UNIVERSITY OF TORONTO

FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION, DECEMBER 1999

ECE350F – PHYSICAL ELECTRONICS

Exam Type: A

Examiner – Prof. E. H. Sargent

Answer any 5 questions. All questions are of equal value.

Parts (a), (b), etc. of a given question are
interdependent only if so stated explicitly.

Good luck!

1. (a) **(4 marks).** Consider a MOS capacitor in which the semiconductor portion is p-type. Assume that there is no work function difference between the metal and semiconductor and that no interface charge is present. Draw band diagrams for this structure under each of the following conditions:

   (i) unbiased
   (ii) depleted
   (iii) accumulated
   (iv) inverted.

   (b) **(3 marks)** Draw a band diagram, now considering a cross-section *along* the channel instead of across the channel, for a MOSFET:

   (i) in active mode with small source-drain bias;
   (ii) in active mode with source-drain bias less than, but approaching, the saturation voltage;
   (iii) beyond saturation.

   Explain on physical grounds the behaviour of the differential resistance characteristic in each case with reference to your diagrams.

   (c) **(3 marks).** For an ideal metal-oxide-semiconductor structure with oxide thickness 300 Å and silicon p-type doping $1.5\text{x}10^{15}$ cm^{-3}, find the applied gate voltage and the electric field at the interface required to make the silicon surface intrinsic; and the applied gate voltage and the electric field at the interface required to bring about strong inversion.

   Intrinsic carrier density in silicon: $1.5\text{x}10^{10}$ cm^{-3}
   Relative dielectric constant of SiO_2: 3.9
   ln(10) = 2.3
   For any other needed materials parameters, please consult attached sheet "Properties of semiconductor materials."

2. (a) **(5 marks).** A uniformly doped pnp bipolar transistor is biased in saturation.

(i) Write down the general solution to the minority carrier diffusion equation inside the base in the low-level injection approximation.

(ii) State and briefly explain the origins of boundary conditions at the emitter-base and collector-base junctions.

(iii) Show that the excess hole concentration in the base can be expressed as:

$$\delta p(x) = p_{B0}\left\{\left[\exp\left(\frac{qV_{EB}}{kT}\right)-1\right]\left[1-\frac{x}{x_B}\right]+\left[\exp\left(\frac{qV_{CB}}{kT}\right)-1\right]\left[\frac{x}{x_B}\right]\right\}$$

for $\frac{x_B}{L_p} << 1$ where x_B is the neutral base width.

(iv) Show that the minority carrier diffusion current in the base is then given by:

$$J_p = \frac{qD_p p_{B0}}{x_B}\left\{\exp\left(\frac{qV_{EB}}{kT}\right)-\exp\left(\frac{qV_{CB}}{kT}\right)\right\}$$

(v) Show that the total excess minority carrier charge, in C/cm^2, inside the base region is given by:

$$\delta Q_{pB} = \frac{qp_{B0}x_B}{2}\left\{\left[\exp\left(\frac{qV_{EB}}{kT}\right)-1\right]+\left[\exp\left(\frac{qV_{CB}}{kT}\right)-1\right]\right\}$$

(b) **(2 marks).** Briefly discuss (e.g. using point form) how the emitter and base doping, base width, and base recombination in a bipolar junction transistor are best chosen in order to optimize the active-mode performance. Consider both static and dynamic characteristics. What are the practical limitations as to how thin the base can be made?

(c) **(3 marks).** A heterojunction – a junction between two semiconductors of *different* bandgaps and electron affinities – can be used to improve the emitter injection efficiency in a forward-biased pnp bipolar transistor. Draw an approximate band diagram of the pertinent junction for an appropriate choice of emitter material bandgap compared to base material bandgap. By what multiplicative factor, approximately, do you expect the electron base-emitter current to be reduced relative to the homojunction (same bandgap) case consider during the lectures? What should be the resulting new expression for the emitter injection efficiency?

3. (a) **(3 marks)** For an extrinsic semiconductor, find the hole concentration which minimizes the conductivity. Take the intrinsic carrier density and the electron and hole mobilities to be independent of the doping. What is the value of this minimum conductivity? Propose one practical use for your result.

(b) **(3 marks)** A semiconductor has bandgap 1 eV, electron effective mass 1 m_0, and hole effective mass 4 m_0. The sample contains 2×10^{15} cm^{-3} ionized acceptors.

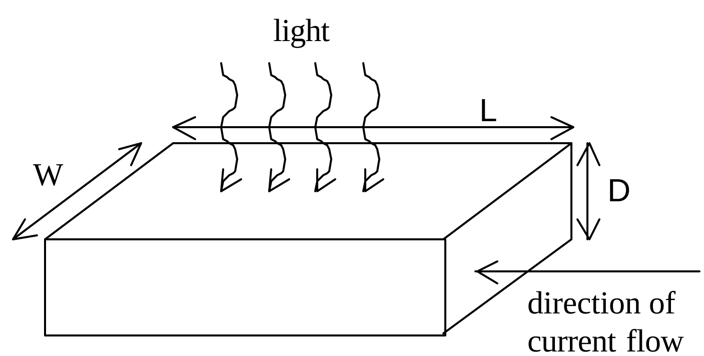
(i) Calculate the position of the room-temperature Fermi level relative to the valence band edge.

(ii) Calculate the position of the room-temperature intrinsic Fermi level relative to the conduction band edge.

(iii) Draw a band diagram and label the band edges, the intrinsic Fermi level position, and the Fermi level position in the doped material.

{ln(10) = 2.3.}

(c) **(4 marks).** A photoconductor (figure) is a device whose conductivity is increased by carriers generated via the absorption of light. The photoconductivity is given by $\Delta \boldsymbol{s} = qG_{ph}\boldsymbol{t}(m_e + m_h)$ where $G_{ph} = \boldsymbol{h}I\boldsymbol{l}/hcD$ is the charge carrier photogeneration rate per second per unit volume and $\boldsymbol{t}$ is the minority carrier lifetime. $\boldsymbol{l}$ is the wavelength and I the intensity of light incident on the sample. $\boldsymbol{h}$ is the number of electron-hole pairs generated per absorbed photon. h is Planck's constant and c is the speed of light.



Derive the above expression for the increase in conductivity under illumination. Explain the origin of each term. Recall that for photons $E = \hbar \boldsymbol{w}$ and that for any wave $f = c/\boldsymbol{l}$, where f represents frequency.

4. (a) **(2 marks).** Starting from the equation for hole current density, use integration with respect to field and hole carrier density to derive the built-in potential across a p-n junction at equilibrium. Express your answer in a form such that the only materials-specific quantity is the ratio of the equilibrium hole carrier densities on either side of the junction. In invoking the Einstein relation, have you made any assumptions or approximations?

(b) **(5 marks)** An abrupt p-n$^+$ junction has 10^{15} cm^{-3} acceptors on the p-side and 10^{19} cm^{-3} donors on the n-side. The intrinsic carrier density of the material is 10^{10} cm^{-3}. The minority carrier recombination times are 4 ns for electrons on the p-side and 10 ns for holes on the p-side. Electrons and holes have the same mobility of 100 cm^2/Vs. The cross-sectional area is 1 mm^2 and the forward voltage applied across the junction is $kT\ln(1000)$.

(i) Under low-level injection, and neglecting recombination and generation in the depletion region, what current will flow? For simplicity, calculate only the current associated with flow of the carrier which dominates the total current. How good an approximation is this?

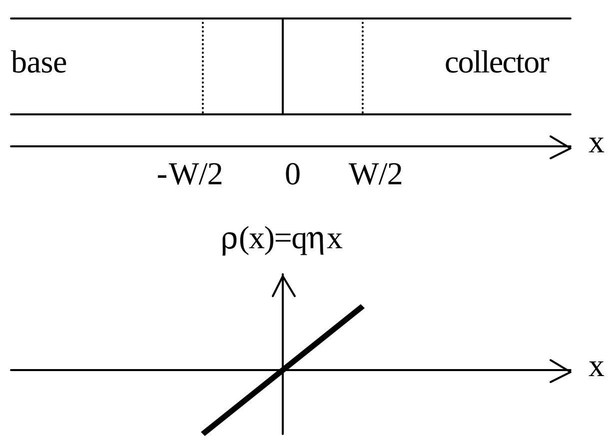
(ii) Explain the dominance of one type of carrier flow with reference to the relative height of energetic barriers. Illustrate using a carefully-drawn band diagram.

(iii) Is the low-level injection approximation justified?

(c) **(3 marks)** In the collector of a *graded* bipolar transistor, the doping profile is such that the net charge density in the junction region can be approximated by:

$$\rho(x) = q\eta x$$

(figure). Find the dependence of the width of the junction on built-in voltage, applied voltage, η, q, and dielectric constant.



5. If one scatters low energy particles from the surface of a solid, the ions do not penetrate significantly. They see only the periodic arrangement of the topmost layer. The resulting effect is described as two-dimensional, or surface, scattering.

(a) **(3 marks)** Consider the case of Argon atoms adsorbed on a smooth graphite substrate and therefore arrayed in the surface plane to form a 2D simple hexagonal lattice. The nearest neighbour spacing is a. Show pictorially that the vectors in the x-y plane:

$$\vec{a}_1 = a\hat{x} \qquad\qquad \vec{a}_2 = \frac{a}{2}\left(\hat{x} + \sqrt{3}\hat{y}\right)$$

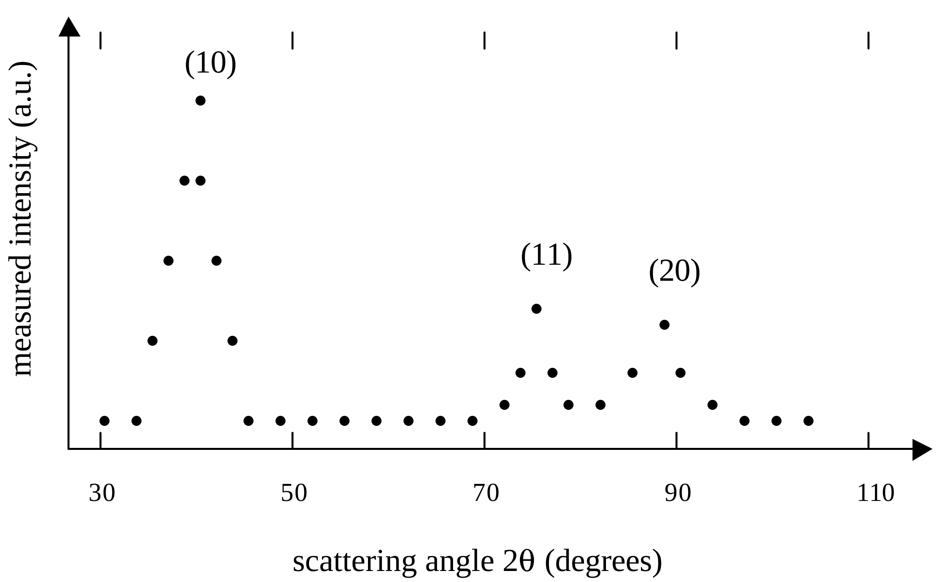
describe a primitive unit cell. Use the requirement $\vec{a}_i \cdot \vec{b}_j = 2\pi\delta_{ij}$ to determine the reciprocal lattice vectors $\vec{b}_j$.

(b) **(7 marks).** A monochromatic beam of neutrons with de Broglie wavelength 2.4 Å is scattered by the adsorbed Ar layer. As you will have found in (a), all reciprocal lattice vectors $\vec{G}$ for the layer of atoms lie in the plane of the surface.

The scattering condition is $\Delta\vec{k}_p = \vec{G}$ where $\Delta\vec{k}_p$ is the change of wave vector in the plane.

For the purposes of this question, assume that the incident and emergent beams are in the plane and the scattering angle is 2θ.

The figure shows data of measured intensity versus scattering angle 2θ. Show that the data are consistent with the simple 2D hexagonal lattice considered in (a). Find the Ar-Ar nearest neighbour distance.



6. (a) **(4.5 marks).** How is each phenomenon below evidenced in a band diagram? Illustrate and explain.

(i) The existence of an electric field inside a semiconductor;
(ii) The presence of a net current flowing inside a semiconductor;
(iii) A conduction band electron with zero kinetic energy;
(iv) A valence band hole with kinetic energy equal to one quarter of the bandgap energy;
(v) An intrinsic semiconductor far from equilibrium.
(iv) Photogeneration of an electron-hole pair.

(b) **(1.5 marks).** An average hole drift velocity of 10^3 cm/s results when 2 V is applied across a 1-cm-long semiconductor bar. What is the hole mobility inside the bar?

(c) **(1 mark).** Describe two important carrier scattering mechanisms inside a semiconductor.

(d) **(3 marks).** A Si bar of length L is nonuniformly doped with donors and maintained under equilibrium conditions at room temperature. The resulting electron carrier concentration has spatial dependence:

$$n(x) = n_i \exp\left(x + a/b\right) \text{ for } 0 \le x \le L$$

where $a = 0.5$ cm, $b = 0.1$ cm, and $L = 1$ cm.

(i) Draw the spatial energy diagram across the structure specifically showing conduction and valence band edge energies, the intrinsic Fermi level, and the Fermi level.
(ii) Make a sketch of electric field inside the bar as a function of position.
(iii) What are the directions (+x or –x) of electron drift, electron diffusion, hole drift, and hole diffusion currents flowing inside the bar? What is the direction of any net current?

7. (a) **(4 marks).** Carefully depict as a function of energy the product of the conduction band density of states with Fermi occupation factor for:

(i) A lightly-doped n-type semiconductor.

(ii) A degenerately-doped ($E_f > E_c$) n-type semiconductor.

Explain how your drawings are related to carrier density.

Discuss both the mathematical and the physical validity of using Maxwell-Boltzmann statistics in each case ((i) and (ii)).

(b) **(3 marks)** F. Bloch wrote,

"When I started to think about it, I felt that the main problem was to explain how the electrons could sneak by all the ions in a metal. . . . By straight Fourier analysis I found to my delight that the wave differed from the plane wave of free electrons only by a periodic modulation."

(i) Relate Bloch's words to the Bloch, or Floquet, theorem.
Under what conditions does the theorem apply?

(ii) Estimate the magnitude of the energy gap in terms of the pertinent Fourier component of the crystal potential.

(c) **(3 marks).** In a semiconductor quantum well formed using a double heterojunction, the spatial distribution of the electron wavefunction may be described using a product of a *carrier*, whose rapid oscillations are determined by the underlying atomic lattice, and an *envelope*, whose behaviour is governed by the macroscopic quantum well potential.

The envelope function is then described by an *envelope Hamiltonian*, in which the macroscopic conduction band edge spatial dependence serves as the potential term.

A *periodic* heterostructure – an infinite concatenation of quantum wells and barriers – is known as a *superlattice*. By analogy with the formation of bands inside *atomically* periodic media, predict the behaviour of an electron which is launched into a *macroscopically* periodic semiconductor superlattice. Extend ideas such as allowed and forbidden bands, propagation and evanescence, and effective mass.