UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING
FINAL EXAMINATION - APRIL 2001
ECE411S - REAL-TIME COMPUTER CONTROL
IV-AEESCBASCB(E), AEESCBASCE, AEELEBASC, AECPEBASC
EXAMINER - R.H. Kwong

Students may use one 8.5"×11" aid sheet in preparing their answers. The aid sheet can be either the type supplied by the Faculty Office, or it can be any other sheet with the above specified dimensions.

(20 points)

1. The various parts of this question are independent of each other.
   (a) In designing real-time software for control applications, one can have a self-targetting system, or one can have a host-target configuration. What is the difference between these 2 configurations? Classify the following real-time software you have encountered in the lab as self-targetting or host-target: Real-time windows target from MATLAB, real-time control software from Opal-RT, and Wincon from Quanser.
   (b) In laboratory experiment 4 for this course, what is the difference between balancing control and self-erecting control of the inverted pendulum on a cart?
   (c) There are 3 periodic tasks: X, Y, and Z, with periods 52, 40, and 30, respectively. Each task deadline is equal to its period. The computation time needed to complete tasks X, Y, and Z are respectively 12, 10, and 10.
      (i) Determine the total utilization $U = \sum \frac{C_i}{T_i}$ where C_i is the computation time required for task i, and T_i is the period of task i. Does U satisfy the bound required for the rate monotonic scheduler to schedule these 3 tasks?
      (ii) Determine the response time of each of these 3 tasks. Based on the response times, are the 3 tasks schedulable by the deadline monotonic schedule (equivalent to the rate monotonic schedule in this case)?
      (Hint: Recall that the response time R_i for task i is given by

$$R_i = C_i + \sum_{j \in hp(i)} \left\lceil \frac{R_i}{T_j} \right\rceil C_j)$$

(20 points)

2. Consider the digital control system described in the following figure:

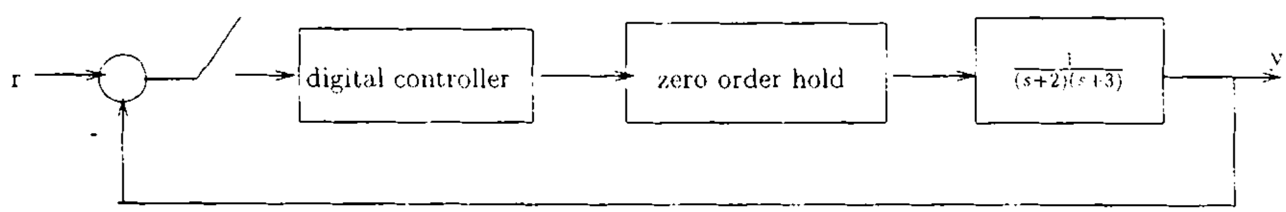


Figure 2.1

The sampling interval $T = 0.2$. The design methodology is to design first a continuous-time controller, and then discretize it.

(a) Write down the first order transfer function approximation for the zero order hold. Use the version with DC gain equals 1.

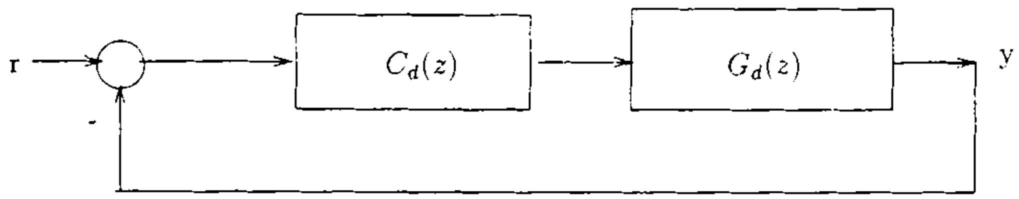
(b) The following controller $C(s)$ is proposed as the continuous-time controller to ensure asymptotic tracking of step reference inputs by the closed-loop system:

$$C(s) = \frac{2}{s}$$

Show that the continuous-time closed-loop system, with the zero order hold replaced by the transfer function approximation from part(a), is stable.
(Hint: For a 4th order monic polynomial with positive coefficients $s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4$, the necessary and sufficient condition for the polynomial to have all roots in the left half plane is $(a_1 a_2 - a_3)a_3 > a_1^2 a_4$.)

(c) Discretize the continuous-time controller $C(s)$ using the bilinear transformation to give $C_d(z)$.

(d) Now put $C_d(z)$ as the digital controller in Figure 2.1. At sampling points, we have a discrete-time feedback control system. It has the structure shown in the following figure.



Determine the pulse transfer function $G_d(z)$ and hence determine explicitly the loop transfer function $L_d(z) = G_d(z)C_d(z)$.

(20 points)

3. Consider the continuous-time linear system described by the state equation

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned}$$

with

$$A = \begin{bmatrix} 1 & 1 \\ -2 & -2 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

This problem asks you to design a reduced-order observer for the discrete-time system arising from sampling the above continuous-time system.

(a) Let the sampling interval be T. The corresponding sampled-data system is of the form

$$\begin{aligned} x(k+1) &= A_d x(k) + B_d u(k) \\ y(k) &= Cx(k) \end{aligned}$$

Determine A_d and B_d in terms of T.

(b) Design a reduced-order observer to estimate the unobserved state variable $x_1(k)$ in the form

$$\hat{x}_1(k+1) = F\hat{x}_1(k) + Gu(k) + Hy(k) + Jy(k+1)$$

Determine the constants F, G, H, and J as a function of T such that the estimation error $e_1(k) = x_1(k) - \hat{x}_1(k)$ satisfies the difference equation

$$e_1(k+1) = 0.8e_1(k)$$

(c) Now eliminate the $y(k+1)$ term from the right hand side of the equation for $\hat{x}_1$ by writing the observer equation in the form

$$\begin{aligned} z(k+1) &= F_1 z(k) + G_1 u(k) + H_1 y(k) \\ \hat{x}_1(k) &= \alpha_1 y(k) + \alpha_2 z(k). \end{aligned}$$

Determine the constants F_1, G_1, H_1, α_1, and α_2 in terms of T.

(20 points)

4. Consider the discrete-time linear system described by the following state equation

$$\begin{aligned} x(k+1) &= \begin{bmatrix} 0 & 0 & 0.4 \\ 1 & 0 & -1.7 \\ 0 & 1 & 2.3 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u(k) \\ y(k) &= \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} x(k) \end{aligned}$$

It is easy to see that this system is controllable and observable. You are asked to design an output feedback compensator for this system using state feedback and full-order observer theory.

(a) Verify that 1 is an eigenvalue of the system matrix so that the open-loop plant is unstable.

(b) Design a state feedback law $u(k) = -Kx(k)$ such that the closed-loop system under state feedback

$$x(k+1) = (A - BK)x(k)$$

has eigenvalues at -0.4, $0.8 \pm 0.6j$. You must use either the procedure involving the matrix which transforms the system to controllable canonical form, or Ackermann's formula to determine K (Note the hint on the next page).

(Hint: $\begin{bmatrix} a & b & 1 \\ b & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -b \\ 1 & -b & b^2 - a \end{bmatrix}$ for any constants a and b)

(c) Design a full-order deadbeat observer to estimate the states of the system (i.e., the poles of the observer should all be at 0).

(d) The controller transfer function from y to u can be written in the form of

$$u = -[H(zI - F)^{-1}G + J]y$$

Determine explicitly the matrices F, G, H, and J.

(20 points)

5. The plant is described by the state equation

$$x(k+1) = \begin{bmatrix} 1 & 0.5 \\ 1 & 1.5 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k)$$

with output equation

$$y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(k)$$

The full state $x(k)$ is available for each k. A feedback controller is to be designed so that the closed-loop system is stable and the output $y(k)$ tracks a reference step input r. Integral control is to be used to achieve this objective. To this end, augment the system with the equation

$$v(k+1) = v(k) + y(k) - r(k)$$

The controller is then of the form

$$u(k) = -Kx(k) - K_I v(k)$$

(a) Let $\xi(k) = \begin{bmatrix} x(k) \\ v(k) \end{bmatrix}$. Write down the state equation for ξ in the form:

$$\xi(k+1) = F\xi(k) + Gu(k) + Hr(k)$$

Identify F, G, and H.

(b) Verify that the pair (F, G) is controllable, and determine the transformation matrix P which brings (F, G) to controllable canonical form.

(c) Determine the feedback gains K and K_I such that the closed loop system has poles at -0.5 and $0.8 \pm 0.6j$.

(Hint: You may use the fact that the following matrices are inverses of each other.

$$\begin{bmatrix} 0 & 0 & 2 \\ 2 & 0 & 2 \\ 4 & 1 & 2 \end{bmatrix}, \quad \begin{bmatrix} -0.5 & 0.5 & 0 \\ 1 & -2 & 1 \\ 0.5 & 0 & 0 \end{bmatrix})$$