UNIVERSITY OF TORONTO
Faculty of Applied Science and Engineering
FINAL EXAMINATION, DECEMBER 2001
Fourth Year
AER 501F -- Advanced Mechanics of Structures
Examiner – J. S. Hansen

*Note: This is a closed book examination. Calculators are allowed. Answer ALL questions.*

MARKS

1. Answer the following as briefly as possible:

(5) (a) The potential energy for a two degree of freedom system is given by

$$\Phi(x) = \frac{1}{2}\left[x^2 - 3xy + y^2\right] - Px - 2Py$$

where $x, y$ are the generalized coordinates. Is the equilibrium configuration for this system stable or unstable. Why?

(5) (b) Describe the essential differences between forced and natural boundary conditions in structural mechanics problems.

(5) (c) Describe the relation between the methods of Galerkin and Ritz in terms of the relation between the Principals of Virtual Work and a Stationary Value of the Total Potential Energy?

(5) (d) When solving for the natural frequencies of a vibrating shell using the finite element method you determine that the least natural frequency is negative. What does this imply?

(5) (e) The rectangular element illustrated in Figure 1 is to be used in a finite element analysis which requires $C^O$ continuous trial functions. The unknown variables are the deflections in the $x, y$ directions $u(x, y), v(x, y)$.

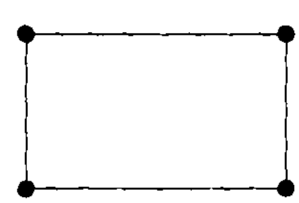

Figure 1

The trail functions are adopted in the form

$$\tilde{u}(x, y) = a_0 + a_1\, x + a_2\, x^2 + a_3\, x^3$$

$$\tilde{v}(x, y) = b_0 + b_1\, y + b_2\, y^2 + b_3\, y^3$$

Are these trial functions acceptable within the context of a finite element analysis using the element shown? Why or why not?

(5) (f) The beam problem illustrated in Figure 2 is to be analyzed using the finite element method. What choice of grid will assist in obtaining an accurate solution? Explain your answer.

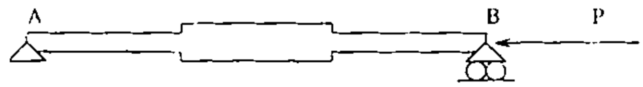


Figure 2

(10) 2. Given the optimization problem

$$F(X_1, X_2) = X_1^3 + 2X_1X_2^2 + 2X_1^2 - X_1X_2 + X_2^2 + 3X_1 - X_2$$

Consider the current design point to be $X_1^\circ = 1; X_2^\circ = 2$. Based on the desire to obtain a minimum value of $F(X_1, X_2)$ determine:

(a) The search direction relative to the current design to be used using the method of steepest descent.

(b) The equations to be solved in the next step using Newton's method.

(40) 3. A curved cantilevered beam is shown in Figure 3. The Total Potential Energy for this problem is

$$\Phi = \frac{EA}{2R}\int_0^{\pi/2} \left[(u + v')^2 + Z(u + u'')^2\right] d\theta - Fv(\pi/2)$$

while the forced boundary conditions are:

$$u(0) = u'(0) = v(0) = 0$$

In the above, primes denote derivatives with respect to $\theta$. In addition, $E, A$ and $R$ are the Young's modulus, the beam cross-sectional area and the radius of the beam centre-line respectively while $Z$ is a shape parameter which is constant.

(a) Using the calculus of variations determine the equilibrium equations and the natural boundary conditions for this problem.

(b) Using the trial functions $\tilde{u}(\theta) = a_1(1 - \cos\theta)$ and $\tilde{v}(\theta) = a_2 \sin\theta$ determine the algebraic equilibrium equations using the method of Ritz.
Note: It is not necessary to solve the resulting equations.

(c) Determine the simplest basis functions which can be used to model $u(\theta), v(\theta)$ within the context of the finite element method.

(20) 4. The symmetric truss shown in Figure 4 is subjected to loads $P_1, P_2$ as illustrated and the response variables $u_1, u_2$ have direction and sign corresponding to these loads. The design variables in the problem are the cross-sectional areas of the bars designated as $A_1, A_2$ and the location of the middle node determined by $Y$. Young's modulus $E$ is the same for all bars and is fixed.

Three load cases are to be considered:

$$\text{Case 1: } P_1^1 = 40; \quad P_2^1 = 0$$
$$\text{Case 2: } P_1^2 = 0; \quad P_2^2 = 25$$
$$\text{Case 3: } P_1^3 = 15; \quad P_2^3 = 15$$

(Note: In the above, the superscript refers to the load case.) The stresses in the bars must satisfy:

$$\text{Bar 1: } -15 \leq \sigma_1 \leq 20$$
$$\text{Bar 2: } -10 \leq \sigma_2 \leq 10$$

Using the displacement approach, formulate the optimal design problem which will yield a minimum weight design. It is not necessary to solve the equations, but the formulation should be completed to a stage where the next step would be to solve the equations.

TOTAL (100)

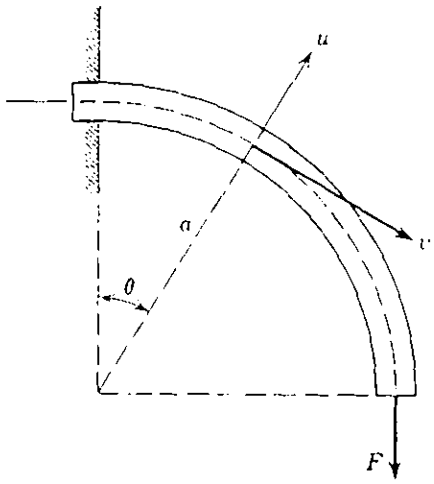


Figure 3

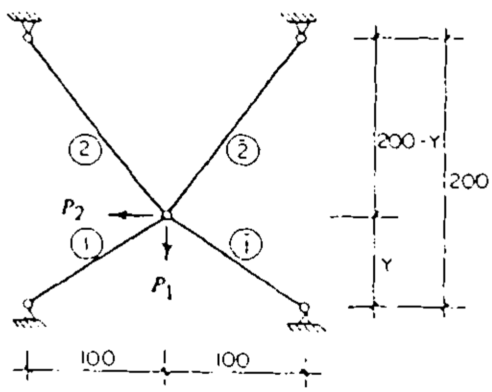


Figure 4