794	1.5	0.9661
0.40	0.4284	1.6	0.9763
0.45	0.4755	1.7	0.9838
0.50	0.5205	1.8	0.9891
0.55	0.5633	1.9	0.9928
0.60	0.6039	2.0	0.9953
0.65	0.6420	2.1	0.9981
0.70	0.6778	2.2	0.9993
0.75	0.7112	2.3	0.9998
0.80	0.7421	2.4	0.9999

Chart for solution of Unsteady Transport Problems

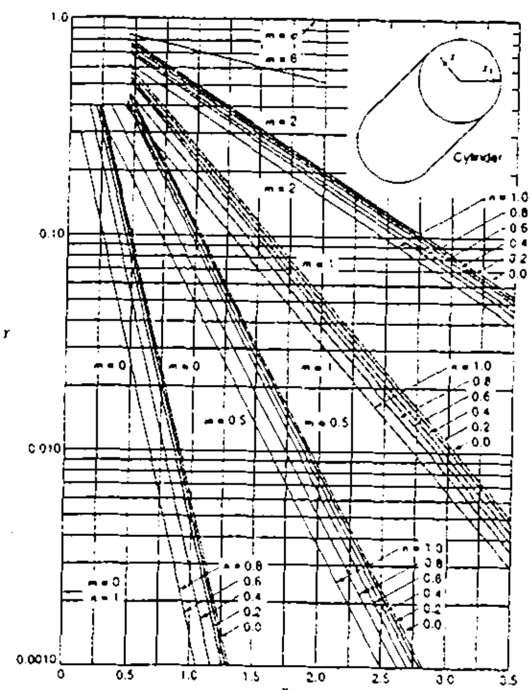
Reproduced from Welty et al., Fundamentals of Momentum, Heat and Mass Transport, 4th edition, Wiley, 2000.

Table F.1 Symbols for Unsteady-State Charts

	Parameter symbol	Molecular mass transfer	Heat conduction
Unaccomplished change, a dimensionless ratio	Y	$\dfrac{c_{A_\infty} - c_A}{c_{A_\infty} - c_{A_0}}$	$\dfrac{T_\infty - T}{T_\infty - T_0}$
Relative time	X	$\dfrac{D_{AB}t}{x_1^2}$	$\dfrac{\alpha t}{x_1^2}$
Relative position	n	$\dfrac{x}{x_1}$	$\dfrac{x}{x_1}$
Relative resistance	m	$\dfrac{D_{AB}}{k_c x_1}$	$\dfrac{k}{h x_1}$

T = temperature
c_A = concentration of component A
x = distance from center to any point
t = time
k = thermal conductivity
h, k_c = convective transfer coefficients
α = thermal diffusivity
D_{AB} = mass diffusivity

Subscripts:
0 = initial condition at time $t = 0$
1 = boundary
A = component A
∞ = reference condition for temperature



Total Mass and Mass Species Balances

$$\frac{\partial \rho}{\partial t} + \nabla \bullet \rho \bar{v} = 0$$ Mass Balance

$$\frac{\partial c_1}{\partial t} = -\nabla \bullet \bar{N}_1 + R_1$$ Molar Species Balance

$$\frac{\partial c_1}{\partial t} + \left[\frac{\partial N_{Ax}}{\partial x} + \frac{\partial N_{Ay}}{\partial y} + \frac{\partial N_{Az}}{\partial z}\right] = R_A \quad \textit{cartesian}$$

$$\frac{\partial c_1}{\partial t} + \left[\frac{1}{r}\frac{\partial}{\partial r}(rN_{Ar}) + \frac{1}{r}\frac{\partial N_{A\theta}}{\partial \theta} + \frac{\partial N_{Az}}{\partial z}\right] = R_A \quad \textit{cylindrical}$$

$$\frac{\partial c_1}{\partial t} + \left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 N_{Ar}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(N_{A\theta}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial N_{A\varphi}}{\partial \varphi}\right] = R_A \quad \textit{spherical}$$

$$\frac{\partial \rho_1}{\partial t} = -\nabla \bullet \bar{n}_1 + r_1$$ Mass Species Balance

Flux at a boundary:

$$\bar{N}_1 = -cD\nabla x_1 + c_1\bar{V}$$ Average molar velocity basis

$$\bar{N}_1 = -cD\nabla x_1 + x_1\sum \bar{N}_i$$

$$\bar{n}_1 = -\rho D\nabla w_1 + c_1\bar{v}$$ Average mass velocity basis

$$\bar{n}_1 = -\rho D\nabla w_1 + w_1\sum \bar{n}_i$$

Expanded Vector Form

$$\frac{\partial c_1}{\partial t} = -\nabla \bullet (-cD\nabla x_1 + c_1\bar{V}) + R_1$$ Average molar velocity basis

$$\frac{\partial c_1}{\partial t} = \nabla \bullet (cD\nabla x_1) - \nabla \bullet (c_1\bar{V}) + R_1$$

$$\frac{\partial \rho_1}{\partial t} = -\nabla \bullet (-\rho D\nabla w_1 + \rho_1\bar{V}) + r_1$$ Average mass velocity basis

Case 1: ρ and D_{12} constant

$$\frac{\partial \rho_1}{\partial t} + \bar{v} \bullet \nabla \rho_1 = -D\nabla^2 \rho_1 + r_1$$

$$\frac{\partial c_1}{\partial t} + \bar{v} \bullet \nabla c_1 = -D\nabla^2 c_1 + R_1$$

Some Vector Operators and Expansions

$$\nabla c_1 = \left(\frac{\partial c_1}{\partial x}, \frac{\partial c_1}{\partial y}, \frac{\partial c_1}{\partial z}\right) \quad \textit{cartesian}$$

$$\nabla c_1 = \left(\frac{\partial c_1}{\partial r}, \frac{1}{r}\frac{\partial c_1}{\partial \theta}, \frac{\partial c_1}{\partial z}\right) \quad \textit{cylindrical}$$

$$\nabla c_1 = \left(\frac{\partial c_1}{\partial r}, \frac{1}{r}\frac{\partial c_1}{\partial \theta}, \frac{1}{r\sin\theta}\frac{\partial c_1}{\partial z}\right) \quad \textit{spherical}$$

$$\nabla^2 c_1 = \frac{\partial^2 c_1}{\partial x^2} + \frac{\partial^2 c_1}{\partial y^2} + \frac{\partial^2 c_1}{\partial z^2} \quad \textit{cartesian}$$

$$\nabla^2 c_1 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial c_1}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 c_1}{\partial \theta^2} + \frac{\partial^2 c_1}{\partial z^2} \quad \textit{cylindrical}$$

$$\nabla^2 c_1 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial c_1}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial c_1}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 c_1}{\partial \varphi^2} \quad \textit{spherical}$$

$$\bar{v} \bullet \nabla c_1 = v_x\frac{\partial c_1}{\partial x} + v_y\frac{\partial c_1}{\partial y} + v_z\frac{\partial c_1}{\partial z} \quad \textit{cartesian}$$

$$\bar{v} \bullet \nabla c_1 = v_r\frac{\partial c_1}{\partial r} + \frac{v_\theta}{r}\frac{\partial c_1}{\partial \theta} + v_z\frac{\partial c_1}{\partial z} \quad \textit{cylindrical}$$

$$\bar{v} \bullet \nabla c_1 = v_r\frac{\partial c_1}{\partial r} + \frac{v_\theta}{r}\frac{\partial c_1}{\partial \theta} + \frac{v_\varphi}{r\sin\theta}\frac{\partial c_1}{\partial z} \quad \textit{spherical}$$

Generalized Mass/Molar Balance (Assume ρ and D constant)

- mass average velocity

$$\frac{\partial c_1}{\partial t} + \bar{v} \bullet \nabla c_1 = D\nabla^2 c_1 + R_1$$

cartesian

$$\frac{\partial c_1}{\partial t} + v_x\frac{\partial c_1}{\partial x} + v_y\frac{\partial c_1}{\partial y} + v_z\frac{\partial c_1}{\partial z} = D\left(\frac{\partial^2 c_1}{\partial x^2} + \frac{\partial^2 c_1}{\partial y^2} + \frac{\partial^2 c_1}{\partial z^2}\right) + R_1$$

cylindrical

$$\frac{\partial c_1}{\partial t} + v_r\frac{\partial c_1}{\partial r} + \frac{v_\theta}{r}\frac{\partial c_1}{\partial \theta} + v_z\frac{\partial c_1}{\partial z} = D\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial c_1}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 c_1}{\partial \theta^2} + \frac{\partial^2 c_1}{\partial z^2}\right) + R_1$$

spherical

$$\frac{\partial c_1}{\partial t} + v_r\frac{\partial c_1}{\partial r} + \frac{v_\theta}{r}\frac{\partial c_1}{\partial \theta} + \frac{v_\varphi}{r\sin\theta}\frac{\partial c_1}{\partial z} = D\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial c_1}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial c_1}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 c_1}{\partial \varphi^2}\right) + R_1$$