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Application of Saturation-height Functions in Integrated Reservoir Description

Paul F. Worthington

Gaffney, Cline & Associates, Alton, Hampshire, U.K.

ABSTRACT

In many integrated field studies, a single reservoir-zonation scheme is not meaningful for all deterministic interpretative applications. These cases require that the concept of reservoir partitioning should evolve to become fit for purpose. Thus, a stratigraphic zonation is used solely for correlation and volumetrics, whereas a petrophysical separation is applied exclusively in core and log analysis for reservoir properties, after which the petrophysical partitioning can be discarded. In such a case, in which the stratigraphic zonation and petrophysical separation do not naturally correspond, the adoption of fit-for-purpose reservoir-partitioning schemes allows saturation-height functions to be much more sharply defined because they are then related to units of fairly uniform physical character. The resulting values of water saturation are subsequently assigned to the stratigraphic zones, using a straightforward petrophysical combining algorithm in which a key parameter is bulk volume water. The nature of the combining equation therefore indicates that the optimum form of saturation-height function should include bulk volume water as predicted variable in reservoirs that require dual partitioning. Such an approach has general application in deterministic studies of water-wet siliciclastic reservoirs. The principal benefit is the controlled reduction of uncertainty in reservoir evaluation.

INTRODUCTION

The application of saturation-height functions forms part of the intersection of geologic, petrophysical, and reservoir engineering practices within integrated reservoir description. These functions are usually applied to the net-reservoir intervals of a 3-D water-wet reservoir model in order to predict water saturation, and thence hydrocarbon saturation, at grid nodes. In this context, the term *net reservoir* relates to acceptable reservoir quality in terms of shale content and porosity.

Saturation-height functions are most effectively used for the deterministic petrophysical evaluation of reservoirs that have a stratigraphically defined architecture in the form of correlatable reservoir zones. Such a deterministic approach is the focus of this paper. The functions commonly draw on other parameters that are part of the contemporary culture of computer mapping. Obvious candidates are porosity and permeability.

Without exception, the application of saturation-height functions requires an input of height before an estimate can be made of water saturation (S_w) and thence of hydrocarbon saturation (S_h). The traditional practice has been to map, geologically, the stratigraphically correlatable zones that comprise the reservoir

architecture and to deduce the midpoint heights of these zones at an array of grid nodes. The application of saturation-height functions then allows water saturation to be evaluated for each zone at each grid node. The underlying assumption is that the stratigraphic zones correspond (lithologically) to the petrophysical rock types used to establish the saturation-height functions themselves. Where they do not, the saturation-height functions should be applied only to the intrazonal petrophysical units. It then becomes necessary to reconcile the two partitioning schemes when the petrophysically interpreted reservoir properties are input to the stratigraphic zonation for purposes of volumetrics.

The concepts of dual partitionings and their subsequent reconciliation are key elements of integrated reservoir description, albeit ones that have received little attention. They constitute the primary subject of this paper. Here the principal thrust is not an analysis of the theory or practice of petrophysical data partitioning. Indeed, those matters have been fully discussed elsewhere (Doveton, 1986, 1994; Moss, 1990, 1997). The emphasis is instead on the methodology for incorporating the petrophysically interpreted reservoir properties, as deliverables of the petrophysical partitioning exercise, within the stratigraphic-zonation scheme. In so doing, particular attention is given to the evaluation of water saturation for incorporation into a 3-D reservoir model. Therefore, the context is one of evaluating water saturation at interwell (grid) locations for use in volumetric computations to determine hydrocarbons in place. Although the algorithm for incorporating petrophysically derived water saturations into a stratigraphic-zonation scheme is specific and readily understood, the form of this algorithm, with bulk volume water (BVW) as an input parameter, conveys important messages for the interwell prediction of water saturation at the reservoir scale. These messages have not been widely appreciated. To focus the ensuing discussion, we will consider only water-wet reservoirs.

INTEGRATED RESERVOIR DESCRIPTION

For years, geoscientists who use deterministic approaches to integrated reservoir description have struggled to reconcile the geologic requirement for a system of correlatable stratigraphic zones, each of which can be mapped confidently, with the petrophysical need for a set of physically distinct units, each of which can be characterized exclusively for well-log analysis. Attempts to forcibly merge these two requirements have sometimes resulted in weak reservoir models, which in turn have drawn the geoscientist toward geostatistical methods in the premature and mistaken belief that the deterministic approach has broken down. A sounder technical strategy is to detach the correlative process of stratigraphic *zonation* from the interpretative practice of petrophysical *separation* into physically discrete units, or *petrofacies*, in the more general case in which the two partitionings do not show a broad equivalence on the basis of lithostratigraphy.

Stratigraphic Zonation

Construction of the reservoir model requires that zonal isopachs be fitted within an overall seismic depth framework. Homogeneity of lithologic or other physical properties of the net-reservoir rock within each reservoir zone is not a prerequisite for a consistent zonation. The important issues are that the zonation scheme has a strong stratigraphic basis and that it be reconcilable with the seismic structural model, which provides the 3-D envelope for the reservoir architecture associated with the stratigraphic zones.

A stratigraphic zone can be made up of different rock types, each of which has a definable petrophysical character. It is necessary to take into account these physically diverse, intrazonal rock types during petrophysical evaluation. In some cases, it may be possible to subdivide the stratigraphic zones on the basis of physically exclusive lithologic characteristics. If such a subdivision is viable fieldwide, the subzones become the basic units for reservoir description. If such a breakdown is not feasible, a distinct petrophysical separation will be needed if the evaluation is to be approached deterministically.

Petrophysical Separation

Petrophysical interpretation comprises a sequence of objectives for inferring reservoir properties (as the predicted variables, or predictands) from measurements of other physical properties (as the predicting variables, or predictors). Each objective draws upon relationships between reservoir and/or other physical properties and quantifies these relationships through interpretative equations, e.g., algorithms for evaluating porosity from the density-log response. These equations have to be calibrated, usually through core measurements of reservoir and other physical properties. The algorithms are established mostly through empirical fits to crossplots of predictor(s) versus predictand. Such crossplots invariably show data scatter. This scatter reflects the uncertainty associated with the regression parameter values that describe the algorithm as it is to be applied to (log-derived) petrophysical properties for reservoir-evaluation purposes. Classic examples of these regression parameters are the Archie porosity exponent **m** and the Archie saturation exponent **n** (Archie, 1942), both of which have geologic significance and exert a profound control on the petrophysical interpretation. The quantification of these governing exponents, and other parameters like them, is here termed petrophysical *characterization*, and the values of the parameters themselves describe petrophysical *character*.

To make the resulting algorithms more definitive, the petrophysicist must separate the reservoir into units of similar petrophysical character. The aim is to evaluate each petrophysical unit throughout the reservoir using the same set of interpretative algorithms, which is distinct from the set used for any other unit. The regression parameters that describe the algorithms do themselves reflect lithologic controls, because they are quantified on the basis of reservoir and other physical properties that are affected by lithology. Petrophysical separation therefore facilitates and optimizes log interpretation in a way that is implicitly related to the reservoir geology. The scheme can be discarded when the log evaluation process has been completed.

The first stage of this process is partitioning into units of similar petrophysical properties, for example, as in the electrofacies approach (Serra and Abbott, 1982). This is an intermediate but important step toward identifying units of similar petrophysical character. Electrofacies allow us to extend the recognition of reservoir facies to uncored intervals, in order to define the reservoir architecture; thus, they honor the relationship between petrophysics and sedimentary geology. Various approaches to electrofacies classification have been proposed (Wolff and Pelissier-Combescure, 1982; Delfiner et al., 1987; Baldwin et al., 1990; Cuddy, 1997). However, Ye and Rabiller (2000) observe that none of these approaches has gained a wide acceptance from the geological community.

Electrofacies are not necessarily petrophysically consistent in terms of their characterizing algorithms. Kapur et al. (1998) expounded the philosophy of electrofacies as an intermediate step toward the definition of petrophysical character. They believed that improved prediction of permeability through better-defined facies-specific porosity-permeability relationships would motivate the development of techniques for facies classification using core and wireline logs. Elphick and Moore (1999) provided an interesting example of this intermediate role for electrofacies. In the Alta de Ceuta field at Lake Maracaibo, Venezuela, cluster-derived electrofacies formed a basis for a partitioning that was expressed in terms of characterizing relationships between porosity and permeability, for the purpose of improving the evaluation of permeability.

Definition of Petrofacies Units

The choice of a name for the petrophysically established units that satisfy the above requirements should reaffirm the link between physics and geology. Cannon (1994) used the term *petrofacies* to describe unique log-derived facies identifiers, each characterized by definitive mean values of porosity, permeability, and water saturation (for a given height above the reference surface of zero capillary pressure). The phrase *rock types* has been used by others, but not always with a consistent meaning. For example, Gunter et al. (1997) describe a petrophysical rock type as "a rock deposited under similar conditions which experienced similar diagenetic processes resulting in an unique porosity-permeability relationship, capillary-pressure profile, and water saturation for a given height above free water in a reservoir." Porras (1998) used the phrase *rock type* as being synonymous with the word *petrofacies*. There, a petrofacies unit was defined on the basis of a range of pore-throat radii (R35), with subunits being defined in terms of ranges of BVW. Each different subunit was characterized by a different porosity-permeability relationship. Watney et al. (1997) adopted a more general definition of petrofacies analysis: "The characterization and classification of pore types and fluid saturations as revealed by petrophysical measures of a reservoir." They used *petrofacies* to describe an "explicit link between petroleum engineers' concerns with pore characteristics as arbiters of reservoir performance, and the facies paradigm of geologists as a methodology for genetic understanding and prediction." Watney et al. (1997) used petrophysical data to map water saturation, BVW, and estimated permeability, in conjunction with capillary-pressure data—parameters that broadly encompass those of greatest interest in this paper.

Here, a petrophysically established unit within a reservoir will be termed a *petrofacies unit*, a phrase that retains its geologic significance and also emphasizes its immediate relevance to petrophysics. Petrofacies units are specified to be of distinct petrophysical character. Therefore, they are not defined solely on the basis of their reservoir properties, which can vary markedly, but instead on the basis of the internal consistency of their interpretative algorithms for well-log analysis. For a given petrophysical objective, such as the evaluation of water saturation from resistivity logs, this consistency is honored only when the pertinent algorithm has the same algebraic form throughout the unit and when the empirically derived values of the constituent coefficients, intercepts, and/or exponents within that algorithm are constant to within acceptable limits of uncertainty. This specification of a petrofacies unit is therefore centered on the process of petrophysical interpretation.

Relationship between Stratigraphic and Petrophysical Partitioning

At each well, the net-reservoir interval of a selected stratigraphic zone will have quantifiable petrofacies components. The fractional representation of each petrofacies unit in this interval must be mapped so that at each grid node this zone has a representation ratio for each constituent petrofacies unit. This requirement is pivotal for traditional deterministic reservoir models, and it can also be implicit within geocellular models, which are not given primary consideration here. A knowledge of fractional petrofacies representation is

especially important in the context of the distribution of water saturation. Saturation-height functions can be peculiar to a given petrofacies unit, requiring that petrofacies-specific estimates of water saturation, S_w, be combined subsequently to furnish a gross estimate of water saturation, $\overline{S_w}$, for the selected stratigraphic zone at each grid node. The feasibility of constructing these grids and the degree of difficulty in doing so are important factors in judging whether coexisting, fit-for-purpose partitioning schemes are viable.

The petrophysical-separation scheme is discarded when the target reservoir properties of net-to-gross, porosity, and hydrocarbon saturation (and perhaps permeability) have been evaluated at each digital level within each well, and average porosity and hydrocarbon saturation have been gridded across all the stratigraphic zones.

PROTOCOL OF VOLUMETRICS

The ordering effects of gravity and sedimentary deposition impart trends to geoscientific data. These data trends traditionally have been depicted through parameter maps, but the requirement for maps has been superseded by digital gridding methods and geocellular modeling. Traditional isochore and parameter maps now serve only to display the gridded data. The data grids are constructed from irregularly spaced observations. Current methodologies for constructing the grids from scattered-input data vary from simple moving-average interpolation to sophisticated algorithms that are appropriate to the observed nature of the data themselves, such as convergent gridding or kriging. The process of generating the data grids should be guided by the available information pertaining to sedimentation, geomorphology, and the like, within the context of structural and depositional history, so that the resulting data distributions honor both regional and local geologic understanding.

Fundamental Role of Stratigraphic Zonation

The grid array is created principally as a basis for volumetric calculations. A variable that is to be gridded must be correlatable across the study area. This is an important requirement because sophisticated gridding algorithms use the calculated gradients between input data points, and it is essential that these gradients be true indicators of the variation of the gridded variable and not the result of random miscorrelations.

Stratigraphic correlation is the primary reference whereby key events, marker beds, or characteristic sequences are used as a basis for generating grids of structural, lithologic, or chronological surfaces that determine the gridded isopachs of the identified reservoir zones. Thus, the basic correlatable unit is the stratigraphically defined reservoir zone, which may or may not correspond to a petrofacies unit within the reservoir.

The stratigraphic zonation, rather than the petrophysical separation, therefore effectively determines how parameters are to be mapped. We will consider two scenarios for reservoir-description purposes: Either the stratigraphic zones are identical to the petrofacies units, because of a strong correspondence between lithostratigraphy and physical properties, or they are significantly different, which is the more general occurrence (Figure 1). Assuming that the stratigraphic zones cannot be subdivided meaningfully in a correlatable manner, the deterministic problem reduces to one of estimating the average water saturation of each zone at locations away from Wells X and Y, such as at point N (Figure 1).

Correspondence of Stratigraphic Zonation and Petrophysical Separation

Where stratigraphic zonation corresponds to petrophysical separation, important reservoir parameters are porosity, for direct input to volumetric calculations, and height above the zero capillary-pressure surface, for inferring water saturation and thence hydrocarbon saturation. Porosity must be gridded using average values calculated in true vertical space across the net-reservoir interval of each stratigraphic zone at each well. Where permeability is to be accommodated, it is usually done through an empirical relationship to porosity. As with all petrophysical algorithms, such a relationship must be scale-compatible, that is, it must be applied at the scale at which it was established empirically. Heights are used to predict water saturation through a set (or sets) of saturation-height functions, each (group) of which is specific to a stratigraphic zone and hydrocarbon phase and is measured from the zero capillary-pressure surface.

Noncorrespondence of Stratigraphic Zonation and Petrophysical Separation

Where stratigraphic zonation does not correspond to petrophysical separation, there are two possible approaches. If the different petrofacies units can be accommodated using the same saturation-height function(s), the zonally averaged porosity is gridded as before. If the different petrofacies units require different saturation-height functions, several parameters must be gridded for each stratigraphic zone. These are the fractional representation of each petrofacies unit within the net-reservoir interval of each stratigraphic zone and the average porosity over the net-reservoir interval of each petrofacies unit within each stratigraphic zone. Note that in a multizonal geocellular model, the requirement for gridding the fractional representations of the petrofacies units is removed, but this modification is achieved only at the expense of having to populate the cells using log-derived data that define the petrofacies units. Heights are used to predict water saturation through a set (or sets) of saturation-height functions, each (group) of which is specific to a petrofacies unit and hydro-

carbon phase, and they are referenced to a zero capillary-pressure surface.

DETERMINATION OF FLUID INTERFACES

Saturation-height functions are usually referenced to the equilibrium-pressure surface between hydrocarbons and water, i.e., the free-fluid level of the wetting phase. This is the level at which the capillary pressure between the wetting and nonwetting phases is zero. Free-fluid levels can be identified from crossplots of true vertical depth versus pore pressure, as measured at several downhole stations by a repeat formation tester. In water-wet reservoirs that contain a single hydrocarbon phase, the free-water level is taken as zero height. In water-wet reservoirs that contain both oil and gas legs, the free-water level is taken as zero height for the oil column. The height in the gas column can be taken as that above a projected equilibrium-pressure surface between gas and water, that is, the level at which the capillary pressure between gas and water can be taken as zero.

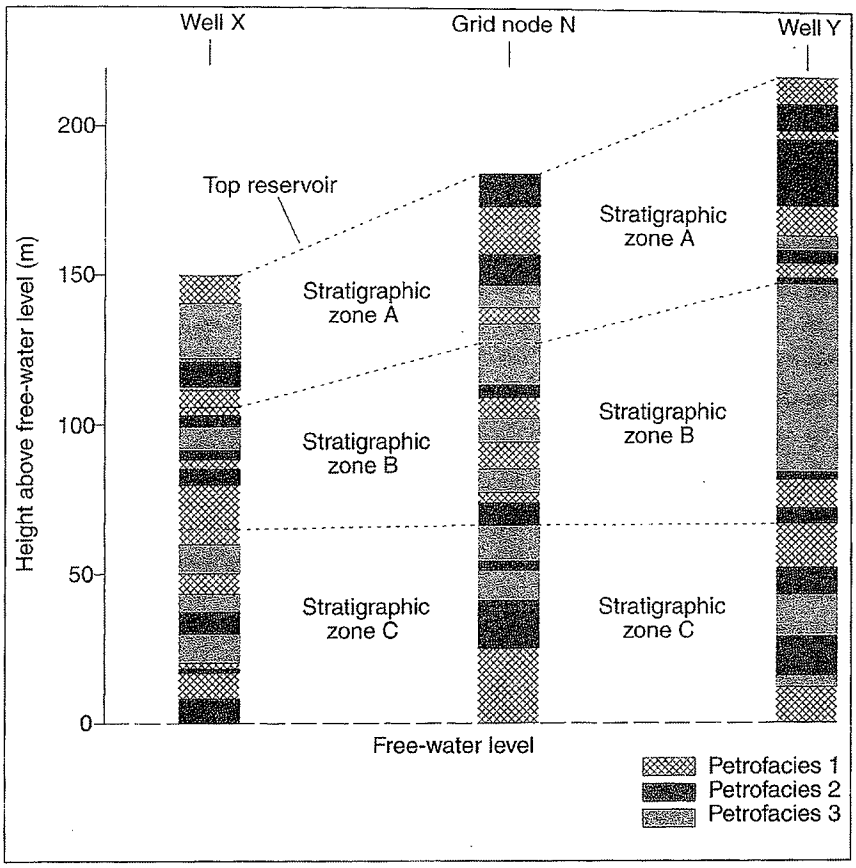

Figure 1. Petrophysical separation overprinted on stratigraphic zonation. In a given stratigraphic unit that is penetrated by a well, a petrofacies unit can be encountered at several places. This model has been designed so that the fractional representation and the average porosity of a given petrofacies unit within a stratigraphic zone can be interpolated linearly (Worthington, 2001).

Unless the reservoir rock is of high quality in terms of porosity and permeability, the free-water level is not the same surface as the hydrocarbon-water contact determined from resistivity logs. There is no reason why these determinations should be the same. One measurement is dynamic, in that it is based on the formation pressure response to fluid withdrawal, whereas the other is static, in that it is based on the electrical properties of an effectively stationary fluid system. The difference corresponds to the threshold pressure that is needed to initiate drainage during capillary-pressure measurements on a representative core sample (Figure 2a).

In some cases, the free-water level is not known because it has not been penetrated by a well within which formation-tester measurements have been made. Where the well logs indicate a water zone, the hydrocarbon-water contact can be adopted with an appropriate correction to free-water level, based on a transformation to height of the threshold capillary pressure (Figure 2b). It is now more difficult to identify a zero capillary-pressure surface between gas and water for application in the gas leg of a three-phase reservoir system.

In cases in which no well has penetrated the free-water level, it may be possible to infer its presence using a predictive method such as the Caplog method (Alger et al., 1989). However, without calibrating this method elsewhere in the same reservoir system where the free-water level is known, the degree of uncertainty can be unacceptably high.

SATURATION-HEIGHT DATA FROM CORE ANALYSIS

The task is to establish a saturation-height function that can be applied to each petrophysical unit and for each hydrocarbon type, oil or gas, that may be present. Although applying these functions necessarily draws on log-derived values of porosity and water saturation (and possibly permeability), many of the functions themselves are based on or calibrated against capillary-pressure data at the core scale. Therefore, the aim is to investigate the capillary properties of each petrophysical unit, as measured on core data. This objective emphasizes the importance of accurate core-to-log depth

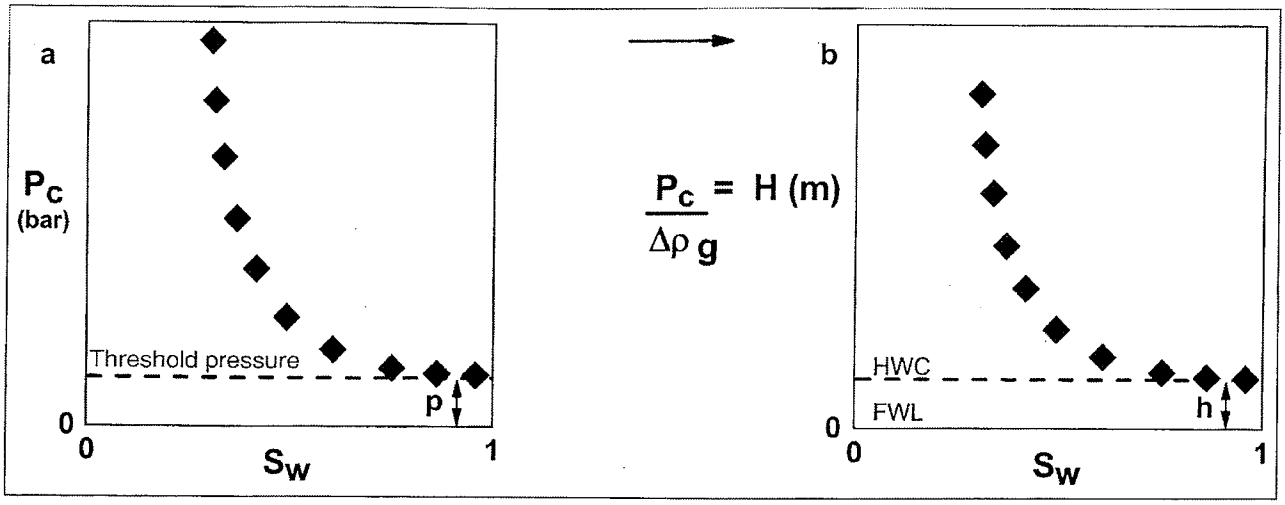

Figure 2. Conversion of (a) laboratory capillary pressure (P_c) at reservoir conditions to (b) height (H) above free-water level, for a two-phase water-wet system. Threshold pressure, p, converts to the height h of the hydrocarbon-water contact (HWC) above the free-water level (FWL). The quantity $\Delta\rho$ is the density difference between the wetting and nonwetting phases.

merging, so that both core and log applications are at the same height scale. It also emphasizes the need to have sufficient data. The characterization of an electrofacies so that it can be classified as a petrofacies unit requires a critical amount of core data. Worthington (1998) has described one way of assessing whether this criterion has been met for a given petrophysical objective. Once the data have been subdivided into candidate petrofacies units, the standard error(s) associated with the characterizing parameter(s) of each unit is evaluated in terms of the known uncertainty associated with each measured predictor, to ensure that there are sufficient data in the subdivided sample to allow a meaningful prediction of the dependent variable.

The following is included here for completeness. The subject matter is described more fully in various standard texts on reservoir properties (e.g., Amyx et. al., 1960; Bass, 1987).

Capillary Pressure

Capillary pressure (P_c) is expressed in terms of the interfacial tension between the wetting and nonwetting phases, σ, and the contact angle between the wetting phase and rock surface, ϑ, as follows:

$$P_c = (2\,\sigma \cos \vartheta) / r \qquad (1)$$

where r is the effective pore radius.

Capillary pressure is usually measured on core plugs in one of three ways: porous-plate desaturation, centrifuge desaturation, and mercury injection. The word *desaturation* relates to drainage from the sample of the wetting phase. The term *imbibition* is used to describe the replacement of a nonwetting phase by the wetting phase. Most porous-plate and centrifuge capillary-pressure data are acquired in drainage mode. Mercury injection is presumed to mimic the drainage process. The experimental procedures have been described by Tiab and Donaldson (1996).

Examples of published oil-brine capillary-pressure data for three water-wet samples of the same rock type in a North Sea reservoir are shown in Figure 3. The measured curves are presented as crossplots of water saturation versus capillary pressure. The curves are distributed according to their porosity and permeability characteristics. Ideally, a multisample crossplot would show several distinct curves that asymptotically approach an irreducible water saturation that increases monotonically with decreasing porosity and permeability. Figure 3 exemplifies the monotonic distribution of capillary-pressure curves, even though the data do not attain asymptotic values.

Conversion of Capillary-pressure Data

Capillary-pressure data can be converted from one fluid system (A) to another (B). An example of a fluid system is oil-brine, where oil is the imbibing fluid and brine is the wetting fluid. The following conversion equation is based on the assumption that effective pore radius is independent of the fluid system:

$$P_{cB} = P_{cA}\,(\sigma \cos \vartheta)_B / (\sigma \cos \vartheta)_A\,. \qquad (2)$$

Conventional values of σ and ϑ for air-brine, oil-brine, and mercury-air systems are listed in Table 1.

Capillary pressure can also be converted from laboratory conditions of interfacial tension and contact angle to (nomi-

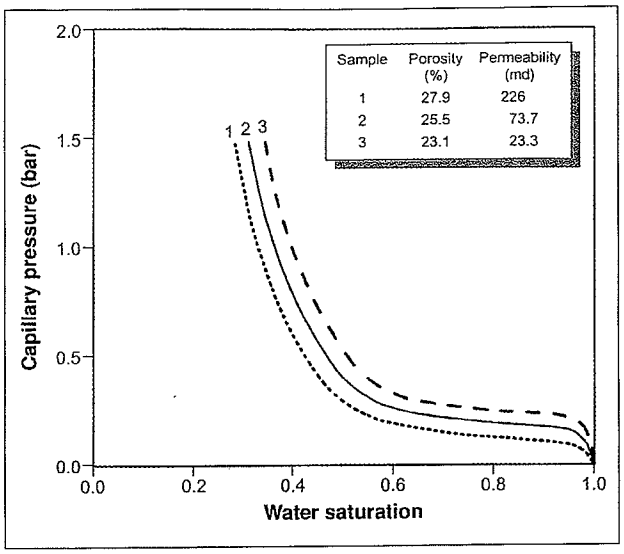

Figure 3. Examples of measured ambient-condition, oil-water capillary-pressure curves for a North Sea reservoir unit (data from Søndenå et al., 1990).

Table 1. Conventional values of contact angle θ and interfacial tension σ (after Core Laboratories, 1982). These values are nominal and should not be allowed to displace experimental data.

Laboratory conditions		
System	*Contact angle θ (degrees)*	*Interfacial tension σ (dynes/cm)*
Air-water	0	72
Oil-water	30	48
Mercury-air	140	480
Generic reservoir conditions		
System	*Contact angle θ (degrees)*	*Interfacial tension σ (dynes/cm)*
Gas-water	0	50*
Oil-water	30	30

* Pressure and temperature dependent; reasonable value to depth of 5000 ft (1500 m).

nal) reservoir conditions. This conversion is described by the following equation, which takes into account only changes in σ and ϑ from the laboratory (LAB) to the reservoir (RES):

$$P_{cRES} = P_{cLAB} (\sigma \cos \vartheta)_{RES} / (\sigma \cos \vartheta)_{LAB}. \quad (3)$$

Here, effective pore radius is again presumed to be invariate, even though the transformation from laboratory to reservoir conditions would appear to imply some stress-induced reduction of effective pore radius, a parameter that is governed by pore-throat size. Conventional values of σ and ϑ for various fluid systems at nominal reservoir conditions are listed in Table 1. Note that in the application of equation (3), the fluid system used in the laboratory need not be the same as that assumed in the reservoir.

Capillary Pressure and Height

A distribution of water saturation versus P_{cRES} can be converted to one of water saturation versus height H in the reservoir by applying the following equation:

$$H = P_{cRES} / g (\rho_1 - \rho_2) \quad (4)$$

where g is acceleration due to gravity, and ρ_1 and ρ_2 are densities of the wetting and nonwetting phases, respectively. Note that H is measured in true vertical space. A schematic core-derived saturation-versus-height distribution is shown in Figure 2b.

Because capillary-pressure characteristics are determined by the distribution of pore-throat size rather than pore size, and because pore throats can close under the influence of applied stress, the dependence of capillary phenomena on effective stress needs to be clearly understood (Søndenå et al., 1990). If there is little or no stress dependence, ambient-condition laboratory data will suffice. Otherwise, the laboratory capillary-pressure data must be measured under simulated reservoir conditions before equations (2) to (4) can be applied meaningfully. Unfortunately, capillary-pressure data at simulated reservoir conditions are the exception rather than the rule. In many cases, the stress dependence of capillary pressure is not investigated at all. This is a major deficiency for data that are intended to provide a reference for field measurements. The shortcoming is sometimes compounded by the irresponsible mixing of data at ambient and reservoir conditions to create a falsely unified data set.

SATURATION-HEIGHT DATA FROM LOG ANALYSIS

Establishing saturation-height functions from log data requires a petrophysical methodology for evaluating water saturation (S_w) from a suite of radiometric, acoustic, and resistivity logs. The procedures for doing this have been fully described elsewhere, deterministically (Dewan, 1983), statistically (Mayer and Sibbitt, 1980) and, more recently, in the enjoining context of petrophysical systemics (Worthington, 1997a).

When the determined values of S_w have been assigned to

depth locations in true vertical space, they must be processed to remove degraded data, such as those affected by imperfect corrections for shaliness, and to eliminate nonreservoir rock. In this respect, an ongoing problem is how to handle data affected by shoulder beds. Blocking and buffering of logs can remove this problem, but rarely without introducing some element of subjectivity. On the other hand, if these degraded data are admitted to the saturation-height exercise—and they usually are—there may be implications for the way in which log-derived reservoir properties are averaged over each petrofacies unit within a given stratigraphic zone, that is, whether the mean, mode, or median is the most representative centroid. These important issues have received comparatively little attention in the literature.

Classically, the data should then be sorted according to characteristics of porosity and, possibly, permeability. However, in this application the downhole estimation of permeability is likely to have been made using log-derived values of porosity in conjunction with a porosity-permeability predictive algorithm founded on core data. The inclusion of permeability as a predictor therefore can add very little to the definition of a saturation-height function that already draws on porosity, and it can cause degradation of data distributions.

The admitted S_w values and the associated porosities are referenced through their corresponding heights in true vertical space either to the free-water level in two-phase water-wet reservoirs or to the projected zero capillary-pressure surface for the gas leg of three-phase water-wet reservoirs.

FORMS OF SATURATION-HEIGHT FUNCTION

Saturation-height functions are mathematical expressions that are used to evaluate water saturation, S_w, from height, H, in true vertical space. They should be established separately for each defined rock type, such as petrofacies unit 1 of Figure 1. They can be grouped into three categories. For clarity, the discussion in the following three subsections assumes only one rock type.

Single-predictor Algorithms

The first category uses only height as a predictor of water saturation (Skelt and Harrison, 1995). Simple examples are:

$$S_w = a\,H^b \quad (5)$$

and

$$S_w = c \log H + d \quad (6)$$

where a, b, c, and d are regression constants. Equations (5) and (6) have often been established as a family, each member of which relates to a specific range of porosity, a porosity "bin." This approach has proved useful in deriving more meaningful coefficients and constant terms. In a gridding context, the procedure is to look up porosity from a porosity grid, to select from the family the saturation-height function appropriate to that porosity value, to look up height from the height grid, and to enter the gridded height into the selected function to compute S_w and thence S_h. As indicated above, there is little benefit in incorporating intergranular permeability, K, into the binning process because at the log-interpretation and gridding stages, K would have to be inferred from a parameter that is measurable downhole, probably porosity itself, through a core-derived empirical expression of the form:

$$K = e\,\phi^f \quad (7)$$

where e and f are regression constants.

The principal drawback of the binning approach is manifested when adjacent porosity-grid values lie immediately on either side of a bin boundary. Despite the proximity of the grids and of the nodal values of porosity, the method will draw on different saturation-height functions, and this selectivity can give rise to an artificially stepped distribution of predicted hydrocarbon saturation. A second disadvantage is that because the saturation-height curves have necessarily been averaged over a predetermined porosity range, they sometimes fail to represent true irreducible conditions.

Multipredictor Algorithms

Multipredictor algorithms incorporate porosity and/or permeability within the saturation-height function. This inclusion avoids stepped artifacts in the mapped distribution of hydrocarbon saturation.

An example that includes porosity is the so-called FOIL method of Cuddy et al. (1993), which relates height to bulk volume water (BVW), the product of porosity and water saturation, as follows:

$$BVW = g\,H^m \quad (8)$$

where g and m are regression constants. The quantity BVW effectively expresses water saturation in terms of the bulk volume of rock rather than the pore volume. The use of the bulk-volume concept can be traced to Heseldin (1974), who also introduced porosity as an additional parameter in the display and analysis of capillary-pressure data. Equation (8) can be written in the mathematically equivalent form

$$S_w = g\,H^m / \phi \quad (9)$$

where ϕ denotes porosity. The procedure is to look up height from the height grid node, to enter the gridded height into the function in order to compute BVW, to look up porosity from a porosity grid, to use this porosity to extract S_w from the predicted BVW, and thence to compute S_h from S_w at the grid

node. Cuddy et al. (1993) used the following alternative form of equation (8):

$$\log BVW = k + m \log H \qquad (10)$$

where k is a regression constant. The corresponding form of equation (9) is:

$$\log S_w = k + m \log H - \log \phi . \qquad (11)$$

Equations (10) and (11) should be used instead of equations (8) and (9), respectively, if the input variables preferentially show a lognormal distribution. To facilitate the immediate discussion, we will assume that this is the case. Where permeability is considered important and can be evaluated meaningfully, equation (11) can be extended empirically to include permeability in a manner reminiscent of a calibrated Caplog method (Alger et al., 1989), albeit one with the reverse application of predicting S_w from H, i.e.,

$$\log S_w = k + m \log H - \log \phi + n \log K \qquad (12)$$

where n is a regression constant. Equation (12) is similar to a rearrangement of equation (9) of Alger et al. (1989), the difference being that porosity is not placed as a logarithmic argument in the Caplog approach. Because permeability cannot be measured directly downhole but porosity can, and given that porosity and permeability are often related through an expression of the form of equation (7), permeability can be substituted in field applications by a porosity function. By following this procedure, Alger et al. (1989) arrived at an equation that contains both porosity and its logarithm as distinct parameters. By combining equations (7) and (12) and rearranging, we have

$$\log BVW = p + m \log H + q \log \phi \qquad (13)$$

where p and q are regression constants, the latter being nonzero. Equation (13) allows us to include permeability through equation (7) and yet to retain BVW as the predictand. This philosophy broadly mirrors that of Heseldin (1974), who for logistic reasons used porosity rather than permeability as a rock-quality indicator and who promoted the advantages of correlating bulk-fluid saturations with porosity, albeit for different purposes.

As presented, equation (13) contains porosity both explicitly as a predictor and implicitly as a factor of the predictand, BVW. The significance of equation (13) will become apparent later when it is considered in the context of mapping applications. For the present, it should be noted that in deterministic mapping, the regressed quantities should strictly be averages taken over a given petrofacies unit within each stratigraphic zone at each well site. This averaging process effectively means that (average) porosity is no longer a factor of the (average) product of porosity and water saturation. Therefore, in the mapping context, log ϕ cannot be extracted from log BVW. Even if this were not the case, over net-reservoir intervals BVW is governed primarily by water saturation rather than porosity, hence its role in saturation-height applications.

A second important point concerns the incorporation of additional variables, such as the inclusion of permeability in equation (12). Where there is a strong relationship between predictor variables, the matrix expression used in the least-squares regression analysis is affected by multicolinearity. This causes an ill-conditioning of the correlation matrix, which might not be invertible, thereby precluding a solution of the normal equations. For a full discussion of this problem, see Dillon and Goldstein (1984).

There is known to be some interrelationship between porosity and permeability, the predictor variables of equation (12). That is why they have been grouped in equation (13). The coefficient q can be set constant during a regression exercise if one wishes to include porosity solely as a notional indicator of permeability. The value chosen should be q = f (from equation [7]), provided that f has been established at the scale of intended application of equation (13). As always, these considerations must be balanced against the predictive benefits that derive from including additional variables. The advantages of using equation (13) rather than equation (10) at the petrofacies scale are demonstrated quantitatively in Worthington (2001).

Johnson (1987) presented an example of a saturation-versus-height function that explicitly includes permeability. A generalized form is

$$\log S_w = r H^{s} + t \log K \qquad (14)$$

where r, s, and t are regression constants. Equation (14) can be written equally in terms of porosity in cases for which an equation of the form of (7) has been established for each petrofacies unit. In this case, the procedure would be to look up height from the height grid node, to look up porosity from a porosity grid, to calculate permeability using a petrofacies-specific porosity-permeability relationship, and to compute S_w and thence S_h at the grid node. Notably, the application of equation (14) to the Argyll field of the central North Sea (Johnson, 1987) did not have the benefit of useful core-derived relationships between porosity and permeability. To overcome this problem, permeability was calculated from water saturation at well sites in a "reverse application" of equation (14). Søndená (1993) proposed an expression for water saturation in terms of permeability and capillary pressure (and thence height); this had a mathematical form

different from equation (14). In a variation on this theme, Smith (1991) predicted downdip water levels by calculating a synthetic capillary-pressure (and thence height) curve from porosity, permeability, and water saturation, using the published relationships of Thomeer (1960, 1983).

Normalized Functions

The third category of saturation-height function uses porosity and permeability grids to generate the input to a J function of the Leverett (1941) type, which can be written in its modified form that takes into account wettability (Rose and Bruce, 1949), for the exemplifying case of a water-wet oil leg, as

$$J = [H (\rho_w - \rho_o) g / \sigma \cos \vartheta] (K/\phi)^{0.5} \qquad (15)$$

where ρ_w and ρ_o are the densities of water and oil, respectively, and J is a dimensionless function of normalized capillary pressure, $[H (\rho_w - \rho_o) g / \sigma \cos \vartheta]$, and pore geometry factor, $(K/\phi)^{0.5}$. Because permeability cannot be mapped reliably, it is usually estimated from porosity though an expression of the form of equation (7).

The procedure is to look up height from the height grid; to look up porosity from the porosity grid; to compute permeability from equation (7); to enter gridded height, gridded porosity, and computed permeability into equation (15); and thence to calculate J using known values of ρ_w, ρ_o, g, σ, and ϑ. This means that the porosity grid values are used twice at each node. Further, the scattered nature of core-derived porosity-versus-permeability crossplots can detract from the theoretical benefits of including permeability. Water saturation is then calculated from an empirical relationship between J and S_w, which is usually established at the core scale and often takes one of the following forms:

$$J = u S_w^{\ v} \qquad (16)$$

or

$$J = y e^{zS_w} \qquad (17)$$

where u, v, y, and z are regression constants. Hydrocarbon saturation follows directly.

CONSIDERATIONS OF SCALE

The choice of saturation-height function usually has been driven by petrophysical preference, with too little regard for the subsequent incorporation of the resulting S_w values into volumetric calculations. An overriding principle of contemporary petrophysics is that an empirical relationship should be applied only at the scale for which it has been established. This principle already has been emphasized in the case of predictive algorithms for permeability (Worthington, 1997b). It is frequently violated in practice.

A detailed analysis of the impact of scale on saturation-height functions is beyond the scope of this paper. Let it suffice to say that if a function is established at the core scale, it should be applied strictly only at the core scale. If agreement between core and log-derived saturations can be demonstrated, the function can be extrapolated to the log scale. However, geologic mapping operates at the scale of stratigraphic zones or their constituent petrofacies units. If, therefore, a saturation-height function is to be applied at the petrofacies scale, it should be established at that scale. Cross-scale application or the introduction of hybrid functions from a scale perspective can cause significant degradation of the resulting evaluation of hydrocarbon saturation. Worthington (2001) has demonstrated this outcome.

FIELD APPLICATION

Any of the saturation-height functions described above can be established for application at grid locations, a procedure that requires proper calibration through core data and the honoring of scales. Each saturation-height function (or group of functions, in the case of porosity bins) relates to a constituent petrofacies unit of the reservoir and to a specific fluid system. Thus, for example, if there are three petrofacies units and two hydrocarbon phases within a water-wet reservoir, a total of six (groups of) functions will be required to characterize the reservoir in terms of saturation-height functions. This number can be reduced for present purposes if any of the petrofacies units does not show exclusivity from the standpoint of saturation-height character.

In theory, the deterministic application of a saturation-height function can be effected in one of two ways. The function can be calibrated and established at the core-to-log scale before being applied at digital levels within the appropriate petrofacies unit at each grid node. Thereafter, zonal averages of water saturation can be calculated for purposes of volumetrics. The difficulty with this approach is that we do not know the precise distribution of a given petrofacies unit within a specified stratigraphic zone at a selected grid node. We know only its mapped fractional representation and estimated midpoint height. Further, where a porosity lookup is required for binning applications, this will not be available on a level-by-level basis at each node.

Alternatively, the log-derived water saturations can be calibrated against core data before being averaged for each petrofacies unit within each stratigraphic zone at each well. These average values are then used to establish a saturation-height function for a given fluid system within a specified petrofacies unit by drawing on the midpoint heights of the intrazonal occurrences of that petrofacies unit at each well. The established function is then applied at the midpoint heights of an intrazonal petrofacies unit at each grid node. In

other respects, this scale-compatible approach overcomes the scaling difficulties exposed in the previous paragraph.

The gridding of fractional petrofacies representation is based on the recognition of geologically controlled data trends within the reservoir. Indeed, this is an unwritten prerequisite for deterministic studies. Where data trends are not evident, recourse might be made to statistical methods of reservoir description, although here too the models need to be constrained by rules based on order.

Where one has had to generate statistical realizations of the reservoir, one can apply saturation-height functions of the type of equations (5) or (6) at grid nodes on a level-by-level basis without recourse to zonal averages. However, if porosity bins are used, the porosity lookup values are more likely to be based on archtypal porosities (e.g., mean values) for the constituent petrofacies units. Such a procedure would introduce a scale disparity. This disparity is retained even where porosity is included within the saturation-height algorithm itself, because the porosity values distributed throughout a geocellular model as a result of the populating exercise are averages within the different petrofacies units. Here the scale disparity can be reduced, but not eliminated, by applying the saturation-height functions to the midpoint heights of the populating cells.

The relationship between theoretical and practical applicability is discerned best by examining how the inferred water saturations are secured and integrated within the stratigraphic reservoir model. There are two stages in this process: the evaluation of S_w at a given height (at a grid node) and the incorporation of the determined values of S_w into the zonal scheme. Each of these stages will be considered.

DISTRIBUTION OF WATER SATURATION WITHIN A 3-D RESERVOIR MODEL

The methodology for the field evaluation of water saturation varies according to the type of saturation-height function to be applied. All functions are specific to a given petrofacies unit and to a given fluid system.

Single-predictor Algorithms

Single-predictor algorithms are used to estimate water saturation from height; each algorithm commonly will relate to a range of (lookup) porosities. Data should be validated at the core-to-log scale, and the functions can then be established at this same scale, for discrete ranges of core porosity corrected to effective reservoir stress and for reservoir fluid conditions. The functions are then applied at grid nodes to those digital levels that correspond to the sampling interval of well-log data. Alternatively, the data can be validated at the core-log scale as before, and the saturation-height functions can then be established by averaging log-derived distributions of water saturation versus height for discrete ranges of log-derived porosity. This process can cause the resulting functions to be transposed to a pseudopetrofacies scale, by loose analogy with the increment method of data averaging that has been used elsewhere in petrophysics (Worthington, 1997b). These functions are subsequently applied at grid nodes to the midpoint heights of intrazonal petrofacies units.

In both cases, the porosity lookup that determines the algorithm to be used is based on mapped values of porosity averaged for each occurrence of a given petrofacies unit within a specified stratigraphic zone. Both approaches therefore introduce a scale inconsistency, in that the lookup porosity is not at the same scale as the porosity used to establish the functions in the first place. Further, where the lookup porosities at adjacent grid nodes lie in different porosity bins, the application of different saturation-height functions can introduce artificial steps in the resulting distributions of average water saturation for a given stratigraphic zone.

Multipredictor Algorithms

Multipredictor algorithms are often used to estimate water saturation from height, porosity, and perhaps permeability. Where permeability is included as a predictor, this is usually represented as a function of porosity, so that porosity becomes the only additional predicting variable. Again, the data should be validated at the core-to-log scale, as described above. However, for purposes of application at grid nodes, a gridded value of porosity is again needed. This cannot be done for each digital level, but it can be achieved fieldwide for a given petrofacies unit within a particular stratigraphic zone. Therefore, the porosity that is input to the saturation-height algorithm is at the petrofacies scale. For this reason, any relationship used to predict permeability from porosity should be established at the petrofacies scale, a condition that is rarely satisfied in practice. To avoid scale disparity, the saturation-height algorithm itself should be established at the petrofacies scale. Although this function could be applied to digital levels at grid nodes, for scale compatibility it should be applied to the gridded midpoint height of a petrofacies unit within a given stratigraphic zone. This approach avoids the introduction of artifacts in the resulting distributions of average water saturation. Figure 4 shows a typical log-derived, scale-compatible, saturation-height data envelope in the form of a bilogarithmic plot of BVW versus height, for the particular case of a gas-water system. The uncertainty in predicted average bulk volume water for this particular petrofacies unit is less than one-quarter of an order of magnitude.

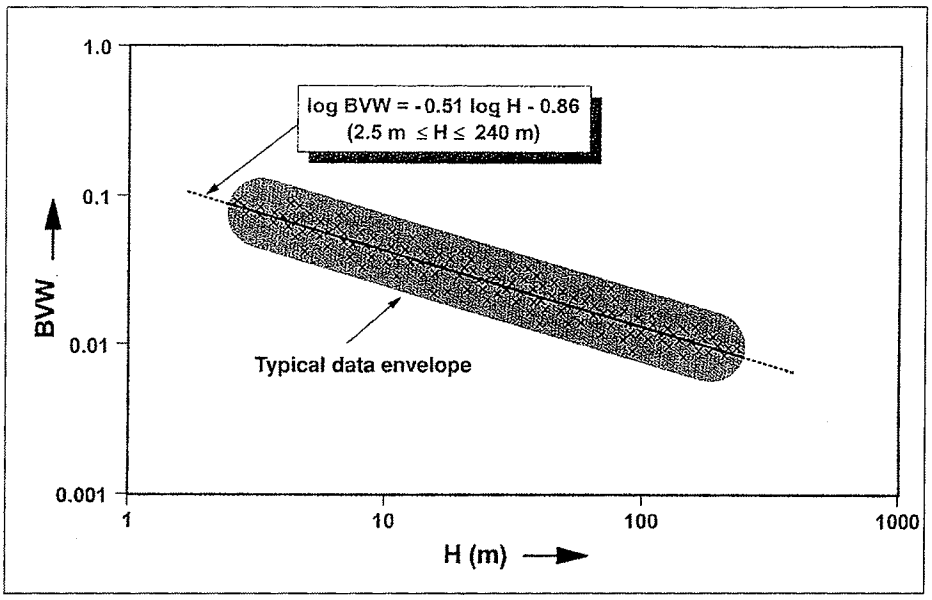

Figure 4. Schematic relationship between bulk volume water (BVW) and height (H), depicted in the form of a log-derived data envelope, for a gas-water system within a given petrofacies unit. The BVW data have been averaged intrazonally for each petrofacies unit at each well. The quantity H represents the midpoint height of an intrazonal petrofacies interval. The data envelope typically defines a predictive range of BVW of one-half an order of magnitude, for a given value of H. The data plot does not account for permeability.

Normalized Functions

A composite normalized capillary-pressure term is used as a predictor of water saturation. The value of the term is computed from height and several other variables. The most commonly applied normalized function draws on the Leverett J-function, as described above.

After validation at the core-to-log scale, a gridded value of porosity is again needed to compute the J-function at a given grid node. This can be mapped across the field for a given petrofacies unit within a particular stratigraphic zone. Therefore, the porosity that is input to the J-function is at the petrofacies scale. To avoid scale disparity, the function itself should be established and applied at the petrofacies scale. Once again, this requires that any relationship used to predict permeability from porosity has been established at the petrofacies scale. Furthermore, for scale compatibility, the predictive algorithm should be applied to a petrofacies unit through its midpoint height within a given reservoir zone. The Leverett J-function is highly exposed to the abuse of scale considerations. However, its use should not lead to artificial discontinuities in the resulting distributions of average water saturation for a given stratigraphic zone.

RECONCILIATION OF PETROFACIES-SPECIFIC WATER SATURATION WITH STRATIGRAPHIC ZONATION

The key step in the entire process of applying saturation-height functions to reservoirs with petrophysically heterogeneous stratigraphic zones is the combination at a grid node of the average water saturation for each petrofacies unit within a given stratigraphic zone into a composite average water saturation for the entire stratigraphic zone at that node. The method of combination is definitive. It indicates the optimum data input for its most effective application.

Calculation of Composite Zonal Water Saturation

The equation for this combination is written below for the specific case in which two petrofacies units (1 and 2) are sufficient to describe fully the admitted net-reservoir intervals of the system:

$$\overline{S_w} = (f_1\ \overline{\phi_1\ S_{w1}} + f_2\ \overline{\phi_2\ S_{w2}}) / (f_1\ \overline{\phi_1} + f_2\ \overline{\phi_2}) \quad (18)$$

where the symbols refer to the net-reservoir interval of a given stratigraphic zone at a particular grid node and are defined as follows:

- $\overline{S_w}$ is the composite average water saturation.
- f_1 and f_2 are the fractional representations of petrofacies units 1 and 2, respectively, within the given stratigraphic zone at the particular grid node.
- $\overline{\phi_1}$ and $\overline{\phi_2}$ are the average porosities of petrofacies units 1 and 2, respectively.
- $\overline{\phi_1\ S_{w1}}$ and $\overline{\phi_2\ S_{w2}}$ are the average bulk volume water terms of petrofacies units 1 and 2, respectively.

Equation (18) can be expanded to accommodate any number of constituent petrofacies units. This equation must be applied as the final stage of the application of saturation-height functions in cases for which the stratigraphic zonation does not correspond to the petrophysical separation. It can be used in conjunction with any of the saturation-height func-

tions described above. Note, however, that equation (18) contains average bulk volume water ($\overline{BVW} = \overline{\phi S_w}$) terms in the numerator. This suggests that a function that predicts BVW from height is especially appropriate to this type of application. Options are the FOIL method of Cuddy et al. (1993) or its extended form that seeks to take some account of permeability, albeit imperfectly (equation 13). The following description of field procedure is set within the context of these two methods.

Practical Application of the Bulk-volume-water Method

Again, this discussion presupposes that the reservoir has two constituent petrofacies units. Each unit has been characterized by an exclusive set of saturation-height functions, which are fluid-specific.

The appropriate saturation-height functions must be calibrated at the core-to-log scale to establish that the log-derived water saturations are indeed substantiated by (reservoir-condition) core data. The functions then should be established at the zonal scale, because this is the scale at which they will be applied. To do this, average values of porosity and bulk-volume water are determined for the net-reservoir interval of each petrofacies unit within each stratigraphic zone at each well. The fractional representations of the constituent petrofacies units are determined for the net-reservoir interval of each stratigraphic zone at each well. The midpoint heights of these representations (H_1 and H_2) are also required. Bivariate regressions of log $\overline{BVW}$ on log H (e.g., as per equation [10]) or trivariate regressions of log $\overline{BVW}$ on log H and log $\overline{\phi}$ (e.g., as per equation [13]) can then be effected at the zonal scale for each petrofacies unit and for each fluid type. This process establishes the saturation-height functions at the zonal scale.

The four reservoir parameters ($\overline{\phi_1}, \overline{\phi_2}, f_1, f_2$) and the two architectural parameters (H_1, H_2) can be gridded for each stratigraphic zone. Values of these parameters are thereby established for each stratigraphic zone at each grid node. The saturation-height functions (at the zonal scale) can then be applied to each petrofacies representation to predict its average bulk volume water within each stratigraphic zone at each grid node. When this has been done, equation (18) can be applied to calculate the composite average water saturation, $\overline{S_w}$, of each stratigraphic zone at each grid node. These data are used in conjunction with the composite average porosity of each stratigraphic zone at each grid node to generate volumetrics. When this has been done, the petrophysical separation of the reservoir can be discarded, just as the stratigraphic zonation itself will give way to hydraulic flow units at the reservoir-simulation stage.

CONCLUSIONS

A strategy has been outlined for applying saturation-height functions in heterogeneous reservoirs, where the stratigraphic reservoir zones themselves are not sufficiently homogeneous and mutually distinct, from the standpoint of physical properties, to serve also as meaningful petrophysical units. Rather than force convergence of incompatible partitioning schemes into a single reservoir model, the stratigraphic and petrophysical requirements of reservoir partitioning have been decoupled and allowed to coexist, with each being fit for its own specific interpretative purpose within integrated reservoir description. The key stage of such a strategy is the assignment of the petrophysical deliverables, in the form of interpreted values of porosity, water saturation, and perhaps permeability, to the different stratigraphic zones as a prelude to computing reservoir volumetrics and identifying flow units. In the case of inferred water saturations, this assignment process uses equation (18).

Equation (18) is informative because it suggests a more fundamental role for bulk volume water in predicting gridded water saturations than does the FOIL approach elucidated by Cuddy et al. (1993). However, the FOIL approach does not take account of permeability. This limitation can be removed by including in the BVW-versus-height function an additional, permeability-driven porosity term, as illustrated in the expansion of equation (10) to equation (13). Whether such an inclusion turns out to be beneficial or whether the latent incorporation of a scattered relationship between porosity and permeability would serve to negate the benefits of an additional predictor in the field estimation of BVW remains an open question that can be answered only through the analysis of several diverse data sets.

In contemporary petrophysics, by far the greatest violation of physical principles occurs through failing to adhere to scale. It is not correct to establish an empirical algorithm at one scale of measurement and then to apply it at another. Saturation-height functions are no exception. However, it is appropriate to calibrate log-derived water saturations against environmentally corrected, core capillary-pressure data and then to gross-up the log-derived reservoir properties to establish a saturation-height function at the intrazonal petrofacies scale, which is the appropriate scale to be used in deterministic mapping for volumetrics.

The general decoupling of stratigraphic zones and petrofacies units has benefits for all tasks that comprise deterministic petrophysical evaluation in integrated field studies. Similarly, the comments on scale adherence are also more generally applicable. With a clear direction on how to integrate petrophysical deliverables into a correlative reservoir

zonation, the door is open to introduce fit-for-purpose geologic and petrophysical partitioning wherever the reservoir data indicate that this would lead to reduced uncertainty in computed hydrocarbons in place and subsequently estimated reserves.

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