UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING
FINAL EXAMINATION, DECEMBER 18, 2001
MIE 301F - KINEMATICS AND DYNAMICS OF MACHINES
Examiner - Professor W.L. Cleghorn

Notes:

- only written/printed aid:
  Course Book entitled *"Mechanics of Machines"* by W.L. Cleghorn, 2001
- answer all questions

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1. (a) A straight spur gear A has 24 teeth. It meshes with an internal gear B. Both gears have 25° pressure angle, 12 inch^{-1} diametral pitch, full depth teeth. The centre-to-centre distance between the gears is 1.50 inches. Determine the:

(14)
   - (i) addendum circle diameter of gear A
   - (ii) number of teeth on internal gear B
   - (iii) base circle radius of internal gear B
   - (iv) thickness of a gear tooth on internal gear B, measured along its pitch circle

   (b) A pinion, has 32 teeth, and has been manufactured using a hob having 20° pressure angle, 8 inch^{-1} diametral pitch, stub teeth. It meshes with a rack. Determine the:

(6)
   - (i) length of action
   - (ii) contact ratio

2. **Figure P.2** shows an inverted slider crank mechanism.

(4) (a) Specify the maximum magnitude of rotational speed of link 3 in the clockwise direction during a complete rotation of link 2, and the corresponding value(s) of θ_2 when this takes place.

(12) (b) Using complex number methods, when

$$\theta_2 = 150°$$

determine rotational velocity of link 3

(4) (c) Using the results of part (b), determine the velocity of point C when

$$\theta_2 = 150°$$

$$\dot{\theta}_2 = 30 \text{ rad/sec CW} \quad \text{(constant)}$$

$$r_{O_2B_2} = 3.0 \text{ cm} \quad ; \quad r_{O_2O_3} = 8.0 \text{ cm} \quad ; \quad r_{O_3C} = 13.0 \text{ cm}$$

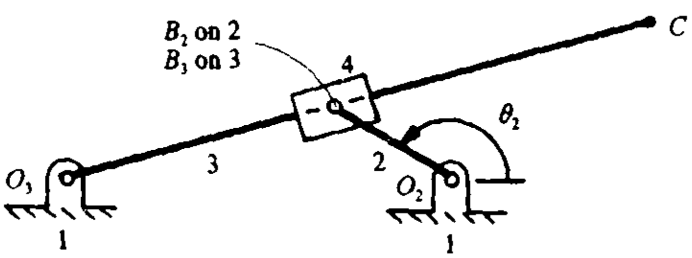

Figure P.2 (not to scale)

$$\frac{d}{dx}\sin^{-1}u = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}\left(-\frac{\pi}{2} \le \sin^{-1}u \le \frac{\pi}{2}\right)$$

$$\frac{d}{dx}\cos^{-1}u = -\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}\left(0 \le \cos^{-1}u \le \pi\right)$$

$$\frac{d}{dx}\tan^{-1}u = \frac{1}{1+u^2}\frac{du}{dx}$$

$$\frac{d}{dx}\cot^{-1}u = -\frac{1}{1+u^2}\frac{du}{dx}$$

3. For the planetary gear train shown in **Figure P.3**:

$$N_1 = 100 \quad ; \quad N_3 = 32 \quad ; \quad N_4 = 45$$

$$N_5 = 150 \quad ; \quad N_6 = 42 \quad ; \quad N_7 = 22$$

$$\omega_7 = 150 \text{ rpm CW} \quad ; \quad \text{mechanical efficiency, } \eta = 0.95$$

$$\text{input torque, } T_i = 100 \text{ N-m}$$

Determine:

(8) (a) speed and direction of rotation of output shaft (gear 7)

(7) (b) speed and direction of rotation of planet carrier (link2)

(5) (c) direction and magnitude of torque required to be applied to the frame (gear 5) in order to keep it stationary

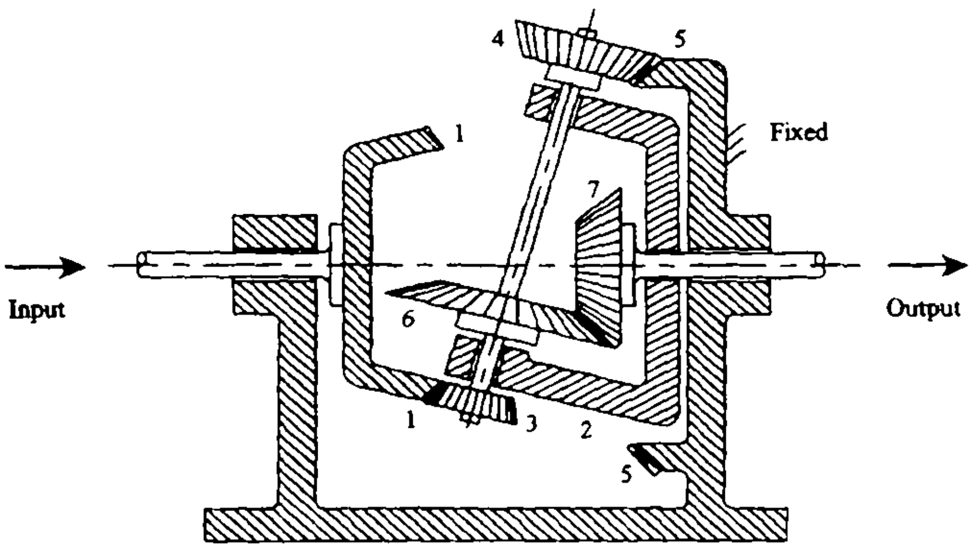

Figure P.3 (not to scale)

4. For the mechanism shown in **Figure P.4**:

(14) (a) Draw the velocity polygon employing pole point O_V and scale 1 cm = 10 cm/sec, if

$$\dot{\theta}_2 = 150 \text{ rpm CCW (constant)}$$

and specify:

(i) velocity of point C

(ii) velocity of point F

(6) (b) Through scaling results that can be obtained from the velocity polygon of part (a), determine the relative Coriolis acceleration of D_3 with respect to D_4, if

$$\dot{\theta}_2 = 450 \text{ rpm CCW (constant)}$$

$$r_{O_2B} = 5.0 \text{ cm} \;\; ; \;\; r_{BE} = 2.0 \text{ cm} \;\; ; \;\; r_{BD_3} = 8.0 \text{ cm}$$

$$r_{BF} = 12.0 \text{ cm} \;\; ; \;\; r_{EC} = 4.0 \text{ cm}$$

Detach and include this sheet as part of your answer

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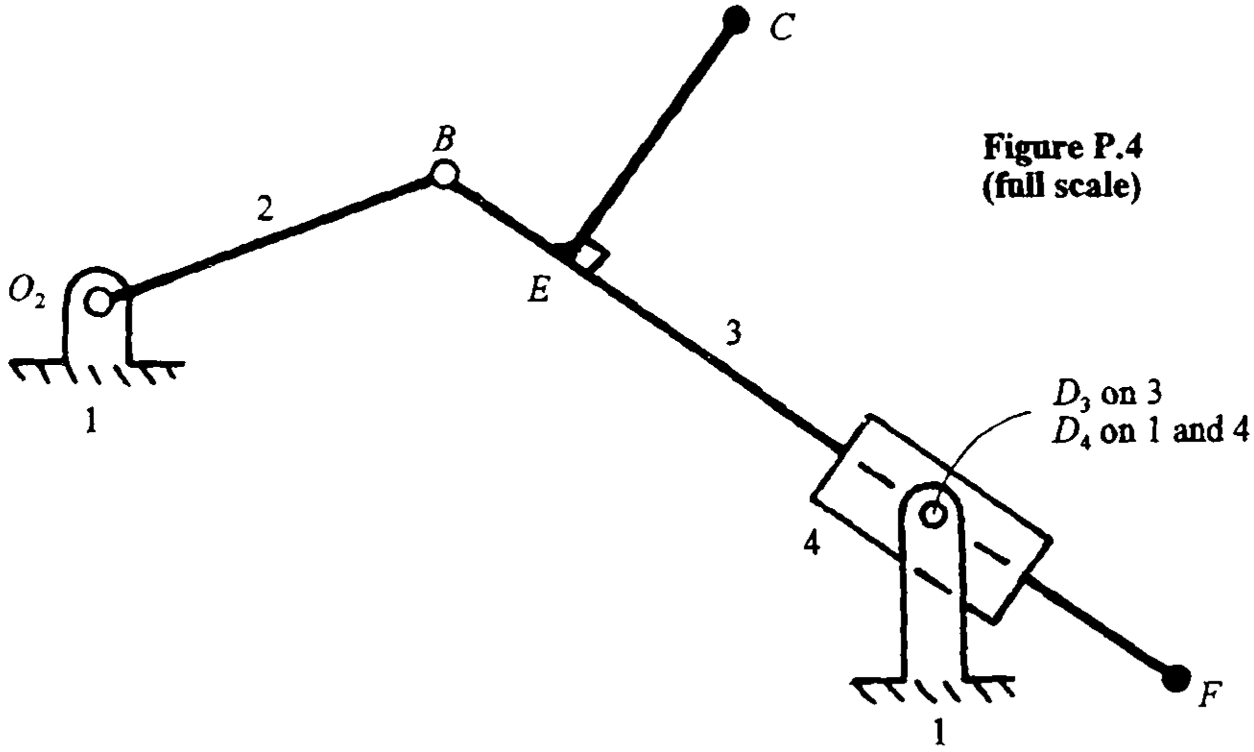

Figure P.4
(full scale)

• O_V

(20) 5. For the given position of the mechanism shown in **Figure P.5**, determine the magnitude and sense of the required torque to be applied to crank O_2B by the base link, to overcome the inertia of link 6.

$$r_{O_2B} = 5.0 \text{ cm} \quad ; \quad m_6 = 50 \text{ grams} \quad ; \quad (I_G)_6 = 4.0 \times 10^{-5} \text{ kg-m}^2$$

$$\bar{a}_{G_6} = 380 \text{ cm/sec}^2 \quad 73^\circ \quad ; \quad \ddot{\theta}_6 = 90 \text{ rad/sec}^2 \text{ CCW}$$

$\overline{(100)}$

Detach and include this sheet as part of your answer

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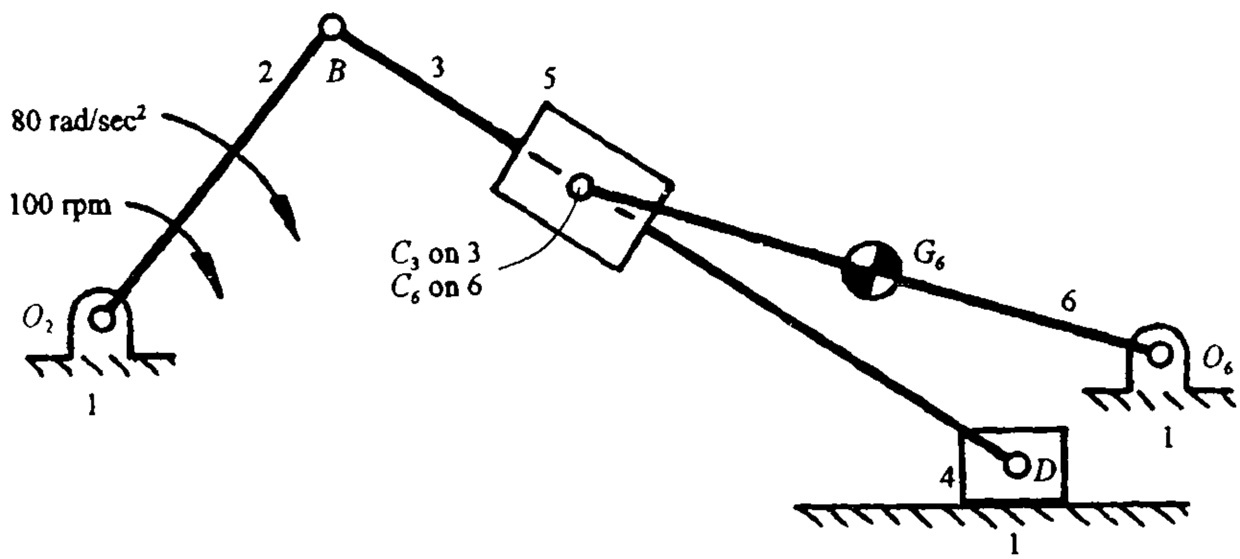

Figure P.5