UNIVERSITY OF TORONTO
Faculty of Applied Science and Engineering
FINAL EXAMINATION, DECEMBER, 2001
Third Year
AER 373F – Advanced Mechanics of Structures
Exam Type: A
Examiner – J. S. Hansen

Note: This is a closed book examination. Programmable calculators are allowed. Answer ALL questions.

MARKS

1. Answer the following as briefly as possible:

(5) (a) What is the characteristic of a strain tensor when it is specified in terms of principal strains.

(5) (b) A plane stress state is specified by the assumptions $\sigma_{zz} = \tau_{yz} = \tau_{xz} = 0$. What are the restrictions on structural geometry which are used to justify these assumptions?

(5) (c) What are the boundary conditions for the problem illustrated in Figure 1? Indicate which boundary conditions are forced (kinematic constraints) or natural (stess constraints).

(5) (d) The constitutive relation for an elastic homogeneous material being analyzed in two dimensions is given as

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{bmatrix} = \begin{bmatrix} A & B & 0 \\ C & D & 0 \\ 0 & 0 & E \end{bmatrix} \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix}$$

What are the restrictions on A, B, C, D, E in order that this be a real material

(10) (e) The beam illustrated in Figure 2 has been analyzed and it has been determined that the least natural frequency is ω_1. In the following situations, what will be the relationship between the new beam least natural frequency ω_1^N relative to ω_1? Justify your answers.

i. The total beam length is fixed, the position of mass M_1 is fixed while the position of M_2 is moved to the left.

ii. The total beam length is fixed, the position of mass M_2 is fixed while the position of M_1 is moved to the right.

iii. The total beam length is increased but the lumped masses M_1, M_2 maintain their position relative to the right hand end.

iv. The total beam length is increased while the lumped masses M_1, M_2 are removed.

(35) 2. The problem illustrated in Figure 3 is modelled using Timoshenko beam theory. Within the assumptions of this theory, the Total Potential Energy is

$$\Phi = \int_0^L \left[\frac{1}{2} EI \left(\frac{d\psi}{dx} \right)^2 + \frac{kGA}{2} \left(\frac{dw}{dx} - \psi \right)^2 - Pw \right] dx$$

where E, I, k, G, A and P are Young's modulus, the second moment of the area, the shear correction factor, the shear modulus, the beam cross-sectional area and the lateral applied force respectively. Also ψ and w are the rotation and the lateral deflection respectively. The forced boundary conditions for the problem are

$$w(x)|_{x=0} = w(x)|_{x=L} = \psi(x)|_{x=L} = 0$$

(a) Using the Principal of a Stationary Value of the Total Potential Energy, determine the equilibrium equations and the natural boundary conditions for this problem.

(b) Based on the use of the method of Ritz and the trial functions:

$$\tilde{w}(x) = a_0 + a_1 x + a_2 x^2 \;\; ; \;\; \tilde{\psi}(x) = b_0 + b_1 x$$

i. Determine the restrictions on a_0, a_1, a_2, b_0, b_1 in order that $\tilde{w}(x), \tilde{\psi}(x)$ be admissible in the method of Ritz.

ii. Using the admissible trial functions from part (i) determine the discretized equilibrium equations for this problem. Note: *It is not necessary to solve the resulting equations.*

(35) 3. Consider the planar pinned truss illustrated in Figure 4. The bars are joined by frictionless pins and each bar satisfies Hooke's law,

$$\frac{f_i}{A_i} = \frac{E_i \delta_i}{L_i}$$

where f_i is the force in the bar, A_i is the bar cross-sectional area, E_i is Young's modulus, δ_i is the elongation of the bar and L_i is the length of the bar. A lumped mass M is located at O. Also, u, v are the deflections of point O in the x, y directions; u, v are measured positive in the positive x, y directions respectively.

In the calculations, assume $A_i = A; E_i = E : i = 1, 4$ Also, note $L_1 = L_2 = 5a; L_3 = L_4 = 3a$.

(a) Determine the Kinetic Energy for this problem.

(b) Determine the the Total Potential Energy for this problem.

(c) Determine the motion equations for this system.

(d) Based on the assumption that the applied forces are zero. Determine the natural frequencies and mode shapes for this problem.

TOTAL

(100)

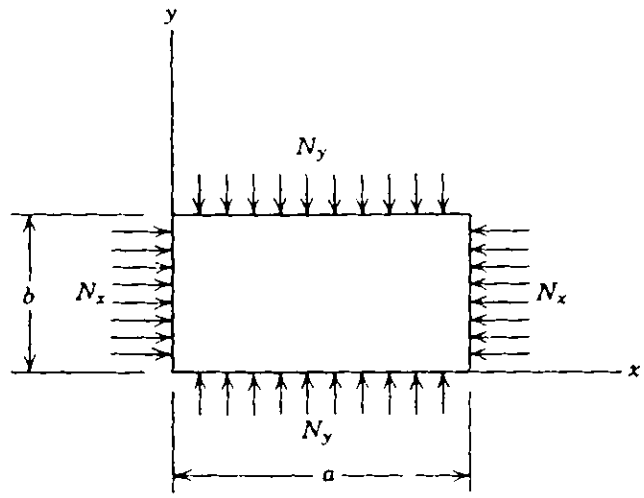

Figure 1

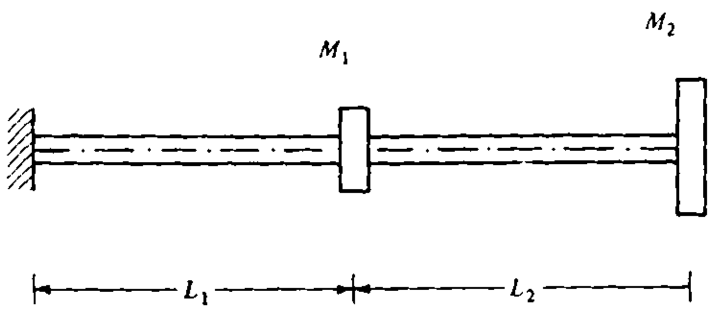

Figure 2

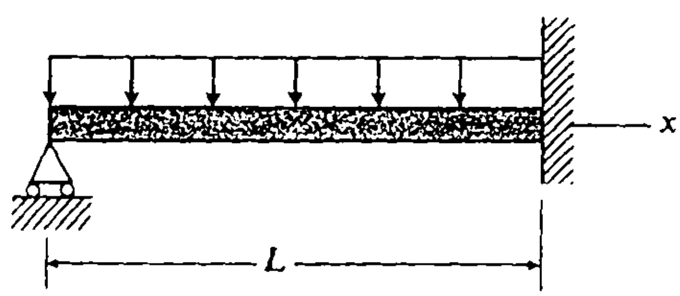

Figure 3

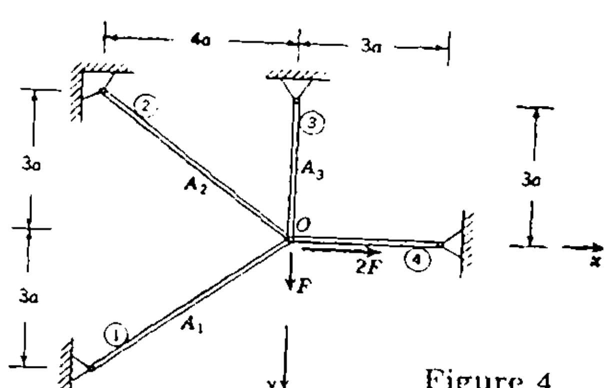

Figure 4