Advanced Petrophysics
PGE 381L, Fall 2023
Unique Number: 20215

Homework Assignment No. 4

September 28, 2023
Due on Tuesday, September 10, 2023, before 11:00 PM

Name: ____

UT EID: ____

Objectives:

a) To practice application of Darcy's law

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Note: Please scan your homework assignment and upload it as one pdf file on the Canvas website before the deadline. Please name your homework document as follows:

PGE381L_2023_Fall _HW04_lastname_name.pdf

Example: PGE381L_2023_Fall_HW04_Heidari_Zoya.pdf

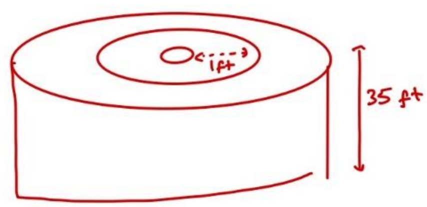
Question 1: A vertical well is drilled in an oil-bearing reservoir. This reservoir can be considered as isotropic and homogeneous, with average porosity and permeability of 15% and 40 mD. Mud-filtrate invasion has caused damage to the near-wellbore region and has decreased the permeability of the invaded zone to 5 mD. Assume that the depth of damage into the reservoir is approximately 1 ft.

Oil viscosity = 4 cp
Reservoir thickness = 35 ft
Wellbore radius = 0.3 ft
External drainage radius = 1000 ft
Reservoir pressure = 5000 psi
Bottom-hole pressure = 2200 psi

Answer the following questions:

a) Estimate the effective horizontal permeability of the reservoir.

b) Estimate the initial rate of production. Report the estimate of production in reservoir bbl per day.



a) $$\overline{K}_H = \frac{\ln(r_e/r_w)}{\sum_{i=1}^{2} \ln(r_i/r_{i-1})/K_i} = \frac{\ln(1000 - 0.3)}{\frac{\ln\left(\frac{1.3}{0.3}\right)}{5} + \frac{\ln\left(\frac{1000}{1.3}\right)}{40}}$$

$$\overline{K}_H = 17.66 \text{ md}$$

b) $$q = \frac{1}{141.2} \frac{kh(P_e - P_w)}{\mu \ln\left(\frac{r_e}{r_w}\right)} = \frac{1}{141.2} \frac{17.66(35)(5000-2000)}{4 \ln\left(\frac{1000}{0.3}\right)}$$

$$q = 377.75 \text{ bbl/day}$$

Question 2: A core prepared for a series of flow experiments consists of a 15 cm long piece of 1 mD rock and a 15 cm long piece of 10 mD rock joined in series (**Figure 1**). Pressure taps are located 7.5 cm from each end of the core. The cross-sectional area of the core is 20 cm^2. The 1 mD core is at the upstream end (where fluid is being injected). The downstream pressure is kept at atmospheric pressure. Brine of viscosity 1 cp is injected into the core at steady-state rate of 5 cm^3/hr. What will be the gauge pressures P_1 and P_2 (in atm) at the pressure taps?

$A = 20\,cm^2$

$q = 5\,cm^3/hr = \frac{5}{3600}\,cm^3/s$

$\mu = 1\,cp$

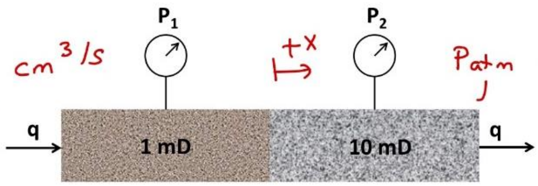


Figure 1: The core setup prepared for flow experiments

$$\boxed{\begin{aligned} P_1 &= 0.625\ atm \\ P_2 &= 0.0521\ atm \end{aligned}}$$

$q = \frac{-kA}{\mu}\frac{dP}{dx}$

$\rightsquigarrow \frac{dP}{dx} = \frac{-\mu q}{kA}$

* For 10 md core: $P = \frac{-\mu q}{kA}x + C_1$

at $x = 15cm \rightsquigarrow P = 1$

$\Rightarrow 1 = -\frac{q\mu}{kA}(15) + C_1$

$\Rightarrow C_1 = 1 + \frac{q\mu}{kA}(15)$

$P(x) = 1 + \frac{\mu q}{kA}(15-x) = 1 + 6.944\times10^{-3}(15-x)$

$\rightsquigarrow P(7.5) = 1+6.944\times10^{-3}(15-7.5) = \underline{1.0521\ atm}$

at $x = 0 \rightsquigarrow P(0) = 1+6.544\times10^{-3}(15-0) = 1.1042\ atm$

* For 1md core: $P = -\frac{\mu q}{kA}x + C_2$

at $x = 15cm \Rightarrow P = 1.1042\ atm$

$\Rightarrow 1.1042 = \frac{-q\mu}{kA}(15) + C_2$

$\Rightarrow C_2 = 1.1042 + \frac{q\mu}{kA}(15)$

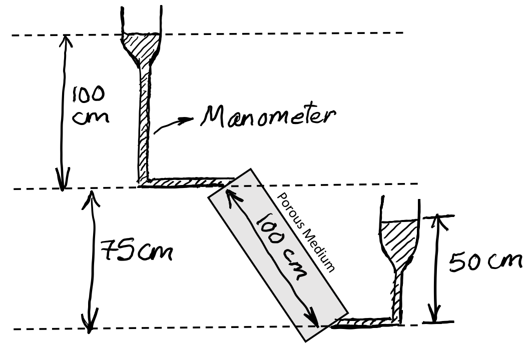
$\rightsquigarrow P(x) = 1.1042 + \frac{q\mu}{kA}(15-x) = 1.1042 + 6.944\times10^{-2}(15-x)$

at $x = 7.5 \Rightarrow P(7.5) = 1.1042 + 6.944\times10^{-2}(15-7.5)$

$\Rightarrow P(7.5) = \underline{1.625\ atm}$

Question 3: The following figure shows an inclined steady-state flow experiment for an incompressible liquid in a porous medium. The rock and fluid properties are as follows:

Absolute permeability = 2D
Density of the liquid = 1.024 g/cc
Cross sectional area = 100 cm^2
Viscosity of the liquid = 1.5 cp
Gravitational acceleration = 981 cm/s^2
Mean grain diameter of the porous medium = 1/16 mm



Answer the following questions:

a) Is there flow through the porous medium? Justify your answer and determine the direction of flow.

b) If there is flow, what is the direction of flow in this porous medium?

c) Estimate the volumetric flow rate.

d) If there is flow, is the flow considered as Darcy or non-Darcy flow? Justify your answer with appropriate calculations.

a) $h_A = 100$ cm $\quad h_B = -25$ cm $\quad h_A > h_B \rightarrow$ FLOW

b) $h_A > h_B \rightarrow$ flow A to B

c) $$q = kA\frac{\Delta h}{L} = \frac{k\rho g}{1.0133 \times 10^6 \mu} A \frac{\Delta h}{L}$$

$$= \frac{2(1.024)(981)}{(1.0133 \times 10^6)(1.5)} (100) \frac{125}{100} = 0.1652 \frac{cm^3}{s}$$

d) $$Re = \frac{\rho v D_p}{\mu} = \frac{(1.024)(0.1652/100)(1/160)}{0.015}$$

$$= 7.05 \times 10^{-4}$$

$Re < 1 \Rightarrow$ Darcy flow

Question 4: Figure 1 shows a falling head permeameter (Domenica and Schwartz, 1990) for determining the permeability of a core using a nonreactive liquid. Answer the following questions:

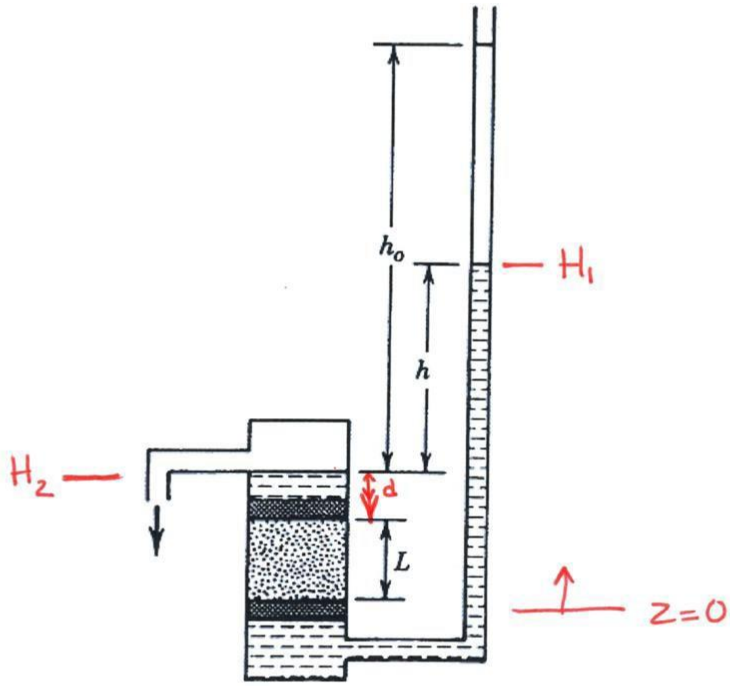


Figure 1: Falling head permeameter (Domenica and Schwartz, 1990)

a) Drive the differential equation for the instantaneous height h in terms of the following system variables:

Cross sectional area of the core	= A
Length of the core	= L
Core permeability	= k
Liquid density	= ρ
Liquid viscosity	= μ
Cross sectional area of eth liquid manometer	= a
Gravitational acceleration	= g
Time	= t
Height at t=0	= h_0

b) Solve the differential equation you derived in part (a).

a) $q = -KA \dfrac{dH}{ds} \rightarrow$ hydraulic head (1)

$H_1 = L + h + d$

$H_2 = L + d$

$$\frac{dH}{ds} = \frac{H_2 - H_1}{S_2 - S_1} = \frac{-(L+h+d) + (L+d)}{L} = \frac{-h}{L} \quad (2)$$

$$(1)\,\&\,(2) \rightarrow q = KA\frac{h}{L}$$

$$\left.\begin{aligned} &\rightarrow q = \frac{k\rho g}{\mu}\frac{Ah}{L} \\ &q = a\left(-\frac{dh}{dt}\right) \end{aligned}\right\} \frac{dh}{dt} = \frac{-k\rho g A}{\mu L a} h$$

b) $$\int \frac{dh}{dt} = -\int \frac{k\rho g A}{\mu L a}\, dt$$

$$\rightsquigarrow \ln h = -\frac{k\rho g A}{\mu L a} t + C' \rightsquigarrow h = Ce^{\frac{-k\rho g A}{\mu L a} t}$$

at $t=0 \Rightarrow h = h_0$

$$\Rightarrow h = h_0 e^{\frac{-k\rho g A}{\underset{1.01325\times 10^{6}}{\hat{\mu}} L a} t}$$

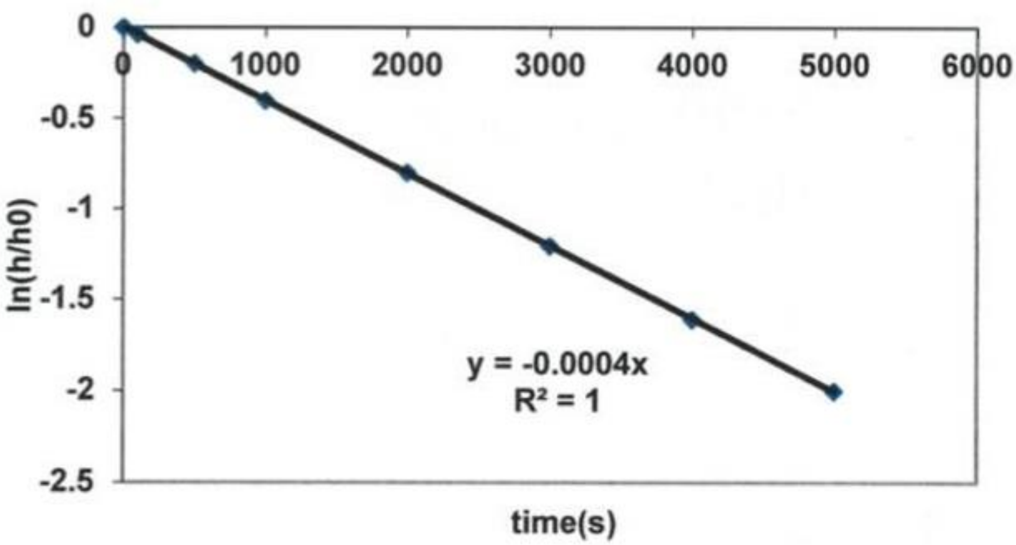
$1.01325 \times 10^6 \longrightarrow$ Unit conv

c) Given the set of h versus t experimental measurements listed in Table 1 and the following information about the rock/fluid samples and experimental setup, determine the absolute permeability of the core sample.

Table 1: Experimental data for Question 1

t (s)	h (cm)
0	100.0
100	96.1
500	82.0
1000	67.0
2000	45.0
3000	30.0
4000	20.0
5000	13.5

Core length = 10 cm
Core diameter = 5 cm
Diameter of manometer = 1 cm
Brine density = 1.02 g/cc
Brine viscosity =1 cp
Gravitational acceleration = 981 cm/s^2



$$h_0 = 100\ \text{cm} \quad ; \quad h = 100\, e^{-0.0004t}$$

$$\frac{k \rho g A}{\mu L a (1.0133 \times 10^{6})} = 0.004$$

$$\Rightarrow \quad k = \frac{0.0004\ (1)\ (10)\ \pi\ (1/2)^2 (1.0133 \times 10^{6})}{(1.02)\ (981)\ \left[\pi\ (5/2)^2\right]}$$

$$k = 0.162\ D = 162\ mD$$

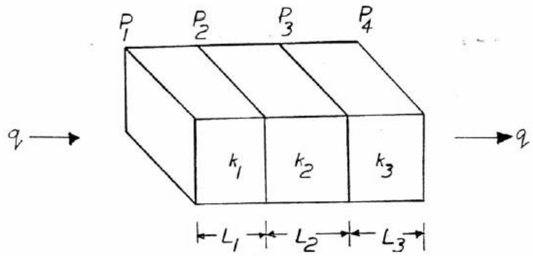
Optional Questions:

You do <u>not</u> need to submit solutions to the following questions. You can solve the following questions for the purpose of practicing. They will <u>not</u> be graded.

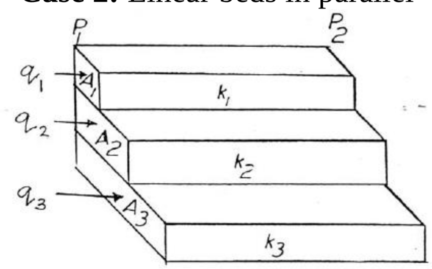
Question 5: Solve question 3.10 in the "Advanced Petrophysics" textbook.

Question 6: Use Darcy's law to estimate average permeability for the following laminated structures.

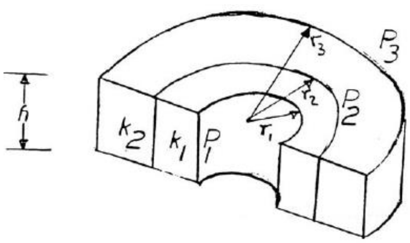
Case 1: Linear beds in series



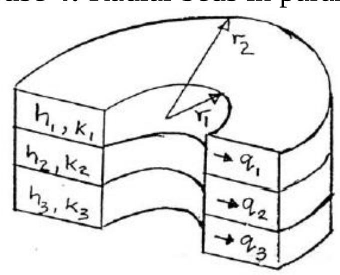
Case 2: Linear beds in parallel



Case 3: Radial beds in series

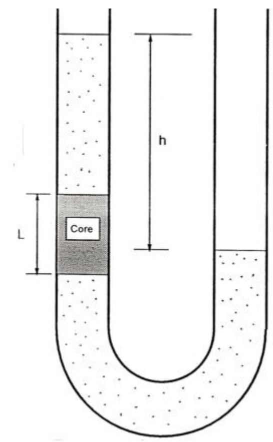


Case 4: Radial beds in parallel



Question 7: Figure 3 shows an apparatus for determining the permeability of a core using a nonreactive liquid. (Question 3.13 in your textbook)

a) Derive the differential equation for the instantaneous height h in terms of the pertinent system variables and parameters. Use the following symbols in your derivation:



Cross sectional area of the core and the U tube	=	A
Length of core	=	L
Core permeability	=	k
Liquid density	=	ρ
Liquid viscosity	=	μ
Gravitational acceleration	=	g
Time	=	t
Height at t = 0	=	h_o

b) Solve the differential equation you derived in part (a) analytically.

c) The following data were obtained in the experiment using brine:

Time (s)	h (cm)
0	100.0
1000	67.0
3000	30.0
5000	13.5

Additional data are as follows:

Length of core	=	10 cm
Core and U-tube diameter	=	2 cm
Brine density	=	1.02 g/cm3
Brine viscosity	=	1 cp
Gravitational acceleration	=	981 cm/s2

Based on the theory you have derived in parts (a) and (b), determine the permeability of the core and state its units.