Advanced Petrophysics
PGE 381L, Fall 2023
Unique Number: 20215

Homework Assignment No. 6

October 26, 2023
Due on Thursday, November 9, 2023, before 11:00 PM

Name: ________________**SOLUTION** ____________________________________

UT EID: ____

Objectives:

- **a)** To practice using Coefficient of Variation, Dykstra-Parsons Coefficient of Variation, and Lorenz Coefficient for heterogeneity quantification
- **b)** To practice calculation of covariance
- **c)** To practice calculation and interpretation of directional variograms
- **d)** To practice interpretation of covariance function and variogram
- **e)** To practice kriging

Note: Please scan your homework assignment and upload it as one pdf file on the Canvas website before the deadline. Please name your homework document as follows:

PGE381L_2023_Fall _HW06_lastname_name.pdf

Example: PGE381L_2023_Fall_HW06_Heidari_Zoya.pdf

Question 1: Download the Excel file "PGE81L_HW_06_Data" including measurements of porosity, permeability, and layer thickness. Please answer the following questions. You can attach an extra page to your solution document and include your plots in that page with appropriate citation in the allotted space after each question.

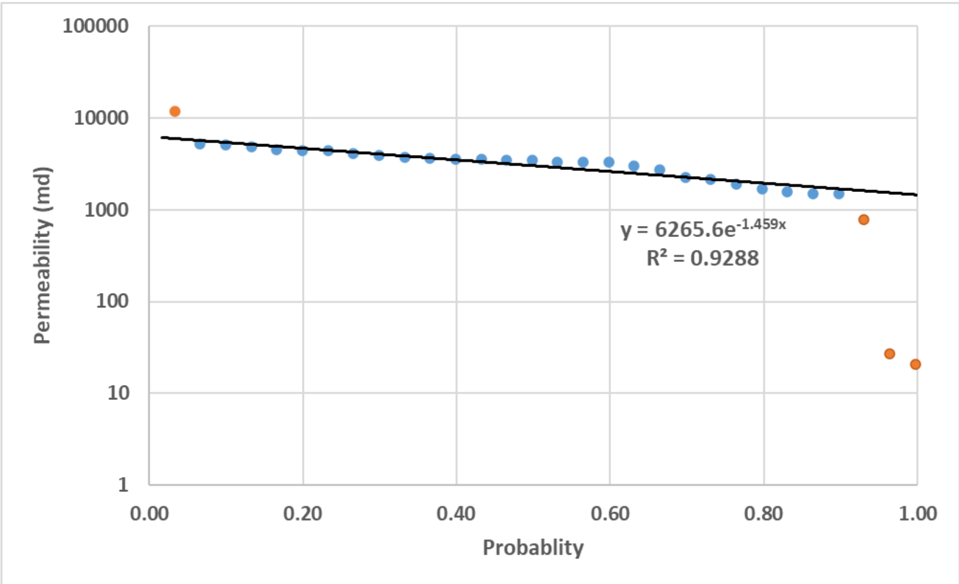
a) Calculate the coefficient of variation. Show the details of your calculation procedure.

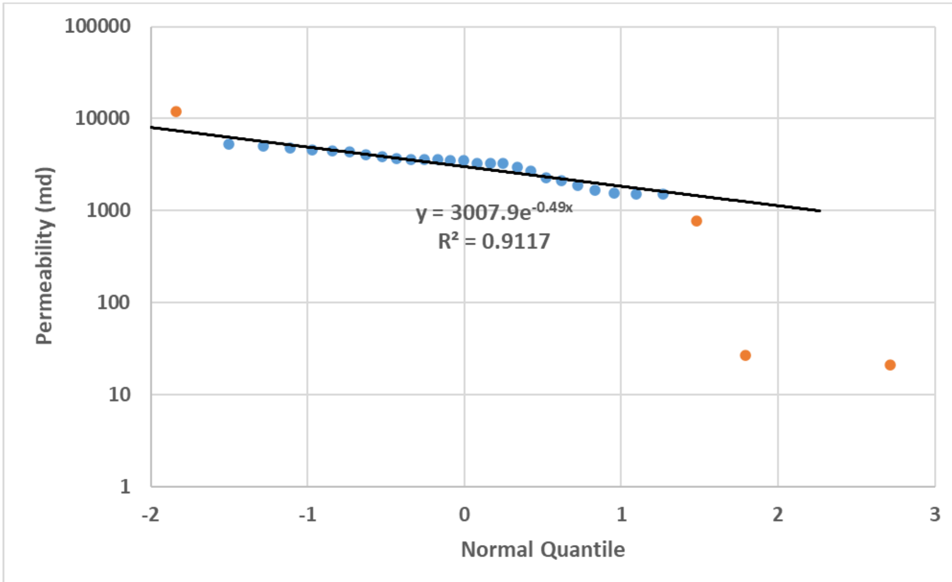
$$C_{v,k} = \frac{\sigma(k)}{\mu(k)} = \frac{2155}{3298} = 0.65$$

$$C_{v,k/\phi} = \frac{\sigma(k/\phi)}{\mu(k/\phi)} = \frac{7313}{10867} = 0.67$$

b) Calculate the Dykstra-Parsons coefficient of variation using the two approaches that we practiced in the class. Plot Permeability vs. normal quantiles and calculate the equation of the line that passes through the data. Show the details of your calculation procedure for both approaches.

Hint: do you see outliers? If there are outliers in the data, separate them and then continue with your calculations.





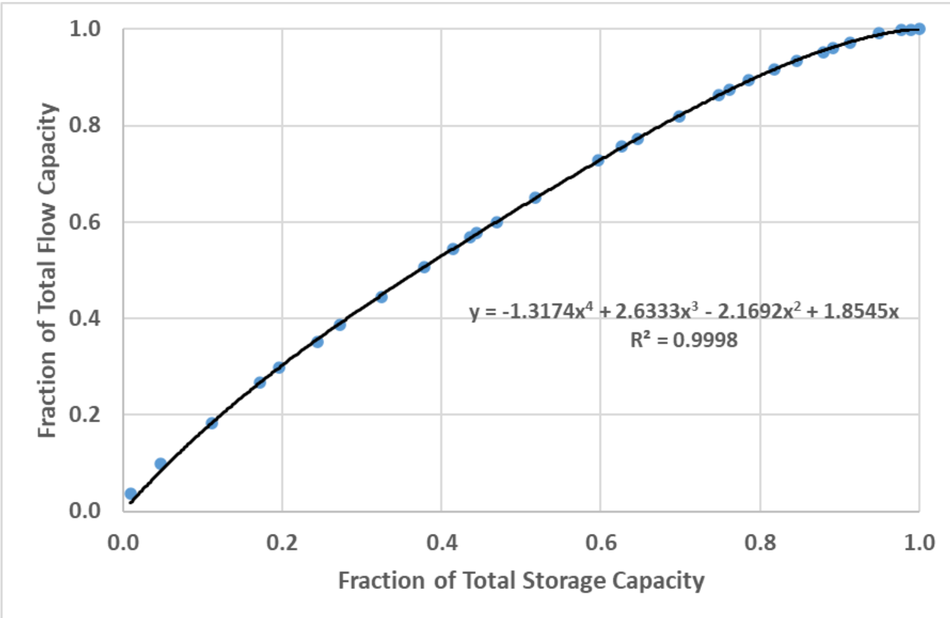
$K = 3008 e^{-0.49x}.$

$K_{50}(x=0) = 3008$

$K_{84.1}(x=1) = 1843.$

$V = \frac{K_{50} - K_{84.1}}{K_{50}} = 0.387.$

c) Calculate the Lorenz coefficient. Show the Lorenz plot, the table of your calculations (similar to what we had in the class), and your calculation procedure to estimate the Lorenz coefficient.



$$LC = \frac{\int_0^1 y\,dx - 0.5}{0.5} = \frac{0.599 - 0.5}{0.5}$$

$$= 0.198$$

d) What is your opinion about the level of heterogeneity in this formation?

Medium to low level of heterogeneity.
LC is more reliable in this case.
LC takes into account porosity, permeability & thickness, while others overlook thickness.

Question 2: Table 1 summarizes two-dimensional distribution of irreducible water saturation in a formation. Please answer the following questions. You can attach an extra page to your solution document and include your plots in that page with appropriate citation in the allotted space after each question. (You can use excel or any programming language of your choice for your calculations in this question)

Table1

		Porosity in West-East Direction (%)						
	Location (ft)	**0**	**10**	**20**	**30**	**40**	**50**	**60**
Porosity in North-South Direction (%)	**0**	20	15	19	21	26	18	19
	10	17	16	23	21	29	25	15
	20	22	17	24	17	26	32	30
	30	20	15	27	23	27	29	20
	40	19	25	32	28	22	24	28

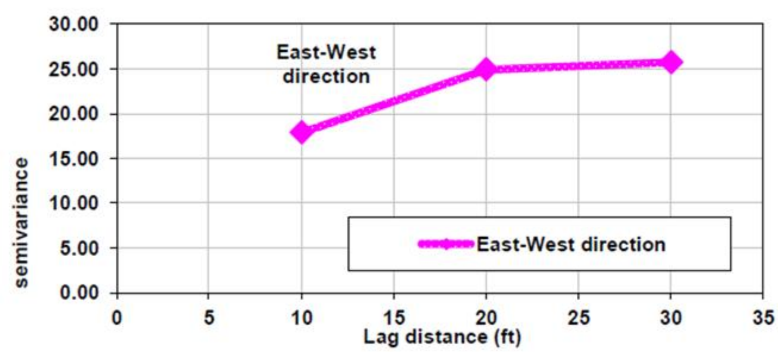
a) Calculate the semivariance values on the experimental variogram in the west-east direction at lag distances 10 ft, 20 ft, and 30 ft. Show your calculations, fill out the following table, and plot the variogrram.

Lag distance (ft)	10	20	30
$\gamma_{W-E}(h)$	17.88	24.91	25.74

lag 10 WE							
25.0	16.0	4.0	25.0	64.0	1.0	h	10
1.0	49.0	4.0	64.0	16.0	100.0	Nh	30
25.0	49.0	56.3	90.3	36.0	4.0	sum	1072.5
25.0	144.0	16.0	16.0	4.0	81.0	G(h)	17.875
36.0	49.0	16.0	36.0	4.0	16.0		

lag 20 WE						
1.0	36.0	49.0	9.0	49.0	h	20
36.0	25.0	36.0	16.0	196.0	Nh	25
4.0	0.3	4.0	240.3	16.0	sum	1245.5
49.0	64.0	0.0	36.0	49.0	G(h)	24.91
169.0	9.0	100.0	16.0	36.0		

lag 30 WE					
1.0	121.0	1.0	4.0	h	30
16.0	169.0	4.0	36.0	Nh	20
30.3	81.0	64.0	182.3	sum	1029.5
9.0	144.0	4.0	9.0	G(h)	25.7375
81.0	9.0	64.0	0.0		



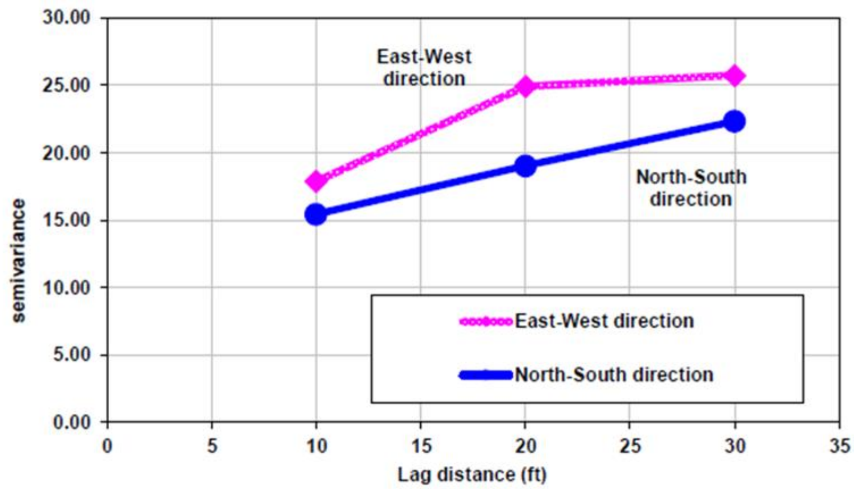
b) Calculate the semivariance values on the experimental variogram in the north-south direction at lag distances 10 ft, 20 ft, and 30 ft. Show your calculations, fill out the following table, and plot the variogrcarm.

Lag distance (ft)	10	20	30
$\gamma_{N-S}(h)$	15.44	19.01	22.32

h	10					
Nh	28					
sum	864.5					
G(h)	15.4375					
lag 10NS						
9.0	1.0	16.0	0.0	9.0	49.0	16.0
25.0	1.0	1.0	20.3	9.0	49.0	225.0
4.0	4.0	9.0	42.3	1.0	9.0	100.0
1.0	100.0	25.0	25.0	25.0	25.0	64.0

h	20					
Nh	21					
sum	798.5					
G(h)	19.01					
lag 20 NS						
4.0	4.0	25.0	20.3	0.0	196.0	121.0
9.0	1.0	16.0	4.0	4.0	16.0	25.0
9.0	64.0	64.0	132.3	16.0	64.0	4.0

h	30					
Nh	14					
sum	625.0					
G(h)	22.3214					
lag 30 NS						
0.0	0.0	64.0	4.0	1.0	121.0	1.0
4.0	81.0	81.0	49.0	49.0	1.0	169.0



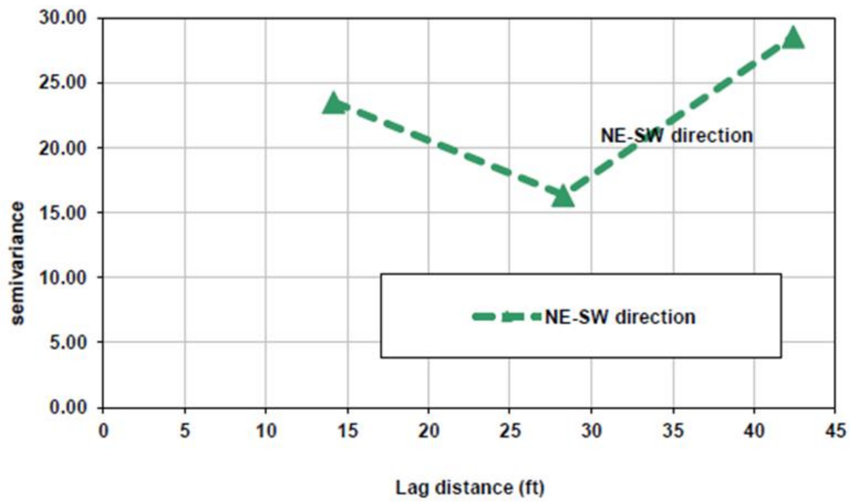
c) Calculate the semivariance values on the experimental variogram in the northeast-southwest direction at lag distances $10\sqrt{2}$ ft, $20\sqrt{2}$ ft, and $30\sqrt{2}$ ft. Show your calculations, fill out the following table, and plot the variogrcarm.

Lag distance (ft)	$10\sqrt{2}$	$20\sqrt{2}$	$30\sqrt{2}$
$\gamma_{NE-SW}(h)$	23.51	16.35	28.56

lag 10 NE-SW							
4.0	9.0	4.0	25.0	121.0	36.0	h	10
36.0	36.0	9.0	156.3	1.0	289.0	Nh	24
9.0	81.0	110.3	9.0	25.0	1.0	sum	1128.5
16.0	4.0	81.0	1.0	49.0	16.0	G(h)	23.51

lag 20 NE-SW						
					h	20
					Nh	15
9.0	16.0	4.0	2.3	49.0	sum	490.5
9.0	36.0	4.0	4.0	144.0	G(h)	16.35
25.0	72.3	36.0	16.0	64.0		

lag 30 NE-SW					
				h	30
				Nh	8
				sum	457.0
1.0	121.0	81.0	16.0	G(h)	28.563
4.0	16.0	49.0	169.0		



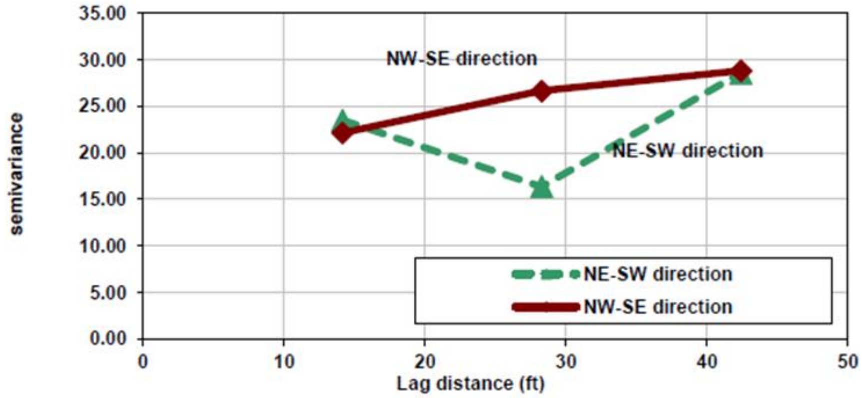
d) Calculate the semivariance values on the experimental variogram in the northwest-southeast direction at lag distances $10\sqrt{2}$ ft, $20\sqrt{2}$ ft, and $30\sqrt{2}$ ft. Show your calculations, fill out the following table, and plot the variogrram.

Lag distance (ft)	$10\sqrt{2}$	$20\sqrt{2}$	$30\sqrt{2}$
$\gamma_{NW-SE}(h)$	22.14	26.65	28.81

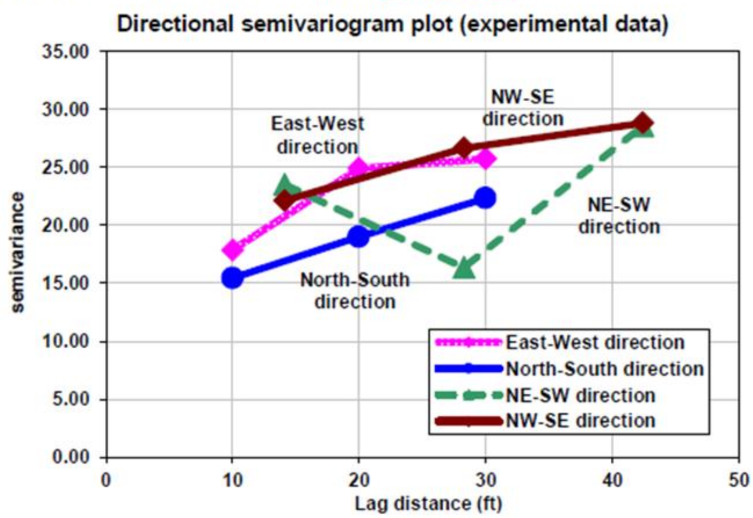
h	10				
Nh	24				
sum	1062.5				
G(h)	22.135				
lag 10 NW-SE					
16.0	64.0	4.0	64.0	1.0	9.0
0.0	64.0	42.3	25.0	9.0	25.0
49.0	100.0	1.0	110.3	9.0	144.0
25.0	289.0	1.0	1.0	9.0	1.0

h	20			
Nh	15			
sum	799.5			
G(h)	26.65			
lag 20NW-SE				
16.0	2.3	49.0	121.0	16.0
100.0	49.0	16.0	64.0	81.0
100.0	121.0	4.0	56.3	4.0

h	30		
Nh	8		
sum	461.0		
G(h)	28.8125		
lag 300 NW-SE			
9.0	144.0	100.0	1.0
121.0	36.0	1.0	49.0



e) Is the variogram isotropic or anisotropic? Anisotropic



Question 3: Consider arrangement of points shown in Figure 1. The volumetric concentration of clay at locations x_1, x_2, and x_3 is measured to be 10%, 30%, and 45%. The theoretical autocovariance is given by:

$$Cov(h) = 6e^{-3|h|/10}$$

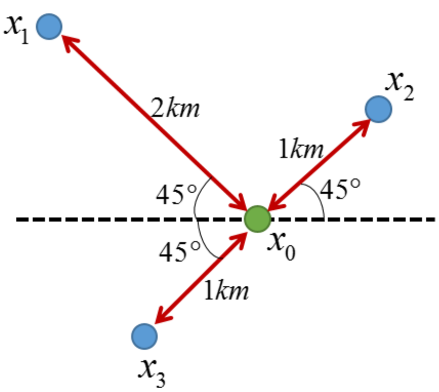


Figure 1: Arrangement of data points in Question 3

a) We plan to use ordinary kriging to estimate volumetric concentration of clay at location x_0. Show your calculations and <u>write the final result in terms of</u> λ_i s.

Please write your equation for calculation of λ_i s **<u>in the following format</u>**. The matrices will be graded.

$$\begin{bmatrix} \gamma(h_{11}) & \gamma(h_{12}) & \cdots & \gamma(h_{1N}) & -1 \\ \gamma(h_{21}) & \gamma(h_{22}) & \cdots & \gamma(h_{2N}) & -1 \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ \gamma(h_{N1}) & \gamma(h_{N2}) & \cdots & \gamma(h_{NN}) & -1 \\ 1 & 1 & \cdots & 1 & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_N \\ \beta \end{bmatrix} = \begin{bmatrix} \gamma(h_{10}) \\ \gamma(h_{20}) \\ \vdots \\ \gamma(h_{N0}) \\ 1 \end{bmatrix}$$

b) Estimate volumetric concentration of clay at location $\boldsymbol{x_0}$. Show your calculations and write the final result below.

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c) Calculate the minimum error variance for estimate of volumetric concentration of clay at location $\boldsymbol{x_0}$. Show your calculations and write the final result below.

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d) Calculate the <u>99% confidence interval</u> for estimate of volumetric concentration of clay at location $\boldsymbol{x_0}$. Show your calculations and write the final result below.

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$$\gamma(h) = Cov(0) - Cov(h)$$
$$= 6\left[1 - e^{-3|h|/10}\right].$$

$$\begin{bmatrix} h_{11} & h_{12} & h_{13} & h_{10} \\ h_{21} & h_{22} & h_{23} & h_{20} \\ h_{31} & h_{32} & h_{33} & h_{30} \end{bmatrix} = \begin{bmatrix} 0 & \sqrt{5} & \sqrt{5} & 2 \\ \sqrt{5} & 0 & 2 & 1 \\ \sqrt{5} & 2 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2.93 & 2.93 & -1 \\ 2.93 & 0 & 2.71 & -1 \\ 2.93 & 2.71 & 0 & -1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \beta \end{bmatrix} = \begin{bmatrix} 2.71 \\ 1.55 \\ 1.55 \\ 1 \end{bmatrix}$$

$\lambda_1 = 0.092 \qquad \lambda_2 = 0.454 = \lambda_3$

$\beta = -0.05$

$$C_Q = \lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3$$
$$= 0.092(10) + 0.454(30) + 0.454(45)$$
$$= 34.96$$

$$\sigma_{e,min}^2 = -(-0.05) + [0.092(2.71) + 0.454(1.55) + 0.454(1.55)]$$
$$= 1.71$$

$$99\%\ C.I = 35 \pm Z_{\alpha/2}\, S.E$$
$$= 35 \pm 2.58\sqrt{1.71} = 35 \pm 3.37$$

Question 5: (Optional) You do not need to submit your solutions for this question.

Consider the TOC data (TOC_Spatial data) provided to you on Canvas website and investigate the impact of following parameters on kriging results and minimum error variance:

- **a)** Variogram model
- **b)** Variogram parameters such as slope, sill, and nugget effect
- **c)** Number of data points used as input to kriging