Advanced Petrophysics
PGE 381L, Fall 2023
Unique Number: 20215

Instructor: Zoya Heidari, Ph.D.

Midterm Exam

October 26, 2023
5:00 PM to 7:00 PM

Name: Solution

UT EID: ____________________

Instructions:

1. Print your name above. Do not put your name anywhere else on the test. Do not fold the examination booklet. If you un-staple the examination booklet, please staple it before submission.
2. This examination is OPEN BOOK, closed notes. The only books you are allowed to use include volumes 1 and 2 of your "Advanced Petrophysics" textbook. You can also bring to the exam one letter-size page only including formulae and notes of your choice. No solution to any question is allowed. The only other resources you are allowed to use are pencil/pen, eraser, calculator, and straightedge.
3. Use of cell phones or electronic communication devices is not permitted.
4. Answers should be provided in an organized manner, with a clear and readable handwriting. There will be no credit for the vague or unreadable answers.
5. When you complete the examination, sign the statement below and turn in the examination booklet.

Total Number of Questions: 7

"I have neither given nor received unauthorized aid on this academic work."

Acknowledged via Your Signature: ____________________

Question 1: (14 points) (6P + 8A)

A synthetic porous medium is made of insulator material and is shaped as a cube of length L. A square of dimension L/2 is drilled through the middle of this cube.

Assuming that this hole is filled with brine of resistivity R_w and that the current will flow perpendicular to the faces with the square holes, calculate the formation resistivity factor (F) for the porous medium. Also, determine the relationship between formation factor and porosity. How does this compare with Archie's equation?

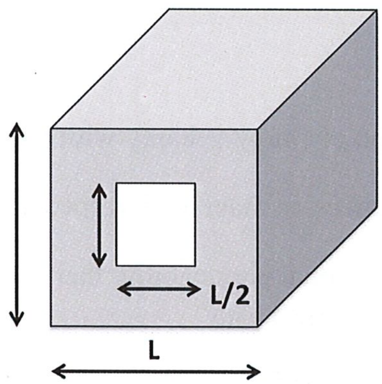


Figure 1: A synthetic porous medium. White and grey regions represent pores and insulator material, respectively.

$$\left.\begin{aligned}\frac{1}{r_o} &= \frac{1}{r_w} + \frac{1}{r_m} \\ r_m &\approx \infty\end{aligned}\right\} \rightarrow r_o = r_w$$

2

$$R_o \frac{L_o}{A_o} = R_w \frac{L_w}{A_w} \rightarrow \frac{R_o L}{L^2} = \frac{R_w L}{(L/2)^2}$$

$$\rightarrow R_o = 4 R_w$$

3

$$\boxed{F = \frac{R_o}{R_w} = 4}$$

2 3

$$\phi = \frac{(L/2)^2 L}{(L^2) L} = \frac{1}{4} \rightsquigarrow \boxed{F = \frac{1}{\phi}}$$

2

$$F = \frac{a}{\phi^m} \rightsquigarrow \boxed{a = m = 1}$$

2

Question 2: (20 points) (10P + 10A)

The permeability tensor for a 2D reservoir in the *x-y* Cartesian coordinate system is given by

$$\bar{\bar{k}}(x,y)=\begin{bmatrix}110 & 50\\ 50 & 110\end{bmatrix} md$$

Viscosity of the reservoir fluid can be assumed to be 1 cp. The potential gradient is given by

$$\nabla\vec{\Phi}=0.4i-0.2j(atm/cm)$$

Answer the following questions:

a) The magnitude and direction of Darcy velocity with respect to *x* coordinate.

b) What angle does the flow direction make with respect to the potential gradient?

c) Estimate the maximum directional permeability that can be observed in this reservoir and its direction with respect to *x* coordinate.

(a) (4A+3P)

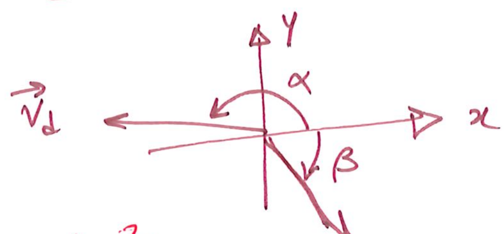
$$\vec{V}_d = -\frac{\bar{\bar{k}}}{\mu}\cdot\vec{\nabla\Phi} = -\begin{bmatrix}110 & 50\\ 50 & 110\end{bmatrix}\begin{bmatrix}0.4\\ -0.2\end{bmatrix}\left(\frac{1}{1000}\right) = \begin{bmatrix}-0.034\\ 0.002\end{bmatrix} cm/s$$

$$|\vec{V}_d| = \sqrt{0.034^2+0.002^2} = 0.03405 \text{ cm/s}$$ 2

$$\cos\alpha = \frac{\vec{V}_d\cdot\hat{i}}{|\vec{V}_d||\hat{i}|} = -\frac{0.034}{0.03405} = -0.9985 \rightarrow \alpha = 176.86^\circ$$ 2

(b) 2A+3P

$$\cos\beta = \frac{\vec{\nabla\Phi}\cdot\hat{i}}{|\vec{\nabla\Phi}|} = \frac{0.4}{\sqrt{0.4^2+0.2^2}} \Rightarrow \beta = 26.57 \text{ with } x \text{ axis}$$



Angle btw $\vec{V}_d$ & $\vec{\nabla\Phi} = \alpha+\beta = 203.43^\circ$

OR 156.6° 2

(c) (4A+4P)

$$|K-\lambda I|=0 \rightarrow \begin{vmatrix}110-\lambda & 50\\ 50 & 110-\lambda\end{vmatrix}=0 \rightarrow \boxed{\lambda_1=160\text{ md}}\ 2 \quad \lambda_2=60\text{ md}$$

$\downarrow$ K_{max}

Direction: $\begin{bmatrix}110-160 & 50\\ 50 & 110-160\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix} \rightarrow x=y$

$\rightarrow u=\begin{bmatrix}1\\ 1\end{bmatrix} \Rightarrow \boxed{\Phi=45^\circ}$ 2

with respect to x axis

Question 3: (24 points)

Figure 2 shows an apparatus for determining the permeability of a core using a nonreactive liquid.

a) Derive the differential equation for the instantaneous height h in terms of the pertinent system variables and parameters. Use the following symbols in your derivation:

Cross sectional area of the core and the U tube	=	A
Length of core	=	L
Core permeability	=	k
Liquid density	=	ρ
Liquid viscosity	=	μ
Gravitational acceleration	=	g
Time	=	t
Height at t = 0	=	h_o

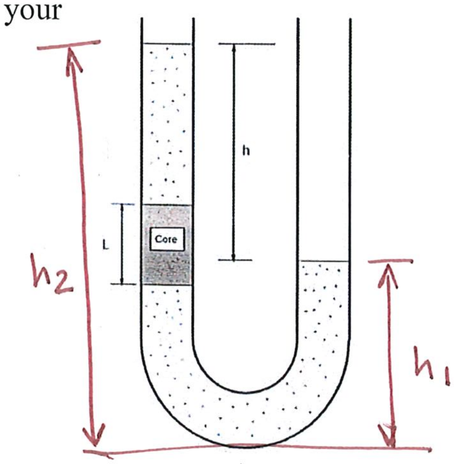


Figure 2

b) Solve the differential equation you derived in part (a) analytically.

c) How can we use the model you developed in part (b) to estimate permeability? What data would you collect for this purpose?

12 (a) (8P+4A)

$q = A\frac{dh_1}{dt}$, $q = -A\frac{dh_2}{dt}$ $\Rightarrow 2q = -A\left[\frac{dh_2}{dt} - \frac{dh_1}{dt}\right] = -A\frac{dh}{dt}$

$\Rightarrow q = -\frac{A}{2}\frac{dh}{dt}$, $q = KA\frac{h}{L}$ $\Rightarrow KA\frac{h}{L} = -\frac{A}{2}\frac{dh}{dt}$

$\Rightarrow \boxed{\frac{dh}{dt} = -\frac{2K}{L}h} \Rightarrow \frac{dh}{h} = -\frac{2K}{L}dt$

8 (6P+2A) (b) $\ln\left(\frac{h}{h_o}\right) = -\frac{2K}{L}t = -\frac{2}{L}\left(\frac{k\rho g}{1.0133\times10^6\mu}\right)t$

(c) h as a function of t

4

$$\boxed{K = -\left[\ln\left(\frac{h}{h_o}\right)\right]\left(\frac{L}{2}\right)\left(\frac{1.0133\times10^6\mu}{\rho g t}\right)}$$

Question 4: (5 points)

We need to measure permeability of a rock sample, in which transport of fine particles can cause challenge. What permeability measurement method would you choose? Gas or liquid steady-state permeability measurement? Explain the reason for your choice in one or two sentences.

GAS, Gas cannot move the fine particles as easy as the water does.

Question 5: (5 points)

Qualitatively plot on **Figure 3** apparent resistivity of water versus formation water resistivity in the cases of (A) absence of clay minerals and (B) high concentration of clay minerals? Label your curves with A and B.

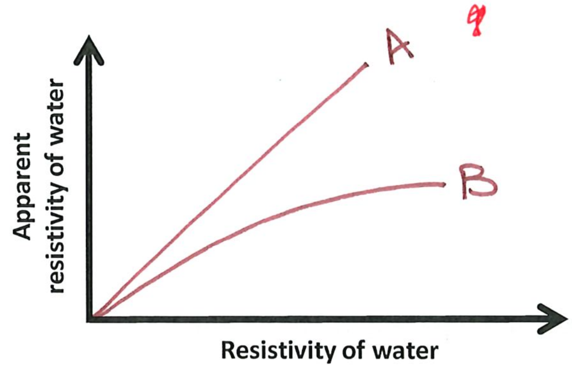


Figure 3: Apparent resistivity of water versus formation water resistivity in the cases of (A) absence of clay minerals and (B) high concentration of clay minerals.

Question 6: (6 points)

In a fining-upward sedimentary sequence, the diameter of spherical grains at the top is half of that at the bottom of the sequence. Assume all the grains are spherical, uniform grain size, and the same packing throughout the sequence. Answer the following questions:

$D_1 \uparrow \phi_1$

$D_2 \quad \phi_2$

$D_1 = \frac{D_2}{2}$

a) Estimate the ratio of formation factor at the top to the formation factor at the bottom of the sequence.

$\phi_1 \simeq \phi_2 \Rightarrow F_1 \simeq F_2 \rightarrow \frac{F_1}{F_2} = 1$

b) Estimate the ratio of permeability at the top to the permeability at the bottom of the sequence.

$K \propto \frac{1}{D^2} \Rightarrow \frac{K_1}{K_2} = \frac{1}{4}$

Question 7: (26 points)

Consider the well logs shown in **Figure 4**. This is an example of well logs acquired in a clastic sedimentary sequence. All depths are given in m TVD (True Vertical Depth) below a reference depth. Archie's parameters are a=1, m=2, and n=2. Neutron and density logs are expressed in apparent water-filled sandstone porosity units. Surface temperature is 80 degF. Formation temperature at 8,500 ft MD is 155 deg F.

a) What is the type of drilling mud? (3 points)

☑ WBM
☐ OBM
☐ Not enough information to respond

b) What is the dominant fluid type at 8122 ft? (3 points)

☐ Mud filtrate
☐ Fresh water
☑ Brine
☐ Hydrocarbon
☐ Uncertain

c) What is the dominant fluid type at 8008 ft? (3 points)

☐ Mud filtrate
☐ Fresh water
☐ Brine
☑ Hydrocarbon
☐ Uncertain

d) What is the porosity at 8122 ft? (3 points)

(1P+2A) SHOW YOUR WORK BELOW 21 %

$$\phi_{N,SS} = \phi_{D,SS} = 0.21 \Rightarrow \phi = 21\%$$

e) What is the porosity at 8008 ft? (3 points)

(2P+2A) SHOW YOUR WORK BELOW 19.6 %

$$\phi = \sqrt{\frac{\phi_{N,SS}^2 + \phi_{D,SS}^2}{2}} = \sqrt{\frac{0.18^2 + 0.21^2}{2}} = 0.196$$

f) What is the depth where you would select for performing calculations to estimate the electrical resistivity of formation water (recall that you will need to calculate formation temperature at this depth)? (3 points)

8122 ft

g) Using the resistivity logs, estimate the electrical resistivity of formation water, Rw. (4 points)

(2P+2A)

SHOW YOUR WORK BELOW Rw = 0.049 Ohm-m

$R_t = 1.1\ \Omega\cdot m$, $a = 1$, $m = 2$

$$\left.\begin{array}{l}\phi_{N,SS} = 0.21 \\ \phi_{D,SS} = 0.21\end{array}\right\} \rightarrow \phi = 0.21$$

$$R_t = R_w \frac{a}{\phi^m} \Rightarrow R_w = (1.1)\left(\frac{0.21^2}{1}\right) = 0.049\ \Omega\cdot m$$

h) What is the hydrocarbon saturation at 8008 ft? (4 points)

(3P + 1A)

SHOW YOUR WORK BELOW $\underline{\quad 67.4 \quad}$ %

$$R_t = R_W \frac{a}{\phi^m S_W^n}$$

$$12 = 0.049 \frac{1}{(0.196)^2 S_W^2} \Rightarrow S_W = 0.326$$

$$\Rightarrow S_{HC} = 1 - 0.326$$