CP8

April 11, 2024

PGE 382 - Numerical Methods in Petroleum and Geosystems Engineering

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CP8 - Apr, 11st

[1]:
```python
from math import pi, sin, cos, exp
import numpy as np
from numpy import linspace, zeros, arange
from numpy import ix_ as ix
np.set_printoptions(threshold=10000, linewidth=10000)
from numpy import exp, linspace, vectorize
import matplotlib.pyplot as plt
plt.style.use('paper.mplstyle')

dx = 0.05 ; dy = dx/2
Tf = 0.1 ; Nt=10
dt = Tf/Nt ; Nt = Nt + 1

X = np.arange(0,1+dx,dx) ; Ni = len(X)
Y = np.arange(0,1+dy,dy) ; Nj = len(Y)
Nij = Ni * Nj

beta = 0.5
Pr = 0.733

# Global index
def _(i,j) : return j + Nj*i

# Calculate F1
def cF1(i, j, U, V, T, Un, Vn, Tn) :
    _ij = _(i,j)
    _0j = _(i-1,j)
    _i0 = _(i,j-1)
    _i1 = _(i,j+1)

    ret = 0
    #1 = T
    ret += beta * T[_ij]
    ret += (1-beta) * Tn[_ij]
    #2 = d2U/dy2
    ret += beta * ( U[_i0] - 2*U[_ij] + U[_i1] ) / dy / dy
    ret += (1-beta) * ( Un[_i0] - 2*Un[_ij] + Un[_i1] ) / dy / dy
    #3 = -dU/dt
    ret += Un[_ij]/dt - U[_ij]/dt
    #4 = -U dU/dx
    ret += -beta     * U[_ij]  * ( U[_ij]  - U[_0j] )/dx
    ret += -(1-beta) * Un[_ij] * ( Un[_ij] - Un[_0j] )/dx
    #5 = -V dU/dy
    ret += -beta     * V[_ij]  * ( U[_ij]  - U[_i0])/dy
    ret += -(1-beta) * Vn[_ij] * ( Un[_ij] - Un[_i0])/dy
    return ret

# Calculate F2
def cF2(i, j, U, V, T, Un, Vn, Tn) :
    _ij = _(i,j)
    _0j = _(i-1,j)
    _1j = _(i+1,j)
    _i0 = _(i,j-1)
    _i1 = _(i,j+1)

    ret = 0
    #1 = 1/Pr d2T/dy2
    ret += beta * (1/Pr) * ( T[_i0] - 2*T[_ij] + T[_i1] ) / dy / dy
    ret += (1-beta) * (1/Pr) * ( Tn[_i0] - 2*Tn[_ij] + Tn[_i1] ) / dy / dy
    #2 = -dT/dt
```

```
    ret += Tn[_ij]/dt - T[_ij]/dt
    #3 = -U dT/dx
    ret += -beta     * U[_ij]  * ( T[_ij]  - T[_0j] )/dx
    ret += -(1-beta) * Un[_ij] * ( Tn[_ij] - Tn[_0j] )/dx
    #4 = -V dT/dy
    ret += -beta     * V[_ij]  * ( T[_ij]  - T[_i0] )/dy
    ret += -(1-beta) * Vn[_ij] * ( Tn[_ij] - Tn[_i0] )/dy
    return ret

# Calculate F3
def cF3(i, j, U, V, T, Un, Vn, Tn) :
    _ij = _(i,j)
    _0j = _(i-1,j)
    _i0 = _(i,j-1)

    ret = 0
    #1 = dU/dx
    ret += beta * ( U[_ij] - U[_0j] ) / dx
    ret += (1-beta) * ( Un[_ij] - Un[_0j] ) / dx
    #2 = dV/dy
    ret += beta * ( V[_ij] - V[_i0] ) / dy
    ret += (1-beta) * ( Vn[_ij] - Vn[_i0] ) / dy
    return ret

# The list of free and prescribed dofs
def build_Df() :
    global DOF_f, Ni, Nj

    DOF_f=[]
    for i in arange(1,Ni) :
        for j in arange(1,Nj-1) :
            DOF_f.append( _(i,j) )


#
# Assign BCs to solution vectors
#
def init_bcs() :
    global Unij, Vnij, Tnij
    Unij = zeros( [Nt,Ni,Nj] )
    Vnij = zeros( [Nt,Ni,Nj] )
    Tnij = zeros( [Nt,Ni,Nj] )

    Unij[0,:,:] = 0 # ic
    Vnij[0,:,:] = 0 # ic
    Tnij[0,:,:] = 0 # ic

    Unij[:,:,0] = 0 # BC , Y=0
    Vnij[:,:,0] = 0 # BC , Y=0
    Tnij[:,:,0] = 1 # BC , Y=0

    Unij[:,:,-1] = 0 # BC , Y=inf
    Vnij[:,:,-1] = 0 # BC , Y=inf
    Tnij[:,:,-1] = 0 # BC , Y=inf

    Unij[:,0,:] = 0 # BC , X=0
    Vnij[:,0,:] = 0 # BC , X=0
    Tnij[:,0,:] = 0 # BC , X=0

#
#
#
#
def build_force() :
    global F1, F2, F3, Ni, Nj, Nij
    # Fx
    F1 = zeros( Nij )
    F2 = zeros( Nij )
    F3 = zeros( Nij )
    for i in arange(1,Ni) :
        for j in arange(1,Nj-1) :
            _ij = _(i,j)
            F1[_ij] += cF1(i,j,Uk,Vk,Tk,Un,Vn,Tn)
            F2[_ij] += cF2(i,j,Uk,Vk,Tk,Un,Vn,Tn)
            F3[_ij] += cF3(i,j,Uk,Vk,Tk,Un,Vn,Tn)
```

```
#
#
#
#
def build_jacobian() :
    global J1U, J1V, J1T, J2U, J2V, J2T, J3U, J3V, J3T

    # DFx/DU
    J1U = zeros([ Nij, Nij ] )
    J2U = zeros([ Nij, Nij ] )
    J3U = zeros([ Nij, Nij ] )
    # DFx/DV
    J1V = zeros([ Nij, Nij ] )
    J2V = zeros([ Nij, Nij ] )
    J3V = zeros([ Nij, Nij ] )
    # DFx/DT
    J1T = zeros([ Nij, Nij ] )
    J2T = zeros([ Nij, Nij ] )
    J3T = zeros([ Nij, Nij ] )

    for i in arange(1,Ni) :
        for j in arange(1,Nj-1) :
            _ij = _(i,j)
            _0j = _(i-1,j)
            _1j = _(i+1,j)
            _i0 = _(i,j-1)
            _i1 = _(i,j+1)

            # F1
            #
            #1 = T
            J1T[_ij,_ij] += beta
            #2 = d2U/dy2
            J1U[_ij,_i0] += beta/dy/dy
            J1U[_ij,_ij] += beta*(-2/dy/dy)
            J1U[_ij,_i1] += beta/dy/dy
            #3 = -dU//dt
            J1U[_ij,_ij] += -1/dt
            #4 = -U dU/dx --
            #            ./dUij = beta*(-2U/dx + U0j/dx)
            #            ./dU0j = beta*U/dx
            J1U[_ij,_ij] += beta * (-2*Uk[_ij] + Uk[_0j])/dx
            J1U[_ij,_0j] += beta * Uk[_ij]/dx
            #5 = -V dU/dy
            #            ./dUij = -beta*V/dy
            #            ./dUj0 = beta*V/dy
            #            ./dVij = -beta*(U-Ui0)/dy
            J1U[_ij,_ij] += -beta * Vk[_ij]/dy
            J1U[_ij,_i0] += beta * Vk[_ij]/dy
            J1V[_ij,_ij] += beta * (-Uk[_ij]+Uk[_i0])/dy

            # F2
            #
            #1 = 1/Pr d2T/dy2
            J2T[_ij,_ij] += beta * (-2) / Pr / dy / dy
            J2T[_ij,_i0] += beta * 1 / Pr / dy / dy
            J2T[_ij,_i1] += beta * 1 /Pr / dy / dy
            #2 = -dT/dt
            J2T[_ij,_ij] += -1/dt
            #3 = -U dT/dx
            #            (3)/dUij = -beta*(T-T0j)/dx
            #            (3)/dTij = -beta*Uij/dx
            #            (3)/dT0j = beta*Uij/dx
            J2U[_ij,_ij] += -beta * ( Tk[_ij]  - Tk[_0j] )/dx
            J2T[_ij,_ij] += -beta * Uk[_ij] /dx
            J2T[_ij,_0j] += beta * Uk[_ij] /dx
            #4 = -V dT/dy
            #            (4)/dVij = -beta*(T-Ti0)/dy
            #            (4)/dTij = -beta*Uij/dy
            #            (4)/dTi0 = beta*Uij/dy
            J2V[_ij,_ij] += -beta * ( Tk[_ij]  - Tk[_i0] )/dy
            J2T[_ij,_ij] += -beta * Vk[_ij] /dy
            J2T[_ij,_i0] += beta * Vk[_ij] /dy
```

```
            # F3
            #
            #1 = dU/dx
            J3U[_ij,_ij] += beta/dx
            J3U[_ij,_0j] += -beta/dx
            #2 = dU/dy
            J3V[_ij,_ij] += beta / dy
            J3V[_ij,_i0] += -beta / dy



#
#
#
#
def linear_solve( ) :
    global Nij
    global J1U, J1V, J1T, J2U, J2V, J2T, J3U, J3V, J3T
    global F1, F2, F3
    global JAC, FORCE

    J_ix = ix( DOF_f, DOF_f )
    F_ix = ix( DOF_f )
    JAC = np.block([[ J1U[J_ix], J1V[J_ix], J1T[J_ix]],
                    [ J2U[J_ix], J2V[J_ix], J2T[J_ix]],
                    [ J3U[J_ix], J3V[J_ix], J3T[J_ix]] ])
    FORCE = np.block([ F1[F_ix], F2[F_ix], F3[F_ix] ])

    dX = np.linalg.solve( JAC, -FORCE )

    # Extract free dofs
    dUk = np.zeros(Nij)
    dVk = np.zeros(Nij)
    dTk = np.zeros(Nij)
    bl = np.shape(F_ix)[1] # block length
    dUk[F_ix] = dX[:bl]
    dVk[F_ix] = dX[bl:2*bl]
    dTk[F_ix] = dX[2*bl:]

    err = np.linalg.norm(dX)

    return dUk, dVk, dTk, err



#
#
#
#  MAIN FLOW
#
#
#

# Global solution vector
init_bcs()
build_Df()

for n in arange(1,Nt) :
    print(f"Solving timestep {n} ...")

    # Solution from the previous TS
    Un = Unij[n-1,:,:].flatten()
    Vn = Vnij[n-1,:,:].flatten()
    Tn = Tnij[n-1,:,:].flatten()
    # Initial guess for newton-raphson
    Uk = Un.copy()
    Vk = Vn.copy()
    Tk = Tn.copy()

    nk = 0 #newton loop index
    while(1) :
        build_jacobian()
        build_force()
        dUk, dVk, dTk, err = linear_solve()
```

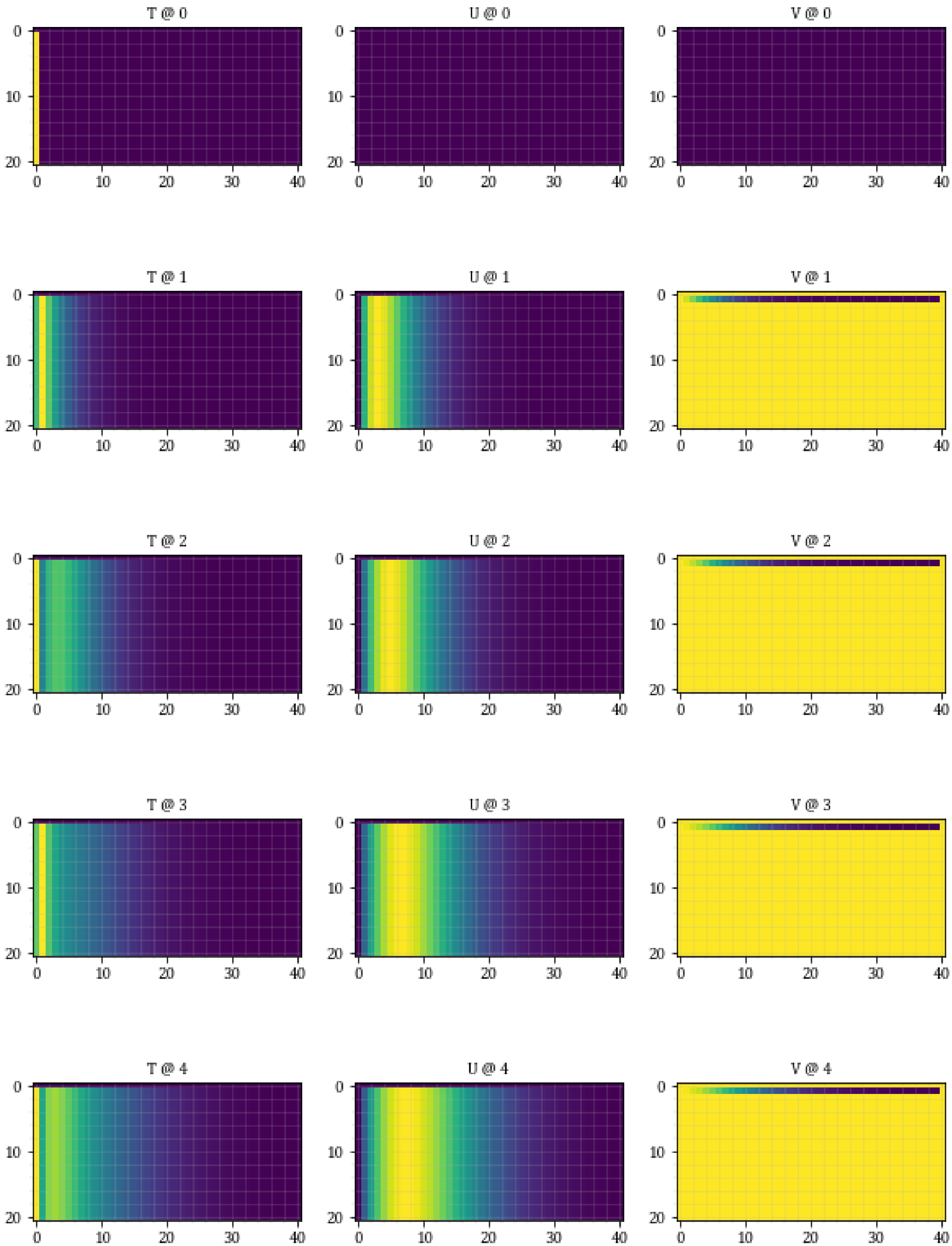
```python
        # Update results
        Uk += dUk
        Vk += dVk
        Tk += dTk

        # Check for convergence
        nk += 1
        print(f"   Newton iteration #{nk} ... (err={err:.3e})")
        if err < 1e-13 : break
        if nk > 50 : break

    Unij[n,:,:] = Uk.reshape(Ni,Nj)
    Vnij[n,:,:] = Vk.reshape(Ni,Nj)
    Tnij[n,:,:] = Tk.reshape(Ni,Nj)
```

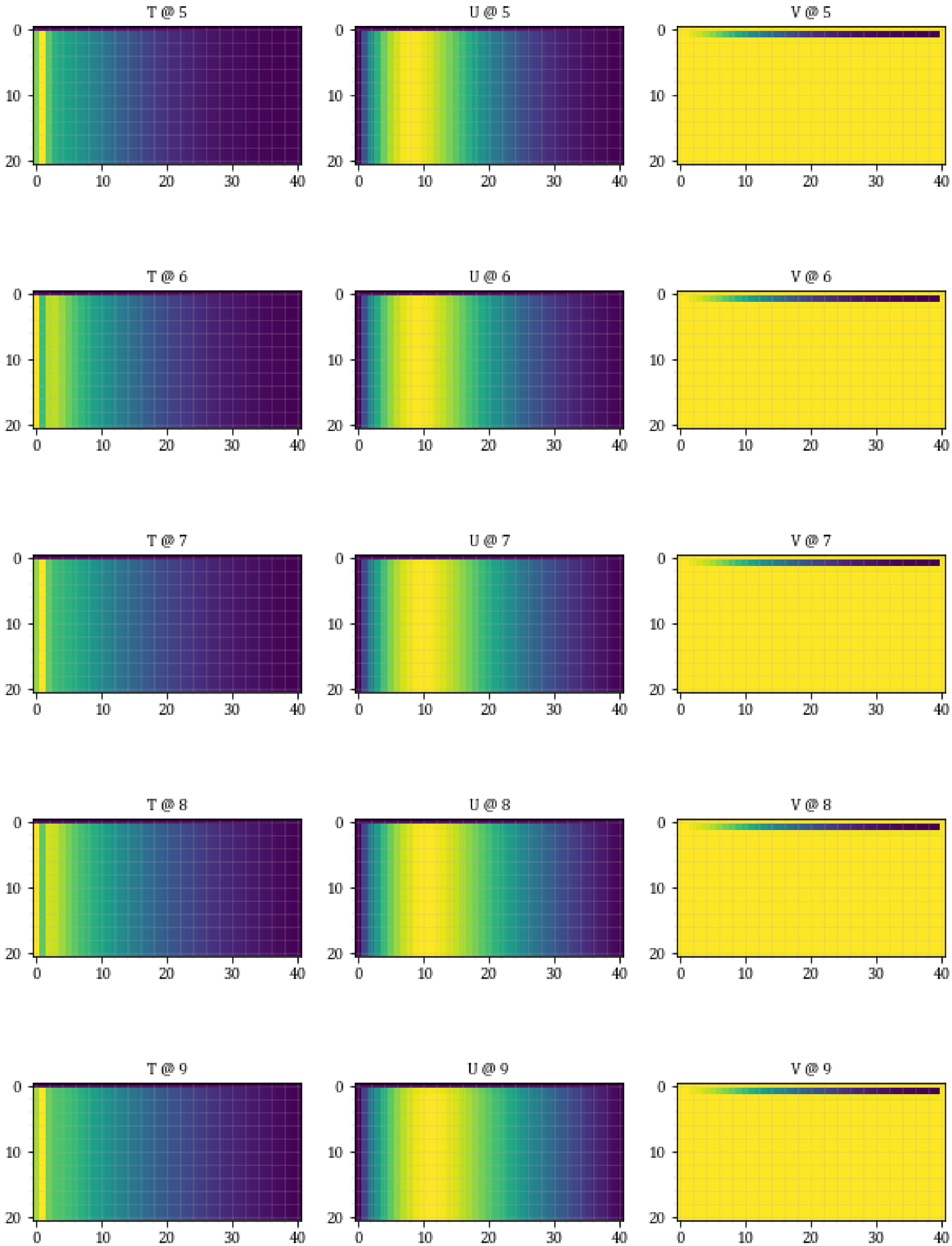
```
Solving timestep 1 ...
   Newton iteration #1 ... (err=9.831e+00)
   Newton iteration #2 ... (err=3.684e-04)
   Newton iteration #3 ... (err=3.624e-11)
   Newton iteration #4 ... (err=1.073e-15)
Solving timestep 2 ...
   Newton iteration #1 ... (err=5.850e+00)
   Newton iteration #2 ... (err=2.300e-04)
   Newton iteration #3 ... (err=3.599e-11)
   Newton iteration #4 ... (err=7.211e-16)
Solving timestep 3 ...
   Newton iteration #1 ... (err=4.467e+00)
   Newton iteration #2 ... (err=1.346e-04)
   Newton iteration #3 ... (err=1.985e-11)
   Newton iteration #4 ... (err=5.838e-16)
Solving timestep 4 ...
   Newton iteration #1 ... (err=3.665e+00)
   Newton iteration #2 ... (err=1.175e-04)
   Newton iteration #3 ... (err=1.534e-11)
   Newton iteration #4 ... (err=1.207e-15)
Solving timestep 5 ...
   Newton iteration #1 ... (err=3.115e+00)
   Newton iteration #2 ... (err=1.001e-04)
   Newton iteration #3 ... (err=1.221e-11)
   Newton iteration #4 ... (err=6.805e-16)
Solving timestep 6 ...
   Newton iteration #1 ... (err=2.704e+00)
   Newton iteration #2 ... (err=9.087e-05)
   Newton iteration #3 ... (err=1.022e-11)
   Newton iteration #4 ... (err=1.107e-15)
Solving timestep 7 ...
   Newton iteration #1 ... (err=2.378e+00)
   Newton iteration #2 ... (err=8.199e-05)
   Newton iteration #3 ... (err=8.633e-12)
   Newton iteration #4 ... (err=7.678e-16)
Solving timestep 8 ...
   Newton iteration #1 ... (err=2.113e+00)
   Newton iteration #2 ... (err=7.478e-05)
   Newton iteration #3 ... (err=7.405e-12)
   Newton iteration #4 ... (err=1.204e-15)
Solving timestep 9 ...
   Newton iteration #1 ... (err=1.890e+00)
   Newton iteration #2 ... (err=6.735e-05)
   Newton iteration #3 ... (err=6.245e-12)
   Newton iteration #4 ... (err=7.698e-16)
Solving timestep 10 ...
   Newton iteration #1 ... (err=1.701e+00)
   Newton iteration #2 ... (err=6.042e-05)
   Newton iteration #3 ... (err=5.206e-12)
   Newton iteration #4 ... (err=1.246e-15)
```

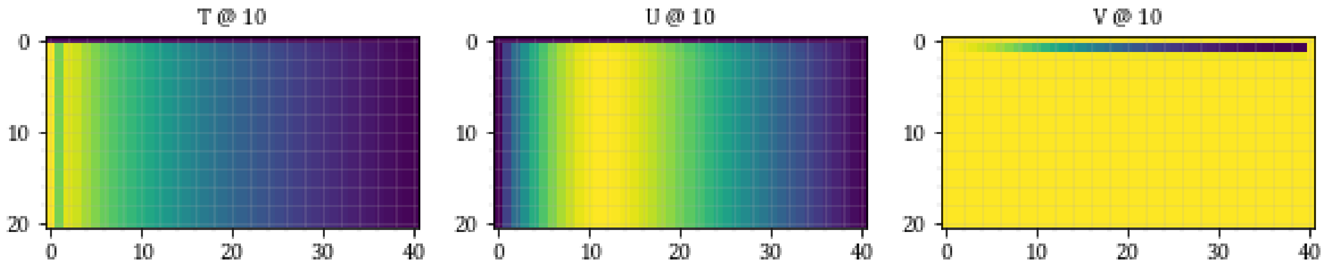
[2]:

```python
#%%
for n in arange(0,Nt) :
    fig, [ax1,ax2,ax3] = plt.subplots(1,3, figsize=[10,5])
    ax1.imshow( Tnij[n,:,:] )
    ax1.set_title(f"T @ {n}")
    ax2.imshow( Unij[n,:,:] )
    ax2.set_title(f"U @ {n}")
    ax3.imshow( Vnij[n,:,:] )
```

```
ax3.set_title(f"V @ {n}")
```




T @ 5
U @ 5
V @ 5
T @ 6
U @ 6
V @ 6
T @ 7
U @ 7
V @ 7
T @ 8
U @ 8
V @ 8
T @ 9
U @ 9
V @ 9



[3]:
```
from math import pi, sin, cos, exp
import numpy as np
from numpy import linspace, zeros, arange
from numpy import ix_ as ix
np.set_printoptions(threshold=10000, linewidth=10000)
from numpy import exp, linspace, vectorize
import matplotlib.pyplot as plt
plt.style.use('paper.mplstyle')

dx = 0.05 ; dy = dx/2
Tf = 0.1 ; Nt=1000
dt = Tf/Nt ; Nt = Nt + 1

X = np.arange(0,1+dx,dx) ; Ni = len(X)
Y = np.arange(0,1+dy,dy) ; Nj = len(Y)
Nij = Ni * Nj

beta = 0.5
Pr = 0.733

# Global index
def _(i,j) : return j + Nj*i

#
# Assign BCs to solution vectors
#
def init_bcs() :
    global Unij, Vnij, Tnij
    Unij = zeros( [Nt,Ni,Nj] )
    Vnij = zeros( [Nt,Ni,Nj] )
    Tnij = zeros( [Nt,Ni,Nj] )

    Unij[0,:,:] = 0 # ic
    Vnij[0,:,:] = 0 # ic
    Tnij[0,:,:] = 0 # ic

    Unij[:,:,0] = 0 # BC , Y=0
    Vnij[:,:,0] = 0 # BC , Y=0
    Tnij[:,:,0] = 1 # BC , Y=0

    Unij[:,:,-1] = 0 # BC , Y=inf
    Vnij[:,:,-1] = 0 # BC , Y=inf
    Tnij[:,:,-1] = 0 # BC , Y=inf

    Unij[:,0,:] = 0 # BC , X=0
    Vnij[:,0,:] = 0 # BC , X=0
    Tnij[:,0,:] = 0 # BC , X=0


#
#
#
#  MAIN FLOW
#
#
#

# Global solution vector
init_bcs()
```

```python
for n in arange(1,Nt) :
    print(f"Solving timestep {n} ...")

    T = Tnij[n-1,:,:]
    U = Unij[n-1,:,:]
    V = Vnij[n-1,:,:]
    # Solve U
    for i in arange(1,Ni) :
        for j in arange(1,Nj-1) :
            Unij[n,i,j] = U[i,j] + dt * (
                                + T[i,j]                                # T
                                + (U[i,j-1]-2*U[i,j]+U[i,j+1])/dy/dy    # Uyy
                                - U[i,j]*(U[i,j]-U[i-1,j])/dx           # - U Ux
                                - V[i,j]*(U[i,j]-U[i,j-1])/dy           # - U Uy
                            )

    # Solve T
    for i in arange(1,Ni) :
        for j in arange(1,Nj-1) :
            Tnij[n,i,j] = T[i,j] + dt*(
                + 1/Pr*(T[i,j-1]-2*T[i,j]+T[i,j+1])/dy/dy               # Tyy/Pr
                - U[i,j]*(T[i,j]-T[i-1,j])/dx                           # -U Tx
                - V[i,j]*(T[i,j]-T[i,j-1])/dy                           # -V Ty
            )

    # Solve V
    for i in arange(1,Ni) :
        for j in arange(1,Nj-1) :
            Vnij[n,i,j] = Vnij[n,i,j-1] - dy/dx*( Unij[n,i,j] - Unij[n,i-1,j])
```

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Solving timestep 1 ...
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[4]:
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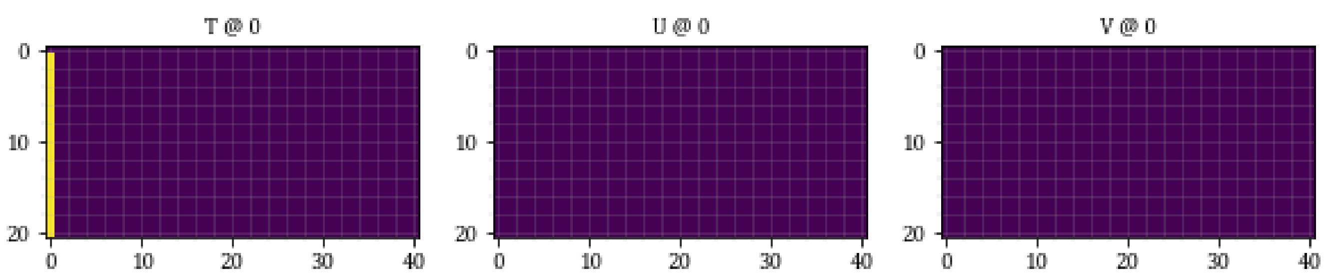
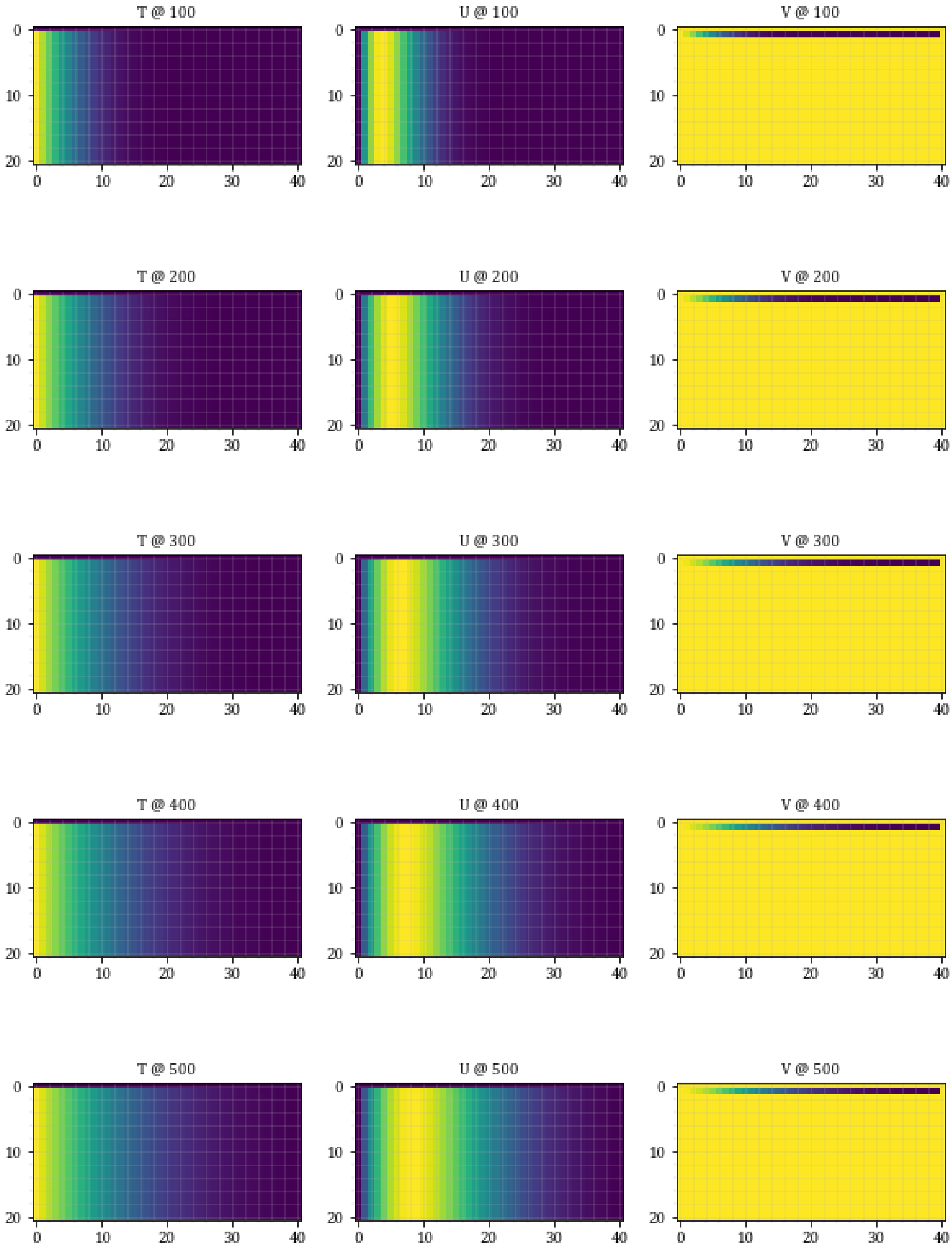
for n in arange(0,Nt,100) :
    fig, [ax1,ax2,ax3] = plt.subplots(1,3, figsize=[10,5])
    ax1.imshow( Tnij[n,:,:] )
    ax1.set_title(f"T @ {n}")
    ax2.imshow( Unij[n,:,:] )
    ax2.set_title(f"U @ {n}")
    ax3.imshow( Vnij[n,:,:] )
    ax3.set_title(f"V @ {n}")
```




T @ 100
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V @ 100
T @ 200
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V @ 200
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V @ 300
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U @ 400
V @ 400
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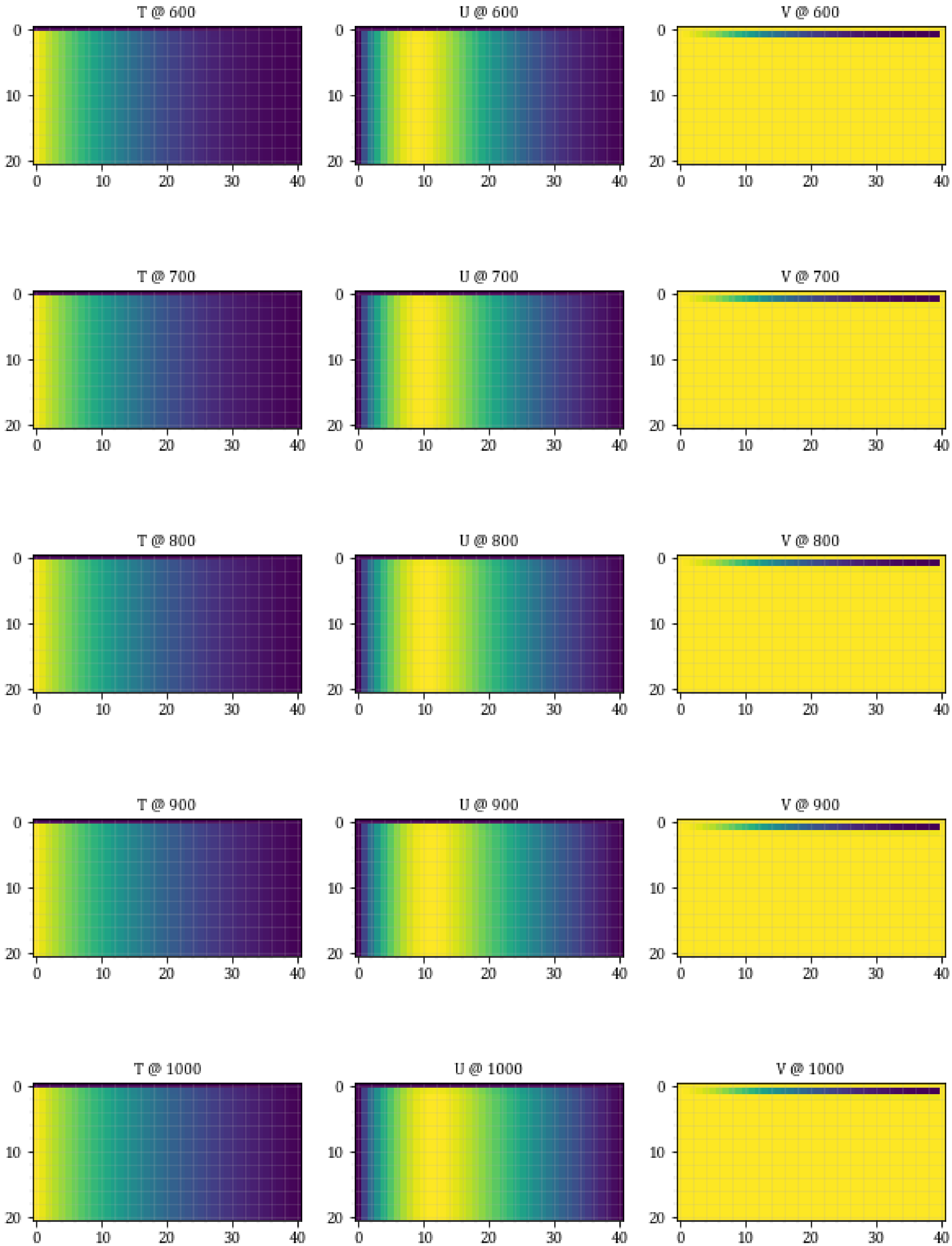
U @ 500
V @ 500


T @ 600
U @ 600
V @ 600
T @ 700
U @ 700
V @ 700
T @ 800
U @ 800
V @ 800
T @ 900
U @ 900
V @ 900
T @ 1000
U @ 1000
V @ 1000