Advanced Petrophysics PGE 381L, Fall 2023
Unique Number: 20215

Homework Assignment No. 6

October 26, 2023
Due on Thursday, November 9, 2023, before 11:00 PM

Name: ____ RENATO ESPIRITO BASSO POLI ____

UT EID: ____ rep2656 ________________________________

Objectives:

- **a)** To practice using Coefficient of Variation, Dykstra-Parsons Coefficient of Variation, and Lorenz Coefficient for heterogeneity quantification
- **b)** To practice calculation of covariance
- **c)** To practice calculation and interpretation of directional variograms
- **d)** To practice interpretation of covariance function and variogram
- **e)** To practice kriging

Note: Please scan your homework assignment and upload it as one pdf file on the Canvas website before the deadline. Please name your homework document as follows:

PGE381L_2023_Fall _HW06_lastname_name.pdf

Example: PGE381L_2023_Fall_HW06_Heidari_Zoya.pdf

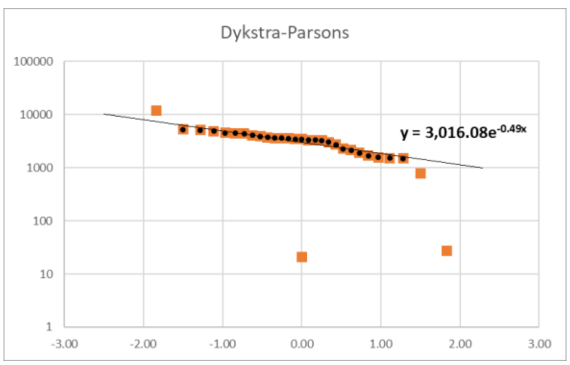
Question 1: Download the Excel file "PGE81L_HW_06_Data" including measurements of porosity, permeability, and layer thickness. Please answer the following questions. You can attach an extra page to your solution document and include your plots in that page with appropriate citation in the allotted space after each question.

a) Calculate the coefficient of variation. Show the details of your calculation procedure.

$$C_v = \frac{\sigma(k/\phi)}{\mu(k/\phi)} = 0.673$$

b) Calculate the Dykstra-Parsons coefficient of variation using the two approaches that we practiced in the class. Plot Permeability vs. normal quantiles and calculate the equation of the line that passes through the data. Show the details of your calculation procedure for both approaches.

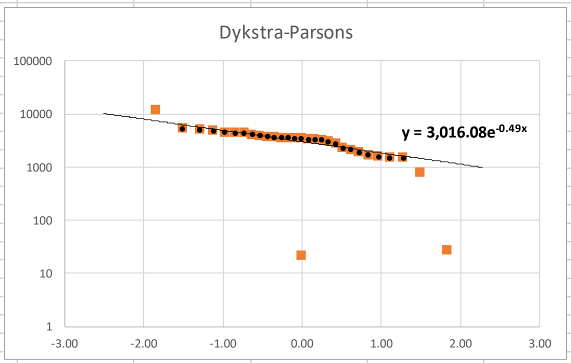
Hint: do you see outliers? If there are outliers in the data, separate them and then continue with your calculations.



METHOD 1 - DIRECT READING		
k50	3482	
k84.1	1549.88	
V_DP=	**0.55489**	

METHOD 2 - NORMAL QUARTILES		
Normal quartile	**K**	
0	3016.1	k_50
1	1851.4	k_84.1
V_DP=	**0.38615**	

	Layer	h (m)		k (md)	(x_i-x_mean)^2	Count	%>K	inv.norm		phi	(x_i-x_mean)^2		k/phi	(x_i-x_mean)^2
	8	0.99		11970	7.52E+07	1	3.3%	-1.83		0.28	2.92E-04		42737	1.02E+09
	16	3.98		5244	3.79E+06	2	6.7%	-1.50		0.28	1.79E-04		18477	5.79E+07
	22	5.45		5075	3.16E+06	3	10.0%	-1.28		0.35	3.08E-03		14391	1.24E+07
	9	5.77		4851	2.41E+06	4	13.3%	-1.11		0.32	3.37E-04		15374	2.03E+07
	28	2.15		4551	1.57E+06	5	16.7%	-0.97		0.33	8.02E-04		13982	9.70E+06
	20	3.92		4441	1.31E+06	6	20.0%	-0.84		0.37	4.64E-03		12157	1.67E+06
	2	2.70		4380	1.17E+06	7	23.3%	-0.73		0.31	1.31E-04		14192	1.11E+07
	5	4.73		4065	5.89E+05	8	26.7%	-0.62		0.34	1.52E-03		12090	1.50E+06
	25	5.20		3876	3.34E+05	9	30.0%	-0.52		0.31	9.34E-05		12632	3.12E+06
	11	3.42		3703	1.64E+05	10	33.3%	-0.43		0.31	1.09E-04		12037	1.37E+06
	7	2.13		3636	1.14E+05	11	36.7%	-0.34		0.30	5.51E-06		12139	1.62E+06
	15	0.82		3600	9.14E+04	12	40.0%	-0.25		0.32	3.89E-04		11360	2.43E+05
	24	2.08		3583	8.14E+04	13	43.3%	-0.17		0.37	4.92E-03		9754	1.24E+06
	17	4.90		3483	3.43E+04	14	46.7%	-0.08		0.29	3.67E-05		11964	1.20E+06
	10	7.30		3482	3.40E+04	15	50.0%	0.00		0.32	7.57E-04		10724	2.03E+04
	18	2.83		3300	5.14E+00	16	53.3%	0.08		0.31	7.70E-05		10786	6.55E+03
	26	1.63		3300	5.14E+00	17	56.7%	0.17		0.36	4.03E-03		9151	2.94E+06
	13	4.60		3276	4.72E+02	18	60.0%	0.25		0.34	1.97E-03		9593	1.62E+06
	14	4.78		3000	8.86E+04	19	63.3%	0.34		0.31	2.04E-04		9632	1.52E+06
	21	1.37		2730	3.22E+05	20	66.7%	0.43		0.28	3.18E-04		9773	1.20E+06
	12	3.05		2277	1.04E+06	21	70.0%	0.52		0.24	3.45E-03		9549	1.74E+06
	4	3.29		2127	1.37E+06	22	73.3%	0.62		0.29	2.06E-05		7268	1.29E+07
	23	3.01		1893	1.97E+06	23	76.7%	0.73		0.28	3.64E-04		6807	1.65E+07
	6	3.42		1686	2.60E+06	24	80.0%	0.84		0.29	7.31E-05		5841	2.53E+07
	30	2.06		1560	3.02E+06	25	83.3%	0.97		0.18	1.34E-02		8605	5.12E+06
	3	2.41		1516	3.17E+06	26	86.7%	1.11		0.27	7.14E-04		5605	2.77E+07
	1	4.31		1500	3.23E+06	27	90.0%	1.28		0.25	2.70E-03		6116	2.26E+07
	27	3.38		779	6.34E+06	28	93.3%	1.50		0.25	1.85E-03		3066	6.09E+07
	29	1.27		27	1.07E+07	29	96.7%	1.83		0.28	1.52E-04		95	1.16E+08
	19	1.54		21	1.07E+07	30	100.0%	#NÚM!		0.20	9.26E-03		104	1.16E+08
SUM				**98932**	**134659044**					**9**	**5.59E-02**		**326000**	**1.55E+09**
N				30						30			30	
MEAN				3298						0.297			10867	
VARIANCE				4.64E+06						1.93E-03			5.35E+07	
STDEV				2154.9						0.0			7312.9	
Cv				0.653						0.148			0.673	



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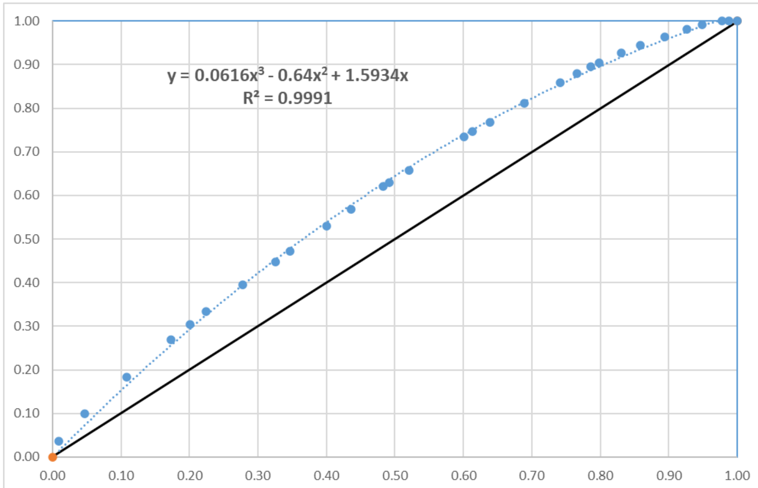
METHOD 2 - NORMAL QUARTILES		
Normal quartile	K	
0	3016.1	k_50
1	1851.4	k_84.1
V_DP=	**0.38615**	

c) Calculate the Lorenz coefficient. Show the Lorenz plot, the table of your calculations (similar to what we had in the class), and your calculation procedure to estimate the Lorenz coefficient.



Lorenz coeff	0.10

Layer	h (m)	k (md)	phi	k/phi	k.h	sum(kihi)	phi.h	sum(phii.	Fraction of total storage capacity	Fractoin of total flow capacity
8	0.99	11970	0.28	42737	11880	11880	0.28	0.28	0.01	0.04
16	3.98	5244	0.28	18477	20876	32756	1.13	1.41	0.05	0.10
9	5.77	4851	0.32	15374	27973	60729	1.82	3.23	0.11	0.18
22	5.45	5075	0.35	14391	27683	88413	1.92	5.15	0.17	0.27
2	2.70	4380	0.31	14192	11818	100230	0.83	5.98	0.20	0.30
28	2.15	4551	0.33	13982	9794	110024	0.70	6.68	0.22	0.33
25	5.20	3876	0.31	12632	20169	130193	1.60	8.28	0.28	0.40
20	3.92	4441	0.37	12157	17418	147611	1.43	9.71	0.33	0.45
7	2.13	3636	0.30	12139	7732	155343	0.64	10.35	0.35	0.47
5	4.73	4065	0.34	12090	19213	174556	1.59	11.94	0.40	0.53
11	3.42	3703	0.31	12037	12660	187216	1.05	12.99	0.44	0.57
17	4.90	3483	0.29	11964	17065	204280	1.43	14.42	0.48	0.62
15	0.82	3600	0.32	11360	2936	207217	0.26	14.68	0.49	0.63
18	2.83	3300	0.31	10786	9326	216543	0.86	15.54	0.52	0.66
10	7.30	3482	0.32	10724	25434	241976	2.37	17.91	0.60	0.73
21	1.37	2730	0.28	9773	3738	245714	0.38	18.29	0.61	0.75
24	2.08	3583	0.37	9754	7445	253159	0.76	19.06	0.64	0.77
14	4.78	3000	0.31	9632	14328	267487	1.49	20.55	0.69	0.81
13	4.60	3276	0.34	9593	15074	282561	1.57	22.12	0.74	0.86
12	3.05	2277	0.24	9549	6944	289505	0.73	22.84	0.77	0.88
26	1.63	3300	0.36	9151	5375	294879	0.59	23.43	0.79	0.90
30	2.06	1560	0.18	8605	3221	298100	0.37	23.81	0.80	0.91
4	3.29	2127	0.29	7268	7008	305108	0.96	24.77	0.83	0.93
23	3.01	1893	0.28	6807	5702	310810	0.84	25.61	0.86	0.94
1	4.31	1500	0.25	6116	6469	317278	1.06	26.67	0.89	0.96
6	3.42	1686	0.29	5841	5760	323038	0.99	27.65	0.93	0.98
3	2.41	1516	0.27	5605	3650	326688	0.65	28.30	0.95	0.99
27	3.38	779	0.25	3066	2630	329318	0.86	29.16	0.98	1.00
19	1.54	21	0.20	104	32	329351	0.31	29.47	0.99	1.00
29	1.27	27	0.28	95	34	329385	0.36	29.83	1.00	1.00

d) What is your opinion about the level of heterogeneity in this formation?

Aside from outliers, the heterogeneity coefficients and plots show a relatively homogeneous formation.

Question 2: Table 1 summarizes two-dimensional distribution of irreducible water saturation in a formation. Please answer the following questions. You can attach an extra page to your solution document and include your plots in that page with appropriate citation in the allotted space after each question. (You can use excel or any programming language of your choice for your calculations in this question)

Table1

		Porosity in West-East Direction (%)						
	Location (ft)	**0**	**10**	**20**	**30**	**40**	**50**	**60**
Porosity in North-South Direction (%)	**0**	20	15	19	21	26	18	19
	10	17	16	23	21	29	25	15
	20	22	17	24	17	26	32	30
	30	20	15	27	23	27	29	20
	40	19	25	32	28	22	24	28

a) Calculate the semivariance values on the experimental variogram in the west-east direction at lag distances 10 ft, 20 ft, and 30 ft. Show your calculations, fill out the following table, and plot the variogram.

Lag distance (ft)	10	20	30
γ_{W-E}	17.6	24.6	25.28

$$\gamma(h) = \frac{1}{2N_h}\sum_1^{N_h}[\phi(x_i) - \phi(x_i + h)]^2$$

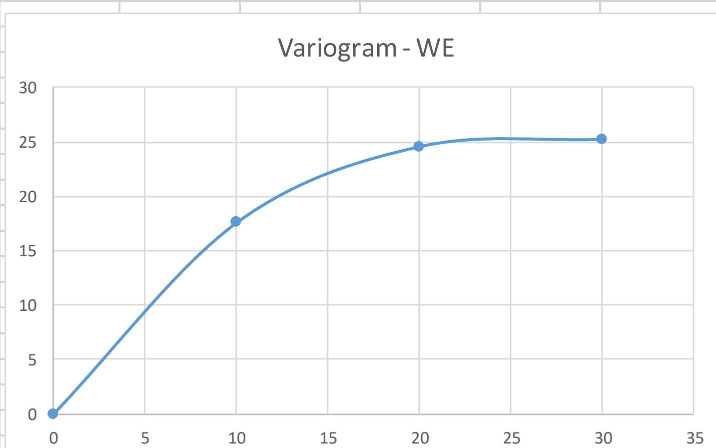
		Porosity in West-East Direction (%)						
	Location (ft)	**0**	**10**	**20**	**30**	**40**	**50**	**60**
Porosity in North-South Direction (%)	**0**	20	15	19	21	26	18	19
	10	17	16	23	21	29	25	15
	20	22	17	24	17	26	32	30
	30	20	15	27	23	27	29	20
	40	19	25	32	28	22	24	28

WEST-EAST 0 0

H=	10	N=	30	\gamma=	17.6
25	16	4	25	64	1
1	49	4	64	16	100
25	49	49	81	36	4
25	144	16	16	4	81
36	49	16	36	4	16

H=	20	N=	25	\gamma=	24.6
1	36	49	9	49	
36	25	36	16	196	
4	0	4	225	16	
49	64	0	36	49	
169	9	100	16	36	

H=	30	N=	20	\gamma=	25.275
1	121	1	4		
16	169	4	36		
25	81	64	169		
9	144	4	9		
81	9	64	0		



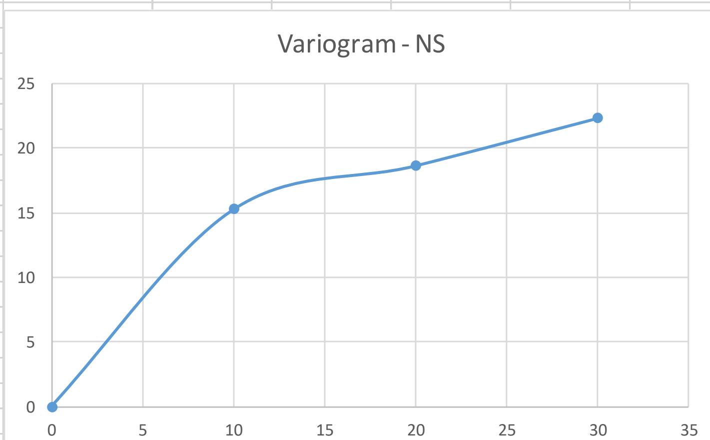
b) Calculate the semivariance values on the experimental variogram in the north-south direction at lag distances 10 ft, 20 ft, and 30 ft. Show your calculations, fill out the following table, and plot the variogram.

Lag distance (ft)	10	20	30
γ_{N-S}	15.25	18.64	22.32

$$\gamma(h) = \frac{1}{2N_h}\sum_1^{N_h}[\phi(x_i) - \phi(x_i + h)]^2$$

		Porosity in West-East Direction (%)						
	Location (ft)	**0**	**10**	**20**	**30**	**40**	**50**	**60**
Porosity in North-South Direction (%)	**0**	20	15	19	21	26	18	19
	10	17	16	23	21	29	25	15
	20	22	17	24	17	26	32	30
	30	20	15	27	23	27	29	20
	40	19	25	32	28	22	24	28

NORTH-SOUTH	0				0	
H=	10	N=	28	\gamma=	15.25	
9	1	16	0	9	49	16
25	1	1	16	9	49	225
4	4	9	36	1	9	100
1	100	25	25	25	25	64
H=	20	N=	21	\gamma=	18.64	
4	4	25	16	0	196	121
9	1	16	4	4	16	25
9	64	64	121	16	64	4
H=	30	N=	14	\gamma=	22.32	
0	0	64	4	1	121	1
4	81	81	49	49	1	169

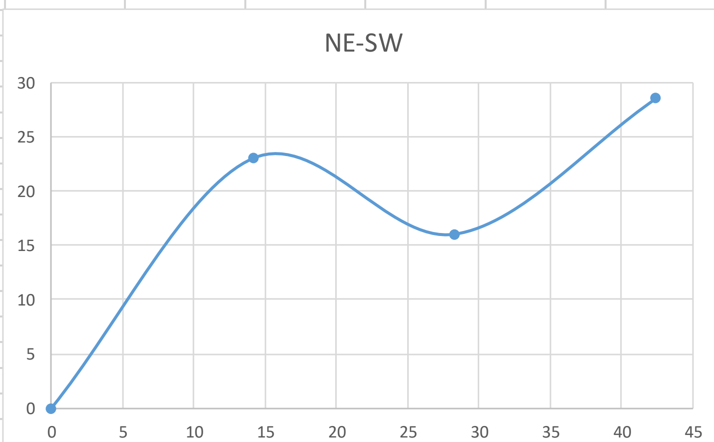


c) Calculate the semivariance values on the experimental variogram in the northeastsouthwest direction at lag distances 10 2 ft, 20 2 ft, and 30 2 ft. Show your calculations, fill out the following table, and plot the variogrram.

Lag distance (ft)	$10\sqrt{2}$	$20\sqrt{2}$	$30\sqrt{2}$
γ_{NE-SW}	23.04	16.03	28.56

		Porosity in West-East Direction (%)						
	Location (ft)	0	10	20	30	40	50	60
Porosity in North-South Direction (%)	0	20	15	19	21	26	18	19
	10	17	16	23	21	29	25	15
	20	22	17	24	17	26	32	30
	30	20	15	27	23	27	29	20
	40	19	25	32	28	22	24	28

NE-SW	0				0	
H=	14.1421356	N=	24	\gamma=	23.04	
4	9	4	25	121	36	
36	36	9	144	1	289	
9	81	100	9	25	1	
16	4	81	1	49	16	
H=	28.2842712	N=	15	\gamma=	16.03	
9	16	4	1	49		
9	36	4	4	144		
25	64	36	16	64		
H=	42.4264069	N=	8	\gamma=	28.56	
1	121	81	16			
4	16	49	169			

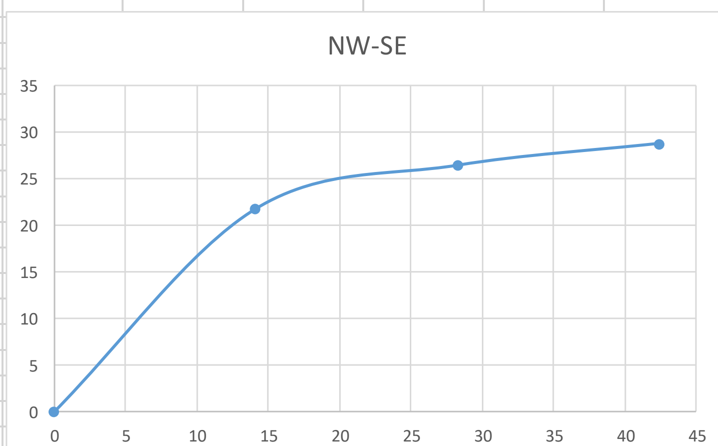


d) Calculate the semivariance values on the experimental variogram in the northwestsoutheast direction at lag distances 10 2 ft, 20 2 ft, and 30 2 ft. Show your calculations, fill out the following table, and plot the variogrargm.

Lag distance (ft)	$10\sqrt{2}$	$20\sqrt{2}$	$30\sqrt{2}$
γ_{NW-SE}	21.79	26.47	28.81

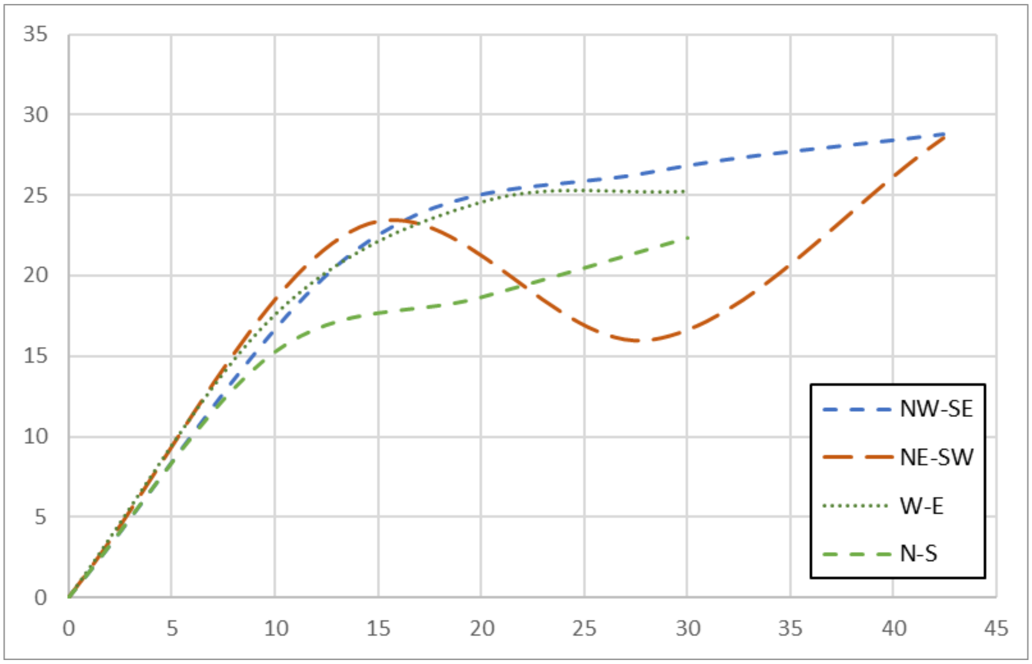
		Porosity in West-East Direction (%)						
	Location (ft)	**0**	**10**	**20**	**30**	**40**	**50**	**60**
Porosity in North-South Direction (%)	**0**	20	15	19	21	26	18	19
	10	17	16	23	21	29	25	15
	20	22	17	24	17	26	32	30
	30	20	15	27	23	27	29	20
	40	19	25	32	28	22	24	28

NW-SE	0				0	
H=	14.1421356	N=	24	\gamma=	21.79	
16	64	4	64	1	9	
0	64	36	25	9	25	
49	100	1	100	9	144	
25	289	1	1	9	1	
H=	28.2842712	N=	15	\gamma=	26.47	
16	4	49	121	16		
100	49	16	64	81		
100	121	4	49	4		
H=	42.4264069	N=	8	\gamma=	28.81	
9	144	100	1			
121	36	1	49			



e) Is the variogram isotropic or anisotropic?

Plotting all variograms in a single grid, we can see less higher correlation lengths and higher variance (sill) in NW-SE and W-E directions. While the data show some degree of anisotropy in porosity, we can see some similarity among the directions analyzed.



Question 3: Consider arrangement of points shown in Figure 1. The volumetric concentration of clay at locations x_1, x_2, and x_3 is measured to be 10%, 30%, and 45%. The theoretical autocovariance is given by:

$$Cov(h) = 6e^{-3|h|/10}$$

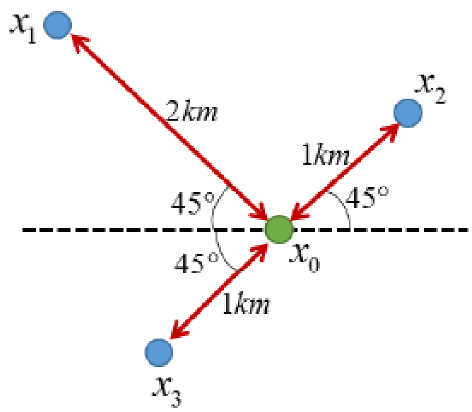


Figure 1: Arrangement of data points in Question 3

a) We plan to use ordinary kriging to estimate volumetric concentration of clay at location x_0. Show your calculations and <u>write the final result in terms of λ_i's</u>.

$$H = \begin{bmatrix} h_{11} & h_{12} & h_{13} & h_{10} \\ h_{21} & h_{22} & h_{23} & h_{20} \\ h_{22} & h_{23} & h_{33} & h_{30} \end{bmatrix} = \begin{bmatrix} 0 & 2.24 & 2.24 & 2 \\ 2.24 & 0 & 2 & 1 \\ 2.24 & 2 & 0 & 1 \end{bmatrix} \text{km}$$

$$h_{13} = h_{21} = \sqrt{1^2 + 2^2} = 2.24 \text{ km}$$

Covariance matrix:

$$C = \begin{bmatrix} 6 & 3.06 & 3.06 & 3.29 \\ 3.06 & 6 & 3.29 & 4.44 \\ 3.06 & 3.29 & 6 & 4.44 \end{bmatrix}$$

System of equations:

$$\begin{bmatrix} 6 & 3.06 & 3.06 & 1 \\ 3.06 & 6 & 3.29 & 1 \\ 3.06 & 3.29 & 6 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \beta \end{bmatrix} = \begin{bmatrix} 3.29 \\ 4.44 \\ 4.44 \\ 1 \end{bmatrix}$$

The solution is:

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \beta \end{bmatrix} = \begin{bmatrix} 0.096 \\ 0.451 \\ 4.51 \\ -0.053 \end{bmatrix}$$

Please write your equation for calculation of λ_i's **<u>in the following format</u>**. The matrices will be graded.

$$\begin{bmatrix} \gamma(h_{11}) & \gamma(h_{12}) & \cdots & \gamma(h_{1N}) & -1 \\ \gamma(h_{21}) & \gamma(h_{22}) & \cdots & \gamma(h_{2N}) & -1 \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ \gamma(h_{N1}) & \gamma(h_{N2}) & \cdots & \gamma(h_{NN}) & -1 \\ 1 & 1 & \cdots & 1 & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_N \\ \beta \end{bmatrix} = \begin{bmatrix} \gamma(h_{10}) \\ \gamma(h_{20}) \\ \vdots \\ \gamma(h_{N0}) \\ 1 \end{bmatrix}$$

b) Estimate volumetric concentration of clay at location $\boldsymbol{x_0}$. Show your calculations and write the final result below.

___34.87%___

$$V_0 = \sum \lambda_i V_i = \begin{bmatrix} 0.096 \\ 0.451 \\ 4.51 \end{bmatrix} \cdot \begin{bmatrix} 10 \\ 30 \\ 45 \end{bmatrix} = 34.87\%$$

```
[2]: import numpy as np
A = np.array( [ [ 6, 3.06, 3.06, 1 ] , [ 3.06, 6, 3.29, 1 ] , [ 3.06, 3.29, 6, 1 ] , [ 1, 1, 1, 0 ] ] )
B = np.array( [ 3.29, 4.44, 4.44, 1 ] )

X = np.linalg.solve( A, B )
print(X)

V=[10, 30, 45, 0]
X@V

[ 0.0961326   0.4519337   0.4519337  -0.05262983]
[2]: 34.85635359116022
```

c) Calculate the minimum error variance for estimate of volumetric concentration of clay at location $\boldsymbol{x_0}$. Show your calculations and write the final result below.

______1.72______

$$\sigma^2_{e,min} = \sigma^2 - \beta - \sum_{i=1}^{N} \lambda_i C(h_{i0}) = 6 + 0.053 - \begin{bmatrix} 0.096 \\ 0.451 \\ 4.51 \end{bmatrix} \cdot \begin{bmatrix} 3.29 \\ 4.44 \\ 4.44 \end{bmatrix} = 1.72$$

d) Calculate the <u>99% confidence interval</u> for estimate of volumetric concentration of clay at location $\boldsymbol{x_0}$. Show your calculations and write the final result below.

____34.87 ± 3.38 %____

$$99\%\text{CI} = 34.87 \pm 2.58\sqrt{1.72} = 34.87 \pm 3.38\ \%$$

Question 5: (Optional) You do not need to submit your solutions for this question.

Consider the TOC data (TOC_Spatial data) provided to you on Canvas website and investigate the impact of following parameters on kriging results and minimum error variance:

a) Variogram model
b) Variogram parameters such as slope, sill, and nugget effect
c) Number of data points used as input to kriging