RELATIVE PERMEABILITY CALCULATIONS FROM PORE SIZE DISTRIBUTION DATA

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ABSTRACT

Formulas for calculating relative permeability from pore size distribution data are derived from basic laws of fluid flow in porous media. The tortuosity factors that appear in the equations are described in terms of the physical properties of the medium and the saturations of the contained fluids. Procedures for calculating and methods for measuring the tortuosity factors are discussed. Values of relative permeability calculated by the formulas are compared with results obtained by experimental measurements.

INTRODUCTION

Recently, several investigators[1,2,3,4] have derived relative permeability equations from Darcy's and Poiseuille's Laws in which some physical factor or factors describing the porous system appear. The factor that has caused the greatest amount of comment, and which is probably least understood, is the tortuosity of the fluid path in the porous sample. Values of the tortuosity factors appearing in the equations range from a constant for all conditions of the fluid flow system to an inverse function of the pore radii.

It is the purpose of this paper to compare some experimentally determined relative permeability curves with those calculated by a method which utilized capillary pressure information. The capillary pressure data were obtained preferably from pore size distribution information as determined by mercury injection but, in some cases, in the absence of suitable mercury injection information, data were taken from measurements made by the porous semi-permeable diaphragm method. The physical significance of certain tortuosity concepts is inferred and the determinations of the tortuosity factors from pore size distribution data and by electrical measurements are discussed.

THEORETICAL BACKGROUND

In previously reported studies on pore size distribution in petroleum reservoir rocks, by the author and co-workers,[5] an equation was derived relating specific permeability, k, to pore entry radii, R_i, porosity, ϕ, tortuosity factor, X_i, and incremental pore volume, V_i. The equation is

$$k = 0.126\, \phi \sum_{o}^{n} \frac{V_i \, \overline{R_i}^4}{X_i^2 \, \overline{R_i}^2} \quad . \quad . \quad . \quad . \quad (1)$$

where $X_i = L_i/L$, L_i being the "effective" length of the flow path in the porous sample, and L the length of the porous sample. The constant, 0.126, is a units conversion factor. (This definition of tortuosity factor $X_i = L_i/L$ does not conform to all such definitions found in the literature.[6])

By similarity, in a two-phase system, the effective permeability of the wetting-phase, k_{ew}, is

$$k_{ew} = 0.126\, S_w \phi \sum_{o}^{n} \frac{V_{iw} \, \overline{R_i}^4}{X_{iw}^2 \, \overline{R_i}^2} \quad . \quad . \quad . \quad . \quad (2)$$

- ●—● ALUNDUM (FRL)
- ⊗—⊗ EAST COALINGA (GATES + TEMPELAAR-LIETZ[8])
- ○—○ LONG BEACH (GATES + TEMPELAAR-LIETZ[8])
- ×—× MEADOWLARK (GATES + TEMPELAAR-LIETZ[8])
- □—□ NELLIE BLY (J.D.HARRINGTON[10])

WETTING PHASE SATURATION (% OF PORE VOLUME)

TORTUOSITY RATIO, X_{rw}
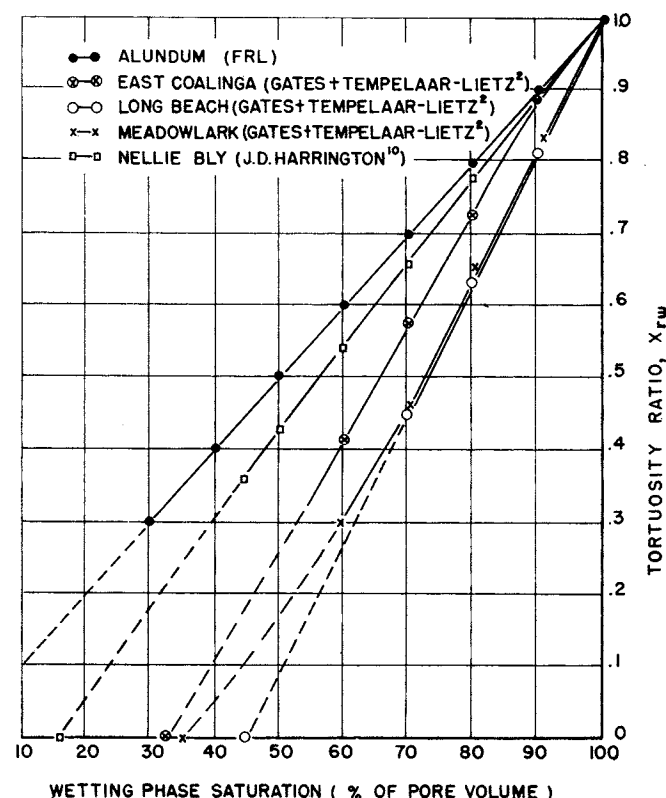
FIG. 1 — CALCULATED TORTUOSITY RATIO CURVES.

[1]References given at end of paper.

where V_{iw} is the incremental pore volume of the wetting-phase, and $X_{iw} = \dfrac{L_{iw}}{L}$, X_{iw} being the tortuosity of the wetting phase and L_{iw} being the "effective" length of the flow path for the wetting-phase in the two-phase system. Let the tortuosity ratio

$$X_{rwi} = \frac{X_i}{X_{iw}} = \frac{L_i}{L_{iw}} \quad \ldots \ldots \ldots \quad (3)$$

or

$$X_{iw} = \frac{X_i}{X_{rwi}} \quad \ldots \ldots \ldots \ldots \quad (4)$$

and, if one assumes that

$$V_{iw} = V_i/S_w \quad \ldots \ldots \ldots \ldots \quad (5)$$

then, substituting Equations (4) and (5) in Equation (2) gives

$$K_{ew} = 0.126\,\phi \sum_{i=o}^{n} \frac{X_{rwi}{}^2 V_i \bar{R_i}{}^4}{X_i{}^2 \bar{R_i}{}^2} \quad \ldots \quad (6)$$

The equation for the wetting-phase relative permeability therefore becomes

$$k_{rw} = \frac{k_{ew}}{k} = \frac{0.126\,\phi}{k} \sum_{i=o}^{n} \frac{X_{rwi}{}^2 V_i R_i{}^4}{X_i{}^2 \bar{R_i}{}^2} \quad \ldots \quad (7)$$

The ratio of tortuosities, X_{rwi}, is the only unknown factor in Equation (7). This equation would otherwise permit the calculation of the wetting-phase relative permeability from pore size distribution or other capillary pressure data. It is assumed

FIG. 3 — RELATIVE PERMEABILITY CURVES. SAMPLE: NELLIE BLY SANDSTONE. GAS PERMEABILITY: 1,080 MD. EFFECTIVE POROSITY: 27.3 PER CENT. FLUIDS: GAS AND OIL. DATA: MEASUREMENTS BY J. D. HARRINGTON.[10] CALCULATIONS FROM MERCURY INJECTION DATA.

that X_i is known from single-phase permeability and pore size distribution measurements.

By similar reasoning the non-wetting phase effective permeability, k_{ew}, in a two-phase system can be shown to be given by the equation,

$$k_{en} = 0.126\,(1 - S_w)\phi \sum_{i=o}^{n} \frac{V_{in} R_i{}^4}{X_{in}{}^2 \bar{R_i}{}^2} \quad \ldots \ldots \quad (8)$$

where V_{in} and X_{in} are respectively the incremental pore volume and the tortuosity for the non-wetting phase. Let

$$X_{rni} = \frac{X_i}{X_{in}} = \frac{L_i}{L_{in}} \quad \ldots \ldots \ldots \quad (9)$$

where X_{rni} is the tortuosity ratio and L_{in} is the "effective" length of the flow path for the non wetting phase.

If one assumes that

$$V_{in} = V_i/(1 - S_w) \quad \ldots \ldots \ldots \quad (10)$$

and if Equations (9) and (10) are substituted in Equation (8),

$$k_{en} = 0.126\,\phi \sum_{i=o}^{n} \frac{X_{rni}{}^2 V_i R_i{}^4}{X_i{}^2 R_i{}^2} \quad \ldots \quad (11)$$

The non-wetting phase relative permeability, k_{rn}, becomes

$$k_{rn} = \frac{k_{en}}{k} = 0.126\,\frac{\phi}{k} \sum_{i=o}^{n} \frac{X_{rni}{}^2 V_i R_i{}^4}{X_i{}^2 R_i{}^2} \quad \ldots \quad (12)$$
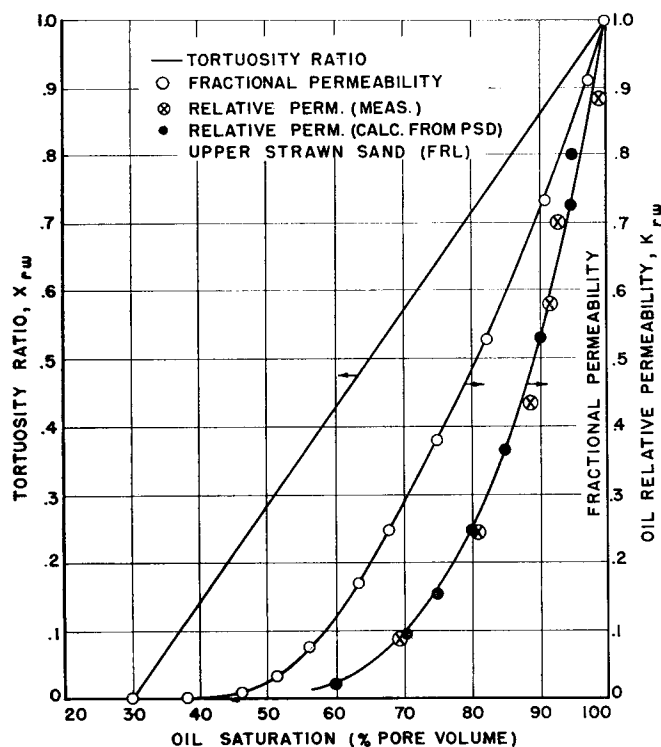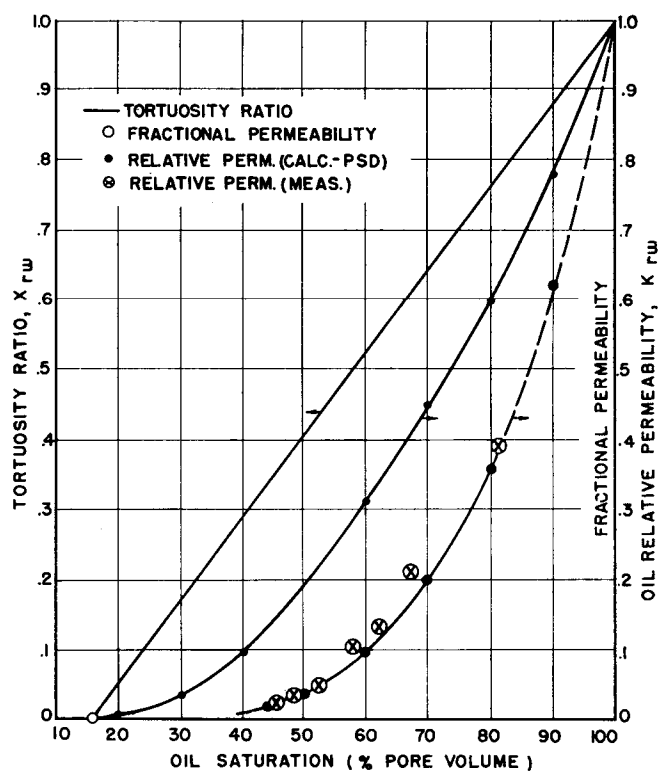

FIG. 2 — RELATIVE PERMEABILITY CURVES. SAMPLE: UPPER STRAWN SAND. GAS PERMEABILITY: 227 MD. EFFECTIVE POROSITY: 21.2 PER CENT. DATA: MEASUREMENTS BY FIELD RESEARCH LABORATORIES CALCULATIONS FROM MERCURY INJECTION DATA.

Equation (12) permits the calculation of non-wetting phase relative permeability from pore size distribution data provided the ratio of tortuosities can be determined.

TORTUOSITY RATIO

Several investigators[1,3,4,6,7,8,9] have studied the tortuosity ratio for multiphase flow in porous media. The large number of variables, resulting from various combinations of fluids and porous media, makes exact solutions to the tortuosity problem highly improbable, but does not eliminate approximate solutions which are within engineering accuracy.

Let Equation (7)

$$K_{rw} = \frac{0.126\,\phi}{K} \sum_{i=o}^{n} \frac{X_{rw\,i}{}^2\,V_i\,\overline{R_i^4}}{X_i{}^2\,\overline{R_i^2}} = \frac{0.126\,\phi\,X_{rw}{}^2}{K} \sum_{i=o}^{n} \frac{V_i\,\overline{R_i^4}}{X_i{}^2\,\overline{R_i^2}} \qquad (13)$$

where X_{rw} is an "effective" wetting phase tortuosity factor. When

$$S_w = 1$$

then, by definition

$$X_{rw} = 1$$

and when

$$S_w = S'_w$$
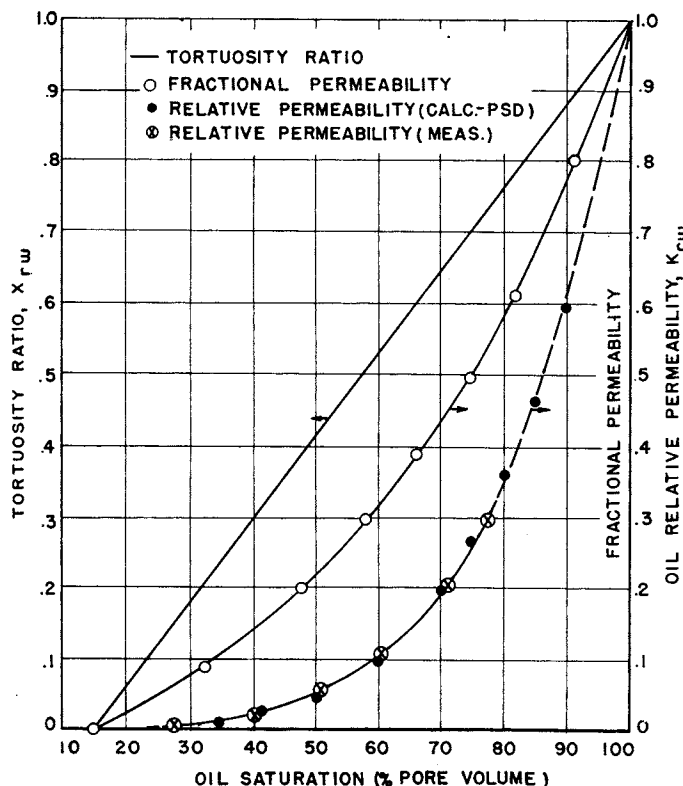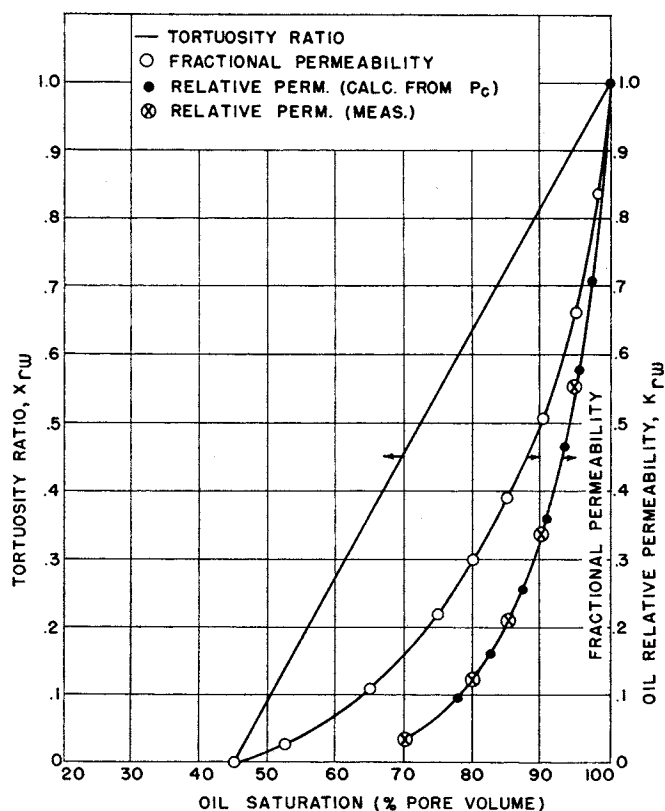
then it is assumed that

$$X_{rw} = 0$$

FIG. 5 — RELATIVE PERMEABILITY CURVES. SAMPLE: LONG BEACH FIELD. GAS PERMEABILITY: 55 MD. EFFECTIVE POROSITY: 18.2 PER CENT. FLUIDS: AIR AND KEROSENE. DATA: MEASUREMENTS BY GATES AND TEMPELAAR-LIETZ.[2] CALCULATIONS FROM POROUS DIAPHRAGM CAPILLARY PRESSURE DATA.

where S'_w is the "irreducible" minimum saturation of the wetting phase. Since theoretical values of X_{rw} are unknown for values of saturation intermediate between the irreducible minimum and one, values of the wetting-phase tortuosity ratio have been calculated at a number of saturations from experimental data obtained in these laboratories and from measurements published by other experimenters.[2,10] Representative results are plotted in Fig. 1. The deviation of these calculated values from a straight line is small, and apparently the deviation is in no preferred direction. As a first approximation, it therefore is assumed throughout the remainder of this paper that X_{rw} is some linear function of the saturation, S_w, such as

$$X_{rw} = A\,S_w + B \qquad (14)$$

where A and B are undetermined constants. By use of the boundary values referred to above, Equation (14) becomes

$$X_{rw} = \frac{1}{1 - S'_w}\,(S_w - S'_w) \qquad (15)$$

APPLICATION OF FORMULAS

In application of Equation (15) in conjunction with Equation (13) for calculating wetting-phase relative permeability from pore size distribution data, the phases (wetting and non-wetting) are assumed to be distributed in the porous matrix in a particular way. The irreducible minimum saturation of the
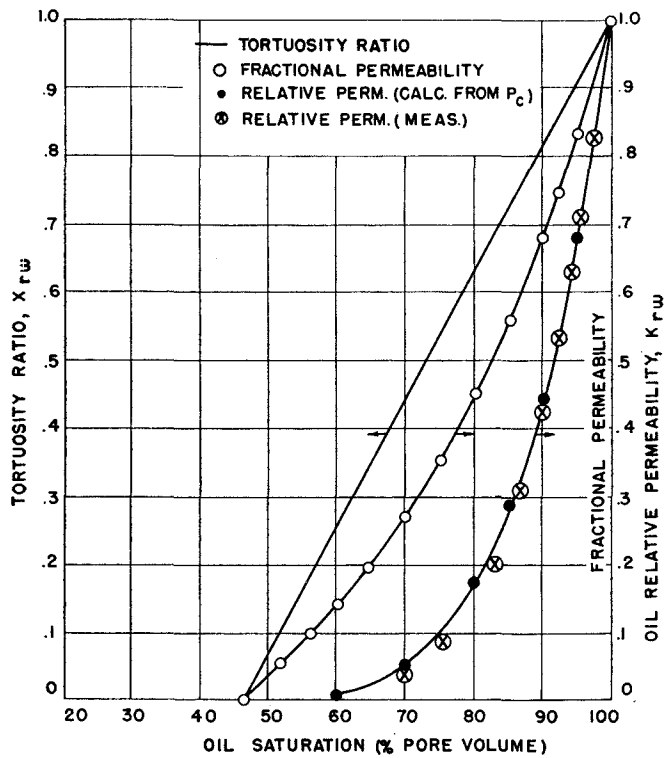
FIG. 4 — RELATIVE PERMEABILITY CURVES. SAMPLE: BASAL BROMIDE SANDSTONE. GAS PERMEABILITY 450 MD. EFFECTIVE POROSITY: 16 PER CENT. FLUIDS: GAS AND OIL. DATA: MEASUREMENTS BY J. D. HARRINGTON.[10] CALCULATIONS FROM MERCURY INJECTION DATA.

FIG. 6 — RELATIVE PERMEABILITY CURVES. SAMPLE: DOMINGUEZ FIELD. GAS PERMEABILITY: 36 MD. EFFECTIVE POROSITY: 26.7 PER CENT. FLUIDS: AIR AND KEROSENE. DATA: MEASUREMENTS BY GATES AND TEMPELAAR-LIETZ.[2] CALCULATIONS FROM POROUS DIAPHRAGM CAPILLARY PRESSURE DATA.
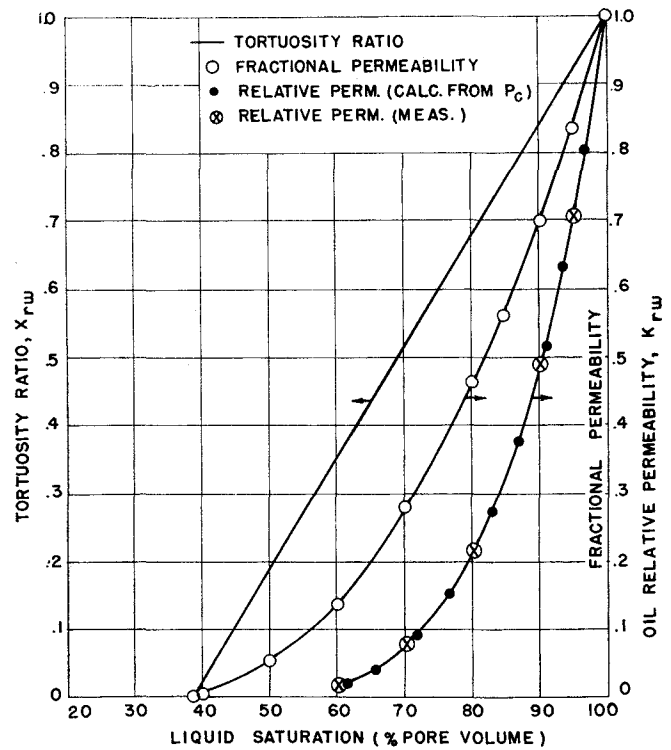
FIG. 7 — RELATIVE PERMEABILITY CURVES. SAMPLE: BEREA CANYON FIELD. GAS PERMEABILITY: 27 MD. EFFECTIVE POROSITY: 15.8 PER CENT. FLUIDS: AIR AND KEROSENE. DATA: MEASUREMENTS BY GATES AND TEMPELAAR-LIETZ.[2] CALCULATIONS FROM POROUS DIAPHRAGM CAPILLARY PRESSURE DATA.
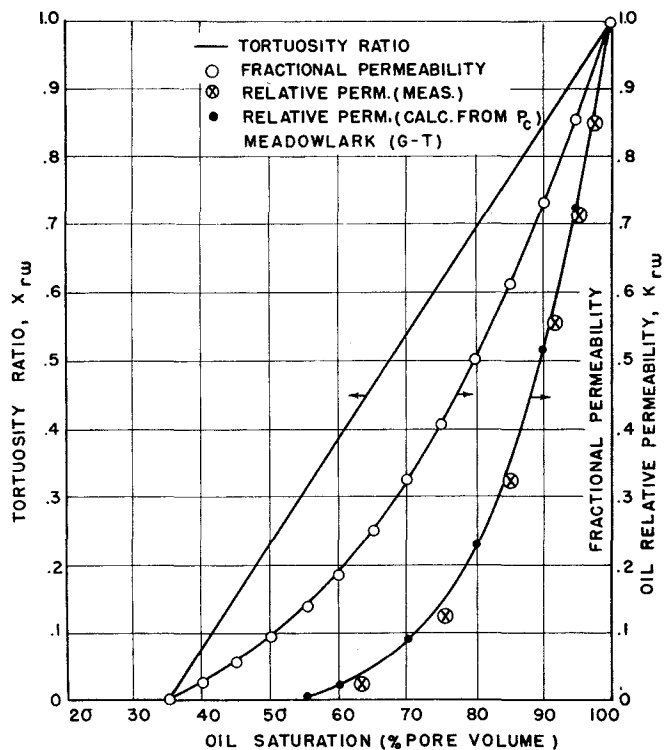
FIG. 8—RELATIVE PERMEABILITY CURVES. SAMPLE: MEADOWLARK FIELD. GAS PERMEABILITY: 370 MD. EFFECTIVE POROSITY: 24.5 PER CENT. FLUIDS: AIR AND KEROSENE. DATA: MEASUREMENTS BY GATES AND TEMPELAAR-LIETZ.[2] CALCULATIONS FROM POROUS DIAPHRAGM CAPILLARY PRESSURE DATA.
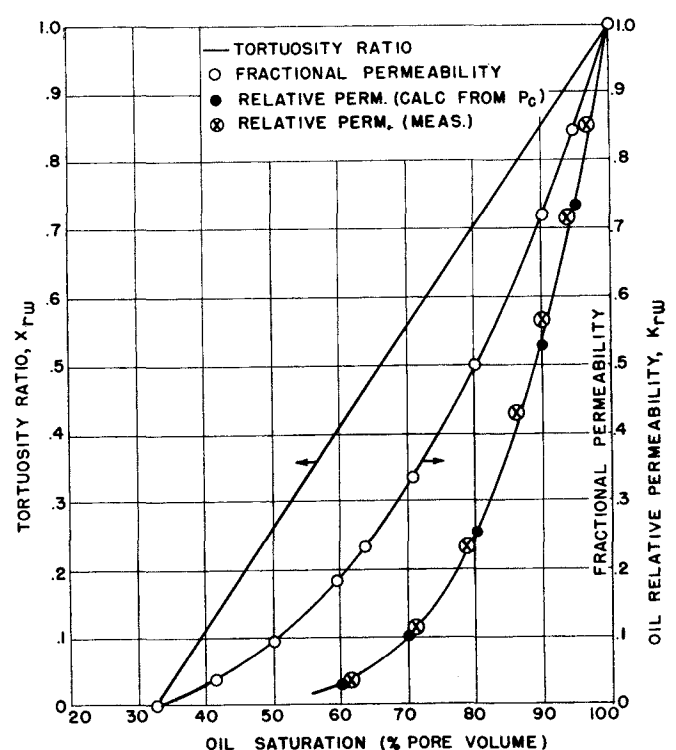
FIG. 9 — RELATIVE PERMEABILITY CURVES. SAMPLE: EAST COALINGA FIELD. GAS PERMEABILITY: 318 MD. EFFECTIVE POROSITY: 27.4 PER CENT. FLUIDS: AIR AND KEROSENE. DATA: MEASUREMENTS BY GATES AND TEMPELAAR-LIETZ.[2] CALCULATIONS FROM POROUS DIAPHRAGM CAPILLARY PRESSURE DATA.
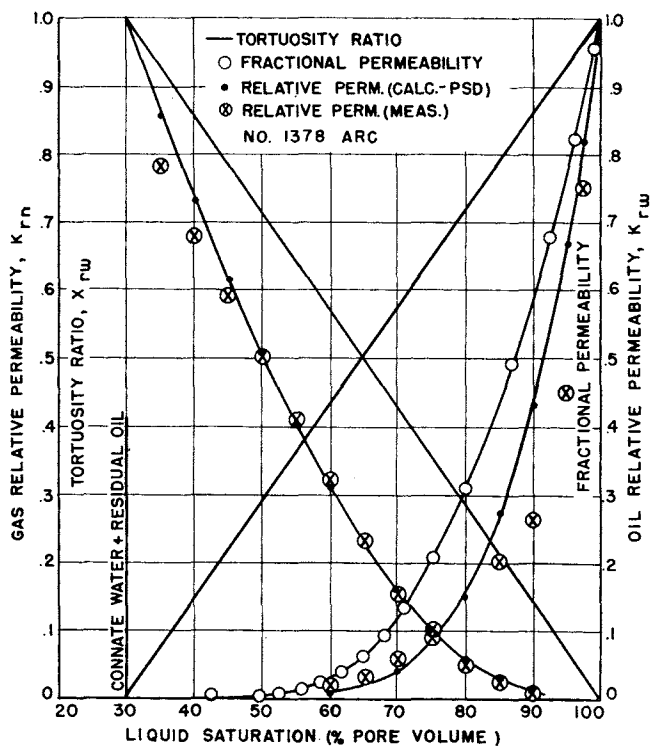
FIG. 10 — RELATIVE PERMEABILITY CURVES. SAMPLE NO. 1378 ATLANTIC REFINING CO. GAS PERMEABILITY: 95 MD. EFFECTIVE POROSITY: 23.8 PER CENT. FLUIDS: GAS, OIL, AND CONNATE WATER. DATA: MEASUREMENTS BY ATLANTIC REFINING CO. CALCULATIONS BY MERCURY INJECTION DATA.
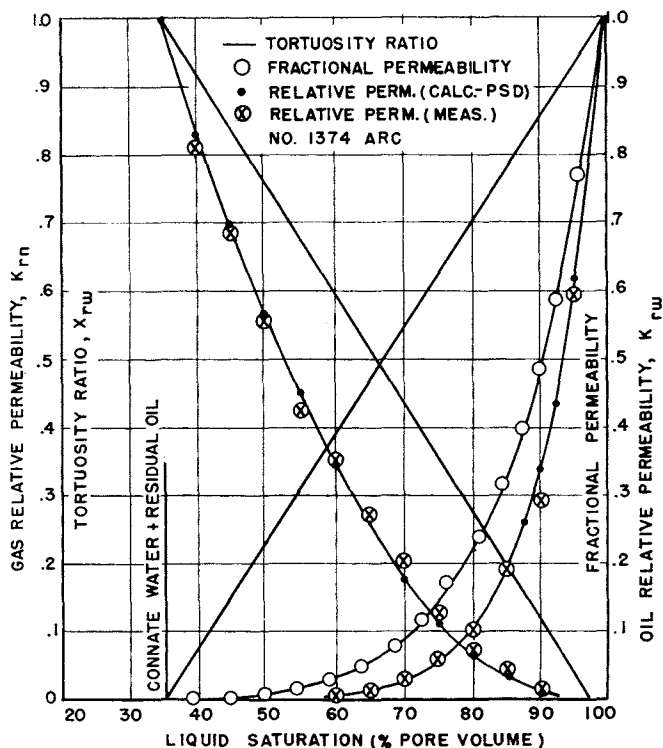
FIG. 11 — RELATIVE PERMEABILITY CURVES. SAMPLE NO. 1374 ATLANTIC REFINING CO. GAS PERMEABILITY: 72 MD. EFFECTIVE POROSITY: 21.9 PER CENT. FLUIDS: GAS, OIL, AND CONNATE WATER. DATA: MEASUREMENTS BY ATLANTIC REFINING CO. CALCULATIONS FROM MERCURY INJECTION DATA.

wetting-phase is assumed to be contained in the smaller pores and the residual non-wetting phase occupies the larger pores according to the mercury injection measurements. For intermediate saturations the permeability to the phase under consideration is adversely affected by the presence of the other phase. This is consistent with the assumption that the wetting-phase surrounds the non-wetting phase which, in turn, occupies the center of the pore space, particularly for media composed of irregular particles, and having crevices, inter-connecting smaller pores, and other irregularities.

The calculation from pore size distribution data of wetting-phase relative permeability in a two-phase system involves the following procedure. The irreducible wetting-phase saturation, S'_w, as determined by the semi-permeable porous diaphragm method or by other capillary pressure methods is substituted in Equation (15). The resulting equation provides a simple linear relation between the tortuosity ratio and the saturation, from which values for the tortuosity ratio may be determined either graphically or by numerical substitution. The single-phase fractional permeability for chosen saturations is then calculated from pore size distribution data. The single-phase fractional permeability is the ratio of the calculated permeability of the pores with radii less than a specified radius to the total permeability of the sample, where the specified radius is determined from the pore size distribution curve for the particular value of fractional volume under consideration. The value of the fractional permeability obtained is multiplied by the square of the tortuosity factor (X_{rw}^2) to give the wetting phase relative permeability. The procedure is repeated for a number of other wetting phase saturations ranging from the irreducible minimum to complete saturation. In case the mer-

cury injection experiment has not been performed on the sample under study, approximate pore size distribution data may be estimated from the capillary pressure curve obtained by the porous semi-permeable diaphragm method. This procedure was followed for some of the examples cited below.

Typical results, calculated and measured, are shown in Figs. 2 to 12 for data taken in Magnolia's Field Research Laboratories, Atlantic Refining Co.'s Laboratory, and for other data reported in the literature.[2,10] The linear tortuosity ratio vs saturation relationship and the calculated fractional permeability vs the saturation are also shown on all figures. In general, the calculated relative permeability values agree very closely with the experimental values for those core samples and fluids studied to date. It must be emphasized that the amount of data now available for correlation purposes is as yet insufficient to arrive at any definite conclusions as to the general validity of the procedure.

Although the major emphasis in this paper is devoted to procedures for calculating wetting phase relative permeability from pore size distribution data, three illustrative examples of calculated non-wetting phase relative permeability are given in Figs. 10, 11, and 12. The procedure for calculating non-wetting phase relative permeability from pore size distribution data is similar to that used for calculating wetting phase relative permeability. The tortuosity ratio for the non-wetting phase (gas) is assumed to have the value of one at the irreducible minimum saturation of the wetting phases consisting of the connate water and the residual oil, and a value of zero at the minimum saturation of the non-wetting phase; this tortuosity ratio is assumed to vary in a linear manner with liquid saturation between these two values corresponding to the limits of
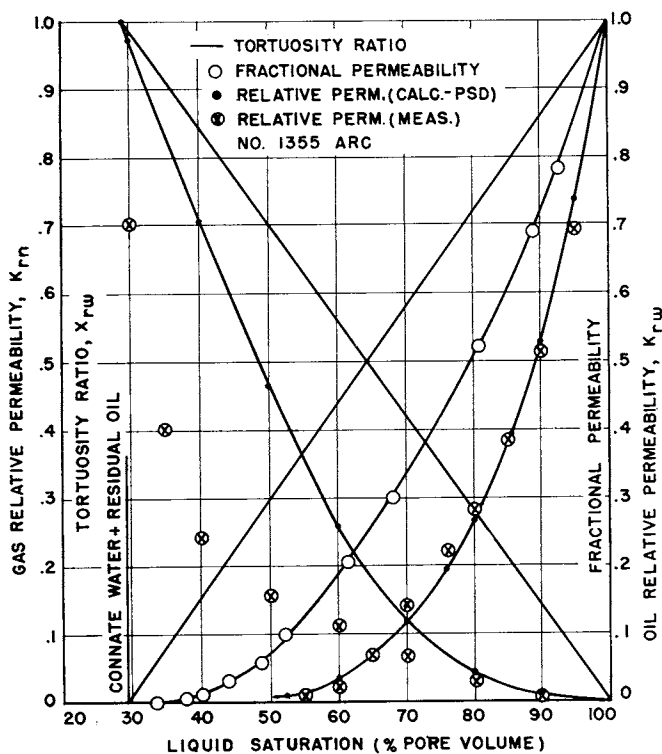
FIG. 12 — RELATIVE PERMEABILITY CURVES. SAMPLE NO. 1355 ATLANTIC REFINING CO. GAS PERMEABILITY: 163 MD. EFFECTIVE POROSITY: 21.3 PER CENT. FLUIDS: GAS, OIL, AND CONNATE WATER. DATA: MEASUREMENTS BY ATLANTIC REFINING CO. CALCULATIONS FROM MERCURY INJECTION DATA.

non-wetting phase saturation. In general, the calculated results are consistent with the measured non-wetting phase relative permeability values.

DETERMINATION OF TORTUOSITY RATIO BY RESISTIVITY MEASUREMENTS

Thornton[6] has assumed that the average lengths of path for fluid flow and electric current flow in a porous medium are equal. If this assumption is valid, the magnitude of the wetting phase tortuosity ratio, X_{rw}, could be determined directly by measurement of electrical resistivity of a brine-saturated sample at various saturations.

According to Thornton[6]

$$I = \frac{1}{S_w X_{rw}} \quad \cdots \cdots \cdots \cdots (16)$$

where I is the resistivity index. Therefore,

$$X_{rw} = \frac{1}{I S_w} \quad \cdots \cdots \cdots \cdots (17)$$

Equation (16) expresses the tortuosity factor as an inverse function of resistivity index and saturation. If Equations (17) and (13) are combined and rearranged with the tortuosity ratio outside of the summation sign, an equation results which is similar to one derived by Wyllie and Spangler:[11]

$$k_{rw} = \frac{0.126 \, \phi}{I^2 S_w^2 k} \sum_{o}^{n} \frac{V_i \, R_i^4}{X_i^2 R_i^2} \quad \cdots \cdots (18)$$

Generally, for partially saturated samples, the experimental results obtained by the author's co-workers indicate that the resistivity index calculated from Equation (18) using measured relative permeability data, is larger than the resistivity index measured by standard methods. One typical example is illustrated in Fig. 13. This could mean that the electrical conductivity of the matrix is greater than zero and, hence, the "effective" ion path is less than the fluid flow path. Further investigations of this phenomenon are in progress.

CONCLUSIONS

A procedure has been outlined and formulas have been developed for calculating wetting and non-wetting phase relative permeability in two phase systems from pore size distribution data. The tortuosity factors which appear in the equations are assumed to be linear functions of saturation. Some consideration has been given to the measurement of the tortuosity ratio by the electrical resistivity index analogy as proposed by other investigators.[6,11] So far, this method of measuring the tortuosity ratios has been unsuccessful.
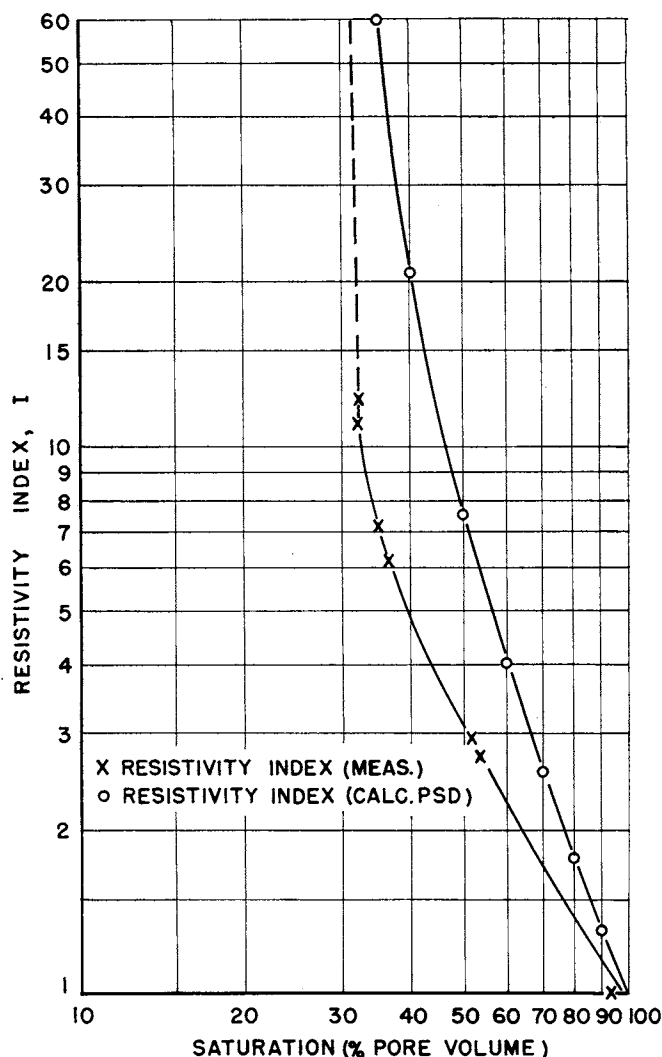
FIG. 13 — RESISTIVITY INDEX CURVES. SAMPLE: MIOCENE SAND. GAS PERMEABILITY: 256 MD. EFFECTIVE POROSITY: 25 PER CENT.

For the samples studied and the fluids used, the calculated relative permeability data agree favorably in most cases with the values measured by standard methods. The results indicate that the calculated data are more consistent and probably contain less maximum error than the measured data. (The error in measurement is unknown.) Of major importance and utility to fluid flow studies in porous media are the speed and ease with which the calculated data may be obtained for either reservoir determinations or for laboratory standards.

Methods and procedures for calculating relative permeability data are being extended and experimental measurements of relative permeability on a variety of samples are being made for correlation purposes. It is emphasized that the results obtained to date are not sufficient for complete evaluation of the method.

ACKNOWLEDGMENTS

NOMENCLATURE

A, B — Constants
I — Resistivity index
k — Permeability (darcys)
k_e — Permeability, effective (darcys)
k_{en} — Permeability, effective non-wetting phase (darcys)
k_{ew} — Permeability, effective wetting phase (darcys)
k_{rn} — Permeability, non-wetting phase relative
k_{rw} — Permeability, wetting phase relative
L — Length, sample (cm)
L_i — Length, "effective" capillary (cm)
L_{in} — Length, non-wetting phase "effective" capillary (cm)
L_{iw} — Length, wetting phase "effective" capillary (cm)
R — Radius, pore (cm)
R_i — Radius, incremental pore entry, $i = 1, 2, 3, \ldots$ (cm)
S_w — Saturation, wetting phase (fractional)
S'_w — Saturation, irreducible minimum (fractional)
V_i — Volume, incremental pore, $i = 1, 2, 3, \ldots$ (fraction of total pore volume)
V_{in} — Volume, non-wetting phase incremental pore (fractional)
V_{iw} — Volume, wetting phase incremental pore (fraction of wetting-phase-saturated pore volume)
X_i — Tortuosity factor (L_i/L)
X_{in} — Tortuosity factor, non-wetting phase (L_{in}/L)
X_{iw} — Tortuosity factor, wetting phase (L_{iw}/L)
X_{rni} — Tortuosity ratio, non-wetting phase (L_i/L_{in})
X_{rwi} — Tortuosity ratio, wetting phase (L_i/L_{iw})
ϕ — Porosity (fractional)
X_{rw} — Tortuosity factor, "effective"

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DISCUSSION

By I. Fatt, California Research Corp., La Habra, Calif., Member AIME

Burdine has presented an interesting modification of one of the equations which has been proposed for calculating relative permeabilities from pore size distribution or capillary pressure data. The equation Burdine uses for calculating relative permeabilities is derived from a very simple model of a porous medium. The model, a bundle of parallel tubes, is so very much over-simplified that calculated and measured relative permeability often do not agree. To bring these into better agreement, the concept of tortuosity has been introduced. Tortuosity, being difficult to evaluate experimentally, is empirically adjusted to improve the fit of calculated data to measured data. An improvement of fit by such empirical adjustment cannot be accepted as making the parallel tube model more valid nor the tortuosity concept more sound.

Burdine's treatment of the tortuosity terms, X_i and X_{rw}, seems to be questionable. The difficulty goes back to his earlier paper[1] where the tortuosity X_i was defined as the tortuosity of tubes of radius R_i. If this definition is adhered to, then X_i cannot be taken out of the summation (in Equation 17 of his earlier paper) when X_i varies with R_i. In extending his equation to the calculation of relative permeability, Burdine has postulated that tortuosity varies with radius and therefore saturation. However, in establishing the empirical tortuosity corrections, he has taken the tortuosity term out of the summation. The tortuosity term is then a complex average for the pores which are conducting fluid. This inconsistency makes Burdine's method of improving the fit of calculated-to-measured relative permeability strictly an empirical one. The relation between Burdine's tortuosity correction term and the concept of varying tortuosity with varying pore radius is only indirect.

Although the agreement between calculated and measured wetting phase relative permeability indicated in this paper seems adequate for engineering purposes, the same cannot be said for the agreement between calculated and measured non-wetting phase relative permeability. Burdine's method does not include the concept of a critical non-wetting phase satura-

tion, although this quantity is of great engineering importance. In Figs. 10 and 12, the critical non-wetting phase saturation is taken as zero. In Fig. 11, a finite critical saturation is shown, but Burdine does not indicate how this could be obtained from capillary pressure data. Engineering calculations require very accurate non-wetting phase permeabilities in the region of the critical saturation; the method of calculating these permeabilities, as proposed by Burdine, does not appear adequate. The reader could better evaluate the method if the non-wetting phase relative permeabilities were plotted as semilogarithmic graphs.

Burdine assumes that mercury injection data and water-oil or water-air capillary pressure data are equally useful for calculating relative permeabilities. If the mercury injection curve differs from the water-oil or water-air capillary pressure curve by only a pressure multiplying factor, Burdine's assumption is correct. There are, however, some data which indicate that for many reservoir samples there is no simple relationship between the mercury injection curve and the water-oil or water-air capillary pressure curve.

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AUTHOR'S REPLY TO MR. FATT

Fatt states that a very simple model has been used to represent a porous system and that, for this reason, agreement between calculated and experimental relative permeability values is often poor. We agree as to the simplicity of the model and to the frequent disagreement observed but point to the significant number of cases in which really remarkable agreement is observed. This very agreement, coupled with the desire, as in all scientific effort, to retain the maximum of simplicity led to the consideration of possible modifications in the use of the capillary tube model which might result in more frequent and superior agreement. That such modifications as have been used required the use of empirical procedures was not unexpected. In the absence of more fundamental approaches, the use of empirical methods seems entirely valid. Whether such empirical procedures are able to make "the parallel tube model more valid" or "the tortuosity concept more sound" depends solely on the predictive capacity of the resulting relationships and we consider this to be a matter as yet undetermined.

Referring to the tortuosity terms, X_i and X_{rwi}, Fatt states correctly that if the original definition of X_i in Equation (17) of an earlier paper is adhered to, then X_i cannot, in general, be removed from the summation sign. If, however, the "effective" capillary lengths of all the pores are known or assumed to be equal, the factors X_i^2 in Equation (17) of reference 1, may be removed from the summation and replaced by a "dividing factor," f, where

$$f = \sum_{i=o}^{n} X_i^2$$

Hence Equation (17) may be rewritten as

$$K = \frac{100 \, \phi}{8Cf} \sum_{i=o}^{n} \frac{V_i \, \overline{R_i^4}}{R_i^2}$$

An analysis of pore size distribution data for petroleum reservoir rock samples indicated that the tortuosity factor, X_i, must be considered a function of the pore radius. This resulted in the formulation of the empirical formula, Equation (18) of reference (1),

$$X_i = 5e^{-0.4 \, R_i} + 1.83$$

Since X_i is an inverse function of the pore radius, R_i, it has been retained inside the summation sign for all single phase and multiphase permeability calculations.

Referring to Equations (7) and (12) of the present paper, the tortuosity ratios are assumed to be functions of the pore radii and, as such, the tortuosity ratio symbols should be written X_{rwi} and X_{rni}, for the wetting and non-wetting phases, respectively. Since no methods, either experimental or theoretical, have been developed to evaluate the tortuosity ratios, X_{rwi} and X_{rni}, an approximate procedure has been employed with Equations (7) and (12) in an attempt to circumvent this difficulty. On this basis Equation (7) has been rewritten in the following manner:

$$K_{rw} = \frac{0.126 \, \phi}{K} \sum_{i=o}^{n} \frac{X_{rwi}^2 \, V_i \, \overline{R_i^4}}{X_i^2 \, \overline{R_i^2}} = $$
$$\frac{0.126 \, \phi \, X_{rw}^4}{K} \sum_{i=o}^{n} \frac{V_i \, \overline{R_i^4}}{\overline{R_i^2}}$$

where X_{rw} may be considered an "effective" wetting phase tortuosity factor that is a function of the saturation only. It is the empirical evaluation of the factor X_{rw} as a function of saturation which is the main subject of the paper.

As pointed out by Fatt, the calculated non-wetting phase relative permeability data presented in the original paper do not show the same order of agreement with experiment as observed in the case of wetting phase data on which was placed, to quote the paper, "the major emphasis." In fact, in the paper no strong claim was made for the use of the method for the calculation of non-wetting phase relative permeability values. It was stated only that results "consistent" with measured data were obtained. Further, contrary to the statement made by Fatt, the method of calculation of non-wetting phase relative permeability does include the concept of critical saturation. It is explicitly stated in the paper that the tortuosity ratio for the non-wetting phase has "a value of zero at the minimum saturation of the non-wetting phase," i.e., at the "critical non-wetting phase saturation" referred to by Fatt. The value of saturation must be determined, of course, by independent means and, insofar as the author is aware, cannot be obtained from capillary pressure data. Such independent data also are required in the case of calculation of either non-aqueous wetting phase relative permeability or non-wetting phase relative permeability in the presence of interstitial water, in which case the minimum non-aqueous wetting phase saturation and the minimum interstitial water content are also required for the calculations (see Figs. 10, 11 and 12 of original paper).

Fatt states that the author has assumed that mercury injection data and water-oil or water-air capillary pressure data are equally useful for calculating relative permeabilities. Actually the paper, by stating that "approximate pore size distribution data may be estimated from the capillary pressure curve obtained by the porous semi-permeable diaphragm method" implies that the mercury injection technique is preferred. The author has not investigated in any detail the use of data obtained by the porous diaphragm method and is unable to generalize on the relative usefulness of the two types of capillary pressure data. It is to be presumed, however, that the data from the porous diaphragm method will prove superior to the data from the mercury method for samples where interaction between fluids and the porous matrix occurs during the course of a relative permeability determination. No experimental verification of this presumption exists, to the best of the author's knowledge.   ★ ★ ★