UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING
MIE232S DIFFERENTIAL EQUATIONS
Final Exam, 23 April 2001
Examiner: Professor L. Chen

- **Closed book exam, no aids permitted**
- **Time allotted: 150 minutes**
- **100 marks in total for 5 problems**

Problem 1 (25 marks)

Solve $\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}$ subject to the following boundary conditions:

$$u(0,t)=0, \quad u(10,t)=0, \quad u_t(x,0)=0, \quad u(x,0)=3\sin 2\pi x - 4\sin\frac{5\pi}{2}x$$

Problem 2 (20 marks)

Solve the following system of differential equations: $\begin{cases} 2\frac{dx}{dt} - \frac{dy}{dt} - 3x + 2y = e^t \\ \frac{dy}{dt} - \frac{dx}{dt} + x - y = 2e^t \ln t \end{cases}$

Problem 3 (20 marks)

Solve $y'' - 2y' + y = r(t)$ given that $y(0)=1$, $y'(0)=0$ and $r(t) = \begin{cases} 3\sin t - \cos t & (0 \le t < 2\pi) \\ 2\sin t + \cos t & (2\pi \le t < 3\pi) \\ 5 & (t \ge 3\pi) \end{cases}$

Problem 4 (15 marks)

Solve the following differential equation: $x^3\frac{dy}{dx} - 2x^2 y + 2x\int_0^x y\,dx = 3x^2 - 6(x+1)[\ln(x+1) - 1]$

Problem 5 (20 marks)

A stationary object with mass of m starts slipping down on a flat from the origin "O", as the flat is slowly rotating about its pivot "Q" within a certain range (see the figure), where θ is a function of time t. The air resistance is proportional to the moving distance (x), which is equal to $4mx$. During moving, friction also needs to be considered with the frictional coefficient, k. Formulate this problem, and find the expression of $x(t)$. (x direction is always along the flat.)

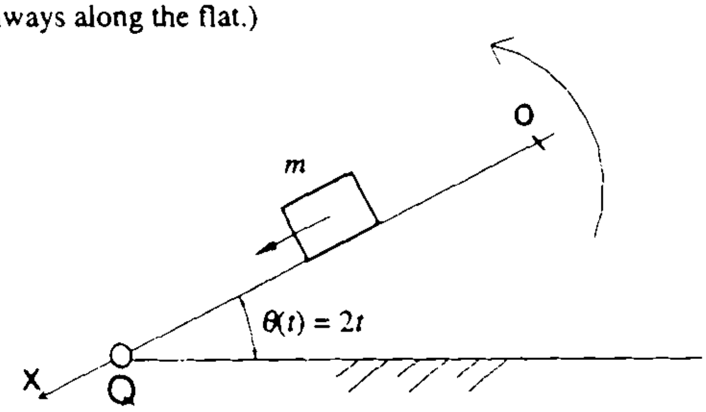

Table of Laplace Transforms

	$F(t)$	$f(s)$
1.	1	$\dfrac{1}{s}$ $\quad s > 0$
2.	t	$\dfrac{1}{s^2}$ $\quad s > 0$
3.	$t^n \quad n = 1, 2, 3, \ldots$	$\dfrac{n!}{s^{n+1}}$ $\quad s > 0$
4.	$t^n \quad n > -1$	$\dfrac{\Gamma(n+1)}{s^{n+1}}$ $\quad s > 0$
5.	e^{at}	$\dfrac{1}{s-a}$ $\quad s > a$
6.	$\sin \omega t$	$\dfrac{\omega}{s^2+\omega^2}$ $\quad s > 0$
7.	$\cos \omega t$	$\dfrac{s}{s^2+\omega^2}$ $\quad s > 0$
8.	$\sinh \omega t$	$\dfrac{\omega}{s^2-\omega^2}$ $\quad s > \|\omega\|$
9.	$\cosh \omega t$	$\dfrac{s}{s^2-\omega^2}$ $\quad s > \|\omega\|$
10.	$e^{at} \sin \omega t$	$\dfrac{\omega}{(s-a)^2+\omega^2}$ $\quad s > a$
11.	$e^{at} \cos \omega t$	$\dfrac{s-a}{(s-a)^2+\omega^2}$ $\quad s > a$
12.	te^{at}	$\dfrac{1}{(s-a)^2}$ $\quad s > a$
13.	$t \sin \omega t$	$\dfrac{2\omega s}{(s^2+\omega^2)^2}$ $\quad s > 0$
14.	$t \cos \omega t$	$\dfrac{s^2-\omega^2}{(s^2+\omega^2)^2}$ $\quad s > 0$
15.	$Y'(t)$	$sy - Y(0)$ where $y = \mathcal{L}\{Y(t)\}$
16.	$Y''(t)$	$s^2y - sY(0) - Y'(0)$
17.	$Y^{(n)}(t) \quad n = 1, 2, 3, \ldots$	$s^ny - s^{n-1}Y(0) - \cdots - Y^{(n-1)}(0)$
18.	$e^{at}F(t)$	$f(s-a)$
19.	$t^nF(t) \quad n = 1, 2, 3, \ldots$	$(-1)^n f^{(n)}(s)$
20.	$\displaystyle\int_0^t F(u)G(t-u)\,du$	$f(s)g(s)$
21.	$\displaystyle\int_0^t F(u)\,du$	$\dfrac{f(s)}{s}$
22.	$F(t-a)H(t-a) = \begin{cases} F(t-a) & t > a \\ 0 & t < a \end{cases}$	$e^{-as}f(s)$