University of Toronto
Faculty of Applied Science and Engineering
Final Examination, December 12, 2000
MIE 372F - CONTROL SYSTEMS
Examiner: C.B. Park

No aids allowed.
For questions #3-#7, use Figure 1 shown in page 1.

1. A system is presented by the following nonlinear differential equation:

$$\ddot{x} = x^2 + b\dot{x} + ku^2/x^2$$

Assume that the operating point is $(x_o, \dot{x}_o, u_o)$.

a) What must the operating point $(x_o, \dot{x}_o, u_o)$ satisfy to be also an equilibrium point (Do not solve the resulting equation). (3 marks)

b) Write a linearized equation about $(x_o, \dot{x}_o, u_o)$ assuming that it is an equilibrium point. (7 marks)

2. Consider a unity feedback control system whose open loop transfer function is:

$$G(s) = \frac{(0.4s + 1)}{s(s + 0.6)}$$

Determine the transient response for unit step input. (15 marks)

3. For the system shown in Figure 1 with $G_c = K$ and $G(s) = 1/[s(s+1)(s+4)]$, use the Routh-Hurwitz criterion to find the limit on K for stability. At this limit, determine the position of the system poles on the imaginary axis of the s-plane. (10 marks)

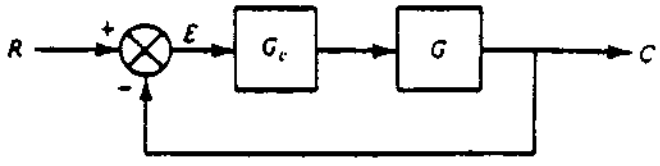

Figure 1

4. In Figure 1, let G(s) be the open loop unstable plant G(s) = 1/(s - 1). Design the simplest possible controller $G_c(s)$ that will satisfy all the following specifications: (15 marks)

a) The steady state error for constant inputs must be zero.
b) The system settling time must be about 4 sec.
c) The system damping ratio should be 0.5.

5. In Figure 1 with $G = 1/[s(s+1)]$:

a) With $G_c = K$ find K for a damping ratio 0.707, and the steady state error for a unit ramp input. Sketch the Root Locus. (10 marks)
b) Repeat the above for $G_c = K\,(10\,s + 1)/(40\,s + 1)$. How does this phase-lag compensator affect the behavior? (10 marks)

6. In Figure 1, G is the transfer function of a field-controlled motor in a position control system:

$$G = \frac{1}{s(0.1s+1)(0.02s+1)}$$

If $G_c = K$, find K for a phase margin of 45° and determine the corresponding gain margin. (15 marks)

7. In Figure 1:

$$G = \frac{1}{s(0.1s+1)(0.01s+1)}$$

design phase-lead compensation to meet the following specifications: (15 marks)

a) The steady-state error for the ramp input may not exceed 2%.
b) The phase margin must be of the order of 45°.
c) To limit noise effects, the ratio of the break frequencies of the compensator should not be larger than necessary.