UNIVERSITY OF TORONTO

FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION — 12 December ~~1995~~ 6

FOURTH YEAR GASDYNAMICS — AER–410 F

Examiner: Professor J. J. Gottlieb

Instructions: *(a)* open book test (D) — notes and textbook permitted,
(b) answer or attempt all questions if you can, and
(c) relative mark values for questions are indicated.

20 % 1. A reservoir of volume V and initial air density ρ_i and pressure p_i receives additional air in the form of a constant mass inflow $\dot{m}$ at a constant stagnation temperature T_0 or sound speed a_0 during some period of time. Start with the integral equations for the conservation of mass and energy and derive separate expressions for the change in density and pressure in the reservoir as a function of time.

20 % 2. A supersonic flow inside a 10-cm-square duct has a pressure $p_1 = 1.5$ atm, flow velocity $u_1 = 900$ m/s and sound speed $a_1 = 300$ m/s. This air flow encounters a slot of width $L = 1$ cm across one wall of the duct, and the air pressure $p_{\text{atm}} = 1$ atm outside of the duct (in the atmosphere). Determine or estimate the percentage of the mass flow that is lost from the duct by means of a jet emanating from the slot. (Note that this jet is produced by a Prandtl-Meyer wave.)

20 % 3. A large reservoir of air with a constant stagnation pressure $p_0 = 1$ atm and temperature $T_0 = 300\ K$ discharges air through a short convergent duct and attached constant-area duct with an average Fanning friction factor $\overline{f} = 0.005$, a diameter $\mathcal{D} = 0.01$ m and a length $L = 20$ m from a spacecraft into Space ($p_{Space} \approx 0$ atm).

a) What is the flow Mach number at the end of the constant-area duct (e.g., M_2 at location 2).

b) What is the flow Mach number at the start of the constant-area duct (e.g., M_1 at location 1).

c) Sketch Fanno lines on a temperature-entropy diagram for the cases of duct lengths of $L = 0$, 5, 10 and 20 m, showing the stagnation state, state at location 1 (start of constant-area duct), and state at location 2 (end of constant-area duct).

20 % *4.* A shock tube is used to produce a shock wave with a thermal pulse of temperature change $\Delta T = T_2 - T_1 = 200$ K and duration $\Delta t_2 = 10$ milliseconds in the test section where thermocouples and hot-film gages are located for calibration purposes. Air ($\gamma = 7/5$, $R = 287$ J/kg·K) is used in both the driver ($T_4 = 290.9$ K) and channel ($T_1 = 290.9$ K, $p_1 = 100$ kPa).

a) Calculate the static pressure p_2, shock Mach number M_s and flow velocity u_2 in region 2 corresponding to the thermal pulse with $\Delta T = T_2 - T_1 = 200$ K that is applied to the thermocouples and hot-film gages.

b) Calculate the distance from the diaphragm that the thermocouples and hot-film gages must be placed so that the flow duration Δt_2 of air in region 2 is 10 milliseconds.

c) Calculate the driver pressure p_4 that is required to produce the thermal pulse with $\Delta T = T_2 - T_1 = 200$ K.

20 % *5.* An intermittant wind tunnel has a vacuum chamber that sucks atmospheric air ($p_{\text{atm}} = 1$ atm, $T_{\text{atm}} = 290$ Kelvin, $\gamma = 7/5$, $R = 287$ J/kg·K) through a convergent-divergent duct (with a throat) to a flow Mach number $M_{ts} = 3.3$ in the test section of area $A_{ts} = 1$ m^2 in front of a vacuum chamber.

a) Calculate the static pressure p_{ts} and temperature T_{ts} in the test section.

b) Explain the process of wind-tunnel 'shut-down,' or eventual flow Mach number disruption in the test section, including references to the type of flow behavior in the vacuum chamber.

c) Calculate the vacuum-chamber pressure p_{vc} when 'shut-down' occurs.

d) Calculate the volume $\mathcal{V}_{vc}$ of the vacuum chamber that enables the wind tunnel to have a test time of 10 seconds, if the original pressure in the vacuum chamber is $p_{vc} = 0.1$ atm. You may use the equation

$$\Delta t = \frac{1}{\gamma} \frac{T_{ts}}{T_{\text{atm}}} \frac{\Delta p_{vc}}{p_{ts}} \frac{\mathcal{V}_{vc}}{\sqrt{\gamma R T_{ts}} M_{ts} A_{ts}}$$

adapted from a handout in the course, in which Δt is time interval for the vacuum-chamber pressure to increase by Δp_{vc} from its initial pressure to the shut-down pressure.