University of Toronto

Faculty of Applied Science and Engineering

Department of Electrical and Computer Engineering

FINAL EXAMINATION, APRIL 1998

Fourth Year - Programs 5ce, 5e, 7, 9 [elective]

ECE 417S - DIGITAL COMMUNICATION

Examiner: *S.Pasupathy*

- A single aid sheet (8.5"x11", handwritten) and a non- programmable calculator are the **only aids allowed.**
- Answer **all** six [6] questions.
- The value of each question is indicated beside each question; total marks = 60.
- Start each new question on a new page.
- If you need to make any assumptions, state them clearly.
- Answers should be clear, crisp and brief; answers without logical reasoning steps showing *all* the work will **not** be given credit.
- Lengthy reproductions of text material should be avoided. Credit is for **solving** the problems.

rks 1. Consider a binary symmetric channel(BSC) with two input symbols

$$x_0 = 0 \text{ and } x_1 = 1$$

and two output symbols

$$y_0 = 0 \text{ and } y_1 = 1.$$

The forward transition probability

$$p(y_0/x_0) = \alpha.$$

The probability of transmitting a zero,

$$p(x_0) = p_0.$$

(a) Draw the transition probability diagram of the BSC.

(b) Express the mutual information of the BSC in terms of the given system parameters. Show that the mutual information is maximized when the **input** symbols are equi-probable. What is the capacity of the BSC?

(c) Sketch a plot of the capacity as a function of the system parameter(s) it depends on.

ks 2. A discrete memoryless source emits six symbols A, B, C, D, E, and F with probabilities

$$P_A = 1/2; P_B = P_C = 1/20; P_D = 1/4; P_E = 1/40; P_F = 1/8.$$

(a) Calculate the source entropy.

(b) Find the amount of information contained in the message ADADDA and compare with the expected amount of information in a six symbol message.

(c) Design a Huffman code for the above source and compute its efficiency.

marks 3. Consider a binary PAM transmission $\sum_{k=0}^{\infty} a_k p(t-kT)$ where $a_k = 0$ or A. The RZ pulse $p(t) = u(t) - u(t-(T/2))$ where $u(t)$ is the unit-step function. Instead of a matched filter, the receiver has a first-order low-pass filter with impulse response $h(t) = Ke^{-bt}\, u(t)$ where $K = b/(1-e^{-bT/2})$.

(a) Assume $a_0 = A$. Find and sketch the the waveform at the receiver output, $A\, p(t) * h(t)$. Where will you sample to detect the transmitted symbol a_0 ? [$*$ denotes convolution.]

(b) Obtain the condition on b such that the ISI due to a_0 does not exceed $0.1A$ at any subsequent sampling time.

4. (a) A Hadamard matrix H_M is an M x M matrix ($M = 2^n$, n an integer) of +1's and -1's such that $HH^T = MI$ where I is an identity matrix. Write the matrices H_2 and H_8. What are the eight Simplex codewords derived from H_8.

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(b) Consider the waveform corresponding to the 4-bit codeword , 0 0 1 0 . Plot the periodic and aperiodic correlations of this codeword making the usual assumptions about the waveform associated with the symbols. Is the code word a PN sequence? a Barker code? Justify your answers.

arks 5. A source emits four messages $\{m_i\}_1^4$ with equal probabilities using the signals $\pm s(t)$, $\pm \hat{s}(t)$ where $s(t)$ is a unit-energy signal and $\hat{s}(t)$ denotes the Hilbert transform. The transmission is over a AWGN channel and the probability of error for a binary transmission $\pm s(t)$ over the same channel equals p.

(a) Represent the four signals in a signal space and determine the optimum decision regions.

(b) What is the symbol error probability for the above 4-ary scheme as a function of p?

(c) What is the minimum-energy signal set corresponding to the four signals?

(d) If a ternary scheme using $s(t)$, $\pm(1/2)\hat{s}(t)$ is used, give a signal-space representation and draw the decision regions. What is the minimum-energy signal set for the ternary scheme? Sketch it in signal-space.

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6. The trellis diagram of a rate 1/2, constraint length-3 convolutional code is shown below. The solid line represents a message input of '0' and the broken line an input of '1'.

   (a) If the received sequence 10 01 10 11 01 , use the Viterbi algorithm to compute the decoded sequence. Clearly show the survivors and path metrics at each stage. What is the corresponding input message sequence?

   (b) How many errors were there in the received sequence?

   (c) If 00000 is the transmitted message sequence, show the erroneous path with minimum distance. What is the minimum free distance, d_f? According to d_f, how many errors can the code correct? Based on your answer, do you think that the decoded sequence in a) is the actual transmitted sequence?