UNIVERSITY OF TORONTO
EDWARD S. ROGERS SR. DEPARTMENT OF ELECTRICAL & COMPUTER ENGINEERING
FINAL EXAMINATION - DECEMBER 2001
ECE410F - CONTROL SYSTEMS
Fourth Year - Program 7
EXAMINER - E.J. Davison

Notes:

1. Please answer all questions.
2. All questions have equal value.
3. Type "B" Examination; an aid sheet is permitted.

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1. The equations of a solenoid problem are given as follows:

$$\dot{x} = v$$
$$\dot{v} = -\frac{D}{M}v - \frac{K}{M}(x - x_A) + \frac{1}{2M}i^2\frac{dL}{dx}(x)$$
$$\dot{i} = -\frac{R}{L(x)}i - \frac{1}{L(x)}\frac{dL}{dx}(x)iv + \frac{1}{L(x)}u$$

where u is the input (voltage), x is the output (displacement), $x_A > 0$ is a constant. $K > 0, D > 0, M > 0, R > 0$ are constants, and $L(x) = \bar{L} + \bar{\beta}x$, where $\bar{L} > 0, \bar{\beta} > 0$ are constants.

(a) Determine the equilibrium point of the system with respect to a constant input voltage $\bar{u}$.

(b) Determine the linearized state model of this system about the equilibrium point obtained in (a).

(c) Assume $\bar{\beta} = 0$. Determine if the linearized system obtained in (b) is controllable.

2. Given the plant

$$u(s) \longrightarrow \boxed{\frac{1}{s^2}} \longrightarrow y(s)$$

find, if possible, constants α, β for the controller

$$u(s) = (\alpha s + \beta) y(s)$$

so that:

(1) The resultant closed loop system is asymptotically stable

(2) The performance index

$$J = \int_0^\infty \{y^2(\tau) + u^2(\tau)\} d\tau$$

is minimized for all values of $y(0), \dot{y}(0)$.

3. (a) Consider the following system:

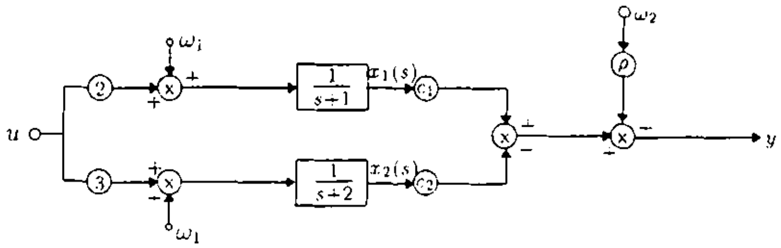
$$\dot{x} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -2 & -\theta \\ 0 & 0 & -3 \end{pmatrix} x + \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} u$$
$$y = \begin{pmatrix} 0 & 1 & 1 \\ c_0 & c_1 & c_2 \end{pmatrix} x$$

For what values of c_0, c_1, c_2, θ is the system

(i) degenerate

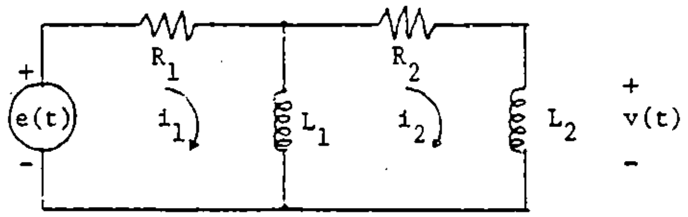
(ii) minimum phase

(b) Consider the system:



where ω_1, ω_2 are uncorrelated white noise sources, $\rho \geq 0$. Here u and y are measurable signals and ω_1, ω_2 are unmeasurable noise disturbances, and it is desired to use a Kalman Filter to estimate the signals $x_1(t) = \mathcal{L}^{-1}(x_1(s)), x_2(t) = \mathcal{L}^{-1}(x_2(s))$. How should the parameters c_1, c_2 be chosen so that x_1, x_2 can be estimated "as best as possible" when $\rho \to 0$?

4. Consider the following circuit:



where the output voltage $v(t)$ and input voltage $e(t)$ can be measured. Assume that $R_1 = 1\Omega, R_2 = 1\Omega, L_1 = 1H, L_2 = 1H$. Design, if possible, an observer which estimates the currents $i_1(t), i_2(t)$ so that the observer has poles $= -\rho, -\rho$ where $\rho = 5$.

5. (a) Given the asymptotically stable system:

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

where $x(0) \neq 0$, assume the input

$$u = \bar{u}e^{\theta t}, \quad t \geq 0, \quad \text{where } \theta > 0$$

is applied at $t = 0$. Determine the output response of the system.

(b) Given the SISO plant $y(s) = g(s)u(s)$ described by:

$$\dot{x} = Ax + bu \qquad y \in R^1, u \in R^1$$
$$y = cx + u$$

assume that the compensator $u(s) = c(s)v(s)$ described by

$$\dot{z} = (A - bc)z + bv$$
$$u = -cz + v$$

is applied to the plant. Show that the output of the resultant controlled system has the property that :

$$y(t) = ce^{(A-bc)t}(x(0) - z(0)) + v(t)$$