Advanced Petrophysics
PGE 381L, Fall 2023
Unique Number: 20215

Homework Assignment No. 8

November 16, 2023
Due on Thursday, December 4, 2023, before 11:00 PM

Name: ____________________**SOLUTION** ____

UT EID: ____

Objectives:

- **a)** To practice application of Young-Laplace equation to estimate capillary pressure
- **b)** To practice assessment and interpretation of capillary pressure
- **c)** To practice saturation-height analysis
- **d)** To understand trapping mechanisms

Note: Please scan your homework assignment and upload it as one pdf file on the Canvas website before the deadline. Please name your homework document as follows:

PGE381L_2023_Fall _HW08_lastname_name.pdf

Example: PGE381L_2023_Fall_HW08_Heidari_Zoya.pdf

Question 1: This question can be considered as an application of Young-Laplace's equation in evaluation of oil migration in porous media. Figure 1 shows an oil blob being displaced by water at the pore scale in a reservoir. The blob has encountered a constriction at a pore throat. In order for the oil blob to pass through the constriction and be produced, a sufficiently high pressure gradient must be applied across the blob. If such a pressure gradient cannot be generated by the water injection, then the blob will be trapped as residual oil. The objective of this exercise is for you to calculate the pressure gradients necessary to mobilize the blob for a variety of situations to determine whether or not such gradients can be created under normal oilfield flow conditions. Answer the following questions:

Wetting phase	=	water
Oil-water interfacial tension	=	σ dynes/cm
Contact angle	=	θ°
Radius of pore body	=	R (cm)
Radius of pore throat	=	r (cm)
Length of blob	=	L (cm)

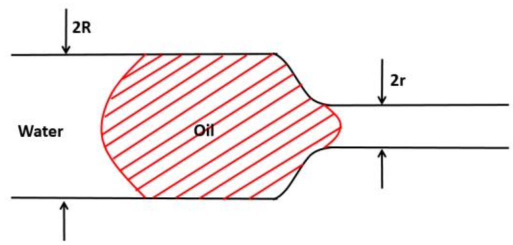


Figure 1

a) Calculate the pressure gradients required for mobilization of the oil blob for an average sand and a very fine sand. Assume the following:

$R = 5r$

$L = R$

$\theta = 0^\circ$

For average sand: $r = 0.005$ cm

For very fine sand: $r = 0.001$ cm

For a normal waterflood: $\sigma = 30$ dynes/cm

1.(a) $\Delta P_A = \frac{2\sigma \cos\theta}{r}$

$\Delta P_B = \frac{2\sigma \cos\theta}{R}$

For mobilization

$\Delta P' = (\Delta P_A - \Delta P_B) \geq 2\sigma\cos\theta\left(\frac{1}{r} - \frac{1}{R}\right)$ (can be equal in limiting case)

Avg. Sand, $\Delta P' = 2 \times 30 \times 1\left(\frac{1}{0.005} - \frac{1}{0.025}\right)$

$= 9600$ dy/cm^2

$\frac{\Delta P'}{L} = 3.84 \times 10^5$ dy/cm^3 $= 169.75$ psi/ft

Fine Sand, $\frac{\Delta P'}{L} = 4243.73$ psi/ft.

b) Calculate the pressure gradients generated in a normal waterflood in rock types A and B (with an order of magnitude difference in permeability) using the following assumptions:

Darcy velocity	=	1 ft/day	
Water viscosity	=	1 cp	
Effective permeability to water in rock type A		=	2 darcies
Effective permeability to water in rock type B		=	500 md

Are these pressure gradients sufficient to mobilize the oil blob of part (a)?

(b) $q = 0.001127 \frac{KA}{\mu B}\frac{\Delta P}{L}$. $u = 1$ ft/day $= \frac{1}{5.615}$ stb/day.

$\left(\frac{\Delta P}{L}\right)_A = \frac{1}{5.615} \times \frac{1 \text{cp} \times 1}{2000 \text{md}} \times \frac{1}{0.001127} = 0.079$ psi/ft < 169.75 psi/ft

$\left(\frac{\Delta P}{L}\right)_B = 0.316$ psi/ft < 169.75 psi/ft.

No mobilization.

c) Repeat the calculations of part (b) for an enhanced waterflood in which a surfactant has been added to the injected water so as to reduce the oil-water interfacial tension to 0.01

dyne/cm. Comment on the effectiveness of the enhanced waterflood under normal oilfield flow conditions for rock types A and B.

(c). $\sigma_{new} = 0.01$ dy/cm.

$\left(\frac{\Delta P}{L}\right)_{avg. sand} = 0.057$ psi/ft < 0.079 psi/ft

$\left(\frac{\Delta P}{L}\right)_{fine\ sand} = 1.415$ psi/ft > 0.316 psi/ft

Mobilization in avg. sand but not in fine sand.

d) Compare the capillary numbers for the ordinary waterflood and the enhanced waterflood for rock types A and B. The capillary number is given by

$$N_c = \frac{\mu_w v}{\sigma}$$

where

N_C	=	capillary number
μ_W	=	water viscosity
v	=	darcy velocity
σ	=	water-oil interfacial tension

(d) $u = q/A = 1$ ft/day $= 0.352 \times 10^{-3}$ cm/s

$\mu = 1$ cp $= 0.01$ dy.s/cm^2

$\sigma = 30$ dy/cm (ordinary)

$\sigma = 0.01$ dy/cm (enhanced)

$N_{c,ord.} = 1.173 \times 10^{-7}$

$N_{c,enh.} = 3.52 \times 10^{-4}$

Doesnot depend on rock type.

Question 2: Figure 2 shows an upward migrating oil bubble from a source rock into a reservoir initially fully saturated with water. The migrating bubble has encountered a restriction at a pore throat of radius r_H. In order for migration to continue, the leading end of the bubble (A) must squeeze through the pore throat. Assume that ends A and B of the bubble are hemispherical with B having a radius r_B.

a) Derive the condition necessary for the oil bubble to pass through the restriction and continue with its migration in terms of the following parameters:

Water density = ρ_W

Oil Density = ρ_O

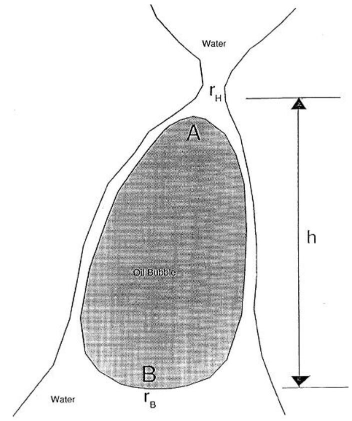
Oil/water interfacial tension = σ

Gravitational acceleration = g

Pore throat radius = r_H

Bubble base radius = r_B

Height of bubble = h



2. (a). $\Delta P_A = P_{oil,A} - P_{w,A} \geq \frac{2\sigma}{r_H}$ for migration (i)

$\Delta P_B = P_{oil,B} - P_{w,B} = \frac{2\sigma}{r_B}$ for equilibrium (ii)

$P_{o,B} = P_{o,A} + \rho_o g h.$ — (iii)

$P_{w,B} = P_{w,A} + \rho_w g h$ — (iv).

Using (iii) & (iv) in (i)

$P_{o,B} - \rho_o g h - P_{w,B} + \rho_w g h \geq \frac{2\sigma}{r_H}$

$\Rightarrow (P_{oB} - P_{wB}) + (\rho_w - \rho_o) g h \geq \frac{2\sigma}{r_H}$

$\Rightarrow (\rho_w - \rho_o) g h \geq 2\sigma\left(\frac{1}{r_H} - \frac{1}{r_B}\right)$ using (ii)

$\Rightarrow h \geq \frac{2\sigma}{(\rho_w - \rho_o) g} \left(\frac{1}{r_H} - \frac{1}{r_B}\right).$

b) Now assume the following quantitative reservoir properties and calculate the height the bubble must achieve in order to continue migration.

$\rho_W = 1.00$ g/cm^3

$\rho_O = 0.8$ g/cm^3

$\sigma = 35.0$ dynes/cm

$g = 981$ cm/s^2

$r_H = 0.0001$ cm

$r_B = 0.01$ cm

1b). $h \geq 3532.1$ cm.

Question 3: Figure 3 shows a microchannel placed horizontally on a chip. Assume that the microchannel has a circular shape with radius of r. Answer the following questions:

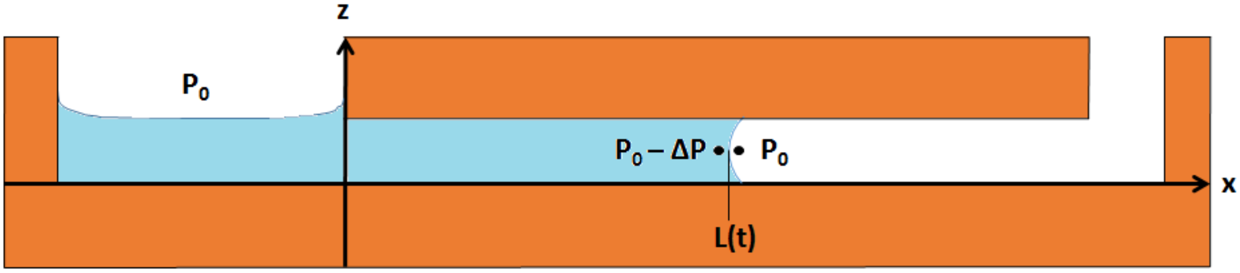


Figure 3: Schematic of a capillary-force micro-scale pump

a) Derive an expression for $L(t)$ as a function of r, t, fluid viscosity (μ), surface tension (σ), and contact angle (θ).

3.(a) $u = \frac{dL}{dt} = \frac{Q}{\pi r^2}$.

$Q = \frac{\pi r^4 \Delta P}{8 \mu L} \Rightarrow \frac{dL}{dt} = \frac{r^2 \Delta P}{8 \mu L}$.

$\Delta P = \frac{2\sigma \cos\theta}{r}$. $\Rightarrow \frac{dL}{dt} = \frac{r \sigma \cos\theta}{4 \mu L}$.

$\Rightarrow L = \sqrt{\frac{r \sigma \cos\theta \, t}{2\mu}}$

b) Derive an expression for the speed of fluid advancement in the horizontal microchannel.

b). $u = \frac{dL}{dt} = \frac{1}{2}\sqrt{\frac{r\sigma\cos\theta}{2\mu t}}$

c) If you are asked to decide about the pump's material type, would you use Platinum or Plexiglas? Which one provides a higher pump efficiency?
HINT: Contact angle in the case of Platinum is smaller than the case of Plexiglas.

(c) $\theta \downarrow \Rightarrow$ lc $\uparrow$

Platinum better Choice.

Question 4: The following data are given for the pore doublet model of Figure 7.73 in your textbook:

v_1 = 1 ft/Day
L = 500 μm
μ = 1 cp
σ = 30 dynes/cm
θ = 0 degrees
r_1 = 50 μm
r_2 = 2.5 μm

Assume the capillaries are cylindrical tubes and that capillary tube 1 (with radius r_1) is larger than capillary tube 2 (with radius r_2). Answer the following questions:

a) Calculate ($P_A - P_B$) across capillary tube 1 in dynes/cm^2. What percentage of this pressure difference is due to viscous pressure drop and what percentage is due to capillary pressure? Are you surprised by the degree of domination of one force over the other at the pore scale?

4. (a). $P_A - P_B = (P_A - P_1) + (P_1 - P_2) + (P_2 - P_B)$

$$= \frac{8\mu v_2 x}{r_2^2} + \left(-\frac{2\sigma\cos\theta}{r_2}\right) + \frac{8\mu v_2 (L-x)}{r_2^2}.$$

$$= \frac{8\mu v_2 L}{r_2^2} - \frac{2\sigma\cos\theta}{r_2}$$

For second tube,

$$P_A - P_B = \frac{8\mu v_1 L}{r_1^2} - \frac{2\sigma\cos\theta}{r_1}.$$

$$= -11999.9 \text{ dy}/\text{cm}^2.$$

$$\frac{\Delta P_{vis}}{|P_A - P_B|} = \frac{0.0564}{11999.9} = 4.7e^{-6}$$

$$\frac{|\Delta P_{cap.}|}{|P_A - P_B|} = \frac{12000}{11999.9} \simeq 1$$

b) Calculate the ratio v_2/v_1. Which capillary tube will be displaced first, the larger tube or the smaller tube in this strongly water-wet medium?

(b). $$-11999.9 = \frac{8\times(0.01)\times\left(v_2 \times \frac{30.48}{24\times3600}\ \text{cm/s}\right)\times 0.05}{(0.00025)^2} - \frac{2\times30\times\cos0^\circ}{(0.00025)}$$

$v_2 = 10098.4$ ft/day.

$v_2/v_1 = 10098.4$ oil displaced in smaller tube first.

Question 4: This is the same data set we used in the class for practice. Please work on this example individually to make sure that you learn the concept we covered in the class.

Download the Excel file “PGE81L_HW_8_Data1” including the capillary pressure curves for the two layers of thickness 100 ft in a conventional reservoir. The water-oil density difference is 8.05 lb/ft3. Assume that the water-oil contact is at the bottom of the lower layer. Calculate the water saturation versus depth measured from the top to the bottom of the reservoir. Present the results of your calculations by plotting a graph of Depth versus Water Saturation, with Depth plotted on the vertical axis and Water Saturation plotted on the horizontal axis.

Solution:

We can calculate the height above FWL via

$$h_{FWL} = \frac{144P_c}{\Delta\rho}$$

Table 1 shows the results. Figures 3 and 4 show capillary pressure vs. water saturation and heights above FWL vs. water saturation.

Sw	Pc (psi)	h above FWL (ft)
1	0.78	13.953
0.4	1.06	18.961
0.385	1.15	20.571
0.369	1.25	22.360
0.354	1.27	22.718
0.338	1.32	23.612
0.323	1.43	25.580
0.308	1.62	28.979
0.285	1.89	33.809
0.269	2.17	38.817
0.254	2.45	43.826
0.238	2.91	52.055
0.235	3	53.665
0.227	3.46	61.893
0.208	4.06	72.626
0.192	6	107.329

Sw	Pc (psi)	h above FWL (ft)
1	1.82	32.557
0.9	1.82	32.557
0.877	1.85	33.093
0.862	1.87	33.451
0.842	1.92	34.345
0.831	1.98	35.419
0.815	2.08	37.207
0.8	2.26	40.427
0.785	2.54	45.436
0.769	2.88	51.518
0.762	3	53.665
0.754	3.23	57.779
0.738	3.83	68.512
0.723	4.62	82.643
0.7	6	107.329
0.677	9	160.994

0.177	9	160.994
0.169	12	214.658
0.168	15	268.323

0.658	12	214.658
0.642	15	268.323

Table 1: h above FWL

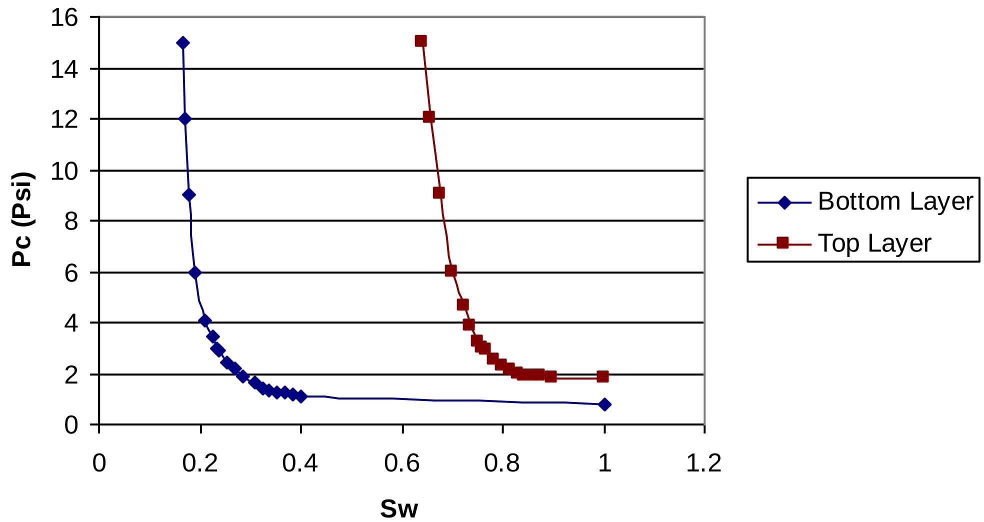


Fig. 3: Capillary pressure vs. water saturation

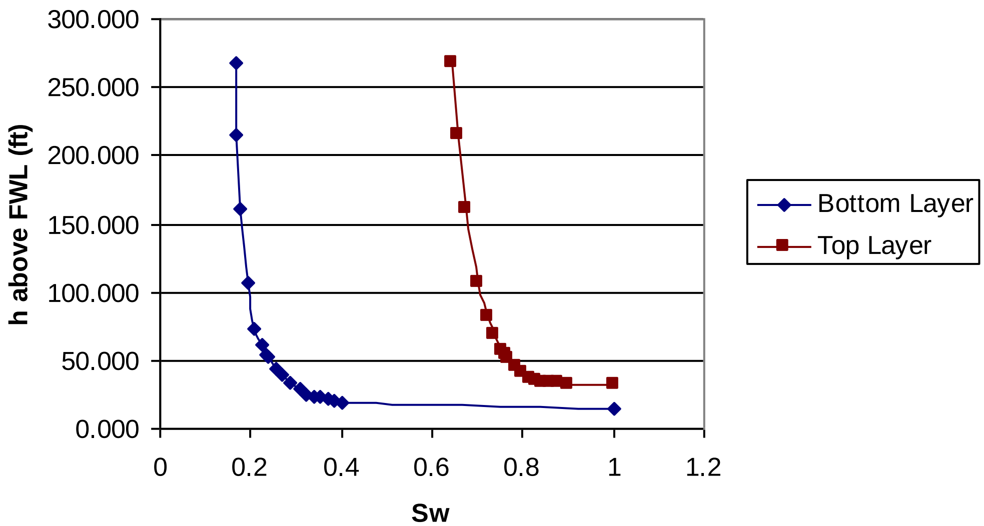


Fig. 4: Height above FWL vs. water saturation

Height above WOC can be calculated via

$$h_{WOC} = h_{FWL} - 13.953$$

Which should be less than 100 for bottom reservoir.

Water saturations and distances from top of reservoir data can be calculated using $h_{top\ of\ reservoir} = 200 - h_{WOC}$

Table 2 summarizes the results. Figure 5 shows the depth versus Water Saturation (S_w).

Sw	**Pc (psi)**	h_{FWL} (ft)	h_{WOC} (ft)	Distance from top of reservoir (ft)
1	0.78	13.953	0.000	200.000
0.4	1.06	18.961	5.009	194.991
0.385	1.15	20.571	6.619	193.381
0.369	1.25	22.360	8.407	191.593
0.354	1.27	22.718	8.765	191.235
0.338	1.32	23.612	9.660	190.340
0.323	1.43	25.580	11.627	188.373
0.308	1.62	28.979	15.026	184.974
0.285	1.89	33.809	19.856	180.144
0.269	2.17	38.817	24.865	175.135
0.254	2.45	43.826	29.873	170.127
0.238	2.91	52.055	38.102	161.898
0.235	3	53.665	39.712	160.288
0.227	3.46	61.893	47.940	152.060
0.208	4.06	72.626	58.673	141.327
0.192	6	107.329	93.376	106.624
0.19		113.953	100	100.000
0.697		113.953	100	100.000
0.677	9	160.994	147.041	52.959
0.658		213.953	200	0

Table 2: Water saturations and distances from top of the reservoir

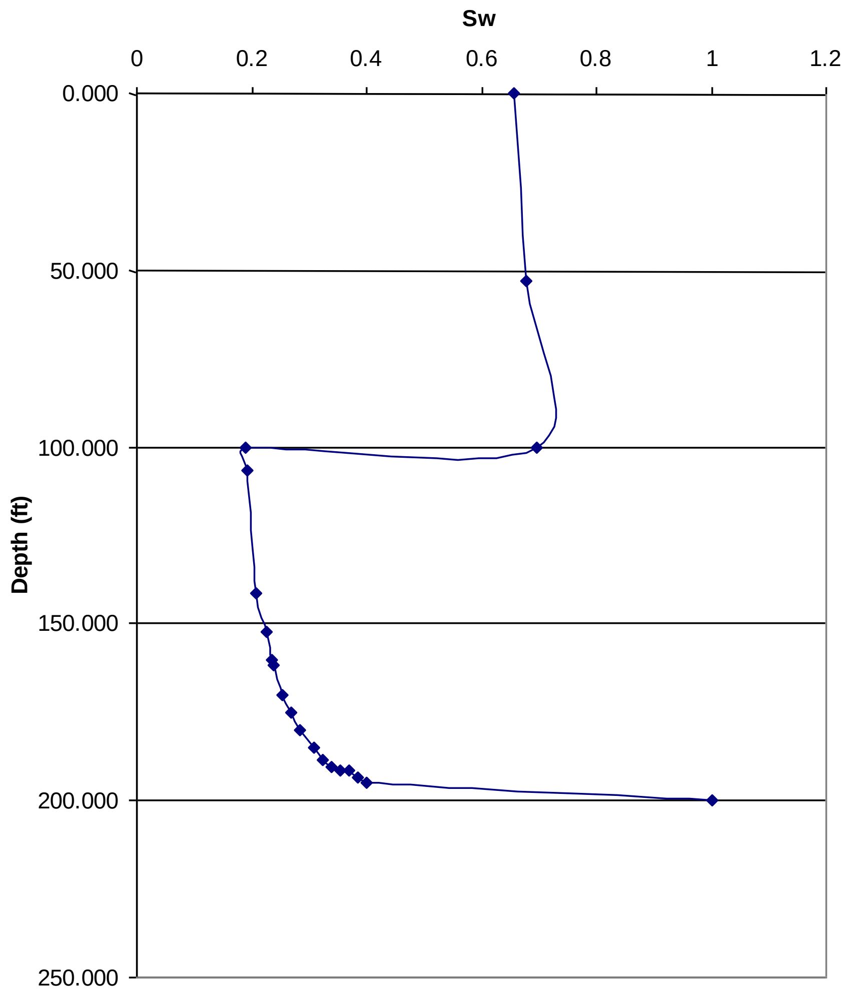


Fig. 5: Graph of Depth versus Water Saturation (S_w)

Question 5: Table 3 summarizes the results of a centrifuge drainage capillary pressure measurement along with other pertinent data about the experiment. Calculate and plot the drainage capillary pressure curve for the sample. Express your capillary pressure in psi.

Table 3: Centrifuge Data for Question 1

d	2.5	cm
L	7.35	cm
k	143	md
ϕ	19.0	%
V_p	7.04	cc
ρ_w	1.036	g/cc
ρ_o	0.822	g/cc
σ_{ow}	40	dynes/cm
θ	0	
r_1	8.25	cm
r_2	15.6	cm

RPM	**V_w (cc)**
520	0.0
800	1
860	1.5
920	2
1000	2.5
1100	2.9
1200	3.2
1400	3.6
1620	3.9
1800	4
2100	4.2

Solution:

The first step is to estimate capillary pressure at the inlet face (P_{c1}):

$$P_c = \frac{\Delta\rho\omega^2}{2}\left(r_2^2 - r_1^2\right)$$

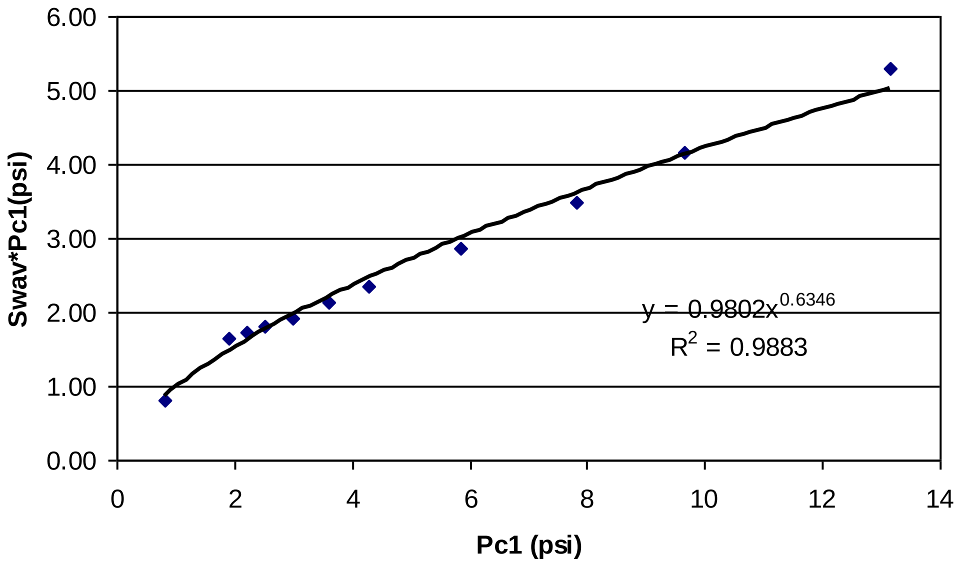
The next step is to calculate the average water saturation S_{wav} :

$$S_{wav} = \frac{V_p - Vw}{V_p}$$

We can estimate the capillary pressure curve by using the following equations:

$$S_{w1} = \frac{d(P_{c1}S_{wav})}{d(P_{c1})} = S_{wav} + P_{c1}\frac{d(S_{wav})}{d(P_{c1})}$$

In order to calculate the derivative $\frac{d(P_{c1}S_{wav})}{d(P_{c1})}$, the exponential curve has been fitted to the initial data and then the estimated model is used for calculations.

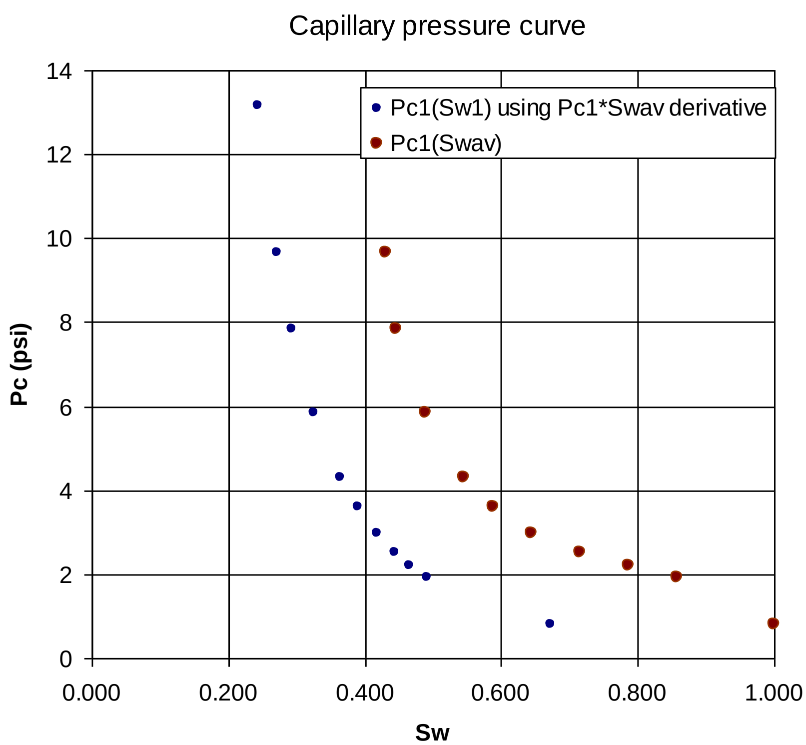


$$S_{w1} = \frac{d(P_{c1}S_{wav})}{d(P_{c1})} = 0.622035(\mathrm{P}_{c1})^{-0.3654}$$

The results of the calculations are presented in the following table.

RPM	Vw (cc)	ω(rad/sec)	Pc1(Pa)	Pc1(psi)	Swav(fraction)	Pc1*Swav (psi)	Sw1
520	0	54.45	5.56E+03	0.81	1.00	0.81	0.673
800	1	83.78	1.32E+04	1.91	0.86	1.64	0.491
860	1.5	90.06	1.52E+04	2.21	0.79	1.74	0.466
920	2	96.34	1.74E+04	2.53	0.72	1.81	0.443

1000	2.5	104.72	2.06E+04	2.98	0.64	1.92	0.417
1100	2.9	115.19	2.49E+04	3.61	0.59	2.12	0.389
1200	3.2	125.66	2.96E+04	4.30	0.55	2.34	0.365
1400	3.6	146.61	4.03E+04	5.85	0.49	2.86	0.326
1620	3.9	169.65	5.40E+04	7.83	0.45	3.49	0.293
1800	4	188.50	6.66E+04	9.67	0.43	4.17	0.272
2100	4.2	219.91	9.07E+04	13.16	0.40	5.31	0.243



Question 6: Download the Excel file "PGE81L_HW_08_Data2" including mercury injection capillary pressure data obtained on a core sample. The core sample has a permeability of 3.86 md and a porosity of 13.2%. Answer the following questions.

a) Plot S_w and S_{nw} versus pore throat radius on the same graph.

Solution:

S_w is calculated using $S_w = 1 - S_{nw}$.

Pore throat radius is calculated via

$$R[cm] = \frac{2\sigma[dynes/cm]\cos\theta}{P_c\left[dunes/cm^2\right]}$$

S_w and S_{nw} versus pore throat radius are presented on the same graph in Fig 6.

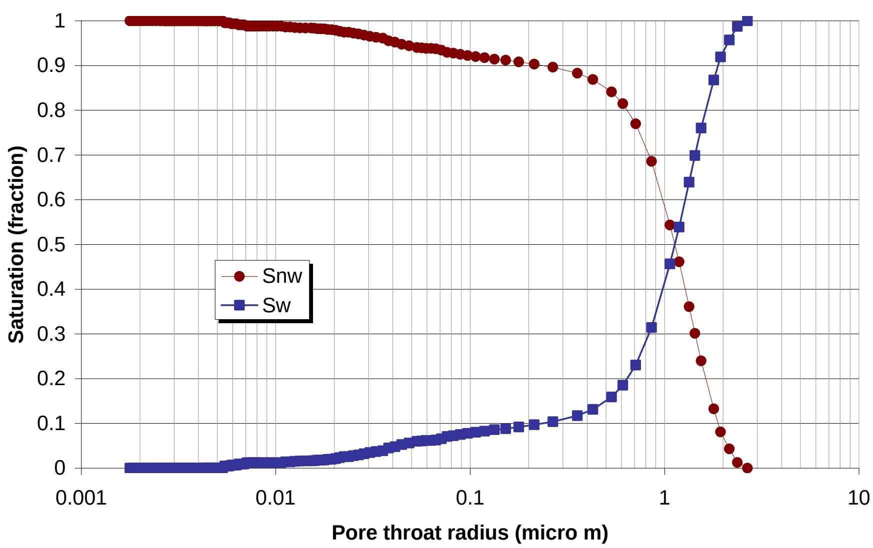


Fig. 6: S_w and S_{nw} versus pore throat radius

b) Calculate the probability density function for the pore volume distribution and plot it on the same graph of S_w and S_{nw} versus pore throat radius. Use the 5-point central difference formula for calculating all derivatives.

Solution:

Probability density function for the pore volume distribution can be calculated using following equation:

$$f(R) = -\frac{dS_{nw}}{dR} = \frac{dS_w}{dR}$$

In which ________________

For calculating all derivatives, we can use the 5-point central difference formula, as following:

$$\left(\frac{dy}{dx}\right)_{x_3} = \frac{|m_4 - m_3| m_2 + |m_2 - m_1| m_3}{|m_4 - m_3| + |m_2 - m_1|}$$

We assume that for mercury, $\sigma = 480 dynes/cm$ and $\theta = 140^{\circ}$

Figure 7 shows the plot of probability density function for the pore volume distribution on the same graph of S_w and S_{nw} versus pore throat radius.

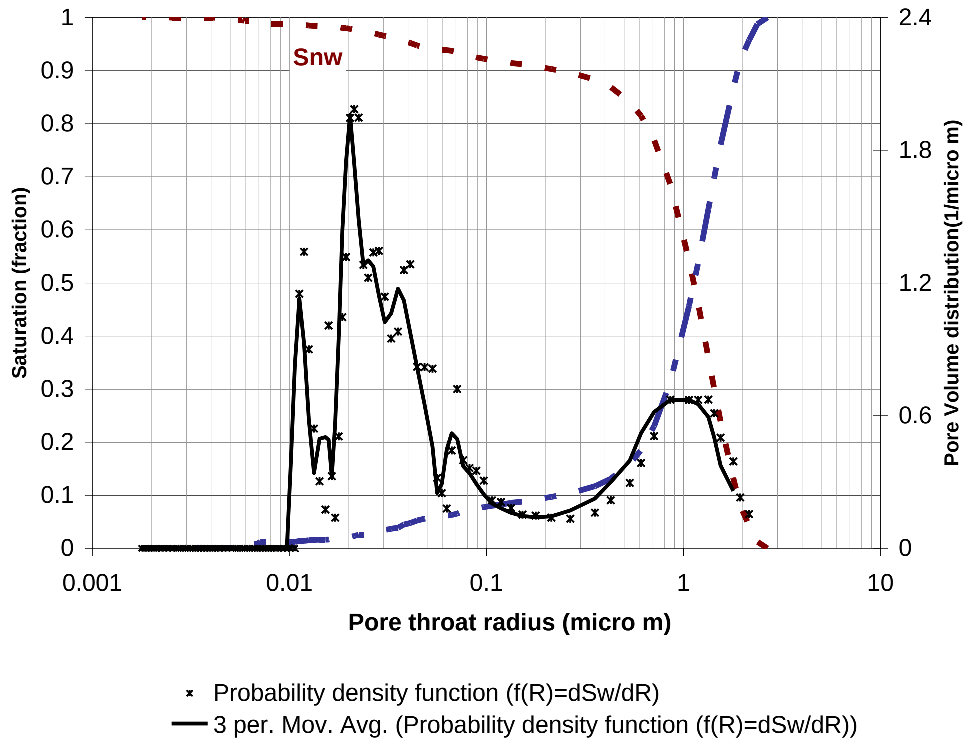


Fig. 7: plot of probability density function for the pore volume distribution on the same graph of S_w and S_{nw} versus pore throat radius

c) Calculate the probability density function for the pore radius distribution assuming a bundle of capillary tubes model for the porous medium. Compare this distribution with the pore volume distribution by plotting them on the same graph versus pore throat size. Comment on the pore structure of this sample.

Solution:

We can calculate probability density function via:

$$\delta(R) = \frac{\bar{R}^2}{R^2}\frac{dS_w}{dR}$$

We calculated $\frac{dS_w}{dR}$ and $\frac{1}{R^2}$ in part (b). Now we should calculate $\bar{R}^2$.

For calculating $\bar{R}^2$ we know that $\int_0^\infty \delta(R)dR = 1$.

$$\int_0^\infty \frac{\bar{R}^2}{R^2}\frac{dS_w}{dR}dR = 1 \qquad \rightarrow \qquad \bar{R}^2 = \frac{1}{\int_0^\infty \frac{1}{R^2}\frac{dS_w}{dR}dR}$$

The integral $\int_0^\infty \frac{1}{R^2}\frac{dS_w}{dR}dR$ can be calculated using numerical methods (trapezoidal method).

Figure 8 shows the plot of probability density function for the pore radius distribution versus pore throat size.

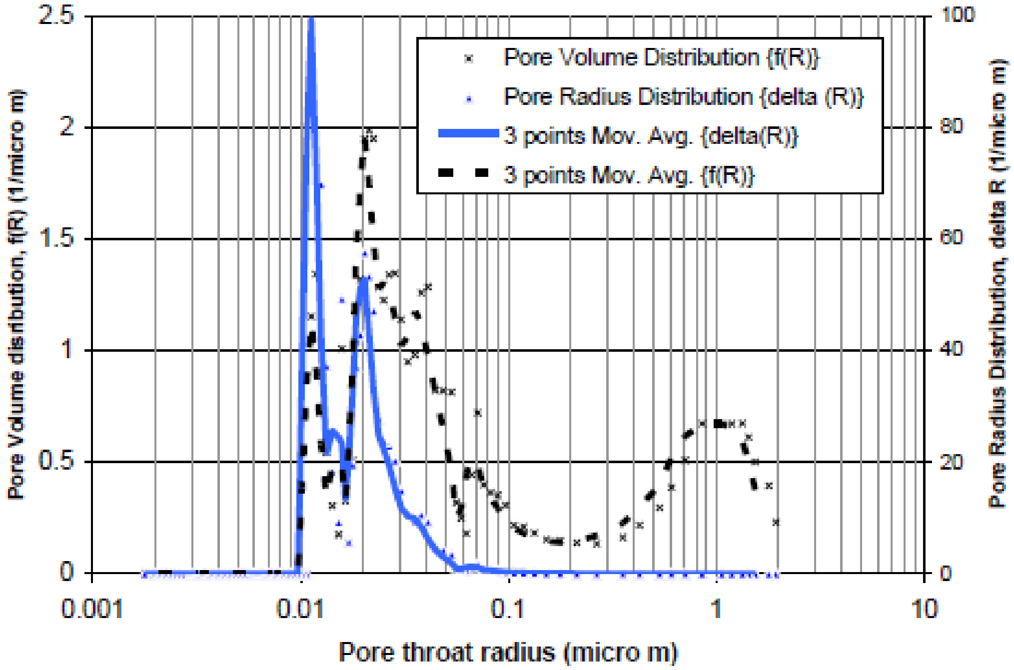


Fig. 8: Plot of probability density function for the pore radius distribution on the same graph of probability density function for the pore volume distribution versus pore throat radius

d) Estimate the absolute permeability of the sample from the mercury injection data. How does it compare with the lab measured permeability?

Solution:

For calculating permeability, we have (Assuming and "):

$$k = 1.441\times10^{6} F_1 \phi \int_0^1 \frac{dS_w}{P_c^2}$$

We can calculate $\int_0^1 \frac{dS_w}{P_c^2}$ numerically using trapezoidal method. Then by assuming $F_1 = 0.216$ we have:

$$k = 1.441\times10^{6}(0.216)(0.132)(0.000158973)$$

$$\boxed{k \cong 6.532\text{md}}$$

The result is approximately twice the lab measured permeability.