UNIVERSITY OF TORONTO
DEPARTMENT OF ELECTRICAL & COMPUTER ENGINEERING
FINAL EXAMINATION - DECEMBER 10, 1998
ECE355F - SYSTEM AND SIGNAL ANALYSIS I
Third Year - Programs 5ce, 5e, 5bm
EXAMINER - W.M. Wonham

No aids permitted, other than a calculator.
PLEASE ANSWER EACH OF THE THREE MAIN QUESTIONS IN A SEPARATE BOOKLET.
Marking scheme: Each of the three main questions is worth 1/3 of the total mark.

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1. Let $f(t) = e^{-\alpha|t|}$, $-\infty < t < \infty$, where $\alpha > 0$. Define

$$g(t) := \sum_{k=-\infty}^{\infty} f(t+2k), \qquad -\infty < t < \infty$$

1.1 Calculate $g(0)$.

1.2 Show that g is periodic.

1.3 Calculate the complex exponential Fourier series (CEFS) of g.

1.4 As usual, define the Nth partial sum of CEFS as

$$g_N := \sum_{n=-N}^{N} \ldots$$

and the corresponding approximation error as $e_N(t) := g(t) - g_N(t)$. Estimate the maximum error magnitude

$$\max(e_N) := \max\{|e_N(t)| \ : \ -\infty < t < \infty\}$$

as a function of N and α, when N is large (i.e. $N\pi >> \alpha$).

1.5 Estimate a value of N sufficient to guarantee that $\max(e_N)/g(0) < 0.001$. Sketch the dependence of your estimate on α.

1.6 Calculate the average power of g at zero frequency (i.e. 'd.c. power' of g).

1.7 Estimate a value of N sufficient to guarantee that the ratio

$$\frac{\text{power of } e_N}{\text{d.c. power of } g}$$

is less than 0.001. Sketch the dependence of your estimate on α.

2. Let $f(t)$, $g(t)$ $(-\infty < t < \infty)$ be complex-valued signals having Fourier transforms $\hat{f}(\omega)$, $\hat{g}(\omega)$ $(-\infty < \omega < \infty)$. Let $h(t) = f(t)g(t)$ $(-\infty < t < \infty)$, and assume that $\hat{h}(\omega)$ exists $(-\infty < \omega < \infty)$.

2.1 Show carefully that

$$\hat{h}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{g}(\alpha)\hat{f}(\omega - \alpha)d\alpha$$

In your derivation you may assume the validity of Fourier integral representation, and such integration operations as may be needed.

2.2 Suppose that $g(t)$ is the gating function

$$g_T(t) := \begin{cases} 1, & |t| \leq T \\ 0, & |t| > T \end{cases}$$

for some $T > 0$. Show that

$$\hat{h}(\omega) = \int_{-\infty}^{\infty} K_T(\alpha - \omega)\hat{f}(\alpha)d\alpha$$

where

$$K_T(x) = \frac{1}{\pi}\frac{\sin(Tx)}{x}, \qquad -\infty < x < \infty$$

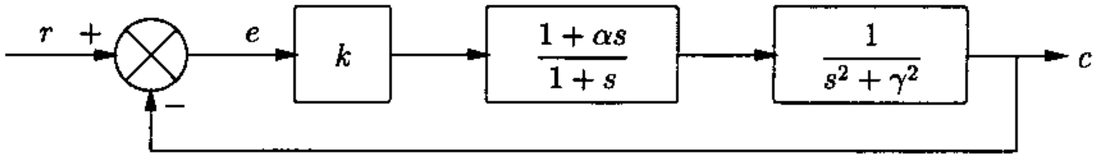
Interpret these results with a discussion and sketch.

2.3 Specialize the results in 2.2 when

$$f(t) = \cos(\omega_0 t) \qquad (-\infty < t < \infty),$$

to obtain a simple explicit formula for $\hat{h}(\omega)$. Note that $\hat{f}(\omega)$ can be expressed in terms of δ-functions. Sketch $\hat{h}(\omega)$ and interpret the effect of varying T. Can you suggest any general implications for the time-gating of a strictly bandlimited signal?

3. A feedback system designed to control an undamped harmonic oscillator has the block diagram shown below. Assume $k > 0$, $\alpha > 0$, $\gamma > 0$.



3.1 Calculate the transfer function

$$\hat{h}(s) := \hat{e}(s)/\hat{r}(s)$$

and identify the characteristic polynomial $p(s)$.

3.2 What condition on the roots of $p(s)$ is necessary and sufficient for BIBO stability? Obtain an equivalent condition in terms of the parameters α, γ, k.

3.3 Determine the critical frequency ω_c at which the system may oscillate on the boundary between stability and instability.

3.4 Suppose the reference input $r(t) = \sin(\omega t)$, $t \geq 0$. Assuming the system is stable, explain how to calculate the frequency, amplitude and phase of the error $e(t)$ for large t. Obtain an approximate (but reasonable) formula for this amplitude $A(\omega)$, when $|\omega - \gamma|$ is small. What can you conclude about the design parameters k, α if good tracking is required for frequencies ω near γ?

3.5 Suppose the reference input $r(t) = 1$, $t \geq 0$. Assuming stability, what can you say about $e(t)$ as $t \to \infty$?

3.6 Suppose a sensor that introduces a small delay is placed in the feedback path connecting c to the comparator $\otimes$. On intuitive grounds, what effect would you expect this to have on the admissible range of k for stability? Justify your answer.