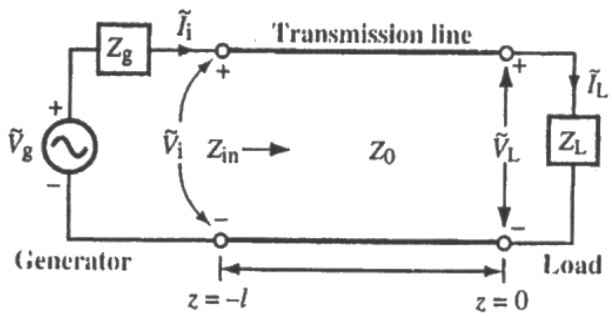
Question 1.

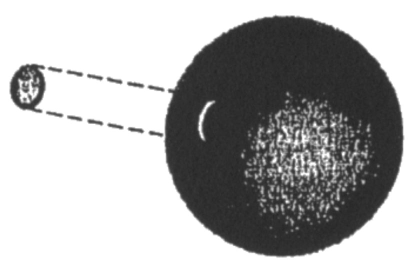
Consider a 130-MHz VHF transmitter, which generates a voltage of $\tilde{V}_g = 120V\angle 0°$. The transmitter (generator) has an impedance of $Z_g = 70\ \Omega$. It feeds an antenna with a feed-point impedance of $Z_L = (90 - j50)\ \Omega$ through a 70-Ω coaxial transmission line, which has a physical length of 20.1923 m and an electrical length of 13.125λ. The whole setup is illustrated below. Find

(a) an expression for the voltage $\tilde{V}(z)$ on the line;
(b) the load voltage $\tilde{V}_L$;
(c) the time-average power P_L absorbed by the VHF antenna;
(d) the time-average power P_g dissipated in the generator impedance Z_g.



Question 2.

Consider a spherical shell of charge of radius $a = 80\ mm$ and surface charge density $\rho_S = 70.832\ nC/m^2$. The shell is surrounded by free space but the relative dielectric permittivity of the medium inside the shell is $\varepsilon_{rs} = 4$. A small circular piece of radius $b = 3.4\ mm$ is removed from the spherical shell as illustrated below. Find the direction and magnitude of the electric field E at the center of the aperture just outside the sphere (i.e. at $r = a^+$) and just inside the sphere (i.e at $r = a^-$). Verify that the dielectric/free-space interface condition is satisfied by the electric field. The free space permittivity is $\varepsilon_0 = 8.854 \times 10^{-12}\ F/m$.



Question 3.

A conducting cylinder of radius r is covered with a dielectric shell of thickness Δr. The left half of the dielectric shell has a relative permittivity of ε_r. The other half of the dielectric shell has a relative permittivity of $4\varepsilon_r$. The outer surface of the dielectric is coated with a metallic film. A voltage is applied between the inner conductor and the outer foil. The inner conductor has a potential that is positive with respect to the foil's potential.

(a) Sketch a diagram of the geometry described above and indicate the location and polarity of the free (or excess) charge density and bound charge density. Label your diagram carefully.

(b) Derive an expression for the capacitance per unit length for this structure. **Hint**: The radial electric field in both dielectric regions is the same.

Question 4.

A cylindrical metallic shell (e.g. copper) has an outer radius of 1 m, an inner radius of 0.5 m and a length of 1 m. A second concentric shell has a length of 1 m, an inner radius of 1 m and an outer radius of 2 m. The outer surface of the second shell is covered with a metallic film that is a very good conductor. The second shell is a moderate conductor and has an electrical conductivity given by $\sigma = a\,r$, where a is a constant and r is the radial distance from the center axis. A conducting cable is used to connect the positive terminal of a 10 V battery to a metallic connector located at r = 0.5m. Another conducting cable is used to connect the negative terminal of the battery to the metallic film. An ammeter placed in series with the battery indicates a current of 10 A.

(a) Sketch a diagram of the geometry described above.

(b) Calculate the value of the constant a.

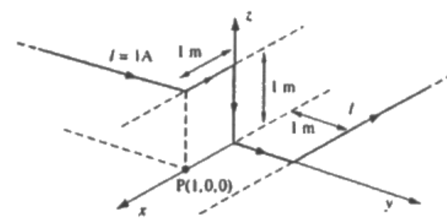
Question 5.

(a) Consider a short, straight filamentary wire segment aligned with the y-axis. The filament carries a current of magnitude I in the positive y-direction. The ends of the wire are at $y = -b$ and $y = +a$.

Derive an expression for the magnetic flux density **B** (magnitude and direction) at a point P that is a radial distance d from the y-axis.

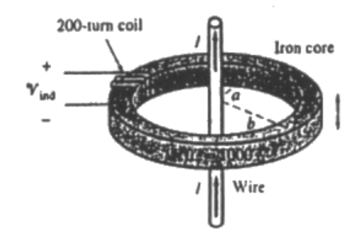
(b) An infinitely long wire carrying a current of magnitude I = 1 A has three sharp 90° bends, 1 m apart, as shown in the accompanying figure.

Determine the magnetic flux density B (magnitude and direction) at the point P (1,0,0).



Question 6.

The figure to the right shows a long straight wire carrying a time varying current I(t). This wire coincides with the principal symmetry axis of a rectangular toroidally-shaped iron core of inner radius a, outer radius b, thickness t, and constant relative permeability μ_r. A coil having N turns is wrapped around the core. One terminal of the coil is marked +ve, the other -ve, as shown in the figure.



(a) The current I(t) has a triangular waveform with a period T, an average value (i.e. DC bias) of I_{DC}, and peak values of $\pm I_0$ relative to the average. Develop an expression for the peak value of the induced emf $|V_{ind}|$.

(b) When the current I(t) is increasing, is the induced emf positive or negative with respect to the polarity shown in the figure? Provide reason(s) to support your answer.

(c) Let I_{DC} = 5 A, I_0 = 10 A, T = 500 ms, a = 6 cm, b = 8 cm, t = 3 cm, μ_r = 1000 and N = 200.

On a common time axis, sketch accurate waveforms for I(t) and V_{ind}(t). Clearly label all axes and show the numerical values of all maximum and minimum waveform magnitudes.