10/3/23

Non-Darcy Flow:

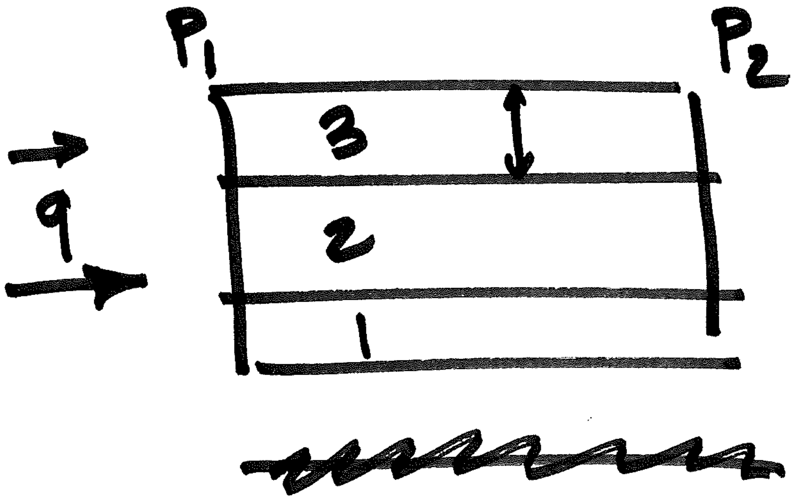
↳ high-vel. flow

$$-\frac{dP}{dx} = \underbrace{\frac{\mu v}{k}}_{\text{Viscous component}} + \beta \rho v^2$$

$\beta \rightarrow$ vel. coef.

$\rightarrow f(k, \phi)$

Example:

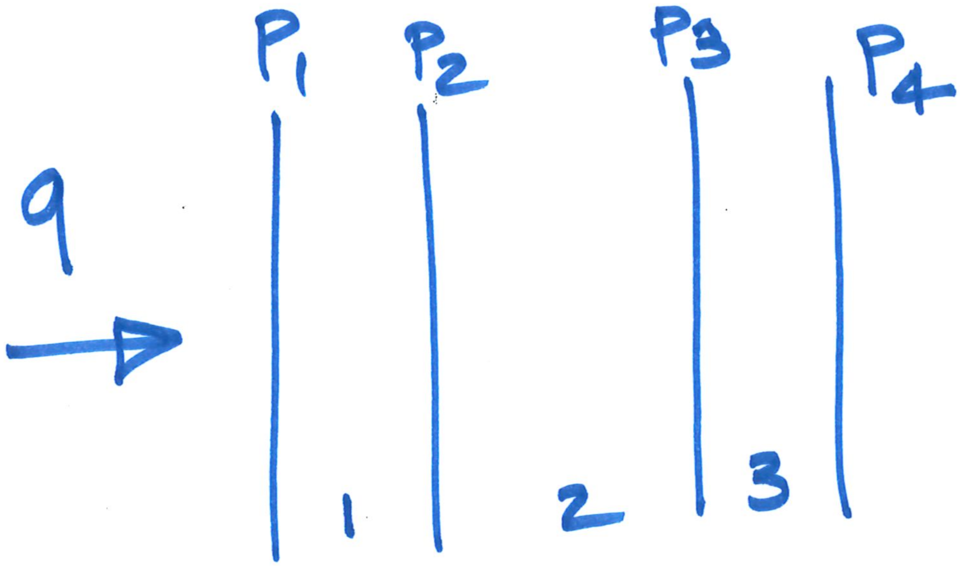


$$\bar{q}_t = q_1 + q_2 + q_3$$

$$K_{avg} A \frac{\Delta P}{L\mu} = k_1 A_1 \frac{\Delta P}{L\mu} + k_2 A_2 \frac{\Delta P}{L\mu} + K_3 A_3 \frac{\Delta P}{L\mu}$$

$$\rightsquigarrow \quad K_{avg} = \frac{\sum k_i A_i}{A}$$

$$= \frac{\sum k_i h_i}{\sum h_i}$$



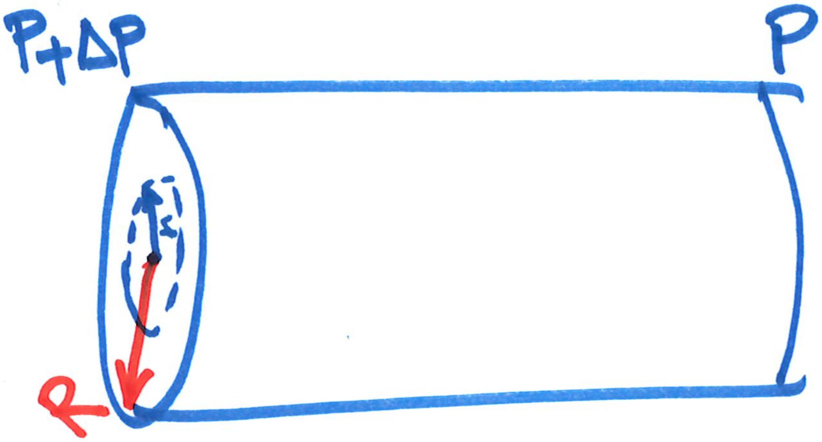
$$(P_4 - P_1) = (P_4 - P_3) + (P_3 - P_2) + (P_2 - P_1)$$

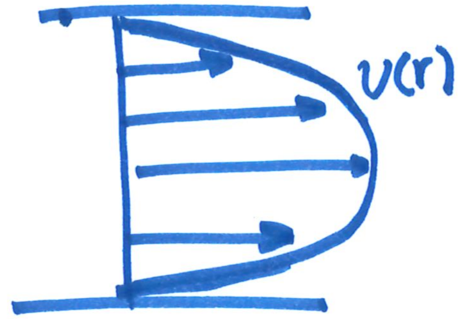
$$\downarrow$$

$$\frac{\mu q}{A}\frac{h}{k_{avg}} = \frac{\mu q}{A}\left[\frac{h_3}{k_3} + \frac{h_2}{k_2} + \frac{h_1}{k_1}\right]$$

$$\Longrightarrow \quad k_{avg} = \frac{\sum h_i}{\sum h_i / k_i}$$

Hagen-Poiseuille's Law





$$F_p = \Delta P(\pi r^2)$$

$$F_{viscous} = -\mu(2\pi r L)\frac{dv}{dr}$$

$$F_p + F_v = 0$$

$$\Delta P(\pi r^2) - \mu(2\pi r L)\frac{dv}{dr} = 0$$

$$\Rightarrow \frac{dv}{dr} = \frac{\Delta P(\cancel{\pi} r^2)}{\mu(2\cancel{\pi} r L)} = \frac{\Delta P}{2\mu L}r$$

at $r = R \rightsquigarrow v = 0$

at $r = 0 \rightsquigarrow \frac{dv}{dr} = 0$.

$$\int_v^0 dv = \frac{\Delta P}{2\mu L} \int_r^R r\,dr$$

$$\rightsquigarrow v(r) = \frac{\Delta P}{2\mu L} \frac{1}{2} r^2 \Big|_r^R$$

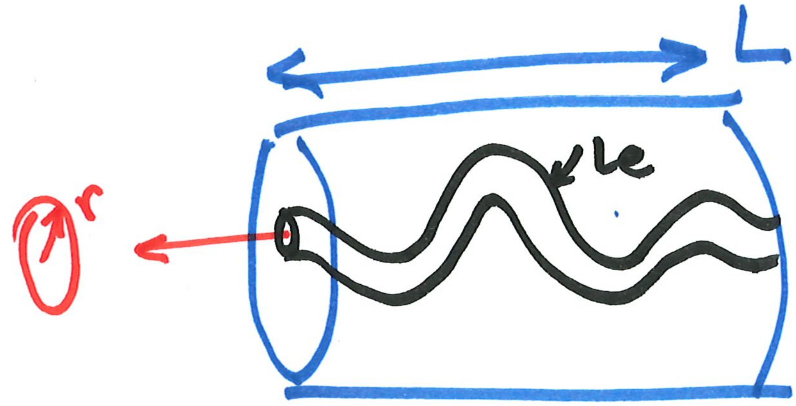
$$\rightsquigarrow v(r) = \frac{\Delta P}{4\mu L} \left[R^2 - r^2\right]$$

$$q = \frac{dV}{dt} = \int v\,dA$$

$$= \int_0^R \frac{\Delta P}{4\mu L} \left[R^2 - r^2\right] 2\pi r\,dr$$

$$\Longrightarrow \boxed{q = \frac{\pi R^4}{8\mu} \frac{\Delta P}{L}}$$

5



$$q_T = \frac{n\pi r^4}{8\mu}\frac{\Delta P}{L_e} \qquad \text{for } n \text{ tubes}$$

Darcy: $$q = \frac{k A_T}{\mu}\frac{\Delta P}{L}$$

$$\frac{n\pi r^4}{8\cancel{\mu}}\frac{\cancel{\Delta P}}{L_e} = \frac{k A_T}{\cancel{\mu}}\frac{\cancel{\Delta P}}{L}$$

$$\Rightarrow k = \frac{n\pi r^4}{8 A_T}\frac{L}{L_e}$$

$$\phi = \frac{V_p}{V_b} = \frac{n\pi r^2 L_e}{A_T L} \Rightarrow A_T = \frac{n\pi r^2 L_e}{\phi L}$$

$$\rightsquigarrow K = \frac{n\pi r^4}{8\,\frac{n\pi r^2 L_e}{\phi L}}\,\frac{L}{L_e}$$

$$\left(\frac{L_e}{L}\right)^2 = \tau$$

$$\rightsquigarrow \boxed{K = \frac{r^2\phi}{8\tau}}$$

Carman-Kozeny Eq.

S_p : Wetted surface area of pores per unit pore volume

$$S_p = \frac{2\pi r L_e\, n}{\pi n r^2 L_e} = \frac{2}{r}$$

circular pores

$$\left.\begin{array}{l} K = \dfrac{r^2\phi}{8\tau} \\ S_p = \dfrac{2}{r} \end{array}\right\} \rightarrow K = \frac{\phi}{2\tau S_p^2}$$

For non-circular:

$$K = \frac{\phi}{K_o \tau S_p^2}$$

K_o → empirical factor

$(K_o \tau)$ → Kozeny constant

For granular materials:

$$K_o \tau \simeq 5$$

Define:

S ⇝ Wetted surface area per unit bulk vol. of the porous medium

$$\left.\begin{aligned} S &= \overset{V_p/V_B}{\phi}\, \overset{WSA/V_p}{S_p} \\ k &= \frac{\phi}{k_o \tau S_p^2} \end{aligned}\right\} \rightarrow k = \frac{\phi^3}{k_o \tau S^2}$$

S_s ⇝ Wetted surface area per unit grain vol.

$$\left.\begin{aligned} S &= (1-\phi) S_s \\ k &= \frac{\phi^3}{k_o \tau S^2} \end{aligned}\right\} \rightarrow k = \frac{\phi^3}{k_o \tau (1-\phi)^2 S_s^2}$$

* Granular Media

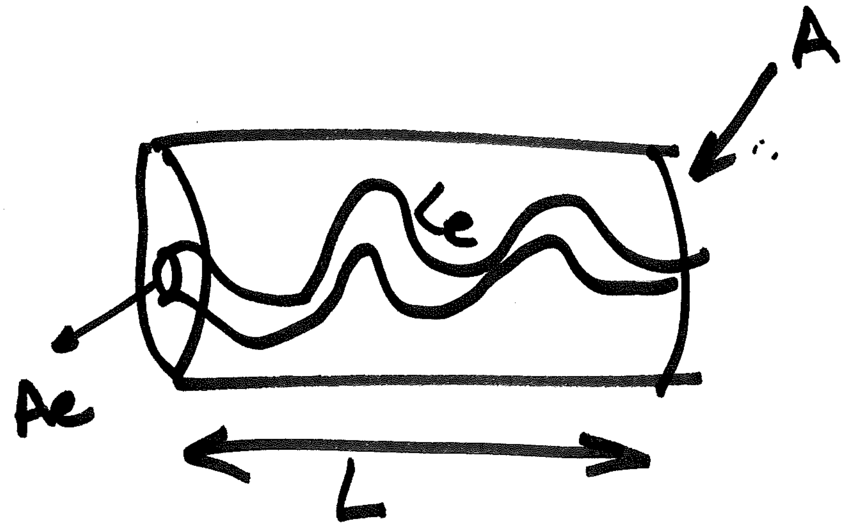
$$S_s = \frac{4\pi (D/2)^2}{\frac{4}{3}\pi (D/2)^3} = \frac{6}{D} \rightarrow \text{Grain Diameter}$$

$$\rightsquigarrow K = \frac{\phi^3}{k_0 \tau S_s^2 (1-\phi)^2}$$

$$\hookrightarrow K = \frac{D^2 \phi^3}{36 k_0 \tau (1-\phi)^2}$$

$$k_0 = 2 \rightsquigarrow \boxed{K = \frac{D^2 \phi^3}{72 \tau (1-\phi)^2}}$$

Example:



$$F \stackrel{?}{=} f(\tau, \phi)$$

$$F = \frac{R_o}{R_w} \qquad \tau = \left(\frac{L_e}{L}\right)^2$$