UNIVERSITY OF TORONTO
DEPARTMENT OF ELECTRICAL & COMPUTER ENGINEERING

ECE 416F
Communication Systems I
Final Exam

December 13, 2001
Time: 2:00 - 4:30 PM

Examiner: Prof. Elvino S. Sousa

Instructions:

1. Type A exam: No aids allowed except for a non-programmable calculator.
2. Answer all of the 6 questions.
3. All 6 questions are of equal value.
4. Begin the answer to each question on a new page.
5. Answer questions 1,2,3 on one book and questions 4,5,6 on a second book.

1) Consider the following signal

$$x(t) = m_1(t)\cos(\omega_c t + \theta) + m_2(t)\sin(\omega_c t + \theta)$$

where the signals $m_1(t)$ and $m_2(t)$ are two low-pass signals with bandwidth B Hz, and $\omega_c \gg 2\pi B$.

a) If $x(t)$ is input to an envelope detector, what is the output of the envelope detector?

b) If $x(t)$ is input to a coherent demodulator with local reference signal equal to $\cos(\omega_c t + \psi)$ what is the output of the demodulator?

c) If $x(t)$ is input to an ideal FM demodulator with center frequency ω_c, what is the output of the FM demodulator?

d) What is the average power of $x(t)$ if $m_1(t)$ is a 1 KHz sinusoidal signal with amplitude 2, and $m_2(t)$ is a 1 KHz triangular wave signal with zero mean and amplitude equal to 1.

2) An FM modulator has the following characteristics. If the input message signal input is grounded, the modulator output is $A_c\cos(2\pi \times 10^8 t + \theta)$. If the input message signal is equal to -2 V (d.c.) the output of the FM modulator is $A_c\cos(2\pi \times 99980000t + \theta)$.

a) Determine the frequency deviation constant for the FM modulator.

b) Give the expression for the above FM modulator output if the message signal is a 5 KHz sinusoidal signal with amplitude equal to 3 volts and the FM signal has a power equal to 10 watts.

c) Give the approximate value for the bandwidth of the FM signal (i.e. the modulator output) in part b) above.

3) A random process is given as follows:

$X(t) = \sum_{i=1}^{10} X_i \cos(\omega_i t + \theta_i)$, where the X_i's are independent Gaussian random variables with zero mean and variance equal to $\sigma^2 = 4$, and the θ_i's are independent random variables with uniform density in $[-\pi, \pi]$. The ω_i's are distinct frequency constants.

a) Is the random process $X(t)$ stationary?

b) Determine the autocorrelation function for the process.

c) Determine the power spectral density for the process

d) What is the average power of the process?

e) If the process is input into a differentiator ($H(\omega) = j\omega$) find the power spectral density of the output process if $\omega_i = i\omega_0$ and $\omega_0 = 2\pi \times 1000$.

4) A PCM system is used to transmit an audio signal with bandwidth equal to 15 KHz. The system utilizes a 12 bit A/D converter.

a) Determine the resulting bit rate for the digitized signal, assuming the minimum sampling rate.

b) Determine the SNR for the reconstructed signal assuming that a uniform quantizer is used and that the audio signal has the uniform probability density function.

c) If the signal is to be transmitted on a baseband channel, determine the minimum bandwidth required for this channel if we use binary transmission.

d) Determine the minimum bandwidth required if we use 8-ary transmission (i.e. we transmit symbols with 8 possible values).

e) If the signal is transmitted using the 8-ary scheme in d) together with QAM modulation, what is the bandwidth of the channel required?

5) In the generation of Gaussian random processes (in the computer lab) a number of steps where followed. Answer the following questions:

a) In the generation of a uniform random variables we plotted a histogram. If the number of samples is 20,000 and the number of bins is 40, what is the expected number of samples per bin?

b) Two Gaussian random variables where obtained from two independent random variables with uniform probability density function (on [0, 1]) by transformations. If the variance of the Gaussian random variables is σ^2 give the transformation equations to obtain two Gaussian random variables X_1 and X_2 from two uniform random variables U_1 and U_2.

c) Suppose that we have a function that generates a random variable U with uniform probability density function on $[0, a]$. We wish to generate a random variable Y with probability density function $f(y) = \lambda e^{-\lambda y}$, where λ is a positive constant. Give the required transformation.

d) In the computer lab we demonstrated the action of filtering a random process by filtering the process using two different filters (i.e. two examples). Give the power spectral density of the process that we were filtering. Give the impulse response of the two filters that were considered.

e) In the computer lab we used two different techniques to generate a Gaussian band-pass random process. Describe these two techniques. Describe clearly the computation procedure for each technique in high level terms so that a good programmer could implement it completely.

6) A cellular system used for telephony utilizes two blocks of radio spectrum of 5 MHz each (one for transmission from base stations and one for transmission from mobile terminals).

a) Determine the total number of full duplex RF channels that can be created if FM modulation with 25 KHz bandwidth is used in the system.

b) Determine the number of full duplex RF channels that can be created if we assume that the voice signal has a 3.5 KHz bandwidth, we use SSB modulation (lower sideband), and we need a guard band of 0.5 KHz between RF channels channels.

c) In the above case of FM modulation determine the number of channels that we can have per cell if the frequency re-use cell cluster size is equal to 4.

d) A DBS broadcast satellite system transmits signals in the frequency band 12.2 - 12.7 GHz. In the receiver, in order to reduce losses in the cable (from the dish on top of the house to the set-top receiver in the home) these signals are converted to a frequency band 950 - 1450 MHz using a frequency converter that is located in the dish antenna. Determine the local oscilator frequency for this frequency converter. Give the block diagram for the frequency converter. Is it necessary to have a filter in the converter if the cable loss increases greatly for frequencies much higher than 1 GHz? Explain.