Department of Electrical and Computer Engineering
University of Toronto

ECE 431 (ECE431H1F), Digital Signal Processing (DSP); Prof. S. Mann

Thursday December 13, 2001, 9:30am, in room HA-410

Type X ("aids allowed"); there are five questions worth equal weight.

Note: Please write your answers in the answer book only. All answers should be properly explained by providing appropriate detailed arguments, reasoning, formulas, diagrams, etc..

Q1a Let $x = [1, 2, 3, 4]$; calculate the 4 point discrete Fourier transform, X=fft(x). Compute the real part, X_r of X, the imaginary part, X_i of X, the even part, X_e of X, and the odd part, X_o of X. Hint: each of these four answers should contain four samples.

Q1b Compute the 2 point DFT of the following signals: X_{02} being the DFT of [1, 3] and X_{13} being the DFT of [2, 4].

Q1c Is the 2 point DFT of a real signal of length 2 always real, and if so, why?

Q1d Show how $X[k]$ can be reconstructed using the results in Q1b. Hint: write each sample of $X[k]$ as a linear combination of the samples of X_{02} and X_{13}.

Q1e Now consider $g \in \mathbb{R}$, where g is an odd function. (Recall that for an odd function, $g[0] = 0$ and $g[n] = -g[N - n]\ \forall 1 < n < N$, where N is the number of samples in g.) If g has a length of four samples, how many degrees of freedom are there in g?

Q2a Consider a **real** signal h having an arbitrary length N. We desire to find the N point DFT of h, as defined by: $H[k] = \sum_{n=0}^{N-1} h[n] e^{\frac{-2\pi jnk}{N}}$, using a single M point DFT. What's the minimum M required, given no additional knowledge of h.

Q2b Show how you can compute $H[k]$ using a single $M < N$ point DFT. Hint: consider a decimation-in-time or decimation-in-frequency approach, combined with some thoughts on question 1.

Q2c In situations where h is an odd function, what is the minimum M required to obtain the N DFT of h from the M point DFT of h.

Q2d Show how you would obtain the N point DFT of a real and odd function using a single $M < N$ point DFT, where M is as found in Q2c.

Q3 Design a single pole causal lowpass filter, $H(z)$ having real coefficients $h[n]$ and having -40dB gain at 22.05kHz. You are to use the bilinear transform, with $T = 1$. For your final design, provide both $H(z)$ and its impulse response $h[n]$.

Q4a Explain how you would rescale an array having dimension 400 (down) by 600 (across) up to an array having dimensions 480 by 640. Hint: recall the resampling (resizing) done in lab 2.