University of Toronto
FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATIONS, DECEMBER 2000
First Year - Programs 1,2,3,4,6,7,8,9

MAT 186H1F
Calculus I

SURNAME _____

GIVEN NAME _____

STUDENT NO. _____

SIGNATURE _____

Examiners

P. Bubenik

D. Burbulla

S. Homayouni

K. Kaveh

INSTRUCTIONS:

Non-programmable calculators permitted.

Answer all questions.

Present your solutions in the space provided; use the back of the **preceding** page if more space is required.

TOTAL MARKS: 100

The value for each question is shown in parentheses after the question number.

MARKER'S REPORT	
Q1	
Q2	
Q3	
Q4	
Q5	
Q6	
Q7	
Q8	
TOTAL	

1. [16 marks] Find the following:

(a) [5 marks] $\int_0^5 \frac{x}{\sqrt{x^2+16}}\,dx$

(b) [5 marks] $\int_0^\infty \left(e^{-x}+e^{-2x}\right)\,dx$

(c) [6 marks] $\lim_{x\to 0}\left(\frac{1}{2}+\frac{e^{3x}}{2}\right)^{\csc x}$

2.(a) [7 marks] At which point(s) on the circle with equation $x^2+y^2=1$ does the tangent line pass through the point $(3,0)$?

2.(b) [7 marks] Find y as a function of x if $\frac{dy}{dx} = y(2x+1)$ and $y=-3$ when $x=0$.

3. [14 marks] The position x of a particle at time t is given by $x = 15t^2 - 5t^3 - 6$, for $0 \leq t \leq 3$. Find the following:

(a) [4 marks] the average velocity of the particle on the interval $0 \leq t \leq 3$.

(b) [5 marks] the maximum speed of the particle on the interval $0 \leq t \leq 3$.

(c) [5 marks] the average speed of the particle on the interval $0 \leq t \leq 3$.

4.(a) [6 marks] If Newton's method is used to approximate a solution to the equation $x^3 + x - 1 = 0$, and $x_1 = 0.5$, what is the value of x_3?

4.(b) [6 marks] Find the equations of all horizontal or vertical asymptotes (if any) to the graph of $y = \dfrac{x^2 - 1}{x^2 + x - 2}$.

5. [12 marks; 6 marks for each part] Let R be the region in the plane bounded by the curves $y = x^2$ and $y = 2x - x^2$. (Draw a picture!) Find:

(a) the volume of the solid obtained by revolving R about the x-axis.

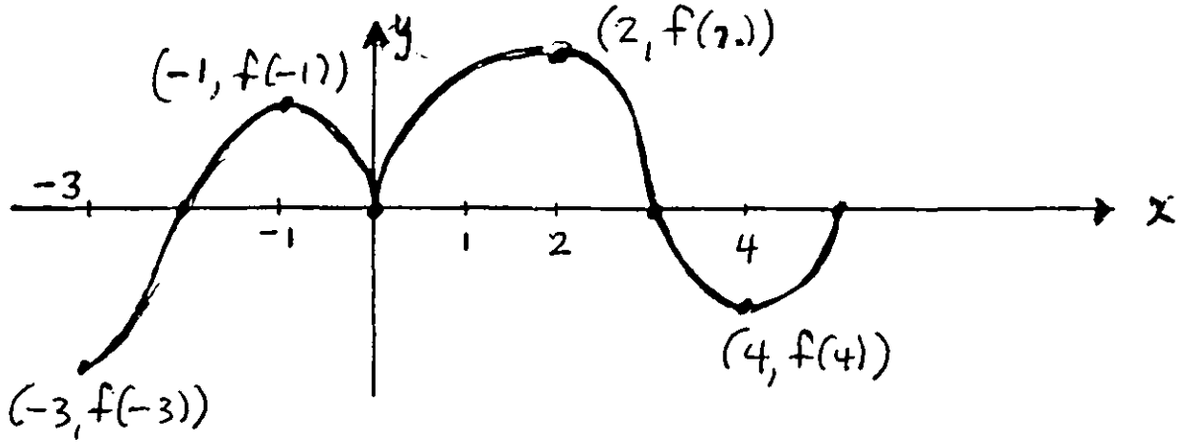
(b) the volume of the solid obtained by revolving R about the line $x = -1$.

6. [12 marks] A wire of length 100 cm is to be cut into two pieces. One piece will be bent into the shape of a circle; the other piece will be bent into the shape of a square. How should the wire be cut to

   (a) maximize the combined area of the two shapes?

   (b) mimimize the combined area of the two shapes?

7. [10 marks] The graph of the function $f(x)$ for $-3 \leq x \leq 5$ is:



Sketch the graph of $F(x) = \int_{-3}^{x} f(t)\,dt$ for $-3 \leq x \leq 5$, given that $F(0) = F(-3)$.

8. [10 marks] A tank filled with water of density $\rho = 1000$ kg/m^3 has the shape of an inverted right circular cone, with radius at the top 2 m and with vertical height 3 m. Find the work done in pumping all of the water out of the tank and up to a horizontal pipe 1 m above the top of the tank. (Use acceleration due to gravity $g = 9.8$ m/sec^2.)