CAPILLARY PRESSURES – THEIR MEASUREMENT USING MERCURY AND THE CALCULATION OF PERMEABILITY THEREFROM

W. R. PURCELL, JUNIOR MEMBER AIME, SHELL OIL COMPANY, HOUSTON, TEXAS

ABSTRACT

An apparatus is described whereby capillary pressure curves for porous media may be determined by a technique that involves forcing mercury under pressure into the evacuated pores of solids. The data so obtained are compared with capillary pressure curves determined by the porous diaphragm method, and the advantages of the mercury injection method are stated.

Based upon a simplified working hypothesis, an equation is derived to show the relationship of the permeability of a porous medium to its porosity and capillary pressure curve, and experimental data are presented to support its validity.

A procedure is outlined whereby an estimate of the permeability of drill cuttings may be made with sufficient acuracy to meet most engineering requirements.

INTRODUCTION

The nature of capillary pressures and the role they play in reservoir behavior have been lucidly discussed by Leverett[1], Hassler, Brunner, and Deahl[2], and others. As a result of these publications the value of determining capillary pressure curves for cores has come to be generally recognized within the oil industry. While considerable attention has been directed toward the subject in an effort to provide a reliable method of estimating percentages of connate water,[3,4,5] it has been recognized that capillary pressure data may prove of value in other equally important applications.

This paper describes a method and procedure for determining capillary pressure curves for porous media wherein mercury is forced under pressure into the evacuated pores of the solids. The pressure-volume relationships ob-

tained are reasonably similar to capillary pressure curves determined by the generally accepted porous diaphragm method. The advantages of the method lie in the rapidity with which the experimental data can be obtained and in the fact that small, irregularly shaped samples, e.g., drill cuttings, can be handled in the same manner as larger pieces of regular shape such as cores or permeability plugs.

Based upon a simplified working hypothesis, a theoretical equation will be derived which relates the capillary pressure curve to the porosity and permeability of a porous solid, and experimental data will be presented to support its validity. This relationship applied to capillary pressure data obtained for drill cuttings by the procedure described provides a means for predicting the permeability of drill cuttings.

METHODS FOR DETERMINING CAPILLARY PRESSURES

Several techniques have so far been employed in determining capillary pressure curves and these fall into two principal categories:

(1) Liquid is removed from, or imbibed by, the core through the medium of a high displacement pressure porous diaphragm[3,4,5,6].

(2) Liquid is removed from the core which is subjected to high centrifugal forces in a centrifuge[4,6].

There are, however, certain limitations inherent in both methods.

The greatest capillary pressure which can be observed by method (1), above, is determined by the maximum displacement pressure procurable in a permeable diaphragm which at the present time appears to be less than 100 psi. An even more serious limitation of the diaphragm method is imposed by the fact that several days may be required to reach saturation equilibrium at a given pressure; hence, the time re-

quired to obtain a well-defined curve may be measured in terms of weeks. Furthermore, to date, no suitable technique for handling relatively small, irregularly shaped pieces of rock, such as drill cuttings, has been reported and, therefore, measurements must be made, in general, on cores, or portions thereof.

The centrifuge method offers the distinct advantage over the porous diaphragm method of arriving at saturation equilibrium in a relatively short time by virtue of the elimination of the transfer medium for the liquid. The calculation of capillary pressures from centrifuge speeds is somewhat tedious[6], however, and the equipment required is fairly elaborate. While there exists the possibility that this method might be adaptable to the determination of the capillary pressures of cuttings, this particular ramification has not been investigated, as far as is known.

In view of the limitations of the two principal methods for determining capillary pressures, the apparatus described in the following sections has been devised in order that difficulties previously encountered might be circumvented.

MERCURY INJECTION METHOD FOR DETERMINING CAPILLARY PRESSURES
Theory

The methods described above for determining capillary pressures are characterized by the fact that one of the fluids present within the pore spaces of the solid is a liquid which "wets" the solid, i.e., the contact angle which the liquid forms against the solid is less than 90° as measured through that phase.

For these "wetting" liquids the action of surface forces is such that the fluid spontaneously fills the voids within the solid. These forces likewise oppose the withdrawal of the fluid from the pores of the solid.

There is, however, a second type of system which may be considered in the study of capillary pressures. This system involves the porous solid and a single "non-wetting" fluid (mercury) which forms a contact angle of greater than 90° against the solid. In this case the action of the surface forces involved opposes the entrance of the liquid into the solid and pressure must be applied to the liquid to cause penetration of the pores of the solid.

This type of system has been employed by Drake and Ritter[7] in studying the pore size distribution of catalysts, and the apparatus to be described below has been developed in order that similar techniques might be applied to materials exhibiting pore sizes of the order of that found in naturally occurring rock formations.

Apparatus and Procedure

An apparatus suitable for determining capillary pressures of porous media is shown in Fig. 1. The essential components of the apparatus are a mercury displacement pump A, a sample holder B, both shown in detail, and a manifold system C, shown schematically, wherein the gas pressure may be varied from small absolute values (high vacuum) to about 2000 psi, gauge.

The mercury pump consists of a pis-

such that one turn of the driving mechanism moves the piston through a distance sufficient to displace one cubic centimeter. The volume of liquid displaced from the pump is determined by successive readings of the scale D, and vernier E which is attached to the hub of the hand wheel.

The sample holder consists of two parts, both of which carry a lucite window, G, of frusto-conical shape which is cemented into the body of the holder and held rigidly in place by bushings. The displacement pump is connected to the sample holder and manifold by means of diametral conduits through the two lucite plugs. Reference marks, H, are incorporated in these conduits at about the midway point of the lucite windows and may be viewed through the openings in the supporting bushings.

The manifold is connected, as shown, to both a vacuum system and a high pressure (2000 psi) nitrogen bottle. To this manifold are also connected a manometer and pressure gauges suitable for measuring gas pressures ranging from a few millimeters of mercury, absolute, to 2000 psi, gauge.

In operation, one or more plugs drilled from a core, or a number of drill cuttings, which have been extracted and dried, are placed in the cavity, F, of the sample holder. The top

gasket makes the seal pressure tight.

With the mercury level somewhat below the reference line of the lower lucite window, a vacuum is drawn on the system until an absolute pressure of 0.005 mm. of mercury, or less, is registered by the McLeod gauge. The mercury level is then accurately positioned at the lower reference mark by advancing the piston of the displacement pump. The scales attached to the volumetric pump are set at zero following which the piston is further advanced until the mercury meniscus reaches the reference mark in the top lucite window. At this point a scale reading is made which indicates the amount of mercury required to fill the cell with the sample in place. This quantity is subtracted from the known volume of the sample holder (between the reference marks) to provide a measure of the bulk volume of the sample under test.

The vacuum pump is isolated from the manifold and gas admitted to the system in increments, thereby increasing the pressure on the mercury surrounding the sample. The entrance of mercury into the pores of the core or cuttings is indicated by a recession of the mercury-gas interface from the upper reference line, and the degree of penetration is determined by advancing the displacement pump piston until the mercury meniscus returns to this reference mark.

The procedure of alternately building up the pressure to cause recession of the mercury meniscus and advancing the pump piston to return the meniscus to the reference mark, thereby determining the amount of mercury injected into the porous solid under various pressures, is repeated until the pressure of the nitrogen cylinder is reached.

A pressure-volume correction curve is established for the apparatus by carrying out a run as described above without a sample in the holder. The volume readings obtained when testing cores or cuttings are corrected by subtracting amounts as determined by this blank run at corresponding pressures.

The pressure on the mercury entering the sample is taken as the pressure in the gas phase plus a hydrostatic head due to the weight of the column of
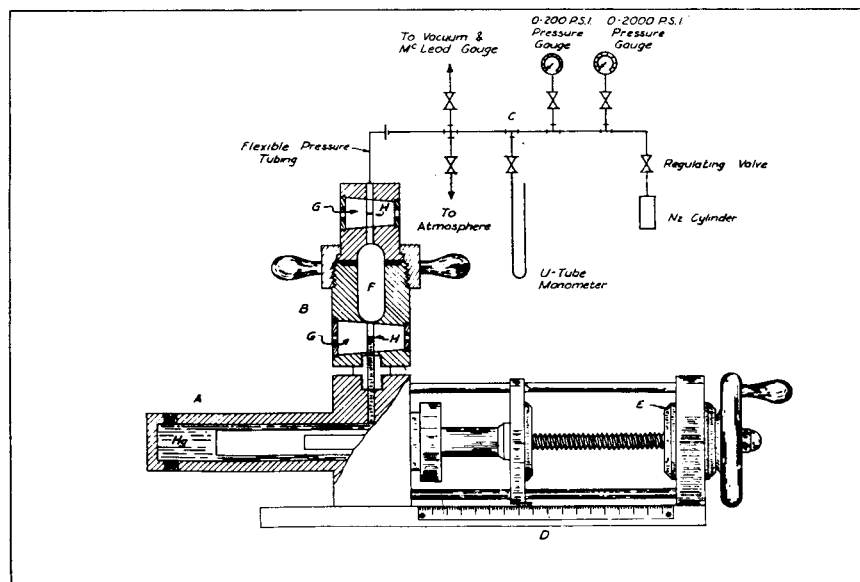

FIG. 1 — APPARATUS FOR DETERMINING MERCURY CAPILLARY PRESSURES

ton-cylinder arrangement, the former being moved by means of an accurately machined screw, the pitch of which is

portion of the sample holder is positioned and the two parts brought together by a make-up nut. A suitable

mercury between the upper reference line and the midway point of the sample. This hydrostatic head may be determined by direct measurement.

Inasmuch as saturation equilibrium is reached very rapidly at any particular pressure, an entire curve may be determined in from 30 to 60 minutes. Temperature fluctuations of the system are ordinarily not sufficiently great during this time to require corrections for thermal expansion or contraction.

Experimental Results

In Figs. 2 to 8, inclusive, (Pages 44 and 45), mercury capillary pressure curves determined in the manner just described are compared with curves determined with water* and air using the porous diaphragm method. The air permeabilities and total porosities of the samples are shown on the graphs.

In comparing these curves it must be recalled that the magnitudes of the capillary pressures are proportional to the product of the surface tension of the liquid being used and the cosine of its angle of contact against the solid. While the surface tensions of the liquids involved can be measured with fair accuracy, the uncertainty in values of the contact angles makes it difficult to predict in advance the exact ratio which should exist between mercury and water/air capillary pressures at corresponding saturations. As a first approximation, however, the following values can be assumed:

   Surface tension of water: 70 dynes per cm.

   Surface tension of mercury: 480 dynes per cm.[7]

   Contact angle of water against the solid: 0°

   Contact angle of mercury against the solid: 140°[7]

The required ratio is then,

$$\frac{\text{(Mercury Capillary Pres.)}}{\text{(Water/Air Capillary Pres.)}}\Bigg|_{\substack{\text{corresponding} \\ \text{saturations}}} =$$

$$\frac{-(480)\ (\cos 140°)}{(70)\ (\cos 0°)} \cong 5.$$

It will be seen that the curves of Figs. 2 to 8 have been plotted on scales such that this approximate ratio of 5 to 1

---

* The water used in all tests referred to in this paper contained 5 per cent by weight of sodium chloride.

is taken into account and hence they may be easily compared qualitatively by visual observation.

As evidenced by these curves, which are typical of those so far obtained, relatively close agreement has been found between mercury and water/air capillary pressure curves for the various types of formations studied and over the range of permeabilities and porosities encountered.

Fig. 9 (P.45) is included to show a capillary pressure curve over the entire saturation range (from 0% to 100%). With the apparatus just described it is possible to measure mercury capillary pressures of the order of 2000 psi. which corresponds approximately to a water/air capillary pressure of 400 psi. The apparatus could be adapted for measurements at higher pressure if this were deemed advisable.

The advantages of the mercury injection method over those previously used are that an entire curve of as many as twenty to thirty points may be determined in about an hour's time and small irregularly shaped pieces can be handled in the same manner as larger portions of regular shape. In addition, the range of capillary pressures that can be observed is considerably greater than for the porous diaphragm method.

CALCULATION OF PERMEABILITY FROM CAPILLARY PRESSURE CURVES

In reservoir analysis and production practice, the importance of that property of rock formations which is referred to as permeability has long been recognized by the exploitation engineer. The determination of air permeability has, for some time, been a routine core analysis test, but in order to obtain the required measurements for this determination, it is first necessary to procure a sample of regular shape and of appreciable dimensions. To accomplish this end, the expensive operation of coring is generally resorted to.

In the course of drilling, cuttings are usually available which, although too small to be suitable for permeability measurements, provide satisfactory samples with which to determine other important characteristics of the rock

formation from which they were cut. An apparatus has been described in the preceding sections which enables the measurement of capillary pressure curves for cuttings. An equation will now be developed which provides a means for calculating the permeability of a porous medium from capillary pressure data. This, in turn, of course, makes possible the estimation of the permeability of drill cuttings.

Theory

The rate of flow Q/t, of a fluid of viscosity, μ, through a single cylindrical tube or capillary of length, L, and internal radius, R, is given by Poiseuille's equation:

$$\frac{Q}{t} = \frac{\pi R^4 P}{8 \mu L} . \qquad \cdots \cdots \cdots (1)$$

wherein P is the pressure drop across the tube. Since the volume, V, of this capillary is $\pi R^2 L$, equation (1) may be written as,

$$\frac{Q}{t} = \frac{V R^2 P}{8 \mu L^2} , \qquad \cdots \cdots (2)$$

The capillary pressure for this single tube, is given by the Pressure of Displacement Equation,

$$P_c = \frac{2\sigma \cos \theta}{R} \cdot \qquad \cdots \cdots (3)$$

where P_c, the capillary pressure, is the minimum pressure required to displace a wetting liquid $(\theta < 90°)$ from or inject a non-wetting liquid $(\theta > 90°)$ into a capillary of radius R when the surface or interfacial tension at the interface is σ and the angle of contact which this interface forms with the solid of the capillary is θ.

Equation (3) indicates that capillary pressure is inversely proportional to pore radius and hence may be used as a measure of capillary size. Substituting equation (3) in (2) we have,

$$\frac{Q}{t} = \frac{(\sigma \cos \theta)^2 V P}{2 \mu L^2 (P_c)^2} \qquad \cdots \cdots (4)$$

Consider now a system composed of a large number, N, of parallel, cylindrical capillaries of equal length but random radii, each tube being identical in all respects except internal area to the capillary under discussion above. The total rate of flow $(Q/t)_s$ through this system must be equivalent to the sum of the contributions made by each of the N single tubes or capillaries.

The flow through each individual tube is given by equation (4); the total flow, therefore, may be represented as follows:

$$(Q/t)_s = \frac{(\sigma \cos \theta)^2 P}{2\mu L^2} \sum_{i=1}^{N} \frac{V_i}{(P_c)_i{}^2} \cdot \quad (5)$$

On the other hand, the rate of flow $(Q/t)_s$ through this same system of capillaries is also given by Darcy's Law:

$$(Q/t)_s = \frac{K A P}{\mu L}, \quad \ldots \ldots (6)$$

where K is the permeability of the system, L is the length of the tubes, A is the total cross-sectional area of the system, and P is the pressure differential causing flow.

By equating the right-hand members of the equations (5) and (6) the following is obtained:

$$K = \frac{(\sigma \cos \theta)^2}{2AL} \sum_{i=1}^{N} \frac{V_i}{(P_c)_i{}^2} \ldots \ldots (7)$$

To simplify equation (7) the volume, V_i, of each capillary may be expressed as a percentage, S_i, of the total void volume, V_T, of the system, i.e.,

$$\frac{V_i}{V_T} \times 100 = S_i$$

Furthermore, since AL is the bulk volume of the system we may introduce the per cent porosity, f, where

$$f = \frac{V_T}{AL} \times 100 ,$$

and equation (7) becomes

$$K = \frac{(\sigma \cos \theta)^2 f}{2 \times 10^4} \sum_{i=1}^{N} \frac{S_i}{(P_c)_i{}^2} \ldots \ldots (8)$$

Equation (8) relates the permeability of a system of parallel cylindrical capillaries of equal lengths, but various radii, to the porosity of the system and to the capillary pressures and volumes of its component parts.

Equation (8) is derived for a porous medium composed of non-interconnected capillaries of circular cross-section and equal length. Certainly no such system obtains in naturally occurring rock formations. The path by which a fluid may travel through a rock

is circuitous, for the pore spaces within these materials are usually interconnected to a greater or lesser degree. Furthermore, this path is neither uniform nor circular in cross-section. It is necessary, therefore, to modify equation (8) by introducing a so-called lithology factor, F, to account for differences between the flow in the hypothetical porous medium for which equation (8) was derived and that in naturally occurring rocks. Equation (8) becomes

$$K = \frac{F (\sigma \cos \theta)^2 f}{2 \times 10^4} \sum_{i=1}^{N} \frac{S_i}{(P_c)_i{}^2} \cdot \quad (9)$$

The amount of variation in the factor F (to be discussed below) for samples of equivalent and of different lithology will determine the practical utility of equation (9).

The evaluation of the quantity

$$\sum_{i=1}^{N} \frac{S_i}{(P_c)_i{}^2}$$

can best be presented by reference to Fig. 10, which shows a typical capillary pressure curve for a naturally occurring porous media such as a reservoir sandstone.

Consider the change in saturation, $\Delta\rho$, that occurs when the pressure is increased from $(P_c)_1$ to $(P_c)_2$. This change in saturation is a result of liquid either entering (non-wetting liquid) or receding from (wetting liquid) all pores having capillary pressures lying between $(P_c)_1$ and $(P_c)_2$. All pores in this interval may be treated as if they exhibited some intermediate average capillary pressure, $(P_c)_{av.}$, where,

$$(P_c)_1 \; (P_c)_{av.} \; \text{VI} \; (P_c)_2$$

If a number, r, of such intervals are chosen, and if n_j is the number of pores in the jth interval, then

$$\sum_{i=1}^{n_j} \frac{S_i}{(P_c)_i{}^2} = \left[\frac{\Delta\rho}{(P_c)_{av.}^2} \right]_j$$

Likewise,

$$\sum_{i=1}^{N} \frac{S_i}{(P_c)_i^2} = \sum_{j=1}^{r} \sum_{i=1}^{n_j} \frac{S_i}{(P_c)_i^2} =$$

$$\sum_{j=1}^{r} \left[\frac{\Delta\rho}{(P_c{}'_{av.}^2} \right]_j ,$$

and

$$\lim_{\substack{\Delta\rho \to 0 \\ r \to \infty}} \sum_{j=1}^{r} \left[\frac{\Delta\rho}{(P_c)_{av.}^2} \right]_j = \int_{\rho=0}^{\rho=100} \frac{d\rho}{(P_c)^2}$$

$$= \sum_{i=1}^{N} \frac{S_i}{(P_c)_i{}^2} \cdot$$

It is seen, therefore, that the quantity

$$\sum_{i=1}^{N} \frac{S_i}{(P_c)_i{}^2} \quad \text{is equal to}$$

the integral of the reciprocal of the square of the capillary pressure expressed as a function of per cent liquid saturation. Such a function is shown in Fig. 10 (P.45). This integral may be determined by planimetering or, more readily, by applying Simpson's Rule.

In the calculation of permeability by by equation (9) from mercury capillary pressure data a surface tension of 480 dynes per cm. and a contact angle of $140°$[7] are assumed. Equation (9) then reduces to

$$K = 0.66 F f \int_{\rho=0}^{\rho=100} \frac{d\rho}{(P_c)^2}, \quad (9a)$$

where K is the permeability in millidarcies, f is per cent porosity, ρ is per cent of total pore space occupied by the liquid, and P_c the capillary pressure expressed in atmospheres.

Experimental Results

In an attempt to determine experimentally the utility of equation (9a), mercury capillary pressure curves have been obtained for numerous rock samples of two types of formations, the Upper Wilcox of Eocene age and the Paluxy of Cretaceous age. Samples of such size were used that their air permeabilities could be measured directly and these observed permeabilities were employed in determining the magnitude of, and the variation in, the factor F of equation (9a). The data obtained

TABLE 1

Observed Values of F

Sample No.	Factor F, Eq. (9a) Required to Make Calculated and Observed Permeabilities Identical	Permeability Calculated from Eq. (9a) Using an Average F of 0.216	Observed Air Permeability (md)
Upper Wilcox			
1	0.085	3.04	1.2
2(1)	0.122	21.2	12.0
3	0.168	17.3	13.4
4	0.149	53.5	36.9
5	0.200	61.9	57.4
6	0.165	91.6	70.3
7	0.257	92.3	110
8	0.256	97.5	116
9	0.191	163	144
10	0.107	680	336
11 (Fig. 6)	0.216	430	430
12	0.273	348	439
13	0.276	388	496
14(1)	0.185	902	772
15	0.282	816	1070
16	0.363	865	1459
Paluxy			
17	—	0.003	<0.1
18	—	0.10	<0.1
19	0.182	42.2	35.7
20	0.158	54.9	40.2
21	0.231	172	184
22	0.276	183	235
23	0.215	308	307
24	0.163	422	320
25	0.284	383	506
26	0.272	502	634
27 (Fig. 5)	0.338	734	1150
	Av. 0.216		

(1) "Cuttings"

with typical samples are recorded in Table I.

It will be seen in Table I that the factors F for the Upper Wilcox and Paluxy sands are of the same general order of magnitude. However, there is some indication that this factor may vary with permeability from a minimum of about 0.1 for samples of low permeability to about 0.4 for samples of high permeability.*

It may be possible, with the further accumulation of data, to establish values of the factor F for various permeability ranges. Likewise, it may be found that the factor F will be different for samples of widely divergent lithology** since the two formations studied in detail and reported here do not vary greatly in lithology. The Upper Wilcox is in general a poorly sorted sandstone while the Paluxy is a well sorted sandstone. The establishment of more precise values of F, however, will serve merely to increase the precision of the method for estimating permeabilities from capillary pressure data. For the moment, with the information at hand, quite satisfactory accuracy may be obtained by merely taking an arithmetical average of the factors F over the entire permeability range.

* It may be shown both theoretically and experimentally[8] that for close-packed spheres of uniform size the length of the path by which a fluid may pass through such a system is $\pi/2$ times as great as the length of the pack. Since the length of the path enters into the denominator of equation (6) as a squared term, it can be shown that the factor $_F$ for this pack is $(2/\pi)^2$ or about 0.4. The difference between this value and those actually observed (0.1 to 0.4) may be due in part to inaccuracies in the assumptions of values for σ and θ, but is probably chiefly the result of the difference in pore structure between that of the ideal pack of uniform spheres and that of naturally occurring rock structures. It is interesting to note, however, that the observed values for $_F$ of natural rocks are of the same order of magnitude as, but somewhat less than, that which can be calculated for an ideal "sand" pack.

** While $_F$ values have been determined extensively for only the Upper Wilcox and Paluxy sandstones, it is interesting to note that the following values are obtained from the data presented in Figs. 2, 3, 4, 7, and 8.

Fig. No.	Formation	Factor, $_F$	Air Permeability (Md.)
2	Frio Sand	0.073	23
3	Frio Sand	0.271	170
4	Frio Sand	0.276	950
7	San Andres Lime	0.133	35
8	San Andres Lime	0.150	43

Although these values are in line with those reported in Table I for samples of comparable permeability, additional data must be obtained before it can be concluded that all four types of formations exhibit similar $_F$ factors.

Table I gives observed air permeabilities together with permeabilities as calculated from equation (9a) using an average value of the factor F of 0.216. These values have been plotted in Fig. 11.

In Table I, samples 2 and 14 are designated as "cuttings". These samples were prepared by crushing cores of known air permeability to obtain a number of small pieces in approximately the size and shape of drill cuttings. Mercury capillary pressure curves were determined for these sets of "cuttings" in exactly the same manner as employed for the larger single pieces. It will be noted in Fig. 11 that the data for these "cuttings" fit the average line as well as the data for the cores.

The procedure for estimating the permeability of drill cuttings (or any porous medium, regardless of size or shape) consists of determining the capillary pressure curve by the method described above, calculating a permeability from the curve by means of equation (9) using a predetermined value of F, and then reading the corresponding air permeability from a plot such as that shown in Fig. 11. The spread of points of Fig. 11 is such as to indicate that the permeability so determined will be sufficiently accurate for most engineering requirements.

SUMMARY AND CONCLUSIONS

A method of determining capillary pressures for porous media wherein mercury is forced under pressure into the pores of the evacuated solid has been found to yield results which are reasonably similar to those obtained using the porous diaphragm technique. This method offers the following advantages over previously decribed procedures:

1. An entire capillary pressure curve consisting of as many as 20 to 30 points can be determined in a matter of hours rather than weeks.

2. Small, irregularly shaped pieces, such as drill cuttings, can be handled in exactly the same manner as larger, regularly shaped samples such as cores or permeability plugs.

3. The range of capillary pressures which can be observed is 5 to 10 times that of conventional methods.
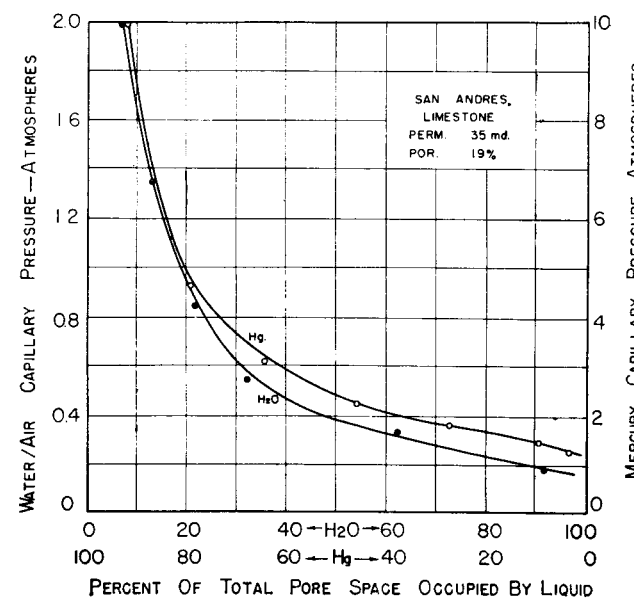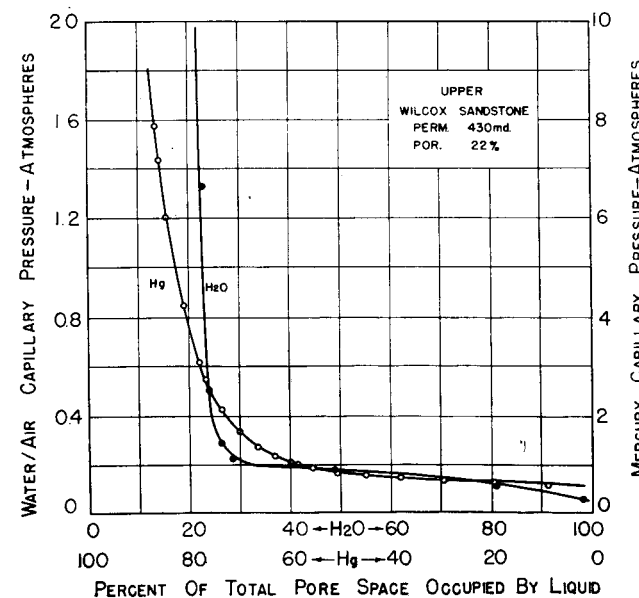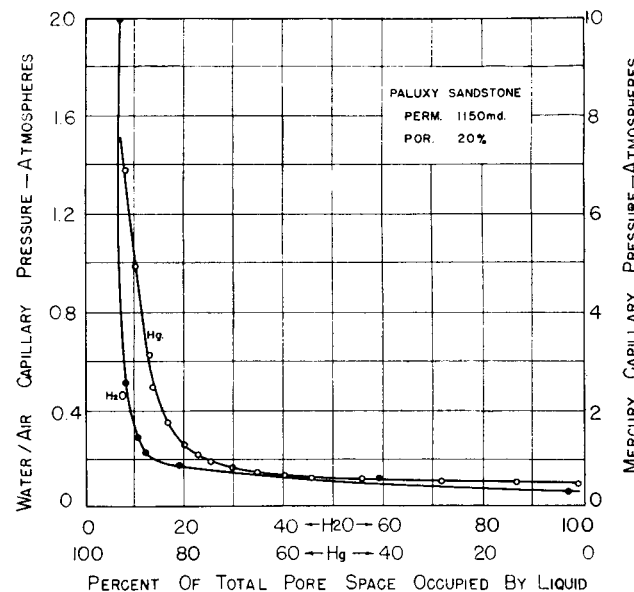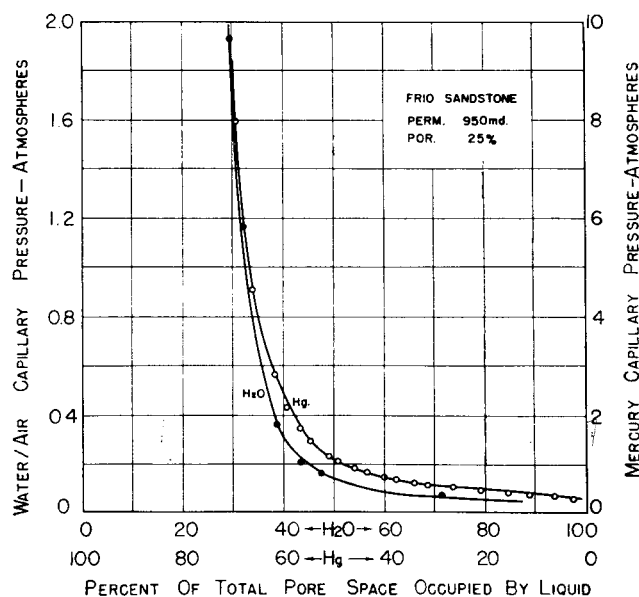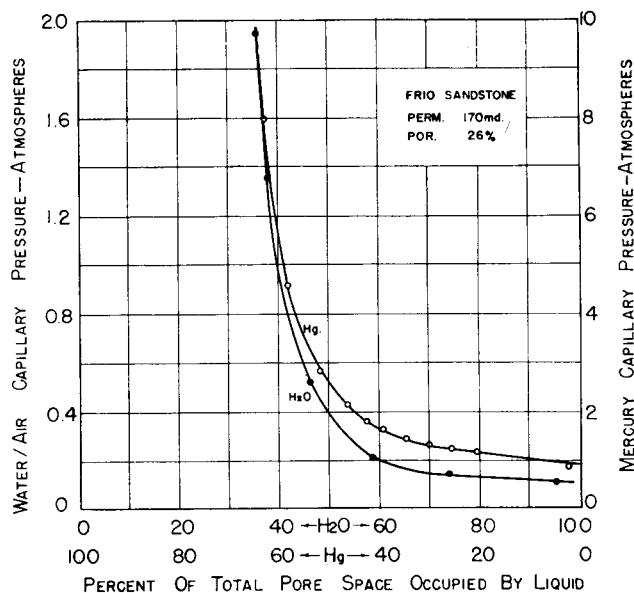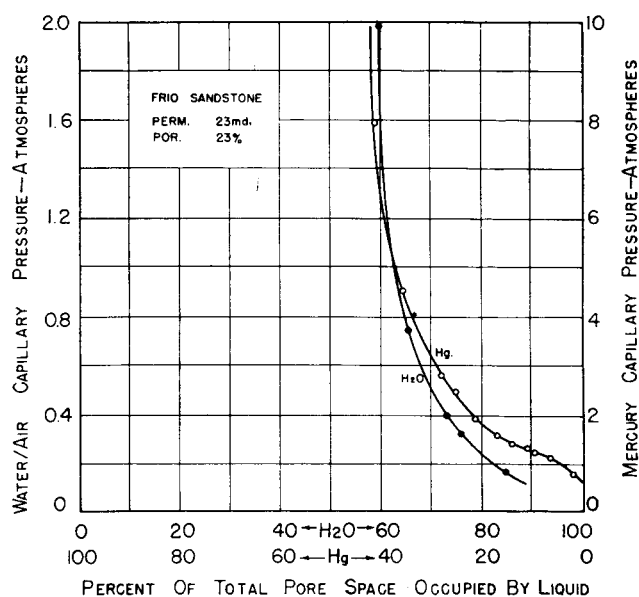
FIGUES 2 THROUGH 7, L TO R, TOP, CENTER, BOTTOM — COMPARISON OF MERCURY AND WATER/AIR CAPILLARY PRESSURE CURVES.
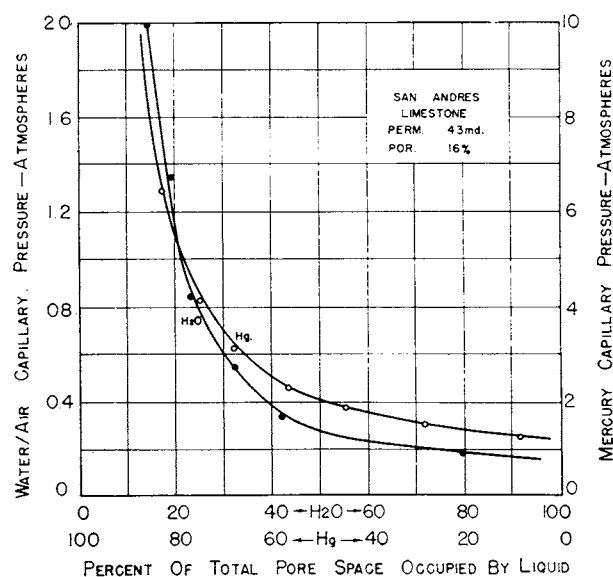
FIG. 8 – COMPARISON OF MERCURY AND WATER/AIR CAPILLARY PRESSURE CURVES
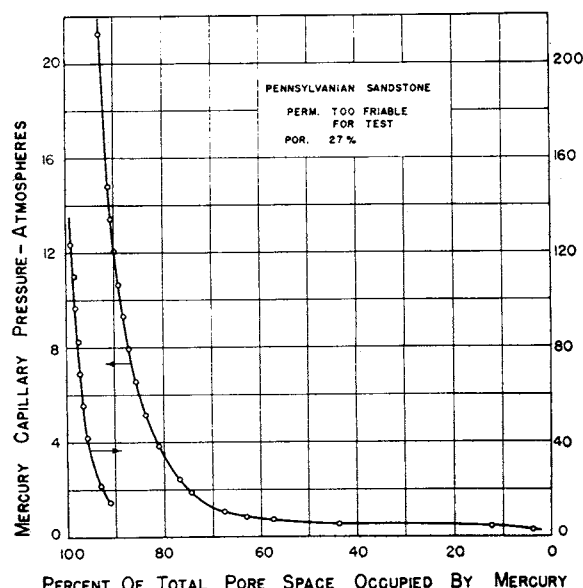

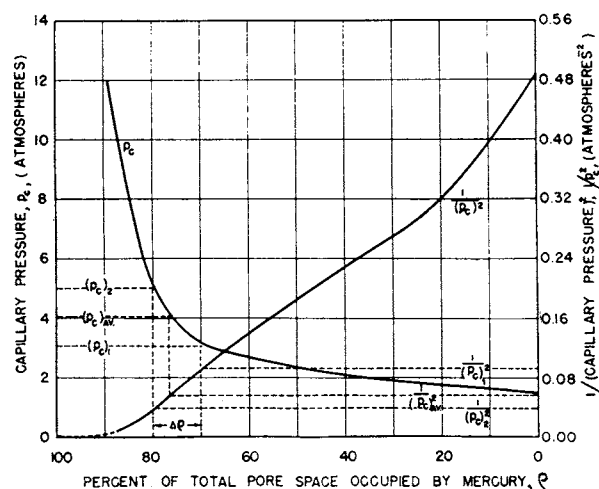FIG. 9 – MERCURY CAPILLARY PRESSURE CURVE OVER THE ENTIRE SATURATION RANGE

FIG. 10 – PERCENT SATURATION AS A FUNCTION OF CAPILLARY PRESSURE AND RECIPROCAL OF SQUARE CAPILLARY PRESSURE
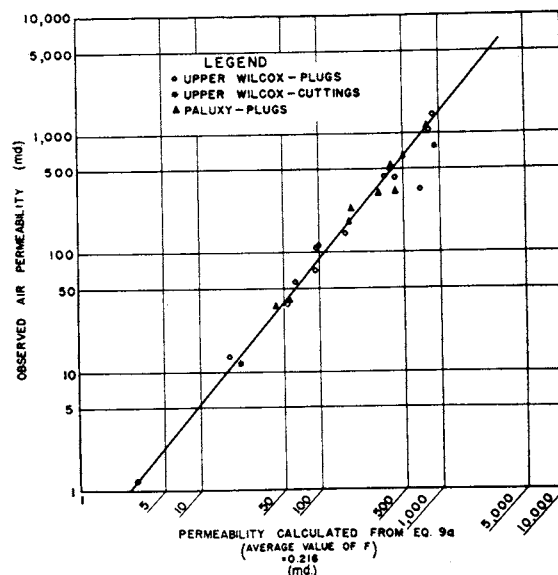

·FIG. 11 – OBSERVED AIR PERMEABILITY AS A FUNCTION OF PERMEABILITY CALCULATED FROM EQUATION 9A.

An equation has been derived which indicates theoretically the relationship which should exist between the permeability of a porous medium and its capillary pressure curve. Experimental data have been presented to show that this equation provides a fairly reliable method of calculating permeability from capillary pressure data.

A combination of the method for measuring the capillary pressure curve of drill cuttings and the equation relating permeability to the capillary pressure curve so determined makes possible the estimation of the permeability of those cuttings.

ACKNOWLEDGMENT

The author wishes to express his appreciation for the aid and cooperation of Mr. G. E. Archie in the formulation of the problem, for the assistance rendered by Messrs. M. A. Westbrook and C. F. Blankenhorn in designing and assembling the apparatus, and for the many valuable suggestions offered by Mr. J. P. Murphy during the preparation of the manuscript.

REFERENCES

1. M. C. Leverett: Capillary Behavior

in Porous Solids. Trans. AIME 142, 152, (1941).

2. G. L. Hassler, E. Brunner, and T. J. Deahl: The Role of Capillarity in Oil Production. Trans. AIME 155, 155, (1944).

3. O. F. Thornton and D. L. Marshall: Estimating Interstitial Water by the Capillary Pressure Method. Trans. AIME 170, 69, (1947).

4. J. J. McCullough, F. W. Albaugh, and P. H. Jones: Determination of Interstitial-Water Content of Oil and Gas Sand by Laboratory Tests of Core Samples: Drilling and Production Practice (A.P.I.) 180 (1944).

5. W. A. Bruce and H. J. Welge: Re-stored-State Method for Determination of Oil in Place and Connate Water. *Oil and Gas Journal*, 46, 223, (1947).

6. G. L. Hassler and E. Brunner: Measurement of Capillary Pressures in Small Core Samples. *Trans.* AIME 160, 114, (1945).

7. R. L. Drake and L. C. Ritter: Pore-Size Distribution in Porous Materials. *Ind. Eng. Chem.*, An. Ed. 17, 782, (1945), and Macropore-Size Distributions in Some Typical Porous Substances. *Ibid* 17, 787, (1945).

8. F. E. Bartell and H. J. Osterhof: Pore Size of Carbon and Silica Membranes. *J. Phys. Chem.* 32, 1553, (1928).

DISCUSSION

☆

By Walter Rose, Gulf Research and Development Company, Pittsburgh, Pa.

It is appropriate certainly to compliment Mr. Purcell on the interesting manner in which he has called attention to another laboratory method for measuring capillary pressure phenomena, and to another useful application for capillary pressure data. lt appears that conventional methods for measurement have not been utilized fully to serve the purposes of routine laboratory core analysis because of the instrumentation and associated difficulties which have been encountered; and also it appears that the heretofore emphasized application for capillary pressure data to evaluate initial fluid phase distributions in virgin petroleum reservoirs can be discredited to some extent due to theoretical uncertainties. However, it is not to be expected that the results of Mr. Purcell's work immediately provide solutions for all the problems related to the evaluation and application of capillary pressure phenomena. This is because the data obtained by the mercury penetration method is not equivalent entirely to that obtained by conventional procedures, as perhaps implied by Purcell, in ways now to be discussed. Elsewhere it will be shown, however, that the data obtained by the mercury penetration method indeed lead to the other useful results suggested by Purcell.

It is to be assumed that when a difference in pressure is established at the interfaces of contact between immiscible fluids saturating the interstices of porous media, this capillary pressure (as an explicit function of fluid saturation and an implicit function of fluid distribution) is in effect a measure of the interstitial pore widths containing these interfaces of contact, assuming conditions of static equilibrium obtain. From this standpoint then it is apparent that Purcell's method and conventional methods of capillary pressure measurement all yield potentially the same kinds of data. To obtain the exact equivalence of these data by the various methods, however, requires that consideration be given to the well-known hysteretic possibilities, that the configurational character of the interstitial spaces not be variably affected by the method of measurement employed, and that the dynamic mechanism of wetting and non-wetting fluid flow (as static equilibruim is approached) be unrelated to the method of measurement employed. Presumably, the first requirement has been satisfied by Purcell's experimentation for he attempts comparison of data obtained only by the capillary drainage mechanism (i.e. where the invading non-wetting phase permits only drainage and no intermediate imbibition of the wetting phase) so that hysteretic uncertainties need not be considered. Regarding the second requirement, it is felt that only in the case of "inert" porous media will the character of the interstitial spaces be unrelated to the physico-chemical properties of the saturating fluids. As an example, the presence of interstitial surface clay coating in certain types of naturally occurring porous media will certainly result in a different pore structure when mercury and rarified gas are substituted for oil and brine as the saturating fluids, due to the dependence of the clay swelling effects on the nature of the fluids contacting the clays. Thus, even in conventional capillary pressure experimentation it has been recognized that use of reservoir-like fluids is to be preferred so that the data will truly reflect on the pore character representative of the natural reservoir condition. This refinement evidently cannot be employed in the use of Purcell's method. However, it is the inevitable disre-

gard for the third requirement as above stated which most seriously limits the possibility of attaining equivalence between the results of Purcell's method and conventional methods, but which nonetheless leads to the potential usefulness for Purcell type experimentation as described in his paper. For it can be anticipated on theoretical grounds, and indeed it can be established from an examination of Purcell's data (c.f. Purcell's Figures 2 through 8), that "irreducible" minimum wetting phase saturations (reflecting on zero wetting phase relative permeability due to the attainment of pendular configurations) will not be a result of the mercury penetration method, since this procedure involves a compression rather than a flow of the wetting phase as desaturation occurs. It is evident then that the data reported by Purcell reflect principally on a complete distribution of pore radii, as originally advanced by Ritter and Drake (c.f. Reference 7 of Purcell's paper), providing information not directly derivable from conventional capillary pressure curves and leading to a method for approximating permeabilities and lithology characteristics of porous media. However, it must be emphasized that exact equivalence between conventional capillary pressure data and that obtained by the mercury penetration method is not to be expected as a common result.

The interesting feature about Purcell's equation for permeability is that it can be arrived at by several processes, leading to a more complete interpretation of the lithology factor, F. For instance, in a recent paper (not yet generally available)* an expression for permeability is given as:

$$k = \frac{\left[\lim_{\rho_w \to 1} j(\rho_w)\right]^2 \left[\sigma \cos \theta\right]^2 f}{P_D^2 \times 10^2}$$

Eq.(1)

where ρ_w is the fractional wetting phase saturation, $j(\rho_w)$ is Leverett's capillary pressure function, P_D is the displacement pressure as commonly defined, and where the other notation follows that as employed by Purcell. In this cited reference it is shown that the limiting (minimum) value for the capil-

---

lary pressure function can be identified with the "rock" textural constant, t, appearing in the Kozeny equation for streamline fluid flow through porous media, as:

$$\lim_{\rho_w \to 1} j(\rho_w) = \left(\frac{1}{t}\right)^{1/2} \qquad \text{Eq.(2)}$$

Now, it is observed that the constant, t, as used in Eq.(2) refers to its maximum value, which has been established as being greater than 5 (and sometimes, for instance, as great as 500) as related to the various types of unconsolidated and consolidated porous media encountered in natural reservoir rock. This implies that Purcell's lithology factor generally will be less than 0.4, as indeed predicted and observed, for it is seen on combination of Eq.(1) above and Purcell's Eq.(9) or (9a) that:

$$\frac{F}{2} > \left(\lim_{\rho_w \to 1} j(\rho_w)\right)^2, \qquad \text{Eq.(3)}$$
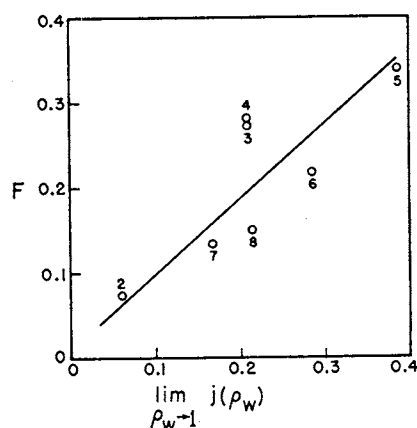
since:

$$\int_0^1 \frac{d\rho}{P_C^2} < \frac{1}{P_D^2}. \qquad \text{Eq.(4)}$$

Moreover, it can be shown from these equations that the lithology factor, F, is defined fundamentally as:

$$F = \frac{2}{\displaystyle\int_0^1 \frac{d\rho}{j(\rho_w)^2}} = \frac{2}{t_{min}}. \qquad \text{Eq.(5)}$$

The above considerations imply that in texturally similar porous media the lithology factor, F, and the limiting value of the j function will be related

to each other by some constant factor characteristic of the media, and that in texturally dissimilar media a rough proportionality will still obtain between these parameters. This is clearly shown by Figure 1, constructed from Purcell's data where F has been plotted versus $\lim_{\rho_w \to 1} j(\rho_w)$. It would appear,

therefore, that Eq.(1) above has the same sort of validity and usefulness as Purcell's Eq.(9a), and its use is to be preferred since from the experimentation standpoint it is easier to evaluate. For only the initial segment of the capillary pressure curve need be measured (actually, only the displacement pressure need be measured) to solve Eq.(1) and to provide values reflecting on lithology or permeability characters of porous media equivalent to those obtained by Purcell. In this connection, a plot of F versus the logarithm of the corresponding permeability value is also given in Figure 1 and the linearity obtained by this plot suggests that in the classification of texturally dissimilar porous media a knowledge of permeability alone suffices sometimes to predict qualitatively lithological differences.

Figure 2 shows the results of some experimentation conducted recently to verify that the mercury penetration method of capillary pressure measurement would yield data equivalent to that obtained by conventional methods for inert porous media. Thus, Curve A of Figure 2 shows the capillary retention replot of the original Ritter and
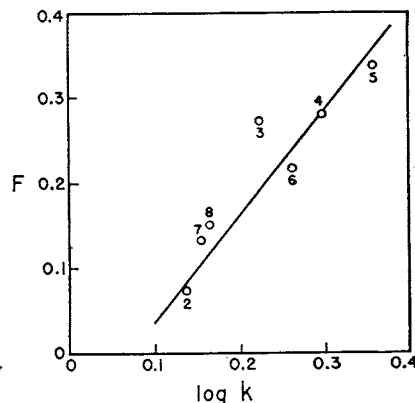
Drake data characterizing pyrex fritted glass media. Curve B is a similar plot of data obtained on other fritted glass media by the conventional capillary pressure technique, and the close conformance between these two curves can be taken as evidence that both methods have provided a measurement of the same phenomena, at least at the saturation levels where the wetting phase is continuous throughout the interstitial spaces (i.e. at saturations greater than those obtaining when the wetting phase configuration is pendular). These results are offered to support further the validity of the experimentation described by Purcell, for it is regarded that Purcell's technique for laboratory
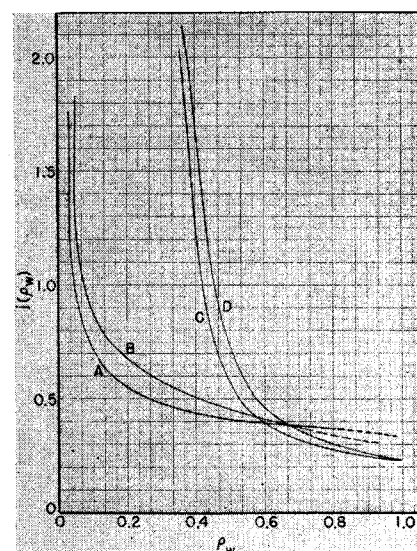
FIGURE 2
CAPILLARY RETENTION CURVES

CURVE A IS A PLOT OF THE RITTER AND DRAKE DATA.

CURVE B IS A PLOT OF DATA CONVENTIONALLY OBTAINED ON POROUS MEDIA TEXTURALLY SIMILAR TO THAT USED BY RITTER AND DRAKE.

CURVE C IS A REPLOT OF PURCELL'S FIG. 3.

CURVE D IS A REPLOT OF PURCELL'S FIG. 4.

measurement must have high order accuracy and precision. This is shown also by the replot of the Frio core data as Curves C and D in Figure 2 from Purcell's Figures 3 and 4, since essentially the same capillary retention curve has been obtained for these two cores of widely different permeability. For when capillary pressure data yield the same capillary retention plot for different porous media, a condition of textural similarity between these media

FIGURE 1
CORRELATION BETWEEN LITHOLOGY FACTOR, F, AND THE LIMITING VALUE FOR THE CAPILLARY PRESSURE FUNCTION, j(ρ_w), (C.F. LEFT DIAGRAM), OR THE LOGARITHM OF PERMEABILITY, LOG K, (C.F. RIGHT DIAGRAM). NUMBERS ASSOCIATE DATA POINTS WITH PURCELL'S FIGURE NUMBERS.

is implied, as already was established in the case of the Frio cores by the observed approximate equivalence of the measured lithology factor.*

To conclude, it is apparent that Mr. Purcell, in applying his method for capillary pressure evaluation, has accomplished probably as much as will be forthcoming from experimentation of this sort. That is, he has developed suitably a rapid and precise method to make valid measurements of capillary pressure on small rock fragments reflecting on the permeability and the lithological character of porous media. Such data undoubtedly will be found to be useful for correlation purposes in the interpretation of the results of other core analysis experimentation. However, it is noted that for some purposes at least conventional experimentation still will be required, and this should not be neglected because of the features of simplicity associated with the mercury penetration type experimentation, especially because it appears that Mr. Purcell's objections to use of conventional methods can be obviated somewhat through suitable instrumentation improvements. For instance, use of cellophane type capillary barriers permits measurements of capillary pressure in excess of 300 p.s.i., which is well above the values required to produce "irreducible" minimum saturations of the wetting phase in petroleum reservoir rock of practical interest. Also, it is noted that the equilibrium values of saturation can be obtained by extrapolation of rate of displacement data, thereby decreasing the time required for conventional laboratory experimentation. Finally, it is noted that the recently proposed "multi-core" procedure for capillary pressure evaluation (c.f. Rose and Bruce paper, loc. cit.) has a potential use for the study of small core fragments similar to that characterizing Purcell's method of evaluation. In this connection, however, it must be realized that no method presently described in the literature will provide satisfactory capillary pressure data in those

instances where the core fragments are so small that their total bulk surface area approaches the total interstitial surface area, thereby imposing a lower limit on these possibilities.

The consideration presented in this discussion have been directed to bear on the significance of the results of Mr. Purcell's excellent paper. It is to be recognized, however, that Mr. Purcell's paper, independent of these considerations, is a most valuable contribution to the literature, and the author is to be congratulated for his developments which contribute so significantly to our knowledge of capillary pressure phenomena.

Author's reply to Mr. Walter Rose:

It is indeed gratifying that Mr. Rose, through his discussion, has provided a link between the theoretical approach to the problem which is exemplified by his recent paper entitled "Theoretical Generalizations Leading to the Evaluation of Relative Permeability"† and the experimental approach of the author. It is believed that through a close intermeshing of these two general types of investigation our knowledge of capillary phenomena should increase substantially in the future.

Mr. Rose has very aptly pointed out certain conditions which must be satisfied if exact equivalence between mercury capillary pressures and those obtained by other means is to be obtained; likewise he has indicated that all of these conditions are not necessarily fulfilled. It would seem appropriate, therefore, to state that the author does not intend to imply, as indicated by Mr. Rose, that the mercury penetration method is exactly equivalent to conventional procedures but instead has chosen to show experimentally (as evidenced by Figures 2 to 8, inc.) that for the various types of formations studied and over the range of permeabilities and porosities encountered a reasonable similarity exists between mercury and water/air capillary pres-

sures. In the paper no conclusions are drawn from the comparison tests other than the one of similarity between the two types of curves and this same conclusion has been obtained by Mr. Rose for his fritted glass plates. Furthermore, it should be pointed out that only one application of capillary pressure data, namely that of estimating permeability, is discussed.

Mr. Rose's confirmation of equation 9 by the derivation outlined in his discussion adds materially to the subject. The calculation of permeability from but a single point of the capillary pressure curve, namely the displacement pressure, should perhaps be used with caution, although it must be admitted indications are that the results so obtained would compare favorably with those obtained by an integration of the entire curve. In this connection, it might be pointed out that an accurate determination of the displacement pressure is exceedingly difficult by means other than mercury penetration.

It is stated in the discussion that difficulties usually encountered in conventional methods of determining capillary pressures, can be alleviated by suitable instrumentation. While it is apparent that the range of pressures can be extended through the use of cellophane membranes, this may be accomplished, perhaps, only through the aggravation of a second difficulty, namely that of the length of the time involved. Yuster** has recently stated that the extrapolation method of estimating equilibrium saturations from rates of displacement data gave accurate results in only about 25 per cent of the tests for which it was employed; hence, it may be concluded that this method of reducing the time involved in conventional tests should be used with reservation. It is to be hoped that the paper of Rose and Bruce referred to in the discussion will soon be made generally available for it is believed that any method of determining capillary pressure curves for cuttings will find immediate application.   ★ ★ ★

---

* Note, from Curves C and D of Figure 2 the validity of the definition presented as Eq.(5) above is readily verified.

† *Jnl. of Petr. Tech.*, in press (1949).

** Producer's Monthly, December 1948, p. 24.