Arguably the most common stable distribution parameterization has characteristic function given by

$$\varphi(t;\alpha,\beta,c,\mu) = \exp(it\mu - |ct|^\alpha(1 - i\beta\,\mathrm{sgn}(t)\Phi)), \qquad \Phi = \begin{cases} \tan(\frac{\pi\alpha}{2}), & \text{if } \alpha \neq 1 \\ -\frac{2}{\pi}\log|t|, & \text{if } \alpha = 1 \end{cases}$$

For simplicity, we force the scale $c = 1$ and the location $\mu = 0$, yielding

$$\varphi(t;\alpha,\beta) = \exp(-|t|^\alpha(1 - i\beta\,\mathrm{sgn}(t)\Phi))$$

with Φ defined as before.

PDF Integrand Derivation

Probability density functions are the Fourier transforms of their respective characteristic functions, so

$$\begin{aligned} f(x) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} \varphi(t)e^{-ixt}\,dt \\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty} \exp(-|t|^\alpha(1 - i\beta\,\mathrm{sgn}(t)\Phi))\exp(-ixt)\,dt \end{aligned}$$

By Euler's formula and ignoring strictly complex terms (since the PDF is real-valued),

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \exp(-|t|^\alpha)[\cos(|t|^\alpha\beta\,\mathrm{sgn}(t)\Phi)\cos(xt) + \sin(|t|^\alpha\beta\,\mathrm{sgn}(t)\Phi)\sin(xt)]\,dt$$

Lastly, by symmetry,

$$f(x) = \frac{1}{\pi}\int_0^{\infty} \exp(-t^\alpha)[\cos(t^\alpha\beta\Phi)\cos(xt) + \sin(t^\alpha\beta\Phi)\sin(xt)]\,dt$$

Note that the oscillatory terms $\cos(xt)$ and $\sin(xt)$ should be incorporated into a weighting function in QUADPACK routines.

Transforming into the S_0 parameterization of Nolan amounts to computing $f(x+\beta\tan(\pi\alpha/2))$ instead of $f(x)$ when $\alpha \neq 1$.

CDF Integrand Derivation

Due to an inversion formula by Gil-Pelaez, we have

$$F(x) = \frac{1}{2} - \frac{1}{\pi}\int_0^{\infty} \frac{\mathrm{Im}[\varphi(t)e^{-ixt}]}{t}\,dt$$

Rewriting to make use of $t \geq 0$ yields

$$= \frac{1}{2} - \frac{1}{\pi}\int_0^{\infty} \frac{\mathrm{Im}[\exp(-t^\alpha(1 - i\beta\Phi))\exp(-ixt)]}{t}\,dt$$

and applying Euler's formula gives us

$$F(x) = \frac{1}{2} - \frac{1}{\pi}\int_0^{\infty} \frac{\exp(-t^\alpha)[-\cos(t^\alpha\beta\Phi)\sin(xt) + \sin(t^\alpha\beta\Phi)\cos(xt)]}{t}\,dt$$

Note that the oscillatory terms $\cos(xt)$ and $\sin(xt)$ should be incorporated into a weighting function in QUADPACK routines.

Transforming into the S_0 parameterization of Nolan amounts to computing $F(x+\beta\tan(\pi\alpha/2))$ instead of $F(x)$ when $\alpha \neq 1$.