UNIVERSITY OF TORONTO
DEPARTMENT OF ELECTRICAL & COMPUTER ENGINEERING

ECE 416F
Communication Systems I
Final Exam

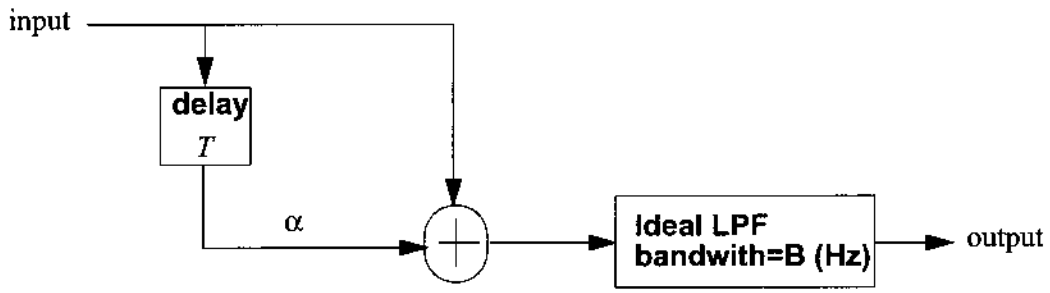
December 8, 1998
Time: 9:30 - 12:00

Examiner: Prof. Elvino S. Sousa

Instructions:

1. **Type A exam: No aids allowed except for a non-programmable calculator.**
2. **Answer all of the 6 questions.**
3. **All 6 questions are of equal value.**
4. **Begin the answer to each question on a new page.**
5. **Answer questions 1,2,3 on one book and questions 4,5,6 on a second book.**

1) A wireless communications channel is modelled according to the following block diagram where the parameter α represents an attenuation for the delay path relative to the line of sight path:



a) Find the impulse response of the channel.

b) Find the transfer function of the channel.

c) If the attenuation parameter has the value $\alpha = 1$ determine the range of values of T (in terms of the bandwith B) such that the channel does not contain a null in the spectum; i.e. $H(\omega) \neq 0$ for all frequencies in the channel band.

2) A band-pass signal has the following form

$$x(t) = m_1(t)\cos(\omega_c t) + m_2(t)\cos(\omega_c t + \theta) + m_3(t)\sin(\omega_c t)$$

where θ is a constant phase angle. The signals $m_i(t)$ $(i = 1, 2, 3)$ are baseband signals with bandwidth $B \ll \frac{\omega_c}{2\pi}$.

a) If $x(t)$ is input to an envelope detector, such as one designed to receive AM signals with carrier ω_c, determine the output of the envelope detector.

b) Give a circuit diagram for such an envelope detector assuming ideal circuit components.

c) Determine the average power of the signal $x(t)$ in terms of the powers of the baseband signals P_i $(i = 1, 2, 3)$.

3) A time division multiplexing system is designed to transmit 20 speech signals using PCM. The bandwidth of each of the speech signals is equal to 10 KHz. A 10 bit A/D converter is used to digitize the speech signals. Also with each encoded sample of the speech signal an extra bit is added for word synchronization.

a) Determine the overall bit rate of the TDM system.

b) If the TDM signal is transmitted using binary transmission determine the minimum bandwidth required to transmit the signal as a baseband signal.

c) If the baseband signal in b) is now transmitted as a band-pass signal using DSB modulation determine the bandwidth of the system.

d) If the baseband signal in b) is now transmitted as a band-pass signal using SSB determine the bandwidth of the system.

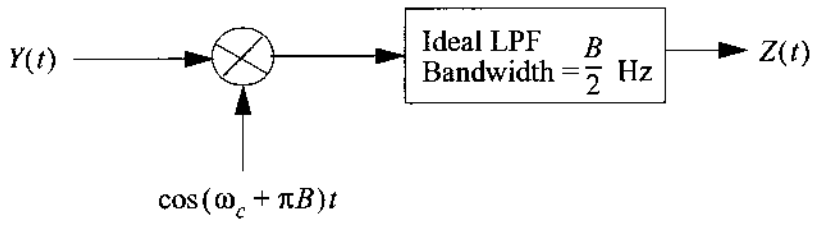
4) A white Gaussian noise process with power spectral density $\frac{N_0}{2}$ is input to a bandpass filter with center frequency ω_c and bandwidth B (Hz), where $\omega_c \gg 2\pi B$. The transfer function of the filter is an even function of ω. For positive frequencies it is specified as follows: $H(\omega) = \sqrt{1 - \frac{|\omega - \omega_c|}{\pi B}}$ for $|\omega - \omega_c| \le \pi B$ and zero elsewhere. The output of the filter is a random process $Y(t)$.

a) Plot the transfer function $H(\omega)$.

b) Give the power spectral density of the process $Y(t)$.

c) Give the autocorrelation function of the process $Y(t)$.

d) The process $Y(t)$ is input to the following demodulator:



Find the power spectral density of the process $Z(t)$.

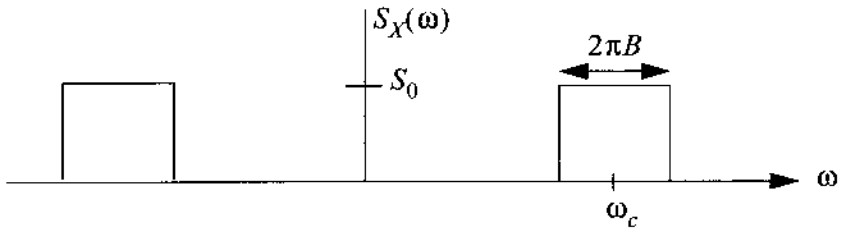
e) Find the average power of the process $Z(t)$.

5)

a) A random variable X has a cumulative distribution function given as follows:

$F_X(x) = 1 - e^{-\lambda x}$ where λ is a parameter. This distribution is known as the exponential distribution. Suppose we have a subroutine in a numerical package that generates random number with uniform distribution in the range [0,1]. Describe how you can generate random variables with an exponential distribution.

b) Describe a procedure to generate a Gaussian random process $X(t)$ with power spectral density function given as follows:



c) We now input the process generated in b) ($X(t)$) into an envelope detector which produces an output $a(t)$. Now, for a fixed t $a(t)$ is a random variable. Give its probability density function. Plot a typical shape for this probability density function.

6)

a) An FM modulator has the following characteristics. When the modulating signal input is zero the output is a sinusoidal signal with frequency equal to 100 MHz. When the modulating signal input is a 2 volt d.c. signal the output is a sinusoidal signal with frequency 100.4 MHz. Now, if the modulating signal is $m(t)$ (volts) give an expression for the output of the modulator.

b) Give the circuit diagram to implement a narrowband FM modulator using only multipliers.

c) In an AM superheterodyne receiver the intermediate frequency is 450 KHz. Suppose we wish to tune to a radio station with carrier frequency equal to 950 KHz. Determine the frequency that we should tune the local oscillator to. Determine the image frequency for this radio station.