Advancements and Perspectives in Embedded Discrete Fracture Models (EDFM)

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ABSTRACT

The Embedded Discrete Fracture Model (EDFM) has emerged as a prominent technology for embedding the hydraulic behavior of rock joints in reservoir numerical models. This paper critically reviews the latest developments and opportunities for further research. The literature is extensive regarding novel algorithms attempting to reach more accurate and computationally effective estimates. While hydraulic fracture models seem suitable for their purposes, their assumptions are excessively simplistic and unrealistic when assessing naturally fractured reservoirs. The paper begins by examining fractures as physical characteristics and the key mechanisms to be considered when integrating them into numerical simulators. The use of the EDFM technique shows promising for simulating capillary continuity and buoyancy effects in a multiphase, multicomponent case. However, there are significant limitations that hinder its widespread adoption in field-scale for reservoir performance evaluation. Lastly, there is a lack of public-domain realistic benchmarks to validate and compare the potential of each method.

Keywords: EDFM, Naturally Fractured Reservoirs, Embedded fractures, Discrete Fracture Models, Reservoir flow simulation, Numerical methods

INTRODUCTION

Significant hydrocarbon reserves are produced from Naturally Fractured Reservoirs (NFR) and tight plays developed with Hydraulically Fractured (HF) wells. Using numerical models to characterize, forecast, and support design decisions is currently common practice. Numerical models are tools for engineers and geoscientists to test hypotheses and increase understanding of reservoir behavior in an effort to drive sensible decisions and communicate with stakeholders. The modeling technique depends on data availability and the physical understanding of the problem under analysis (Starfield and Cundall, 1988). The design tradeoff in discrete fracture modeling is preventing excessive complexity without denying the existence and relevance of the joints. The design starts from the physical description of the joints, which is unique for each accumulation. For instance, Nelson (2001) classified fractures as essential for drainage (type 1), as permeability enhancement features (type 2 or 3), or as a negative impact fluid flow as barriers creating strong flow anisotropy (type 4).

The fundamental classification of fractures regards their genesis, whether they are originally present in the field as Natural Fractures (NFs) or artificially man-made features as Hydraulic Fractures (HF). While NFs are sparsely distributed on the reservoir domain, Hydraulic Fractures (HF) are stimulation techniques to enhance the well-reservoir coupling. The literature is comprehensive in both cases. We emphasize the importance of considering HF and NF in distinct ways because of their unique spatial position and geometry with respect to the field pressure gradient distribution.

There is reasonable information on fracture attributes for HF modeling: their geometries are anticipated as a design parameter, and their permeabilities are a consequence of the proppant or can be estimated from the residual aperture. While dealing with NFs, however, the fracture network geometry and hydraulic conductivity are highly uncertain, fundamentally based on geosciences conceptualization and analogs. Unlike intact rock, it is impractical to sample fractures or keep their in situ conditions behavior on the way to the laboratory. Hence, numerical calibration eventually relies on production data that cannot characterize individual joints but rather the overall behavior of the network as an equivalent continuous. The lack of physical meaning of the parameters may lead history-matching into unpredictive results Sahimi (1993).

Intuitively, one cannot expect a single model to fit all possible NF scenarios. A thorough geologic understanding of the genesis of the fractures is the definite starting point. A multidisciplinary team must investigate and promote sensible data acquisition from production instruments, logs, laboratory, and well-testing. Data interpretation and statistical analysis must be used sensibly together as drawing conclusions over deterministic or long single runs is misleading, as they offer little or no predictability (Nelson, 2001). Hence, probabilistic models, sensitivity analyses, and history matching are preferred and demand fast optimization loops.

The challenge is, therefore, to navigate existing technologies to enable physically consistent fracture-aware workflows, providing fast assessment cycles while coping with high uncertainties. While we understand each technology has its particular application niche, we focus our attention on recent progress in developing Embedded Discrete Fracture Models (EDFM). EDFM has received significant attention in the past decade, with significant advancements reported. The idea to embed fractures as nonconforming entities into existing models has proven to accurately solve field-scale models with notable performance (Yu et al., 2021).

The paper is organized as follows: the next section briefs the physical conceptualization of fractures and rock joints and the relevant phenomena to model in the numerical counterparts. Then, a critical review of the history and the state-of-the-art of fracture models and EDFM is presented, acknowledging recent progress and identifying open issues and areas of research interest. While geological, geophysical, and data acquisition workflows and techniques are essential to a complete understanding of the topic, they are outside the scope of this manuscript.

Fractures as physical features

Fractures are defined as *breaks or mechanical discontinuities in rock that consist of two rough surfaces in partial contact (...) complex in shape and often filled with mineral precipitates or transport materials* (Viswanathan et al., 2022). Their occurrence is linked to deformation after mechanical stress or to physical diagenesis (Nelson, 2001). If connected void spaces are present along the fracture, they may comprise low porosity fluid conduits, potentially being a primary drainage mechanism of the reservoir. However, when cracks are filled with fine impermeable material, the fluid transmissibility is significantly penalized perpendicularly to its faces.

A Naturally Fractured Reservoir (NFR) is defined as a reservoir in which *naturally occurring fractures either have or are predicted to have a significant effect on reservoir fluid flow* (Nelson, 2001). Narr et al. (2006) classifies NFRs according to the relevance of the fractures to the fluid flow: Type 1 - fractures provide essential porosity and permeability; Type 2 - fractures provide essential permeability; Type 3 - fractures provide permeability assistance. Identifying the NFR type is the first step to optimally translate the fractures into the models. In type 1, for example, the fracture network controls the overall reservoir drainage, likely with fluid channeling and slow matrix imbibition; in type 3, on the other limit, fluid flow is likely to occur more homogeneously, as matrix-matrix flow is non-negligible.

Man-made HFs are engineering-designed and positioned across the well to enhance well productivity or injectivity. They are mostly considered tensile in nature and are supported either by added proppant or wall roughness in case of nonuniform acidizing. The uncertainties in HF geometry are bounded by operational parameters, analog wells, well logs, and pressure tests. It is reasonable to assume planar geometry, although some authors show HF is rarely planar and is, in fact, a complex network to be calibrated to an effective planar representation (Fisher et al., 2005; Gale et al., 2024; Guerrero et al., 2022; Manchanda et al., 2020).

Natural Fractures (NF) studies present absolutely distinct challenges. NF genesis is multifactorial, resulting in wide ranges of uncertainties and site-dependent behavior (Lee et al., 2001; Bourbiaux et al., 2002). NF can be cemented or open, span from seismic to millimetric scales, be interconnected or standalone, etc., so that rules of thumb do not apply (Aguilera, 1998). Authors have tried for many years to find correlations to understand the hydraulic behavior through estimates of fracture aperture, rugosity, contact area, and geometry with limited success (Viswanathan et al., 2022; Candela et al., 2012). Nevertheless, the lubrication theory (also known as the *cubic law*) is often used to characterize fluid flow inside a NF, even though the complexity of NF networks departs by far from the assumptions behind the equations. In fact, recent work shows that the cubic law estimates result in up to 100 times excessive transmissibility and cannot forecast the varied degrees of flow anisotropy seen in a typical fracture network (Frash et al., 2019; Oliveira et al., 2019).

As HF are designed to be major fluid channels to the well, large pressure gradients are expected from the fracture to the surrounding matrix. In this case, cubic law may approximate reasonably and viscous fluid transport is likely to control the fracture-matrix fluid exchange. In NF, on the other hand, pressure gradients are typically not expected to be significant across matrix blocks due to the small pressure gradient along the surrounding fractures. In these cases, capillary continuity, buoyancy, and multiphase interaction physics become more relevant than Darcy's straightforward viscous flow. Designers must now consider various mechanisms, like counter- and co-current flows and capillary continuity. (Blunt, 2017; Firoozabadi, 2000; Bina et al., 2020). Although intrinsically scale-dependent, building up a solid laboratory test program is the key to understanding the reservoir's fundamental drainage mechanisms, and results must be used after upscaling considerations (Horie et al., 1990;

Lemonnier and Bourbiaux, 2010).

Fractures as numerical entities

Mathematical models of fractured reservoirs date back to the 1960s (Barenblatt et al., 1960; Warren and Root, 1963). The early models represented fractures as a collection of joints with regular geometry approximated by an effective continuum. Barenblatt et al. (1960) understood the need to deal with the fracture collection as a whole, considering it impractical to model each fracture individually. At the time, the models were targeted to transient well test interpretations of naturally fractured conventional reservoirs, and authors could define two numerical parameters to describe the network dynamics: a characteristic time and a characteristic distance. After that, following the evolution and commercial adoption of flow simulators, more interest was seen in numerical models for fractures, using the idealized concepts as a basis for the framework.

Long et al. (1982) discussed the scale of the fractures and their interconnectivity in high permeability large-scale paths. From the author's perspective, single fractures are only relevant as segments of large features and won't significantly impact global system drainage unless combined into large-scale effective elements. Kazemi et al. (1976) and Gilman and Kazemi (1983) extended the model to multiphase to account for capillary forces, which were not considered so far, and formulated the multiphase mathematical background still used today in modern simulators.

The multiphase physics was further investigated by Hinkley and Davis (1986) and more recently by Elputranto and Yucel Akkutlu (2020), who was concerned with capillary end effects in fractured tight rocks. The issue is that the capillary pressure discontinuity, as idealized in early models, may mislead the analysis. As natural fractures are distributed irregularly, much more capillary continuity is expected than in idealized geometries. That means one can expect a significant contribution from gravity-driven co-current flows and an environment more favorable to recovery factor (Firoozabadi, 2000; Cardwell Jr and Parsons, 1949; Labastie, 1990; Horie et al., 1990).

A Discrete Fracture Network (DFN) is a geometrical description of a set of fractures and their interconnections, explicitly and individually. While the literature is not uniform in classifying the different numerical approaches to modeling a DFN and its impact on drainage, this work divides fracture models into two major classes: (1) conforming and (2) non-conforming. In conforming fracture models, the features are explicitly represented in the numerical mesh. In contrast, in non-conforming models, they are embedded as an equivalent continuum with no impact in the original mesh.

Conforming meshes algorithms represent the fractures by local grid refinements (LGR) (Henn et al., 2000; Bourbiaux et al., 2002) or as elements reduced (or mixed) in dimensions – i.e., 2D elements in a 3D environment or 1D elements in a 2D environment. In LGR, the low porosity and high permeability of the narrow elements representing the fractures have a high computational cost and are impractical for most applications. Improvements and practical aspects given simulation time and numerical limitations such as effective fracture aperture are discussed in detail by Reiss (1980).

Using elements of reduced dimensions benefits from the fact that the fracture aperture is orders of magnitude smaller than its length and height and that no significant pressure drop or flow is expected inside the fracture along the thin axis. Hence, this dimension may be reduced analytically. Still, this approach is only suitable for small domains due to the high computational when many fractures are to be mapped. Moreover, just like any conforming algorithm, any modification in fracture geometry requires full re-meshing and costly model re-processing, which harms iterative workflows.

Non-conforming approaches are preferred in field scale models where performance is critical, and fracture geometries carry high uncertainty. In this case, fractures are embedded in the continuum either as an effective medium or by introducing additional Representative Elementary Volumes (REV) for each fracture, such as in classic dual porosity and dual permeability models (DKDP) or embeded models, like EDFM.

DKDP was proposed originally by Barenblatt et al. (1960) and applied to well-testing interpretation by Warren and Root (1963). The idea was to include two equivalent media (matrix and fractures) with the same gridblock distribution and size. Transfer functions (or shape factors) were originally proposed by Gilman and Kazemi (1983) and Kazemi (1969) to quantify the communication between the two media.

In all these cases, upscaling processes must be used to find the effective continua parameters for the DFN (mainly porosity, permeability and shape factors), which are later calibrated during data assimilation (interchangeably known as history matching), as by Long et al. (1982); Oda (1985); Ahmed Elfeel and Geiger (2012).

Accurate results have been reported in conforming and non-conforming models for large-scale fractures with known geometries. In the case of diffuse fracture geometries, many features are present, and discrete fracture models may add too many degrees of freedom to the model. As the approach works for small models, field-scale models may become overcomplex and counterproductive, and design teams may lose intellectual control of the model. In this case, embedding fractures in the

Table 1. Relevant fracture models published in the literature.

Non-conforming	Effective continuum model	(Wu, 1999)
	Warren and Root	(Barenblatt et al., 1960; Warren and Root, 1963)
	Multiporosity	(Bai et al., 1993; Abdassah and Ershaghi, 1986)
	MINC	(Pruess and Narasimhan, 1985)
	EDFM	(Moinfar et al., 2013)
	pEDFM	(Ţene et al., 2017)
	cEDFM	(Chai, 2018)
	XFEM	(Fumagalli and Scotti, 2012)
Conforming	Lower-dimensional elements	(Karimi-Fard et al., 2004; Oda, 1986)
	Local Grid Refinement (LGR)	(Henn et al., 2000; Bourbiaux et al., 2002)

continua either by classic models or upscaled effective discrete models is generally more sensible.

For completeness, Table 1 lists popular fracture models, but details of each are beyond the scope of this paper. Recent benchmarks on them are found in Berre et al. (2021) and Flemisch et al. (2018).

EDFM formulation

This section reviews the EDFM history and common grounds. Extensive discussion and validation of the technique are found in Sepehrnoori et al. (2020); Moinfar et al. (2013); Xu et al. (2017).

To our knowledge, the first use of the EDFM principles was proposed by Hearn et al. (1997) and, a few years later, by Lee et al. (2001) and Li and Lee (2008). At the time, formulations were limited to Cartesian meshes and simplified fracture geometries. Moinfar et al. (2013) expanded the idea for 3D with inclined fractures and proposed the EDFM naming for the method family. In the coming years, many researchers expanded EDFM in different directions by incorporating more complex mesh setups and fracture geometries, chemo-thermo-mechanics, and new formulations in various target applications.

EDFM workflow starts by characterizing the fracture network as a set of discrete lower dimensional entities (2D in a 3D model or 1D in a 2D model). Additional Representative Elementary Volumes (REV) are assigned for each effective discrete entity and the new REVs are then analytically connected to the geometrically co-located elements by redefining element connectivity. As the fracture REVs are not present in the mesh of matrix elements, special non-neighboring connections link the fracture REVs to the original matrix ones. Conventional well-established reservoir flow simulators can then solve the new model in such a way that the original EDFM engines can be seen as a preprocessor.

The elementary entity of the method is a fracture segment, defined as the slice of a given fracture intersecting a matrix block. The segment will be associated with a new REV, with an assigned fluid volume and connectivity to the matrix blocks. The REV porosity represents the fluid volume and is defined as

$$\phi_f = \frac{wS}{V_b}\,, \tag{1}$$

where w is the fracture aperture, S is the surface area of the segment, and V_b is the bulk volume of the numerical element in which the segment will be represented.

We remark that, in field scale models, w is in the order of millimeters, whereas V_b is typically in the $10^2 - 10^6 m^3$ range. Hence, ϕ_f will tend to be small ($< 0.01\%$) and drive the solver into numerical issues. It is a common practice to set a lower bound to ϕ_f to avoid numerical issues.

The additional connectivity (or transmissibility) to be added to the model must account for (i) the fluid flow throughout the interconnected fracture segments T_F; (ii) the fluid flow between intersecting fractures T_I; and (iii) the fluid flow from the fracture to the surrounding matrix block T_{FM}.

The flow equations formulation for two segments of the same fracture follows a two-point flux approximation that is

$$T_F = \frac{T_i + T_j}{T_i T_j}\,, \qquad T_i = \kappa_f \frac{A_i}{d_i}\,, \qquad T_j = \kappa_f \frac{A_j}{d_j}\,, \tag{2}$$

where T_i and T_j are the transmissibility of each segment to their contact, κ_f stands for the fracture permeability, A_i is the contact area between the two segments, and d_i is the distance from the fracture centroid to the common face.

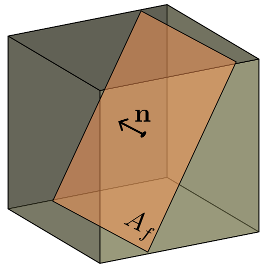

Figure 1. A fracture segment crossing a matrix block.

In case of fracture intersection, the transmissibility is

$$T_I = \kappa_f \frac{w_i L_i}{d_i j}, \qquad d_{ij} = \frac{\int_{S_i} x_n dS_i + \int_{S_j} x_n dS_j}{S_i + S_j}, \tag{3}$$

where w_i is the fracture aperture, L_i is the length of the intersecting segment, and d_{ij} is the distance from the segment centroid to the intersection line j.

Finally, the transmissibility from the fracture to the surrounding matrix block is geometrically estimated as

$$T_{FM} = \frac{2A_f(\kappa_M \cdot \mathbf{n}) \cdot \mathbf{n}}{d_{FM}}, \qquad d_{FM} = \frac{\int_V x_n dV}{V}, \tag{4}$$

where A_f is the area of the fracture segment open to flow, $\mathbf{n}$ is the vector normal to the fracture surface, κ_M is the matrix permeability tensor, d_{FM} is the normal average distance between the fracture and the matrix, x_n is the shortest distance measured from each infinitesimal volume in V to the fracture plane (see Fig. 1).

As described, the original developments in EDFM could represent planar fractures in 2D Cartesian meshes. Later, the technology was deployed towards more complex mesh descriptions, namely corner point (Xu et al., 2019) and unstructured grids (Xu et al., 2020). The novel formulations enabled mixed representations: that is, fractures can now be represented by lower dimensional and embedded elements on the same numerical framework. Moreover, preexisting models may now be used as bases for EDFM studies, avoiding costly mesh conversions.

Limitations on fracture geometry were overcome by (Xu et al., 2017). The authors validated EDFM on the representation of complex natural fracture networks and complex networks of hydraulic fractures whose geometry was estimated by numerical simulators.

Low-permeability fractures

As discussed previously, fracture simulation techniques were originally thought of to incorporate additional flow permeability into the existing matrix REVs. However, natural fractures appear in different flavors: while continuous open joints enhance fluid flow, cemented fractures are restrictions.

The ability to deal with low-permeability fractures using EDFM was proposed by Ţene et al. (2017) by the name of Projection-based EDFM (pEDFM). The idea is to penalize the original Matrix-to-Matrix transmissibilities at the cell interfaces (Fig. 2a). pEDFM adds value to history matching and uncertainty assessment, as the overall mesh is kept static, enabling fast optimization loops even with extreme anisotropy ratios.

pEDFM projects the fracture path at the matrix cells' interfaces along each dimension x_e, with surface areas $A_{if\perp x_e}$. The M-M transmissibilities are

$$T_{ii_e} = \kappa_{ii_e} \frac{A_i i_e - A_{if\perp x_e}}{\Delta x_e} \tag{5}$$

where κ_{ii_e} is the fluid mobility between the matrix cell i and its neighbor in direction x_e. The fracture transmissibilities enhancements are similar to the base EDFM technique, except that they are projected to the matrix cell interfaces as

$$T_{i_e f} = \kappa_{i_e f} \frac{A_{if\perp x_e}}{d_{i_e} f}. \tag{6}$$

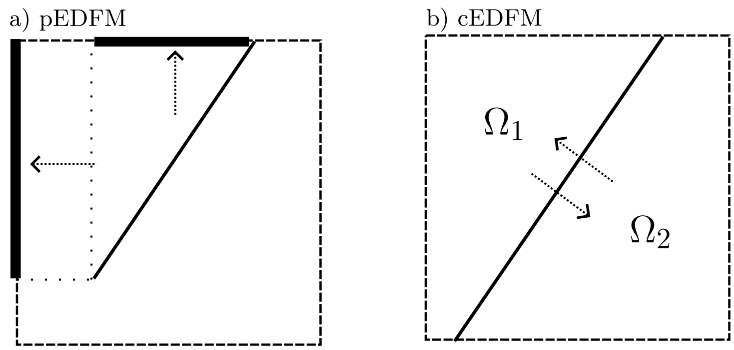

Figure 2. A fracture segment crossing a matrix block.

The limiting condition of a fluid-blocking fracture would represent a split between the reservoir volumes (compartmentalization). As it makes sense, a significant error emerges if large cells are used. One might also consider that volumes might be confined between two blocking fractures if they are mapped to opposing faces of the volume, which would not be physical or would drop excessive volume apart. Such an approach suggests that pEDFM requires fine meshes near the fluid-blocking fractures, for which Li et al. (2023) investigates a solution with adaptive re-meshing.

Chai (2018) advances in that direction with the Compartmental EDFM (cEDFM), which enables unstructured cell volume split (2b). In cEDFM, the matrix REV is split into separate domains, one on each fracture side, following unstructured subgrids. The splitting process conceptualizes a connection map in which the REVs are represented as nodes and the transmissibilities as edges. Details of the cEDFM formulation follow the same fundamentals of EDFM and are out of the scope of this manuscript.

Although cEDFM was validated in a proprietary simulator with the freedom to modify the core code, it seems possible to implement it as a preprocessor of commercial simulators, following the original EDFM strategy. However, the number of new volume domains increases with the network's complexity. Upscaling techniques starting from the cEDFM connectivity graph look promising, but we are unaware of any work on the subject.

Enhanced transmissibility calculation

Significant effort has been devoted to improving transmissibility calculations. It is the case in the Integrally EDFM (iEDFM) (Shao and Di, 2018), where the fracture network inside a given block is considered altogether in a simple yet powerful upscaling technique. The advantage is to reduce the number of new REVs and connectivity as neighboring fractures are merged. Similarly, there has also been progress in using multiscale simulations to compute the transmissibility calculations, allowing for the numerical incorporation of complex physics into the field-scale models. Losapio and Scotti (2023) developed an algorithm in that direction. However, the solution is costly and significantly increases the design optimization loops. We agree with the author that this might be an interesting research topic for machine learning and artificial intelligence algorithms.

One weakness of most EDFM transmissibility calculation strategies is disregarding multiphase aspects and relying on highly uncertain data. After all, field data reveals the connectivity among the joints only globally and at a later time when the field has already been developed. Pursuing highly accurate characterization without reliable data for calibration is pointless. One alternative is to build up local fine-grained and physically rich models, more suitable for addressing phenomenological approaches. The idea would be to enhance overall physical understanding and constrain the uncertainties of large-scale models. This looks like a fruitful research topic to be explored, especially considering data-driven multiscale strategies.

Geological considerations

Fracture modeling has poor and highly uncertain calibration data regarding quality and spatiality. The predictability of a fracture model eventually relies on the physical understanding of the problem derived from geoscience studies. That means that the quality of a model is directly linked to its ability to enhance multidisciplinary communication and implement and test high-level understandings of the fracture network attributes. Hence, design cycles and numerical methodologies must include the geosciences and human interventions, both in adding new interpretations to the models and getting feedback from them.

It is challenging or even impossible to build a generic correlation between fracture geometry and hydraulic behavior, and the simplistic idea of fracture flow controlled by average apertures and planar geometries is unrealistic. A geometrical description must include, besides the fracture large-scale geometry path, at least its roughness, aperture, contact area, and history (Viswanathan et al., 2022). Frash et al. (2019) and Pyrak-Nolte et al. (1987) show, for example, that the lubrication theory in parallel plates (cubic law) is not applicable for natural fractures, as a geometry-to-transmissibility correlation is complex and the lubrication theory assumptions are excessively restrictive. Besides, laboratory investigations have important scale and mechanical limitations that usually invalidate the findings.

Hence, the design team must agree on the nature of the fracture network and essential attributes before assigning fracture conductivity. Even though EDFM provides a framework to model discrete fractures, one must be aware of detaching the geometry of the fracture from its numerical counterpart. As much as W&R models map complex natural fracture networks as a continuum by assuming the equivalent geometry of regularly spaced cubic blocks, discrete fractures must be interpreted as numerical representations of interconnected multiscale joints. The best approach to our knowledge is to seek higher-level parameters in geosciences to characterize the fractured medium as effective features.

Multiphase flow

The drainage mechanisms in a flow simulator are the forces in the domain that control the rock and fluid interactions. The major forces to consider in a given porous media are gravitational, viscous, and capillary. Their preponderance in the overall equilibrium varies with the composition of the materials, the geometry of the pore structure, the height of the reservoir, and the pressure gradients imposed, etc.

We note that most research papers that address fluid flow inside fractures ignore capillary equilibrium. When calculations are performed in idealized geometries and multiphase flow is left open for the reader's discretion, the authors suggest that considering capillary pressure and relative permeability as a function of fluid saturation is enough to embed multiphase physics into the NFR numerical models. However, in naturally fractured reservoirs, discussions on capillary continuity, spontaneous imbibition, counter-current and co-current flow patterns, and their numerical consistency remain open (Saidi, 1983; Labastie, 1990; Van Golf-Racht, 1996; Lemonnier and Bourbiaux, 2010).

As relative permeability embeds capillary-controlled behavior into larger scale Darcy's viscous flow formulation (Cense and Berg, 2009), its use in NFR is not straightforward. When derived from laboratory tests, relative permeabilities map the behavior *along* the matrix. Similarly, *along* the fractures, capillary pressure is routinely considered negligible, and relative permeabilities are set as linear, close to an X-shape (Romm and Blake, 1966). However, work by Pieters and Graves (1994) shows this is untrue even for idealized parallel plates. When capillary forces are considered, Firoozabadi and Hauge (1990) and Karimi-Fard et al. (2004) show additional deviation, with a bias to produce over-pessimistic recovery estimates when the X-shape relative permeability is used. After all, multiphase flow in the fracture and porous medium depends on complex physics and fluid-rock interactions, and predicting relative permeabilities is not straightforward and likely impossible with current technology.

The imbibition dynamics of NFRs subject to waterflooding enhance oil recovery in water-wet formations; it reduces oil recovery in strongly oil-wet formations; and, in mixed wet rocks, the recovery varies and can be even more favorable than the WW scenarios (Karimi-Fard et al., 2004; Blunt, 2017; Alhammadi et al., 2017). It must be underlined that although laboratory testing of fractured rock is not trivial, describing the imbibition mechanisms in the pore scale is fundamental. In fact, the assumption that fluid flow in an NFR occurs mainly inside fractures breaks down when multiphase effects are considered (Wu, 2015): as the capillary pressure inside fractures is usually non-negligible, capillary continuity throughout a fracture set is probably greater than anticipated, gravity and co-current flow play a role as primary drainage mechanisms and imbibition of the natural fractures delay the water breakthrough. Fracture models must go beyond simplistic idealized geometries to find representative realistic estimates.

Similar issues arise in the evaluation of gas injection and carbon storage. March et al. (2018) presents strategies in a dual porosity model concerning the non-wetting nature of the injected fluid and the need for specialized transfer functions. Machado et al. (2023) explores a similar idea comparing EDFM, LGR, and dual porosity models, additionally considering chemical reactions. In all cases, the presence of fractures cannot be neglected when estimating the optimal injection strategy and the site's storage capacity.

Wu et al. (2004) further discusses potential numerical issues. Finite difference flow simulators use upstream approximations for the derivatives as a numerical stabilization technique. This implies that the fracture's relative permeability dominates the process when fluid flows from the fracture to the matrix. This is unrealistic, as the permeability of the most restrictive block controls the equivalent permeability between two blocks with contrasting permeability. The proposed solution is to model the fracture-matrix interface physics instead of relying on matrix and fracture parameters alone. While EDFM looks like an interesting approach to controlling capillary continuity and buoyancy of each numerical effective feature, a clear recommendation is still an open research area.

Finally, benchmarks and validation procedures for multiphase flow in NFRs are scarce, as work found in the literature tends to focus on the assessment of accuracy and performance of algorithms in single-phase flow scenarios (Berre et al., 2021; Flemisch et al., 2018). The strategy is indeed suitable for estimates related to stimulated wells and hydraulic fractures but eventually misleads and oversimplifies the analysis for naturally fractured reservoirs.

Thermo-Hydro-Chemo-Mechanical (THCM)

Most of the advancements seen in EDFM lately relate to applications beyond hydraulic flow. Pei (2022) has recently used EDFM to model fully coupled mechanics and hydraulic flow, which can handle time-dependent aspects of fracture transmissibility as a response to stress and pressure dynamics. Ren and Younis (2019) presented an algorithm to model isothermal complex fracture propagation in a hybrid XFEM-EDFM strategy. One key aspect is the ability to track the fracture path, an important degree of freedom in this class of algorithms. Despite the historical research effort, strategies for fracture tracking in THM-coupled processes in 3D are not yet consolidated.

EDFM has also been extended to Enhanced Geothermal Systems (EGS), in which the coupling between energy exchange and fluid flow are primary for the design (Murphy, 1978; Ghassemi, 2012; Sun et al., 2021; Rao et al., 2022; **?**). We note the relevance of not only the energy exchange along the reservoir but also the thermal stresses the system is subject to. Fracture understanding in EGS application differs significantly from stimulation jobs. As the latter are operated for a few hours with generally neglected localized thermal stresses, EGS operates for years, and thermally induced stresses likely propagate fractures in non-planar geometries. It is unclear how to map secondary fractures as effective discrete fractures and numerical simulation of long-term propagation is still an area of active development.

Most simulators dealing with mechanical coupling restrict the analysis to linear time-dependent transmissibilities as a response to linear elasticity. That is, the fracture conductivity reduces as effective stresses rise due to depletion or cooling (de Sousa Junior et al., 2016; Pei, 2022). Proppant distribution and crushing due to elevated effective stresses have also been studied, for example, by Yu et al. (2015). However, fundamental nonlinear phenomena, such as plasticity, creep, hysteresis, and large strains, are often neglected or left as future work.

Recently, with the emergent interest in carbon capture and storage (CCS), workflows are challenged to model reactive transport (Machado et al., 2023). EDFM has been computationally effective and could capture reactive phenomena inside each fracture individually. Most importantly, EDFM could be used as a preprocessor, taking advantage of the existing commercial simulator engines for thermodynamically intensive computation.

Data assimilation under uncertainties

Fracture-aware workflows and optimization loops must cope with a highly uncertain characterization environment. Narrowing data uncertainty is a long-term ongoing effort, although the physical attributes of the problem limit the range of success. The data assimilation and sensitivity analysis approach must consider probabilistic tools to enhance understanding and provide reliable results under uncertainty. Estimating the uncertainty degree of each piece of information is challenging, but awareness is crucial to avoid time-consuming, over-accurate optimization processes that are impossible to calibrate.

In fact, the optimization method in use cannot assume the fracture network or individual fracture conductivities are known. A better approach is to consider every simulation run as a probabilistic realization to be interpreted as an intermediate result to feed the global probabilistic interpretation. Moreover, designers must avoid the excessive number of parameters.

Depending on the scale of analysis, a fracture network may consist of thousands to millions of fractures. There is a need to control higher-level parameters – like fracture genesis, large-scale path, anisotropy, or global transmissibility multipliers – to get control of the optimization process. In these loops, we must establish upscaling methods to control the fracture network in each realization. This open research field may take advantage of recent AI and multiscale data assimilation (Yu et al., 2018; Kim et al., 2021; Liu et al., 2023).

The evolution of such algorithms and automatic history matching, e.g., by Canchumuni et al. (2021), shows the potential to reduce human interaction times. However, optimization workflows still demand visualization, assessment, and validation in postprocessing, which is limiting for many applications of novel methods. Work by de Sousa Junior et al. (2016) is one example where visually interpreting coupled geomechanical processes in fractures was central.

We acknowledge that the use of EDFM in standard industry workflows is progressing as the tools become more user-friendly, fast, reliable, and integrated into existing workflows. For now, pre- and post-processing tools and optimization loops are likely the limiting ones. Consistent results visualization is essential for calibrating, extracting information, and communicating with geoscientists and stakeholders.

Performance and accuracy

Reservoir flow models are currently built of as many as 100 million active elements. As models grow, performance becomes an issue even for modern computers. EDFM has proven to be a computationally efficient framework to model discrete fractures, preventing local grid refinement while maintaining accuracy. One must pay special attention to the grid sensitivity of each algorithm. As EDFM and related models are supposed to oppose local grid refinement, it loses value as the grid is refined. Effective techniques must be tested and validated for grid sensitivity and present accurate results with sufficient large meshes.

Yu et al. (2021) has successfully stressed the idea of a large model with a million fractures. In single-phase benchmarks, it has also been competitive in accuracy and computational cost (Berre et al., 2021; Flemisch et al., 2018). We emphasize that, unlike most other methods, EDFM can use commercial simulator engines, meaning that multiphase behavior, complex thermodynamics, well control, pre and post-processing, and uncertainty analysis framework are available with no extra development.

As methodologies are proposed, benchmarking against simplistic approaches must be avoided, while we prefer to see commercial standards as references. For example, single-porosity or local grid refined (LGR) are rarely used in real-life simulations to model fractures. Instead, commercial software has consolidated dual-porosity W&R workflows, which work well for dense sets of NF. While there are known accuracy and performance limitations, we understand those as the reference to be improved by novel tentative algorithms.

To the best of our knowledge, there are no consistent public-domain benchmarks for multiphase flow assessment in fractured porous media. As discussed in previous sections, while single-phase tests are enough for Hydraulic Fractured Wells, that is certainly not true for Naturally Fractured Reservoirs. Proposing comprehensive, public-domain NFR benchmarks and reference metrics for accuracy and computational performance is an important research area.

Finally, the validation of a given model must assess the ability of the model to *predicting the behavior of the reservoir*, in opposition to simply *history matching* the data. This may be true even for well-established dual-porosity models, in which many parameters are offered as degrees of freedom to history matching but may lack physical meaning and hence, predictability (Sahimi, 1993).

Closure

This paper reviewed the latest development of EDFM frameworks, willing to identify important open issues and research opportunities. While progress in distinct directions of the technique is evident, fundamental issues remain open, especially related to the incorporation and validation of multiphase flow dynamics, physical description of natural fractures, large-scale problems, and uncertainty management. One of the reasons for that is the topic's complexity, multidisciplinarity, and notable distanciation between geoscientists, petrophysicists, simulation engineers, and computer scientists.

The significant differences in drainage mechanisms between Naturally Fractured Reservoirs and Hydraulic Fractured Wells suggest that building oversimplified models or seeking a single formulation for both cases is counterproductive. A missing step in assessing state-of-the-art fracture modeling in its various flavors is settling on a comprehensive set of public-domain benchmarks covering a wide range of complex fracture networks, revealing each methodology's actual scalability and accuracy.

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