University of Toronto
FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION – DECEMBER 2001

MAT196F – Calculus A Duration: 2 1/2 hours

Examiners: S. Abou-Ward, E. Soprounova, S. Uppal

SURNAME _____

GIVEN NAME _____

STUDENT NO. _____

SIGNATURE _____

INSTRUCTIONS:
No Aids of Any Kind.
Answer all questions.
Present your solutions in the space provided; use the back of the **preceding** page if more space is required.
TOTAL MARKS: 100

For Marker Use Only	
Question	Mark
1	/22
2	/19
3	/14
4	/14
5	/16
6	/15
TOTAL	/100

1. (a) Let $f(x) = \ln|x|$ if $x < 0$.

(i) **(3 marks)** Show that f has an inverse, and denote this inverse by g.

(ii) **(3 marks)** What is the domain of g?

(iii) **(3 marks)** Find a formula for computing $g(y)$ for each y in the domain of g.

(iv) **(3 marks)** Find $g'(0)$.

(v) **(2 marks)** Sketch the graph of g.

(b) Evaluate the following:

(i) **(4 marks)** $\displaystyle\int \frac{\ln|x|}{x\sqrt{1+\ln|x|}}dx.$

(ii) **(4 marks)** $f'(\pi)$ if $f(x) = x^2(1+x^4)^{-7}\cos x$.

2. (a) A spring has a natural length of 1 meter. A force of 100 newtons compresses it to 0.9 meter.

(i) **(5 marks)** How many joules of work are required to compress it to half its natural length? (P.S. - Hooke's law is given by $F(x) = kx$).

(ii) **(5 marks)** What is the length of the spring when 20 joules of work have been expended?

(b) Calculate the following limits and explain which limit theorems you are using.

(i) **(3 marks)** $\lim_{x \to 1} \dfrac{\sin(x^2 - 1)}{(x - 1)}$

(ii) **(3 marks)** $\lim_{x \to 0} x \sin(1/x)$.

(iii) **(3 marks)** $\lim_{n \to \infty} \dfrac{\sqrt{1} + \sqrt{2} + \ldots + \sqrt{n}}{n\sqrt{n}}$

3. (a) **(6 marks)** Let R be the plane figure bounded below by the x-axis, above by the graph of $y = e^{-x^2}$, and on the sides by the vertical lines at $x = 0$ and $x = 1$. Find the volume generate by rotating R about the y-axis.

(b) Evaluate the following integrals:

(i) **(4 marks)** $\int \frac{e^{3x}}{e^x + e^{-x}} dx$

(ii) **(4 marks)** $\int \frac{2x^2}{(1+x^3)^{1/5}} dx$

4. (a) **(10 marks)** Sketch the graph

$$f(x) = \frac{\ln x}{x}, \qquad x > 0 .$$

(b) **(4 marks)** Find the critical numbers and classify the extreme values of $f(x) = \frac{x}{4 + x^2}$, $x \in [-3, 1]$.

6. **(15 marks)** Given a functin g, continuous everywhere, such that $g(1) = 5$ and $\int_0^1 g(t)dt = 2$. Let

$$f(x) = \frac{1}{2}\int_0^x (x-t)^2 g(t)dt$$

Show that

$$f'(x) = x\int_0^x g(t)dt - \int_0^x tg(t)dt\ ,$$

then compute $f''(1)$ and $f'''(1)$.

5. (a) **(7 marks)** Let $f(x) = x - x^2$, $g(x) = ax$. Determine a so that the region above the graph of g and below the graph of f has area $9/2$.

(b) If *pi* is defined to be the area of a unit circular disk, that is,

$$\pi = 2\int_{-1}^{1} \sqrt{1 - x^2}dx$$

Use properties of the integral to compute in terms of π:

(i) **(4 marks)** $\int_{-3}^{3} \sqrt{9 - x^2}dx$

(ii) **(5 marks)** $\int_{-2}^{2} (x - 3)\sqrt{4 - x^2}dx$

4. Digital camera manufacturers are willing to supply high-end digital cameras according to the following supply (Q_S) and demand (Q_D) schedules:

$$Q_S = 10.5(p + 12{,}000) \qquad Q_D = 63{,}000 + 10(10{,}000\text{-}p)$$

where p is the price per digital camera and Q_S and Q_D is the quantity supplied and demanded respectively (units per year). In a free market, how many of these cameras will producers actually sell and what price will consumers pay? [5]

5. A profitable company is considering acquiring more warehousing space. It is evaluating a site north of Toronto. This site has a building that is suitable and the building and property are selling for $1,000,000. In the case of this property, the land is valued at 40% of the purchase price and the building's value is the remainder of the purchase price. The building is constructed of brick. The company's MARR is 15%. The company's tax rate is 40%. What is the after-tax present value cost of the purchase of this site? (For this question only, ignore the CCA "half-year" rule and assume that the building will be kept for a considerable length of time.) [5]

Question 2 [20 marks]

1. An investor is considering a new issue of a 10-year Nortel bond with a face value of $100,000 and a bond rate of 8% per annum payable semi-annually. The bond is currently selling for $98,000. She plans on owning the bond for six full years (receiving the interest payment at the end of Year 6) at which time she will sell it. Her expected annual rate of return is 10.25%. Her reinvestment rate is 6% compounded semi-annually. Use a before-tax analysis.

   (a) What must she sell the Nortel bond for to realize her expected rate of return? [7]
   (b) Do you think it is likely that she will achieve her required rate of return? Support your answer with an appropriate financial analysis. [5]

2. A $433,000 revamping project is underway for the receiving dock of Home Depot's distribution centre. The completed project is expected to provide future benefits for which the Home Depot would have been willing to pay up to $500,000 if necessary. However, new information has been received that the entire facility will be shut down when its lease expires, leading to a new estimate that the future benefits of the revamp are worth only $300,000. If the revamping project is curtailed quickly, Home Depot can save the last $150,000 of its cost. Should Home Depot abandon or continue the project? (Explain your recommendation clearly and concisely stating your assumptions in support of your answer. Quantify the value of your recommendation.) [4]

3. The current ratio and the acid-test ratio are important indicators of the financial health of a company. What do these ratios measure and why are they important? By way of a numerical example for a hypothetical company scenario, clearly illustrate a situation where the acid-test ratio would provide a better measure or interpretation of a company's financial health than would the current ratio. [4]

Question 3 [20 marks]

The Ministry of Transportation pays the Great Lakes Company to ferry cars across a bay that effectively splits a large city in the province. The Great Lakes Company has submitted a bid of $5 million per year (to be paid by the province) for 15 years to provide the service since their current contract will run out shortly. The benefit associated with the ferry in time savings, reduction in gasoline consumption, etc., is estimated at $3 per car. A low-level bridge can be constructed that will increase the user benefits to $4.50 per car, mainly due to the reduced time to cross the bay. The low-level bridge will have a first cost of $30 million and annual maintenance charges of $1.8 million. A second bridge design will cost $37 million to implement with $2.5 million annual maintenance costs. The user benefits are estimated at $5.60 per car with the second bridge design. There are no other costs or salvage values, and a 15-year bridge life has been assumed. The Ministry uses an 8 percent interest rate and for planning purposes has estimated that 2 million cars per year will use either the ferry or a bridge. The Ministry uses B/C ratio analysis in its evaluation of public projects.

(a) Make a recommendation to the Ministry as to how they should proceed. Use an annual worth approach. **[12]**

(b) The provincial government has a chronic funding shortfall problem and has decided to use the "opportunity cost" approach to select the appropriate interest rate in evaluating its public works projects. Discuss this approach and how it will affect the interest rate that the government will use in its evaluation of public projects? **[3]**

(c) Use the "opportunity cost" approach that you outlined in part (b) to re-evaluate your recommendation in part (a). At what approximate interest rate (the nearest percentage point) will your recommendation no longer be in the public interest? **[5]**

Question 4 [25 marks]

The Brute Force Construction Company (BFCC) has been a successful construction company for 25 years. Current economic conditions, however, have forced the company to examine all practical opportunities for reducing costs. As part of a company-wide assessment, the Engineering Department was asked to consider whether or not the company could save money by having various activities performed by third parties outside the company (i.e., subcontracting). BFCC is subject to a 40% income tax rate and uses a MARR of 20%.

In one case, the Engineering Department was investigating whether a bulldozer, purchased 3 years ago for $50,000 should be sold and the work normally done by this machine subcontracted over the next 4 years. As part of its analysis, the Engineering Department identified that the work done by this machine has generated a Gross Revenue of $45,000 per year and was expected to do so for the remaining 4 years whether or not the machine was retained or the services subcontracted. The operating costs of $22,000 per year would also continue over the same period if the machine were to be retained. If the machine were to be sold, the subcontractor's service charges would be $38,000 per annum.

Use the relevant provisions of the Canadian Income Tax Act in your analysis. Because BFCC will always have construction equipment, any terminal loss considerations may be ignored.

(a) Given a depreciation rate of 20% for construction equipment, what book value should be used to establish a benchmark value for considering any offer received for the purchase of the bulldozer if BFCC decided to subcontract? **[3]**

(b) If the bulldozer were to be retained for the remaining 4 years of its economic life, what is the present value of all of the after-tax cash flows assuming a salvage value of $5,000 at the end of the remaining 4 years? **[10]**

(c) If the subcontract option were to be implemented, what is the present value of all of the after-tax cash flows for the remaining 4 years assuming that the bulldozer was sold for a price equal to the book value obtained in part (a)? **[6]**

(d) Which alternative is the most economic for BFCC? **[1]**

(e) What is the minimum price at which BFCC would have to sell the bulldozer to make the subcontracting option attractive? **[5]**