UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING
FINAL EXAMINATION - APRIL 1999
ECE356S - SYSTEM AND SIGNAL ANALYSIS II
Third Year - Programs 5bm(e), 5ce, 5e
Examiner - R.H. Kwong

tudents may use one 8.5"×11" aid sheet in preparing their answers. Questions re not of equal weight.

10 points)

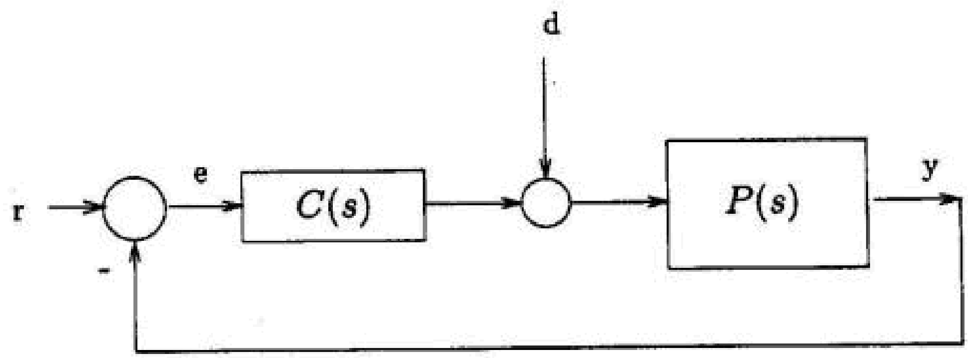
(a) It is known that the matrix

$$A = \begin{bmatrix} 1 & 0 & -42 \\ 0 & 1 & 12 \\ 0 & 0 & -5 \end{bmatrix}$$

has eigenvectors $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 7 \\ -2 \\ 1 \end{bmatrix}$. Determine the matrix exponential e^{At} using the diagonalization method.

10 points)

(b) Consider the feedback system



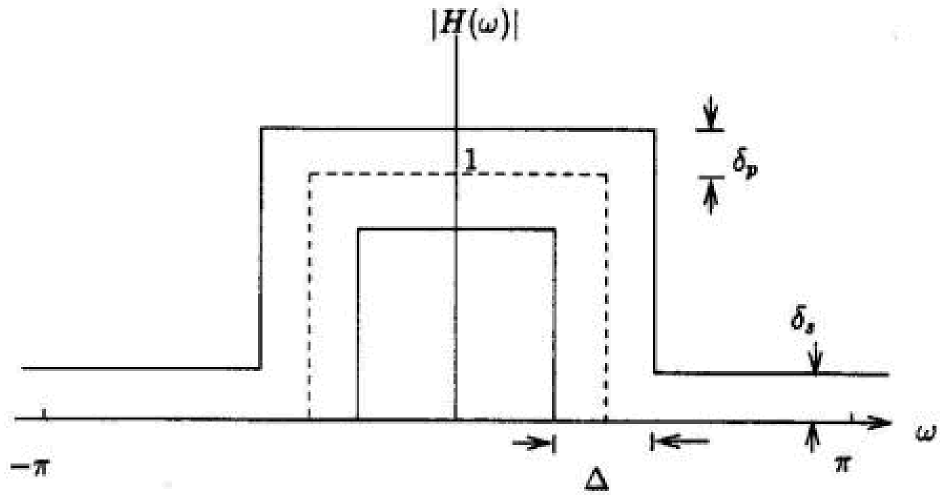
with $r(s) = \frac{1}{s}$, $d(s) = \frac{1}{s^2+1}$, and $P(s) = \frac{1}{s-1}$. The objective is to design a controller $C(s)$ so that the output y tracks the unit step reference input r and that the effects of the disturbance d asymptotically do not appear in the output y. The following controller is proposed:

$$C(s) = \frac{2(s+1)^2(s+2)}{s(s^2+1)}$$

Will this controller achieve the design specifications? Explain your answer.

10 points)

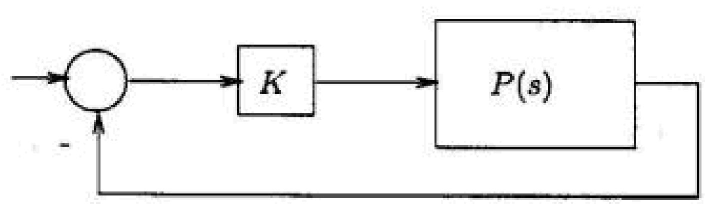
1.(c) An FIR low pass digital filter is to be designed using the Kaiser window to satisfy the specifications given in the following figure.



with $\delta_p = 0.00115$, $\delta_s = 0.01$, and $\Delta = 0.1\pi$. Note that the passband ripple specification is different from the stopband ripple specification. Determine the parameters N and α for the Kaiser window needed to satisfy the given specifications.

(25 points)

2. In the following feedback system



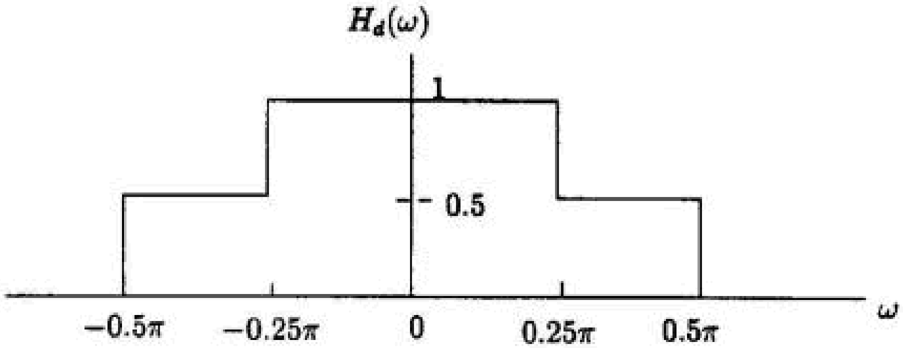
$P(s)$ is given by

$$P(s) = \frac{1 + (\frac{s}{2})}{s^2[1 + (\frac{s}{4})][1 + (\frac{s}{6})]}$$

and K is a positive constant gain. Sketch the Nyquist plot of P. Based on the Nyquist plot, determine the range of values of the gain K for which the closed loop system is stable.

(20 points)

3. Suppose we would like to design a 7th order FIR filter ($N = 3$) based on Hamming windows to approximate the following ideal multiband filter



i.e.,

$$\begin{aligned} H_d(\omega) &= 1, && \text{for} \quad |\omega| \le 0.25\pi, \\ &= 0.5, && \text{for} \quad 0.25\pi < |\omega| \le 0.5\pi, \\ &= 0 && \text{otherwise} \end{aligned}$$

Determine the weights h_k of the Hamming FIR filter

$$y_k = \sum_{j=-3}^{3} h_j u_{k-j}$$

(Recall the Hamming window weights are given by $w_k = 0.54 + 0.46\cos\dfrac{\pi k}{N}$, $|k| \le N$.)

(25 points)

4. Design a low-pass Butterworth digital filter based on the bilinear transformation $s = \dfrac{1-z^{-1}}{1+z^{-1}}$ to satisfy the following specifications:

   (i) The cutoff (3-db) frequency ω_c for the digital filter is 0.4688π.

   (ii) For $\omega \ge 0.664\pi$, the attenuation for $|H(e^{i\omega})|$ is at least 15 db.

   If there are several factors, you do not need to multiply them out. You are to follow these steps.

   (a) Determine the parameters N and ω_{ac} for the analog Butterworth filter

$$|H_a(\omega)|^2 = \frac{1}{1 + \left(\dfrac{\omega}{\omega_{ac}}\right)^{2N}}$$

(b) Determine $H_a(s)$ using the results of (a). You may also, if you wish, use the following table to determine the left half plane roots of $1+\left(\frac{s}{i}\right)^{2N}$.

N	Roots
2	$(-0.7071 \pm i7071)$
3	$-1, -0.5 \pm i0.866$
4	$-0.3827 \pm i0.9239; -0.9239 \pm i0.3827$

Be careful, however, on how to use the table in your calculations.

(c) Determine $H(z)$.

(Hint: For $s = \frac{1-z^{-1}}{1+z^{-1}}$, $\frac{1}{s^2+as+b} = \frac{(1+z^{-1})^2}{1+a+b+(2b-2)z^{-1}+(1+b-a)z^{-2}}$.)