Advanced Petrophysics:
Quantification of Heterogeneity, Spatial Data Analysis, and Geostatistics
Part 2: Variogram Analysis and Kriging

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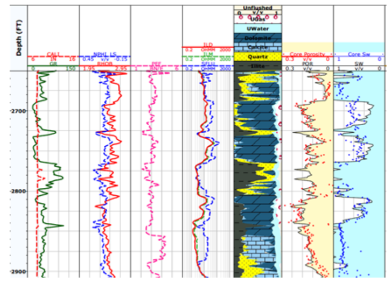
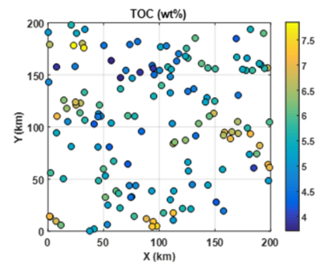
PGE381L Outline

- Introduction to petrophysics, geology, and formation data
- Porosity
- Fluid saturations
- Permeability
- Quantification of heterogeneity, spatial data analysis, and geostatistics
- Interfacial phenomena and wettability
- Capillary pressure
- Relative permeability
- Dispersion in porous media
- Introduction to petrophysics of unconventional reservoirs

What do we Learn in this Lecture?

- Spatial Analysis and Modeling
  - Quantify spatial continuity
  - Estimating reservoir properties where measurements are not available
    - Kriging

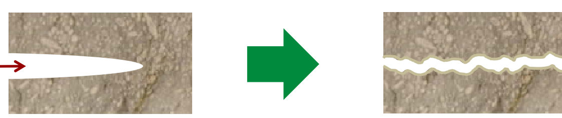




Why does Spatial Analysis Matter?

Acid Fracturing

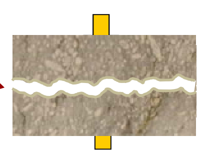
Acid



Etched Surface

Under Closure Stress:

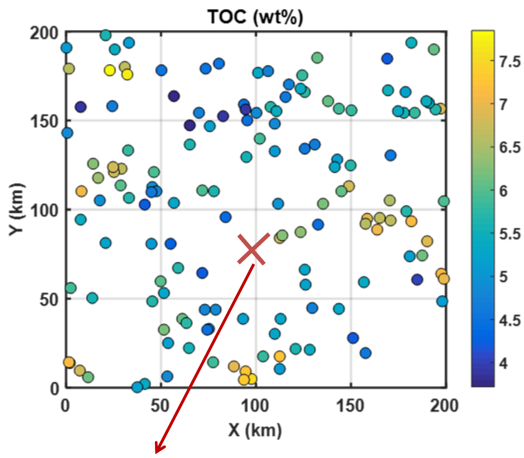
Conductive Channel



Spatial Distribution of Petrophysical & Compositional Properties

Source: Saneifar and Heidari, 2014

Why does Spatial Analysis Matter?



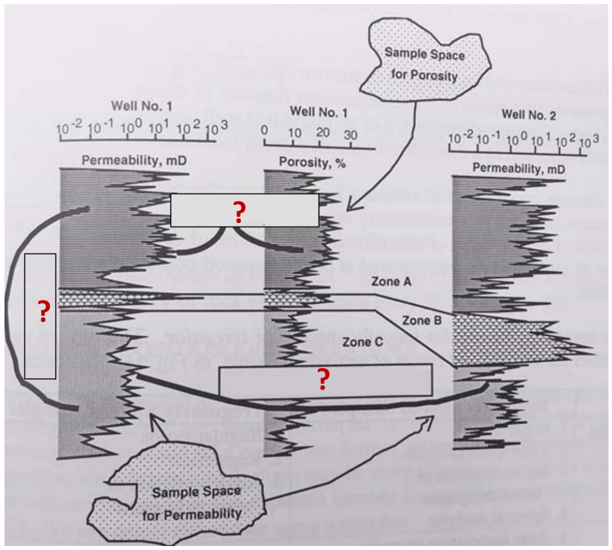
How to estimate TOC at this location?

What is the best location for drilling a new well?

Measures of Spatial Continuity

- **Measures of Spatial Continuity**
  - **Covariance function (Autocovariance function)**
  - **Correlation coefficient function (Autocorrelation function)**
  - **Variogram (Semivariogram)**

Correlation, Autocorrelation, and Crosscorrelation



Source: Jensen, J. R., Lake, L. W., Corbett P. M. W., and Goggin, D. J., 2000, Statistics for Petroleum Engineers and Geoscientists, Elsevier.

Autocovariance and Autocorrelation

The covariance function provides the strength of the linear relationship between $\Phi(x)$ and $\Phi(x+h)$

Autocovariance

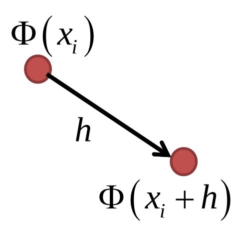
$$\mathrm{cov}(h) = C(h) = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}\left[\Phi(x_i) - \bar{\Phi}\right]\left[\Phi(x_i + h) - \bar{\Phi}\right]$$

Value of the property of interest at location x_i

Mean value

Lag distance

Number of data pairs separated by distance h



Value of the property of interest at location x_i+h

Autocorrelation

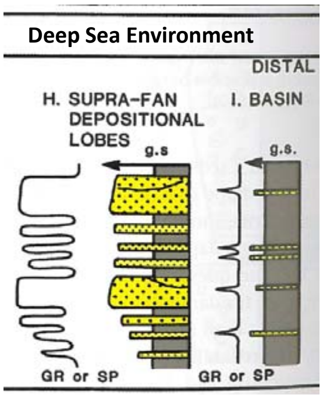
$$\mathrm{corr}(h) = \rho(h) = \frac{\mathrm{cov}(h)}{\mathrm{cov}(0)} = \frac{\mathrm{cov}(h)}{\mathrm{var}(\Phi)} \qquad -1 \le \mathrm{corr}(h) \le 1$$

Examples

- **Please see the examples uploaded on the canvas website and discussed in the class.**

Example

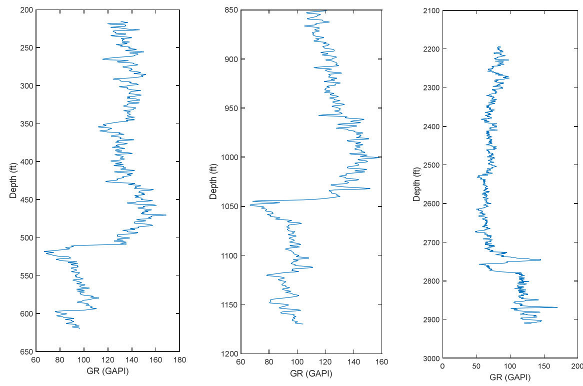
- **What is your expectation of autocorrelation coefficient as a function of lag distance in the following geologic environment?**



Source: Rider, M. and Kennedy, M., 2011, The Geological Interpretation of Well Logs

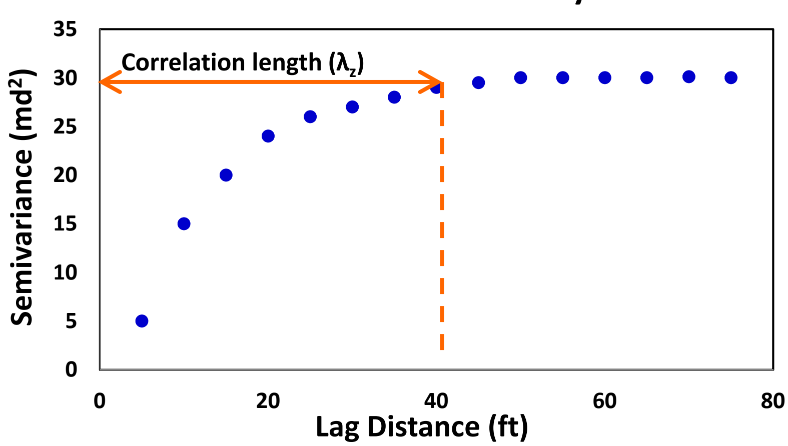
Example

- **What is your expectation of crosscorrelation as a function of lag distance for the well log data in the following three wells?**



Variogram

A variogram describes the spatial dependency/continuity of a given parameter in a one or multidimensional space. It quantifies the directions and scales of continuity.



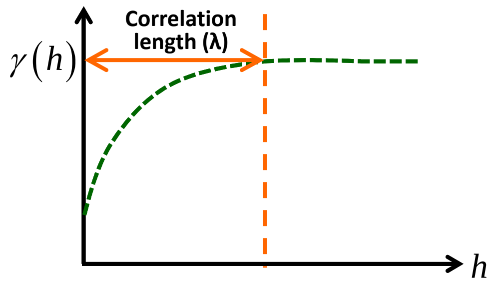
Variogram

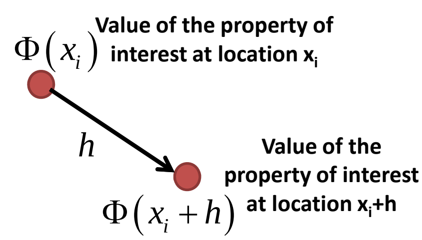
Semivariance:

$$\gamma(h) = \frac{1}{2N_h}\sum_{i=1}^{N_h}\left[\Phi(x_i) - \Phi(x_i + h)\right]^2$$

Lag distance

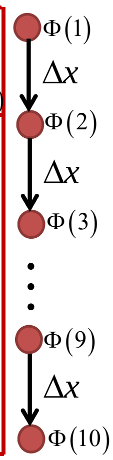
Number of data pairs separated by distance h





Step-by-Step Procedure

$h=0$, $N_h=10$, $\Phi(x_i)\Phi(x_i+h)$	$h=\Delta x$, $N_h=9$, $\Phi(x_i)\Phi(x_i+h)$	$h=2\Delta x$, $N_h=8$, $\Phi(x_i)\Phi(x_i+h)$	$h=3\Delta x$, $N_h=7$, $\Phi(x_i)\Phi(x_i+h)$	$h=4\Delta x$, $N_h=6$, $\Phi(x_i)\Phi(x_i+h)$	$h=5\Delta x$, $N_h=5$, $\Phi(x_i)\Phi(x_i+h)$
$\Phi(1)\ \Phi(1)$	$\Phi(1)\ \Phi(2)$	$\Phi(1)\ \Phi(3)$	$\Phi(1)\ \Phi(4)$	$\Phi(1)\ \Phi(5)$	$\Phi(1)\ \Phi(6)$
$\Phi(2)\ \Phi(2)$	$\Phi(2)\ \Phi(3)$	$\Phi(2)\ \Phi(4)$	$\Phi(2)\ \Phi(5)$	$\Phi(2)\ \Phi(6)$	$\Phi(2)\ \Phi(7)$
$\Phi(3)\ \Phi(3)$	$\Phi(3)\ \Phi(4)$	$\Phi(3)\ \Phi(5)$	$\Phi(3)\ \Phi(6)$	$\Phi(3)\ \Phi(7)$	$\Phi(3)\ \Phi(8)$
$\Phi(4)\ \Phi(4)$	$\Phi(4)\ \Phi(5)$	$\Phi(4)\ \Phi(6)$	$\Phi(4)\ \Phi(7)$	$\Phi(4)\ \Phi(8)$	$\Phi(4)\ \Phi(9)$
$\Phi(5)\ \Phi(5)$	$\Phi(5)\ \Phi(6)$	$\Phi(5)\ \Phi(7)$	$\Phi(5)\ \Phi(8)$	$\Phi(5)\ \Phi(9)$	$\Phi(5)\ \Phi(10)$
$\Phi(6)\ \Phi(6)$	$\Phi(6)\ \Phi(7)$	$\Phi(6)\ \Phi(8)$	$\Phi(6)\ \Phi(9)$	$\Phi(6)\ \Phi(10)$	
$\Phi(7)\ \Phi(7)$	$\Phi(7)\ \Phi(8)$	$\Phi(7)\ \Phi(9)$	$\Phi(7)\ \Phi(10)$		
$\Phi(8)\ \Phi(8)$	$\Phi(8)\ \Phi(9)$	$\Phi(8)\ \Phi(10)$			
$\Phi(9)\ \Phi(9)$	$\Phi(9)\ \Phi(10)$				
$\Phi(10)\ \Phi(10)$					



$$\gamma(h) = \frac{1}{2N_h}\sum_{i=1}^{N_h}\left[\Phi(x_i) - \Phi(x_i + h)\right]^2$$

Example

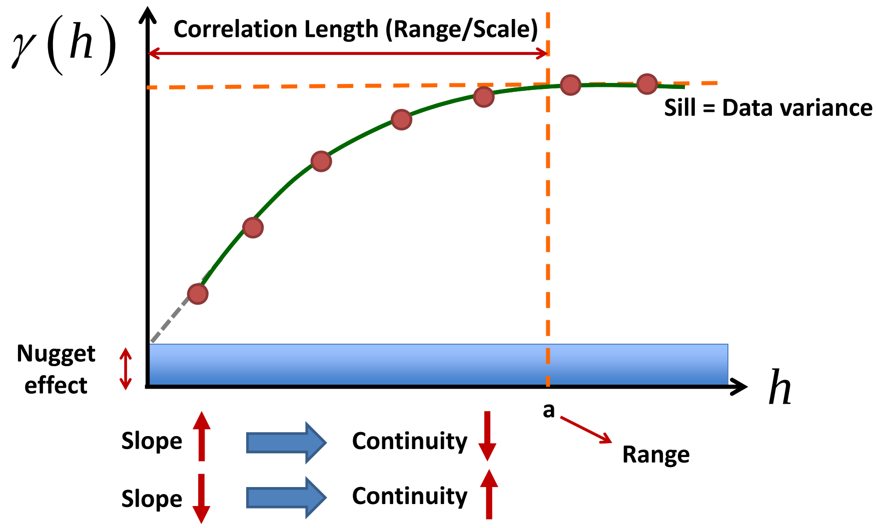
Variography

Do we have the true variogram of the spatial data in geosciences? Why?

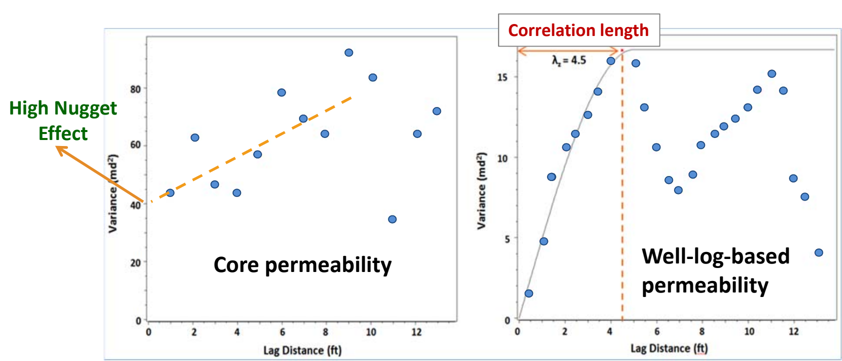
We have to estimate it from limited observations.

Variography

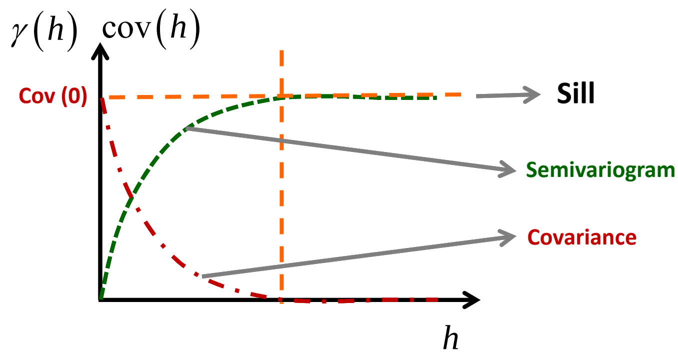
How to Analyze a Variogram?



How to Analyze a Variogram?



Covariance vs. Semivariogram



$$\gamma(h) = \mathrm{cov}(0) - \mathrm{cov}(h)$$

If the mean is constant and the covariance is independent of location

Example

- **See the 2D example, uploaded on the canvas website**
- **Take notes**

18	14	21	23	27	19	21
18	18	26	23	31	22	20
26	19	25	17	29	31	32
21	20	28	21	25	27	22
23	29	31	27	24	26	30

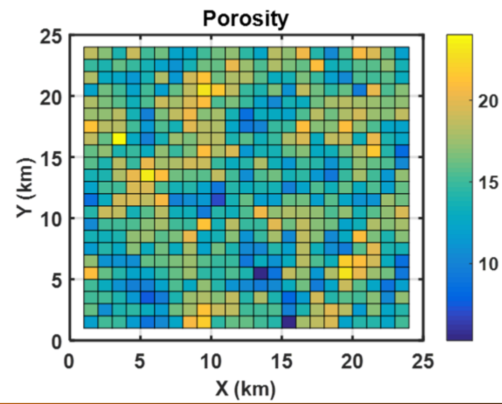
Example

Download the Porosity data set including equally-spaced porosity values in X and Y directions.

Task 1: Plot the data.

Task 2: Plot the variogram of the porosity data in X and Y directions. How do you interpret them?

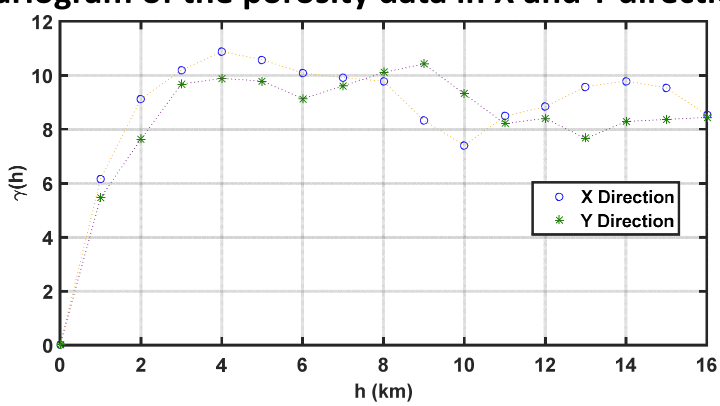
Task 3: Estimate the correlation length.



Example

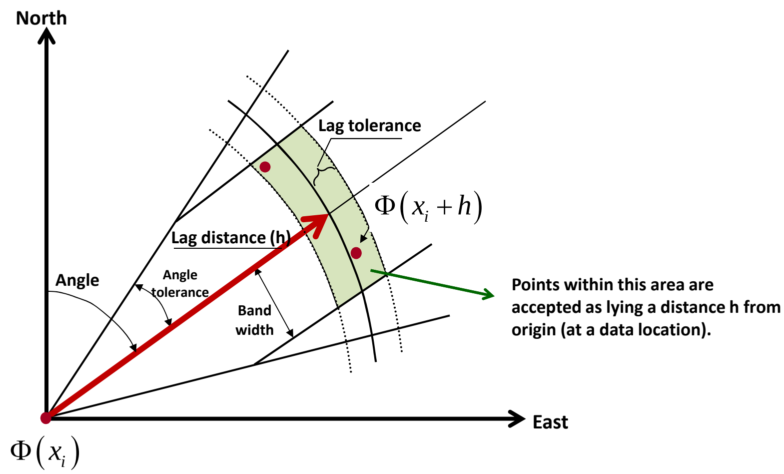
Solution:

Variogram of the porosity data in X and Y directions



Run the uploaded Matlab codes and take notes in the class.

Variogram in a Given Direction



Variogram of Irregularly-Spaced Data

Please take notes!

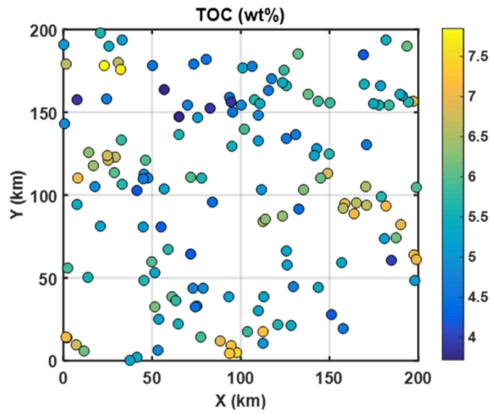
Example

Download the TOC_Spatial data set including TOC values (i.e., Z value) at given locations (i.e., X and Y).

Task 1: Plot the data.

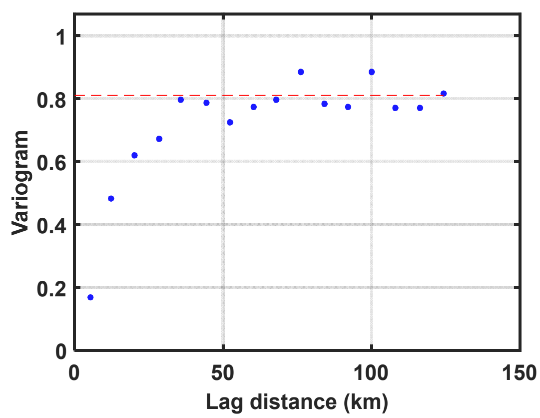
Task 2: Plot the variogram of the TOC data.

Task 3: Estimate the correlation length.



Example

Solution:



Run the uploaded Matlab codes and take notes in the class.

Analytical Variogram Models

Spherical

$$\gamma_{sph}(h) = C_0 + \begin{cases} C\left[\frac{3}{2}\left(\frac{|h|}{a}\right) - \frac{1}{2}\left(\frac{|h|}{a}\right)^3\right] & |h| \le a \\ C & |h| > a \end{cases}$$

Exponential

$$\gamma_{\exp}(h) = C_0 + C\left[1 - \exp\left(-\frac{3|h|}{a}\right)\right]$$

Gaussian

$$\gamma_G(h) = C_0 + C\left[1 - \exp\left(-\frac{3h^2}{a^2}\right)\right]$$

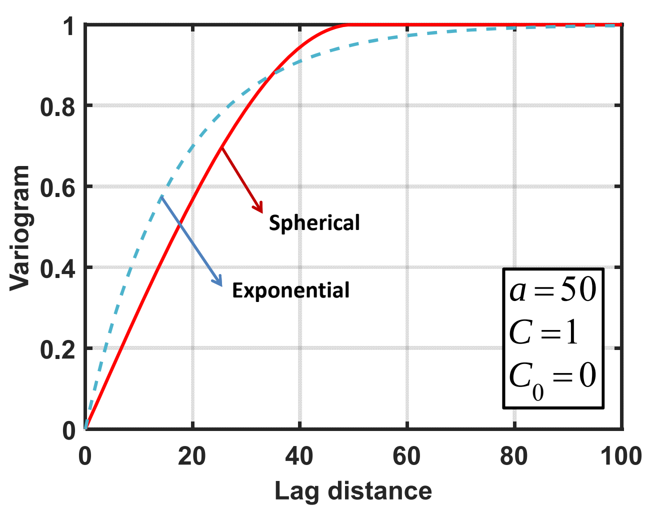
Power

$$\gamma(h) = C_0 + C|h|^{\alpha} \qquad \alpha \le 2$$

Cardinal Sine Model

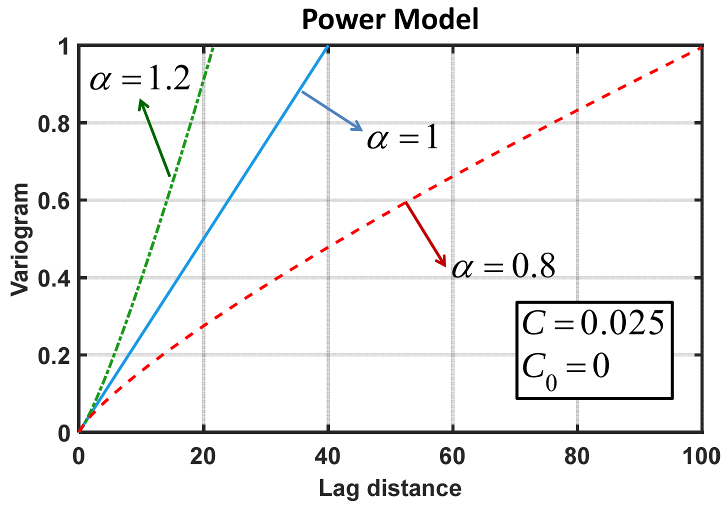
$$\gamma(h) = C_0 + C\left[1 - \frac{\sin(h/a)}{|h/a|}\right]$$

Example: Analytical Variogram Models



Use the uploaded Matlab codes and investigate the impact of different parameters on the variogram model

Example: Analytical Variogram Models



Use the uploaded Matlab codes and investigate the impact of different parameters on the variogram model

Example

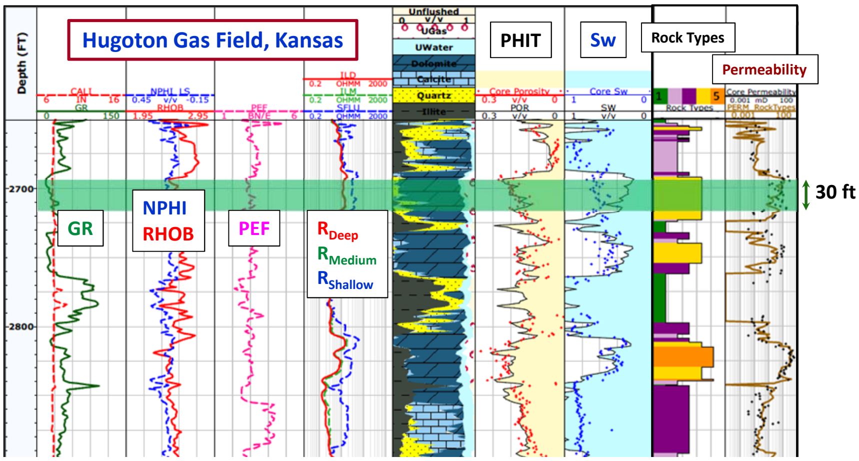
Download the TOC_Spatial data set including TOC values (i.e., Z value) at given locations (i.e., X and Y) and plot the variogram of the TOC data.

Task: Plot different analytical variogram models on top of the experimental variogram.

Solution:

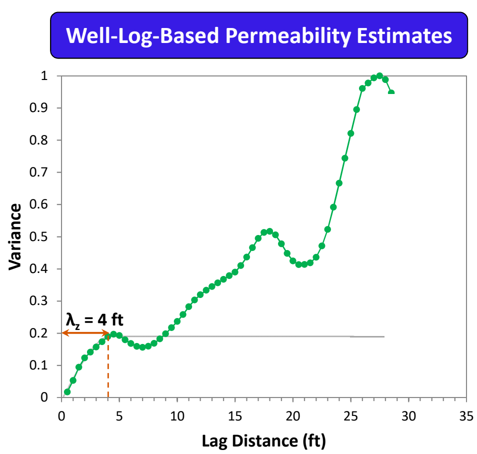
Run the uploaded Matlab codes and take notes in the class.

Example: Geostatistcal Analysis



Source: Saneifar, M., Heidari, Z., and Hill, A. D. 2015. Application of Conventional Well Logs to Characterize Spatial Heterogeneity in Carbonate Formations Required for Prediction of Acid Fracture Conductivity. *SPE Production and Operations Journal* **30** (1)

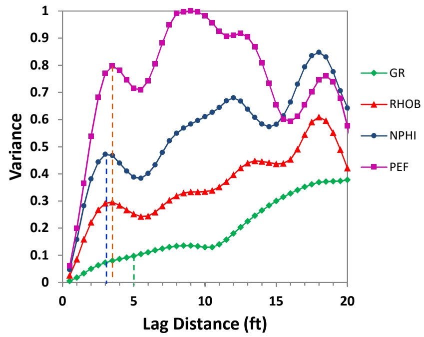
Variogram of Permeability



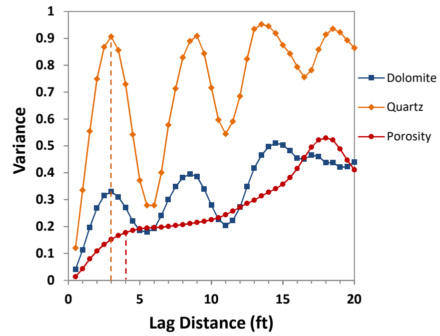
Source: Saneifar, M., Heidari, Z., and Hill, A. D. 2015. Application of Conventional Well Logs to Characterize Spatial Heterogeneity in Carbonate Formations Required for Prediction of Acid Fracture Conductivity. *SPE Production and Operations Journal* **30** (1)

Variogram of Well Logs vs. Interpretation Results

Well Logs



Estimated Porosity & Mineralogy



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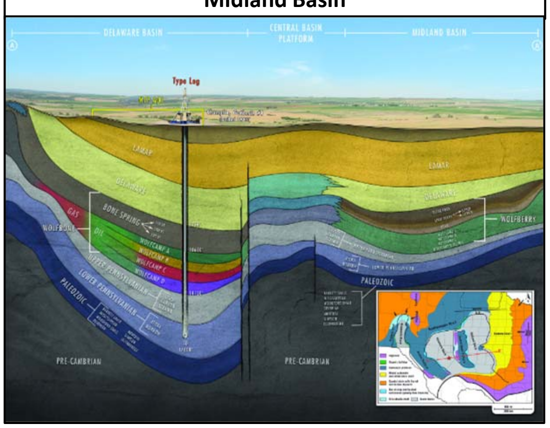
Example from the Permian Basin

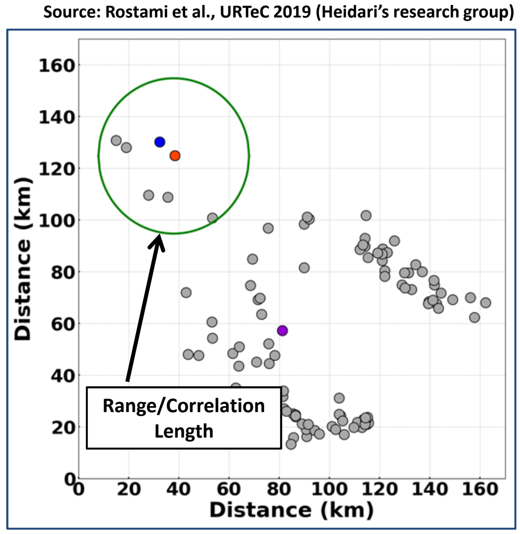
Application of formation evaluation workflows to non-cored wells

Source: Rostami et al., URTeC 2019 (Heidari's research group)

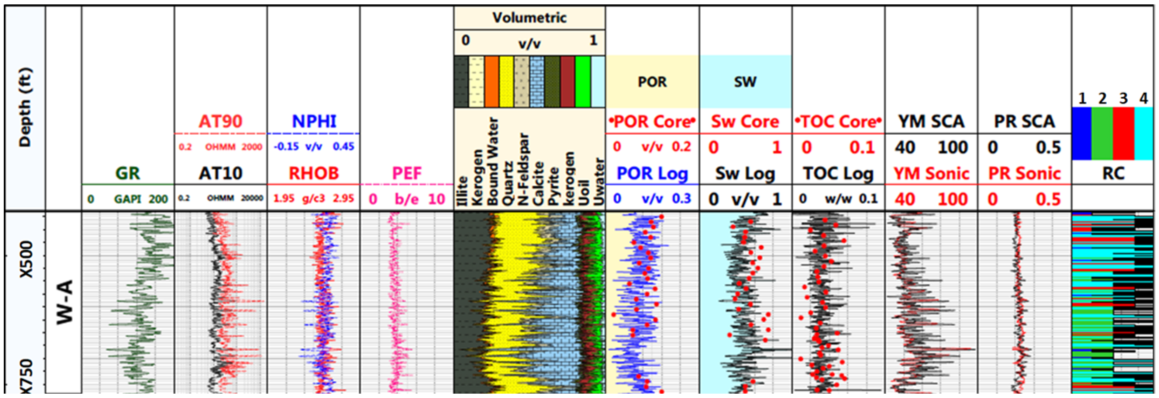
Core and Log Data from 20 Counties in Midland Basin



Source: http://tarka.com/permian-basin/

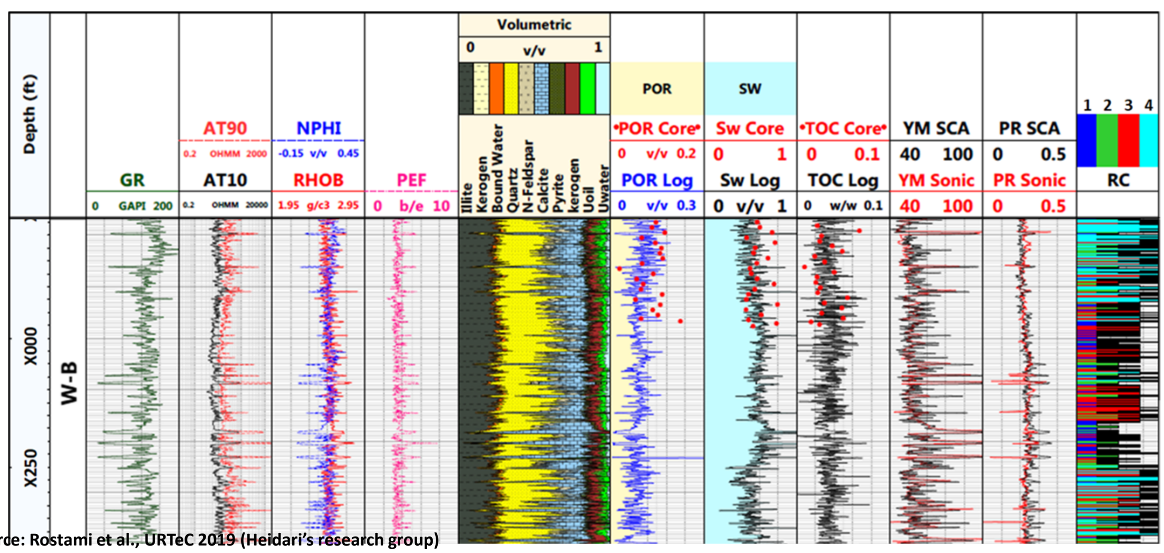


Formation Evaluation Results in a Cored Well



Source: Rostami et al., URTeC 2019 (Heidari's research group)

Formation Evaluation Results in a Cored Well



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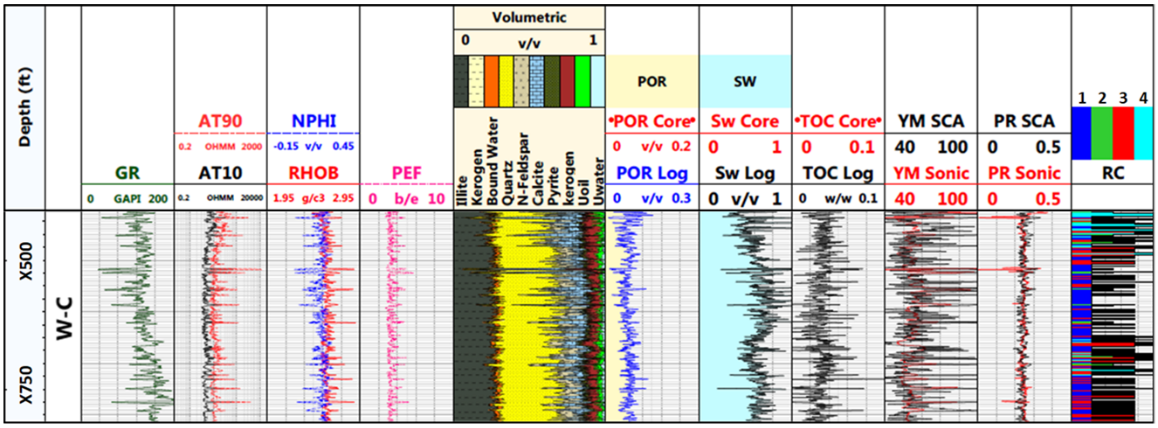
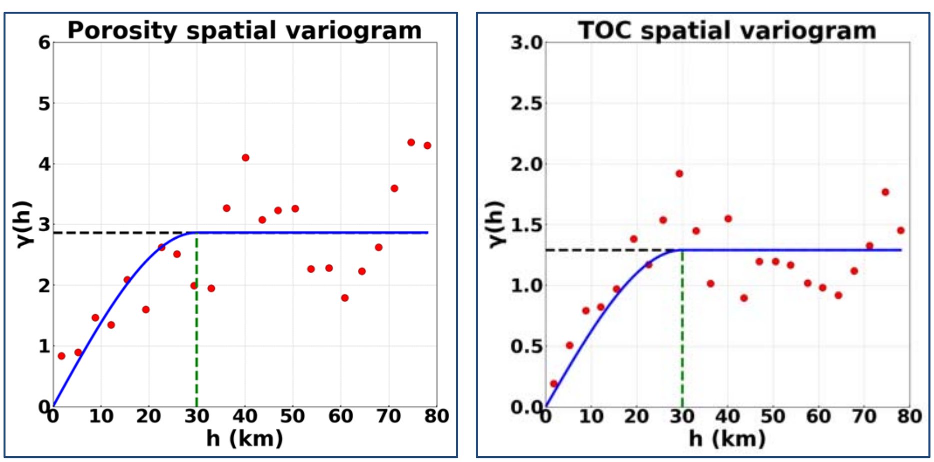
Formation Evaluation Results in a Cored Well



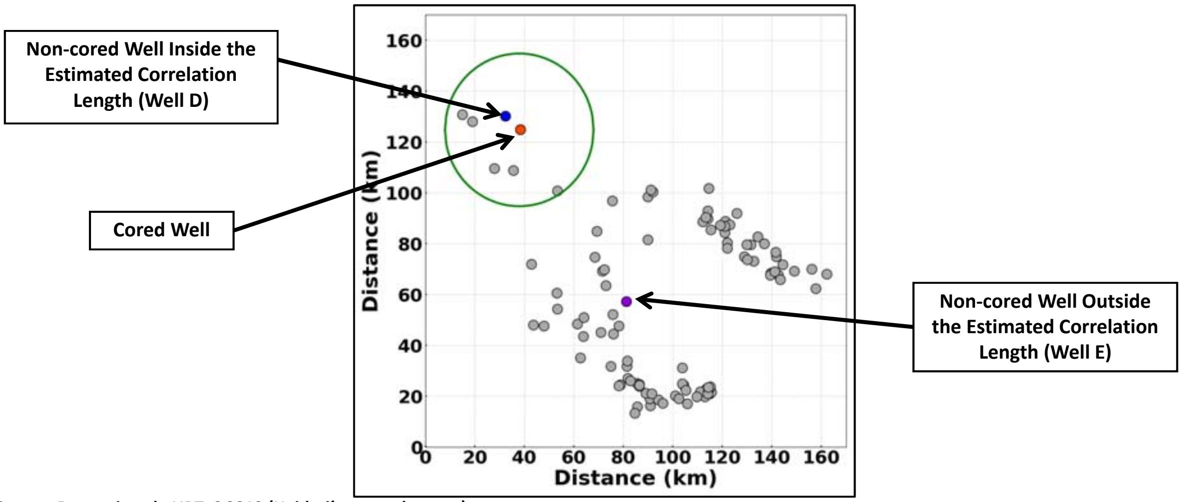
Source: Rostami et al., URTeC 2019 (Heidari's research group)

Variogram Analysis

Analyzed 106 wells, located in 4 different counties (i.e., Crockett, Glasscock, Reagan, and Upton) in the Permian basin

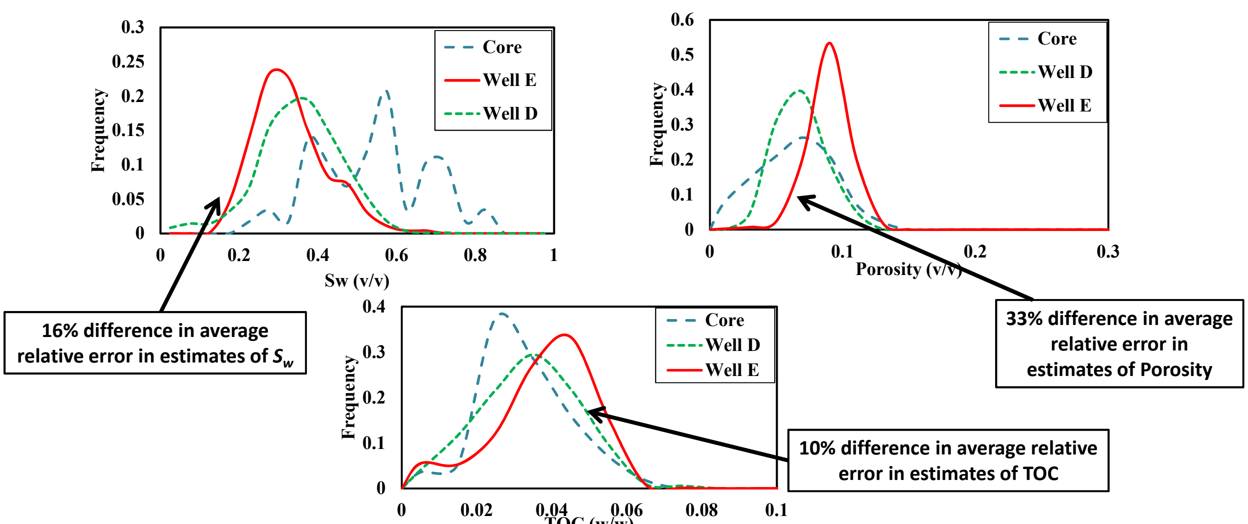


Application of Variogram Analysis in Petrophysical Evaluation of Non-cored Wells



Source: Rostami et al., URTeC 2019 (Heidari's research group)

Quantification of the Impact

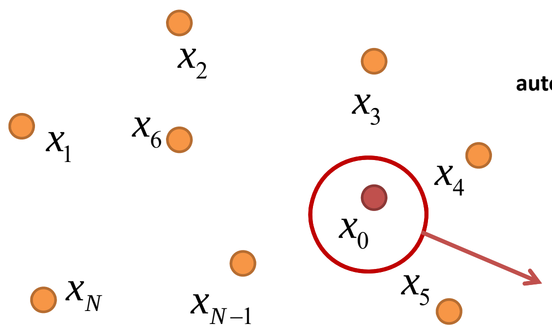


Source: Rostami et al., URTeC 2019 (Heidari's research group)

Kriging

- **Kriging is an interpolation technique for spatial data to estimate values at unsampled locations.**

We are going to use the previously discussed autocovariance/semivariogram models.



The location at which an estimation is to be made.

Kriging

- **Different types of Kriging:**
  - **Ordinary kriging**
  - **Simple kriging**
  - **Universal kriging**
  - **Block kriging**
  - **Lognormal kriging**
  - **Indicator kriging**
  - **Disjunctive kriging**
  - **Co-kriging**
  - **…**

Ordinary Kriging

A brief overview of the technique:

$$Z^*(X = x_0) = \sum_{i=1}^{N} \lambda_i Z(X = x_i)$$ **Subject to:** $$\sum_{i=1}^{N} \lambda_i = 1$$

Objective:

Minimize: $$\sigma_e^2 = E\left[\left(Z^*(x_0) - Z(x_0)\right)^2\right]$$ **Minimum error variance**

$$\sigma_e^2 = \sigma^2 + \sum_{i=1}^{N}\sum_{j=1}^{N} \lambda_i \lambda_j C(h_{ij}) - 2\sum_{i=1}^{N} \lambda_i C(h_{i0})$$

OR

$$\sigma_e^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} \lambda_i \lambda_j \gamma(h_{ij}) + 2\sum_{i=1}^{N} \lambda_i \gamma(h_{i0})$$

Subject to: $$\sum_{i=1}^{N} \lambda_i - 1 = 0$$

Ordinary Kriging

$$Z^*(X = x_0) = \sum_{i=1}^{N} \lambda_i Z(X = x_i)$$ **Subject to:** $$\sum_{i=1}^{N} \lambda_i = 1$$

Calculate λ_i in terms of autocovariance function:

$$\begin{bmatrix} C(h_{11}) & C(h_{12}) & \cdots & C(h_{1N}) & 1 \\ C(h_{21}) & C(h_{22}) & \cdots & C(h_{2N}) & 1 \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ C(h_{N1}) & C(h_{N2}) & \cdots & C(h_{NN}) & 1 \\ 1 & 1 & \cdots & 1 & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_N \\ \beta \end{bmatrix} = \begin{bmatrix} C(h_{10}) \\ C(h_{20}) \\ \vdots \\ C(h_{N0}) \\ 1 \end{bmatrix} \Rightarrow \lambda_i$$

Minimum error variance: $$\sigma_{e,\min}^2 = \sigma^2 - \beta - \sum_{i=1}^{N} \lambda_i C(h_{i0})$$

Ordinary Kriging

$$Z^*\left(X = x_0\right) = \sum_{i=1}^{N} \lambda_i Z\left(X = x_i\right)$$

Subject to: $\sum_{i=1}^{N} \lambda_i = 1$

Calculate λ_i in terms of variogram:

$$\begin{bmatrix} \gamma(h_{11}) & \gamma(h_{12}) & \cdots & \gamma(h_{1N}) & -1 \\ \gamma(h_{21}) & \gamma(h_{22}) & \cdots & \gamma(h_{2N}) & -1 \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ \gamma(h_{N1}) & \gamma(h_{N2}) & \cdots & \gamma(h_{NN}) & -1 \\ 1 & 1 & \cdots & 1 & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_N \\ \beta \end{bmatrix} = \begin{bmatrix} \gamma(h_{10}) \\ \gamma(h_{20}) \\ \vdots \\ \gamma(h_{N0}) \\ 1 \end{bmatrix} \Rightarrow \lambda_i$$

Minimum error variance: $\sigma_{e,\min}^2 = -\beta + \sum_{i=1}^{N} \lambda_i \gamma\left(h_{i0}\right)$

Example

Please take notes!

Example

Download the TOC_Spatial data set including TOC values (i.e., Z value) at given locations (i.e., X and Y).

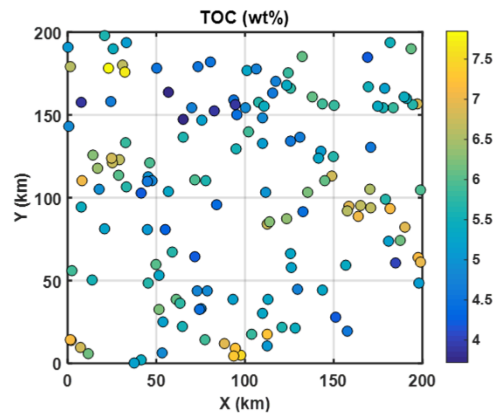
Task 1: Plot the data.

Task 2: Estimate the 2D map of TOC

Task 3: Estimate the 2D map of minimum error variance

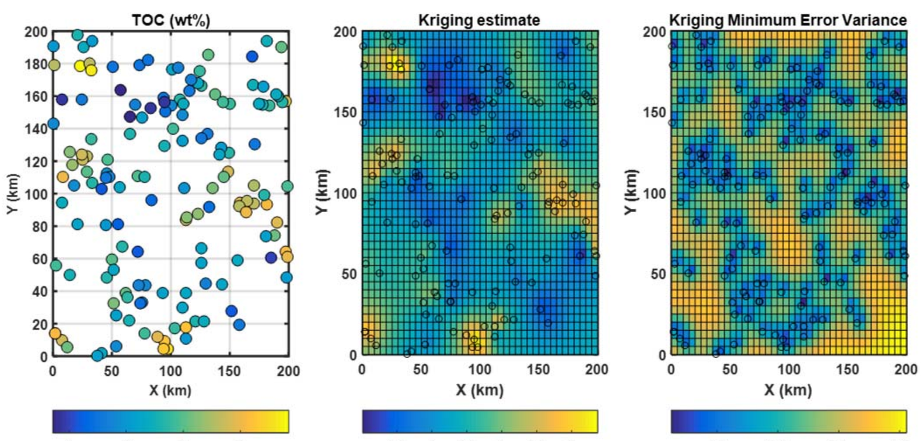
Task 4: Estimate TOC at (100,70)km

Task 5: Estimate the 95% confidence interval on the estimate of TOC at (100,70)km



Solution

Solution:



Run the uploaded Matlab code and take notes in the class.

Impact of Variogram Model on Kriging Results

- Use the data of previous example to investigate the impact of following parameters on kriging results:
  - Variogram model
  - Variogram parameters such as slope and sill
  - Nugget effect
  - Number of data points

- How can we evaluate kriging accuracy?

Complementary References

- Peters, E. J., 2012, Advanced Petrophysics. Live Oak Book Company. Chapter 4

- Jensen, J. R., Lake, L. W., Corbett P. M. W., and Goggin, D. J., 2000, Statistics for Petroleum Engineers and Geoscientists, Elsevier. Chapter 11