University of Toronto

FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION, DECEMBER 1996

Third Year – Program III-5(a), 5bme, 5env, 5p

APM 384 F - Partial Differential Equations

Exam Type: C

Examiner: R. Ross

Instructions: *All questions have equal value*

1. Solve for $u(r,\theta)$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)+\frac{\partial^2 u}{\partial \theta^2} = 0, \quad 0<\theta<\frac{\pi}{2}, \quad 0<r<a$$

where

$$\frac{\partial u}{\partial \theta}(r,0) = 0, \quad u(r,\frac{\pi}{2}) = 0, \quad 0<r<a$$

and $u(a,\theta)=f(\theta)$.

2. Find the steady-state temperature $u(r,z)$ in the cylinder bounded by

$$z=0,\ z=\ell, \quad 0<r<a$$

and $r=a, \quad 0<z<\ell$ where

$$u(r,0) = 0, \quad u_z(r,\ell)=f(r), \quad 0\le r<a \quad \text{and}$$
$$u_r(a,z) = 0, \quad 0<z<\ell.$$

3. Solve for $u(x,t)$

$$\frac{\partial u}{\partial t}-k\frac{\partial^2 u}{\partial x^2} = e^{-\alpha t}, \quad 0<x<\ell, \quad t>0$$

if $u(0,t) = T_1, \quad u(\ell,t) = T_2, \quad u(x,0) = a.$

4. Solve $u_{xx}+u_{yy}-c^2u = 0$, for $u(x,y)$, where $0<x<\infty, \quad 0<y<\ell$, and $u_x(0,y)=0, \quad u(x,0) = 0, \quad u_y(x,\ell) = g(x).$

5. Solve

$$\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0, \quad 0 < x < \infty, \quad t > 0,$$

for $u(x,t)$ where

$$\begin{aligned} u(0,t) &= 0, \quad t > 0 \quad \text{and} \\ u(x,0) &= 0, \quad 0 < x < \infty \quad \text{and} \\ u_t(x,0) &= v\delta(x-a), \quad a > 0, \quad 0 < x < \infty \end{aligned}$$