UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING

MIE 222S - MECHANICS OF SOLIDS I
FINAL EXAMINATION - APRIL 18, 2001
PROFESSOR S.A. MEGUID

This is a closed book examination.
Useful formulae are provided on pages 6 and 7
Calculators may be used.
Attempt all questions

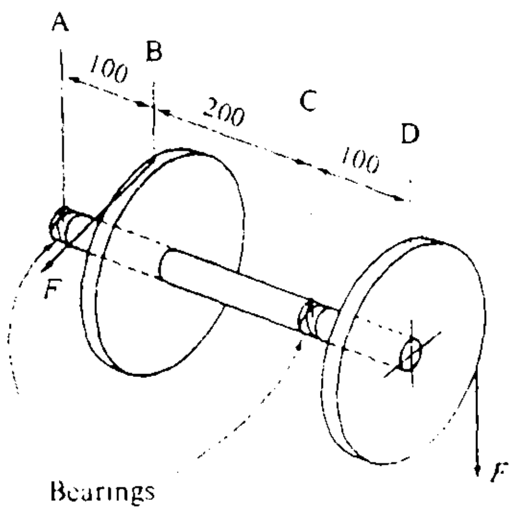
Question #1 [20 Marks]

Explain why we cannot use the formula $\tau = \frac{T\ r}{J}$ for non-circular cross sections.

Figure Q1 shows two pulleys B and D of diameter 200 mm attached to a 40 mm diameter solid shaft, which is supported by two bearings at A and C.

i) Sketch the shear force and bending moment diagrams.
ii) If the maximum allowable shear stress is limited to 40 MPa, calculate the largest magnitude of the applied forces **F**.

You may use the maximum shear yield criterion (Tresca) for your design.



All dimensions are in mm

Fig. Q1

Question #2 [20 Marks]

For the beam with the cross section shown in figure Q2, discuss the limitations of using the shear formula $\tau = \dfrac{VQ}{It}$ at section A.

The cantilever beam shown in figure Q2 is made of a C section with the dimensions indicated below. If a vertical force **F** = 40 kN is applied through its shear centre at the free end of the beam, calculate:

(i) the location of the shear center "e", and
(ii) the shear stress at section B.

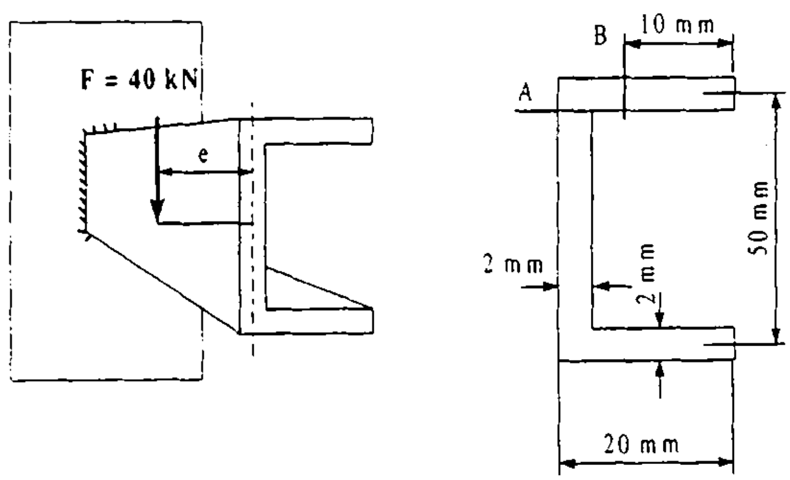


Fig. Q2

Question #3 [20 Marks]

Explain what is meant by Poisson's ratio.

The cylinder shown in figure Q3, is subjected to an internal pressure P = 2 MPa and a twisting moment M_t. Given that the mean radius of the cylinder r_m = 200 mm and the wall thickness t = 2 mm, calculate:

i) the max twisting torque M_t that can be applied, using the maximum shear stress theory (Tresca), and

ii) the principal strains.

You may assume that Young's modulus of the material E = 200 GPa, Poisson's ratio ν = 0.28 and the allowable shear stress τ_{all} = 70 MPa.

Treat as a closed cylinder.

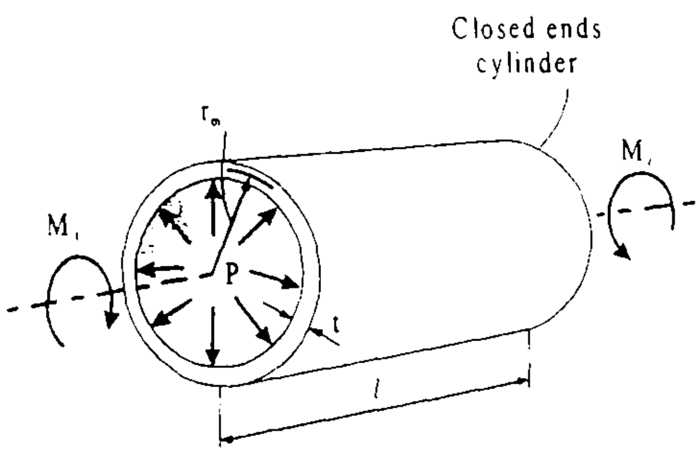


Fig. Q3

Question #4 [20 Marks]

List the methods and state the principles used in determining the deflection of beams. Why is that aspect of stress analysis important in the design of mechanical components?

Starting from a suitable expression for load intensity or the bending moment at any point, find the deflection curve for the uniform beam of a flexural rigidity EI, shown in figure Q4.

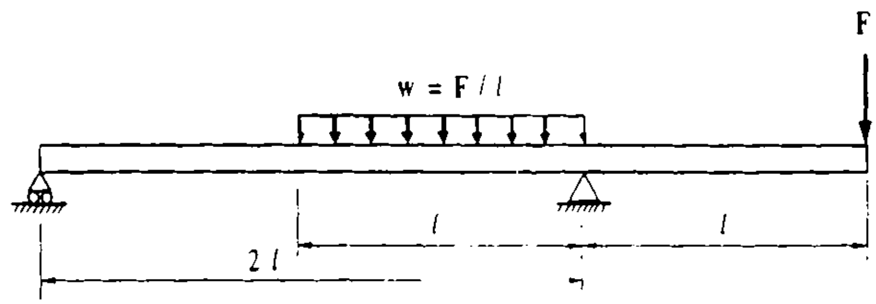


Fig. Q4

Question #5 [20 Marks]

Explain what is meant by statically indeterminate structures.

A copper tube 20 cm long and having a cross-sectional area of 2×10^3 mm^2 is placed between two very rigid caps made of Invar, which shows no strain with temperature change as shown in figure Q5. Two 20 mm diameter steel bolts are arranged parallel to the axis of the tube and are lightly tightened.

Find the stress in the tube if the temperature of the assembly is raised from 20 ^{0}C to 70 ^{0}C.

Take E_{cu} = 100 GPa , $\alpha_{cu} = 2 \times 10^{-5}\ {}^0C^{-1}$, E_S = 200 GPa , $\alpha_S = 1.2 \times 10^{-5}\ {}^0C^{-1}$

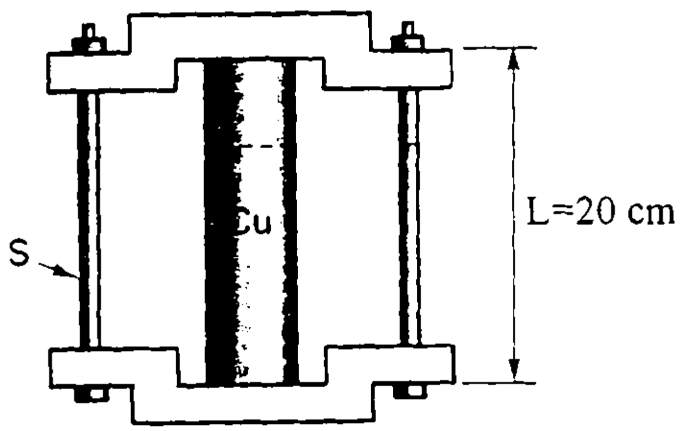


Fig. Q5

USEFUL FORMULAE

$$\Delta = \int \frac{Pdx}{AE}$$

$$\phi = \int \frac{Tdx}{JG}$$

$$\phi\, r = \gamma\, l$$

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu(\sigma_y + \sigma_z)\right)$$

$$\varepsilon_y = \frac{1}{E}\left(\sigma_y - \nu(\sigma_x + \sigma_z)\right)$$

$$\varepsilon_z = \frac{1}{E}\left(\sigma_z - \nu(\sigma_x + \sigma_y)\right)$$

$$\varepsilon_{thermal} = \alpha \Delta T$$

$$\gamma_{xy} = \frac{1}{G}\tau_{xy} \quad , \quad \gamma_{xz} = \frac{1}{G}\tau_{xz} \quad , \quad \gamma_{yz} = \frac{1}{G}\tau_{yz}$$

$$G = \frac{E}{2(1+\nu)}$$

$$\sigma = \frac{F}{A}$$

$$\sigma = \frac{My}{I}$$

$$\tau = \frac{Tr}{J}$$

$$\tau = \frac{VQ}{It}$$

$$\tau = \frac{q}{t}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2_{xy}}$$

For a circular section:

$$A = \pi R^2 \quad , \quad I = \frac{\pi R^4}{4} \quad , \quad J = \frac{\pi R^4}{2}$$

For a rectangular section:

$$A = bh\, , \quad I = \frac{bh^3}{12}$$

For thin-walled cylinders

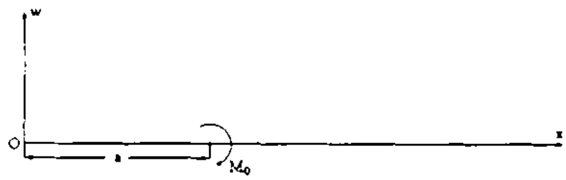
$$\sigma_\theta = \frac{p\,r}{t}$$

$$\sigma_z = \frac{p\,r}{2t}$$

For beam bending, the singular functions are:

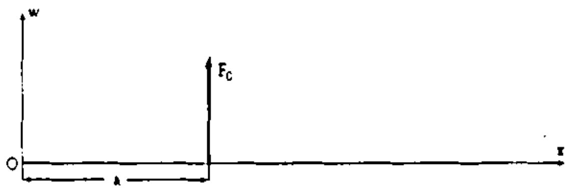
For concentrated moment

$$w(x) = M_0 \langle x - a \rangle^{-2}$$



For concentrated force

$$w(x) = F_0 \langle x - a \rangle^{-1}$$



For uniformaly distributed load

$$w(x) = w_0 \langle x - a \rangle^{0}$$