

UNIVERSITY OF TORONTO  
FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION, DECEMBER 2001

Third Year – Program 6

CHE 321F – STATISTICS

Exam Type: C

Examiner – S.T. Balke

Notes:

- i. This is a Type C Exam: One aid sheet using the supplied 8.5" x 11" paper is permitted.
- ii. All non-programmable calculators permitted.
- iii. Answer all questions to four significant figures.
- iv. Attempt all questions.
- v. Marks total 100.

Marks

- 20      1. Given the following measurements of density of a new foamed plastic product:

0.811, 0.814, 0.812, 0.856, 0.854

Note: you may assume that the observations originate from a population that is distributed  $N(\mu, \sigma)$ .

Calculate the following quantities (be careful to present only physically reasonable values):

- a. the best point estimate of the population mean
- b. the best point estimate of the population standard deviation
- c. the 99% upper confidence bound on the population mean
- d. the 90% two sided prediction interval for a single additional measurement
- e. the 98% two sided confidence interval on the population standard deviation
- f. completely explain what the quantity that you calculated in part c. above means
- g. the probability of obtaining a value of an observation between observation values of 0.811 and 0.812.

## Marks

15

2. Given that the two sided 95% confidence interval on the population mean from a sample of 5 observations is 10.0,15.0 and assuming that the observations,  $x$ , are distributed  $N(\mu, \sigma)$ , sketch your best estimate for a box plot for the sample. On the plot, name and show the value of all relevant quantities defining the plot, including the value of upper and lower fences. Assume that the value of the smallest observation in the sample is  $0.80 \cdot \mu$  and that the largest is  $1.25 \cdot \mu$ . State any additional assumptions that you require.

35

3. A rupture disk is manufactured to fail at a mean value of 21.00 MPa. Six replicate observations of disk failures provided the data shown in Table 3.1.

**Table 3.1**  
**Disk Failure Pressures**  
**[Mpa]**

16.829  
25.423  
22.325  
24.391  
25.124  
26.657

The customer is concerned that, on the average, a disk does not rupture at pressures lower or higher than that mean value. In particular, a failure at 26.86 MPa would be too late: the reactor would be damaged by the excess pressure. You set the probability of an alpha error as 0.025. Assume that the population of observations of failure pressure follow a  $N(\mu, \sigma)$  distribution.

- i. Write the null hypothesis. (Based on the assumption that the product meets the requirement.)
- ii. Write the alternate hypothesis. (Based on the assumption that the product does not meet the requirement.)
- iii. Define the  $H_0$  rejection region for the observed sample mean.
- iv. Define alpha in general terms.
- v. Write the equation(s) defining alpha (or alpha/2) for the specific problem.
- vi. Decide whether or not to accept the null hypothesis based upon the criterion (or criteria) values and show your calculations.
- vii. Decide whether or not to accept the null hypothesis based upon the p value of alpha. You may not be able to calculate the exact p value. However, you can make a statement about it. Also, write the Excel formula with numerical arguments that would enable the p value to be calculated.
- viii. Calculate the most important value to be plotted on the y axis of the OC curve based upon the information provided in the question.

4. The rate of reaction in a batch reactor is being retarded by the presence of an impurity. Data collected was rate,  $R$ , [moles/(liter second)], versus concentration,  $c$ , [moles/liter] of the impurity. Table 4.1 below shows both the input and a portion of the output of the linear regression carried out using Excel 2000.

TABLE 4.1: LINEAR REGRESSION INPUT AND OUTPUT

	A	B	C	D	E	F	G	H	I	
1	INPUT									
2										
3	R	c								
4	mol/(liter second)	mol/liter								
5	1.63	0.8								
6	1.65	0.8								
7	1.69	0.8								
8	1.72	0.8								
9	1.72	0.8								
10	1.37	1								
11	1.54	1								
12	1.58	1								
13	1.1	1.2								
14	1.26	1.2								
15	1.42	1.2								
16										
17	SUMMARY OUTPUT									
18										
19	Regression Statistics									
20	Multiple R	0.89374674								
21	R Square									
22	Adjusted R Square									
23	Standard Error									
24	Observations	11								
25										
26	ANOVA									
27			df	SS	MS	F	Significance F			
28	Regression		1	0.334096212	0.334096212		0.000208211			
29	Residual		9	0.084158333	0.009350926					
30	Total		10	0.418254545						
31										
32			Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%	Lower 90.0%	Upper 90.0%
33	Intercept		2.52416667		14.75205391	1.3637E-07	2.13709744	2.911235893		
34	X Variable 1		-1.04583333	0.174956141		0.00020821	-1.441634543	-0.650332123		
35										
36										
37										
38	RESIDUAL OUTPUT									
39										
40	Observation	Predicted Y	Residuals	Standard Residuals		Percentile				
41	1	1.6875	-0.0575	-0.626785992		4.545454545				1.1
42	2	1.6875	-0.0375	-0.408773473		13.63636364				1.26
43	3					22.72727273				1.37
44	4	1.6875	0.0325	0.354270343		31.81818182				1.42
45	5	1.6875	0.0325	0.354270343		40.90909091				1.54
46	6	1.47833333	-0.108333333	-1.180901144		50				1.58
47	7	1.47833333	0.061666667	0.672205266		59.09090909				1.63
48	8	1.47833333	0.101666667	1.08230304		68.18181818				1.65
49	9	1.26916667	-0.169166667	-1.844022555		77.27272727				1.69
50	10	1.26916667	-0.009166667	-0.099927404		86.36363636				1.72
51	11	1.26916667	0.150833333	1.644177746		95.45454545				1.72

X Variable 1 Line Fit Plot

Y

X Variable 1

Y

Predicted Y

Question 4 continued on Page 4.

Question 4 continued from Page 3.

<b>Marks</b>	4. Referring to Table 4.1:
15	i. Calculate the quantities belonging in the following cells of Table 4.1 (Be sure to clearly indicate which cell you are calculating): B21 B22 B23 E28 C33 D34 H34 B43 C43
5	ii. A probability plot of the residuals can be used to obtain an estimate of the population quantities. Calculate the value on the y axis of the point on the probability plot corresponding to the residual value of 0.1017 (cell C48) plotted on the x axis, and state the expected value of the slope of this plot if a straight line is obtained.
5	iii. Assuming that the fit is adequate, calculate the 95% two sided prediction interval for the value that you expect to obtain when you average 7 new measurements of the value of y at $x=1.70$ .
5	iv. There is something seriously wrong in the “goodness of the fit”. State what this is and using symbols, not calculated quantities, show the objective function that you would minimize using “Solver” to overcome the problem. Explain your answer.

# APPENDIX

## B

## Tables

Table B.1  
Random Digits

12159	66144	05091	13446	45653	13684	66024	91410	51351	22772
30156	90519	95785	47544	66735	35754	11088	67310	19720	08379
59069	01722	53338	41942	65118	71236	01932	70343	25812	62275
54107	58081	82470	59407	13475	95872	16268	78436	39251	64247
99681	81295	06315	28212	45029	57701	96327	85436	33614	29070
27252	37875	53679	01889	35714	63534	63791	76342	47717	73684
93259	74585	11863	78985	03881	46567	93696	93521	54970	37601
84068	43759	75814	32261	12728	09636	22336	75629	01017	45503
68582	97054	28251	63787	57285	18854	35006	16343	51867	67979
60646	11298	19680	10087	66391	70853	24423	73007	74958	29020
97437	52922	80739	59178	50628	61017	51652	40915	94696	67843
58009	20681	98823	50979	01237	70152	13711	73916	87902	84759
77211	70110	93803	60135	22881	13423	30999	07104	27400	25414
54256	84591	65302	99257	92970	28924	36632	54044	91798	78018
36493	69330	94069	39544	14050	03476	25804	49350	92525	87941
87569	22661	55970	52623	35419	76660	42394	63210	62626	00581
22896	62237	39635	63725	10463	87944	92075	90914	30599	35671
02697	33230	64527	97210	41359	79399	13941	88378	68503	33609
20080	15652	37216	00679	02088	34138	13953	68939	05630	27653
20550	95151	60557	57449	77115	87372	02574	07851	22128	39189
72771	11672	67492	42904	64647	94354	45994	42538	54885	15983
38472	43379	76295	69406	96510	16529	83500	28590	49787	29822
24511	56510	72654	13277	45031	42235	96502	25567	23653	36707
01054	06674	58283	82831	97048	42983	06471	12350	49990	04809
94437	94907	95274	26487	60496	78222	43032	04276	70800	17378

(continued)

Table B.1  
Random Digits (*continued*)

97842	69095	25982	03484	25173	05982	14624	31653	17170	92785
53047	13486	69712	33567	82313	87631	03197	02438	12374	40329
40770	47013	63306	48154	80970	87976	04939	21233	20572	31013
52733	66251	69661	58387	72096	21355	51659	19003	75556	33095
41749	46502	18378	83141	63920	85516	75743	66317	45428	45940
10271	85184	46468	38860	24039	80949	51211	35411	40470	16070
98791	48848	68129	51024	53044	55039	71290	26484	70682	56255
30196	09295	47685	56768	29285	06272	98789	47188	35063	24158
99373	64343	92433	06388	65713	35386	43370	19254	55014	98621
27768	27552	42156	23239	46823	91077	06306	17756	84459	92513
67791	35910	56921	51976	78475	15336	92544	82601	17996	72268
64018	44004	08136	56129	77024	82650	18163	29158	33935	94262
79715	33859	10835	94936	02857	87486	70613	41909	80667	52176
20190	40737	82688	07099	65255	52767	65930	45861	32575	93731
82421	01208	49762	66360	00231	87540	88302	62686	38456	25872

Reprinted from *A Million Random Digits with 100,000 Normal Deviates*, RAND (New York: The Free Press, 1955).  
Copyright © 1955 and 1983 by RAND. Used by permission.

Table B.2  
Control Chart Constants

$m$	$d_2$	$d_3$	$c_4$	$A_2$	$A_3$	$B_3$	$B_4$	$B_5$	$B_6$	$D_1$	$D_2$	$D_3$	$D_4$
2	1.128	0.853	0.7979	1.880	2.659		3.267		2.606		3.686		3.267
3	1.693	0.888	0.8862	1.023	1.954		2.568		2.276		4.358		2.575
4	2.059	0.880	0.9213	0.729	1.628		2.266		2.088		4.698		2.282
5	2.326	0.864	0.9400	0.577	1.427		2.089		1.964		4.918		2.114
6	2.534	0.848	0.9515	0.483	1.287	0.030	1.970	0.029	1.874		5.079		2.004
7	2.704	0.833	0.9594	0.419	1.182	0.118	1.882	0.113	1.806	0.205	5.204	0.076	1.924
8	2.847	0.820	0.9650	0.373	1.099	0.185	1.815	0.179	1.751	0.388	5.307	0.136	1.864
9	2.970	0.808	0.9693	0.337	1.032	0.239	1.761	0.232	1.707	0.547	5.394	0.184	1.816
10	3.078	0.797	0.9727	0.308	0.975	0.284	1.716	0.276	1.669	0.686	5.469	0.223	1.777
11	3.173	0.787	0.9754	0.285	0.927	0.321	1.679	0.313	1.637	0.811	5.535	0.256	1.744
12	3.258	0.778	0.9776	0.266	0.886	0.354	1.646	0.346	1.610	0.923	5.594	0.283	1.717
13	3.336	0.770	0.9794	0.249	0.850	0.382	1.618	0.374	1.585	1.025	5.647	0.307	1.693
14	3.407	0.763	0.9810	0.235	0.817	0.406	1.594	0.399	1.563	1.118	5.696	0.328	1.672
15	3.472	0.756	0.9823	0.223	0.789	0.428	1.572	0.421	1.544	1.203	5.740	0.347	1.653
20	3.735	0.729	0.9869	0.180	0.680	0.510	1.490	0.504	1.470	1.549	5.921	0.415	1.585
25	3.931	0.708	0.9896	0.153	0.606	0.565	1.435	0.559	1.420	1.806	6.056	0.459	1.541

This table was computed using Mathcad.

Table B.3  
Standard Normal Cumulative Probabilities

$$\Phi(z) = \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{t^2}{2}\right) dt$$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2297	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641



Table B.3  
Standard Normal Cumulative Probabilities (*continued*)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9773	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9983	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

This table was generated using MINITAB.

Table B.4  
t Distribution Quantiles

$\nu$	$Q(.9)$	$Q(.95)$	$Q(.975)$	$Q(.99)$	$Q(.995)$	$Q(.999)$	$Q(.9995)$
1	3.078	6.314	12.706	31.821	63.657	318.317	636.607
2	1.886	2.920	4.303	6.965	9.925	22.327	31.598
3	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.849
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
$\infty$	1.282	1.645	1.960	2.326	2.576	3.090	3.291

This table was generated using MINITAB.

Table B.5  
Chi-Square Distribution Quantiles

$\nu$	$Q(.005)$	$Q(.01)$	$Q(.025)$	$Q(.05)$	$Q(.1)$	$Q(.9)$	$Q(.95)$	$Q(.975)$	$Q(.99)$	$Q(.995)$
1	0.000	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.143	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.290	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.653	40.647	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.195	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.994
29	13.121	14.256	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
31	14.458	15.655	17.539	19.281	21.434	41.422	44.985	48.232	52.192	55.003
32	15.134	16.362	18.291	20.072	22.271	42.585	46.194	49.480	53.486	56.328
33	15.815	17.074	19.047	20.867	23.110	43.745	47.400	50.725	54.775	57.648
34	16.501	17.789	19.806	21.664	23.952	44.903	48.602	51.966	56.061	58.964
35	17.192	18.509	20.569	22.465	24.797	46.059	49.802	53.204	57.342	60.275
36	17.887	19.233	21.336	23.269	25.643	47.212	50.998	54.437	58.619	61.581
37	18.586	19.960	22.106	24.075	26.492	48.364	52.192	55.668	59.893	62.885
38	19.289	20.691	22.878	24.884	27.343	49.513	53.384	56.896	61.163	64.183
39	19.996	21.426	23.654	25.695	28.196	50.660	54.572	58.120	62.429	65.477
40	20.707	22.164	24.433	26.509	29.051	51.805	55.759	59.342	63.691	66.767

This table was generated using MINITAB.

For  $\nu > 40$ , the approximation  $Q(p) \approx \nu \left( 1 - \frac{2}{9\nu} + Q_z(p) \sqrt{\frac{2}{9\nu}} \right)^3$  can be used.

Table B.6A  
F Distribution .75 Quantiles

$\nu_2$ (Denominator Degrees of Freedom)	$\nu_1$ (Numerator Degrees of Freedom)																		
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	$\infty$
1	5.83	7.50	8.20	8.58	8.82	8.98	9.10	9.19	9.26	9.32	9.41	9.49	9.58	9.63	9.67	9.71	9.76	9.80	9.85
2	2.57	3.00	3.15	3.23	3.28	3.31	3.34	3.35	3.37	3.38	3.39	3.41	3.43	3.43	3.44	3.45	3.46	3.47	3.48
3	2.02	2.28	2.36	2.39	2.41	2.42	2.43	2.44	2.44	2.44	2.45	2.46	2.46	2.46	2.47	2.47	2.47	2.47	2.47
4	1.81	2.00	2.05	2.06	2.07	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08
5	1.69	1.85	1.88	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.88	1.88	1.88	1.87	1.87	1.87
6	1.62	1.76	1.78	1.79	1.79	1.78	1.78	1.78	1.77	1.77	1.77	1.76	1.76	1.75	1.75	1.75	1.74	1.74	1.74
7	1.57	1.70	1.72	1.72	1.71	1.71	1.70	1.70	1.69	1.69	1.68	1.68	1.67	1.67	1.66	1.66	1.65	1.65	1.65
8	1.54	1.66	1.67	1.66	1.66	1.65	1.64	1.64	1.64	1.63	1.62	1.62	1.61	1.60	1.60	1.59	1.59	1.58	1.58
9	1.51	1.62	1.63	1.63	1.62	1.61	1.60	1.60	1.59	1.59	1.58	1.57	1.56	1.56	1.55	1.54	1.54	1.53	1.53
10	1.49	1.60	1.60	1.59	1.59	1.58	1.57	1.56	1.56	1.55	1.54	1.53	1.52	1.52	1.51	1.51	1.50	1.49	1.48
11	1.47	1.58	1.58	1.57	1.56	1.55	1.54	1.53	1.53	1.52	1.51	1.50	1.49	1.49	1.48	1.47	1.47	1.46	1.45
12	1.46	1.56	1.56	1.55	1.54	1.53	1.52	1.51	1.51	1.50	1.49	1.48	1.47	1.46	1.45	1.45	1.44	1.43	1.42
13	1.45	1.55	1.55	1.53	1.52	1.51	1.50	1.49	1.49	1.48	1.47	1.46	1.45	1.44	1.43	1.42	1.42	1.41	1.40
14	1.44	1.53	1.53	1.52	1.51	1.50	1.49	1.48	1.47	1.46	1.45	1.44	1.43	1.42	1.41	1.41	1.40	1.39	1.38
15	1.43	1.52	1.52	1.51	1.49	1.48	1.47	1.46	1.46	1.45	1.44	1.43	1.41	1.41	1.40	1.39	1.38	1.37	1.36
16	1.42	1.51	1.51	1.50	1.48	1.47	1.46	1.45	1.44	1.44	1.43	1.41	1.40	1.39	1.38	1.37	1.36	1.35	1.34
17	1.42	1.51	1.50	1.49	1.47	1.46	1.45	1.44	1.43	1.43	1.41	1.40	1.39	1.38	1.37	1.36	1.35	1.34	1.33
18	1.41	1.50	1.49	1.48	1.46	1.45	1.44	1.43	1.42	1.42	1.40	1.39	1.38	1.37	1.36	1.35	1.34	1.33	1.32
19	1.41	1.49	1.49	1.47	1.46	1.44	1.43	1.42	1.41	1.41	1.40	1.38	1.37	1.36	1.35	1.34	1.33	1.32	1.30
20	1.40	1.49	1.48	1.47	1.45	1.44	1.43	1.42	1.41	1.40	1.39	1.37	1.36	1.35	1.34	1.33	1.32	1.31	1.29
21	1.40	1.48	1.48	1.46	1.44	1.43	1.42	1.41	1.40	1.39	1.38	1.37	1.35	1.34	1.33	1.32	1.31	1.30	1.28
22	1.40	1.48	1.47	1.45	1.44	1.42	1.41	1.40	1.39	1.39	1.37	1.36	1.34	1.33	1.32	1.31	1.30	1.29	1.28
23	1.39	1.47	1.47	1.45	1.43	1.42	1.41	1.40	1.39	1.38	1.37	1.35	1.34	1.33	1.32	1.31	1.30	1.28	1.27
24	1.39	1.47	1.46	1.44	1.43	1.41	1.40	1.39	1.38	1.38	1.36	1.35	1.33	1.32	1.31	1.30	1.29	1.28	1.26
25	1.39	1.47	1.46	1.44	1.42	1.41	1.40	1.39	1.38	1.37	1.36	1.34	1.33	1.32	1.31	1.29	1.28	1.27	1.25
26	1.38	1.46	1.45	1.44	1.42	1.41	1.39	1.38	1.37	1.37	1.35	1.34	1.32	1.31	1.30	1.29	1.28	1.26	1.25
27	1.38	1.46	1.45	1.43	1.42	1.40	1.39	1.38	1.37	1.36	1.35	1.33	1.32	1.31	1.30	1.28	1.27	1.26	1.24
28	1.38	1.46	1.45	1.43	1.41	1.40	1.39	1.38	1.37	1.36	1.34	1.33	1.31	1.30	1.29	1.28	1.27	1.25	1.24
29	1.38	1.45	1.45	1.43	1.41	1.40	1.38	1.37	1.36	1.35	1.34	1.32	1.31	1.30	1.29	1.27	1.26	1.25	1.23
30	1.38	1.45	1.44	1.42	1.41	1.39	1.38	1.37	1.36	1.35	1.34	1.32	1.30	1.29	1.28	1.27	1.26	1.24	1.23
40	1.36	1.44	1.42	1.40	1.39	1.37	1.36	1.35	1.34	1.33	1.31	1.30	1.28	1.26	1.25	1.24	1.22	1.21	1.19
60	1.35	1.42	1.41	1.38	1.37	1.35	1.33	1.32	1.31	1.30	1.29	1.27	1.25	1.24	1.22	1.21	1.19	1.17	1.15
120	1.34	1.40	1.39	1.37	1.35	1.33	1.31	1.30	1.29	1.28	1.26	1.24	1.22	1.21	1.19	1.18	1.16	1.13	1.10
$\infty$	1.32	1.39	1.37	1.35	1.33	1.31	1.29	1.28	1.27	1.25	1.24	1.22	1.19	1.18	1.16	1.14	1.12	1.08	1.00

This table was generated using MINITAB.

Table B.68  
F Distribution .90 Quantiles

$\nu_2$ (Denominator Degrees of Freedom)	$\nu_1$ (Numerator Degrees of Freedom)																		
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	$\infty$
1	39.86	49.50	53.59	55.84	57.24	58.20	58.90	59.44	59.85	60.20	60.70	61.22	61.74	62.00	62.27	62.53	62.79	63.05	63.33
2	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.39	9.41	9.42	9.44	9.45	9.46	9.47	9.47	9.48	9.49
3	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24	5.23	5.22	5.20	5.18	5.18	5.17	5.16	5.15	5.14	5.13
4	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94	3.92	3.90	3.87	3.84	3.83	3.82	3.80	3.79	3.78	3.76
5	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30	3.27	3.24	3.21	3.19	3.17	3.16	3.14	3.12	3.10
6	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96	2.94	2.90	2.87	2.84	2.82	2.80	2.78	2.76	2.74	2.72
7	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72	2.70	2.67	2.63	2.59	2.58	2.56	2.54	2.51	2.49	2.47
8	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56	2.54	2.50	2.46	2.42	2.40	2.38	2.36	2.34	2.32	2.29
9	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44	2.42	2.38	2.34	2.30	2.28	2.25	2.23	2.21	2.18	2.16
10	3.28	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	2.32	2.28	2.24	2.20	2.18	2.16	2.13	2.11	2.08	2.06
11	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27	2.25	2.21	2.17	2.12	2.10	2.08	2.05	2.03	2.00	1.97
12	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21	2.19	2.15	2.10	2.06	2.04	2.01	1.99	1.96	1.93	1.90
13	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16	2.14	2.10	2.05	2.01	1.98	1.96	1.93	1.90	1.88	1.85
14	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12	2.10	2.05	2.01	1.96	1.94	1.91	1.89	1.86	1.83	1.80
15	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	2.06	2.02	1.97	1.92	1.90	1.87	1.85	1.82	1.79	1.76
16	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06	2.03	1.99	1.94	1.89	1.87	1.84	1.81	1.78	1.75	1.72
17	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03	2.00	1.96	1.91	1.86	1.84	1.81	1.78	1.75	1.72	1.69
18	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00	1.98	1.93	1.89	1.84	1.81	1.78	1.75	1.72	1.69	1.66
19	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98	1.96	1.91	1.86	1.81	1.79	1.76	1.73	1.70	1.67	1.63
20	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	1.94	1.89	1.84	1.79	1.77	1.74	1.71	1.68	1.64	1.61
21	2.96	2.57	2.36	2.23	2.14	2.08	2.02	1.98	1.95	1.92	1.87	1.83	1.78	1.75	1.72	1.69	1.66	1.62	1.59
22	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	1.90	1.86	1.81	1.76	1.73	1.70	1.67	1.64	1.60	1.57
23	2.94	2.55	2.34	2.21	2.11	2.05	1.99	1.95	1.92	1.89	1.84	1.80	1.74	1.72	1.69	1.66	1.62	1.59	1.55
24	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	1.88	1.83	1.78	1.73	1.70	1.67	1.64	1.61	1.57	1.53
25	2.92	2.53	2.32	2.18	2.09	2.02	1.97	1.93	1.89	1.87	1.82	1.77	1.72	1.69	1.66	1.63	1.59	1.56	1.52
26	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	1.86	1.81	1.76	1.71	1.68	1.65	1.61	1.58	1.54	1.50
27	2.90	2.51	2.30	2.17	2.07	2.00	1.95	1.91	1.87	1.85	1.80	1.75	1.70	1.67	1.64	1.60	1.57	1.53	1.49
28	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87	1.84	1.79	1.74	1.69	1.66	1.63	1.59	1.56	1.52	1.48
29	2.89	2.50	2.28	2.15	2.06	1.99	1.93	1.89	1.86	1.83	1.78	1.73	1.68	1.65	1.62	1.58	1.55	1.51	1.47
30	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	1.82	1.77	1.72	1.67	1.64	1.61	1.57	1.54	1.50	1.46
40	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76	1.71	1.66	1.61	1.57	1.54	1.51	1.47	1.42	1.38
60	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71	1.66	1.60	1.54	1.51	1.48	1.44	1.40	1.35	1.29
120	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68	1.65	1.60	1.55	1.48	1.45	1.41	1.37	1.32	1.26	1.19
$\infty$	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63	1.60	1.55	1.49	1.42	1.38	1.34	1.30	1.24	1.17	1.00

This table was generated using MINITAB.

Table B.6C  
F Distribution .95 Quantiles

$\nu_2$ (Denominator Degrees of Freedom)	$\nu_1$ (Numerator Degrees of Freedom)									
	1	2	3	4	5	6	7	8	9	10
1	161.44	199.50	215.69	224.57	230.16	233.98	236.78	238.89	240.55	241.89
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.39	19.40
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91
$\infty$	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83

Table B.6C  
F Distribution of .95 Quantiles (continued)

$\nu_2$ (Denominator Degrees of Freedom)	$\nu_1$ (Numerator Degrees of Freedom)								
	12	15	20	24	30	40	60	120	$\infty$
1	243.91	245.97	248.02	249.04	250.07	251.13	252.18	253.27	254.31
2	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
$\infty$	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

Thus table was generated using MINITAB.

Table B.6D  
F Distribution .99 Quantiles

$\nu_2$ (Denominator Degrees of Freedom)	$\nu_1$ (Numerator Degrees of Freedom)									
	1	2	3	4	5	6	7	8	9	10
1	4052	4999	5403	5625	5764	5859	5929	5981	6023	6055
2	98.51	99.00	99.17	99.25	99.30	99.33	99.35	99.38	99.39	99.40
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51
19	8.19	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47
$\infty$	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32



Table B.6D  
F Distribution of .99 Quantiles (continued)

$\nu_2$ (Denominator Degrees of Freedom)	$\nu_1$ (Numerator Degrees of Freedom)								
	12	15	20	24	30	40	60	120	$\infty$
1	6107	6157	6209	6235	6260	6287	6312	6339	6366
2	99.41	99.43	99.44	99.45	99.47	99.47	99.48	99.49	99.50
3	27.05	26.87	26.69	26.60	26.51	26.41	26.32	26.22	26.13
4	14.37	14.20	14.02	13.93	13.84	13.75	13.65	13.56	13.46
5	9.89	9.72	9.55	9.47	9.38	9.29	9.20	9.11	9.02
6	7.72	7.56	7.40	7.31	7.23	7.14	7.06	6.97	6.88
7	6.47	6.31	6.16	6.07	5.99	5.91	5.82	5.74	5.65
8	5.67	5.52	5.36	5.28	5.20	5.12	5.03	4.95	4.86
9	5.11	4.96	4.81	4.73	4.65	4.57	4.48	4.40	4.31
10	4.71	4.56	4.41	4.33	4.25	4.17	4.08	4.00	3.91
11	4.40	4.25	4.10	4.02	3.94	3.86	3.78	3.69	3.60
12	4.16	4.01	3.86	3.78	3.70	3.62	3.54	3.45	3.36
13	3.96	3.82	3.66	3.59	3.51	3.43	3.34	3.25	3.17
14	3.80	3.66	3.51	3.43	3.35	3.27	3.18	3.09	3.00
15	3.67	3.52	3.37	3.29	3.21	3.13	3.05	2.96	2.87
16	3.55	3.41	3.26	3.18	3.10	3.02	2.93	2.84	2.75
17	3.46	3.31	3.16	3.08	3.00	2.92	2.83	2.75	2.65
18	3.37	3.23	3.08	3.00	2.92	2.84	2.75	2.66	2.57
19	3.30	3.15	3.00	2.92	2.84	2.76	2.67	2.58	2.49
20	3.23	3.09	2.94	2.86	2.78	2.69	2.61	2.52	2.42
21	3.17	3.03	2.88	2.80	2.72	2.64	2.55	2.46	2.36
22	3.12	2.98	2.83	2.75	2.67	2.58	2.50	2.40	2.31
23	3.07	2.93	2.78	2.70	2.62	2.54	2.45	2.35	2.26
24	3.03	2.89	2.74	2.66	2.58	2.49	2.40	2.31	2.21
25	2.99	2.85	2.70	2.62	2.54	2.45	2.36	2.27	2.17
26	2.96	2.81	2.66	2.58	2.50	2.42	2.33	2.23	2.13
27	2.93	2.78	2.63	2.55	2.47	2.38	2.29	2.20	2.10
28	2.90	2.75	2.60	2.52	2.44	2.35	2.26	2.17	2.06
29	2.87	2.73	2.57	2.49	2.41	2.33	2.23	2.14	2.03
30	2.84	2.70	2.55	2.47	2.39	2.30	2.21	2.11	2.01
40	2.66	2.52	2.37	2.29	2.20	2.11	2.02	1.92	1.80
60	2.50	2.35	2.20	2.12	2.03	1.94	1.84	1.73	1.60
120	2.34	2.19	2.03	1.95	1.86	1.76	1.66	1.53	1.38
$\infty$	2.18	2.04	1.88	1.79	1.70	1.59	1.47	1.32	1.00

This table was generated using MINITAB.

Table B.6E  
F Distribution .999 Quantiles

$\nu_2$ (Denominator Degrees of Freedom)	$\nu_1$ (Numerator Degrees of Freedom)									
	1	2	3	4	5	6	7	8	9	10
1	405261	499996	540349	562463	576409	585904	592890	598185	602359	605671
2	998.55	999.01	999.23	999.26	999.29	999.38	999.40	999.35	999.45	999.41
3	167.03	148.50	141.11	137.10	134.58	132.85	131.58	130.62	129.86	129.25
4	74.14	61.25	56.18	53.44	51.71	50.53	49.66	49.00	48.48	48.05
5	47.18	37.12	33.20	31.08	29.75	28.83	28.16	27.65	27.24	26.92
6	35.51	27.00	23.70	21.92	20.80	20.03	19.46	19.03	18.69	18.41
7	29.24	21.69	18.77	17.20	16.21	15.52	15.02	14.63	14.33	14.08
8	25.41	18.49	15.83	14.39	13.48	12.86	12.40	12.05	11.77	11.54
9	22.86	16.39	13.90	12.56	11.71	11.13	10.70	10.37	10.11	9.89
10	21.04	14.91	12.55	11.28	10.48	9.93	9.52	9.20	8.96	8.75
11	19.69	13.81	11.56	10.35	9.58	9.05	8.66	8.35	8.12	7.92
12	18.64	12.97	10.80	9.63	8.89	8.38	8.00	7.71	7.48	7.29
13	17.82	12.31	10.21	9.07	8.35	7.86	7.49	7.21	6.98	6.80
14	17.14	11.78	9.73	8.62	7.92	7.44	7.08	6.80	6.58	6.40
15	16.59	11.34	9.34	8.25	7.57	7.09	6.74	6.47	6.26	6.08
16	16.12	10.97	9.01	7.94	7.27	6.80	6.46	6.19	5.98	5.81
17	15.72	10.66	8.73	7.68	7.02	6.56	6.22	5.96	5.75	5.58
18	15.38	10.39	8.49	7.46	6.81	6.35	6.02	5.76	5.56	5.39
19	15.08	10.16	8.28	7.27	6.62	6.18	5.85	5.59	5.39	5.22
20	14.82	9.95	8.10	7.10	6.46	6.02	5.69	5.44	5.24	5.08
21	14.59	9.77	7.94	6.95	6.32	5.88	5.56	5.31	5.11	4.95
22	14.38	9.61	7.80	6.81	6.19	5.76	5.44	5.19	4.99	4.83
23	14.20	9.47	7.67	6.70	6.08	5.65	5.33	5.09	4.89	4.73
24	14.03	9.34	7.55	6.59	5.98	5.55	5.23	4.99	4.80	4.64
25	13.88	9.22	7.45	6.49	5.89	5.46	5.15	4.91	4.71	4.56
26	13.74	9.12	7.36	6.41	5.80	5.38	5.07	4.83	4.64	4.48
27	13.61	9.02	7.27	6.33	5.73	5.31	5.00	4.76	4.57	4.41
28	13.50	8.93	7.19	6.25	5.66	5.24	4.93	4.69	4.50	4.35
29	13.39	8.85	7.12	6.19	5.59	5.18	4.87	4.64	4.45	4.29
30	13.29	8.77	7.05	6.12	5.53	5.12	4.82	4.58	4.39	4.24
40	12.61	8.25	6.59	5.70	5.13	4.73	4.44	4.21	4.02	3.87
60	11.97	7.77	6.17	5.31	4.76	4.37	4.09	3.86	3.69	3.54
120	11.38	7.32	5.78	4.95	4.42	4.04	3.77	3.55	3.38	3.24
$\infty$	10.83	6.91	5.42	4.62	4.10	3.74	3.47	3.27	3.10	2.96

Table B.6E  
F Distribution .999 Quantiles (continued)

$\nu_2$ (Denominator Degrees of Freedom)	$\nu_1$ (Numerator Degrees of Freedom)								
	12	15	20	24	30	40	60	120	$\infty$
1	610644	615766	620884	623544	626117	628724	631381	634002	636619
2	999.46	999.40	999.44	999.45	999.47	999.49	999.50	999.52	999.50
3	128.32	127.37	126.42	125.94	125.45	124.96	124.47	123.97	123.47
4	47.41	46.76	46.10	45.77	45.43	45.09	44.75	44.40	44.05
5	26.42	25.91	25.40	25.13	24.87	24.60	24.33	24.06	23.79
6	17.99	17.56	17.12	16.90	16.67	16.44	16.21	15.98	15.75
7	13.71	13.32	12.93	12.73	12.53	12.33	12.12	11.91	11.70
8	11.19	10.84	10.48	10.30	10.11	9.92	9.73	9.53	9.33
9	9.57	9.24	8.90	8.72	8.55	8.37	8.19	8.00	7.81
10	8.45	8.13	7.80	7.64	7.47	7.30	7.12	6.94	6.76
11	7.63	7.32	7.01	6.85	6.68	6.52	6.35	6.18	6.00
12	7.00	6.71	6.40	6.25	6.09	5.93	5.76	5.59	5.42
13	6.52	6.23	5.93	5.78	5.63	5.47	5.30	5.14	4.97
14	6.13	5.85	5.56	5.41	5.25	5.10	4.94	4.77	4.60
15	5.81	5.54	5.25	5.10	4.95	4.80	4.64	4.47	4.31
16	5.55	5.27	4.99	4.85	4.70	4.54	4.39	4.23	4.06
17	5.32	5.05	4.78	4.63	4.48	4.33	4.18	4.02	3.85
18	5.13	4.87	4.59	4.45	4.30	4.15	4.00	3.84	3.67
19	4.97	4.70	4.43	4.29	4.14	3.99	3.84	3.68	3.51
20	4.82	4.56	4.29	4.15	4.01	3.86	3.70	3.54	3.38
21	4.70	4.44	4.17	4.03	3.88	3.74	3.58	3.42	3.26
22	4.58	4.33	4.06	3.92	3.78	3.63	3.48	3.32	3.15
23	4.48	4.23	3.96	3.82	3.68	3.53	3.38	3.22	3.05
24	4.39	4.14	3.87	3.74	3.59	3.45	3.29	3.14	2.97
25	4.31	4.06	3.79	3.66	3.52	3.37	3.22	3.06	2.89
26	4.24	3.99	3.72	3.59	3.44	3.30	3.15	2.99	2.82
27	4.17	3.92	3.66	3.52	3.38	3.23	3.08	2.92	2.75
28	4.11	3.86	3.60	3.46	3.32	3.18	3.02	2.86	2.69
29	4.05	3.80	3.54	3.41	3.27	3.12	2.97	2.81	2.64
30	4.00	3.75	3.49	3.36	3.22	3.07	2.92	2.76	2.59
40	3.64	3.40	3.14	3.01	2.87	2.73	2.57	2.41	2.23
60	3.32	3.08	2.83	2.69	2.55	2.41	2.25	2.08	1.89
120	3.02	2.78	2.53	2.40	2.26	2.11	1.95	1.77	1.54
$\infty$	2.74	2.51	2.27	2.13	1.99	1.84	1.66	1.45	1.00

This table was generated using MINITAB.

Table B.7A  
Factors for Two-Sided Tolerance Intervals for Normal Distributions

$n$	95% Confidence			99% Confidence		
	$p = .90$	$p = .95$	$p = .99$	$p = .90$	$p = .95$	$p = .99$
2		36.519	46.944		182.720	234.877
3	8.306	9.789	12.647	18.782	22.131	28.586
4	5.368	6.341	8.221	9.416	11.118	14.405
5	4.291	5.077	6.598	6.655	7.870	10.220
6	3.733	4.422	5.758	5.383	6.373	8.292
7	3.390	4.020	5.241	4.658	5.520	7.191
8	3.156	3.746	4.889	4.189	4.968	6.479
9	2.986	3.546	4.633	3.860	4.581	5.980
10	2.856	3.393	4.437	3.617	4.294	5.610
11	2.754	3.273	4.282	3.429	4.073	5.324
12	2.670	3.175	4.156	3.279	3.896	5.096
13	2.601	3.093	4.051	3.156	3.751	4.909
14	2.542	3.024	3.962	3.054	3.631	4.753
15	2.492	2.965	3.885	2.967	3.529	4.621
16	2.449	2.913	3.819	2.893	3.441	4.507
17	2.410	2.868	3.761	2.828	3.364	4.408
18	2.376	2.828	3.709	2.771	3.297	4.321
19	2.346	2.793	3.663	2.720	3.237	4.244
20	2.319	2.760	3.621	2.675	3.184	4.175
25	2.215	2.638	3.462	2.506	2.984	3.915
30	2.145	2.555	3.355	2.394	2.851	3.742
35	2.094	2.495	3.276	2.314	2.756	3.618
40	2.055	2.448	3.216	2.253	2.684	3.524
50	1.999	2.382	3.129	2.166	2.580	3.390
60	1.960	2.335	3.068	2.106	2.509	3.297
80	1.908	2.274	2.987	2.028	2.416	3.175
100	1.875	2.234	2.936	1.978	2.357	3.098
150	1.826	2.176	2.859	1.906	2.271	2.985
200	1.798	2.143	2.816	1.866	2.223	2.921
500	1.737	2.070	2.721	1.777	2.117	2.783
1000	1.709	2.036	2.676	1.736	2.068	2.718
$\infty$	1.645	1.960	2.576	1.645	1.960	2.576

This table was computed using Mathcad.

Table B.7B  
Factors for One-Sided Tolerance Intervals for Normal Distributions

$n$	95% Confidence			99% Confidence		
	$p = .90$	$p = .95$	$p = .99$	$p = .90$	$p = .95$	$p = .99$
2						
3	6.155	7.656	10.553	14.006	17.372	23.896
4	4.162	5.144	7.042	7.380	9.083	12.388
5	3.407	4.203	5.741	5.362	6.578	8.939
6	3.006	3.708	5.062	4.411	5.406	7.335
7	2.755	3.399	4.642	3.859	4.728	6.412
8	2.582	3.187	4.354	3.497	4.285	5.812
9	2.454	3.031	4.143	3.240	3.972	5.389
10	2.355	2.911	3.981	3.048	3.738	5.074
11	2.275	2.815	3.852	2.898	3.556	4.829
12	2.210	2.736	3.747	2.777	3.410	4.633
13	2.155	2.671	3.659	2.677	3.290	4.472
14	2.109	2.614	3.585	2.593	3.189	4.337
15	2.068	2.566	3.520	2.521	3.102	4.222
16	2.033	2.524	3.464	2.459	3.028	4.123
17	2.002	2.486	3.414	2.405	2.963	4.037
18	1.974	2.453	3.370	2.357	2.905	3.960
19	1.949	2.423	3.331	2.314	2.854	3.892
20	1.926	2.396	3.295	2.276	2.808	3.832
25	1.838	2.292	3.158	2.129	2.633	3.601
30	1.777	2.220	3.064	2.030	2.515	3.447
35	1.732	2.167	2.995	1.957	2.430	3.334
40	1.697	2.125	2.941	1.902	2.364	3.249
50	1.646	2.065	2.862	1.821	2.269	3.125
60	1.609	2.022	2.807	1.764	2.202	3.038
80	1.559	1.964	2.733	1.688	2.114	2.924
100	1.527	1.927	2.684	1.639	2.056	2.850
150	1.478	1.870	2.611	1.566	1.971	2.740
200	1.450	1.837	2.570	1.524	1.923	2.679
500	1.385	1.763	2.475	1.430	1.814	2.540
1000	1.354	1.727	2.430	1.385	1.762	2.475
$\infty$	1.282	1.645	2.326	1.282	1.645	2.326

This table was computed using Mathcad.