University of Toronto
Department of Civil Engineering
Faculty of Applied Science and Engineering

Final Examination
December 11, 2001

Civ261F
Engineering Mathematics I
Examiner: B. E. Sleep

Type C: Aid sheet allowed, calculators must be non-programmable

Name:_____
Student Number:_____

Question Number	Possible Marks	Earned Marks
1	10	
2	15	
3	15	
4	15	
5	10	
6	10	
7	10	
8	15	
Total	100	

Instructions:

Attempt all 8 questions.

Answers are to be submitted on the question sheets. The backs of pages may be used if necessary, but question numbers must be clearly indicated.

All work must be **neat and legible**, any formulae used must be given, and **all steps** in calculations must be shown.

Name:_____

1. Use a double integral to find the volume of the object created by rotating the area bounded by $x=y^2$, $x=2$, $y=0$, $y \geq 0$ about the line $y = 6$. **(10 marks)**

Name:_____ _____

2. Find the second moment of the area bounded by $y=(x-2)^2$ and $y = x$ about the y-axis. **(15 marks)**

Name:_____

3. a) Determine dz/dt and $\partial z / \partial t$ for $z(u,v,t) = u^2 - 2v^3 + t^3$, $u(x,y,t) = 2xe^{-t}$, $v(x,y,t) = xy$, $x(t) = \sin t$, $y(t) = \cos t$. (**8 marks**)
   b) For $h(x,y) = ye^{-x} + 3x^2y^3$, determine the magnitude and direction (given as a bearing) of the steepest slope at $(x,y) = (1,1)$, and determine the slope in the northwest direction (**7 marks**).

Name:_____

4. Show that the following integral is independent of path and evaluate the integral:

$\int_C xye^x dx + (xe^x - e^x)dy$, C is the region bounded by $x = y^2$, $x = 3$, and $y = 0$. **(15 marks)**

Name:_____

5. Using two-point Gaussian Legendre evaluate:

   a) $\int_0^2 \frac{dx}{1+x}$ **(5 marks)**

   b) $\int_{-1}^{1}\int_{-1}^{1} xydxdy$ **(5 marks)**

   The following may be useful:

$$\int_a^b f(x)dx = \frac{b-a}{2}\int_{-1}^{1} f\left(\frac{b+a}{2}+\frac{b-a}{2}z\right)dz \qquad I = f\left(\frac{-1}{\sqrt{3}}\right)+f\left(\frac{1}{\sqrt{3}}\right)$$

Name:_____

6. Using **two** iterations (not counting initial guesses) of the secant method with initial guesses of $x = -3$ and $x = -2$, estimate the root of: $f(x) = x^3 + e^{-x} + 3$. **(10 marks)**

Name:_____

7. Approximate $f(0.25)$ given the following data and the first, second and third order Newton divided difference formulae:

x	0.0	0.1	0.3	0.5
$f(x)$	0.5	0.75	1.1	1.5

(10 marks)

Name:_____

8. The following data were thought to be described by $y = a/(b+x)$. Use linear regression to determine the best fit values of a and b. Present your intermediate calculations in a table with the appropriate columns.

x	1.0	2.0	3	4
y	0.52	0.39	0.37	0.26

(15 marks)