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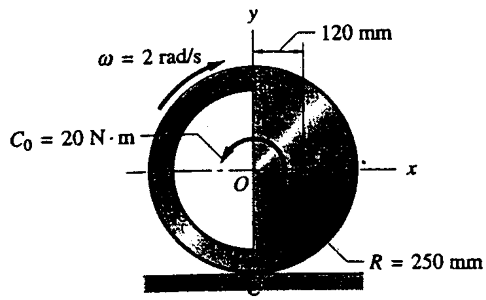
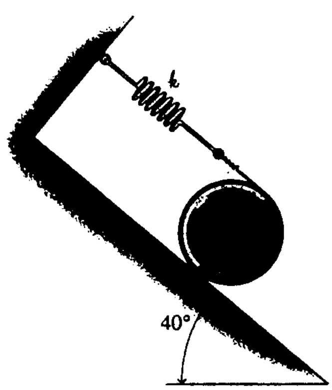
Question 1 [20%]
The 40-kg unbalanced wheel in the figure is rolling without slipping under the action of a counterclockwise moment $C_0 = 20$ Nm. When the wheel is in the position shown, its angular velocity is $\omega = 2$ rad/s, clockwise. The radius of gyration of the wheel about its mass center G is k = 200mm. For the position shown :

1. Calculate the angular acceleration α.
2. Calculate the forces exerted on the wheel at C by the rough horizontal plane.



Question 2 [20%]
A 15-kg uniform cylinder 800mm in diameter rolls without slipping on an inclined surface as shown in the figure. A light cord wrapped around the cylinder is attached to a spring having k = 150 N/m. If the cylinder is released from rest when the spring is stretched 1m, determine :

1. The speed v and the angular velocity ω of the cylinder when the stretch in the spring is 0.5m.
2. The stretch in the spring when the cylinder is again at rest.



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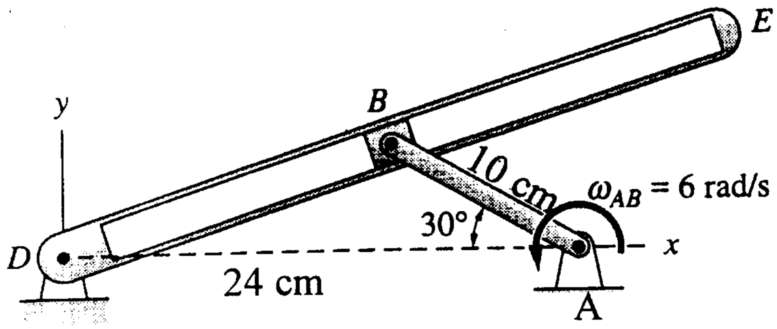
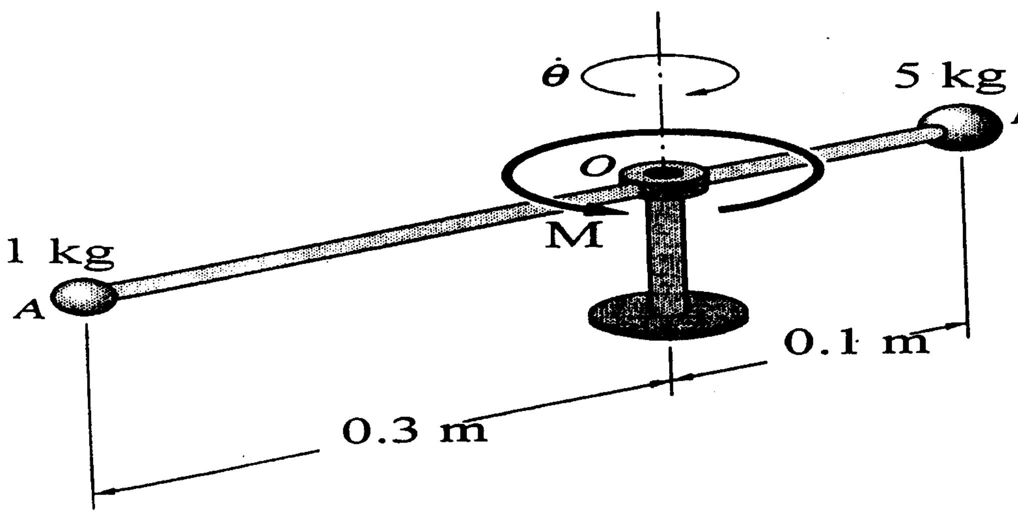
Question 3 [20%]
Crank AB of the quick return mechanism shown in the figure rotates clockwise with a constant angular velocity ω_{AB} = 6 rad/s. When the mechanism is in the position shown calculate

1. the velocity of the slider B relative to arm DE,
2. the acceleration of the of the slider B relative to arm DE,
3. the angular velocity of arm DE,
4. and angular acceleration of arm DE



Question 4 [20%]
The rigid assembly, consisting of two masses attached to a massless rod rotates about the vertical axis at 0. The assembly is initially rotating freely at the angular velocity $\dot{\theta}_o$ = 120 rad/s. when a moment of M = 2 Nm acting against the motion is applied. Find :

1. The time required to stop the assembly
2. The number of revolutions made by the assembly before coming to rest.



Question 5 [20%]
For a spring-mass-damper system with m = 50 kg and k = 5000 N/m.

1. Find the critical damping constant c_c.
2. Find the damped natural frequency when $c = c_c/2$.
3. Find the logarithmic decrement.

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Question 2

When the mechanism is in the position shown, the angular velocity of the gear is $\omega = 2$ rad/sec clockwise and its angular acceleration is $\alpha = 4$ rad/sec^2 counterclockwise.

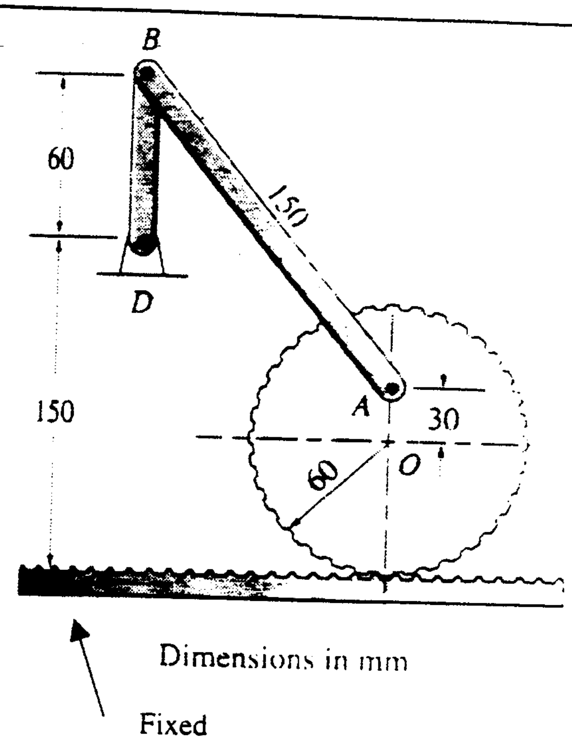
Find:

1. Angular velocities of links AB and BD
2. Angular accelerations of links AB and BD

in this position.

Hints:

A is not at the centre of the disk.

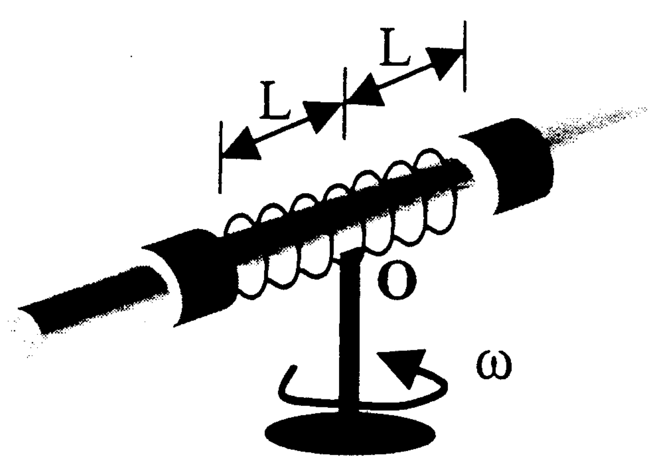


Question 1

Two collars, each of mass m = 2 kg, are joined by a spring having a relaxed length of 3m. The collars slide on a horizontal, massless rod rotating about O. Initially the collars are each held at L=1m by pins, while the rod rotates at $\omega_i = 18$ rad/s. The pins are then removed, and the collars slide to a final equilibrium position at L=3m, i.e. they end up being 6 m apart.

Assuming that no friction acts, find:

(a) The final angular velocity of the rod, ω_f
(b) The spring constant, k.



Note the spring joining the two collars.

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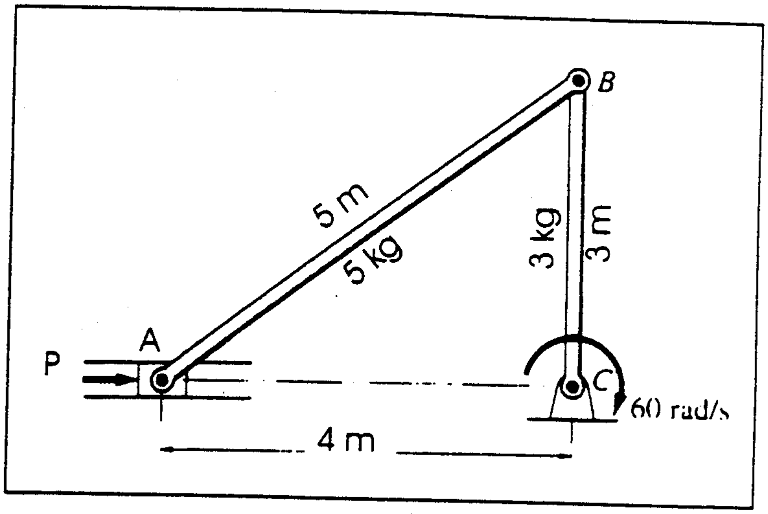
Please write answers in spaces below

Question 3



The mechanism consists of two homogeneous bars of the masses shown and the piston A of negligible weight. A varying horizontal force P acting on the piston maintains a constant angular velocity $\omega_{BC} = 60$ rads/sec.

Radii of gyrations are:

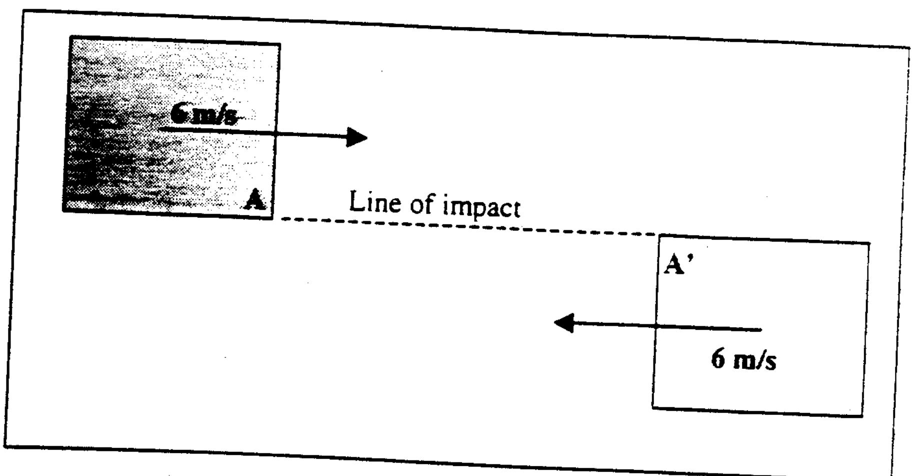
k_{AB}=1.44 m about centre of mass of AB
k_{BC}=0.87 m about centre of mass of BC

Neglect friction.

Find:

1. Angular acceleration α_{AB}
2. Acceleration of point A, a_A
3. Acceleration of mass center of AB, a_G
4. Magnitude and direction of force P (give as magnitude of force and direction (left or right))

Question 4



Two identical square plates (each of mass 10 kg, side length 2 m) slide towards each other on a frictionless tabletop. Each plate has velocity 6 m/s. Corners A and A' just collide with one another, and the plates stick together at these corners.

(a) Show that the corners A and A' have zero velocity after the collision.
(b) Compute the angular velocity of the plates immediately after the collision.

For a square plate of side L and mass M, the mass moment of inertia about the plate centroid is

$$\bar{I} = \frac{1}{6}ML^2.$$

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Question 5

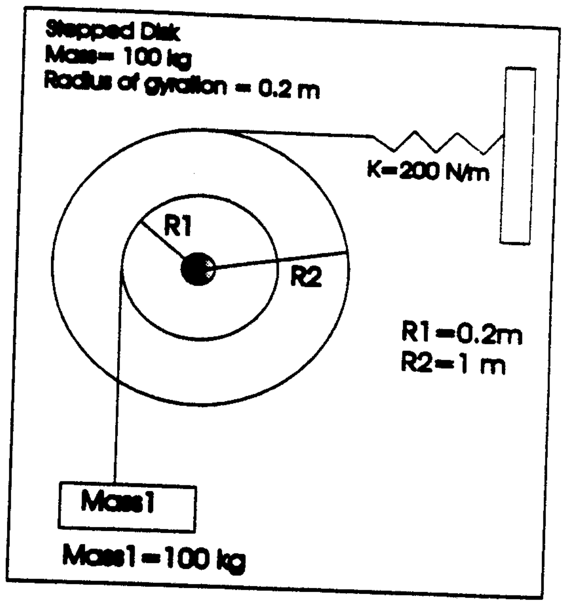
A stepped disk (mass stepped disk = 100 kg, radius of gyration k = 0.2 m about the centre of mass) supports a mass (Mass1=100 kg) while being held by a linear spring (K=200 N/m).

The disk is displaced from its static equilibrium condition by $\theta 1 = 0.1$ radians clockwise and released from rest.

Assume that the cord holding mass1 is inextensible and remains taut.

Find:

1. An expression for θ at any time after the motion begins for the disk
2. The displacement of the disk at t = 1 second.



Hint: *Do not neglect the mass.*

1. The 2 kg collar is released from rest at A and slides down the inclined fixed rod in the vertical plane, Fig.1. The coefficient of kinetic friction is 0.4. Calculate *(a)* the velocity v of the collar as it strikes the spring and *(b)* the maximum deflection x of the spring.

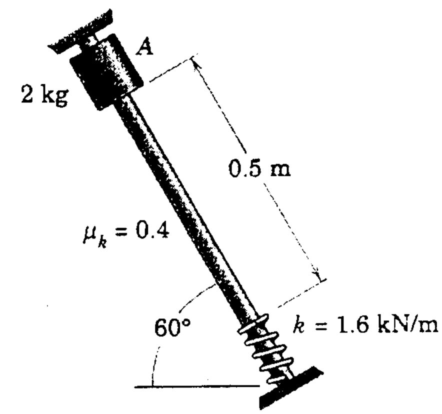


Figure 1

2. Motion of link ABC is controlled by the horizontal movement of the piston rod of the hydraulic cylinder D and by the vertical guide for the pinned slider at B, Fig.2. For the instant when $\theta = 45°$, the piston rod is retracting at the constant rate $v_c = 180\,mm/s$. For this instant determine (a) angular acceleration of bar AC, and (b) the acceleration of point A.

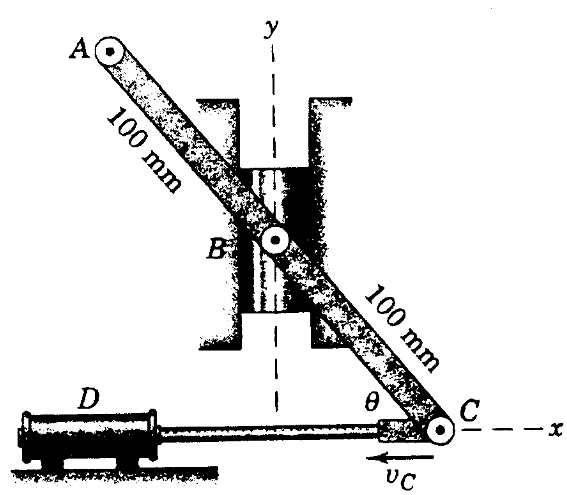


Figure 2

3. The semicircular disk having a mass of $10\,kg$, Fig.3, is rotating at $\omega = 4\,rad/s$ at the instant $\theta = 60°$. If the coefficient of static friction at A is $\mu_s = 0.5$. Determine if the disk slips at the instant. $\bar{I} = 0.51168\ kg.m^2$ is moment of inertia of the semicircular about its mass center.

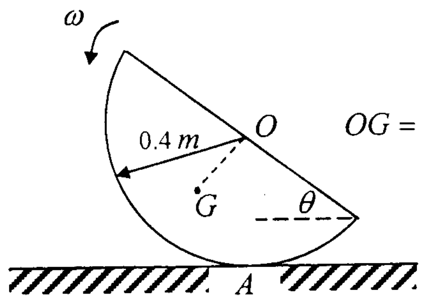
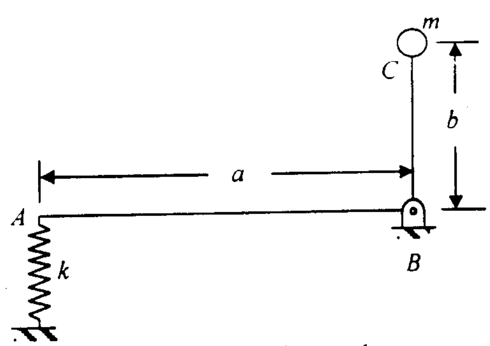


Figure 3

4. The ball has a mass of $8\,kg$ and radius $r = 100\,mm$ and rolls without slipping on the horizontal surface at $v_G = 6\,m/s$, Fig.4. Determine the angular velocity of the ball and the normal force the ball exerts on the track when it reaches the position $\theta = 70°$. Take $R = 500\,mm$. The moment of inertia of the ball about its mass center is $\frac{2}{5}mr^2$.

Figure 4

5. Find the natural frequency of the system shown in Fig.5.



ABC is a rigid member

Figure 5

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Rectilinear Motion

$a=\frac{dv}{dt}=\frac{d^2x}{dt^2}=v\frac{dv}{dx}$ $x=x_0+v_0t+\frac{1}{2}at^2$ $v=v_0+at$ $v^2=v_0^2+2a(x-x_0)$

Curvilinear Motion

$\vec{v}=\frac{d\vec{r}}{dt}$ $\vec{r}=x\vec{i}+y\vec{j}+z\vec{k}$ $\vec{e}_n=\frac{d\vec{e}_t}{d\theta}$ $\dot{\vec{e}}_r=\dot{\theta}\vec{e}_\theta$ $\dot{\vec{e}}_\theta=-\dot{\theta}\vec{e}_r$

$\vec{a}=\frac{d\vec{v}}{dt}$ $\vec{v}=\dot{x}\vec{i}+\dot{y}\vec{j}+\dot{z}\vec{k}$ $\vec{v}=v\vec{e}_t$ $\vec{v}=\dot{r}\vec{e}_r+r\dot{\theta}\vec{e}_\theta$

$\vec{a}=\ddot{x}\vec{i}+\ddot{y}\vec{j}+\ddot{z}\vec{k}$ $\vec{a}=\frac{dv}{dt}\vec{e}_t+\frac{v^2}{\rho}\vec{e}_n$

$\vec{a}=\left(\ddot{r}-r\dot{\theta}^2\right)\vec{e}_r+\left(r\ddot{\theta}+2\dot{r}\dot{\theta}\right)\vec{e}_\theta$

$\vec{r}_B=\vec{r}_A+\vec{r}_{B/A}$ $\vec{v}_B=\vec{v}_A+\vec{v}_{B/A}$ $\vec{a}_B=\vec{a}_A+\vec{a}_{B/A}$

Kinetics of Particles

$\sum\vec{F}=m\vec{a}$

$\sum F_x=ma_x$ $\sum F_t=ma_t$ $\sum F_r=ma_r$

$\sum F_y=ma_y$ $\sum F_n=ma_n$ $\sum F_\theta=ma_\theta$

$\sum F_z=ma_z$ $\sum F_z=ma_z$

$V_e=\frac{1}{2}kx^2$ $V_g=mgh$ $V_g=-\frac{mgR^2}{r}$ $T=\frac{1}{2}mv^2$

$T_1+U_{1-2}=T_2$ $T_1+V_1=T_2+V_2$ Power $=\vec{F}\cdot\vec{v}$

$\vec{L}=m\vec{v}$ $\sum\vec{F}=\dot{\vec{L}}$ $\int_1^2\sum\vec{F}\,dt=\vec{L}_2-\vec{L}_1$

$\vec{H}_O=\vec{r}\times m\vec{v}$ $\sum\vec{M}_O=\dot{\vec{H}}_O$ $\int_1^2\sum\vec{M}_O\,dt=\vec{H}_{O_2}-\vec{H}_{O_1}$

Systems of Particles

$\sum\vec{F}=m\vec{a}$ $\vec{L}=\sum m\vec{v}=m\vec{v}$ $\sum\vec{F}=\dot{\vec{L}}$ $\int_1^2\sum\vec{F}\,dt=\vec{L}_2-\vec{L}_1$

$\vec{H}=\sum\vec{r}\times m\vec{v}$ $\sum\vec{M}_O=\dot{\vec{H}}_O$ $\sum\vec{M}_G=\dot{\vec{H}}_G$ $\int_1^2\sum\vec{M}_O\,dt=\left(\vec{H}_O\right)_2-\left(\vec{H}_O\right)_1$

Kinematics of Rigid Bodies

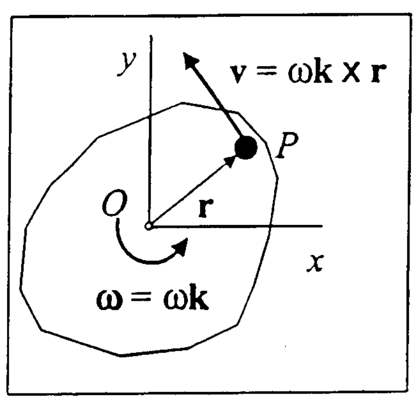
$\omega=\frac{d\theta}{dt}=\dot{\theta}$

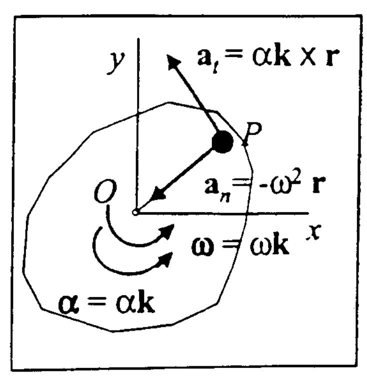
$\alpha=\frac{d\omega}{dt}=\dot{\omega}$ or $\alpha=\frac{d^2\theta}{dt^2}=\ddot{\theta}$

$\omega\,d\omega=\alpha\,d\theta$ or $\dot{\theta}\,d\dot{\theta}=\ddot{\theta}\,d\theta$

$v=r\omega$

$a_n=r\omega^2$ $a_t=r\alpha$

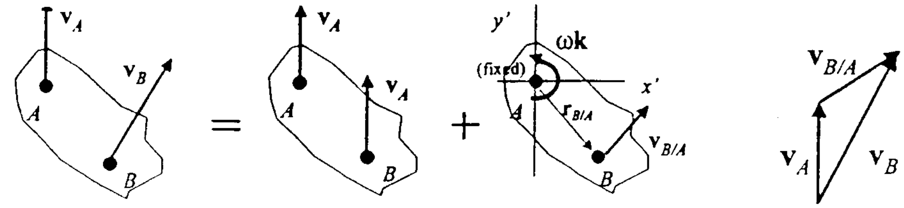




$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$

$v_{a/B} = r\omega$

$\vec{v}_{A/B} = \vec{\omega} \times \vec{r}$

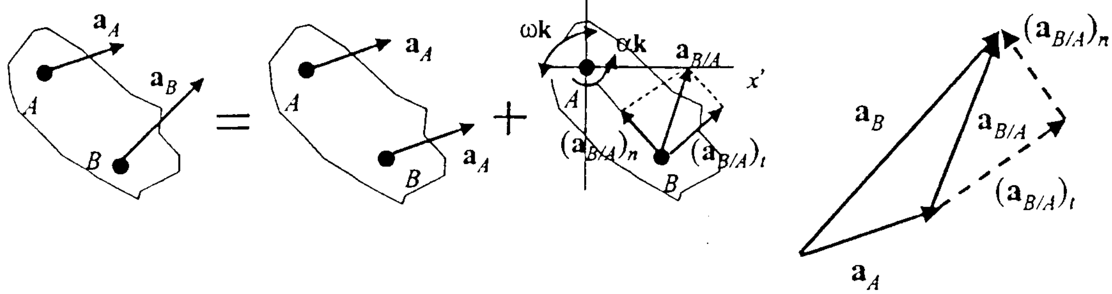


$\vec{a}_A = \vec{a}_B + \vec{a}_{A/B}$

$\vec{a}_A = \vec{a}_B + (\vec{a}_{A/B})_n + (\vec{a}_{A/B})_t$

$(a_{A/B})_n = \frac{v_{A/B}^2}{r} = r\omega^2$

$(a_{A/B})_t = \dot{v}_{A/B} = r\alpha$



Kinetics of Rigid Bodies

Equations of Motion

$\Sigma F_x = m\,\bar{a}_x \qquad \Sigma F_y = m\,\bar{a}_y \qquad \Sigma M_G = \bar{I}\,\alpha \qquad \Sigma M_o = I_o\alpha$

Energy

$T = \frac{1}{2} I_o\omega^2 \qquad T = \frac{1}{2} m\,v_G^2 + \frac{1}{2} I_G\omega^2 \qquad T_1 + \Sigma U_{1\text{-}2} = T_2$

Impulse and Momentum

$\vec{L} = m\vec{v} \qquad \sum \vec{F} = \vec{\dot{L}} \qquad \int_1^2 \vec{F}\,dt = \vec{L}_2 - \vec{L}_1$

$H_O = I_O\omega \qquad \sum \vec{M}_O = \vec{\dot{H}}_O \qquad \int_1^2 \sum \vec{M}_O\,dt = I_O(\omega_2 - \omega_1)$

$H_G = \bar{I}\omega \qquad \sum \vec{M}_G = \vec{\dot{H}}_G \qquad \int_1^2 \sum \vec{M}_G\,dt = (\vec{H}_G)_2 - (\vec{H}_G)_1$

Free Vibration

$$m\ddot{x} + c\dot{x} + kx = 0 \qquad \omega_n = \sqrt{\frac{k}{m}} \qquad c_c = 2m\sqrt{\frac{k}{m}} = 2m\omega_n \qquad \omega_d = \omega_n\sqrt{1 - \left(\frac{c}{c_c}\right)^2}$$

$$m\lambda^2 + c\lambda + k = 0 \qquad \lambda_1 = -\frac{c}{2m} + \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}} \qquad \lambda_2 = -\frac{c}{2m} - \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}$$

$c > c_c$ Overdamped $x = Ae^{\lambda_1 t} + Be^{\lambda_2 t}$ $\qquad c = c_c$ Critically damped $x = (A + Bt)e^{-\omega_n t}$

$c < c_c$ Underdamped $\quad x = D[e^{-(c/2m)t}\sin(\omega_d t + \phi)]$

log decrement $\qquad \delta = \ln\left(\frac{x_1}{x_2}\right) = \frac{2\pi\left(\frac{c}{c_c}\right)}{\sqrt{1 - \left(\frac{c}{c_c}\right)^2}}$

Forced Vibration

$m\ddot{x} + c\dot{x} + kx = P_m\sin(\omega_f t) \qquad x_p = X\sin(\omega t - \phi)$

$$M = \frac{X}{P_m/k} = \frac{1}{\sqrt{[1 - (\omega_f/\omega_n)^2]^2 + [2(\frac{c}{c_c})(\omega_f/\omega_n)]^2}}$$

$$\phi = \tan^{-1}\left[\frac{2(\frac{c}{c_c})\omega_f/\omega_n}{1 - (\omega_f/\omega_n)^2}\right]$$