UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING
FINAL EXAMINATION, APRIL 1995
Third Year - Programs 05bme, 05ce, 05e
ECE356S - SYSTEM AND SIGNAL ANALYSIS II
Examiner - B.A. Francis

Instructions: Aid sheet permitted. Attempt all problems.

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1. The figure below is a schematic diagram of a crane. The cart of mass M_1 rolls without friction on a track, its distance from a stationary point denoted by x. The pendulum, of length l and point mass M_2, can swing (again without friction) in the plane indicated.

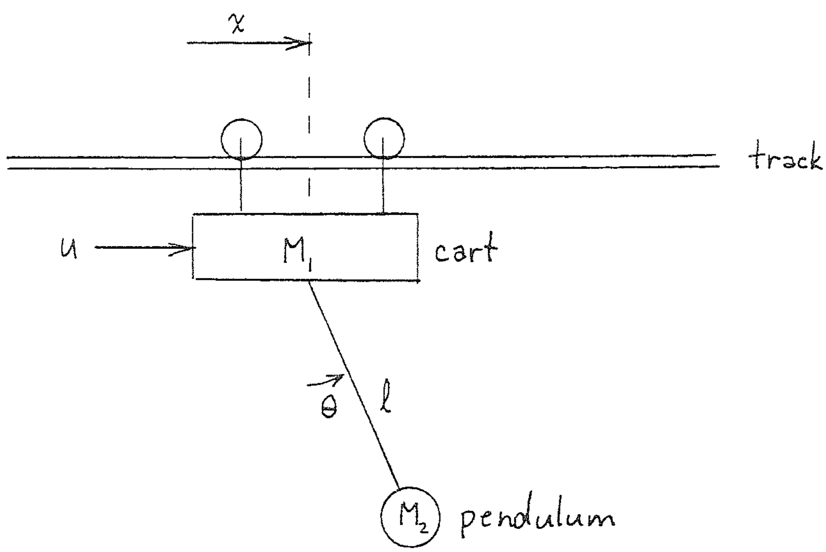
   Consider this system as a control system with input the external force u applied to the cart and output the angle θ.

   (a) (10 marks) Letting f denote the tension in the pendulum, derive the following nonlinear equations of motion:

   $$\begin{aligned} -M_1\ddot{x} &= u + f\sin\theta \\ M_2(\ddot{x} + l\ddot{\theta}\cos\theta - l\dot{\theta}^2\sin\theta) &= -f\sin\theta \\ -M_2 l(\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta) &= M_2 g - f\cos\theta \end{aligned}$$

   (b) (8 marks) Linearize about an operating point where $\theta = 0$ and find the transfer function from Δu to $\Delta\theta$.

   (c) (2 marks) Is this linearized system stable?



2. (a) (6 marks) Let $\delta(k)$ and $1(k)$ denote the unit impulse and step functions in discrete time. Plot the graphs of

$$\delta(k-2), \quad \delta(3k-2), \quad 1(-3k-2).$$

(b) (4 marks) Plot the graph of the convolution of $1(k-2)$ and $1(k+3)$.

(c) (10 marks) In the following initial-value problem, solve for the z-transform of y and its ROC. Outline how to obtain $y(k)$ for $k \geq 0$.

$$\begin{aligned} y(k) - \frac{5}{2}y(k-1) + y(k-2) &= x(k), \quad k \geq 0 \\ y(k) &= \begin{cases} 0, & k < -2 \\ -1, & k = -2 \\ 1, & k = -1 \end{cases} \\ x(k) &= 1(k-2). \end{aligned}$$

3. In Exercise 26 you studied the compressor

$$\xrightarrow{x(k)} \boxed{C} \xrightarrow{y(k)}$$

where the input-output equations in the time and frequency domains are

$$\begin{aligned} y(k) &= x(2k) \\ \hat{y}(e^{j\omega}) &= \frac{1}{2}\hat{x}(e^{j\omega/2}) + \frac{1}{2}\hat{x}(e^{j(\omega-2\pi)/2}), \end{aligned}$$

and the expander

$$\xrightarrow{x(k)} \boxed{E} \xrightarrow{y(k)}$$

where the input-output equations are

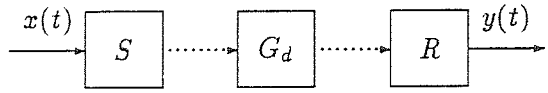
$$\begin{aligned} y(k) &= \begin{cases} x(k/2), & k \text{ even} \\ 0, & k \text{ odd} \end{cases} \\ \hat{y}(e^{j\omega}) &= \hat{x}(e^{j2\omega}). \end{aligned}$$

(a) (10 marks) Find the matrix representations of the two series connections EC and CE (only one of them is time-invariant):

$$\longrightarrow \boxed{C} \longrightarrow \boxed{E} \longrightarrow \qquad \longrightarrow \boxed{E} \longrightarrow \boxed{C} \longrightarrow$$

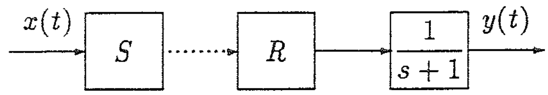
(b) (10 marks) For the system C (a discrete-time sampler), under what conditions can the input be reconstructed from the output? (Hint: Look at the Fourier transform of the output.) What system will reconstruct the input from the output?

4. (a) (10 marks) Consider the system



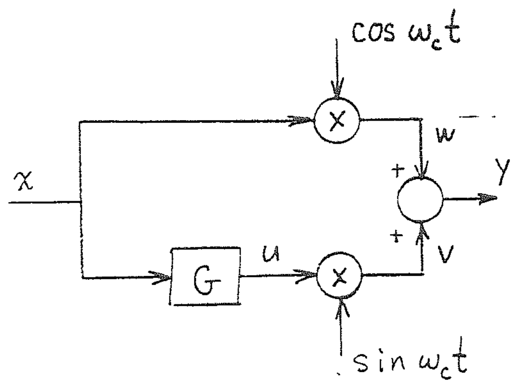
where S is the ideal sampler, R the ideal reconstructor (as in the sampling theorem), and G_d is LTI. Assuming $x(t)$ is bandlimited to frequencies less than 100 rad/s, find a suitable sampling period h and frequency response $\hat{g}_d(e^{j\theta})$ so that the input-output relationship is $\dot{y}(t) + y(t) = x(t)$.

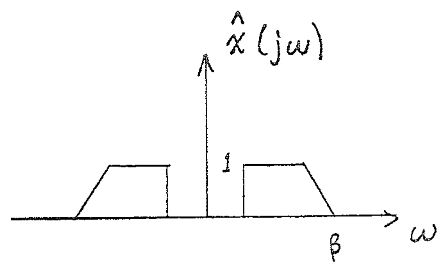
(b) (10 marks) Now consider



Here $x(t) = \cos 2t$, $-\infty < t < \infty$, and the sampling period is $h = 1$. Find $y(t)$.

5. (20 marks) Consider the following modulation system:





The Fourier transform of the input is as shown. The system G has frequency response

$$\hat{g}(j\omega) = \begin{cases} j, & \omega < 0 \\ -j, & \omega \geq 0. \end{cases}$$

The carrier frequency ω_c is greater than β. Sketch the graphs of $\hat{w}(j\omega)$, $\hat{u}(j\omega)$, $\hat{v}(j\omega)$, and $\hat{y}(j\omega)$.

6. (a) (10 marks) Find a state model for the system with transfer function

$$\frac{s^2+1}{2s^2-s-1}.$$

(b) (10 marks) Find a state model for the LTI continuous-time system whose impulse response is

$$g(t) = \begin{cases} 0, & t < 0 \\ 2 - 3e^{-2t} + e^{3t}, & t \geq 0. \end{cases}$$