UNIVERSITY OF TORONTO

FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION — 17 April 2001

FOURTH YEAR GASDYNAMICS — AER-310 S

Examiner: Professor J. J. Gottlieb

Instructions: open book test (D) — course notes permitted.

20 % 1. Do the following short questions.

(a) Calculate the sound speed of diatomic hydrogen (H_2) at a temperature of 500 K.

(b) What are the main assumptions used to derive the energy equation in the form $a_o^2 = a^2 + \frac{\gamma - 1}{2} v^2$ for a steady flow of a gas in a duct with a changing area, in which the symbols a_o, a, v and γ are the stagnation sound speed, static sound speed, flow velocity and ratio of the specific heats, respectively.

(c) A supersonic air flow at an initial flow Mach number $M_1 = 2.0$ is deflected by a sharp-edged wedge. Determine the wedge angle that makes the flow Mach number M_2 sonic behind the oblique shock wave. Determine the wedge angle that just makes the oblique shock detached. Use graphs from notes.

(d) A storage reservoir contains gaseous methane (R = 558.3 J/kg-mass·K, $\mathcal{M}$ = 16.04 kg-mass/kg-mole, $\gamma = 1.32$) at a constant pressure and temperature of p_{tank} = 140 kPa and T_{tank} = 280 K, surrounded by the atmosphere ($p_{\rm atm}$ = 100 kPa and $T_{\rm atm}$ = 280 K). If a valve on this tank is opened to release the gas to the atmosphere, what would be the flow Mach number of the free jet.

(e) Consider the steady flow of a compressible gas from location (1) to another location (2) farther along a duct, between which there can occur a change in area, heat transfer, viscous losses and shaft work. Derive the expression

$$\frac{p_{02}}{p_{01}} \frac{T_{01}^{1/2}}{T_{02}^{1/2}} \frac{\mathcal{A}_2}{\mathcal{A}_1} = \frac{M_1}{M_2} \left(\frac{1 + \frac{\gamma - 1}{2} M_2^2}{1 + \frac{\gamma - 1}{2} M_1^2} \right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

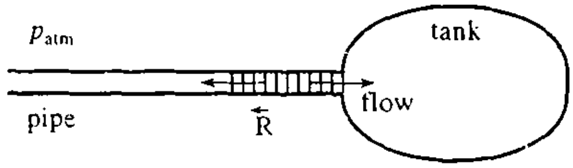
for mass conservation in terms of the stagnation pressure (p_0) and temperature (T_0), flow Mach number (M) and area $\mathcal{A}$ at locations (1) and (2).

(f) Two smoke puffs are located 10 cm apart on the axis of a constant-area duct filled with air at an initial pressure p_1 = 100 kPa and sound speed a_1 = 310 m/s. After a shock wave passes over these two puffs they are only 4 cm apart (behind the shock front). What is the density ratio ρ_2/ρ_1 across the shock front. Determine the pressure ratio p_2/p_1 across the shock front and the velocity V_s of the shock wave.

16 % 2. A tank of internal volume $\mathcal{V}_{\text{tank}}$ and inner surface area $\mathcal{A}_{\text{tank}}$ is evacuated completely ($p_{\text{tank}} = 0$ Pa initially). A long pipe with an internal area $\mathcal{A}_{\text{pipe}}$ is connected to the tank, and the pipe entrance to the tank is initially closed (plugged) at the tank, where $\mathcal{A}_{\text{pipe}} \approx \mathcal{A}_{\text{tank}}/500$. When the plug is removed suddenly, an unsteady rarefaction wave in the pipe accelerates air from the pipe (initially at a pressure p_{atm}) into the tank.

(a) Describe the filling process in terms of choked and unchoked inflow to the tank, the types of jets inside the tank, and the pressure rise in the tank. At what approximate average pressure in the tank does the flow become unchoked?

(b) Derive an expression for the average temperature T_{tank} of the air inside the vessel during the filling process, that is during the time in which the flow is choked.



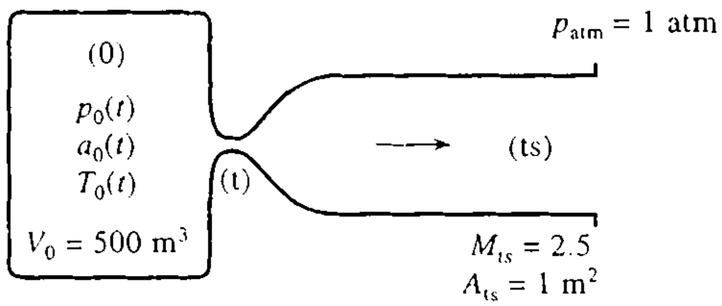
Hints for this problem: You can ignore heat transfer and frictional effects in tank and pipe, and you might assume that the tank air is always thoroughly mixed. For derivations, you should use the integral expressions for the accumulation of mass and energy accumulation within the vessel as a function of the incoming flow properties.

16 % 3. The blow-down wind tunnel shown below delivers air from its reservoir (volume $\mathcal{V}_0 = 400$ m^3 and $p_{0_i} = 15$ atm and $a_{0_i} = 350$ m/s initially) to the test section at the designed area $\mathcal{A}_{ts} = 1$ m^2 and supersonic Mach number $M_{ts} = 2.5$.

(a) Determine the throat area $\mathcal{A}_{\text{throat}}$.

(b) Determine the type of free jet at the duct exit to the atmosphere, when $p_0(t = 0) = p_{0_i} = 15$ atm.

(c) Determine the test section pressure when the supersonic flow in the test section is shutdown by a shock wave advancing into the test section.



10 % 4. A conventional shock tube has air in driver at a pressure $p_4 = 5$ atm and sound speed $a_4 = 340$ m/s, and has air in the channel at $p_1 = 1$ atm and $a_1 = 340$ m/s).

(a) Determine the shock strength p_2/p_1 after the diaphragm is broken.

(b) Determine the test time 4 m from the diaphragm for which an experiment can be done with the shocked air between the shock and contact surface.

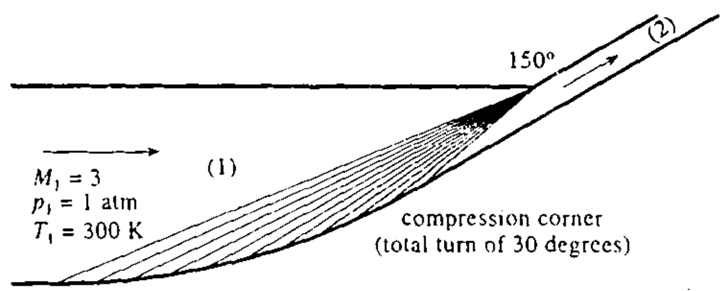
16 % 5. A supersonic flow of air with an initial flow Mach number $M_1 = 3.0$ is turned 30 degrees by a compression corner in a duct, as illustrated in the figure.

(a) Determine the flow Mach number after the turn, in region (2).

(b) Determine the area ratio A_2/A_1 and compression ratio p_2/p_1

(c) Determine the stagnation temperatures T_{01} and T_{02} and explain why they are equal or differ.

(d) The compression wave is focussed on the top corner. Explain why there is no wave reflected from this corner.



16 % 6. A subsonic gas flow inside a constant-area duct of diameter D absorbs heat by radiation from $x = 0$ (location 1) to $x = L$ (location 2). The heat addition is given by $\dot{q} = \dot{q}_o \sin\left(\frac{\pi x}{L}\right)$, with $0 \leq x \leq L$, where $\dot{q}_o$ has the units of J/m^3·s in terms of m^3 of duct volume.

(a) Sketch the temperature-entropy diagram for this process, showing states (1), (2) and (01) and (02).

(b) Derive the following expression for the spatial distribution of stagnation temperature

$$\frac{T_{0x}}{T_{01}} = 1 + \frac{\dot{q}_o D^2 L}{4\dot{m} C_p T_{01}} \left[1 - \cos\left(\frac{\pi x}{L}\right)\right] \qquad 0 \leq x \leq L,$$

where $\dot{m}$ is the mass flow rate of gas, C_p is the gas specific heat at constant pressure, and T_{0x} and T_{01} are the stagnation temperatures at locations x and 1, respectively.

(c) Explain with the aid of relevant equations how you would obtain the spatial distributions of both the flow Mach number M_x and static pressure p_x. Consider all flow conditions at location 1 as known.