UNIVERSITY OF TORONTO
DEPARTMENT OF ELECTRICAL & COMPUTER ENGINEERING

ECE 110S
Electrical Fundamentals

Final Exam

April 25, 1997
Time: 2:00 - 4:30 PM

Examiners

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Instructions:

1. **Type A exam: No aids allowed except for a non-programmable calculator as specified by the faculty.**
2. **Answer all questions for each of the 4 parts.**
3. **Each of the four parts is of equal weight.**
4. **You may use the back of the preceeding page for rough work in a given solution.**
5. **Place your final answers in the corresponding boxes.**

Part	Marks
I	
II	
III	
IV	
Total	

Part I

Electricity and Magnetism - Quick work (5 parts, 5 marks each)

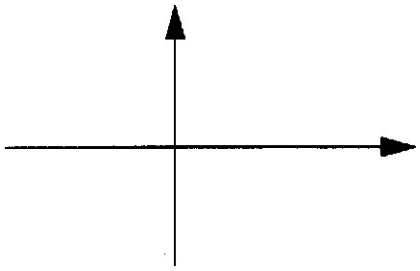
(Note: $e = 1.6 \times 10^{-19}$ C, $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m, $\mu_0 = 4\pi \times 10^{-7}$ Tm/A)

a) A charge $Q_1 = 10$ mC is located on the x-y plane at (0,1m) and a charge $Q_2 = -2$ mC is located at (1m, 3m).

i) Clearly label the coordinate system provided and show the location of the two charges.

ii) Find the force on Q_1 and the force on Q_2 and indicate the two forces (using vectors) on the diagram of part (i).

iii) Give the components of these two forces.



Components of F_1 ________________

Components of F_2 ________________

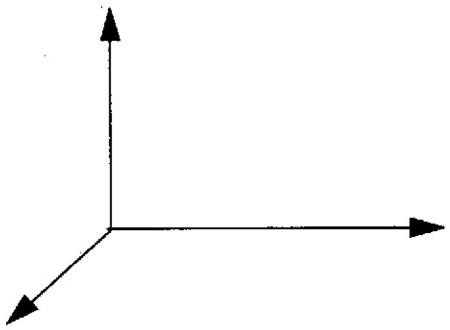
b) Assume a uniform electric field exists throughout space, $E = E_0\hat{\imath}$, and $E_0 = 1$ V/m ($\hat{\imath}$ represents a unit vector along the x-axis). Find the work required to move a charge $Q = 1$ mC from the origin to the point (1m, 2m, 3m).

Work required to move the charge:

c) Assume a uniform electric field and a uniform magnetic field co-exist throughout space. We have $\vec{E} = 1\hat{\imath}$ (V/m) and $\vec{B} = 1\hat{k}$ [T] ($\hat{\imath}$ and $\hat{k}$ are unit vectors along the x and y-axis, respectively). A long straight copper wire is stretched along the y-axis. It carries a current of 1A. The positive sense of this current is that of the positive sense of the y-axis.

i) Clearly label the 3-d Cartesian coordinate system provided and indicate the electric and magnetic fields by one representative arrow for each.

ii) Find the force on the conductor, per 1m length. Give the components of this force and indicate this force on the drawing of part (i).

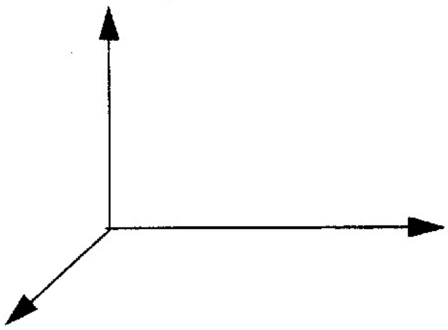


Components of force: ____________________

d) A straight copper wire 50 m long is stretched along the x-axis. The centre of the wire is at the origin. The wire carries a current of 1A. The positive sense of this current is that of the positive sense of the x-axis. A circular copper wire loop of radius 1m lies in the y-z plane. The centre of the loop is at the origin. The loop carries a current of 5A, clock-wise, when viewed along the x-axis; the eye looking in the positive x direction.

i) Clearly label the 3-d Cartesian coordinate system provided and draw the straight wire and the loop.

ii) Find the force on the wire and on the loop, give its components, explain.



Components of force: ____________________

e) Assume a uniform magnetic field exists throughout space. The field changes with time according to: $\vec{B} = B_{x0}\cos(\omega_x t + \phi_x)\hat{\imath} + B_{y0}\cos(\omega_y t + \phi_y)\hat{\jmath} + B_{z0}\cos(\omega_z t + \phi_z)\hat{k}$. Where B_{x0}, ω_x, ϕ_x, B_{y0}, ω_y, ϕ_y, B_{z0}, ω_z, and ϕ_z are constants and $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ are unit vectors of the x, y, z-axis, respectively. A circular copper wire loop of radius R lies in the x-y plane with its centre at the origin. Find the voltage induced in the loop. (Hint: Draw a diagram showing the field components and the loop).

The induced voltage: ________________

Part II

Both questions a) and b) refer to the following circuit.

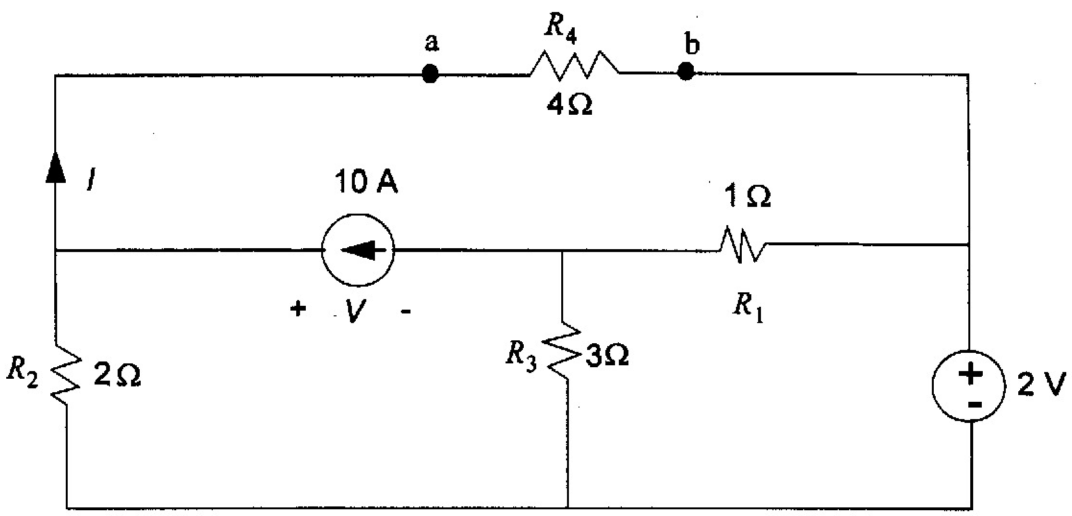


Figure 1

a) Nodal Analysis

i) How many nodes are there in the circuit?

Number of nodes =

ii) Write a set of equations whose solution yields the node voltages. Label the nodes V_a, V_b, V_c, etc. Specify a reference node.

Set of Equations

iii) Determine the voltage $\dot{V}$.

$V =$

iv) Determine the current I.

$I =$

v) Is the current source supplying or absorbing power and how much?

$P =$

b) Thevenin Equivalent Circuit

i) Remove the R_4 resistor from terminals (a) and (b) in the circuit shown in Figure 1. Find the Thevenin equivalent circuit for the circuit as seen from terminals (a) and (b). Draw the Thevenin equivalent circuit.

Thevenin Equivalent Circuit

ii) Connect the resistor $R_4 = 4\Omega$ to terminals (a) and (b) of the Thevenin equivalent circuit found above and determine the current through the resistor R_4.

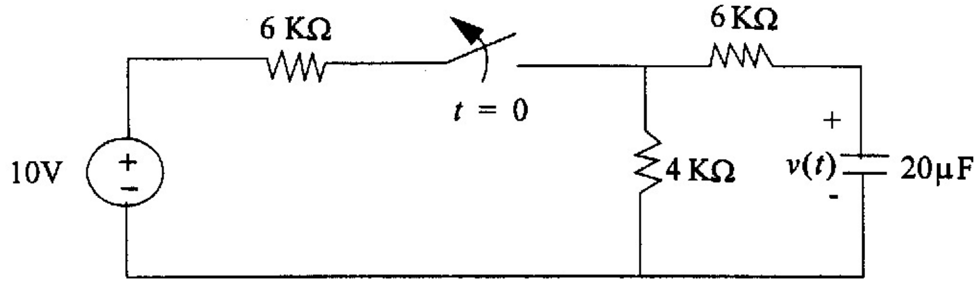
I =

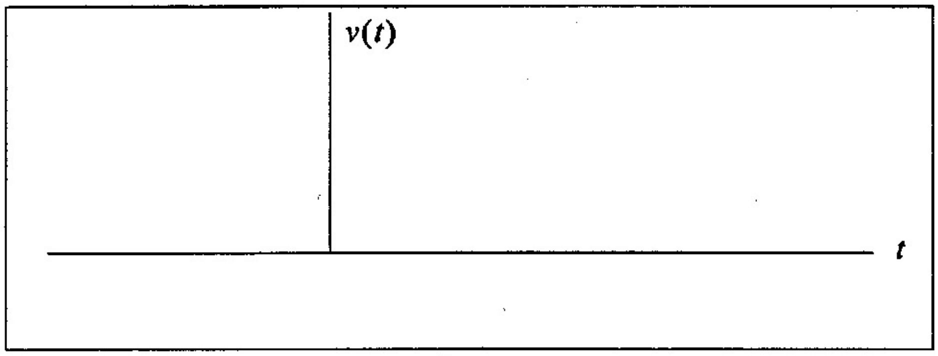
iii) Connect a resistor R_L to terminals (a) and (b) of the Thevenin equivalent circuit. For what value of R_L will the circuit deliver maximum power? Find the maximum power delivered to R_L.

R_L =
P_{MAX} =

PART III

a) In the following circuit the switch is opened at time $t = 0$ after being closed for a long time. Determine the voltage accross the capacitor $v(t)$ for all time $-\infty < t < \infty$. Plot your result and clearly indicate the various expressions. In particular give values for $v(t)$ at $t = -\infty$, $t = 0-$ (just before the switch is opened), $t = 0+$ (just after the switch is opened), and $t = \infty$.





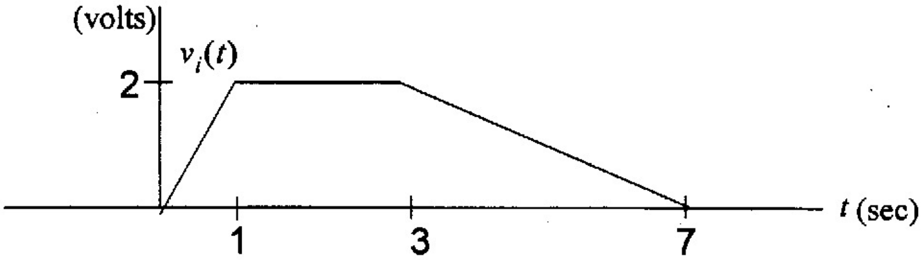
$v(-\infty) =$

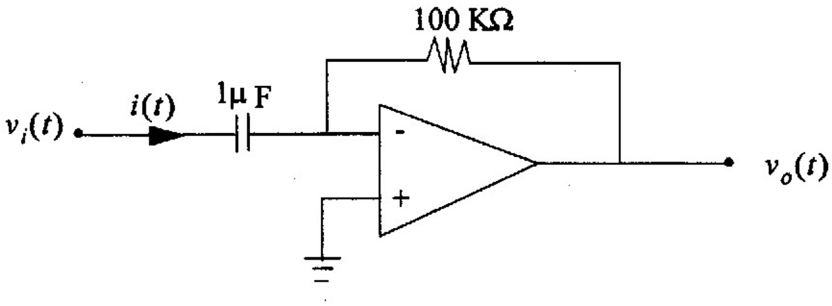
$v(0-) =$

$v(0+) =$

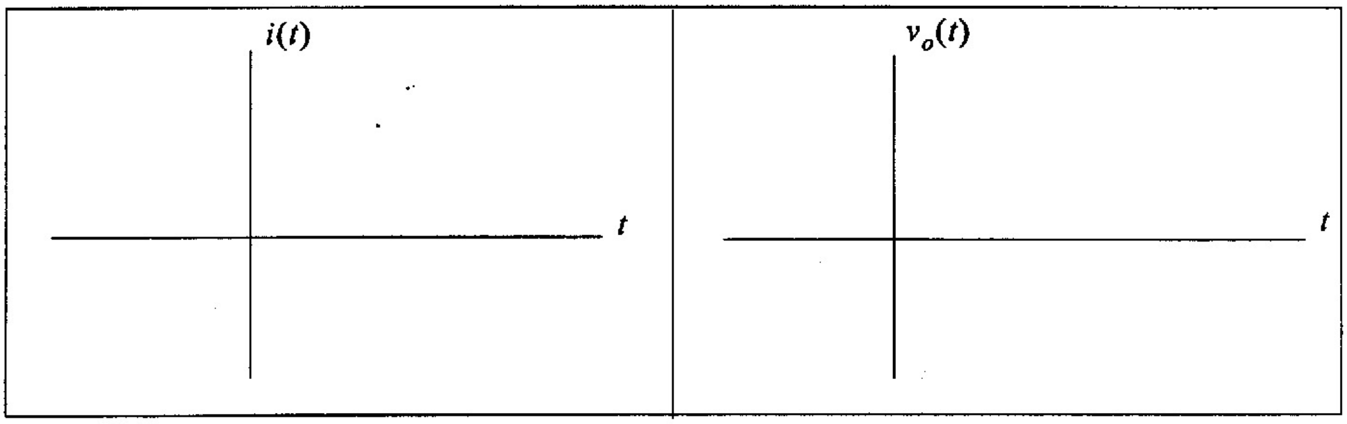
$v(\infty) =$

b) The signal $v(t)$ is the input to the following Op-Amp circuit:





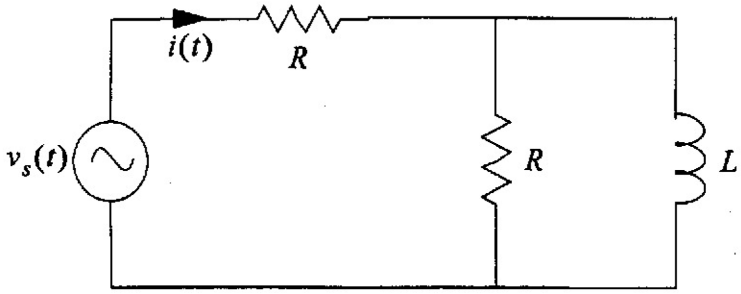
Find the current through the capacitor $i(t)$. Find the output voltage $v_o(t)$. What is the maximum energy E_{MAX} that is stored in the capacitor?



$E_{MAX} =$

Part IV

The circuit shown is driven by a sinusoidal voltage $v_s(t) = V\cos(\omega t) = 100\cos(\omega t)$.



Both resistors are of an identical value, $R = 100\ \Omega$. Inductance $L = 0.1$ H.

i) Find an expression for the phasor **I** in terms of the circuit parameters V, R, L, and ω.

I =

ii) Find a numerical value of the phasor **I** at the frequency $\omega = \dfrac{R}{2L}$ [rad/sec].

I =

iii) Express **I** in the rectangular and in the polar (exponential) form.

Rectangular: **I** =
Polar: **I** =

iv) What is the phase difference (ϕ (in degrees) between the phasor **V** representing the input voltage, and the phasor **I** representing the current into the circuit at the frequency $\omega = \frac{R}{2L}$?

ϕ =

v) Compute the average power P_{AVE} supplied into the circuit by the voltage source $v_s(t)$ at frequency $\omega = \frac{R}{2L}$ [rad/sec].

$P_{AVE} =$