Advanced Petrophysics
PGE 381L, Fall 2023
Unique Number: 20215

Homework Assignment No. 3

September 14, 2023
Due on September 28, 2023, before 11:00 PM

Name: ______________________ **Solution** ______________________________

UT EID: ____

Objectives:

a) To practice assessment of porosity and fluid saturations in-situ condition
b) To practice calculation of hydrocarbon reserves
c) To practice assessment of fluid saturation in the presence of shale

Note: Please scan your homework assignment and upload it as one pdf file on the Canvas website before the deadline. Please name your homework document as follows:

PGE381L_2023_Fall _HW03_lastname_name.pdf

Example: PGE381L_2023_Fall_HW03_Heidari_Zoya.pdf

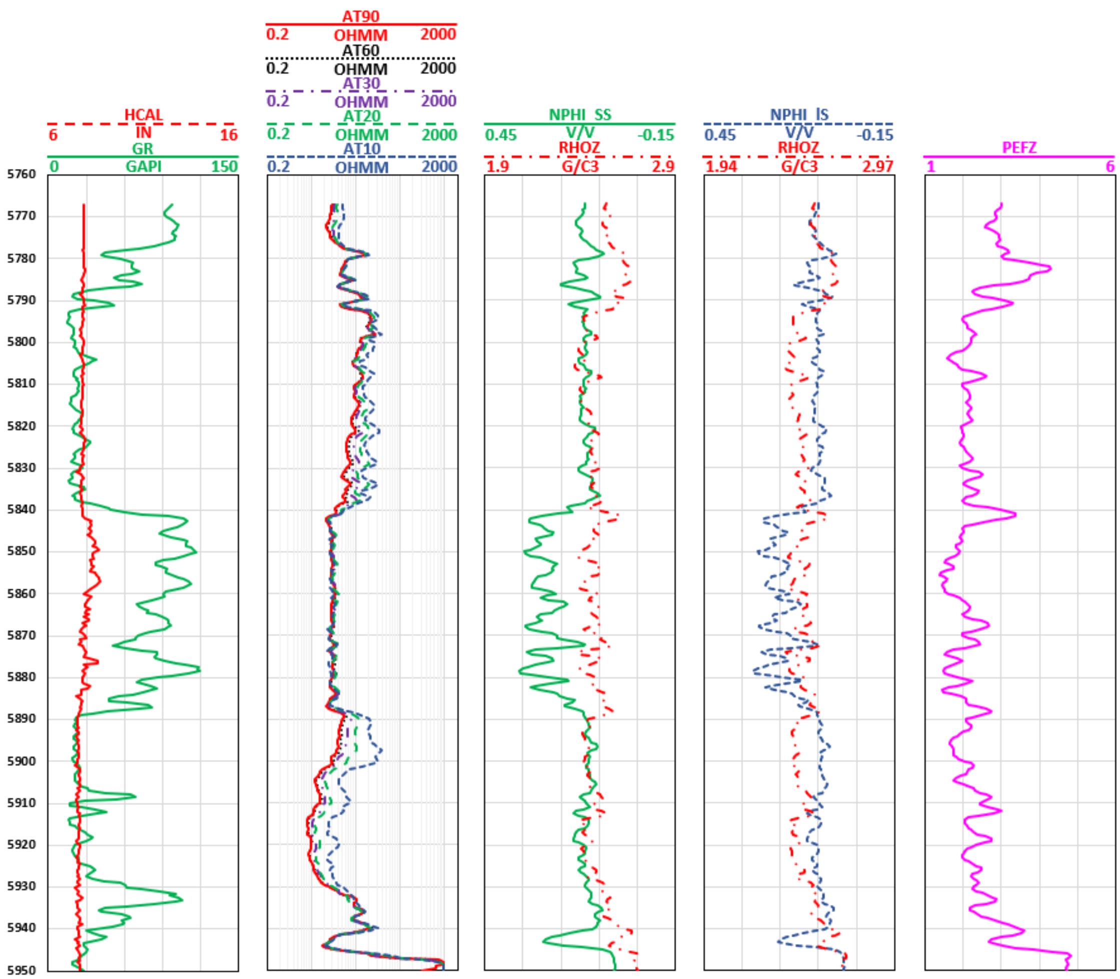
Question 1: Consider the well logs provided to you in Excel format, named "Question1_data." Answer the following questions:

Note: Please do not use any well-log interpretation software.

You can assume a = 1, m =2, n = 2

a) Plot the well logs similar to the example distributed in the class (Example 1). Pay attention to the scale.

   Question to think about (will not be graded): Do you know the reason behind the choice of scale for plotting bulk density, when plotted on top on neutron porosity in sandstone units versus the case it is plotted against neutron porosity in sandstone units?

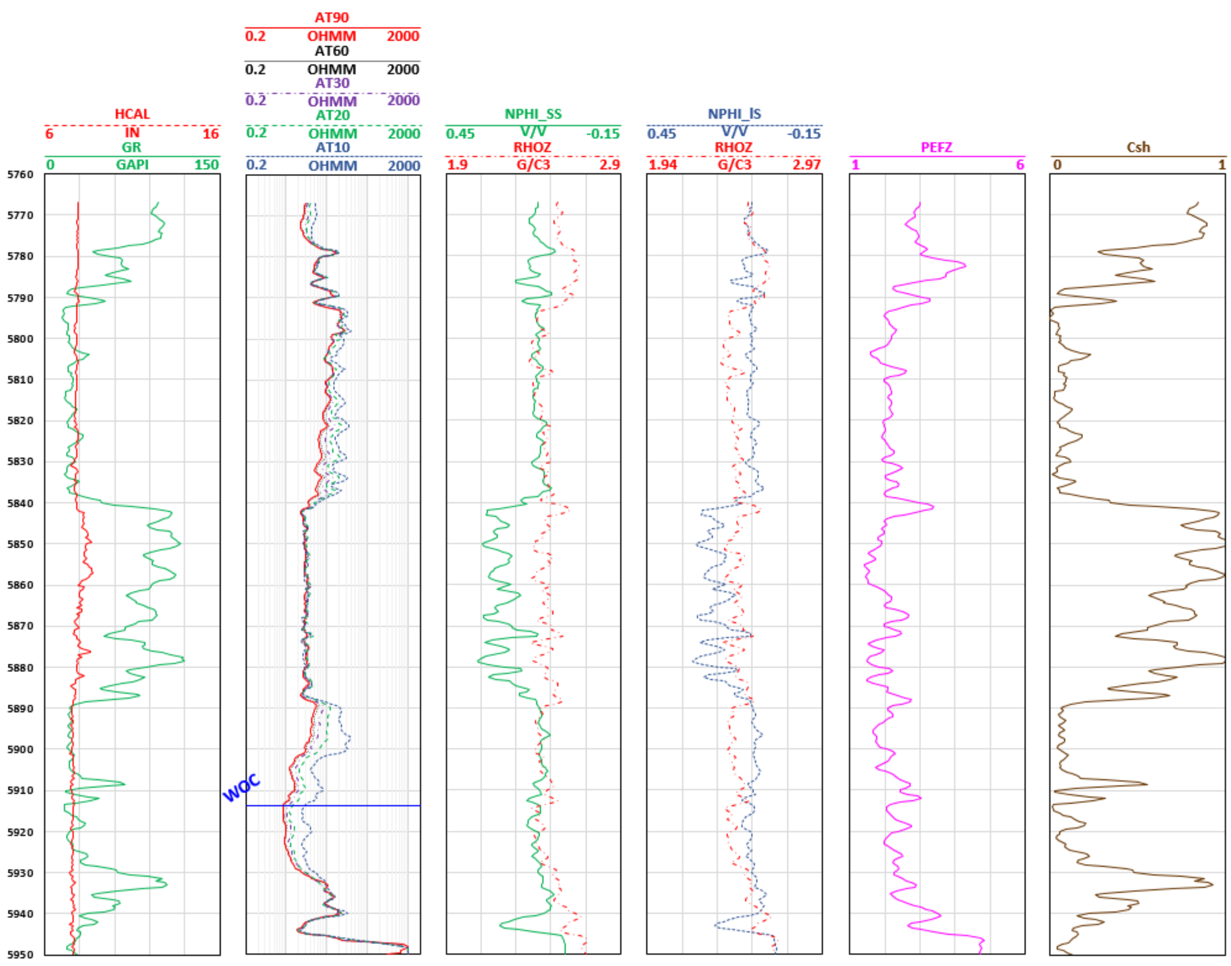


b) Calculate the volumetric concentration of shale at each depth via

$$C_{sh} = \frac{GR - GR_s}{GR_{sh} - GR_s},$$

where GR_s and GR_{sh} are the gamma ray readings in clay-free sand and pure shale, respectively.

GR_s ~15 GAPI
GR_{sh} ~ 121 GAPI.



c) Using the density log, calculate the porosity at each depth. Make appropriate assumptions for matrix and fluid densities. Plot the estimated depth-by-depth water saturation next to your well logs. Consider 85% Illite and rest Chlorite clay type in shale. You can assume that 70% of the matrix in shale is composed of clay and the rest is silt.
What type of porosity are you calculating using the density log? Total or effective?

Density of illite – 2.8 g/cc (anything in the range of 2.6-2.9 is acceptible)

Density of chlorite – 2.89 g/cc (anything in the range of 2.6-3.3 is acceptible)

$$\rho_{sh} = 0.3 \times \rho_{silt} + 0.7 \times (0.85 \times \rho_{illite} + 0.15 \times \rho_{chlorite})$$

Density of shale ρ_{sh} – 2.75 g/cc

Density of sandstone ρ_s – 2.65 g/cc

Calculate the porosity of the shale zone:

$$\phi_{sh} = \frac{\rho_b - \rho_{sh}}{\rho_f - \rho_{sh}}$$

Calculate the density of the sand zone:

$$\phi_s = \frac{\rho_b - \rho_s}{\rho_f - \rho_s}$$

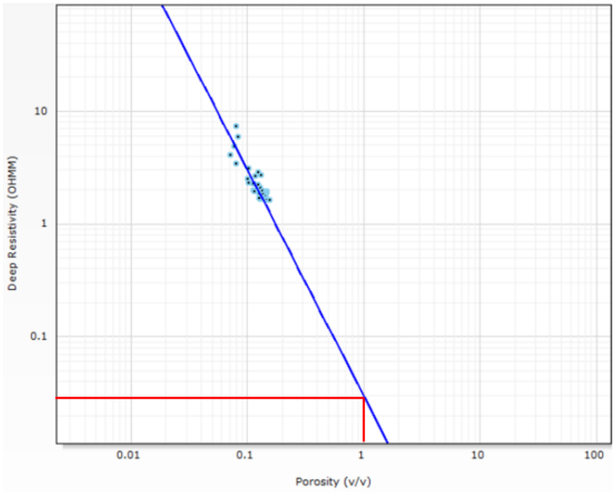
Fluid density - 1g/cc (fresh water mud invasion)

d) Identify the water-oil contact.

~5915 ft

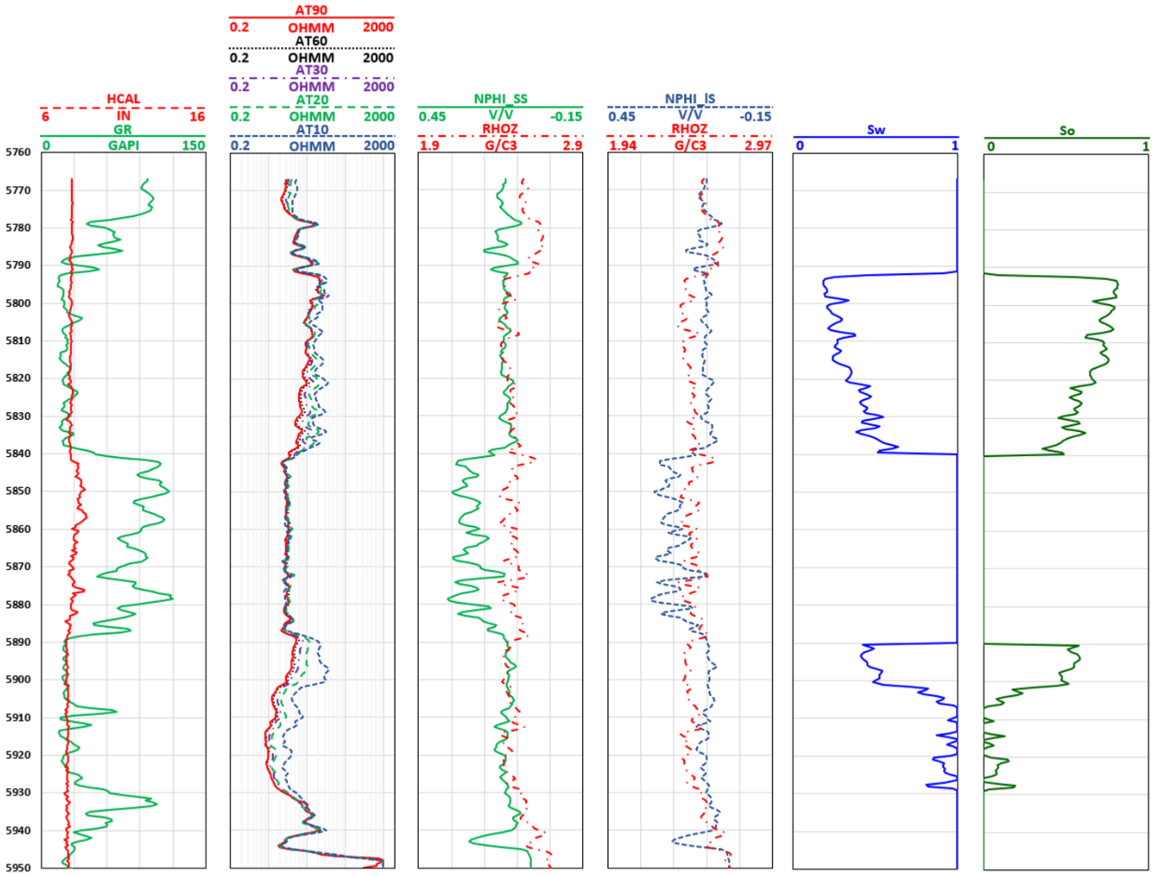
e) Figure 1 shows deep resistivity measurements versus porosity for the water-saturated zone in this formation. This plot is called Pickett plot. Estimate the formation water resistivity using the data provided Figure 1.

R_w = 0.03 OHMM



f) Estimate water/hydrocarbon saturation at each depth. Plot the estimated depth-by-depth water saturation next to your well logs. Plot the estimated depth-by-depth oil saturation next to your well logs. Note that shales are fully water saturated. You do not need to perform calculations for shale zones.

$$S_w = \sqrt{\frac{R_w}{R_t \times \phi^2}}$$



g) Estimate total hydrocarbon reserves in place (in bbls/unit area).

$$N_{HC} = \sum_{i=1}^{n} 7758 \times h_i \times \phi_i \times (1 - S_{wi})$$

$$h = 0.5\ ft$$

Upper interval (5791 ft -5839.5 ft)

$$N_{HC} = 30111.9 \frac{Bbls}{Acre}$$

Lower interval (5890.5 ft -5914.5 ft)

$$N_{HC} = 6857.87 \frac{Bbls}{Acre}$$

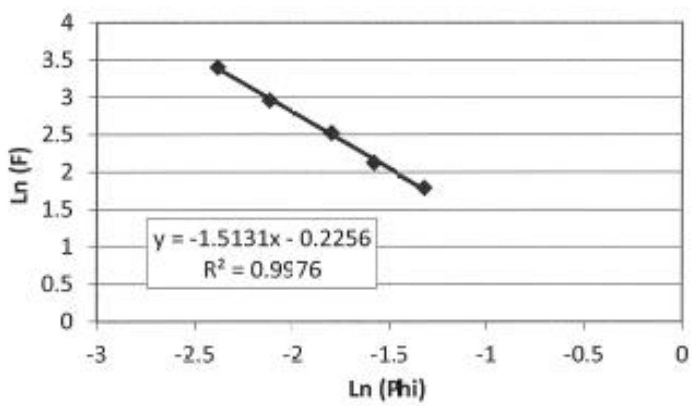
$$N_{HCtotal} = 36969.8 \frac{Bbls}{Acre}$$

h) **Bonus Question:** Estimate movable hydrocarbon saturation at each depth. Plot your results. What are the uncertainties associated with your interpretation?

Question 2: A series of core measurements from a well provided the formation factor versus porosity data as listed in the following table:

Formation Resistivity Factor	Porosity
30	0.092
19.3	0.12
12.5	0.165
8.4	0.205
6	0.268

A thick salt water bearing layer of the reservoir is encountered in an offset well with a resistivity of 1.79 ohm-m. If the resistivity of the salt water is 0.076 ohm-m, estimate the porosity of the water bearing layer.



$$F = \frac{a}{\phi^m}$$

$$\ln F = \ln a - m \ln \phi$$

$$m = 1.513, a = 0.798$$

$$\phi = 10.8\%$$

Question 3: Figure 2 shows a synthetic porous medium which is made of insulator material and is shaped as a cube of length L. Three cylindrical shape tubes of radius *r* are drilled through the middle of this cube. You can assume:

$r / L = 1 / \left(3\sqrt{\pi}\right)$

Two of the cylindrical tubes are filled with brine of resistivity R_w, and the third one is filled with oil. Answer the following questions:

a) Calculate the formation resistivity factor (F), and the resistivity index (I) for the porous medium in the direction parallel to the length of the cylindrical tubes.
b) Determine the relationship between formation factor and porosity.
c) How does this compare with Archie's equation?

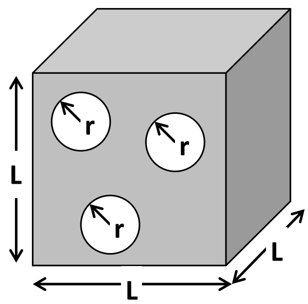


Figure 2: A synthetic porous medium. White and grey regions represent pores and insulator material, respectively.

(a)

$$\frac{1}{r'_o} = \frac{1}{r'_w} + \frac{1}{r'_m}, r'_m \approx \infty, r'_o = r'_w$$

$$r' = R\frac{L}{A}, R_o \frac{L}{L^2} = R_w \frac{L}{3\times\left(\pi r^2\right)}$$

$$F = \frac{R_o}{R_w} = 3$$

$$R_t \frac{L}{L^2} = R_w \frac{L}{2\times\left(\pi r^2\right)}, \frac{R_t}{R_w} = \frac{9}{2}$$

$$I = \frac{R_t}{R_o}, \frac{R_t}{R_w} = I \times F, I = 1.5$$

(b) $$\phi = \frac{V_p}{V_b} = \frac{3\pi r^2 L}{L^3} = \frac{1}{3}$$
$$F = 3 = \frac{1}{\phi}$$

$$F = \frac{1}{\phi} = \frac{a}{\phi^m}$$
$$a = 1, m = 1$$
(c) $$S_w = \frac{2}{3}$$
$$I = \frac{3}{2} = \frac{1}{S_w^n}$$
$$n = 1$$

Question 4: We know that negative charge on the surface of clay minerals can cause variation (from surface of the clay mineral to the center of the pore) in electrical conductivity of water. **Figure 2** is a simplified illustration to model this variation in electrical conductivity of water in porous media by assuming two parallel layers of water. You can assume the electrical conductivity of clay-bound water layer is σ_{wb} and the electrical conductivity of formation water layer is σ_w. Saturation of clay-bound water is equal to S_{wb}. Answer the following questions:

a) Estimate equivalent electrical conductivity of water, $\sigma_{w,avg}$, in the presence of clay minerals. You can assume the pore space is fully saturated with water.

b) Update Archie's equation by taking into account your updated electrical conductivity of water. You can assume the pore space is fully saturated with water.

c) How would you update your resistivity model in part (b), if there is hydrocarbon in the system? You can assume total saturation of water and hydrocarbon are S_{WT} and S_{HC}, respectively. Write your new model.

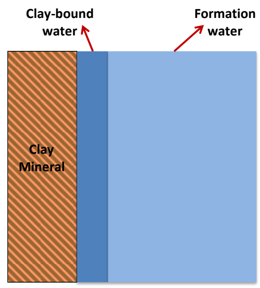


Figure 2

(a) $\overline{\sigma}_w = \sigma_{wb} S_{wb} + (1 - S_{wb})\sigma_w$

$$\sigma_T = \overline{\sigma}_w \left(\frac{\phi^m}{a}\right) S_w^n$$

(b) $S_w = 1$

$$\sigma_T = \overline{\sigma}_w \left(\frac{\phi^m}{a}\right)$$

$$\overline{\sigma}_w = \left[\sigma_{wb}\frac{S_{wb}}{S_{wt}} + \sigma_w\left(1 - \frac{S_{wb}}{S_{wt}}\right)\right]$$

(c) $\sigma_T = \overline{\sigma}_w \left(\frac{\phi^m}{a}\right) S_w^n = \left[\sigma_{wb}\frac{S_{wb}}{S_{wt}} + \sigma_w\left(1 - \frac{S_{wb}}{S_{wt}}\right)\right]\left(\frac{\phi^m}{a}\right) S_{wt}^n$

$$\sigma_T = \frac{\phi^m S_{wt}^n}{a}\left[\sigma_w + \frac{S_{wb}}{S_{wt}}(\sigma_{wb} - \sigma_w)\right]$$

Question 5: Consider the well logs in Example 2, of the well-log package which is distributed in the class. This is a laminated shaly-sand formation. Answer the following questions for depth 5522 ft. You can consider the following readings for well-log measurements:
Resistivity = 2.8 ohm-m (This is equivalent to horizontal resistivity).

a) Estimate hydrocarbon saturation at 5522 ft.

b) Estimate density of hydrocarbon at 5522 ft.

c) Estimate hydrocarbon pore volume per unit depth.

d) In the last three parts, it is expected that you take into account shale laminations in your calculations. Now assume that you use the measured resistivity without taking into account the impact of shale. What would be the relative error in estimate of hydrocarbon saturation, compared to what you obtained in part (a)?

(a)

$$PureShale:$$
$$GR_{sh} = 94, GR_s = 25$$
$$\phi_{D,sh} = 26.5\%(ss)$$
$$\phi_{N,sh} = 45\%(ss)$$
$$R_{sh} = 0.9\Omega.m$$
$$@5522\ ft$$
$$Csh = \frac{GR - GR_s}{GR_{sh} - GR_s} = 30.4\%$$
$$\phi_N = 23.5\%$$
$$\phi_D = 39\%$$
$$\phi_N^{(sh)} = \frac{\phi_N - C_{sh}(\phi_N)_{sh}}{1 - C_{sh}} = 14.09\%$$
$$\phi_D^{(sh)} = \frac{\phi_D - C_{sh}(\phi_D)_{sh}}{1 - C_{sh}} = 44.47\%$$
$$\phi s = \sqrt{\frac{14.09^2 + 44.47^2}{2}} = 33\%$$

Find a fully water saturated zone

$$R_t = R_w \frac{a}{\phi^m S_w^n}, R_w = 0.0536\Omega.m$$
$$\frac{1}{R_t} = \frac{C_{sh}}{R_{sh}} + \frac{1 - C_{sh}}{R_s}, R_s = 35.94\Omega.m$$
$$R_s = R_w \frac{a}{\phi^m S_w^n}, S_w = 0.12, S_{hc} = 0.88$$

(b)

$$\left(\phi_{D,ss}\right)_{sh} = 0.265 = \frac{\rho_{sh} - 2.65}{1 - 2.65}, \rho_{sh} = 2.21 g / cc$$

$$\rho_b = \rho_s\left(1 - C_{sh}\right) + \rho_{sh} C_{sh}, \rho_s = 1.916 g / cc$$

$$\phi s = \frac{\rho_s - \rho_m}{\rho_f - \rho_m}, \rho_f = 0.42 g / cc$$

$$\rho_{hc} = 0.35 g / cc$$

(c)

$$HPV = S_{hc}\phi_s(1 - C_{sh}) = 0.88(0.3298)(1 - 0.304)$$

$$= 0.202 / ft$$

(d)

$$R_t = R_w \frac{a}{\phi^m S_w^n}, 2.8 = 0.0536 \frac{1}{0.3298^2 S_w^2}$$

$$S_w = 0.42, S_{hc} = 0.58$$

$$error\% = \left|\frac{0.88 - 0.58}{0.88}\right| \times 100 = 34.1\%$$