UNIVERSITY OF TORONTO

FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION, APRIL 17, 2001

Second Year -- Engineering Science

PHY281S -- Quantum Physics

Exam Type: C

Examiners: Profs. Wei and Trischuk

You are required to answer all four questions. Please read the questions carefully.

The questions start on the next page (page 1)
A table of constants & integrals is given on the final page (page 3)

1. Answer all parts of this question briefly, but with full sentences.

   a) Write down the three-dimensional time-dependent Schrodinger equation.
   b) Explain how this equation reproduces classical mechanics. In what limit?
   c) Define the "Classically Forbidden Region". You may want to give an example potential.
   d) Choose two observations and explain why they forced physicists to develop quantum mechanics in the first part of the 20^{th} century. Keep your answers concise and limit yourself to one or two equations in describing the physics of each example.
   e) If a bound state in three dimensions has zero angular momentum, what can be said about its wavefunction?
   f) Why do we normalize the wavefunction so that the integral of $|\Psi|^2$ over all space is 1?
   g) The photon energy for the n_i to n_f transition in hydrogen is given by:
   $$E_{ph} = \frac{e^2}{2a_0}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right).$$
   What is the ionization energy of hydrogen?
   h) Rydberg transitions from the 101^{st} to 100^{th} levels of hydrogen have been observed in very dilute inter-stellar gases. How large are such atoms?

2. A particle of mass m is confined within an infinite rectangular well along the z-direction, and bound by simple harmonic oscillation in the x-y plane, according to the following potential:

$$V(x,y,z):\left\{\begin{array}{ll} =0 & \text{for } |z|\le c; \\ =\infty & \text{for } |z|>c; \\ =ax^2/2 & \text{for all x;} \\ =by^2/2 & \text{for all y;} \end{array}\right\}$$

where a, b, c are constants.

   a) Write down and separate the time-independent Schrödinger equation in Cartesian coordinates.
   b) Write down an eigenfunction for the ground state (show reasoning and how to normalize).
   c) Write down a general expression for the energy eigenvalues, in terms of m, a, b, c and $\hbar$.
   d) If m were quadrupled and c reduced by a factor of $\sqrt{2}$, how would the ground-state energy change?
   e) Letting $b=9a$ and freezing the z-motion, find the lowest energy which has degenerate states.

3. A wavepacket is localized in space with the following wave-vector distribution:

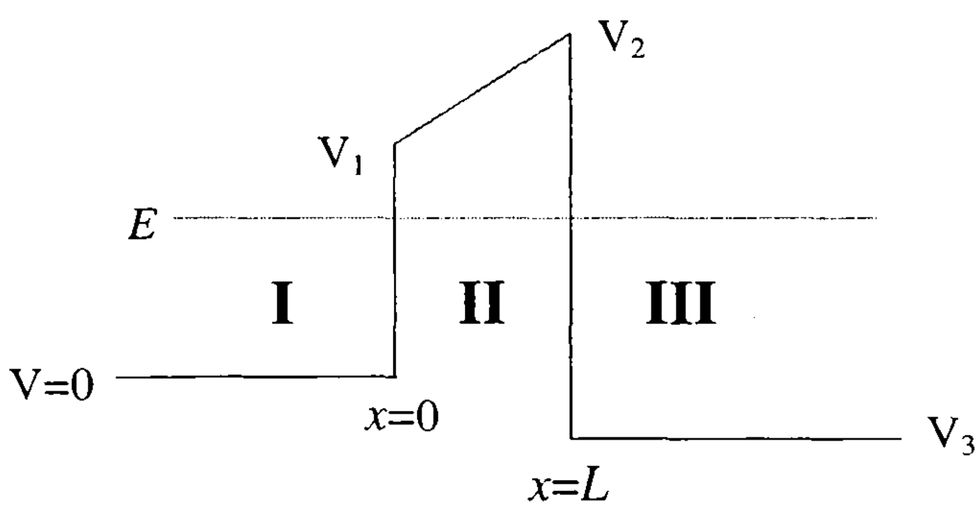
$$B(k) = \cos\left(\frac{\pi}{2}\frac{k-k_0}{\Delta k}\right)$$

for $k_0 - \Delta k \le k \le k_0 + \Delta k$ and zero otherwise.

a) Sketch the momentum space representation of this wavefunction.
b) What is the velocity of this wave-packet?
c) Calculate the wavefunction $\Psi(x,t=0)$.
d) What is the asymptotic behaviour of the wavepacket's probability density as $|x| \to \infty$?
e) Sketch $|\Psi(x,t=0)|^2$ as a function of x.
f) By making plausible choices for the extent of the wavepacket in physical space and momentum space show how this packet obeys Heisenberg's uncertainty principle.
g) Explain the physical consequences of Heisenberg's uncertainty principle.

4. Consider a beam of particles with mass m and energy E ($0<E\le V_1$) incident from the left on a generalized potential barrier shown below, with width L, heights V_1 and V_2, and base V_3 on the right:

a) Letting $V_1=V_2$ and $V_3<0$, such that the barrier is rectangular but asymmetric, write down the steady-state wavefunction for each region (I, II, III) and state the boundary conditions.
b) For this rectangular barrier, find an expression for the probability current $J(x,t)$ in region II.
c) For this rectangular case, define the transmission coefficient T. How is J in region II related to T?
d) Letting $V_1<V_2$ and $V_3=0$, such that the barrier is trapezoidal, derive an expression for T in terms of: E, V_1, V_2, m, L, $\hbar$. (hint: approximate trapezoid as sum of infinitesimal low-transmission rectangles).
e) Now letting $E=V_1$ for this trapezoidal case, is the barrier transmission symmetric with respect to the direction of incidence? That is, if the beam were incident from the right, how would T be different?



Some Useful Constants:

Atomic Mass Unit:	1.661×10^{-27} kg
	10^{-24} g
	931.5 MeV/c^2
Electron Mass:	9.109×10^{-31} kg
	10^{-28} g
	0.511 MeV/c^2
Elementary Charge:	1.602×10^{-19} C
	4.803×10^{-10} esu
Planck's Constant:	6.626×10^{-34} J - s
	10^{-27} erg - s
	4.136×10^{-15} eV - s
Speed of Light:	2.998×10^{8} m/s
	10^{10} cm/s

Conversion Factors

1 eV = 1.602×10^{-19} J = 1.602×10^{-12} erg

1 amu = 1.492×10^{-10} J = 1.492×10^{-3} erg = 931.5 MeV/c^2

1 Angstrom = 10^{-10} m = 10^{-8} cm

Useful Integrals

$$\int e^{ax}\sin(bx)dx = e^{ax}\frac{(a\sin(bx) - b\cos(bx))}{a^2+b^2}$$

$$\int e^{ax}\cos(bx)dx = e^{ax}\frac{(a\cos(bx) + b\sin(bx))}{a^2+b^2}$$

$$\int e^{ax}\sin^2(bx)dx = e^{ax}\sin(bx)\frac{(a\sin(bx) - 2b\cos(bx))}{a^2+4b^2} + 2b^2\int e^{ax}dx$$

$$\int e^{ax}\cos^2(bx)dx = e^{ax}\cos(bx)\frac{(a\cos(bx) + 2b\sin(bx))}{a^2+4b^2} + 2b^2\int e^{ax}dx$$