NAME	STUDENT NUMBER

University of Toronto
Faculty of Applied Science and Engineering
Department of Mechanical and Industrial Engineering

First Year – MIE100S - Dynamics
Final Examination

April 23, 2001
9:30 a.m. – Noon

INSTRUCTIONS

1. The exam has five questions.
2. Answer all questions. All rough work must be **neatly** shown to earn full credit for each question.
3. Type B exam.
4. Marks for each problem shown with question.
5. This exam is worth 65% of the total marks of the course.
6. Put your name and student number on all pages in the space provided.
7. You may write on the back of the paper.

Question 1 (20 marks)	
2 (20 marks)	
3 (20 marks)	
4 (20 marks)	
5 (20 marks)	
Total	

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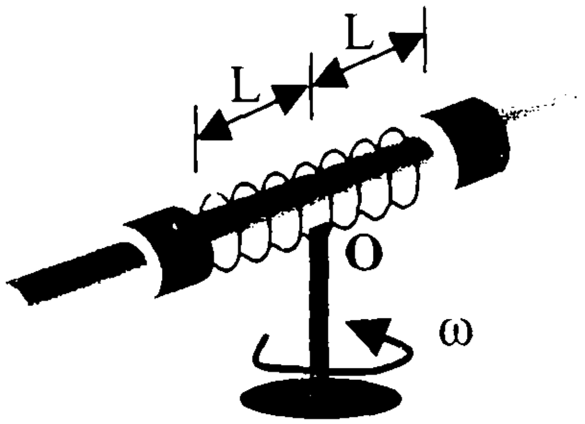
Question 1

Two collars, each of mass m = 2 kg, are joined by a spring having a relaxed length of 3m. The collars slide on a horizontal, massless rod rotating about O. Initially the collars are each held at L=1m by pins, while the rod rotates at ω_i = 18 rad/s. The pins are then removed, and the collars slide to a final equilibrium position at L=3m, i.e. they end up being 6 m apart.



Note the spring joining the two collars.

Assuming that no friction acts, find:

(a) The final angular velocity of the rod, ω_f
(b) The spring constant, k.

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ω_f =	k =

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Question 2

When the mechanism is in the position shown, the angular velocity of the gear is $\omega = 2$ rad/sec clockwise and its angular acceleration is $\alpha = 4$ rad/sec^2 counterclockwise.

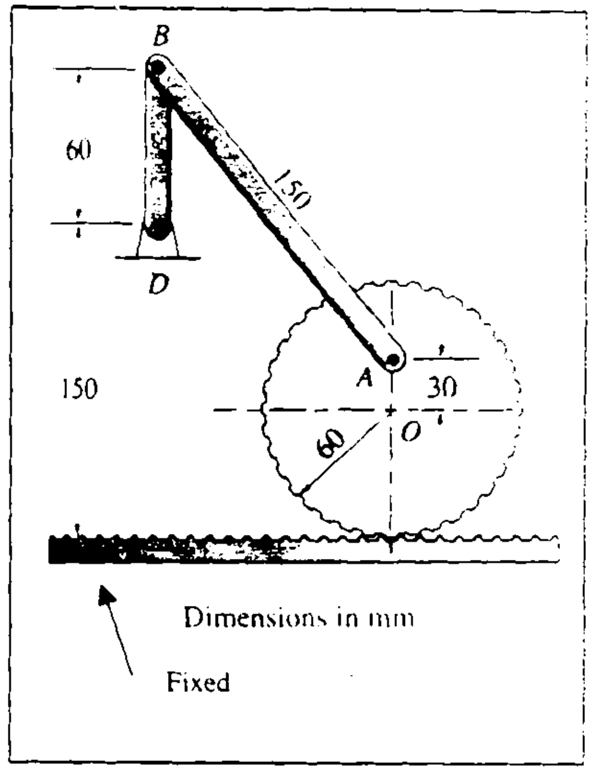
Find:

1. Angular velocities of links AB and BD
2. Angular accelerations of links AB and BD

in this position.

Hints:

A is not at the centre of the disk.



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Please write answers in spaces below

$\omega_{AB} =$	$\alpha_{AB} =$
$\omega_{BD} =$	$\alpha_{BD} =$

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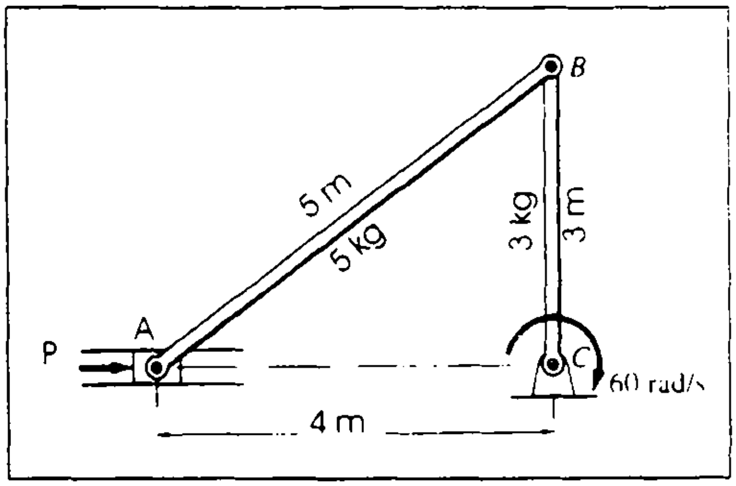
Question 3

The mechanism consists of two homogeneous bars of the masses shown and the piston A of negligible weight. A varying horizontal force P acting on the piston maintains a constant angular velocity $\omega_{BC} = 60$ rads/sec.



Radii of gyrations are:

k_{AB}=1.44 m about centre of mass of AB
k_{BC}=0.87 m about centre of mass of BC

Neglect friction.

Find:

1. Angular acceleration α_{AB}
2. Acceleration of point A, a_A
3. Acceleration of mass center of AB, a_G
4. Magnitude and direction of force P (give as magnitude of force and direction (left or right))

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$\alpha_{AB} =$	$a_A =$
$a_G =$	$P =$

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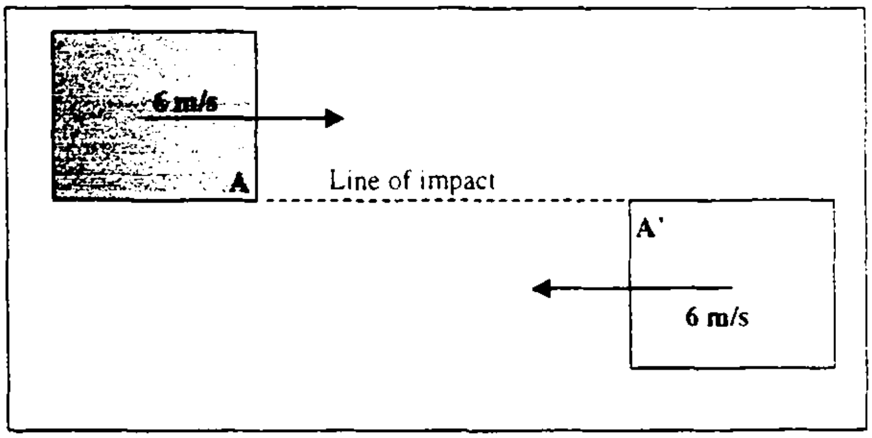
<u>Question 4</u>

Two identical square plates (each of mass 10 kg, side length 2 m) slide towards each other on a frictionless tabletop. Each plate has velocity 6 m/s. Corners A and A' just collide with one another, and the plates stick together at these corners.



(a) Show that the corners A and A' have zero velocity after the collision.

(b) Compute the angular velocity of the plates immediately after the collision.

For a square plate of side L and mass M, the mass moment of inertia about the plate centroid is

$$\bar{I} = \frac{1}{6} ML^2$$

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Angular velocity just after impact =

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Question 5

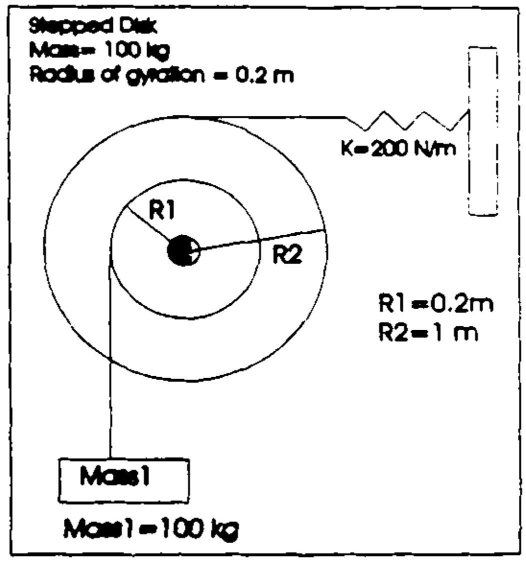
A stepped disk (mass stepped disk = 100 kg, radius of gyration k = 0.2 m about the centre of mass) supports a mass (Mass1=100 kg) while being held by a linear spring (K=200 N/m).

The disk is displaced from its static equilibrium condition by $\theta 1 = 0.1$ radians clockwise and released from rest.

Assume that the cord holding mass1 is inextensible and remains taut.

Find:

1. An expression for θ at any time after the motion begins for the disk
2. The displacement of the disk at t = 1 second.



Hint: *Do not neglect the mass.*

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$\theta =$
Displacement at t= 1 second

Rectilinear Motion

$a=\frac{dv}{dt}=\frac{d^2x}{dt^2}=v\frac{dv}{dx}$ $\quad x=x_0+v_0t+\frac{1}{2}at^2$ $\quad v=v_0+at$ $\quad v^2=v_0^2+2a(x-x_0)$

Curvilinear Motion

$\vec{v}=\frac{d\vec{r}}{dt}$ $\quad \vec{r}=x\vec{i}+y\vec{j}+z\vec{k}$ $\quad \vec{e}_n=\frac{d\vec{e}_t}{d\theta}$ $\quad \overrightarrow{\dot{e}_r}=\dot{\theta}\vec{e}_\theta$ $\quad \overrightarrow{\dot{e}_\theta}=-\dot{\theta}\vec{e}_r$

$\vec{a}=\frac{d\vec{v}}{dt}$ $\quad \vec{v}=\dot{x}\vec{i}+\dot{y}\vec{j}+\dot{z}\vec{k}$ $\quad \vec{v}=v\vec{e}_t$ $\quad \vec{v}=\dot{r}\vec{e}_r+r\dot{\theta}\vec{e}_\theta$

$\vec{a}=\ddot{x}\vec{i}+\ddot{y}\vec{j}+\ddot{z}\vec{k}$ $\quad \vec{a}=\frac{dv}{dt}\vec{e}_t+\frac{v^2}{\rho}\vec{e}_n$

$\vec{a}=\left(\ddot{r}-r\dot{\theta}^2\right)\vec{e}_r+\left(r\ddot{\theta}+2\dot{r}\dot{\theta}\right)\vec{e}_\theta$

$\vec{r}_B=\vec{r}_A+\vec{r}_{B/A}$ $\quad \vec{v}_B=\vec{v}_A+\vec{v}_{B/A}$ $\quad \vec{a}_B=\vec{a}_A+\vec{a}_{B/A}$

Kinetics of Particles

$\sum\vec{F}=m\vec{a}$

$\sum F_x=ma_x$ $\quad \sum F_t=ma_t$ $\quad \sum F_r=ma_r$

$\sum F_y=ma_y$ $\quad \sum F_n=ma_n$ $\quad \sum F_\theta=ma_\theta$

$\sum F_z=ma_z$ $\quad \sum F_z=ma_z$

$V_e=\frac{1}{2}kx^2$ $\quad V_g=mgh$ $\quad V_g=-\frac{mgR^2}{r}$ $\quad T=\frac{1}{2}mv^2$

$T_1+U_{1-2}=T_2$ $\quad T_1+V_1=T_2+V_2$ $\quad$ Power $=\vec{F}\cdot\vec{v}$

$\vec{L}=m\vec{v}$ $\quad \sum\vec{F}=\dot{\vec{L}}$ $\quad \int_1^2\sum\vec{F}\,dt=\vec{L}_2-\vec{L}_1$

$\vec{H}_O=\vec{r}\times m\vec{v}$ $\quad \sum\vec{M}_O=\dot{\vec{H}}_O$ $\quad \int_1^2\sum\vec{M}_O\,dt=\vec{H}_{O_2}-\vec{H}_{O_1}$

Systems of Particles

$\sum\vec{F}=m\bar{a}$ $\quad \vec{L}=\sum m\vec{v}=m\bar{v}$ $\quad \sum\vec{F}=\dot{\vec{L}}$ $\quad \int_1^2\sum\vec{F}\,dt=\vec{L}_2-\vec{L}_1$

$\vec{H}=\sum\vec{r}\times m\vec{v}$ $\quad \sum\vec{M}_O=\dot{\vec{H}}_O$ $\quad \sum\vec{M}_G=\dot{\vec{H}}_G$ $\quad \int_1^2\sum\vec{M}_O\,dt=\left(\vec{H}_O\right)_2-\left(\vec{H}_O\right)_1$

Kinematics of Rigid Bodies

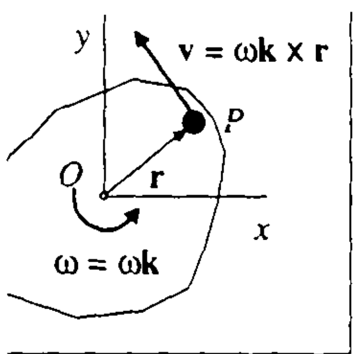
$\omega=\frac{d\theta}{dt}=\dot{\theta}$

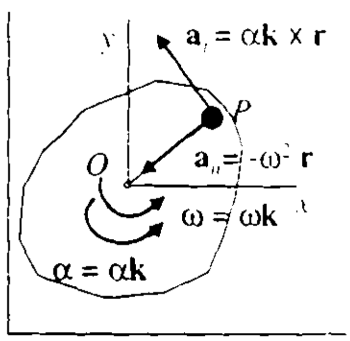
$\alpha=\frac{d\omega}{dt}=\dot{\omega}$ $\quad$ or $\quad \alpha=\frac{d^2\theta}{dt^2}=\ddot{\theta}$

$\omega d\omega=\alpha d\theta$ or $\quad \dot{\theta}\,d\dot{\theta}=\ddot{\theta}\,d\theta$

$v=r\omega$

$a_n=r\omega^2 \qquad a_t=r\alpha$





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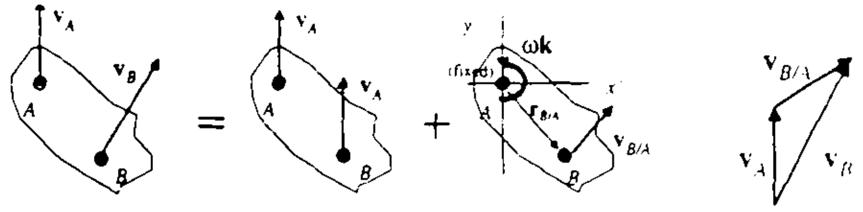
$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$

$v_{a/B} = r\omega$

$\vec{v}_{A/B} = \vec{\omega} \times \vec{r}$

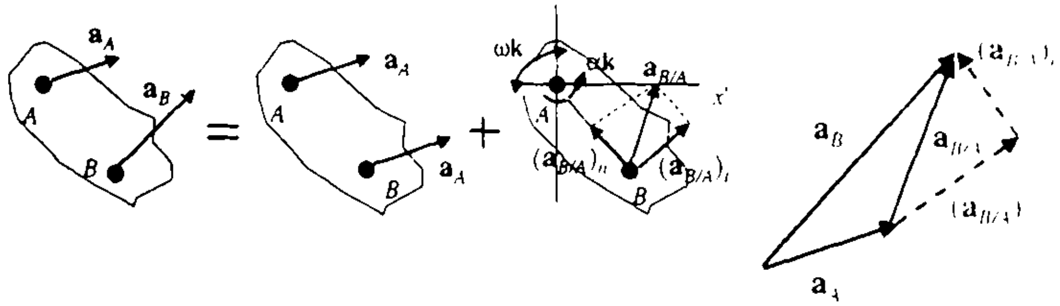


$\vec{a}_A = \vec{a}_B + \vec{a}_{A/B}$

$\vec{a}_A = \vec{a}_B + (\vec{a}_{A/B})_n + (\vec{a}_{,}$

$(a_{A/B})_n = \frac{v_{A/B}^2}{r} = r\omega^2$

$(a_{A/B})_t = \dot{v}_{A/B} = r\alpha$



Kinetics of Rigid Bodies

Equations of Motion

$\Sigma F_x = m\bar{a}_x \quad \Sigma F_y = m\bar{a}_y \quad \Sigma M_G = \bar{I}\alpha \quad \Sigma M_o = I_o\alpha$

Energy

$T = \frac{1}{2} I_o\omega^2 \quad T = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G\omega^2 \quad T_1 + \Sigma U_{1-2} = T_2$

Impulse and Momentum

$$\vec{L} = m\vec{v} \qquad \sum \vec{F} = \vec{\dot{L}} \qquad \int_1^2 \vec{F}\,dt = \vec{L}_2 - \vec{L}_1$$

$$H_O = I_O\omega \qquad \sum \vec{M}_O = \vec{\dot{H}}_O \qquad \int_1^2 \sum \vec{M}_O\,dt = I_O(\omega_2 - \omega_1)$$

$$H_G = \bar{I}\omega \qquad \sum \vec{M}_G = \dot{\vec{H}}_G \qquad \int_1^2 \sum \vec{M}_G\,dt = (\vec{H}_G)_2 - (\vec{H}_G)_1$$

Free Vibration

$$m\ddot{x} + c\dot{x} + kx = 0 \quad \omega_n = \sqrt{\frac{k}{m}} \quad c_c = 2m\sqrt{\frac{k}{m}} = 2m\omega_n \quad \omega_d = \omega_n\sqrt{1 - \left(\frac{c}{c_c}\right)^2}$$

$$m\lambda^2 + c\lambda + k = 0 \quad \lambda_1 = -\frac{c}{2m} + \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}} \quad \lambda_2 = -\frac{c}{2m} - \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}$$

$c > c_c$ Overdamped $x = Ae^{\lambda_1 t} + Be^{\lambda_2 t}$ $\quad c = c_c$ Critically damped $x = (A + Bt)e^{-\omega_n t}$

$c < c_c$ Underdamped $\quad x = D[e^{-(c/2m)t}\sin(\omega_d t + \phi)]$

log decrement $\quad \delta = \ln\left(\frac{x_1}{x_2}\right) = \frac{2\pi\left(\frac{c}{c_c}\right)}{\sqrt{1 - \left(\frac{c}{c_c}\right)^2}}$

Forced Vibration

$m\ddot{x} + c\dot{x} + kx = P_m\sin(\omega_f t) \quad x_p = X\sin(\omega t - \phi)$

$$M = \frac{X}{P_m/k} = \frac{1}{\sqrt{[1 - (\omega_f/\omega_n)^2]^2 + [2(\frac{c}{c_c})(\omega_f/\omega_n)]^2}}$$

$$\phi = \tan^{-1}\left[\frac{2(\frac{c}{c_c})\omega_f/\omega_n}{1 - (\omega_f/\omega_n)^2}\right]$$