UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATIONS, APRIL 2001
MAT 188 S – LINEAR ALGEBRA. FIRST YEAR: T-PROGRAM
EXAMINER: FELIX J. RECIO

INSTRUCTIONS:

1. ATTEMPT ALL QUESTIONS.
2. SHOW AND EXPLAIN YOUR WORK IN ALL QUESTIONS.
3. GIVE YOUR ANSWERS IN THE SPACE PROVIDED. USE BOTH SIDES OF PAPER, IF NECESSARY.
4. DO NOT TEAR OUT ANY PAGES.
5. USE OF NON-PROGRAMMABLE POCKET CALCULATORS, BUT NO OTHER AIDS ARE PERMITTED.
6. THIS EXAM CONSISTS OF EIGHT QUESTIONS. THE VALUE OF EACH QUESTION IS INDICATED (IN BRACKETS) BY THE QUESTION NUMBER.
7. THIS EXAM IS WORTH 50% OF YOUR FINAL GRADE.
8. TIME ALLOWED: 2 ½ HOURS.
9. PLEASE WRITE YOUR NAME, YOUR STUDENT NUMBER, AND YOUR SIGNATURE IN THE SPACE PROVIDED AT THE BOTTOM OF THIS PAGE.

PLEASE DO NOT WRITE HERE

QUESTION NUMBER	QUESTION VALUE	GRADE
1	30	
2	15	
3	15	
4	15	
5	15	
6	20	
7	20	
8	20	
9	20	
TOTAL:	170	

NAME: ______________________ , ______________________
(FAMILY NAME. PLEASE PRINT.) (GIVEN NAME.)

STUDENT No.: ______________________ SIGNATURE: ______________________

1. Consider the points $A(2,0,1)$, $B(2,1,0)$, and $C(1,1,k)$.

   a) (5 marks) Find the values of k, if any, for which the line that passes through the points A and C is parallel to the plane $2x-5y+3z=1$.

   b) (5 marks) Find the values of k, if any, for which the line that passes through the points A and C contains the point $(4,-2,8)$.

   c) (5 marks) Find the values of k, if any, for which the plane that passes through the points A, B, and C is perpendicular to the line with parametric equations $x=-1+3t$, $y=-2t$, and $z=5-2t$.

   d) (5 marks) Find the values of k, if any, for which the angle between the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ is $\pi/6$.

   e) (5 marks) Find the values of k, if any, for which the volume of the parallelepiped generated by the vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$, and $\overrightarrow{OC}$ is 7.

   f) (5 marks) Find the values of k, if any, for which the distance from the point C to the line that passes through the points A and B is 2.

2. (15 marks) Solve the linear system: $\begin{cases} x_1 + 2x_3 + x_4 = 3 \\ x_2 + x_4 + x_5 = 1 \\ x_1 - x_2 + 2x_3 - x_5 = 2 \\ x_1 + x_2 + 2x_3 - x_5 = -2 \end{cases}$

3) (15 marks) Consider the linear system: $\begin{cases} x + az = 1 \\ y - 2z = a \\ -x + 2z = -1 \\ 2x + ay = 6 \end{cases}$

Find the values of the constant a, if any, for which:

a) The system has no solutions.

b) The system has exactly one solution.

c) The system has exactly two solutions.

d) The system has infinitely many solutions.

4. (15 marks) Consider the matrices $A = \begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 0 \\ 0 & 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 3 & -2 \\ -2 & 2 & 2 \\ -2 & 3 & 1 \end{pmatrix}$.

Find all matrices M, if any, for which $A^T - 2M = B - AM$.

5. (15 marks) Let $A = \begin{pmatrix} 5 & 1 & 1 & 0 \\ 1 & 5 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 5 \end{pmatrix}$ and let M be another 4×4 matrix such that $\det(A M^3) = 1$.

Compute $\det M$.

6. Let $\mathbf{S}$ be the subspace of $\mathbf{R}^4$ generated by the vectors $\mathbf{v}_1 = (1, 0, -1, 2)$, $\mathbf{v}_2 = (0, -1, 0, 1)$, $\mathbf{v}_3 = (1, 2, -1, 0)$, $\mathbf{v}_4 = (-1, 1, 1, -3)$, and $\mathbf{v}_5 = (1, 0, -1, 0)$.

a) (10 marks) Determine the dimension of the subspace $\mathbf{S}$ and find a basis for $\mathbf{S}$.

b) (5 marks) Is the vector $\mathbf{v}_5$ a linear combination of the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$, and $\mathbf{v}_4$? Why or why not?

c) (5 marks) Is the vector $\mathbf{v} = (1, -3, -1, 1)$ one of the vectors in $\mathbf{S}$? Why or why not?

7. (20 marks) Let $\mathbf{C}[-1, 1]$ denote the inner product space consisting of all real valued functions which are continuous on the interval $[-1, 1]$, with the inner product defined as $(\mathbf{f}, \mathbf{g}) = \int_{-1}^{1} \mathbf{f}(x)\mathbf{g}(x)\,dx$. Find an orthonormal basis for the subspace of $\mathbf{C}[-1, 1]$ spanned by the set $\{1, 3 + x, 2x + 3x^2\}$.

8. (20 marks) Given the matrix $A = \begin{pmatrix} -4 & 1 & 2 \\ 2 & -3 & -2 \\ -4 & 2 & 2 \end{pmatrix}$. Find an invertible matrix P and a diagonal matrix D such that $P^{-1} A P = D$.

9. Determine, in each of the following cases, whether the given proposition is true or false. Give and briefly explain your reasons in each case.

   a) (5 marks) If A is any 3×3 matrix such that $\det A = 3$, then $\det(\operatorname{Adj} A) = 27$.

   b) (5marks) If M is any 5×7 matrix such that the rank of M is 3, then the dimension of the solution space of $A\mathbf{x} = \mathbf{0}$ is 2.

   c) (5 marks) The set consisting of all polynomials $\mathbf{p}(x) = a + bx + cx^2$ such that $a = bc$ is a subspace of $\mathbf{P}_2$.

   d) (5 marks) If λ is an eigenvalue of the square matrix A, then λ^2 is an eigenvalue of the matrix A^2.