Advanced Petrophysics
PGE 381L, Fall 2023
Unique Number: 20215

Homework Assignment No. 4

September 28, 2023
Due on Tuesday, September 10, 2023, before 11:00 PM

Name: ____

UT EID: ____

Objectives:

a) To practice application of Darcy's law

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Note: Please scan your homework assignment and upload it as one pdf file on the Canvas website before the deadline. Please name your homework document as follows:

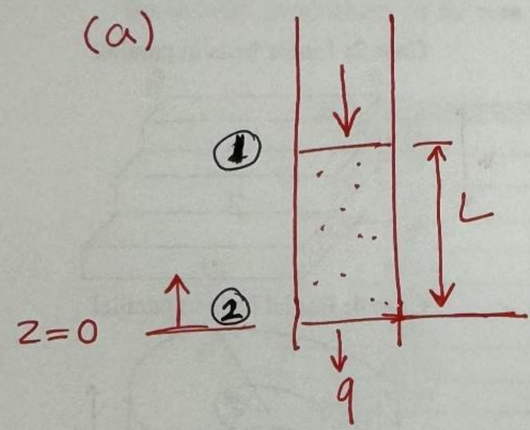
PGE381L_2023_Fall _HW04_lastname_name.pdf

Example: PGE381L_2023_Fall_HW04_Heidari_Zoya.pdf

Optional Questions:

You do <u>not</u> need to submit solutions to the following questions. You can solve the following questions for the purpose of practicing. They will <u>not</u> be graded.

Question 5: Solve question 3.10 in the "Advanced Petrophysics" textbook.

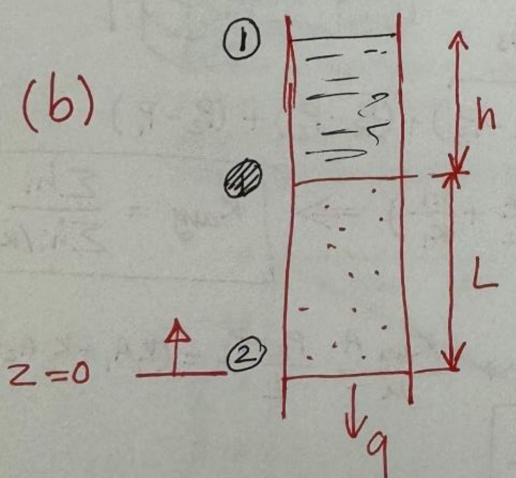


$$q = -KA\frac{dh}{ds} = -KA\frac{h_2 - h_1}{\Delta s}$$

$$\left.\begin{matrix} h_1 = L \\ h_2 = 0 \end{matrix}\right\} \rightarrow q = KA\frac{L-0}{L}$$

$$\Rightarrow q = KA = \frac{k\rho g}{1.0133\times10^{6}\mu}A$$

$$\Rightarrow \boxed{k = \frac{1.0133\times10^{6}\mu q}{\rho g A}}$$



$$h_1 = L + h$$

$$h_2 = 0$$

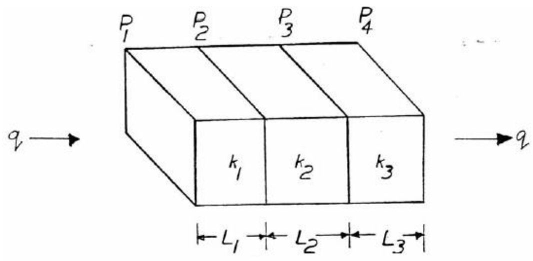
$$\Rightarrow q = KA\frac{L+h}{L}$$

$$\Rightarrow q = \frac{k\rho g}{1.0133\times10^{6}\mu}A\left(\frac{L+h}{L}\right)$$

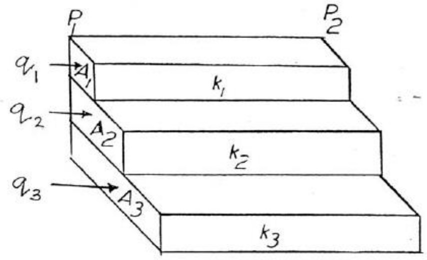
$$\Rightarrow \boxed{k = \frac{1.0133\times10^{6}\mu q}{\rho g A\left(\frac{L+h}{L}\right)}}$$

Question 6: Use Darcy's law to estimate average permeability for the following laminated structures.

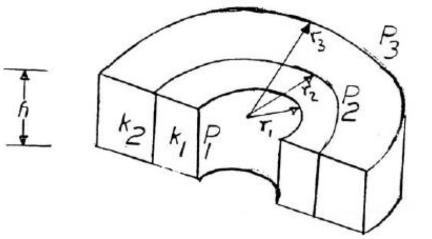
Case 1: Linear beds in series



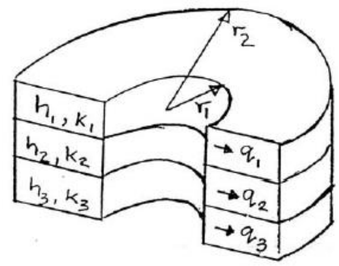
Case 2: Linear beds in parallel



Case 3: Radial beds in series



Case 4: Radial beds in parallel



Case 1: $(P_4-P_1) = (P_4-P_3)+(P_3-P_2)+(P_2-P_1)$

$$\Rightarrow \frac{\mu q h}{K_{avg} A} = \frac{\mu q}{A}\left(\frac{h_3}{k_3}+\frac{h_2}{k_2}+\frac{h_1}{k_1}\right) \Longrightarrow \boxed{K_{avg} = \frac{\sum h_i}{\sum h_i/k_i}}$$

Case 2: $q_T = q_1+q_2+q_3 \Rightarrow \frac{K_{avg}A}{\mu}\frac{P_2-P_1}{L} = (k_1A_1+k_2A_2+k_3A_3)\frac{\Delta P}{L\mu}$

$$\Longrightarrow \boxed{K_{avg} = \frac{\sum k_i A_i}{\sum A_i}}$$

Case 3: $(P_3-P_1) = (P_3-P_2)+(P_2-P_1)$

$$\Rightarrow \frac{q\ln\left(\frac{r_3}{r_1}\right)}{2\pi h K_{avg}}\mu = \frac{q\ln\left(\frac{r_3}{r_2}\right)}{k_2 2\pi h}\mu + \frac{q\ln\left(\frac{r_2}{r_1}\right)}{k_1 2\pi h}\mu$$

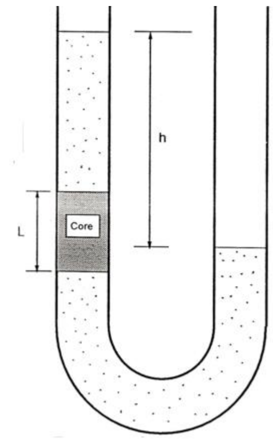
$$\Rightarrow \boxed{K_{avg} = \frac{\ln(r_e/r_w)}{\sum_{i=1}^{n}\frac{\ln(r_i/r_{i-1})}{k_i}}}$$

Case 4: $\frac{k_{avg}}{\mu}2\pi h\Delta P/\ln\left(\frac{r_e}{r_w}\right) = \frac{k_1}{\mu}2\pi h_1\Delta P/\ln\left(\frac{r_e}{r_w}\right) + \frac{k_2}{\mu}2\pi h_2\Delta P/\ldots + \frac{k_3}{\mu}2\pi h_3\Delta P/\ldots$

$$\Rightarrow \boxed{K_{avg} = \frac{\sum k_i h_i}{\sum h_i}}$$

Question 7: Figure 3 shows an apparatus for determining the permeability of a core using a nonreactive liquid. (Question 3.13 in your textbook)

a) Derive the differential equation for the instantaneous height h in terms of the pertinent system variables and parameters. Use the following symbols in your derivation:



Cross sectional area of the core and the U tube	=	A
Length of core	=	L
Core permeability	=	k
Liquid density	=	ρ
Liquid viscosity	=	μ
Gravitational acceleration	=	g
Time	=	t
Height at t = 0	=	h_o

b) Solve the differential equation you derived in part (a) analytically.

c) The following data were obtained in the experiment using brine:

Time (s)	h (cm)
0	100.0
1000	67.0
3000	30.0
5000	13.5

Additional data are as follows:

Length of core	=	10 cm
Core and U-tube diameter	=	2 cm
Brine density	=	1.02 g/cm3
Brine viscosity	=	1 cp
Gravitational acceleration	=	981 cm/s2

Based on the theory you have derived in parts (a) and (b), determine the permeability of the core and state its units.

(a) $V_s = q/A = -K \dfrac{dh}{ds}$.

(b) $K = \dfrac{k \rho g}{1.0133 \times 10^6 \, \mu}$

$dh = (d) - (h + d) = -h$.

$ds = L$.

$\therefore q = \dfrac{k \rho g}{1.0133 \times 10^6 \, \mu} \dfrac{h}{L} . A$. —— (1)

and. $q = -A . (dh_1/dt)$ —— (2)

where $\dfrac{dh_1}{dt} = \dfrac{dh}{dt} - \dfrac{dh_2}{dt}$.

As we have similar surfaces, area at both sides we can write

$\dfrac{dh_1}{dt} = \dfrac{dh_2}{dt} \longrightarrow \left(\dfrac{dh_1}{dt}\right) = \dfrac{1}{2}\left(\dfrac{dh}{dt}\right)$

$\therefore q = -\dfrac{1}{2} A \left(\dfrac{dh}{dt}\right)$ —— (3)

Now. equating (1) & (3)

$\dfrac{1}{2} A \left(\dfrac{dh}{dt}\right) = - \dfrac{k \rho g}{1.0133 \times 10^6 \, \mu} \dfrac{h}{L} . A$.

$\Rightarrow \left(\dfrac{dh}{dt}\right) = - \dfrac{2 k \rho g}{1.0133 \times 10^6 \, \mu} . \left(\dfrac{h}{L}\right)$

(b) $$\left(\frac{dh}{dt}\right) = -\frac{2K\rho g}{1.0133\times10^{6}\mu}\cdot\left(\frac{h}{L}\right).$$

Assume at $t = t_0$ we have $h = h_0$

$$\int_{h_0}^{h}\left(\frac{dh}{h}\right) = -\frac{2K\rho g}{1.0133\times10^{6}\mu L}\int_{t_0}^{t} dt$$

$$\Rightarrow \ln h\Big|_{h_0}^{h} = -\frac{2K\rho g}{1.0133\times10^{6}\mu L}(t-t_0)$$

(c). At $t_0 = 0$ we have $h_0 = 100$ cm

$$\ln\left(\frac{h}{100}\right) = -\frac{2K\rho g}{1.0133\times10^{6}\mu L}\,t$$

$$\frac{2K\rho g}{1.0133\times10^{6}\mu L} = 0.0004$$

$$\Rightarrow \frac{2K(1.02)(981)}{1.0133\times10^{6}(1)(10)} = 0.0004$$

$$\Rightarrow K = 2.025\ \text{D} \quad \text{or} \quad 2025\ \text{mD}$$