CP7

Residual

$$R = -uu_x + \mu u_{xx} - uu_y + \mu u_{yy} - uu_z + \mu u_{zz} - u_t$$

reorgonize

$$R = (-u)\underbrace{\left(u_x + u_y + u_z\right)}_{1} + \underbrace{\mu\left(u_{xx} + u_{yy} + u_{zz}\right)}_{2} - \underbrace{u_t}_{3}$$

1 $$(-u)\left(\frac{u - u_{i-1}}{\Delta x} + \frac{u - u_{j-1}}{\Delta y} + \frac{u - u_{k-1}}{\Delta z}\right)$$

$$\boxed{u^2\left(-\frac{1}{\Delta x} - \frac{1}{\Delta y} - \frac{1}{\Delta z}\right) + u\left(\frac{u_{i-1}}{\Delta x} + \frac{u_{j-1}}{\Delta y} + \frac{u_{k-1}}{\Delta z}\right)}$$

2 $$\mu\frac{u_{i-1} - 2u + u_{i+1}}{\Delta x^2} + \mu\frac{u_{j-1} - 2u + u_{j+1}}{\Delta y^2} + \mu\frac{u_{k-1} - 2u + u_{k+1}}{\Delta z^2}$$

$$\boxed{\mu(-2u)\left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} + \frac{1}{\Delta z^2}\right) + \frac{\mu}{\Delta x^2}\left(u_{i-1} + u_{i+1}\right) + \frac{\mu}{\Delta y^2}\left(u_{j-1} + u_{j+1}\right) + \frac{\mu}{\Delta z^2}\left(u_{k-1} + u_{k+1}\right)}$$

3 $$-\frac{u^{n+1} - u^n}{\Delta t}$$

Calculate the jacobian:

$$\frac{\partial R}{\partial u} = (-2u)\left(\frac{1}{\Delta x}+\frac{1}{\Delta y}+\frac{1}{\Delta z}\right)+\left(\frac{u_{i-1}}{\Delta x}+\frac{u_{j-1}}{\Delta y}+\frac{u_{k-1}}{\Delta z}\right)+$$

$$+(-2\mu)\left(\frac{1}{\Delta x^2}+\frac{1}{\Delta y^2}+\frac{1}{\Delta z^2}\right)-\frac{1}{\Delta t}$$

$$\frac{\partial R}{\partial u_{i-1}} = \frac{u}{\Delta x}+\frac{\mu}{\Delta x^2}$$

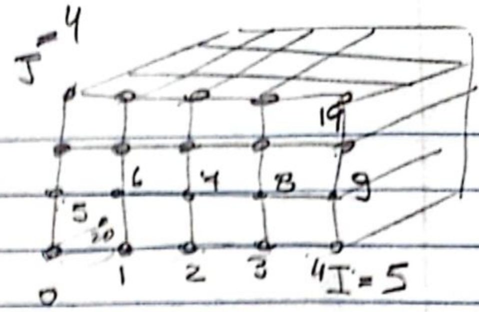
$$\frac{\partial R}{\partial u_{i+1}} = \frac{\mu}{\Delta x^2}$$

$$\frac{\partial R}{\partial u_{j-1}} = \frac{u}{\Delta y}+\frac{\mu}{\Delta y^2}$$

$$\frac{\partial R}{\partial u_{j+1}} = \frac{\mu}{\Delta y^2}$$

$$\frac{\partial R}{\partial u_{k-1}} = \frac{u}{\Delta z}+\frac{\mu}{\Delta z^2}$$

$$\frac{\partial R}{\partial u_{k+1}} = \frac{\mu}{\Delta z^2}$$



Jacobian stencil

$$\boxed{\xi = i + Ij + (I \times J)k}$$

$20 = 0 + 0 + (4 \times 5 \times 1)$.

$$\boxed{\text{def } \xi(i,j,k): \text{return } i + Ij + (IJ)k}$$

—x—

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for n in (1, NT) :
    U_K = U_N (n-1)

    while (1) :
        for i = 0 : N_I
        for j = 0 : N_J
        for k = 0 : N_K
            ξ = ξ(i,j,k)

            J[ξ, ξ] = δR/δu
            J[ξ, ξP00] = δR/δu_{i+1}
            J[ξ, ξN00] = δR/δu_{i-1}
            J[ξ, ξ0P0] = δR/δu_{j+1}
            ⋮
            F[ξ] += f(U_K, U_{N-1}, ξ)

        dU_K ← solve(J, -f)
        U_K += dU_K
        stop?
    U_N = U_K .
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