University of Toronto

Faculty of Applied Science and Engineering

MIE 467 - Advanced Operational Research

Final Examination - December 12, 2001

Examiner: J.S. Rogers

Type C

Note: Explain fully the rationale for your procedures. Full marks will **NOT** be given for just numbers
Marks as indicated [ ]

(1) Explain your understanding of and the significance in applications of the following terms

(a) [4] The significance to management of the following. At the optimal solution of an LP problem using GAMS, for a given variable X[J] we have X.M[J] > 0 and X.L[J]>0.
(b) [4] The type of solutions arising from maximizing a concave function over a closed convex set (typically formed from linear inequalities).
(c) [3] The features of a GAMS LP solution that indicate alternate optima.
(d) [4] The effects of degeneracy on the optimal solution of a LP model

(2) Consider

$$\begin{aligned} \max \quad & 21 x_1 + 11 x_2 \\ \text{subj to} \quad & 7 x_1 + 4 x_2 \le 16 \\ & x_2 \le 2 \\ & x_1, x_2 \ge 0 \end{aligned}$$

(a) [15] Solve by the matrix version of the simplex method

(b) [10] Write a GAMS program that will solve the problem. Label the variables X(J) and the rows ROW(I)

(c) [10] Using the optimal LP solution from part (a), find the values of X.L(2), X.M(2), ROW.L[1] and ROW.M[2]. If row 1 represents skilled labour, row 2 represents raw materials and the x's represent processes, interpret these results in a form useful to management.

(3) [10]Consider a situation in which an auction market like that in chapter two of the text is used to coordinate the way in which 2 suppliers (producing S1 and S2 respectively) compete to supply 2 customers consuming (D1 and D2 respectively)

Customer 1's problem is
Max [21x11 + 11x21]
subj to 7x11 + 4x21 <= D1
x21 <= 2
all xij >= 0

Customer 2's problem is
Max [21x12 + 13x22]
subj to 7x12 + 4x22 <= D2
x22 <= 2

Supplier 1's problem is
Max [21u11 + 11u21]
subj to 7u11 + 4u21 <= 13
u11 + u21 >= S1
all uij >= 0

Supplier 2's problem is
Max [21u12 + 13u22]
subj to 7u12 + 4u22 <= 13
u12 + u22 >= S2

Formulate a Complementarity Programming model that can be used to find a Nash Equilibrium solution.

(4) A firm must decide how many personnel to hire over the next 4 periods. The requirements in terms of person-hours are 8000, 9000, 7000, 10000.
It takes one month of training before each person can be useful. During that month each trainee requires 100 hours of supervision by experienced personnel taken away from regular duties. Each experienced person can work up to 150 hours during a month and 60 experienced persons are available at the start . There are no layoffs but 10% of the experienced personnel leave at the end of each month. Experienced persons are paid $4000 per month and trainees $2000.

(a) [5] Formulate a linear programming model that can be used to find the minimum cost hiring policy.

(b) [10] Write the dual problem and describe how the values of the dual variables could best be used by management.

(c) Suppose that the firm is uncertain about the business climate from period 3 onward. It assigns a probability p to the requirements stated above and thinks that with probability q (=1-p) the requirements in periods 3,4 could be 2000 higher.

(i) [10] Formulate a model that will minimize expected cost of hiring over the 4 periods.

(ii) [10] Suppose the firm can, at the start of period one, buy options to acquire experienced personnel in period 3 after the requirements are known. The premium is K per person and the exercise price is E. The salary is the $4000/mo. Formulate an optimization model (to minimize cost) that distinguishes between how many options are bought and how many are exercised.

(iii) [5]Formulate the dual to your answer in (ii) and interpret the dual equations.