UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING
FINAL EXAMINATIONS, APRIL 2001
SECOND YEAR - PROGRAM 6
CHE 222H1S - APPLIED DIFFERENTIAL EQUATIONS
Examiner - R. Luus

Answer all questions. All questions are of equal value.

Name_____

Student Number_____

Marks report

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Total_____

Total number of pages = 7.
Please check that all pages are in your exam booklet.
Do not unstaple the pages.

1.(a) Show how the following set of 4 equations in 4 unknowns can be arranged into two groups, one consisting of 3 simple equations and the other group consisting of a single difficult equation.

$$x_1 + x_2 - 2x_3 + 5x_4 = 7$$
$$\cos x_2 + x_3 + e^{x_4} - \sin x_1 = 2$$
$$x_2^2 + 2x_4 - x_1^3 = 5$$
$$x_2 + x_1^2 - \sin x_1 = 1.$$

(b) Give an efficient procedure for solving this set of equations. Be as explicit as possible, giving an algorithm that can be converted readily into a computer program. Include the choice of the starting conditions.

2. Integrate the following differential equations

(a) $(\sin y - y\sin x)dx + (\cos x + x\cos y - y)dy = 0$

(b) $ydx + x(\ln x - \ln y - 1)dy = 0$

(c) $y(xy^3 - 1)dx - dy = 0$

(d) $(y^2 + 1)dx + [3y^5 x - 6y\exp(-\frac{3y^2}{2})]dy = 0$

3. (a) Solve $x^3 \frac{d^3y}{dx^3} + x^2 \frac{d^2y}{dx^2} = 1 + 2x \div 4x^2$

(b) By using Laplace transform, solve, with the initial condition $y(0) = 0$, $y'(0) = 2$, the differential equation $y'' - 6y' \div 13y = 26$.

4. In a batch reactor the following reaction is occurring $A \sim 2B - 3C$. At time $t = 0$ the concentrations of the three chemical species are $c_A = 1$ mol/L, $c_B = c_C = 0$. Under suitable assumptions such as first order reactions, etc., the equations describing this system are

$$\frac{dc_A}{dt} = -k_1 c_A$$

$$\frac{dc_B}{dt} = k_1 c_A - 2k_2 c_B.$$

(a) Obtain expressions for c_B as a function of time t under the following conditions

(i) $k_1 \neq 2k_2$

(ii) $k_1 = 2k_2$

(b) Suppose $k_1 = 1.00\ s^{-1}$ and $k_2 = 0.500\ s^{-1}$, find the concentration c_B at $t = 1.5$ s. What is the maximum value of c_B?

5. (a) By using the initial condition $\mathbf{x}(0) = [\ 1 \quad 2\ ]^T$, solve the differential equation $d\mathbf{x}/dt = A\mathbf{x}$ for

$$A = \begin{bmatrix} 4 & -2 \\ 1 & 1 \end{bmatrix}$$

(b) Find the transition matrix $\exp(At)$ for $A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 5 & 1 & 1 \end{bmatrix}$

6. By feeding a diet of goat milk, carrots and viagra to specially cloned rabbits, a researcher found that the breeding rate exceeded the breeding rate of rabbits predicted by the Fibonacci series. She found that the number of these rabbits gave the series:

1, 2, 5, 12, 29, 70, ...

(a) What is the next term in this series?

(b) Obtain an expression for the nth term y_n, giving the number of rabbits after n years.

(c) Suppose the lab can hold only 10,000 rabbits; how many years will it take to reach this maximum number of rabbits?

(d) Find the limit as $n \to \infty$ of the ratio y_{n+1} / y_n.