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UNIVERSITY OF TORONTO
Faculty of Applied Science and Engineering

FINAL EXAMINATION, April 27 1998
First Year – Program 7,9

ECE115S – Electricity and Magnetism
Exam Type: A
Examiners – M.L.G. Joy
T.E. van Deventer

Closed book.

Only the following calculators will be allowed: Casio 991; Sharp 520; Texas Instruments 30.

Answer the questions in the spaces provided or on the facing page.

All questions have equal weight.

For numerical answers specify units.

1	2	3	4	5	6	7	TOTAL

$e = 1.6 \times 10^{-19}\,[C]$ $\qquad$ $m_e = 9.11 \times 10^{-31}\,[kg]$

$\mu_0 = 4\pi \times 10^{-7}\left[\frac{T \cdot m}{A}\right]$ $\qquad$ $\epsilon_0 = 8.85 \times 10^{-12}\left[\frac{C^2}{N \cdot m^2}\right]$

$\int \frac{x\,dx}{(x^2+a^2)^{3/2}} = \frac{1}{(x^2+a^2)^{1/2}}$ $\qquad$ $\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2 (x^2+a^2)^{1/2}}$ Question 5!

$\int \frac{dx}{\sqrt{x^2+a^2}} = ln\left(x + \sqrt{x^2 + a^2}\right)$ $\qquad$ $F = \frac{1}{4\pi\epsilon_0}\frac{|q_1||q_2|}{r^2}$

$\boldsymbol{\tau}_E = \mathbf{p} \times \mathbf{E}$ $\qquad$ $E = \frac{1}{2\pi\epsilon_0}\frac{p}{z^3}\,(dipole)$

$E = \frac{1}{4\pi\epsilon_0}\frac{qz}{(z^2+R^2)^{3/2}}\,(ring)$ $\qquad$ $E = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{z}{\sqrt{z^2-R^2}}\right)(disk)$

$\epsilon_0 \oint \mathbf{E} \cdot d\mathbf{A} = q_{enc}$(free space) $\qquad$ $E = \frac{\sigma}{\epsilon_0}$ (conducting surface)

$E = \frac{\sigma}{2\epsilon_0}$ (insulating surface) $\qquad$ $\Delta V = V_f - V_i = -\frac{W}{q} = -\int_i^f \mathbf{E} \cdot ds$

$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$ $\qquad$ $E_x = -\frac{\partial V}{\partial x},\quad E_y = -\frac{\partial V}{\partial y},\quad E_z = -\frac{\partial V}{\partial z}$

$q = CV$ $\qquad$ $C = \frac{\epsilon_0 A}{d}$ (plates)

$C = 2\pi\epsilon_0 L \,/\, ln\,(b/a)$ (cylinder) $\qquad$ $C = 4\pi\epsilon_0\, ab \,/\, (b-a)$ (spherical capacitor)

$C = 4\pi\epsilon_0 R$ (sphere) $\qquad$ $\epsilon_0 \oint K\mathbf{E} \cdot d\mathbf{A} = q_{enc}$(dielectric)

$I = \frac{dq}{dt}$ $\qquad$ $R = \frac{V}{I}$

$\mathbf{E} = \rho \mathbf{J}$ $\qquad$ $P = VI$

$EMF = \frac{dW}{dq}$ $\qquad$ $W = \frac{1}{2}Vq$

$\boldsymbol{\tau}_B = \boldsymbol{\mu} \times \mathbf{B}$ $\qquad$ $\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$

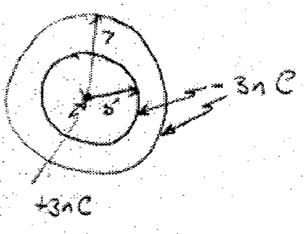
$\mathbf{F}_B = I\,\mathbf{L} \times \mathbf{B}$ $\qquad$ $d\mathbf{B} = \frac{\mu_0}{4\pi} I ds \times \mathbf{r} \,/\, r^3$

$B = \mu_0 I\,\Phi \,/\, 4\pi R$ (arc) $\qquad$ Question 5 $\qquad$ $B = \mu_0 I\, n$ (solenoid)

$B = \mu_0 I \,/\, 2\pi R$ (wire) $\qquad$ $\oint B \cdot ds = \mu_0 I_{enc}$

$EMF = \oint \mathbf{E} \cdot ds = -\frac{d\Phi}{dt}$ $\qquad$ $\Phi_B = \int \mathbf{B} \cdot d\mathbf{A}$

1. An insulating spherical shell with inside radius $5cm$ and outside radius $7cm$ has a point charge of $+3nC$ at its centre. Charges of $-3nC$ are uniformly distributed over the inner and over the outer surface of the shell.



(a) Compute the charge density, σ, on the inner surface.

$\sigma = \frac{Q}{4\pi R^2}$ where $r = 0.05m$

$Q = -3\times10^{-9}C$

Ans. $\sigma = \frac{-3\times10^{-9}}{4\pi(0.05)^2} = 9.55\times10^{-8}\ C/m^2$ //

(b) Derive an expression for the electric field, $E(r)$, as a function of the distance, r, from the centre of the shell. Take the outward direction as positive.

Gauss' Law: Gaussian Surface is a sphere radius r.

$\therefore\ E(r)\cdot 4\pi r^2 = Q_{encl}/\varepsilon_0$

$$\vec{E}(r) = \begin{cases} +3\times10^{-9}/4\pi\varepsilon_0 r^2 = 27/r^2 & 0<r<5cm \\ 0/4\pi\varepsilon_0 r^2 = 0 & 5cm<r<7cm \\ -3\times10^{-9}/4\pi\varepsilon_0 r^2 = -27/r^2 & 7cm<r \end{cases}$$

(c) Evaluate the magnitude and direction of the electric field, $E(r)$, just inside and just outside the outer surface.

From (b) Just inside $5<r<7$ so $E = 0$ //

Just outside $7<r$ so $E = -27/(.07+\epsilon)^2 \quad \epsilon \ll 7$

$= \frac{-27}{(0.07)^2} = -5{,}510\ V/m$ //

(ie Directed inward)

(d) Compute the electric potential difference, V_{IO}, at the inner surface with respect to the outer surface.

$V_{IO} = 0$ Since $V_{IO} = -\int_O^I \vec{E}\cdot\vec{dl}$ + $\vec{E} = 0$

(e) Suppose (for this part only) that a $+2pC$ point charge is placed outside the shell but close to its outer surface. Compute the magnitude and direction of the force on this point due to the other point charge and charge distributions.

Force is inwards $= EQ = -5{,}510\times2\times10^{-12} = -11.0\ nN$ //

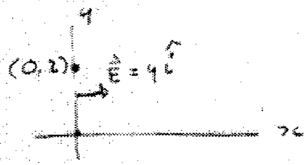
(f) If the $+3nC$ point charge inside the shell was moved a small distance away from the centre and released, would it be attracted to the negative inner surface? Justify your answer.

No // The spherical distribution of charge produce No field inside the cavity. NB the $-6nC$ only acts as if it were at the center for $r>7cm$

2. An electric field $\mathbf{E} = y\,\hat{\mathbf{i}} - x\,\hat{\mathbf{j}}\ \left[\frac{V}{m}\right]$. The field does not vary in the $\hat{\mathbf{k}}$ direction.

(a) Compute the following line integrals, $\int \mathbf{E} \cdot d\mathbf{s}$, along a straight line.

1. From the origin to point $(0,2)\,[m]$.



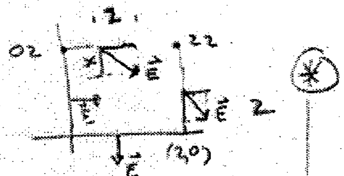
On this line $x = 0$ $\therefore \vec{E} = y\,\hat{i}$

$\therefore \vec{E}$ is $\perp$ the line.

$\therefore \vec{E} \cdot \vec{\Delta L} = 0$

$\therefore \int_{(0,0)}^{(0,2)} \vec{E} \cdot d\vec{l} = 0$

2. From point $(0,2)\,[m]$ to point $(2,2)\,[m]$.



On this line $y = 2$

$\therefore E = 2\hat{i} - x\hat{j}$

$\vec{\Delta l} = \Delta s\,\hat{i} + 0\,\hat{j}$

$\vec{E} \cdot \vec{\Delta l} = 2\Delta s$. ie The component of E along the line.

$\therefore \int_{02}^{22} \vec{E} \cdot d\vec{l} = \int_0^2 2\,ds = 2\left[s\right]_0^2 = 4$

(b) How much work would you have to do to move a point charge q around a square closed path from $(0,0)$ to $(0,2)$ to $(2,2)$ to $(2,0)$ and back to $(0,0)$?

[This was a very hard part and only 2 or 3 students got it!]

Method. $-q\vec{E}\cdot d\vec{l}$ is the work you do to move $+1\,C$ along this path. $\therefore$ compute $-q\int \vec{E} \cdot d\vec{l}$. Since E may not be conservative (Chapter 31) we cannot assume this is zero! From the diagram (*) it can be seen that $\vec{E} \cdot d\vec{l}$ is either 2 or zero on each segment of the path.

Thus Work $= -q\oint \vec{E} \cdot d\vec{l} = -q\left[0 + 4 + 4 + 0\right] = -q \cdot 8\ [J]$

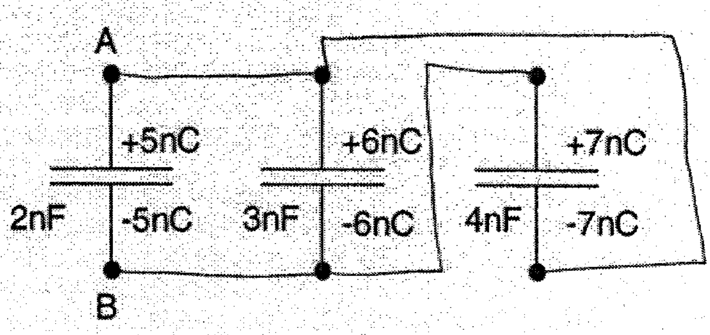
(c) Is the field $\mathbf{E}$ produced by a charge distribution at rest (*i.e.* stationary)? Justify your answer.

If it is produced by a stationary charge distribution then $\oint \vec{E} \cdot d\vec{l} = 0$ (always). If it is not then $\oint \vec{E} \cdot d\vec{l}$ may not equal zero. From (b) we see $\oint E \cdot dl \neq$ zero

$\therefore$ The field $\underline{E}$ cannot be produced by stationary charges. Answer is NO//

3. Consider three capacitors of $2nF$, $3nF$ and $4nF$.

(a) The capacitors are charged with $5nC$, $6nC$ and $7nC$, as shown in the diagram..



1. Show, in the diagram above, how to connect the three charged capacitors in parallel so that the voltage, V_{AB}, across the combination is minimized.
2. Compute this minimum voltage, V_{AB}.

The charge on plate connected to Node A is

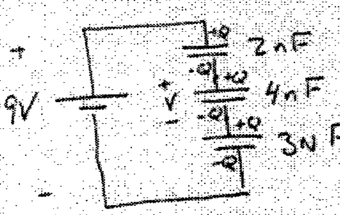
$+5+6-7 = +4\ nC$

The Equivalent capacitance is $(2+3+4) = 9nF = C_{eq}$.

$V_{AB} = Q/C_{eq} = \frac{4\ nC}{9\ nf} = 4/9\ [V] = 0.4\dot{4}\ [V]$

(b) The three capacitors of part 3(a) are now discharged and then connected, in series, to a 9 volt battery.

1. Compute the charge on the plates of the $3nF$ capacitor.



It does not matter what order you use, the answer for Q will be the same! / $C_{eqs} = 0.923\ nF$

$Q = C_{eqs} \cdot 9V = 8.31\ nC$ //

2. Compute the voltage across the $4nF$ capacitor.

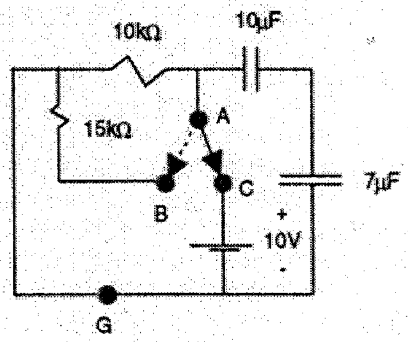
Again $Q = 4nF \cdot V \quad \therefore V = \frac{8.31\ nC}{4\ nF} = 2.077\ [V]$

3. Does the order of the capacitors in the series connection affect the answer to parts 3(b)1 and 3(b)2? Justify your answer.

See above Answer $\boxed{\text{No}}$ because the charge on each capacitor in series is identical!

4. Consider the circuit shown in the following diagram:

This was used in Quiz L0201 March 22



(a) The switch has been in position C for a long time:

1. What is the voltage, V_A, of node A?

   10V

2. What is the current, i, through the $10k\Omega$ resistor?

   1 mA

3. What is the power dissipated in the $10k\Omega$ resistor?

   10 mW

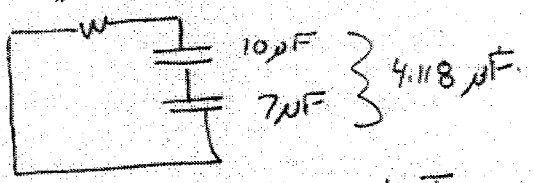
4. What is the total energy stored in the two capacitors?

   206×10^{-6} J

(b) At time $t = 0$ the switch is moved to position B.

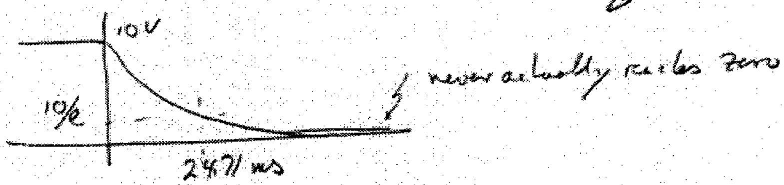
1. Draw the equivalent single loop circuit that exists for $t > 0$.

   $10 \| 15 = 6\,k\Omega$



∴ Time const $= 6 \times 10^3 \times 4.118 \times 10^{-6}$
$= 24.71$ ms

2. Sketch $V_A(t)$ for both positive and negative t.

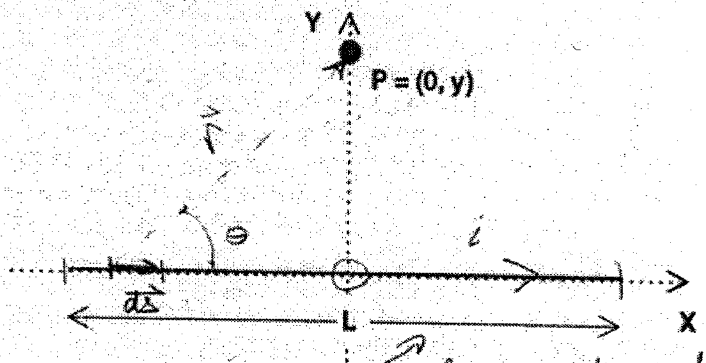


3. How long does it take for $V_A(t)$ to reach $1V$?

   $V_A(t) = 10e^{-t/24.71} = 1$ for $t = 57$ ms.

Solve with logs $\quad -t/24.71 = \log_e(0.1)$ . . .

5. In the figure below a straight wire segment of length L carries a current i. You are to derive the formula for the magnitude of the magnetic flux density, $\mathbf{B}(y)$, arising, at a point P, from the current in the wire segment. Assume P is at a distance y from the segment and on its perpendicular bisector.



for consistency define i left →

(a) If the current flows from left to right in the diagram, what is the direction of the magnetic flux density, $\mathbf{B}(y)$, at point P?

Use right hand rule. $\vec{B}(0,y)$ will be out of the page //

(b) Using the Biot-Savart relation,

$$d\mathbf{B} = \frac{\mu_o}{4\pi}\frac{i d\mathbf{s}\times\mathbf{r}}{r^3},$$

set up the integral that will give the formula for $\mathbf{B}(y)$.

$$\vec{dB} = \frac{\mu_0}{4\pi}\frac{i\, dx\, |\vec{r}| \sin\theta}{|\vec{r}|^3} \qquad \sin\theta = \frac{y}{|\vec{r}|} \qquad |\vec{r}|^2 = x^2+y^2$$

$$\therefore\ B(y) = \int_{-L/2}^{+L/2} \frac{\mu_0 i}{4\pi}\frac{y}{(x^2+y^2)^{3/2}}\, dx \quad //$$

See front page a

(c) Calculate $\mathbf{B}(y)$.

$$B(y) = \frac{\mu_0 i y}{4\pi}\int_{-L/2}^{+L/2}\frac{dx}{x^2+y^2} = \frac{\mu_0 i y}{4\pi}\left[\frac{x}{y^2(x^2+y^2)^{\frac{1}{2}}}\right]_{-L/2}^{+L/2}$$

$$= \frac{\mu_0 i y}{4\pi}\frac{1}{y^2}\left[\frac{x}{(x^2+y^2)^{3/2}}\right]_{-L/2}^{+L/2} \quad //$$

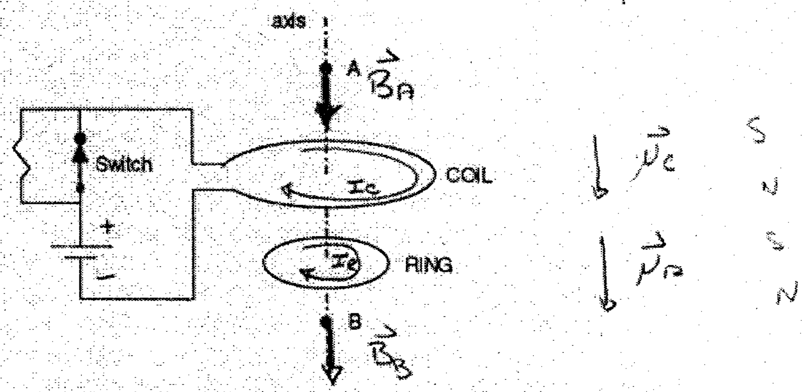
(d) Check your result in part 5(c) against the formula for a wire of infinite length.

if $L \to \infty$ then

$$B_y = \frac{\mu_0 i}{4\pi}\frac{1}{y}\left[1-(-1)\right] = \frac{\mu_0 i}{2\pi}\frac{1}{y}. \quad //$$

This agrees with Formula on front page $R = y$!

6. A thin metallic ring lies below a resistive coil which is connected to a circuit consisting of a battery, a resistor and a switch, as shown below.

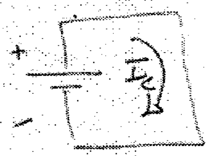


For each question below, justify your answer and mention which physical "rule" or "law" describes the given phenomenon.

(a) The switch has been closed for a long time:

1. Show, with an arrow on the diagram, the direction of current flow in the coil, I_C.

See above
Ohms Law



2. Show, with an arrow on the diagram, the direction of the magnetic flux density at points A and B on the axis of the coil, $\mathbf{B}_A$ and $\mathbf{B}_B$.

Right hand rule.

(b) The switch is opened at time $t = 0$. The current in the coil continues to flow in the same direction as it did before the switch was opened and gradually drops.

1. Show, with an arrow on the diagram, the direction of induced current flow in the ring, I_R, for $t > 0$. (31.4 part 2)

Lenz's law, Downward flux will be decreasing so Ring will provide downward flux to oppose this change. ∴ Induced current I_R will flow in direction shown

2. When the switch is opened, is the ring attracted to (or repelled by) the coil. Justify your answer. (31.4 part 1) $\vec{\mu}_C + \vec{\mu}_R$

The magnetic dipole moments of coil + ring are alligned. There will therefore be an attractive force on the ring //

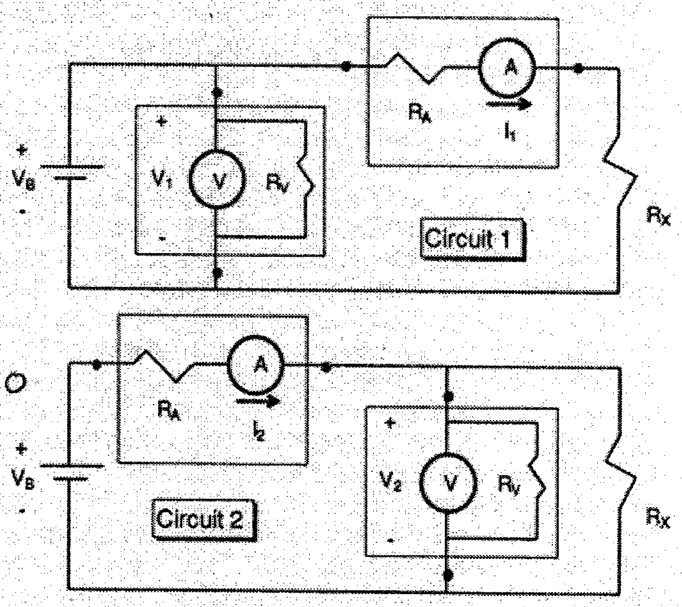
7. Two digital multimeters are used to measure the resistance, R_X, of an *unknown* resistor. One multimeter, A, is set up as an ammeter and has an internal resistance, R_A. The other multimeter, V, is set up as a voltmeter and has an internal resistance, R_V.

Two circuits are set up as shown in the diagram below. The circles in the diagram represent "ideal" meters.

Notes ✱

① $V_1 = V_B$

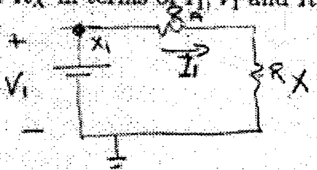
② $I_2 R_A + V_2 - V_B = 0$



The multimeter readings, (I_1, V_1) and (I_2, V_2), are recorded. The battery voltage, V_B, is the same for both circuits.

(a) For Circuit 1, derive an equation for R_X in terms of I_1, V_1 and R_A.

use Loop method ∴



$I_1(R_A + R_X) = V$

$\therefore R_X = \dfrac{V_1}{I_1} - R_A$ //

(b) For Circuit 2, derive an equation for R_X in terms of I_2, V_2 and R_V.

By Ohms Law $\quad V_2 = I_2 (R_V \| R_x)$

$\therefore V_2 \left[\frac{1}{R_V} + \frac{1}{R_x}\right] = I_2$

$\dfrac{1}{R_x} = \dfrac{I_2}{V_2} - \dfrac{1}{R_A}$ //

(c) Show that the resistance, R_A, can be computed from V_1, V_2 and I_2.

See notes ✱ above. from ② $R_A = \dfrac{V_2 - V_B}{I_2} = \dfrac{V_2 - V_1}{I_2}$ //

from ①

(d) In Circuit 1, under what condition is $R_X = \frac{V_1}{I_1}$? Is this realistic?

If $R_A \ll R_X$. // Because $\dfrac{V_1}{I_1} = R_A + R_X$

(e) In Circuit 2, under what condition is $R_X = \frac{V_2}{I_2}$? Is this realistic?

If $R_V \gg R_X$ // Because. $\dfrac{I_2}{V_2} = \dfrac{1}{R_V} + \dfrac{1}{R_X}$