UNIVERSITY OF TORONTO

FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION, DECEMBER 2000

Third Year - Program 5A, MS

AER373H1 - MECHANICS OF SOLIDS AND STRUCTURES

Examiner - W.D. Morison

Type D Examination

Candidates may use only the class notes, quiz solutions, problem set solutions and type 1 calculators. Candidates may not use worked solutions of problems that are not part of the course problem sets as examination aids.

ANSWER ALL QUESTIONS.
MARKS ARE SHOWN BESIDE EACH QUESTION

Candidates are expected to provide complete solutions for each problem that demonstrate an understanding of the method required to obtain the correct solution.

1) The two member frame shown in Figure 1 is subjected to a vertical force W at node 4. Given that E and A are the same for both members,
   a) Show that the magnitude and sense of the member forces are
      $F_1 = \sqrt{2}W/(\sqrt{3}-1)$ *tension* and $F_2 = 2W/(\sqrt{3}-1)$ *compression*. [10 marks]
   b) Use Castigliano's Second Theorem to show that the vertical displacement of the common node is
      $\delta_4 = \dfrac{3\sqrt{2}+4\sqrt{3}}{3(2-\sqrt{3})}\dfrac{WL}{EA}$. [10 marks]
   c) To increase the stiffness of the frame, a third member of matching E and A is added between nodes 3 and 4; however, this member is short by an amount equal to δ_4 above. Show that this lack of fit alone produces a force in the new member equal to $\dfrac{3\sqrt{2}+4\sqrt{3}}{3(6+\sqrt{2}+\sqrt{3})}W$ *tension*. [10 marks]

2) The fully idealized cross-section shown in Figure 2 has the following properties:
   $I_{xx} = I_{yy} = 6 \times 10^6$ mm^4, $I_{xy} = 3 \times 10^6$ mm^4, E = 10 GPa, $A_1 = A_2 = A_3 = 100$ mm^2. If this section used in a 10 m. long cantilever beam that supports 10 N tip loads in both the –y and –x-directions through the shear centre, calculate
   a) The location and value of the maximum axial stress in each boom. [10 marks]
   b) The maximum and minimum slope of the neutral axis. [10 marks]
   c) The vertical and horizontal deflections of the centroid. [10 marks]

3) The fully idealized cross-section shown in Figure 3 has the following properties:
   $I_{xx} = 4 \times 10^6$ mm^4, $I_{yy} = 16 \times 10^6$ mm^4, $I_{xy} = 0$, G = 10 GPa, $A_1 = A_2 = A_3 = A_4 = 100$ mm^2, t = 1 mm except web 13 where $t_{13} = 2$ mm.
   a) Cut the structure on 1-2 and 1-3 and calculate the closed section shear flow distribution. [10 marks]
   b) Calculate the maximum shear stress. [10 marks]
   c) Calculate the rate of twist in rad/mm. [10 marks]

4) The thin walled cross-section shown in Figure 4 has the following properties:
   $A_1 = A_2 = 100$ mm^2, $t_{12}^{i} = 5$ mm, $t_{12}^{o} = 1$ mm.
   a) Idealize the cross-section and show that $B_3 = B_3 = 319.02$ mm^2. [5 marks]
   b) Calculate the location of the shear centre relative to the vertical web. [5 marks]

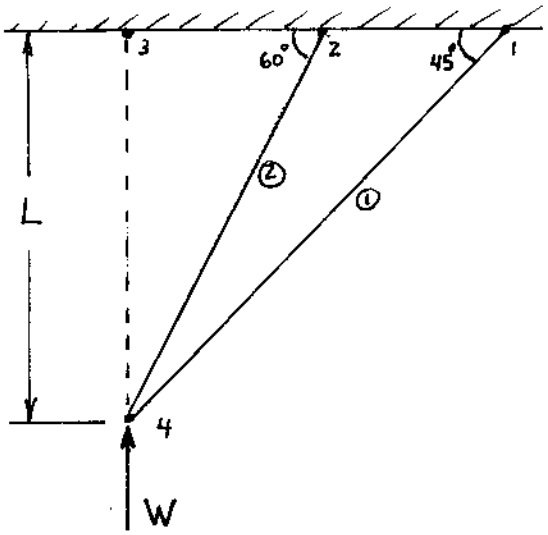

FIGURE 1

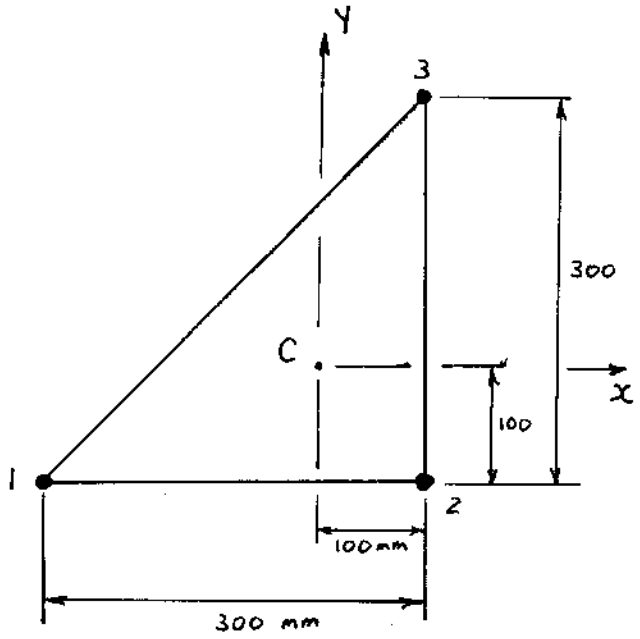

FIGURE 2

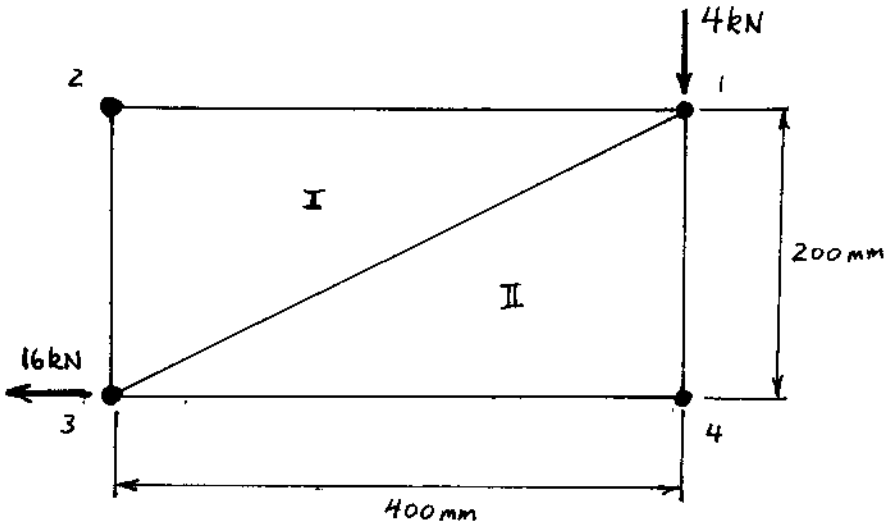

FIGURE 3

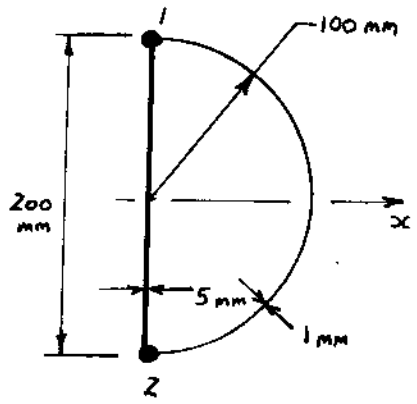

FIGURE 4