UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION, APRIL 1998

First year - Programs: 01, 02, 03, 04, 06, and 08

ECE 110S - ELECTRICAL FUNDAMENTALS

NAME:______________________________
(Last) (First)

STUDENT NO.:______________________________

Question	Mark	Maximum
1		25
2		25
3		25
4		25
Total		100

EXAM TYPE: A

EXAMINERS:
- L. de Windt
- H. Hnik
- B. Wang
- S. Zukotynski - Coordinator

INSTRUCTIONS:
1. Type A examination: No aids allowed.
2. Only non-programmable calculators allowed (models: as specified in the Faculty Calendar).
3. Answer all parts of all four questions.
4. All four questions are of equal weight.
5. All work is to be done on these pages.
6. Place your final answer in the corresponding box. You may use the back of the proceeding page for rough work.
7. Do not unstaple this exam book.

$e=1.6\times10^{-19}$ C, $\epsilon_o = 8.85 \times 10^{-12}$ F/m, $\mu_o = 4\pi \times 10^{-7}$ H/m

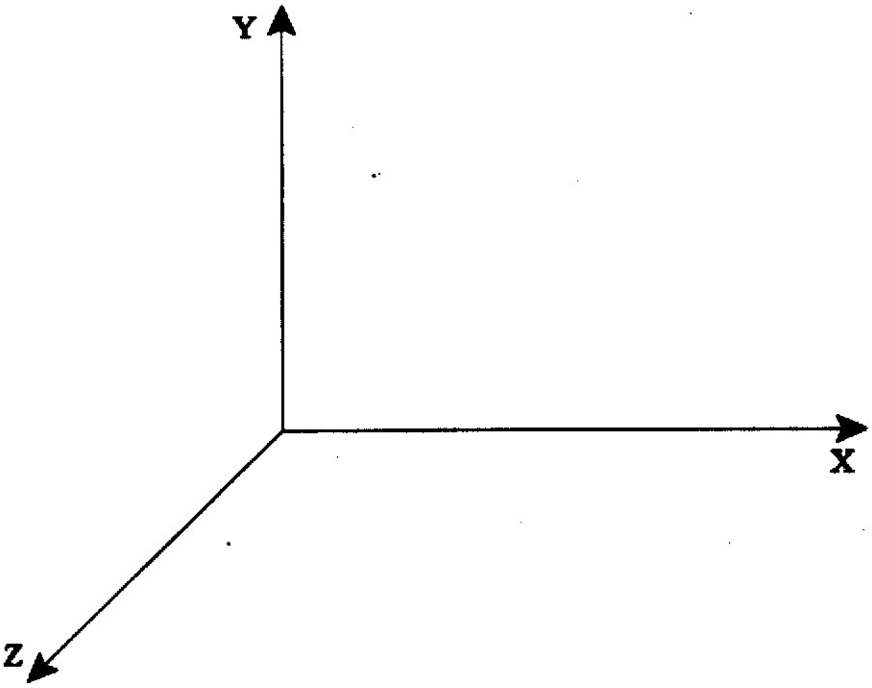
Question 1. Electricity and Magnetism

Part A. The Electric field

Two charges, equal in magnitude, 2 μC, but of opposite sign are 8 cm apart.

i) Place the two charges on the coordinate system below. What is the magnitude and direction of the electric field at the point, P, equidistant from the two charges and 3 cm away from the line connecting the two charges? Give the direction (a unit vector in the direction of $\vec{E}$) in terms of unit vectors $\hat{i}$, $\hat{j}$, $\hat{k}$. Use the coordinate system below to illustrate your answer.

$\lvert E \rvert =$
$\hat{E} =$



ii) What force would act on an electron placed at P? Give both the magnitude and direction of the force. Indicate the force on the figure.

$\lvert F \rvert =$
$\hat{F} =$

Question 1. Electricity and Magnetism

Part B. Capacitors

Two capacitors, $C_1 = 2\ \mu F$ and $C_2 = 8\ \mu F$, are connected in series. A potential difference of 300 V is applied to the pair.

i) What are the charge and potential difference for each capacitor?

$Q_1 =$	$Q_2 =$
$V_1 =$	$V_2 =$

ii) How much energy is stored in each capacitor?

$W_1 =$	$W_2 =$

iii) The capacitors (as charged) are disconnected and reconnected in parallel, with the negative terminals connected together and the positive terminals connected together. What is the charge on each capacitor now and what is the potential across the pair?

$Q_1 =$	$Q_2 =$
$V_1 =$	$V_2 =$

Question 1. Electricity and Magnetism

Part C. Induction

A circular, flat loop of wire 10 cm in diameter is placed in a uniform magnetic field. The magnetic field is normal to the surface of the loop. The loop has a resistance of 1 Ω. The magnetic field is reduced at a uniform rate from 1 T to 0.5 T in 20 s.

i) What is the induced EMF in the loop?

EMF =

ii) What is the current in the loop?

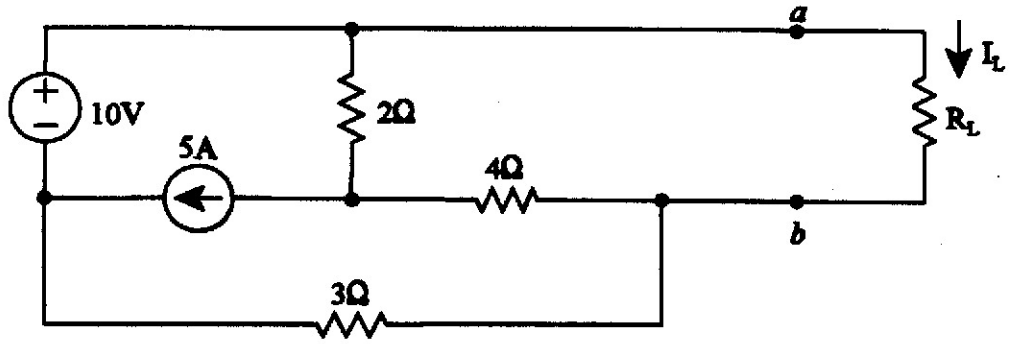
I =

iii) What is the total charge that flows through the loop during the 20 s?

Q =

Question 2. DC Circuit

Both Part (A) and Part (B) refer to the following circuit:



Part A. Nodal analysis

For R_L=3Ω analyze the circuit using nodal analysis.

i) Write a set of equations whose solution yields the nodal voltages. Label the nodes V_1, V_2, V_3, etc.. Specify a reference node.

Set of Equations

Question 2. DC Circuit

ii) Determine the load current I_L.

I_L =

iii) Is the voltage source supplying or absorbing power and how much?

Supplying ☐	Absorbing ☐
P =	

Question 2. DC Circuit

Part B. Thevenin equivalent and superposition

i) Find the Thevenin equivalent circuit to the LEFT of terminals ***a*** and ***b*** using the principle of superposition.

Thevenin equivalent circuit

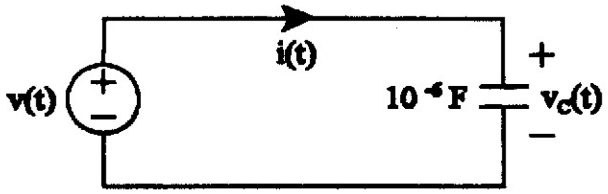
ii) Determine the value of R_L which absorbs the maximum amount of power and find the maximum power.

R_L =
P_{max} =

Question 3. Charging a Capacitor and Transient Analysis

Part A: Charging a capacitor

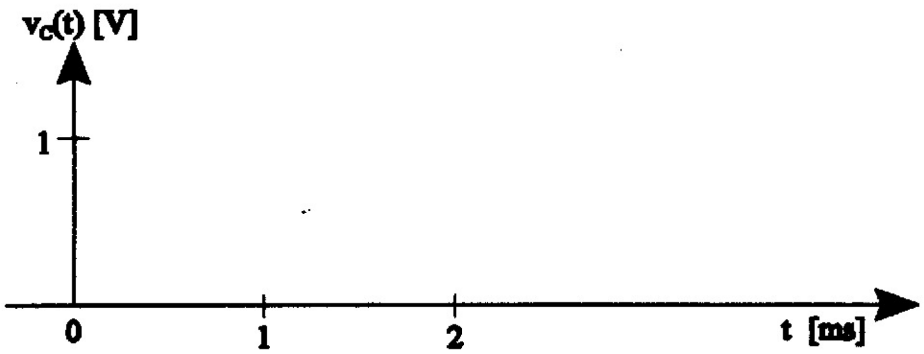
Consider the circuit shown below.



Where the voltage v(t) is specified as follows:

$v(t) = (10^3\ \text{V/s}) \times t$ for $0 \le t < 1$ ms
$v(t) = 1$ V for $1\text{ms} \le t < 2$ ms

i) Sketch the voltage across the capacitor, $v_C(t)$, as a function of time.

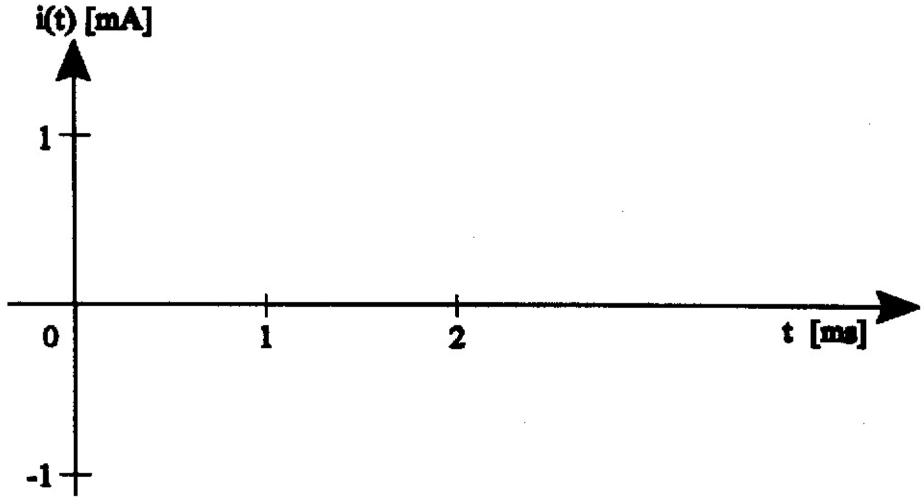


ii) Find the expression for the current, i(t), in the time interval $0 \le t < 2$ms.

i(t) =	$0 \le t < 1$ ms
i(t) =	$1\text{ ms} \le t < 2$ ms

Question 3. Charging a Capacitor and Transient Analysis

iii) Sketch the current in the capacitor, i(t), as a function of time.



iv) How much energy, W, is stored in the capacitor at time t = 1.5 ms.

W =

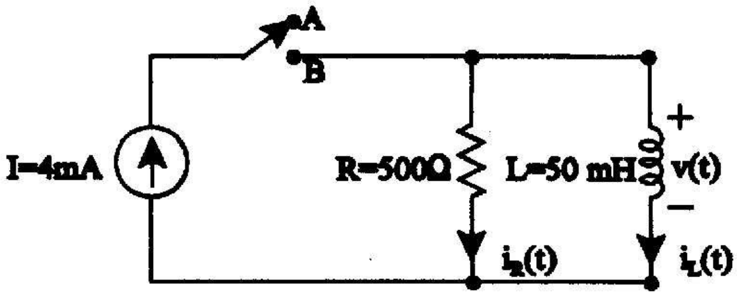
v) What is the instantaneous power, P, supplied by the voltage source at time t = 1.5 ms.

P =

Question 3. Charging a Capacitor and Transient Analysis

Part B. Transient analysis

Consider the circuit below.
Assume that the switch is open (at position A) for a long time. At time $t = 0$ the switch is closed (moved to position B).



i) Find the value of the time constant, τ, for the circuit.

$\tau =$

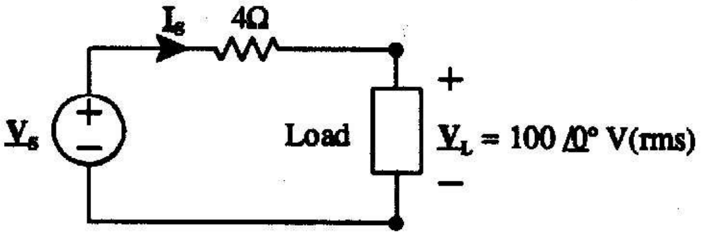
ii) Find the expression for the currents $i_R(t)$ and $i_L(t)$ and the voltage $v(t)$ in terms of the variables I, R, and L for $t > 0$. (Do not substitute numerical values for I, R, and L.)

$i_R(t) =$
$i_L(t) =$
$v(t) =$

Question 4. AC Circuits

Part A. Phasors and impedance

For the circuit below, the load absorbs 1kW at a 0.8 lagging power factor when the voltage across the load is $100\angle 0^\circ$ V(rms). (Note: the term lagging power factor means the current in the load lags the voltage across the load.)



i) Determine the phasor current $\mathbf{I_s}$.

$\mathbf{I_s}$ =

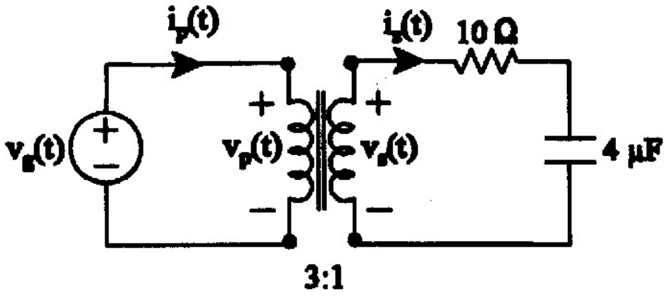
ii) The circuit is operating at a frequency of 60 Hz. Select the appropriate series model to represent the load: R in series with C or R in series with L. Find R and C or L.

R =
C =   or   L =

Question 4. AC Circuits

Part B. Transformer

For the circuit shown below the voltage source is sinusoidal with a magnitude of $90\sqrt{2}$ V peak, an angular frequency of 2.5×10^4 rad/s and a phase angle of 0°. Assume the transformer is ideal.



i) Draw the equivalent circuit in the frequency domain (using phasors). Fully label all the voltages, currents and circuit elements.

ii) Determine the current phasor $\mathbf{I}_p$. Note: $\mathbf{I}_p$ is the phasor representing $i_p(t)$.

$\mathbf{I}_p$ =