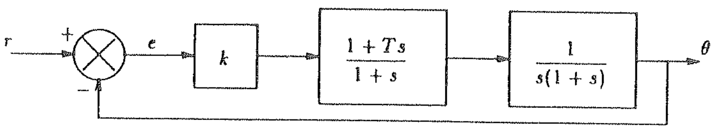
UNIVERSITY OF TORONTO
DEPARTMENT OF ELECTRICAL & COMPUTER ENGINEERING
FINAL EXAMINATION - DECEMBER 1996
ECE355F - SYSTEM AND SIGNAL ANALYSIS I
Third Year - Programs 5bme, 5ce, 5e
EXAMINER - W.M. Wonham

No aids permitted, other than a calculator.
PLEASE ANSWER EACH OF THE THREE MAIN QUESTIONS IN A SEPARATE BOOKLET.
Marking scheme: Each of the three main questions is worth 1/3 of the total mark.

1.



In the feedback system shown, k and T are positive parameters.

1.1 Calculate the transfer function $\hat{h}(s) = \hat{e}(s)/\hat{r}(s)$. Call the system with this transfer function (i.e. with r as input, e as output) SYS. Assume from now on that $r(t) = 0$ for all $t < 0$.

1.2 Determine the conditions on k and T for SYS to be BIBO stable. Write an equation for the stability boundary relating the critical values (k_c, T_c) in the (k, T)-plane (in the first quadrant $k, T > 0$). Display this boundary with a careful sketch, and label the regions of stability and instability.

1.3 For a suitable range of T (obtained in 1.2) determine the critical frequency ω_c at which SYS will oscillate, when (k, T) lies on the stability boundary. Express ω_c as a function of T_c, and sketch its graph.

1.4 Suppose the input is $r(t) = 1 - t$ $(t > 0)$. Calculate $\hat{e}(s)$. Under what condition will it be true that $e(\infty) := \lim e(t)$ $(t \to \infty)$ exists and is finite? Under this condition, calculate $e(\infty)$. Under what condition on T will it be possible to reduce $|e(\infty)|$ to less than $\frac{1}{3}$ by a suitable choice of k? When could $e(\infty)$ be made arbitrarily small?

1.5 Suppose $r(t) = t^2$ $(t \geq 0)$ and the SYS is stable. Without working out $e(t)$ in detail, describe the general behavior of $e(t)$ as $t \to \infty$.

1.6 Suppose $r(t) = \cos(\omega t)$ $(t \geq 0)$ for some $\omega > 0$. Under what condition would you be safe in assuming that $e(t)$ is also sinusoidal with frequency ω as $t \to \infty$? Under this condition, calculate the approximate amplitude of $e(\cdot)$ as a function of ω, (i) when ω is small, (ii) when ω is very large.

2. 2.1 A signal $f(t)$ is defined for $t \geq 0$ by $f(t) = t/\pi$ $(0 \leq t < \pi)$, and by periodic extension for $t \geq \pi$. Calculate the complex exponential Fourier series (CEFS) of f. From the CEFS derive the standard trigonometric Fourier series of f.

2.2 Consider the linear time-invariant causal filter, say **FIL**, with transfer function $\hat{h}(s) = s(s-1)/(s+1)^2$. Calculate the impulse response function $h(u)$.

2.3 Write down the complex frequency response function and calculate the amplitude response function $A(\omega)$, of **FIL**. Sketch the graph of A for $0 < \omega < \infty$.

2.4 Suppose the signal f (as in 2.1) is input to **FIL**. Using the previous results, show that the steady-state output of **FIL** can be written as an infinite series of real sinusoids, and specify their amplitudes and frequencies (but not their phases).

2.5 Write an infinite series for the average power of the steady-state output of **FIL** under input f.

2.6 Assume that t is measured in sec. To a good approximation, calculate the least frequency $\nu_{\min}$ (in Hz.), such that the total output power in the frequency range $\nu > \nu_{\min}$ is no greater than 0.1% of the total average power of f.

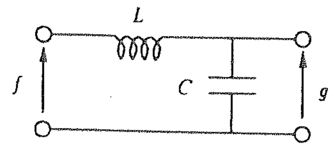
3. Consider a general system $\mathbf{S} : \mathcal{F} \to \mathcal{G}$, where $\mathcal{F}$ and $\mathcal{G}$ are suitable classes for the input and output signals respectively, assumed defined for $-\infty < t < \infty$.

3.1 Provide mathematical definitions of the properties that S is (i) causal, (ii) time-invariant, (iii) BIBO stable. Note that in case (iii) you will first need to define a suitable norm for signals in $\mathcal{F}$ and $\mathcal{G}$. Along with each mathematical definition, provide a short (one-sentence) intuitive interpretation.

3.2 Provide an example of a system S that is stable and time-invariant and non-causal.

3.3 Repeat 3.2 for the property unstable and (not time-invariant) and causal.

3.4 Consider the network NET displayed below, with L and C constant, and $LC = 1/4$. The signals f and g are the input and output terminal voltages as shown.



Calculate (by any method) the transfer function $\hat{h}(s)$ and impulse response function $h(u)$ of NET.

3.5 Assuming the initial conditions on NET are zero, write an expression for the output $g(t)$ $(t > 0)$ in terms of the input $f(t')$ for $0 \leq t' \leq t$.

3.6 Fix t in 3.5. Suppose you'd like to maximize $g(t)$ over the set of input signals f that satisfy $|f(t')| \leq 1$, $0 \leq t' \leq t$. Show exactly how this could be done when $t = \pi$, and sketch the maximizing signal f. More generally, calculate the maximum value of $g(t)$, say $g^*(t)$, when $t = k\pi$ $(k = 1, 2, \ldots)$. What can you say about the sequence of numbers $g^*(k\pi)$ as $k \to \infty$? What, if anything, does this imply about the BIBO stability of NET?