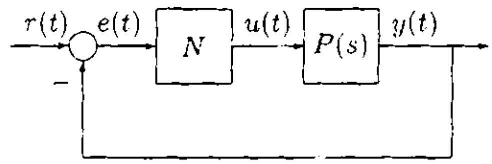
UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING
FINAL EXAMINATION, APRIL 2001
Third Year - Programs 05bme, 05ce, 05e
ECE356S - SYSTEM AND SIGNAL ANALYSIS II
Examiner - B.A. Francis

Aid sheet and non-storage calculator permitted. Attempt all five problems.

---

1. Consider the feedback system



consisting of a nonlinear actuator N characterized by the equation $u = 2e + e^3$, and a linear plant with transfer function

$$P(s) = \frac{s+1}{s(s+2)}.$$

(a) [4 marks] Derive a nonlinear state model for the overall system of the form

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, r), \quad y = g(\mathbf{x}, r).$$

(b) [1 mark] Find all equilibrium points for the nonlinear model.

(c) [3 marks] Linearize about one equilibrium point, ending up with a linear state model of the form

$$\dot{\Delta \mathbf{x}} = \mathbf{A}\Delta\mathbf{x} + \mathbf{B}\Delta r, \quad \Delta y = \mathbf{C}\Delta\mathbf{x} + D\Delta r.$$

(d) [2 marks] Find the transfer function from Δr to Δy.

2. (a) [3 marks] Consider the system model $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$ with

$$\mathbf{A} = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}.$$

Find necessary and sufficient conditions on the real numbers a, b so that the origin is stable (in the sense of Lyapunov). Find necessary and sufficient conditions on a, b so that the origin is asymptotically stable.

(b) [3 marks] Consider the system model $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$ with

$$\mathbf{A} = \begin{bmatrix} 0 & 2 & 0 \\ -2 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}.$$

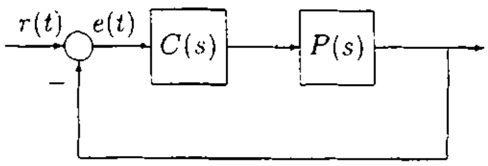
Does there exist an $\mathbf{x}(0) \neq \mathbf{0}$ such that $\mathbf{x}(t) \to \mathbf{0}$ as $t \to \infty$? If so, find one; if not, prove it.

(c) [2 marks] The function

$$y(t) = \begin{cases} 1, & 0 \le t \le 5 \\ 0, & t > 5 \end{cases}$$

is an example of a signal of *finite duration*, whereas $y(t) = \sin(t)$, $y(t) = e^t$, and $y(t) = e^{-t}$ are examples of signals that do not have this property. Write in logic format the statement that $y(t)$ is not of finite duration. (Take the time range to be $0 \le t < \infty$.)

3. Consider the feedback control system



(a) [3 marks] Take $P(s) = 10/(s-1)$. Find a proper controller transfer function $C(s)$ such that the feedback system is stable and such that $e(t) \to 0$ as $t \to \infty$ when $r(t)$ is the unit step.

(b) [3 marks] Take $P(s) = 10/(s-1)$ and $C(s) = K$. Find the minimum $K > 0$ such that the feedback system is stable and the steady-state absolute error $|e(t)|$ is less than or equal to 0.1 for all inputs of the form

$$r(t) = \cos(\omega t), \quad 0 \le \omega \le 2.$$

(c) [5 marks] Take

$$P(s) = \frac{s^2+2}{(s+1)(s^2-2)}.$$

Sketch the Nyquist plot of $P(s)$.

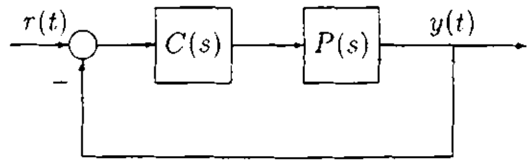
(d) [3 marks] State the Principle of the Argument. Apply this principle to determine how many unstable closed-loop poles there are if $P(s)$ is as in (c) and $C(s) = 2$.

4. (a) [2 marks] Consider the system with transfer function

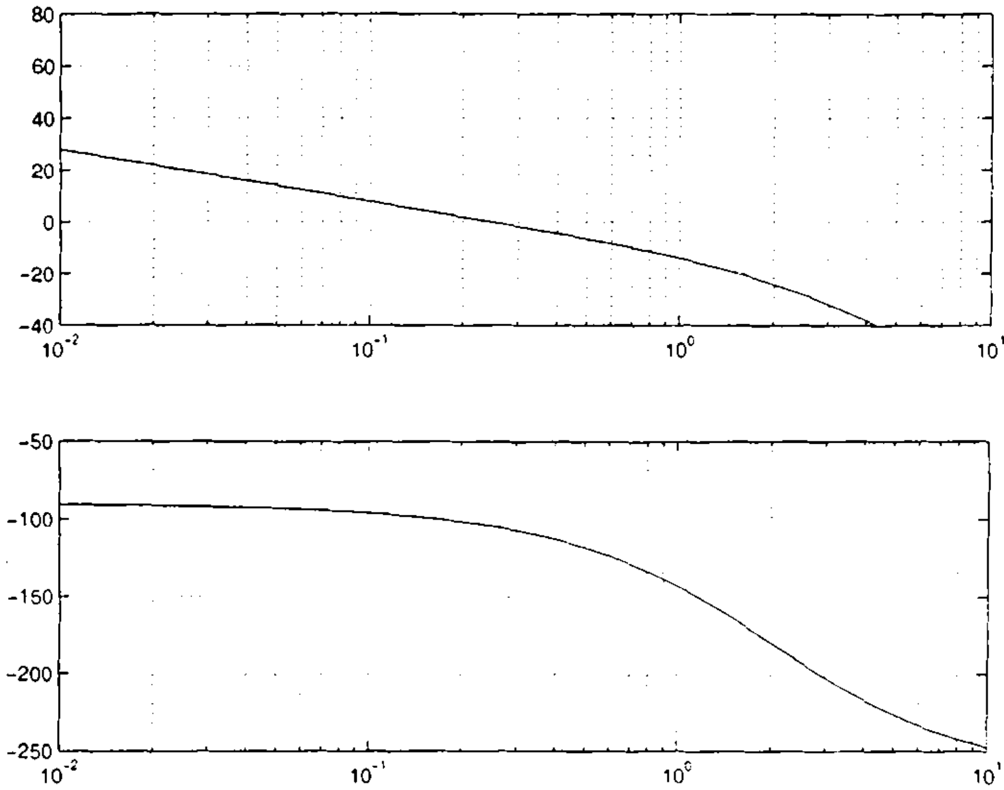
$$G(s) = \frac{s^2+1}{s^2+2s+2},$$

input $u(t)$, and output $y(t)$. What is $y(t)$ in steady state if $u(t) = \cos(2t)$? (You don't have to take any inverse transforms.)

(b) [4 marks] Consider the feedback system



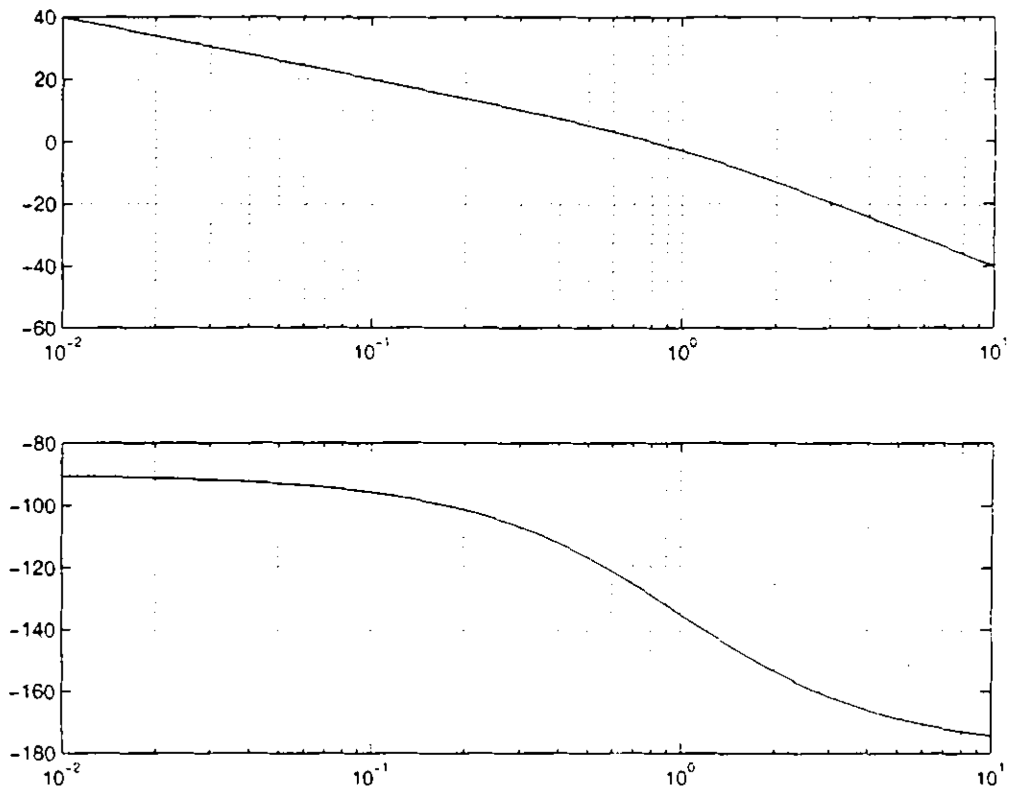
Assume the feedback system is stable and the Bode plot of $P(s)C(s)$ is as below (magnitude in dB, phase in degrees). What is $y(t)$ in steady state if $r(t) = \cos(2t)$?



(c) [4 marks] Same block diagram. The plant transfer function is

$$P(s) = \frac{1}{s(s+1)}$$

and its Bode plot is shown below (magnitude in dB, phase in degrees). Design a lead compensator $C(s)$ to achieve a phase margin of 50° and a gain crossover frequency of 2 rad/s.



5. (a) [3 marks] The signal

$$x(t) = \begin{cases} \cos(2t), & t \geq 0 \\ 0, & t < 0 \end{cases}$$

is sampled at 10 Hz, producing the discrete-time signal $y[k]$. Derive the $z-$transform of $y[k]$, including its region of convergence. [Hint: Convert $y[k]$ to exponential form using Euler's formula.]

(b) [3 marks] Consider the discrete-time system with input $u[k]$ and output $y[k]$ modeled by

$$\begin{aligned} y[0] &= 0 \\ y[k] &= \frac{1}{2}y[k-1] + u[k] - u[k-1], \quad k \geq 1 \end{aligned}$$

Find the transfer function of the system.

(c) [3 marks] Continuing from (b), find the output z-transform $Y(z)$ when $u[k] = 2^k$. Take the inverse z-transform to get an expression for $y[k]$.

(d) [3 marks] Let G denote the series connection of the zero-order hold $D2C$, followed by the system with transfer function $P(s) = \frac{1}{s^2}$, followed by the sampler $C2D$. Assume $D2C$ and $C2D$ are synchronized and of the same period $T = 0.5$. Find the z-transfer function of G.