

UNIVERSITY OF TORONTO  
FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION, APRIL 16, 2001

Third Year - Program: Engineering Science

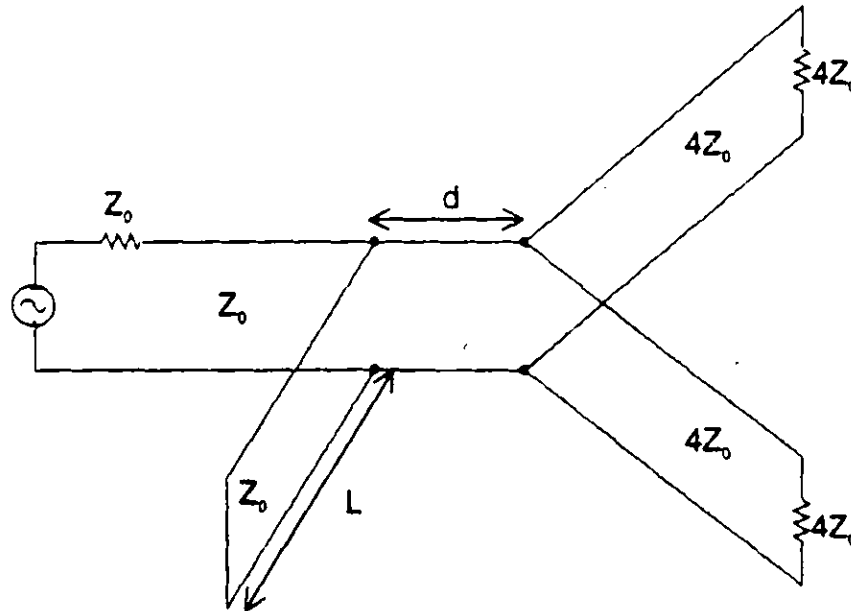
ECE357 - ELECTROMAGNETIC FIELDS

Exam Type: A

Examiner: A. Lüttgen

**Problem 1: (20/120 Marks)**

Use the single-stub matching method to match the two matched  $4Z_0$  lines to the  $Z_0$  line (see figure below). Find  $d$  and  $L$  in terms of  $\lambda$ . All lines are lossless.



**Problem 2: (25/120 Marks)**

A uniform plane wave, represented by the phasor

$$\mathbf{E}_i = E_{i0}(\hat{x}\cos(\theta_i) - \hat{z}\sin(\theta_i) + j\hat{y})e^{-j(x\sin(\theta_i) + z\cos(\theta_i))}$$

for its electrical field strength, propagates in air and is incident on glass ( $\epsilon_r=4$ ) at  $z=0$  with an angle of incidence  $\theta_i = 45^\circ$ . Determine its polarization. Find the electric field phasors and their instantaneous expressions for the transmitted and reflected waves and determine their polarization.

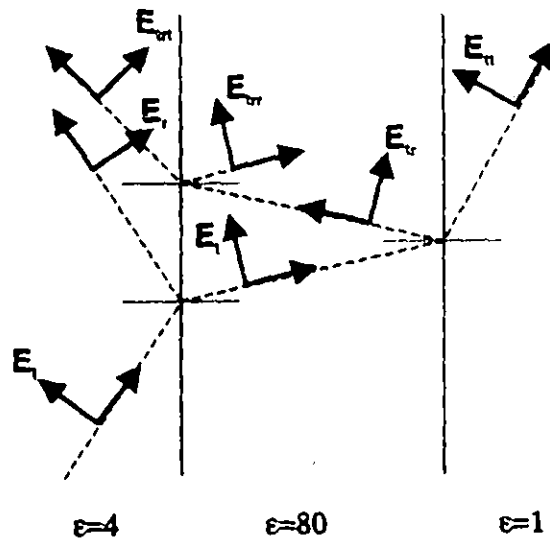
**Problem 3: (25/120 Marks)**

A parallel-polarized uniform plane wave with the following phasor representation for its electric field

$$E_i = E_{i0}(\hat{x}\cos\theta_i - \hat{z}\sin\theta_i)e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

propagates in semi-infinite medium 1 with  $\epsilon_r=4$  and is incident on the boundary with medium 2 ( $\epsilon_r=80$ ) at  $z = -4\pi\beta_1\sqrt{1+\epsilon_1/\epsilon_2}$  (boundary 1). The transmitted wave is incident on the boundary with semi-infinite medium 3 ( $\epsilon_r=1$ ) at  $z=0$  (boundary 2). The angle of incidence in medium 1 is given by  $\sin\theta_i = 1/(\sqrt{1.05})$ .

- Find the Brewster angles and critical angles of no reflection for all transitions in the figure below.
- Find the angle  $\theta_i$  for transmission from medium 1 to medium 2 at boundary 1 and the associated reflection and transmission coefficients for the incident wave.
- Find reflection and transmission coefficients at boundary 2 for the wave transmitted into medium 2.
- Find reflection and transmission coefficients at boundary 1 for the wave reflected from boundary 2.
- Find the amplitudes of all electric field vectors in the figure below.



**Problem 4: (25/120 Marks)**

At low frequencies the Earth-Ionosphere system acts as a parallel-plate waveguide. Assume the electron density in the Ionosphere to be a step function of the altitude with  $N=0$  below 70km,  $N=1.2 \cdot 10^{10} \text{ m}^{-3}$  between 70 and 90 km,  $N=9.6 \cdot 10^{10} \text{ m}^{-3}$  between 90 and 130 km,  $N=4.8 \cdot 10^{11} \text{ m}^{-3}$  between 130 and 210 km,  $N=1.3 \cdot 10^{12} \text{ m}^{-3}$  between 210 and 300 km, and  $N=2 \cdot 10^{12} \text{ m}^{-3}$  above 300 km. The ground has a relative permittivity of  $\epsilon_r=10$  and a conductivity of  $\sigma=10^{-2} \text{ S/m}$ . Determine at approximately what frequency this system fails to act as a waveguide and at what frequencies there might be changes to its wave guiding properties.  $((2\pi f_p)^2 = (Ne^2)/(m_e \epsilon_0))$

**Problem 5: (25/120 Marks)**

Find the array factor and plot the normalized polar radiation pattern of an array of seven isotropic elements spaced by  $d$  and having excitation amplitude ratios 1:2:3:4:3:2:1 for the following two cases:

(a)  $d=\lambda/2$  and  $\xi=0$ .

(b)  $d=\lambda/4$  and  $\xi=-\pi/2$ .

**Constants:**

$$\epsilon_0 = 8.8542 \cdot 10^{-12} \left[ \frac{\text{As}}{\text{Vm}} \right]$$

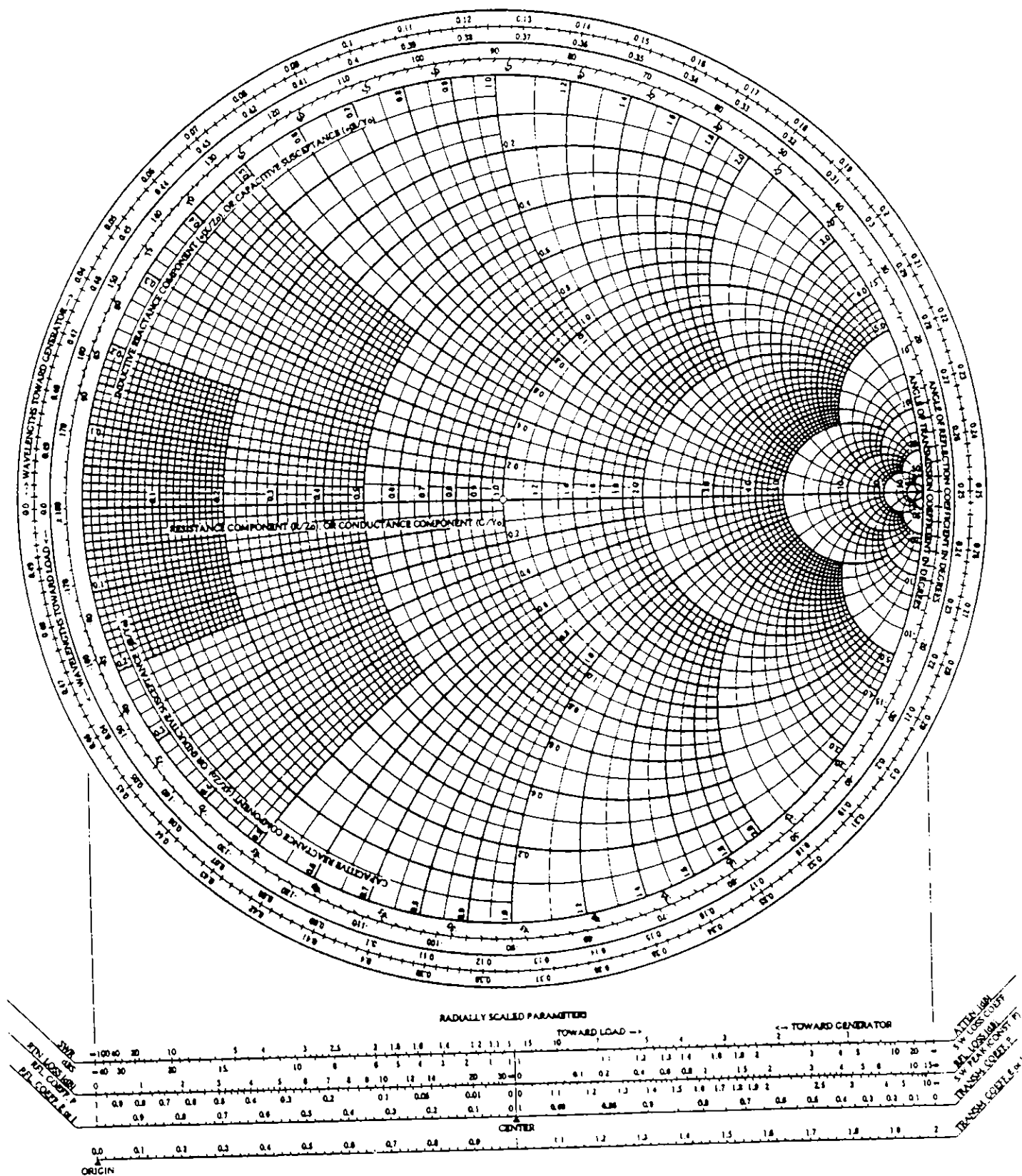
$$\mu_0 = 4\pi \cdot 10^{-7} \left[ \frac{\text{Vs}}{\text{Am}} \right]$$

$$e = 1.602 \cdot 10^{-19} [\text{C}]$$

$$m_e = 9.11 \cdot 10^{-31} [\text{kg}]$$

# The Complete Smith Chart

## Black Magic Design



# The Complete Smith Chart

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