THE UNIVERSITY OF TEXAS AT AUSTIN

EM 397, Stabilized and Variational Multiscale Methods in Computational Fluid Dynamics (14670).

CSE 397, 1-Multiscale Methods in Computational Fluid Dynamics (62310).

(CSE 397 is the same course as EM 397. I don't know why the course names are different.)

Spring 2024

TIME: Mondays and Wednesdays, 3:00 pm - 4:30 pm

PLACE: ETC 2.102, Engineering Teaching Center building, 281 E. Dean Keaton Street



UNIQUE NUMBER: **14670** (EM397), **62310** (CSE397)

INSTRUCTOR: Tom Hughes, POB 5.430A, hughes@oden.utexas.edu

TEACHING ASSISTANTS:

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PREFERRED MODE OF COMMUNICATION: Email!

WEB PAGE: UT Canvas

BACKGROUND:

First and foremost: This will be a mathematics class. If you do not like math, in particular functional analysis, you will not like this course, so please don't take it.

Here are some background and topics to be covered. This is roughly the plan as of the moment (October 26^{th} , 2023).

Background -- Diverse problems of engineering and physics are governed by partial differential equations in space and time. Real-world problems also involve very complicated geometrical objects such as airplanes, automobiles, trains, bridges, buildings, machines, semiconductor devices, the human anatomy, etc. The finite element method was the first general approach developed for solving broad classes of partial differential equations on complicated geometries of a real-world nature. Despite this generality, and the success of the finite element method on a variety of problem classes, shortcomings have been noted in several areas of considerable practical importance, most notably fluid mechanics. The origin of the problem is with the variational formulation that has been the generally accepted basis of finite element discretizations, that is, the Galerkin method. This method is a member of a class of so-called "weighted residual methods" and it possesses optimal properties of approximation for specific symmetric, positive-definite operators such as those arising in heat conduction and elasticity. However, when applied to problems in which the operators are dominantly skew-symmetric, such as those arising in many fluid mechanical applications, the optimal properties of the Galerkin method are lost, and often, very poor approximation properties are observed.

It has been noted that the deficiency of the Galerkin method in fluid mechanical applications can be traced to its very poor numerical stability behavior for skew-symmetric operators. Classical ad hoc procedures based on artificial viscosity and upwinding technique improve stability, but seriously degrade accuracy. For many years the fundamental open problem in computational fluid dynamics was to develop procedures that combined good accuracy and stability properties. A general solution to this problem was achieved by early renditions of what has become known as "stabilized methods." The theory of stabilized methods begins with the Galerkin method but incorporates certain additional terms that preserve the weighted residual format, thus retaining underlying accuracy, while at the same time enhancing stability. In recent years, it has been shown that stabilized methods can be derived from a multiscale formulation of the exact variational equations, a somewhat surprising result. The upshot is a framework for constructing robust and general methods applicable to a wide variety of physical problems including, but not limited to, fluid mechanics.

Topics to be covered -- This course will focus on advective-diffusive equations, the Boltzmann equation, and compressible and incompressible Navier-Stokes flows. It will be mathematically oriented and convergence results and error estimates will be presented. The pedagogical approach taken will be to start with Galerkin discretizations and take them as far as they can go. Then, the variational multiscale method will be employed to create stabilized methods. The emphasis in this course will be on the Boltzmann equation, and its relationship with the Navier-Stokes equations of compressible and incompressible flows. Our goal is to understand the relationship of the Boltzmann approach to fluid mechanics to the standard continuum compressible and incompressible Navier-Stokes equations.

LEVEL:

This is an advanced course aimed at PhD students in engineering, computer science, mathematics, and the physical sciences interested in developing new insights and skills that they may apply to their research.

PREREQUISITES:

There are no formal prerequisites, but the course will require mathematical maturity, including an acquaintance with linear algebra, advanced calculus, elementary functional analysis, finite differences and finite elements, and ordinary and partial differential equations, things all CSEM and EM students should know.

TEXT:

Notes will be made available to enrolled students and some research papers will be provided.

HOMEWORK POLICY:

There will be frequent assignments. Homework will be due at the end of the class on the due date. The homework will be graded.

TESTING AND EXAMINATION PLAN AND POLICIES:

There will be no tests or exams.

GRADING POLICY:

Grades will be based on the homework.

CLASS FORMAT:

Lectures.

ATTENDANCE:

This course will be taught live. Attendance at lectures is expected. I may also record the lectures, but I may not. My experience with recording classes post-Covid means only half the class shows up.

EVALUATION:

The course and instructor will be evaluated at the end of the semester using the approved procedure.

COMPUTER:

Any programming language or software may be used for this class. CLASS WEBSITES AND STUDENT PRIVACY:

Web-based, password-protected class sites are associated with all academic courses taught at The University. Syllabi, handouts, assignments and other resources are types of information that may be available within these sites. Site activities could include exchanging e-mail, engaging in class discussions and chats, and exchanging files. In addition, electronic class rosters will be a component of the sites. Students who do not want their names included in these electronic class rosters must restrict their directory information in the Office of the Registrar, Main Building, Room 1. For information on restricting directory information see:

http://registrar.utexas.edu/students/records/restrictmyinfo

DROPPING COURSES:

January 31. Last day to drop a class without permission. Last day a graduate student may, with required approvals, add a class.

April 15. Last day a graduate student may change registration in a class to, or from, the credit/no credit basis.

April 29. Last day a graduate student may, with the required approvals, drop a class or withdraw from the University.

An engineering student must have the Dean's approval to add or drop a course after the fourth class day of the semester or after the second class day of a summer term. Adds and drops are not approved after the fourth class day except for good cause. "Good cause" is interpreted to be documented evidence of extenuating nonacademic circumstances (such as health or personal problems) that did not exist on or before the fourth class day. Applications for approval to drop a course after the fourth class day should be made in the Office of Student Affairs, Ernest Cockrell, Ir. Hall 2.200.

SPECIAL NOTES:

The University of Texas at Austin provides upon request appropriate academic adjustments for qualified students with disabilities. For more information, contact the Office of the Dean of Students at 471-6259, 471-4641 TDD or the college of Engineering Director of Students with Disabilities at 471-4321.