

UNIVERSITY OF TORONTO  
Department of Civil Engineering

FINAL EXAMINATION, DECEMBER 2001

**CIV 529S - ROCK ENGINEERING**

Examiner: J.H. Curran

Duration of examination: 2.5 hours

There are five questions of equal weight

No aids are permitted other than a calculator

### Closed Form Solution - Cylindrical Hole in an Infinite Elastic Medium

The stresses  $\sigma_r$ ,  $\sigma_\theta$  and  $\tau_{r\theta}$  for a point at polar coordinate  $(r, \theta)$  near the cylindrical opening of radius 'a' (Figure 1.1) are given by:

$$\sigma_r = \frac{p_1 + p_2}{2} \left(1 - \frac{a^2}{r^2}\right) + \frac{p_1 - p_2}{2} \left(1 - \frac{4a^2}{r^2} + \frac{3a^4}{r^4}\right) \cos 2\theta$$

$$\sigma_\theta = \frac{p_1 + p_2}{2} \left(1 + \frac{a^2}{r^2}\right) - \frac{p_1 - p_2}{2} \left(1 + \frac{3a^4}{r^4}\right) \cos 2\theta$$

$$\tau_{r\theta} = -\frac{p_1 - p_2}{2} \left(1 + \frac{2a^2}{r^2} - \frac{3a^4}{r^4}\right) \sin 2\theta$$

The radial (outward) and tangential displacements (see Figure 1.1), assuming conditions of plane strain, are given by:

$$u_r = \frac{p_1 + p_2}{4G} \frac{a^2}{r} + \frac{p_1 - p_2}{4G} \frac{a^2}{r} \left[4(1 - \nu) - \frac{a^2}{r^2}\right] \cos 2\theta$$

$$u_\theta = -\frac{p_1 - p_2}{4G} \frac{a^2}{r} \left[2(1 - 2\nu) + \frac{a^2}{r^2}\right] \sin 2\theta$$

where G is the shear modulus and  $\nu$  is the Poisson ratio.

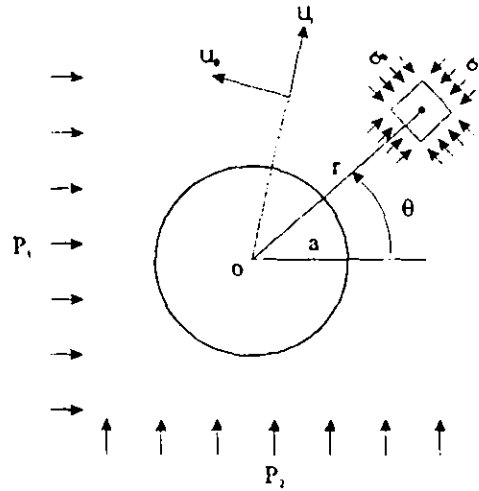


Fig 1.1 Cylindrical hole in an infinite elastic medium

### Closed Form Solution – Cylindrical Hole in an Infinite Mohr-Coulomb Medium

The yield zone radius,  $R_0$ , is given analytically by a theoretical model based on the solution of Salencon (1969):

$$R_0 = a \left( \frac{2}{K_p + 1} \frac{P_0 + \frac{q}{K_p - 1}}{P_i + \frac{q}{K_p - 1}} \right)^{1/(K_p - 1)}$$

where  $a$  = radius of hole,  $c$  = cohesion,  $\phi$  = friction angle,  $q = 2c \tan(45 + \phi/2)$ ,  $P_0$  = initial in situ stress,  $P_i$  = internal pressure and

$$K_p = \frac{1 + \sin \phi}{1 - \sin \phi}$$

The radial stress at the elastic-plastic interface is

$$\sigma_{re} = \frac{1}{K_p + 1} (2P_0 - q)$$

The stresses and radial displacement in the elastic zone are

$$\sigma_r = P_0 - (P_0 - \sigma_{re}) \left( \frac{R_0}{r} \right)^2$$

$$\sigma_\theta = P_0 + (P_0 - \sigma_{re}) \left( \frac{R_0}{r} \right)^2$$

where  $r$  is the distance from the field point  $(x, y)$  to the center of the hole. The stresses in the plastic zone are

$$\sigma_r = -\frac{q}{K_p - 1} + \left( P_i + \frac{q}{K_p - 1} \right) \left( \frac{r}{a} \right)^{(K_p - 1)}$$

$$\sigma_\theta = -\frac{q}{K_p - 1} + K_p \left( P_i + \frac{q}{K_p - 1} \right) \left( \frac{r}{a} \right)^{(K_p - 1)}$$

### Mohr-Coulomb Strength Criterion

$$\sigma_1 = \frac{2c \cos \phi}{1 - \sin \phi} + \frac{1 + \sin \phi}{1 - \sin \phi} \sigma_3$$

### Hoek-Brown Criterion

$$\sigma_1 = \sigma_3 + \sigma_c \left( m_b \frac{\sigma_3}{\sigma_c} + s \right)^a$$

1. Based on the back analysis of pillar failures in underground coal mines in South Africa, Salamon and Munroe determined that the factor of safety ( $FS$ ) of a coal pillar could be expressed as:

$$FS = \frac{Kh^\alpha w^\beta (1-e)}{\gamma z}$$

where  $K$  is the strength of a unit cube of coal,  $h$  is the pillar height,  $w$  is the pillar width,  $\alpha$  and  $\beta$  are constants,  $e$  is the extraction ratio,  $\gamma$  is the unit weight of rock and  $z$  is the depth below surface.

- a) If the parameters  $K$ ,  $w$ ,  $e$ ,  $\alpha$ ,  $\beta$ ,  $\gamma$  and  $z$  have the following values:

$K=93.0$ ;  $w=8.0\text{m}$ ;  $e=0.6$ ,  $\alpha=0.46$ ,  $\beta=0.46$ ,  $\gamma=2.7$  and  $z=100$

and  $h$  is given in the form of the histogram shown in Fig. 1 complete the table below for  $h$  and  $FS$ .

Random Number	$h$	$FS$
0.05		
0.20		
0.40		
0.70		
0.90		

- b) What is the probability of pillar failure from part a. Do you think your answer would change if you carried the analysis out for a 100 samples?
- c) Discuss why this type of risk assessment is better suited to the analysis of pillar failure and rock slope stability than it is to the stability of underground excavations

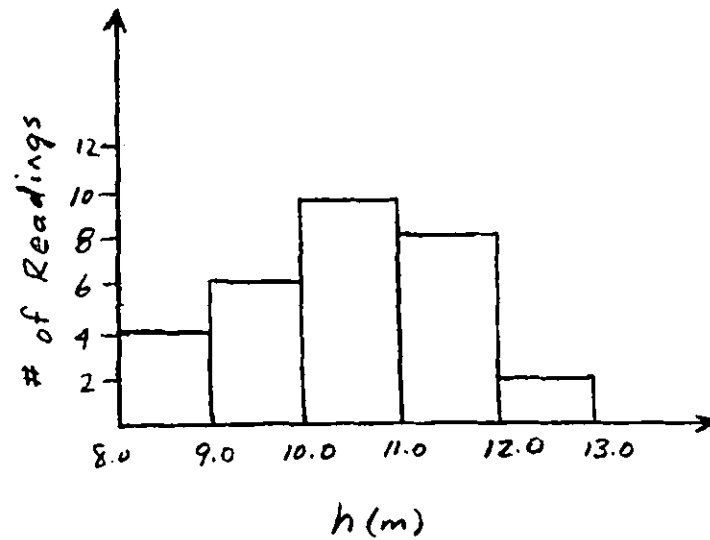


Figure 1

2. As a member of a geotechnical design team you are requested to carry out a preliminary design for an underground powerhouse in weak rock located near the toe of a steep slope. Briefly discuss the influence of each of the following factors:
  - a) cavern location with respect to the toe of a slope
  - b) cavern orientation
  - c) pillar size between the main cavern (powerhouse) and the transformer gallery
  - d) cavern shape
  - e) excavation sequence
  - f) type of support.
  
3. A long opening of circular cross section is located 1000 m below ground surface. In the plane perpendicular to the tunnel axis, the field principal stresses are vertical and horizontal. The vertical stress  $p$  is equal to the depth stress, and the horizontal stress is defined by  $0.28p$ . The unit weight of the rock mass is  $0.027 \text{ kN/m}^3$ , the compressive strength is defined by a Mohr-Coulomb criterion with  $c = 20 \text{ MPa}$ ,  $\phi = 25^\circ$ , and the tensile strength by  $T_0 = 0$ .
  - a) Predict the response of the excavation peripheral rock to the given conditions.
  - b) Propose an alternative design for the excavation.
  
4. A vertical hydraulic fracture was initiated in a packed off section of a vertical borehole at a depth of 800 m. The water pressure was first raised to 15.5 MPa and then it was not possible to raise it further. When pumping stopped, the water pressure fell to a value 10 MPa (the "shut-in" pressure). After a day, the pressure was raised again, but it could not be pumped to a value higher than 12 MPa. Estimate the horizontal stresses

at the site of measurement, the tensile strength of the rock, and the vertical pressure at the site. The unit weight of the rock mass is  $0.027 \text{ MN/m}^3$ .

5. The ground reaction curve for a 12 m diameter tunnel is shown in Fig. 5.1. The tunnel is to be constructed at a depth of 60 meters in a rock mass whose strength is defined by the Mohr-Coulomb criterion with  $c = 0.1 \text{ MPa}$  and  $\phi = 22^\circ$ .
  - a) What would the shape of the ground reaction curve be if the rock mass behaves elastically?
  - b) How is the ground reaction curve (required support line) calculated?
  - c) Is the excavation stable without the addition of support? Why?
  - d) What will be the maximum deformation (mm) of the tunnel be if a support system consisting 34 mm rock bolts on a 1 m x 1 m pattern is applied 3 m from the tunnel face (see Fig. 5.2). Note: 34 mm rock bolts on a 1 m x 1 m pattern can provide a maximum support pressure of 0.36 MPa at a strain of 0.2%. What is the factor of safety for the rockbolts?
  - e) What is the maximum deformation (mm) of the tunnel if the support system in part d) is applied 1 m from the tunnel face?

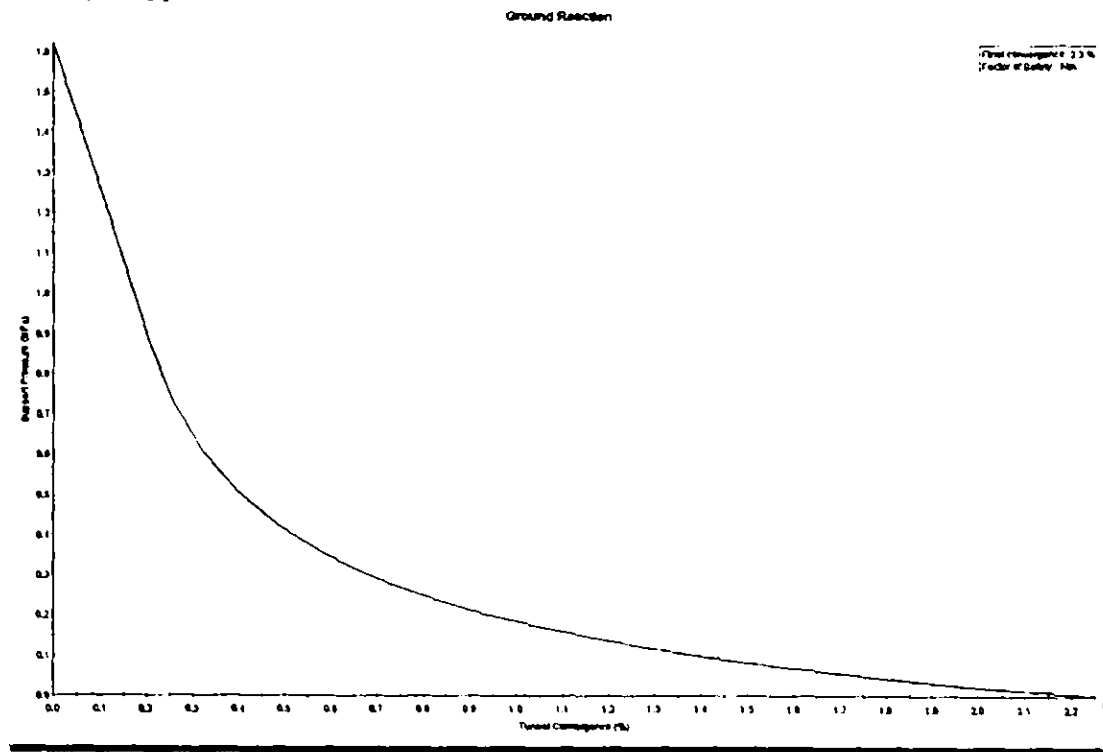


Figure 5.1 Ground Reaction Curve - Support Pressure (MPa) vs Tunnel Convergence (%)

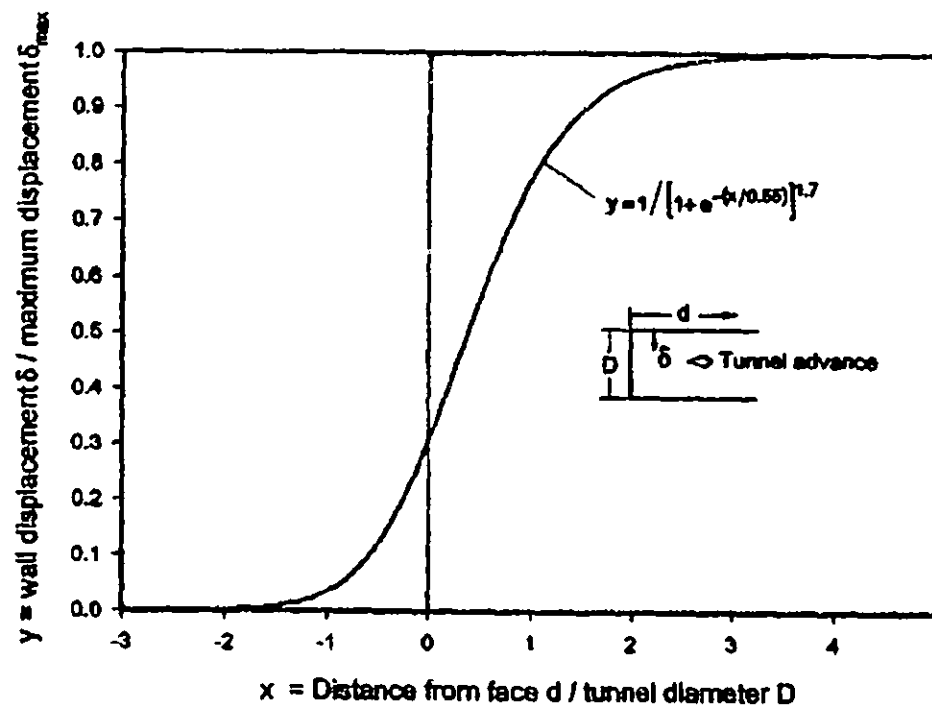


Figure 5.2 Tunnel wall displacement as a function of distance form face