University of Toronto

Faculty of Applied Science and Engineering

Department of Electrical and Computer Engineering

FINAL EXAMINATION, APRIL 1997

Fourth Year - Programs 5ce, 5e, 7, 9 [elective]

ECE 417S - DIGITAL COMMUNICATION

Examiner: *S.Pasupathy*

- A single aid sheet (8.5"x11", handwritten) and a non- programmable calculator are the **only aids allowed.**
- Answer **all** six [6] questions.
- The value of each question is indicated beside each question; total marks = 60.
- Start each new question on a new page.
- If you need to make any assumptions, state them clearly.
- Answers should be clear, crisp and brief; answers without logical reasoning steps showing *all* the work will **not** be given credit.
- Lengthy reproductions of text material should be avoided. Credit is for **solving** the problems.

0 marks

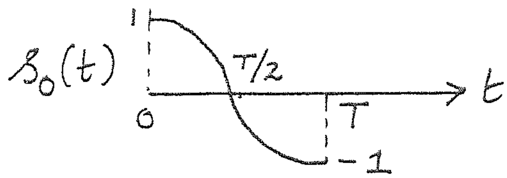
1. A digital source emits six messages $\{m_i, i = 1, \ldots, 6\}$ with probabilities

$$0.3,\ 0.25,\ 0.15,\ 0.12,\ 0.1 \text{ and } 0.08,$$

respectively. Find the **4-ary (quaternary)** Huffman code. Determine its average word length and its efficiency.

5 marks

2. A Minimum Shift Keying modulation scheme uses the waveform $s_0(t)$ to indicate the lower-frequency signal corresponding to '0' bit.



Sketch $s_1(t)$ representing the '1' bit.
Sketch clearly the waveform corresponding to the input data

$$1\ 0\ 0\ 0\ 1\ 1.$$

12 marks

3. Consider a (6,2) block code generated by the generator matrix

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

   (a) Construct the code table [i.e. all the message words and the corresponding code words] for this code.

   (b) What is the minimum distance for this code? What is the minimum Hamming weight of this code ? Are they the same or different? Explain.

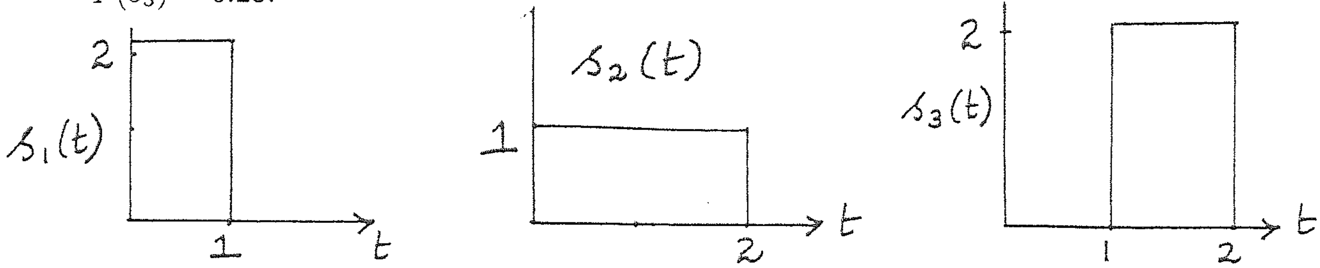
   (c) How many errors can this code correct (and why) ?

   (d) How many errors can this code detect (and why) ?

   (e) What is the parity-check matrix, H , for this code ?

   (f) If a received word is = 1 1 0 0 0 0 , what is the decoded codeword ? Explain.

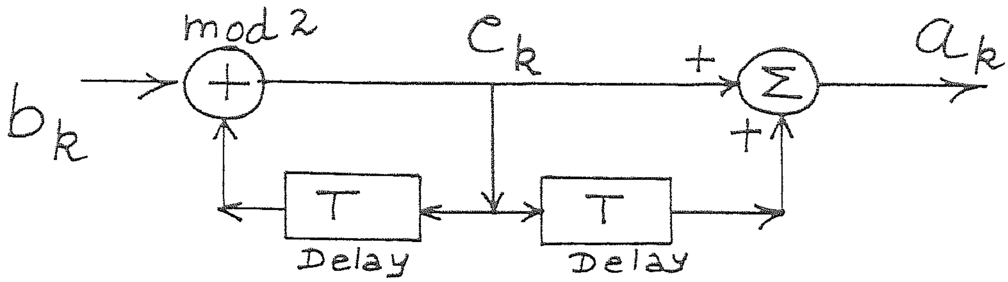
0 marks 4. A ternary signalling scheme (M = 3) uses the three waveforms $s_1(t), s_2(t)$ and $s_3(t)$ shown. The probabilities of the three messages are $P(s_1) = 0.25$ $P(s_2) = 0.5$ and $P(s_3) = 0.25$.



(a) Determine the minimum-energy signal set and sketch the waveforms. Show the set in a suitable signal-space diagram.

(b) Compute the mean energies of the signal set shown and its minimum - energy equivalent set found in part (a).

10 marks 5. A Partial Response Signalling equivalent model is shown below.



Assume an input $\{b_k\}_{k=1}^{8}$ sequence of

$$0\;1\;1\;1\;1\;0\;0\;0\,.$$

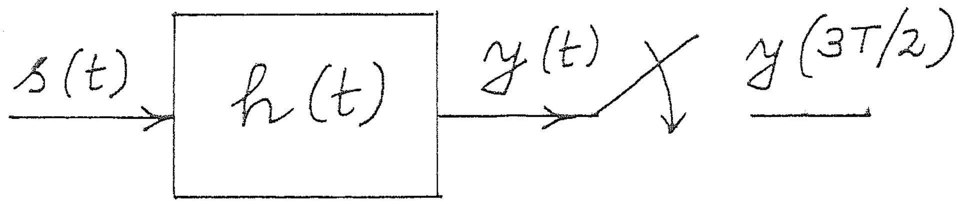
(a) What is $\{a_k\}_1^8$? Assume that $c_0 = 1$. From your answer, deduce a decoding rule, assuming the usual type of noise model.

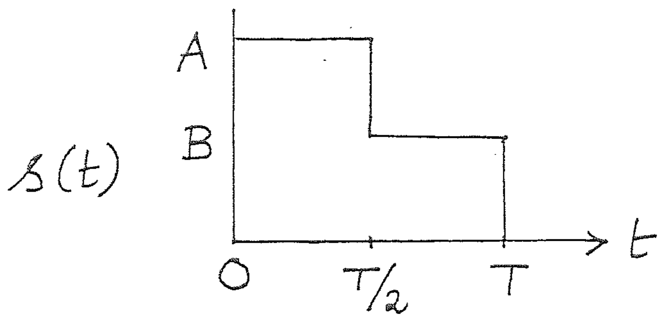
(b) What is $\{a_k\}_1^8$, assuming $c_0 = 0$. Does your answer lead to a change in the decoding rule found in the previous part ? If so, how should it change ? Explain briefly.

2 marks

6. (a) $x(t)$ is the real impulse response of a baseband channel which satisfies Nyquist's criterion for zero ISI. The symbol rate is $1/T$.(The channel is known to be bandlimited with a bandwidth more than the Nyquist's minimum ; otherwise nothing else is known .) An engineer wants to build a parallel transmission scheme using $\hat{x}(t)$ (the Hilbert transform of $x(t)$). Does $\hat{x}(t)$ satisfy Nyquist's criterion ? Prove any assertions made.

   (b) The signal $s(t)$, shown below, is passed through a filter with impulse response $h(t)$ and then is sampled at t = 3T/2.





Both $||s||$ and $||h|| = 1$. (Assume an appropriate norm definition). Find and sketch an impulse response $h(t)$ such that $y(3T/2) = 1$. Show all your steps CLEARLY leading to the answer.