UNIVERSITY OF TORONTO

FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION, DECEMBER 1999

Fourth Year - Program 4

Exam Type: C

MIE566F - Decision Analysis

Examiner: Eddie Hsu

Total marks of examination 100
This will count for 40% of the final grade.
Non-programmable calculators permitted

15 1. In order for a strategic move to be effective, one must show a commitment to act in a pre-specified manner. This requires credibility on the part of the decision-maker making the strategic move. Give three ways in which credibility can be established and explain how these ways establish credibility in terms of the manipulation of information and payoffs.

2. A ship involved in a salvage operation is in the process of recovering material from a sunken wreck. They are under a time constraint, however, as a storm will hit them shortly. The have five days in which to work before they must leave. The storm will make continued salvage impossible and they will be unable to access the wreck after the storm passes. The captain must decide what procedure they will use to recover material from the wreck. There are three options:

   Option 1) manual recovery with divers (D),

   Option 2) mechanically assisted recovery with cables and winches (W), and

   Option 3) recovery using inflatable balloons (B)

   Manual recovery is slow but will certainly succeed and requires no setup time. It has a constant rate of material recovery of \$600,000 / day.

   Mechanically assisted recovery with winches requires a day to set up before any attempt can be made to lift material. The cables are difficult to deploy and have a tendency to tangle. The captain judges that the recovery rate will be \$1,000,000 / day with some unknown probability p and \$300,000 / day otherwise.

   The third option, using inflatable balloons requires a day to set up the equipment as well. It involves attaching inflatable balloons to the material to be recovered and allowing the balloons to lift the material to the surface. This is always tricky and there is no guarantee that it will work. It is an all or nothing proposition. The captain judges that the balloons will succeed with some probability q. In such an event, the recovery rate will be \$2,000,000 / day. If the balloons fail then no recovery is possible with this method in the time remaining before the storm hits.

   The characteristics of the three options are listed in the following table:

	Set-up Time	Recover Rate (\$100 000s / day)
Manual (M)	0 days	6
Winches (W)	1 day	10 with probability p 3 with probability $1 - p$
Balloons (B)	1 day	20 with probability q 0 with probability $1 - q$

The problem is further complicated by the fact that once the decision is made to use either Option 1 or 2, there is no turning back. The effort required to change the entire ship's operation over to another procedure would take more time than they have. If the balloons are used, however, and fail, then there is still time to use one of the other two procedures.

18 a) Sketch a diagram showing the optimal strategy to select for each combination of p and q.

8 b) Assume p = 70%. Draw a rainbow diagram for all values of q from 0 to 1.

4 c) Suppose the captain thinks that p could range anywhere from 60% to 80% and q could range anywhere from 30% to 50%. Assume that there is sufficient time to get information about one of these probabilities before he must decide on a course of action and that the costs of acquiring such information are negligible. About which probability should he attempt to gather more information? Explain your choice.

3. You are trying to construct a two-attribute utility function from some data that you have. Normally, you would require several data points to produce a utility function that would accurately reflect the decision-maker's preferences. Fortunately, other evidence indicates that the two attributes are mutually utility independent. Thus, you can use the utility function:

$$U(x,y) = k_X U_X(x) + k_Y U_Y(y) + (1 - k_X - k_Y)U_X(x)U_Y(y)$$

6 a) Explain what is meant by utility independence. What does this imply about the relationship between the two utility curves $U(x_1,y)$ and $U(x_2,y)$?

8 b) Presume that you have the following 10 data points:

(x, y)	*U(x, y)*	*(x, y)*	*U(x, y)*
(2,0)	.24	(5,0)	.68
(2,2)	.688	(5,2)	.784
(2,5)	.784	(5,5)	.862
(2,8)	.848	(5,8)	.914
(2,10)	.88	(5,10)	.94

You may additionally assume that U(0,0) = 0 and U(10, 10) = 1.

Using the data, calculate the values of k_X and k_Y. Are attributes x and y complements or substitutes?

8 c) Based on the data in part b), sketch the utility curves, $U_X(x)$ and $U_Y(y)$. Each curve should indicate the utilities for values 0, 2, 5, 8, and 10.

7 4. Explain why randomness may be useful when dealing with an opponent. What is the random component in brinkmanship and how is it used to influence an opponent?

5. An investment firm is thinking of investing in a new fuel cell development. The cell can be used in automobiles, making them more fuel-efficient. Preliminary research shows that the fuel cell will work. The question remains, however, whether automobile makers will adopt the product.

For simplicity, the adoption rate of the fuel cell by automobile makers can be ranked as low (L), medium (M), or high (H). The greater the adoption rate, the greater the return to the investment firm. Analysts in the investment firm assess the probabilities and returns for the different adoption rates as:

	Low	Medium	High
Probability of Occurring	20%	60%	20%
Expected Return to Investment Firm	$5 million	$10 million	$20 million

Once the adoption rate is know, the firm can decide to lobby the government to enact more stringent laws promoting more fuel-efficient vehicles in the hopes that this will result in a greater adoption rate. Lobbying the government costs $2 million. The effectiveness of the lobby depends in part on the adoption rate at the time of the lobby. In general, the greater the current adoption rate, the more likely the government is to pass laws encouraging the use of the fuel cell. Let L+, M+, and H+ denote the adoption rates after lobbying. Analysts in the investment firm, have assessed the conditional probabilities of the lobbying effect as:

$$P(L+\,|\,L) = 60\% \qquad P(M+\,|\,L) = 30\% \qquad P(H+\,|\,L) = 10\%$$

$$P(L+\,|\,M) = 0\% \qquad P(M+\,|\,M) = 60\% \qquad P(H+\,|\,L) = 40\%$$

(Thus, the probability that the adoption rate will be medium after lobbying given an adoption rate of low prior to lobbying is 40%.)

The investment firm also has an alternative project in which they can invest. The expected return of this alternative investment is $10 million.

9 a) Assume that the investment firm is risk-neutral. Say that the firm can get an estimate of the adoption rate before they have to decide whether or not to invest. What is the most the investment firm would pay to determine the adoption rate of the fuel cell prior to any lobbying effort?

9 b) Assume that the investment firm is risk-neutral. The firm has the option of hiring a political consultants group before they decide to lobby. The consultants group has a good track record of predicting the effect of any lobbying effort. They will correctly predict the outcome of the lobbying effort 80% of the time. The other 20% of the time, any of the remaining possible alternatives is equally likely. Hence, if all three adoption levels were possible and the consultants group predicted a medium (M+) adoption rate then,

$$P(L+ \mid \text{consultants group said } M+) = 10\%$$

$$P(M+ \mid \text{consultants group said } M+) = 80\%$$

$$P(H+ \mid \text{consultants group said } M+) = 10\%$$

If only medium and high adoption rates were possible, then the same prediction would result in the conditional probabilities:

$$P(M+ \mid \text{consultants group said } M+) = 80\%$$

$$P(H+ \mid \text{consultants group said } M+) = 20\%$$

The consultants group charges $1 million for their services. Should the investment firm hire them and when should they do so?

8 6. It was been proposed that the way in which we think is constrained to some extent by the limits by which we can express ourselves. Briefly explain how language and graphics can either limit or constrain us from exploring different possibilities. Your explanation should consider both creative and psychological aspects.