University of Toronto

Faculty of Applied Science and Engineering

FINAL EXAMINATION, APRIL 2001

Fourth Year - Program: Electrical and Computer Engg (Elective)

ECE 417S - DIGITAL COMMUNICATION

Exam Type: D

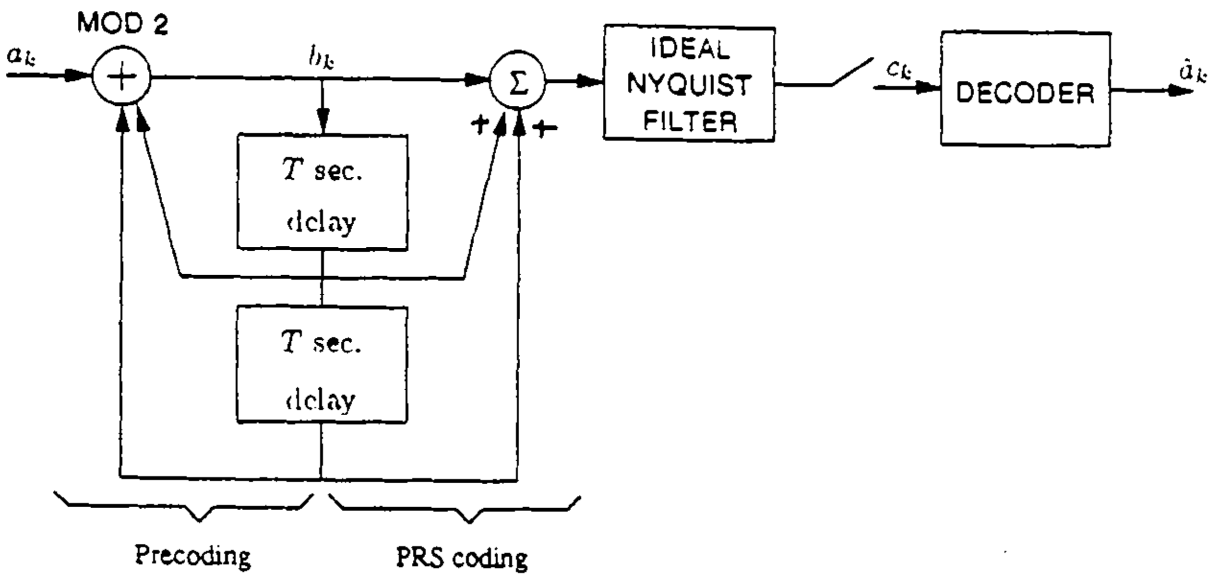
Examiner: *S.Pasupathy*

- A single aid sheet (8.5"x11", two-sided, handwritten) and a non- programmable calculator are the **only aids allowed.**
- Answer **all** five [5] questions.
- The value of each question is indicated beside each question; total marks = 60.
- Start each new question on a new page.
- If you need to make any assumptions, state them clearly.
- Answers should be clear, crisp and brief; answers without logical reasoning steps showing *all* the work will **not** be given credit.
- Lengthy reproductions of text material should be avoided. Credit is for **solving** the problems.

12 points

1. The figure below shows the block schematic of a Partial Response(PRS) $(1 + D + D^2)$ system.

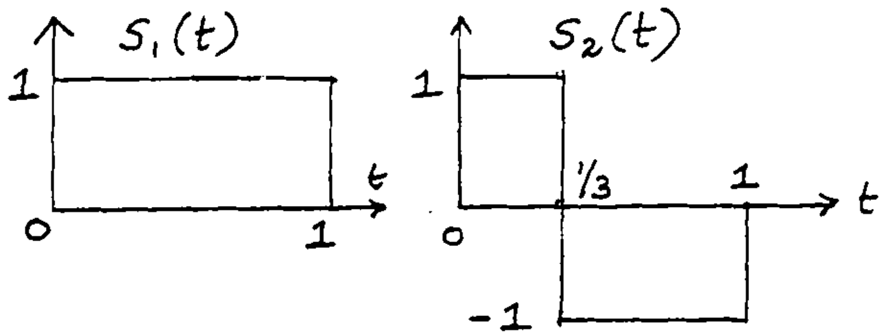
   (a) Write out the mathematical expressions for the PRS coder output b_k, c_k in terms of their respective inputs. (Assume that the channel is noiseless.)



   (b) Assume that a_k, $k = 3, 4, ..., 10$ are given by
   1 0 1 1 0 0 0 1,
   and write out the corresponding b_k, c_k .What is the decoding rule? Write out the corresponding $\hat{a}_k$. assuming first that $b_1 = b_2 = 1$ and then $b_1 = 0; b_2 = 1$. Does the decoded sequence and rule change under the two different assumptions? Explain.

12 marks

2. Consider finite-energy signals, non-zero over the entire interval of (0,1). Consider two signals $s_1(t)$ and $s_2(t)$ in such a space.



Use the usual inner-product for finite-energy signals, time-limited to (0,1).

(a) Find the norm of $s_1(t)$ and $s_2(t)$ and the inner-product of these two signals in signal space.

(b) Find the innerproduct between $s_1(t) + s_2(t)$ and $s_1(t) - s_2(t)$.

(c) Sketch a signal (non-zero over the entire interval(0,1)) that is orthogonal to both $s_1(t)$ and $s_2(t)$.

(d) Use Gram-Schmidt procedure on $s_1(t)$ and $s_2(t)$ and find and sketch two orthonormal signals, $\phi_1(t)$ and $\phi_2(t)$ in the space.

(e) Find and sketch a signal $s_3(t)$ (non-zero over (0,1)) that is in the subspace spanned by $s_1(t)$ and $s_2(t)$ and is orthogonal to $s_1(t)$.

12 marks

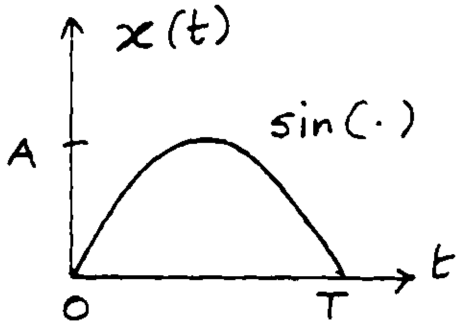
3. For the two signals shown below , assumimg the usual environment of Additive White Gaussian Noise channel transmission (symbol rate=1/T), find the following:

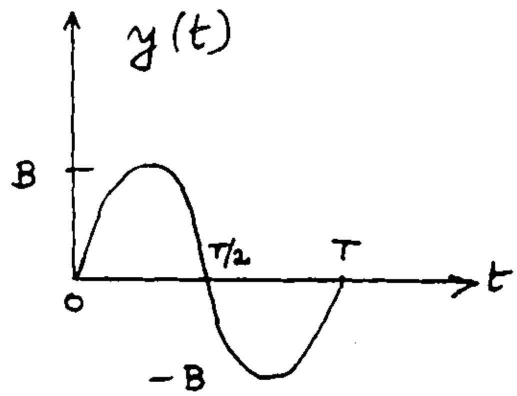
   (a) Sketch the causal matched-filter impulse responses.

   (b) Relate the constants A and B such that both cases give the same peak-signal-to-rms noise ratio at the matched-filter noise output.

   (c) If $\pm x(t)$ are used in equi-probable binary communication, what would be the power spectral density of the signal stream, assuming a PAM transmission at the symbol rate of 1/T .

   (d) Suppose we want to use $x(t)$ and $y(t)$ for Minimum Shift Keying modulation. Sketch the signal for a sample data sequence of 0 0 1 0 1 .





12 marks

4. A discrete memoryless source emits six symbols A, B, C, D, E and F with probabilities

$$P_A = 0.44;\ P_B = 0.28;\ P_C = 0.03;\ P_D = 0.07;\ P_E = 0.04;\ P_F = 0.14.$$

(a) Calculate the source entropy.

(b) The above source is to be encoded using a Huffman code with a binary alphabet $(0, 1)$ and having the average word-length with the minimum variance. Find the code, the average word-length and its variance.

(c) Design another code for the above source using a **ternary** alphabet (0,1,2).

(d) What is the maximum entropy possible for a new digital source with six possible messages? When is this maximum value achieved ?

12 marks

5. A code consists of three information bits m_1, m_2, m_3 and three check bits c_1, c_2, c_3. The transmitted sequence is $m_1c_1m_2c_2m_3c_3$. At the transmiter the check digits are formed from the following equations:

$$c_1 = m_1 \oplus m_2 \oplus m_3$$

$$c_2 = m_2 \oplus m_3$$

$$c_3 = m_1 \oplus m_2$$

(a) How many code words are there in the code?

(b) Write down the H matrix.

(c) Will this code correct single errors? Why?

(d) For the message $m_1 = 0, m_2 = 1, m_3 = 1$, find the transmitted codeword.

(e) Assume that the sequence 1 0 1 1 0 0 is received and that no more than one error has occurred. Decode the sequence. Find the location of the error and the transmitted message $m_1m_2m_3$.