UNIVERSITY OF TORONTO

FACULTY OF APPLIED SCIENCE AND ENGINEERING

DEPARTMENT OF MECHANICAL & INDUSTRIAL ENGINEERING

Third Year - MIE302S

VIBRATIONS

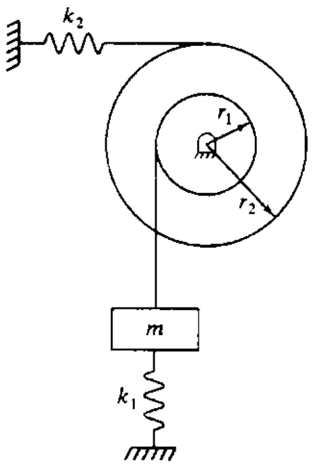
Final Examination

Examiner: J.W. Zu **Date: April 22, 1998** **Time: 2:00-4:30**

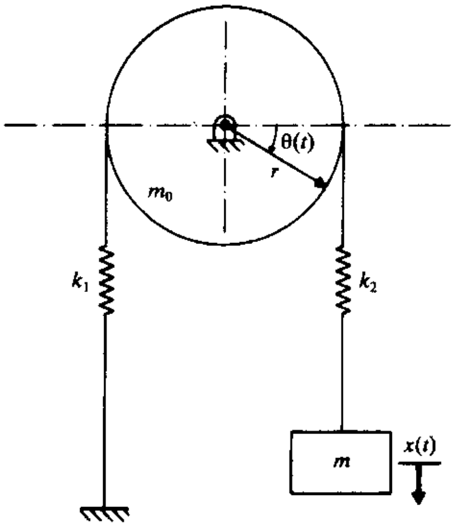
Instructions:

1. Answer all the questions.
2. Aid sheet only.
3. Only non-programmable calculators are allowed.

1. (30%) Establish the equations of motion of the following systems using Lagrange's Equations and find the corresponding $[m]$, $[k]$ matrices. Assuming small displacement. (15% each)

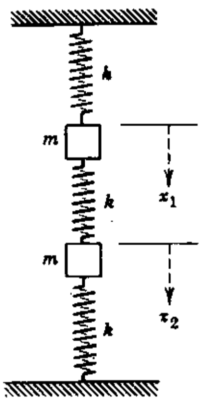


(1)



(2)

2. (15%) Find the flexibility matrix of the system using the definition of the influence coefficient.



3. (20%) Two of the normal modes of a vibrating system are known to be

$$\begin{Bmatrix} 0.275 \\ 0.399 \\ 0.449 \end{Bmatrix} \quad \text{and} \quad \begin{Bmatrix} 0.692 \\ 0.297 \\ -0.339 \end{Bmatrix}$$

and the mass matrix of the system is

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

(a) Find the remaining $[m]$ orthogonal normal mode;
(b) If the stiffness matrix of the system is given by

$$\begin{bmatrix} 6 & -4 & 0 \\ -4 & 10 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

determine all the natural frequencies of the system, using the normal modes in part (a). Do not use the eigenvalue problem to solve for natural frequencies.

4. (35%) For the system shown, $m_1 = 9, m_2 = 1, k_1 = 24$, and $k_2 = 3$.

1) Find the equations of motion and the corresponding $[m]$, $[k]$ matrices;
2) Determine the natural frequencies and normalized normal modes;
3) If free vibrations are initiated by moving m_1 a distance of $1\,mm$ to the right while holding the m_2 in its equilibrium position and releasing the system from rest. determine the subsequent motion;
4) Find steady-state response using mode superposition method if m_1 is subjected to a harmonic excitation $F_1 = 4\sin\omega t$.