University of Toronto
FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATIONS, APRIL 2001
First Year - Programs 1,2,3,4,6,7,8,9

MAT 187H1S
Calculus II

SURNAME _____

GIVEN NAME _____

STUDENT NO. _____

SIGNATURE _____

Examiners

D. Burbulla

C. Lun

G. Maschler

R. Rotman

INSTRUCTIONS:

Non-programmable calculators permitted.

No other aids permitted.

Answer all questions.

Present your solutions in the space provided; use the back of the **same** page if more space is required.

TOTAL MARKS: 100

The value for each question is shown in parentheses after the question number.

MARKER'S REPORT	
Q1	
Q2	
Q3	
Q4	
Q5	
Q6	
Q7	
Q8	
TOTAL	

1. (15 marks: each part is worth 5 marks) Find the following:

(a) $\int \frac{\cot(\ln x)}{x}\, dx$

(b) the length of the curve with parametric equations $x = 4\cos t\,; y = 4\sin t\,; z = 3t$ for $0 \leq t \leq 2\pi$.

(c) $\frac{\partial^2}{\partial x \partial y}\left(\sqrt{1+xy^2}\right)$ at the point $(x, y) = (1, 1)$

2. (15 marks) Find the general solution to each of the following differential equations:

(a) (6 marks) $\frac{dy}{dx} = 2x(y^2 + 4)$

(b) (9 marks) $\frac{dy}{dx} + \frac{2}{x}y = \frac{\sin x}{x}$

3. (15 marks) Let $f(x) = \sum_{n=0}^{\infty} (-4)^n \frac{n+1}{n^2+1} x^{2n} = 1 - 4x^2 + \frac{48}{5}x^4 - \frac{128}{5}x^6 + \frac{1280}{17}x^8 - \ldots$.

Find the following:

(a) (2 marks) $f^{(2)}(0)$

(b) (2 marks) the $6th$ degree Taylor polynomial of $f(x)$ about $x = 0$

(c) (2 marks) $\lim_{x \to 0} \frac{f(x) - 1 + 4x^2}{x^4}$

(d) (4 marks) the radius of convergence for $f(x)$

(e) (5 marks) $\int_0^{0.5} x^2 f(x)\, dx$ correct to within .01

4. (15 marks) Consider the curve in the x-y plane with equation $x^2 - 6x + y^2 = 0$. Find the following:

(a) (5 marks) the polar equation of the curve

(b) (5 marks) the length of the curve

(c) (5 marks) the area of the region within the curve

5. (10 marks) Find the critical points of $f(x,y) = 3x^4 - 6xy^2 - 4y^3$ and at each critical point determine whether f has a relative maximum point, a relative minimum point, or a saddle point.

6. (10 marks) Do the following infinite series converge or diverge? Justify your answer.

(a) (3 marks) $\sum_{n=1}^{\infty} \frac{n+2}{n^3+\sin n}$

(b) (3 marks) $\sum_{n=1}^{\infty} \frac{\ln n}{\sqrt{1+n^2}}$

(c) (4 marks) $\sum_{n=1}^{\infty} \frac{n!}{n^n}$

7. (10 marks; each part is worth 5 marks) Find the following:

(a) the exact sum (*not* a decimal approximation) of $\sum_{n=1}^{\infty} \frac{n}{4^n}$.

(b) the first four (non-zero) terms of the Maclaurin series of $f(x) = \dfrac{3x+7}{x^2+2x-15}$.
What is the radius of convergence for this series?

8. (10 marks) Find $\int_0^\infty \frac{x}{(x+3)(x^2-3x+9)}\,dx$.