Department of Mechanical & Industrial Engineering
University of Toronto

Final Examination – April 2001

Second Year
MIE262 – Operational Research I

Exam Type: A

Examiner: Prof. A.K.S. Jardine

NOTES:

1. Attempt all questions. All questions are of equal value.
2. If doubt exists as to the interpretation of any question, you are urged to submit with your answer a clear statement of any assumption you have made.
3. Show all calculations.

1.a) The management of a coal-fired electric power generating plant is studying the plant's operational setup in order to comply with the latest emission standards. Maximum emission rates are

- maximum sulphur oxide emission: 3000 parts per million (PPM),
- maximum particulate emission (smoke): 12 kilograms/hour (kg/hr).

Coal is brought to the plant then carried by a conveyor belt to the pulverizer unit, where it is pulverized and fed via conveyor directly into the boiler at the desired rate. The heat produced in the boiler is used to make steam to drive the turbines. Two types of coal are used: grade A, which is a hard and clean-burning coal with a low sulphur content; and grade B, which is a cheaper, relatively soft and smoky coal with a high sulfur content, as shown in the Table below.

Emission of pollutants

Coal	Sulphur Oxides in Flue Gases	Particulate (emission/ton)
A	1800 PPM	0.5 kg
B	3800 PPM	1.0 kg

The thermal value in terms of steam produced is higher for coal A than for coal B, namely 24,000 pounds per ton for A against 20,000 pounds per ton for B. Since coal A is a hard coal, the pulverizer unit can handle at most 16 tons of coal A per hour, whereas it can pulverize up to 24 tons of coal B per hour. The conveyor loading system has a capacity of 20 tons per hour regardless of which coal is loaded.

Given the limits on emission of pollutants and the grades of coal available, what is the maximum possible output of electricity of the plant?

The decision variables of the problem are

- the amount of coal A used per hour, denoted by x_1,
- the amount of coal B used per hour, denoted by x_2.

Management's objective is to maximize the output of electricity of the plant which is equivalent to maximizing steam output. Total steam produced /hour: $24x_1 + 20x_2 = z$, in units of 1000 lb.

The contraints are:

Constraint on particle emission

The maximum amount of smoke that the plant is allowed to emit per hour is limited to 12 kg.

$$0.5x_1 + x_2 \le 12$$

Constraint on loading facilities

The conveyor system transporting coal from the stockpiles to the pulverizer has an hourly capacity of 20 tons. Thus, the loading constraint is:

$$x_1 + x_2 \le 20$$

Constraint on pulverizer unit

The pulverizer constraint is:

$$\tfrac{1}{16}x_1 + \tfrac{1}{24}x_2 \le 1$$

Constraint on sulphur oxide emission

The maximum sulphur oxide emission is not to exceed 3000 PPM at any time. The sulfur constraint is:

$$1200x_1 - 800x_2 \ge 0$$

Solve the problem graphically and identify the optimal values of the decision variables x, and x_2. **(8 marks)**

1.b) A metal processing plant receives an order to produce 10,000 casings. The contract specifies a sale price of $14.85 per casing. The products design engineer proposes four alternative designs for the casings, resulting in different machine time usages and material costs. The material costs differ because of varying amounts of wastage.

Input data for casing production

Production Design	Machine Time (in minutes) Cutting	Forming	Welding	Finishing	Material Cost	Rejects Produced
1	0.40	0.70	1.00	0.50	$3.355	3%
2	0.80	1.00	0.40	0.30	$4.150	1%
3	0.35	0.60	1.20	0.75	$3.005	4%
4	0.70	0.80	0.60	0.55	$3.705	2%
Cost/minute	$0.20	$0.30	$0.15	$0.10		

The customer wants to receive delivery within one month of signing of the contract. On the basis of the present production commitments, the operations manager forecasts that the plant has excess capacities of 90 hours of cutting machine time, 140 hours of forming machine time, 154 hours of welding time, and 120 hours of finishing time. The industrial engineer's problem is to determine which designs to use so as to guarantee delivery within the contractual arrangement and maximize profit.

Formulate the LP model and put in standard form letting:

X_1 = number of casings of design 1 scheduled.
X_2 = number of casings of design 2 scheduled.
X_3 = number of casings of design 3 scheduled.
X_4 = number of casings of design 4 scheduled. **(12 marks)**

2.a) The heavy-duty diesel motor of the emergency power generating plant of a factory needs an extensive overhaul job. At the same time, the concrete base, which is cracked, also has to be replaced. Management plans to have this job performed during the annual vacation of the factory when the factory closes for 15 working days. Management would like to know whether the overhaul can be completed in this time. The Table below lists the various tasks.

Diesel motor overhaul (all times shown in days)

Task	Precedence	t_{ij}
A. Dismantle motor	—	2
B. Overhaul motor	A	6
C. Rebuild motor base and cure	A	9
D. Test and adjust motor	B	3
E. Mount motor on base	C, D	2

(i) Construct an activity-on-arrow network using the minimum number of dummy activities. **(3 marks)**

(ii) Calculate the total float for each activity and use this to identify the critical path. **(3 marks)**

(iii) Do you expect the project to be completed on time? Explain your answer. **(2 marks)**

(iv) If a project target date is not going to be met, what remedial actions might you take. **(4 marks)**

2.b) When dealing with three-time estimating in network analysis (PERT/CPM) we use the formula (a + 4m + b)/6 to estimate the mean of the distribution of possible activity deviations.

(i) Describe briefly situations where 3-time estimating would be used. **(4 marks)**

(ii) Explain your understanding of the values a and b, including an estimate of what you believe would be appropriate for the probability of the activity duration being less than a. **(4 marks)**

3. In class we introduced the Laplace criterion for decision making under uncertainty. We examined two decision situations:

    1. Invest in bank or buy lottery ticket
    2. Have operation or not have operation

    For each of these alternatives we defined two states of nature.

    Create your own decision making under uncertainty decision problem having:

    Two alternatives
    Two events or states-of-nature

    Clearly explain the two alternatives and explain unambiguously whether the consequences are dependent on events or states-of-nature. **(10 marks)**

    Solve your problem using the Laplace criterion. **(2 marks)**

    Briefly outline the situation in which the Laplace criterion would be used. Compare this with the situation where the minimax regret criterion would be used. **(8 marks)**

4.a) The following is the initial tableau for a transportation problem having used the N-W corner rule to obtain an initial solution.

		Store					
		A	B	C	D	E	AVAIL
Warehouse	I	12 200	8 300	13 200	15	12	700
	II	7	3	4 100	8 200	2 100	400
	III	0	0	0	0	0 300	300
	Demand	200	300	300	200	400	1,400 1,400

The final solution is:

		Store A	Store B	Store C	Store D	Store E
Warehouse	I	12 200	8 300	13 200	15 2	12 0
	II	7 4	3 4	4 0	8 4	2 400
	III	0 1	0 5	0 100	0 200	0 2

Explain your understanding of the meaning of the circled 5 (i.e. $5) in the above tableau for cell [III, B]. **(5 marks)**

4.b) In probabilistic inventory control models are developed that are termed the (r, q) and (s, S) models.

Explain briefly the difference between these two models. **(5 marks)**

4.c) When improving systems through preventive replacement, a policy frequently adopted is:

age replacement

Outline your understanding of this policy explaining clearly how the data necessary to solve the problem can be obtained. **(5 marks)**

Outline two practical situations where the policy may be employed, clearly justifying your answer. **(5 marks)**

5. The methodology of operational research may be defined by the following steps:

1. Define problem
2. Construct model
3. Test model
4. Obtain solution
5. Implement solution
6. Control solution

In class this methodology was illustrated by means of a classic deterministic inventory control problem that included derivation of the economic order quality formula:

$$Q^* = \sqrt{\frac{2DC_0}{C_h}}$$

Illustrate the six steps of the O.R. methodology beginning with a different problem situation (Step 1) then demonstrate your understanding of the remaining 5 steps through reference to your defined problem. **(20 marks)**