

# CP8

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## PGE 382 - Numerical Methods in Petroleum and Geosystems Engineering

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### CP8 - Apr, 11st

```
[1]: from math import pi, sin, cos, exp
import numpy as np
from numpy import linspace, zeros, arange
from numpy import ix_ as ix
np.set_printoptions(threshold=10000, linewidth=10000)
from numpy import exp, linspace, vectorize
import matplotlib.pyplot as plt
plt.style.use('paper.mplstyle')

XMAX = 100
YMAX = 25
dx = XMAX/10 ; dy = YMAX/10
Tf = 80 ; Nt=160
dt = Tf/Nt ; Nt = Nt + 1
X = np.arange(0,XMAX+dx,dx) ; Ni = len(X)
Y = np.arange(0,YMAX+dy,dy) ; Nj = len(Y)
Nij = Ni * Nj

beta = .5
Pr = 0.733

# Global index
def _(i,j) : return j + Nj*i

# Calculate F1
def cF1(i, j, U, V, T, Un, Vn, Tn) :
    _ij = _(i,j)
    _0j = _(i-1,j)
    _i0 = _(i,j-1)
    _i1 = _(i,j+1)

    ret = 0
    #1 = T
    ret += beta * T[_ij]
    ret += (1-beta) * Tn[_ij]
    #2 = d2U/dy2
    ret += beta * ( U[_i0] - 2*U[_ij] + U[_i1] ) / dy / dy
    ret += (1-beta) * ( Un[_i0] - 2*Un[_ij] + Un[_i1] ) / dy / dy
    #3 = -dU/dt
    ret += Un[_ij]/dt - U[_ij]/dt
    #4 = -U dU/dx
    ret += -beta * U[_ij] * ( U[_ij] - U[_0j] )/dx
    ret += -(1-beta) * Un[_ij] * ( Un[_ij] - Un[_0j] )/dx
    #5 = -V dU/dy
    ret += -beta * V[_ij] * ( U[_ij] - U[_i0] )/dy
    ret += -(1-beta) * Vn[_ij] * ( Un[_ij] - Un[_i0] )/dy
    return ret

# Calculate F2
def cF2(i, j, U, V, T, Un, Vn, Tn) :
    _ij = _(i,j)
    _0j = _(i-1,j)
    _1j = _(i+1,j)
    _i0 = _(i,j-1)
    _i1 = _(i,j+1)

    ret = 0
    #1 = 1/Pr d2T/dy2
    ret += beta * (1/Pr) * ( T[_i0] - 2*T[_ij] + T[_i1] ) / dy / dy
    ret += (1-beta) * (1/Pr) * ( Tn[_i0] - 2*Tn[_ij] + Tn[_i1] ) / dy / dy
```

```

#2 = -dT/dt
ret += Tn[_ij]/dt - T[_ij]/dt
#3 = -U dT/dx
ret += -beta * U[_ij] * ( T[_ij] - T[_0j] )/dx
ret += -(1-beta) * Un[_ij] * ( Tn[_ij] - Tn[_0j] )/dx
#4 = -V dT/dy
ret += -beta * V[_ij] * ( T[_ij] - T[_i0] )/dy
ret += -(1-beta) * Vn[_ij] * ( Tn[_ij] - Tn[_i0] )/dy
return ret

```

*# Calculate F3*

```
def cf3(i, j, U, V, T, Un, Vn, Tn) :
```

```

    _ij = _ (i,j)
    _0j = _ (i-1,j)
    _i0 = _ (i,j-1)

```

```

    ret = 0
    #1 = dU/dx
    ret += beta * ( U[_ij] - U[_0j] ) / dx
    ret += (1-beta) * ( Un[_ij] - Un[_0j] ) / dx
    #2 = dV/dy
    ret += beta * ( V[_ij] - V[_i0] ) / dy
    ret += (1-beta) * ( Vn[_ij] - Vn[_i0] ) / dy
    return ret

```

*# The list of free and prescribed dofs*

```
def build_Df() :
```

```

    global U_doff, V_doff, T_doff, Ni, Nj

```

```

    U_doff=[]
    T_doff=[]
    for i in arange(1,Ni) :
        for j in arange(1,Nj-1) :
            U_doff.append( _ (i,j) )
            T_doff.append( _ (i,j) )

```

```

    V_doff=[]
    for i in arange(1,Ni) :
        for j in arange(1,Nj) :
            V_doff.append( _ (i,j) )

```

*#*

*# Assign BCs to solution vectors*

*#*

```
def init_bcs() :
```

```

    global Unij, Vnij, Tnij
    Unij = zeros( [Nt,Ni,Nj] )
    Vnij = zeros( [Nt,Ni,Nj] )
    Tnij = zeros( [Nt,Ni,Nj] )

```

```

    Unij[0,:,:] = 0 # ic
    Vnij[0,:,:] = 0 # ic
    Tnij[0,:,:] = 0 # ic

```

```

    Unij[:,0] = 0 # BC , Y=0
    Vnij[:,0] = 0 # BC , Y=0
    Tnij[:,0] = 1 # BC , Y=0

```

```

    Unij[:,Nj-1] = 0 # BC , Y=inf
    Tnij[:,Nj-1] = 0 # BC , Y=inf

```

```

    Unij[:,0,:] = 0 # BC , X=0
    Vnij[:,0,:] = 0 # BC , X=0
    Tnij[:,0,:] = 0 # BC , X=0

```

*#*

*#*

*#*

*#*

```
def build_force() :
```

```

    global F1, F2, F3, Ni, Nj, Nij

```

```

    # Fx

```

```

    F1 = zeros( Nij )

```

```

    F2 = zeros( Nij )

```

```

F3 = zeros( Nij )
for i in arange(1,Ni) :
    for j in arange(1,Nj-1) :
        _ij = _(i,j)
        F1[_ij] += cF1(i,j,Uk,Vk,Tk,Un,Vn,Tn)
        F2[_ij] += cF2(i,j,Uk,Vk,Tk,Un,Vn,Tn)
for i in arange(1,Ni) :
    for j in arange(1,Nj) :
        F3[_ij] += cF3(i,j,Uk,Vk,Tk,Un,Vn,Tn)

#
#
#
#
def build_jacobian() :
    global J1U, J1V, J1T, J2U, J2V, J2T, J3U, J3V, J3T

    # DFx/DU
    J1U = zeros([ Nij, Nij ])
    J2U = zeros([ Nij, Nij ])
    J3U = zeros([ Nij, Nij ])
    # DFx/DV
    J1V = zeros([ Nij, Nij ])
    J2V = zeros([ Nij, Nij ])
    J3V = zeros([ Nij, Nij ])
    # DFx/DT
    J1T = zeros([ Nij, Nij ])
    J2T = zeros([ Nij, Nij ])
    J3T = zeros([ Nij, Nij ])

    for i in arange(1,Ni) :
        for j in arange(1,Nj-1) :
            _ij = _(i,j)
            _0j = _(i-1,j)
            _1j = _(i+1,j)
            _i0 = _(i,j-1)
            _i1 = _(i,j+1)

            # F1
            #
            #1 = T
            J1T[_ij,_ij] += beta
            #2 = d2U/dy2
            J1U[_ij,_i0] += beta/dy/dy
            J1U[_ij,_ij] += beta*(-2/dy/dy)
            J1U[_ij,_i1] += beta/dy/dy
            #3 = -dU/dt
            J1U[_ij,_ij] += -1/dt
            #4 = -U dU/dx --
            #
            #         ./dUij = beta*(-2U/dx + U0j/dx)
            #         ./dU0j = beta*U/dx
            J1U[_ij,_ij] += beta * (-2*Uk[_ij] + Uk[_0j])/dx
            J1U[_ij,_0j] += beta * Uk[_ij]/dx
            #5 = -V dU/dy
            #
            #         ./dUij = -beta*V/dy
            #         ./dUj0 = beta*V/dy
            #         ./dVij = -beta*(U-Ui0)/dy
            J1U[_ij,_ij] += -beta * Vk[_ij]/dy
            J1U[_ij,_i0] += beta * Vk[_ij]/dy
            J1V[_ij,_ij] += beta * (-Uk[_ij]+Uk[_i0])/dy

            # F2
            #
            #1 = 1/Pr d2T/dy2
            J2T[_ij,_ij] += beta * (-2) / Pr / dy / dy
            J2T[_ij,_i0] += beta * 1 / Pr / dy / dy
            J2T[_ij,_i1] += beta * 1 / Pr / dy / dy
            #2 = -dT/dt
            J2T[_ij,_ij] += -1/dt
            #3 = -U dT/dx
            #
            #         (3)/dUij = -beta*(T-T0)/dx
            #         (3)/dTij = -beta*Uij/dx
            #         (3)/dT0j = beta*Uij/dx
            J2U[_ij,_ij] += -beta * ( Tk[_ij] - Tk[_0j] )/dx
            J2T[_ij,_ij] += -beta * Uk[_ij] /dx

```

```

J2T[_ij,_0j] += beta * Uk[_ij] /dx
#4 = -V dT/dy
#
# (4)/dVij = -beta*(T-Ti0)/dy
# (4)/dTij = -beta*Uij/dy
# (4)/dTio = beta*Uij/dy
J2V[_ij,_ij] += -beta * ( Tk[_ij] - Tk[_i0] )/dy
J2T[_ij,_ij] += -beta * Vk[_ij] /dy
J2T[_ij,_i0] += beta * Vk[_ij] /dy

for i in arange(1,Ni) :
    for j in arange(1,Nj) :
        _ij = _(i,j)
        _0j = _(i-1,j)
        _1j = _(i+1,j)
        _i0 = _(i,j-1)
        _i1 = _(i,j+1)
        # F3
        #
        #1 = dU/dx
        J3U[_ij,_ij] += beta/dx
        J3U[_ij,_0j] += -beta/dx
        #2 = dU/dy
        J3V[_ij,_ij] += beta / dy
        J3V[_ij,_i0] += -beta / dy

#
#
#
#
def linear_solve( ) :
    global Nij
    global J1U, J1V, J1T, J2U, J2V, J2T, J3U, J3V, J3T
    global F1, F2, F3
    global JAC, FORCE
    global U_doff, V_doff, T_doff

    JAC = np.block([[ J1U[ix(U_doff,U_doff)], J1V[ix(U_doff,V_doff)], J1T[ix(U_doff,T_doff)]],
                     [ J2U[ix(T_doff,U_doff)], J2V[ix(T_doff,V_doff)], J2T[ix(T_doff,T_doff)]],
                     [ J3U[ix(V_doff,U_doff)], J3V[ix(V_doff,V_doff)], J3T[ix(V_doff,T_doff)] ]])
    FORCE = np.block([ F1[ix(U_doff)], F2[ix(T_doff)], F3[ix(V_doff)] ])

    dX = np.linalg.solve( JAC, -FORCE )

    # Extract free dofs
    dUk = np.zeros(Nij)
    dVk = np.zeros(Nij)
    dTk = np.zeros(Nij)
    bl1 = len(U_doff) # block length
    bl2 = len(V_doff) # block length
    bl3 = len(T_doff) # block length
    dUk[ix(U_doff)] = dX[:bl1]
    dVk[ix(V_doff)] = dX[bl1:(bl1+bl2)]
    dTk[ix(T_doff)] = dX[(bl1+bl2):]

    err = np.linalg.norm(dX)

    return dUk, dVk, dTk, err

#
#
#
# MAIN FLOW
#
#
#
# Global solution vector
init_bcs()
build_Df()

for n in arange(1,Nt) :
```

```

print(f"Solving timestep {n} ...")

# Solution from the previous TS
Un = Unij[n-1,:].flatten()
Vn = Vnij[n-1,:].flatten()
Tn = Tnij[n-1,:].flatten()
# Initial guess for newton-raphson
Uk = Un.copy()
Vk = Vn.copy()
Tk = Tn.copy()

nk = 0 #newton loop index
while(1) :
    build_jacobian()
    build_force()
    dUk, dVk, dTk, err = linear_solve()

    # Update results
    Uk += dUk
    Vk += dVk
    Tk += dTk

    # Check for convergence
    nk += 1
    print(f"    Newton iteration #{nk} ... (err={err:.3e})")
    if err < 1e-13 : break
    if nk > 50 : break

Unij[n,:] = Uk.reshape(Ni,Nj)
Vnij[n,:] = Vk.reshape(Ni,Nj)
Tnij[n,:] = Tk.reshape(Ni,Nj)

```

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Solving timestep 1 ...
  Newton iteration #1 ... (err=3.211e-01)
  Newton iteration #2 ... (err=6.590e-05)
  Newton iteration #3 ... (err=4.760e-11)
  Newton iteration #4 ... (err=1.870e-17)
Solving timestep 2 ...
  Newton iteration #1 ... (err=3.266e-01)
  Newton iteration #2 ... (err=2.126e-04)
  Newton iteration #3 ... (err=6.699e-10)
  Newton iteration #4 ... (err=5.655e-17)
Solving timestep 3 ...
  Newton iteration #1 ... (err=3.633e-01)
  Newton iteration #2 ... (err=3.574e-04)
  Newton iteration #3 ... (err=1.959e-09)
  Newton iteration #4 ... (err=6.383e-17)
Solving timestep 4 ...
  Newton iteration #1 ... (err=4.026e-01)
  Newton iteration #2 ... (err=4.686e-04)
  Newton iteration #3 ... (err=3.389e-09)
  Newton iteration #4 ... (err=1.044e-16)
Solving timestep 5 ...
  Newton iteration #1 ... (err=4.366e-01)
  Newton iteration #2 ... (err=5.306e-04)
  Newton iteration #3 ... (err=4.419e-09)
  Newton iteration #4 ... (err=1.086e-16)
Solving timestep 6 ...
  Newton iteration #1 ... (err=4.641e-01)
  Newton iteration #2 ... (err=5.444e-04)
  Newton iteration #3 ... (err=4.754e-09)
  Newton iteration #4 ... (err=1.719e-16)
Solving timestep 7 ...
  Newton iteration #1 ... (err=4.861e-01)
  Newton iteration #2 ... (err=5.235e-04)
  Newton iteration #3 ... (err=4.365e-09)
  Newton iteration #4 ... (err=1.787e-16)
Solving timestep 8 ...
  Newton iteration #1 ... (err=5.037e-01)
  Newton iteration #2 ... (err=4.887e-04)
  Newton iteration #3 ... (err=3.421e-09)
  Newton iteration #4 ... (err=2.168e-16)
Solving timestep 9 ...
  Newton iteration #1 ... (err=5.180e-01)
  Newton iteration #2 ... (err=4.607e-04)

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    Newton iteration #3 ... (err=2.282e-09)
    Newton iteration #4 ... (err=1.762e-16)
Solving timestep 10 ...
    Newton iteration #1 ... (err=5.298e-01)
    Newton iteration #2 ... (err=4.494e-04)
    Newton iteration #3 ... (err=1.616e-09)
    Newton iteration #4 ... (err=3.584e-16)
Solving timestep 11 ...
    Newton iteration #1 ... (err=5.396e-01)
    Newton iteration #2 ... (err=4.505e-04)
    Newton iteration #3 ... (err=1.889e-09)
    Newton iteration #4 ... (err=2.641e-16)
Solving timestep 12 ...
    Newton iteration #1 ... (err=5.478e-01)
    Newton iteration #2 ... (err=4.535e-04)
    Newton iteration #3 ... (err=2.307e-09)
    Newton iteration #4 ... (err=2.495e-16)
Solving timestep 13 ...
    Newton iteration #1 ... (err=5.544e-01)
    Newton iteration #2 ... (err=4.520e-04)
    Newton iteration #3 ... (err=2.347e-09)
    Newton iteration #4 ... (err=3.206e-16)
Solving timestep 14 ...
    Newton iteration #1 ... (err=5.597e-01)
    Newton iteration #2 ... (err=4.458e-04)
    Newton iteration #3 ... (err=2.012e-09)
    Newton iteration #4 ... (err=2.933e-16)
Solving timestep 15 ...
    Newton iteration #1 ... (err=5.638e-01)
    Newton iteration #2 ... (err=4.381e-04)
    Newton iteration #3 ... (err=1.574e-09)
    Newton iteration #4 ... (err=4.601e-16)
Solving timestep 16 ...
    Newton iteration #1 ... (err=5.666e-01)
    Newton iteration #2 ... (err=4.314e-04)
    Newton iteration #3 ... (err=1.374e-09)
    Newton iteration #4 ... (err=4.946e-16)
Solving timestep 17 ...
    Newton iteration #1 ... (err=5.682e-01)
    Newton iteration #2 ... (err=4.261e-04)
    Newton iteration #3 ... (err=1.414e-09)
    Newton iteration #4 ... (err=3.881e-16)
Solving timestep 18 ...
    Newton iteration #1 ... (err=5.688e-01)
    Newton iteration #2 ... (err=4.215e-04)
    Newton iteration #3 ... (err=1.410e-09)
    Newton iteration #4 ... (err=4.399e-16)
Solving timestep 19 ...
    Newton iteration #1 ... (err=5.682e-01)
    Newton iteration #2 ... (err=4.170e-04)
    Newton iteration #3 ... (err=1.274e-09)
    Newton iteration #4 ... (err=6.491e-16)
Solving timestep 20 ...
    Newton iteration #1 ... (err=5.666e-01)
    Newton iteration #2 ... (err=4.125e-04)
    Newton iteration #3 ... (err=1.116e-09)
    Newton iteration #4 ... (err=6.151e-16)
Solving timestep 21 ...
    Newton iteration #1 ... (err=5.639e-01)
    Newton iteration #2 ... (err=4.081e-04)
    Newton iteration #3 ... (err=1.042e-09)
    Newton iteration #4 ... (err=7.127e-16)
Solving timestep 22 ...
    Newton iteration #1 ... (err=5.602e-01)
    Newton iteration #2 ... (err=4.040e-04)
    Newton iteration #3 ... (err=1.008e-09)
    Newton iteration #4 ... (err=7.795e-16)
Solving timestep 23 ...
    Newton iteration #1 ... (err=5.554e-01)
    Newton iteration #2 ... (err=4.001e-04)
    Newton iteration #3 ... (err=9.438e-10)
    Newton iteration #4 ... (err=9.421e-16)
Solving timestep 24 ...
    Newton iteration #1 ... (err=5.495e-01)
    Newton iteration #2 ... (err=3.964e-04)
    Newton iteration #3 ... (err=8.659e-10)

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    Newton iteration #4 ... (err=8.389e-16)
Solving timestep 25 ...
    Newton iteration #1 ... (err=5.426e-01)
    Newton iteration #2 ... (err=3.928e-04)
    Newton iteration #3 ... (err=8.115e-10)
    Newton iteration #4 ... (err=6.900e-16)
Solving timestep 26 ...
    Newton iteration #1 ... (err=5.345e-01)
    Newton iteration #2 ... (err=3.894e-04)
    Newton iteration #3 ... (err=7.711e-10)
    Newton iteration #4 ... (err=8.993e-16)
Solving timestep 27 ...
    Newton iteration #1 ... (err=5.253e-01)
    Newton iteration #2 ... (err=3.862e-04)
    Newton iteration #3 ... (err=7.247e-10)
    Newton iteration #4 ... (err=7.703e-16)
Solving timestep 28 ...
    Newton iteration #1 ... (err=5.148e-01)
    Newton iteration #2 ... (err=3.830e-04)
    Newton iteration #3 ... (err=6.791e-10)
    Newton iteration #4 ... (err=6.570e-16)
Solving timestep 29 ...
    Newton iteration #1 ... (err=5.032e-01)
    Newton iteration #2 ... (err=3.798e-04)
    Newton iteration #3 ... (err=6.427e-10)
    Newton iteration #4 ... (err=8.664e-16)
Solving timestep 30 ...
    Newton iteration #1 ... (err=4.902e-01)
    Newton iteration #2 ... (err=3.765e-04)
    Newton iteration #3 ... (err=6.088e-10)
    Newton iteration #4 ... (err=9.217e-16)
Solving timestep 31 ...
    Newton iteration #1 ... (err=4.758e-01)
    Newton iteration #2 ... (err=3.729e-04)
    Newton iteration #3 ... (err=5.746e-10)
    Newton iteration #4 ... (err=7.662e-16)
Solving timestep 32 ...
    Newton iteration #1 ... (err=4.601e-01)
    Newton iteration #2 ... (err=3.686e-04)
    Newton iteration #3 ... (err=5.432e-10)
    Newton iteration #4 ... (err=7.853e-16)
Solving timestep 33 ...
    Newton iteration #1 ... (err=4.430e-01)
    Newton iteration #2 ... (err=3.632e-04)
    Newton iteration #3 ... (err=5.141e-10)
    Newton iteration #4 ... (err=8.414e-16)
Solving timestep 34 ...
    Newton iteration #1 ... (err=4.244e-01)
    Newton iteration #2 ... (err=3.565e-04)
    Newton iteration #3 ... (err=4.859e-10)
    Newton iteration #4 ... (err=8.231e-16)
Solving timestep 35 ...
    Newton iteration #1 ... (err=4.044e-01)
    Newton iteration #2 ... (err=3.481e-04)
    Newton iteration #3 ... (err=4.599e-10)
    Newton iteration #4 ... (err=9.425e-16)
Solving timestep 36 ...
    Newton iteration #1 ... (err=3.831e-01)
    Newton iteration #2 ... (err=3.378e-04)
    Newton iteration #3 ... (err=4.351e-10)
    Newton iteration #4 ... (err=7.489e-16)
Solving timestep 37 ...
    Newton iteration #1 ... (err=3.605e-01)
    Newton iteration #2 ... (err=3.254e-04)
    Newton iteration #3 ... (err=4.066e-10)
    Newton iteration #4 ... (err=9.176e-16)
Solving timestep 38 ...
    Newton iteration #1 ... (err=3.368e-01)
    Newton iteration #2 ... (err=3.108e-04)
    Newton iteration #3 ... (err=3.707e-10)
    Newton iteration #4 ... (err=8.838e-16)
Solving timestep 39 ...
    Newton iteration #1 ... (err=3.124e-01)
    Newton iteration #2 ... (err=2.937e-04)
    Newton iteration #3 ... (err=3.303e-10)
    Newton iteration #4 ... (err=7.794e-16)

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Solving timestep 40 ...  
   Newton iteration #1 ... (err=2.874e-01)  
   Newton iteration #2 ... (err=2.736e-04)  
   Newton iteration #3 ... (err=2.950e-10)  
   Newton iteration #4 ... (err=1.035e-15)  
 Solving timestep 41 ...  
   Newton iteration #1 ... (err=2.625e-01)  
   Newton iteration #2 ... (err=2.502e-04)  
   Newton iteration #3 ... (err=2.704e-10)  
   Newton iteration #4 ... (err=9.743e-16)  
 Solving timestep 42 ...  
   Newton iteration #1 ... (err=2.380e-01)  
   Newton iteration #2 ... (err=2.236e-04)  
   Newton iteration #3 ... (err=2.481e-10)  
   Newton iteration #4 ... (err=1.614e-15)  
 Solving timestep 43 ...  
   Newton iteration #1 ... (err=2.148e-01)  
   Newton iteration #2 ... (err=1.944e-04)  
   Newton iteration #3 ... (err=2.166e-10)  
   Newton iteration #4 ... (err=1.167e-15)  
 Solving timestep 44 ...  
   Newton iteration #1 ... (err=1.936e-01)  
   Newton iteration #2 ... (err=1.642e-04)  
   Newton iteration #3 ... (err=1.748e-10)  
   Newton iteration #4 ... (err=1.389e-15)  
 Solving timestep 45 ...  
   Newton iteration #1 ... (err=1.751e-01)  
   Newton iteration #2 ... (err=1.348e-04)  
   Newton iteration #3 ... (err=1.309e-10)  
   Newton iteration #4 ... (err=1.068e-15)  
 Solving timestep 46 ...  
   Newton iteration #1 ... (err=1.601e-01)  
   Newton iteration #2 ... (err=1.085e-04)  
   Newton iteration #3 ... (err=9.304e-11)  
   Newton iteration #4 ... (err=8.483e-16)  
 Solving timestep 47 ...  
   Newton iteration #1 ... (err=1.490e-01)  
   Newton iteration #2 ... (err=8.672e-05)  
   Newton iteration #3 ... (err=6.415e-11)  
   Newton iteration #4 ... (err=1.324e-15)  
 Solving timestep 48 ...  
   Newton iteration #1 ... (err=1.416e-01)  
   Newton iteration #2 ... (err=7.038e-05)  
   Newton iteration #3 ... (err=4.308e-11)  
   Newton iteration #4 ... (err=1.007e-15)  
 Solving timestep 49 ...  
   Newton iteration #1 ... (err=1.376e-01)  
   Newton iteration #2 ... (err=5.903e-05)  
   Newton iteration #3 ... (err=2.832e-11)  
   Newton iteration #4 ... (err=1.035e-15)  
 Solving timestep 50 ...  
   Newton iteration #1 ... (err=1.360e-01)  
   Newton iteration #2 ... (err=5.144e-05)  
   Newton iteration #3 ... (err=1.876e-11)  
   Newton iteration #4 ... (err=1.616e-15)  
 Solving timestep 51 ...  
   Newton iteration #1 ... (err=1.357e-01)  
   Newton iteration #2 ... (err=4.630e-05)  
   Newton iteration #3 ... (err=1.272e-11)  
   Newton iteration #4 ... (err=1.050e-15)  
 Solving timestep 52 ...  
   Newton iteration #1 ... (err=1.359e-01)  
   Newton iteration #2 ... (err=4.286e-05)  
   Newton iteration #3 ... (err=8.529e-12)  
   Newton iteration #4 ... (err=1.391e-15)  
 Solving timestep 53 ...  
   Newton iteration #1 ... (err=1.358e-01)  
   Newton iteration #2 ... (err=4.085e-05)  
   Newton iteration #3 ... (err=6.013e-12)  
   Newton iteration #4 ... (err=1.259e-15)  
 Solving timestep 54 ...  
   Newton iteration #1 ... (err=1.350e-01)  
   Newton iteration #2 ... (err=4.008e-05)  
   Newton iteration #3 ... (err=5.640e-12)  
   Newton iteration #4 ... (err=1.061e-15)  
 Solving timestep 55 ...



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Newton iteration #1 ... (err=1.332e-01)
Newton iteration #2 ... (err=4.020e-05)
Newton iteration #3 ... (err=6.156e-12)
Newton iteration #4 ... (err=1.004e-15)
Solving timestep 56 ...
Newton iteration #1 ... (err=1.303e-01)
Newton iteration #2 ... (err=4.068e-05)
Newton iteration #3 ... (err=6.296e-12)
Newton iteration #4 ... (err=8.489e-16)
Solving timestep 57 ...
Newton iteration #1 ... (err=1.263e-01)
Newton iteration #2 ... (err=4.100e-05)
Newton iteration #3 ... (err=5.966e-12)
Newton iteration #4 ... (err=8.502e-16)
Solving timestep 58 ...
Newton iteration #1 ... (err=1.214e-01)
Newton iteration #2 ... (err=4.078e-05)
Newton iteration #3 ... (err=5.563e-12)
Newton iteration #4 ... (err=1.118e-15)
Solving timestep 59 ...
Newton iteration #1 ... (err=1.156e-01)
Newton iteration #2 ... (err=3.985e-05)
Newton iteration #3 ... (err=5.395e-12)
Newton iteration #4 ... (err=1.396e-15)
Solving timestep 60 ...
Newton iteration #1 ... (err=1.092e-01)
Newton iteration #2 ... (err=3.817e-05)
Newton iteration #3 ... (err=5.411e-12)
Newton iteration #4 ... (err=8.696e-16)
Solving timestep 61 ...
Newton iteration #1 ... (err=1.022e-01)
Newton iteration #2 ... (err=3.584e-05)
Newton iteration #3 ... (err=5.375e-12)
Newton iteration #4 ... (err=1.101e-15)
Solving timestep 62 ...
Newton iteration #1 ... (err=9.483e-02)
Newton iteration #2 ... (err=3.297e-05)
Newton iteration #3 ... (err=5.124e-12)
Newton iteration #4 ... (err=1.057e-15)
Solving timestep 63 ...
Newton iteration #1 ... (err=8.726e-02)
Newton iteration #2 ... (err=2.976e-05)
Newton iteration #3 ... (err=4.635e-12)
Newton iteration #4 ... (err=1.245e-15)
Solving timestep 64 ...
Newton iteration #1 ... (err=7.962e-02)
Newton iteration #2 ... (err=2.635e-05)
Newton iteration #3 ... (err=3.973e-12)
Newton iteration #4 ... (err=7.691e-16)
Solving timestep 65 ...
Newton iteration #1 ... (err=7.204e-02)
Newton iteration #2 ... (err=2.291e-05)
Newton iteration #3 ... (err=3.233e-12)
Newton iteration #4 ... (err=1.075e-15)
Solving timestep 66 ...
Newton iteration #1 ... (err=6.461e-02)
Newton iteration #2 ... (err=1.956e-05)
Newton iteration #3 ... (err=2.506e-12)
Newton iteration #4 ... (err=8.903e-16)
Solving timestep 67 ...
Newton iteration #1 ... (err=5.744e-02)
Newton iteration #2 ... (err=1.640e-05)
Newton iteration #3 ... (err=1.854e-12)
Newton iteration #4 ... (err=8.256e-16)
Solving timestep 68 ...
Newton iteration #1 ... (err=5.061e-02)
Newton iteration #2 ... (err=1.351e-05)
Newton iteration #3 ... (err=1.315e-12)
Newton iteration #4 ... (err=1.087e-15)
Solving timestep 69 ...
Newton iteration #1 ... (err=4.419e-02)
Newton iteration #2 ... (err=1.093e-05)
Newton iteration #3 ... (err=8.977e-13)
Newton iteration #4 ... (err=9.297e-16)
Solving timestep 70 ...
Newton iteration #1 ... (err=3.823e-02)

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    Newton iteration #2 ... (err=8.690e-06)
    Newton iteration #3 ... (err=5.941e-13)
    Newton iteration #4 ... (err=1.020e-15)
Solving timestep 71 ...
    Newton iteration #1 ... (err=3.279e-02)
    Newton iteration #2 ... (err=6.790e-06)
    Newton iteration #3 ... (err=3.845e-13)
    Newton iteration #4 ... (err=1.079e-15)
Solving timestep 72 ...
    Newton iteration #1 ... (err=2.791e-02)
    Newton iteration #2 ... (err=5.219e-06)
    Newton iteration #3 ... (err=2.466e-13)
    Newton iteration #4 ... (err=8.786e-16)
Solving timestep 73 ...
    Newton iteration #1 ... (err=2.362e-02)
    Newton iteration #2 ... (err=3.954e-06)
    Newton iteration #3 ... (err=1.566e-13)
    Newton iteration #4 ... (err=9.749e-16)
Solving timestep 74 ...
    Newton iteration #1 ... (err=1.997e-02)
    Newton iteration #2 ... (err=2.959e-06)
    Newton iteration #3 ... (err=9.915e-14)
Solving timestep 75 ...
    Newton iteration #1 ... (err=1.697e-02)
    Newton iteration #2 ... (err=2.197e-06)
    Newton iteration #3 ... (err=6.158e-14)
Solving timestep 76 ...
    Newton iteration #1 ... (err=1.464e-02)
    Newton iteration #2 ... (err=1.624e-06)
    Newton iteration #3 ... (err=3.710e-14)
Solving timestep 77 ...
    Newton iteration #1 ... (err=1.294e-02)
    Newton iteration #2 ... (err=1.200e-06)
    Newton iteration #3 ... (err=2.164e-14)
Solving timestep 78 ...
    Newton iteration #1 ... (err=1.181e-02)
    Newton iteration #2 ... (err=8.906e-07)
    Newton iteration #3 ... (err=1.207e-14)
Solving timestep 79 ...
    Newton iteration #1 ... (err=1.112e-02)
    Newton iteration #2 ... (err=6.707e-07)
    Newton iteration #3 ... (err=6.231e-15)
Solving timestep 80 ...
    Newton iteration #1 ... (err=1.073e-02)
    Newton iteration #2 ... (err=5.250e-07)
    Newton iteration #3 ... (err=2.555e-15)
Solving timestep 81 ...
    Newton iteration #1 ... (err=1.048e-02)
    Newton iteration #2 ... (err=4.436e-07)
    Newton iteration #3 ... (err=1.116e-15)
Solving timestep 82 ...
    Newton iteration #1 ... (err=1.028e-02)
    Newton iteration #2 ... (err=4.126e-07)
    Newton iteration #3 ... (err=1.094e-15)
Solving timestep 83 ...
    Newton iteration #1 ... (err=1.006e-02)
    Newton iteration #2 ... (err=4.103e-07)
    Newton iteration #3 ... (err=1.181e-15)
Solving timestep 84 ...
    Newton iteration #1 ... (err=9.765e-03)
    Newton iteration #2 ... (err=4.161e-07)
    Newton iteration #3 ... (err=1.392e-15)
Solving timestep 85 ...
    Newton iteration #1 ... (err=9.392e-03)
    Newton iteration #2 ... (err=4.172e-07)
    Newton iteration #3 ... (err=1.263e-15)
Solving timestep 86 ...
    Newton iteration #1 ... (err=8.939e-03)
    Newton iteration #2 ... (err=4.078e-07)
    Newton iteration #3 ... (err=1.495e-15)
Solving timestep 87 ...
    Newton iteration #1 ... (err=8.414e-03)
    Newton iteration #2 ... (err=3.871e-07)
    Newton iteration #3 ... (err=1.653e-15)
Solving timestep 88 ...
    Newton iteration #1 ... (err=7.833e-03)

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    Newton iteration #2 ... (err=3.565e-07)
    Newton iteration #3 ... (err=1.625e-15)
Solving timestep 89 ...
    Newton iteration #1 ... (err=7.217e-03)
    Newton iteration #2 ... (err=3.189e-07)
    Newton iteration #3 ... (err=1.354e-15)
Solving timestep 90 ...
    Newton iteration #1 ... (err=6.584e-03)
    Newton iteration #2 ... (err=2.775e-07)
    Newton iteration #3 ... (err=1.439e-15)
Solving timestep 91 ...
    Newton iteration #1 ... (err=5.956e-03)
    Newton iteration #2 ... (err=2.353e-07)
    Newton iteration #3 ... (err=1.353e-15)
Solving timestep 92 ...
    Newton iteration #1 ... (err=5.353e-03)
    Newton iteration #2 ... (err=1.949e-07)
    Newton iteration #3 ... (err=1.026e-15)
Solving timestep 93 ...
    Newton iteration #1 ... (err=4.791e-03)
    Newton iteration #2 ... (err=1.581e-07)
    Newton iteration #3 ... (err=8.992e-16)
Solving timestep 94 ...
    Newton iteration #1 ... (err=4.288e-03)
    Newton iteration #2 ... (err=1.260e-07)
    Newton iteration #3 ... (err=9.977e-16)
Solving timestep 95 ...
    Newton iteration #1 ... (err=3.856e-03)
    Newton iteration #2 ... (err=9.918e-08)
    Newton iteration #3 ... (err=1.110e-15)
Solving timestep 96 ...
    Newton iteration #1 ... (err=3.501e-03)
    Newton iteration #2 ... (err=7.744e-08)
    Newton iteration #3 ... (err=1.192e-15)
Solving timestep 97 ...
    Newton iteration #1 ... (err=3.227e-03)
    Newton iteration #2 ... (err=6.027e-08)
    Newton iteration #3 ... (err=1.120e-15)
Solving timestep 98 ...
    Newton iteration #1 ... (err=3.027e-03)
    Newton iteration #2 ... (err=4.697e-08)
    Newton iteration #3 ... (err=8.645e-16)
Solving timestep 99 ...
    Newton iteration #1 ... (err=2.889e-03)
    Newton iteration #2 ... (err=3.683e-08)
    Newton iteration #3 ... (err=8.474e-16)
Solving timestep 100 ...
    Newton iteration #1 ... (err=2.798e-03)
    Newton iteration #2 ... (err=2.931e-08)
    Newton iteration #3 ... (err=1.360e-15)
Solving timestep 101 ...
    Newton iteration #1 ... (err=2.736e-03)
    Newton iteration #2 ... (err=2.406e-08)
    Newton iteration #3 ... (err=9.369e-16)
Solving timestep 102 ...
    Newton iteration #1 ... (err=2.688e-03)
    Newton iteration #2 ... (err=2.085e-08)
    Newton iteration #3 ... (err=6.995e-16)
Solving timestep 103 ...
    Newton iteration #1 ... (err=2.642e-03)
    Newton iteration #2 ... (err=1.932e-08)
    Newton iteration #3 ... (err=8.737e-16)
Solving timestep 104 ...
    Newton iteration #1 ... (err=2.590e-03)
    Newton iteration #2 ... (err=1.889e-08)
    Newton iteration #3 ... (err=1.093e-15)
Solving timestep 105 ...
    Newton iteration #1 ... (err=2.528e-03)
    Newton iteration #2 ... (err=1.893e-08)
    Newton iteration #3 ... (err=1.242e-15)
Solving timestep 106 ...
    Newton iteration #1 ... (err=2.452e-03)
    Newton iteration #2 ... (err=1.897e-08)
    Newton iteration #3 ... (err=1.042e-15)
Solving timestep 107 ...
    Newton iteration #1 ... (err=2.363e-03)

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    Newton iteration #2 ... (err=1.875e-08)
    Newton iteration #3 ... (err=8.506e-16)
Solving timestep 108 ...
    Newton iteration #1 ... (err=2.261e-03)
    Newton iteration #2 ... (err=1.815e-08)
    Newton iteration #3 ... (err=1.006e-15)
Solving timestep 109 ...
    Newton iteration #1 ... (err=2.149e-03)
    Newton iteration #2 ... (err=1.719e-08)
    Newton iteration #3 ... (err=7.962e-16)
Solving timestep 110 ...
    Newton iteration #1 ... (err=2.027e-03)
    Newton iteration #2 ... (err=1.593e-08)
    Newton iteration #3 ... (err=7.786e-16)
Solving timestep 111 ...
    Newton iteration #1 ... (err=1.900e-03)
    Newton iteration #2 ... (err=1.444e-08)
    Newton iteration #3 ... (err=9.482e-16)
Solving timestep 112 ...
    Newton iteration #1 ... (err=1.769e-03)
    Newton iteration #2 ... (err=1.283e-08)
    Newton iteration #3 ... (err=1.061e-15)
Solving timestep 113 ...
    Newton iteration #1 ... (err=1.637e-03)
    Newton iteration #2 ... (err=1.119e-08)
    Newton iteration #3 ... (err=7.996e-16)
Solving timestep 114 ...
    Newton iteration #1 ... (err=1.507e-03)
    Newton iteration #2 ... (err=9.583e-09)
    Newton iteration #3 ... (err=1.016e-15)
Solving timestep 115 ...
    Newton iteration #1 ... (err=1.380e-03)
    Newton iteration #2 ... (err=8.077e-09)
    Newton iteration #3 ... (err=1.053e-15)
Solving timestep 116 ...
    Newton iteration #1 ... (err=1.258e-03)
    Newton iteration #2 ... (err=6.709e-09)
    Newton iteration #3 ... (err=1.198e-15)
Solving timestep 117 ...
    Newton iteration #1 ... (err=1.143e-03)
    Newton iteration #2 ... (err=5.503e-09)
    Newton iteration #3 ... (err=8.845e-16)
Solving timestep 118 ...
    Newton iteration #1 ... (err=1.036e-03)
    Newton iteration #2 ... (err=4.465e-09)
    Newton iteration #3 ... (err=9.094e-16)
Solving timestep 119 ...
    Newton iteration #1 ... (err=9.368e-04)
    Newton iteration #2 ... (err=3.593e-09)
    Newton iteration #3 ... (err=8.410e-16)
Solving timestep 120 ...
    Newton iteration #1 ... (err=8.472e-04)
    Newton iteration #2 ... (err=2.876e-09)
    Newton iteration #3 ... (err=9.005e-16)
Solving timestep 121 ...
    Newton iteration #1 ... (err=7.672e-04)
    Newton iteration #2 ... (err=2.297e-09)
    Newton iteration #3 ... (err=1.106e-15)
Solving timestep 122 ...
    Newton iteration #1 ... (err=6.966e-04)
    Newton iteration #2 ... (err=1.835e-09)
    Newton iteration #3 ... (err=1.209e-15)
Solving timestep 123 ...
    Newton iteration #1 ... (err=6.354e-04)
    Newton iteration #2 ... (err=1.471e-09)
    Newton iteration #3 ... (err=1.291e-15)
Solving timestep 124 ...
    Newton iteration #1 ... (err=5.829e-04)
    Newton iteration #2 ... (err=1.186e-09)
    Newton iteration #3 ... (err=6.786e-16)
Solving timestep 125 ...
    Newton iteration #1 ... (err=5.385e-04)
    Newton iteration #2 ... (err=9.634e-10)
    Newton iteration #3 ... (err=1.118e-15)
Solving timestep 126 ...
    Newton iteration #1 ... (err=5.014e-04)

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    Newton iteration #2 ... (err=7.893e-10)
    Newton iteration #3 ... (err=8.579e-16)
Solving timestep 127 ...
    Newton iteration #1 ... (err=4.706e-04)
    Newton iteration #2 ... (err=6.533e-10)
    Newton iteration #3 ... (err=9.321e-16)
Solving timestep 128 ...
    Newton iteration #1 ... (err=4.452e-04)
    Newton iteration #2 ... (err=5.474e-10)
    Newton iteration #3 ... (err=1.011e-15)
Solving timestep 129 ...
    Newton iteration #1 ... (err=4.241e-04)
    Newton iteration #2 ... (err=4.654e-10)
    Newton iteration #3 ... (err=1.155e-15)
Solving timestep 130 ...
    Newton iteration #1 ... (err=4.064e-04)
    Newton iteration #2 ... (err=4.025e-10)
    Newton iteration #3 ... (err=1.026e-15)
Solving timestep 131 ...
    Newton iteration #1 ... (err=3.915e-04)
    Newton iteration #2 ... (err=3.548e-10)
    Newton iteration #3 ... (err=9.402e-16)
Solving timestep 132 ...
    Newton iteration #1 ... (err=3.786e-04)
    Newton iteration #2 ... (err=3.189e-10)
    Newton iteration #3 ... (err=1.394e-15)
Solving timestep 133 ...
    Newton iteration #1 ... (err=3.672e-04)
    Newton iteration #2 ... (err=2.918e-10)
    Newton iteration #3 ... (err=9.712e-16)
Solving timestep 134 ...
    Newton iteration #1 ... (err=3.569e-04)
    Newton iteration #2 ... (err=2.712e-10)
    Newton iteration #3 ... (err=1.046e-15)
Solving timestep 135 ...
    Newton iteration #1 ... (err=3.474e-04)
    Newton iteration #2 ... (err=2.552e-10)
    Newton iteration #3 ... (err=1.091e-15)
Solving timestep 136 ...
    Newton iteration #1 ... (err=3.384e-04)
    Newton iteration #2 ... (err=2.422e-10)
    Newton iteration #3 ... (err=1.096e-15)
Solving timestep 137 ...
    Newton iteration #1 ... (err=3.298e-04)
    Newton iteration #2 ... (err=2.314e-10)
    Newton iteration #3 ... (err=1.172e-15)
Solving timestep 138 ...
    Newton iteration #1 ... (err=3.214e-04)
    Newton iteration #2 ... (err=2.219e-10)
    Newton iteration #3 ... (err=9.943e-16)
Solving timestep 139 ...
    Newton iteration #1 ... (err=3.132e-04)
    Newton iteration #2 ... (err=2.133e-10)
    Newton iteration #3 ... (err=7.761e-16)
Solving timestep 140 ...
    Newton iteration #1 ... (err=3.050e-04)
    Newton iteration #2 ... (err=2.053e-10)
    Newton iteration #3 ... (err=8.653e-16)
Solving timestep 141 ...
    Newton iteration #1 ... (err=2.969e-04)
    Newton iteration #2 ... (err=1.976e-10)
    Newton iteration #3 ... (err=9.726e-16)
Solving timestep 142 ...
    Newton iteration #1 ... (err=2.887e-04)
    Newton iteration #2 ... (err=1.901e-10)
    Newton iteration #3 ... (err=1.202e-15)
Solving timestep 143 ...
    Newton iteration #1 ... (err=2.806e-04)
    Newton iteration #2 ... (err=1.826e-10)
    Newton iteration #3 ... (err=9.846e-16)
Solving timestep 144 ...
    Newton iteration #1 ... (err=2.724e-04)
    Newton iteration #2 ... (err=1.752e-10)
    Newton iteration #3 ... (err=7.946e-16)
Solving timestep 145 ...
    Newton iteration #1 ... (err=2.642e-04)

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    Newton iteration #2 ... (err=1.676e-10)
    Newton iteration #3 ... (err=8.778e-16)
Solving timestep 146 ...
    Newton iteration #1 ... (err=2.559e-04)
    Newton iteration #2 ... (err=1.600e-10)
    Newton iteration #3 ... (err=7.902e-16)
Solving timestep 147 ...
    Newton iteration #1 ... (err=2.477e-04)
    Newton iteration #2 ... (err=1.523e-10)
    Newton iteration #3 ... (err=9.553e-16)
Solving timestep 148 ...
    Newton iteration #1 ... (err=2.395e-04)
    Newton iteration #2 ... (err=1.445e-10)
    Newton iteration #3 ... (err=9.574e-16)
Solving timestep 149 ...
    Newton iteration #1 ... (err=2.313e-04)
    Newton iteration #2 ... (err=1.367e-10)
    Newton iteration #3 ... (err=9.319e-16)
Solving timestep 150 ...
    Newton iteration #1 ... (err=2.231e-04)
    Newton iteration #2 ... (err=1.289e-10)
    Newton iteration #3 ... (err=1.262e-15)
Solving timestep 151 ...
    Newton iteration #1 ... (err=2.151e-04)
    Newton iteration #2 ... (err=1.212e-10)
    Newton iteration #3 ... (err=1.055e-15)
Solving timestep 152 ...
    Newton iteration #1 ... (err=2.071e-04)
    Newton iteration #2 ... (err=1.136e-10)
    Newton iteration #3 ... (err=1.095e-15)
Solving timestep 153 ...
    Newton iteration #1 ... (err=1.993e-04)
    Newton iteration #2 ... (err=1.061e-10)
    Newton iteration #3 ... (err=1.337e-15)
Solving timestep 154 ...
    Newton iteration #1 ... (err=1.915e-04)
    Newton iteration #2 ... (err=9.888e-11)
    Newton iteration #3 ... (err=1.117e-15)
Solving timestep 155 ...
    Newton iteration #1 ... (err=1.840e-04)
    Newton iteration #2 ... (err=9.190e-11)
    Newton iteration #3 ... (err=1.280e-15)
Solving timestep 156 ...
    Newton iteration #1 ... (err=1.766e-04)
    Newton iteration #2 ... (err=8.520e-11)
    Newton iteration #3 ... (err=9.097e-16)
Solving timestep 157 ...
    Newton iteration #1 ... (err=1.695e-04)
    Newton iteration #2 ... (err=7.882e-11)
    Newton iteration #3 ... (err=1.209e-15)
Solving timestep 158 ...
    Newton iteration #1 ... (err=1.625e-04)
    Newton iteration #2 ... (err=7.277e-11)
    Newton iteration #3 ... (err=1.037e-15)
Solving timestep 159 ...
    Newton iteration #1 ... (err=1.558e-04)
    Newton iteration #2 ... (err=6.707e-11)
    Newton iteration #3 ... (err=9.900e-16)
Solving timestep 160 ...
    Newton iteration #1 ... (err=1.492e-04)
    Newton iteration #2 ... (err=6.171e-11)
    Newton iteration #3 ... (err=1.035e-15)

```

[2]:

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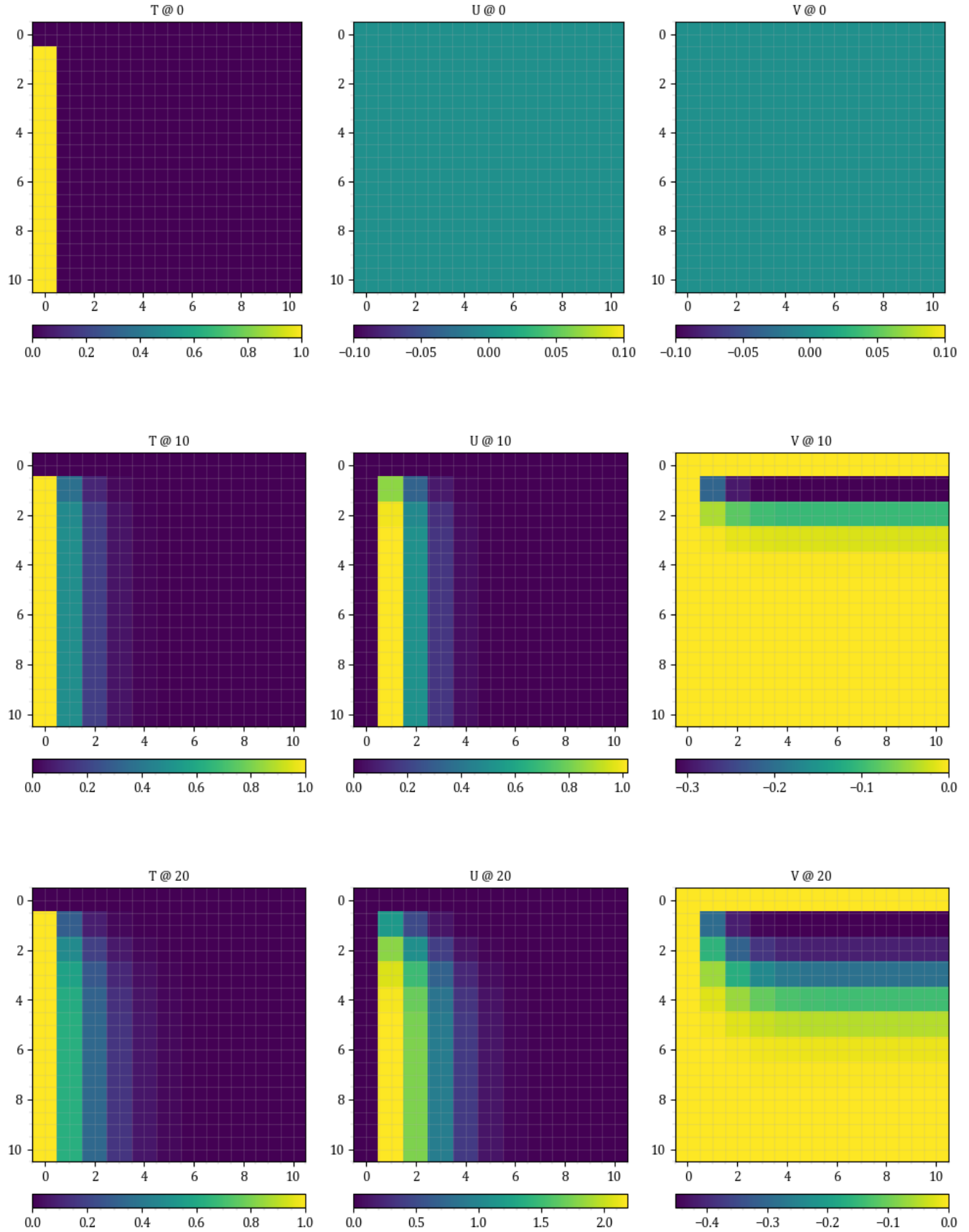
#####
for n in range(0,Nt,10):
    fig, [ax1,ax2,ax3] = plt.subplots(1,3, figsize=[10,5])
    pcm=ax1.imshow( Tnij[n,:,:] )
    cb1 = fig.colorbar(pcm, ax=ax1, location='bottom', pad=.07)
    pcm=ax2.imshow( Unij[n,:,:] )
    cb2 = fig.colorbar(pcm, ax=ax2, location='bottom', pad=.07)
    pcm=ax3.imshow( Vnij[n,:,:] )
    cb3 = fig.colorbar(pcm, ax=ax3, location='bottom', pad=.07)

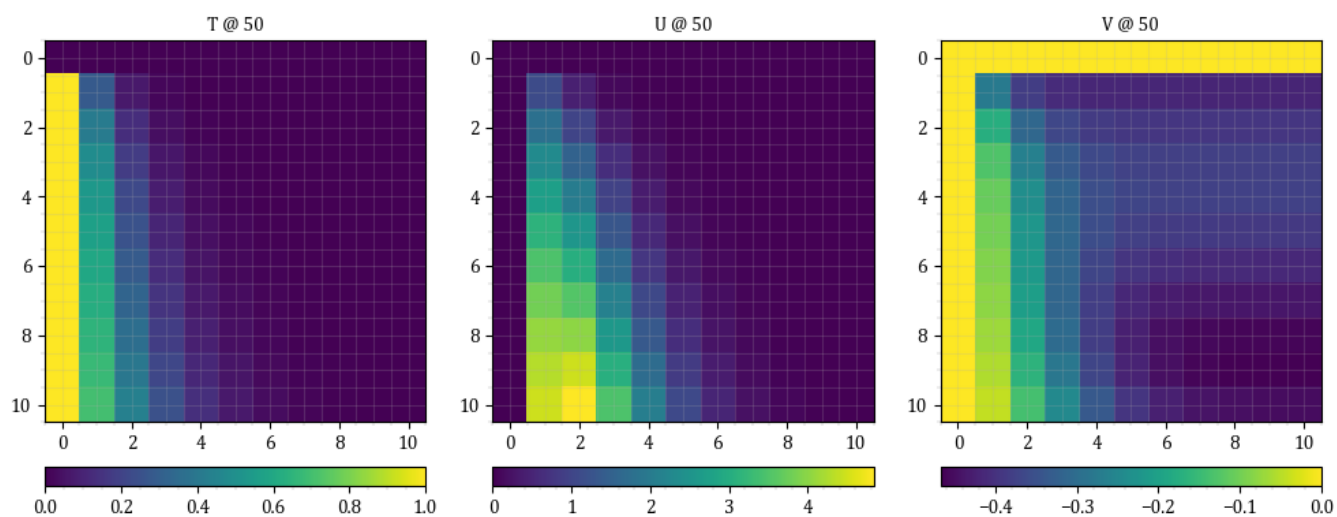
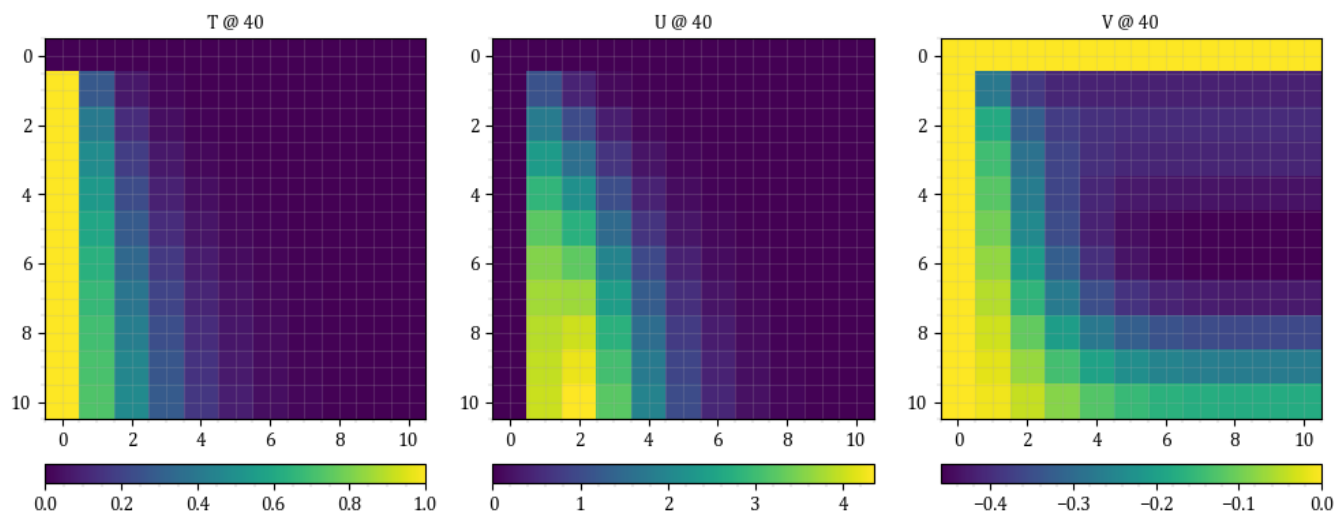
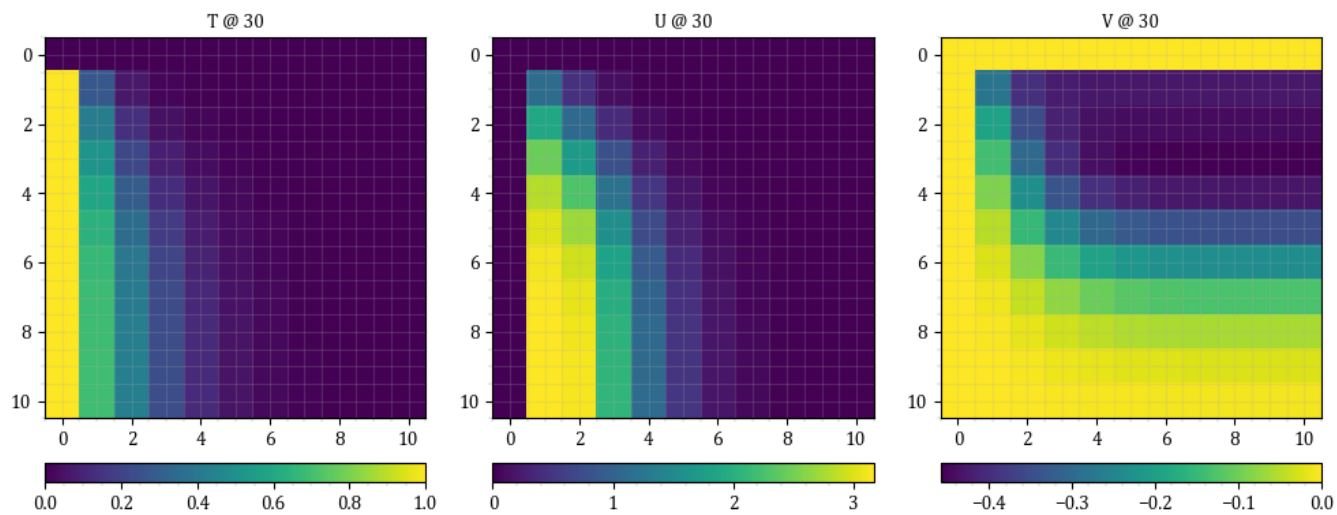
    ax1.set_title(f"T @ {n}")

```

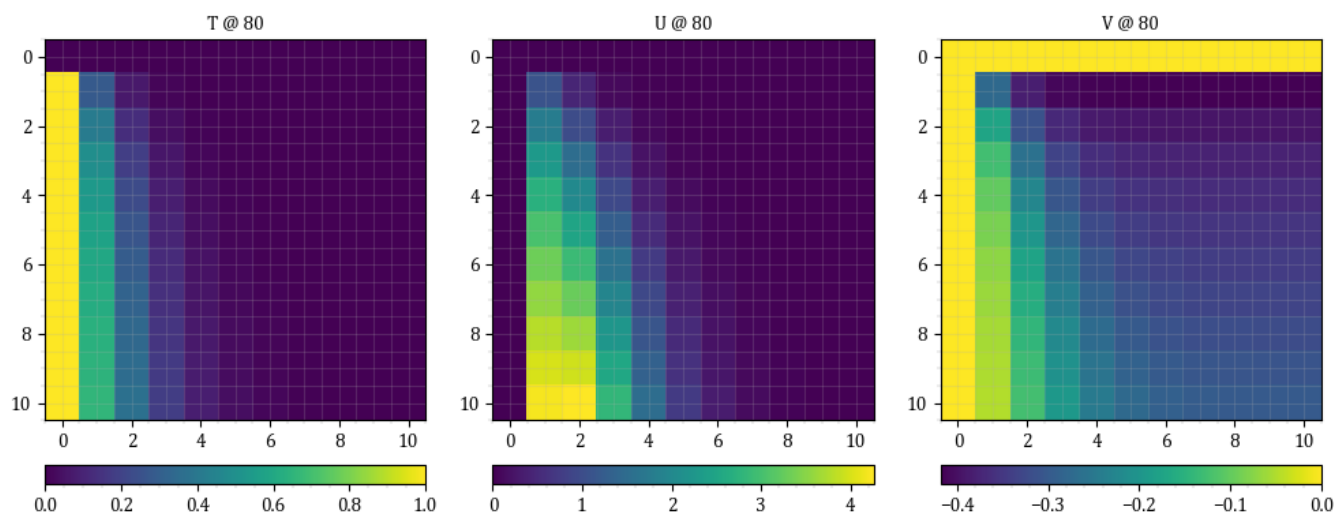
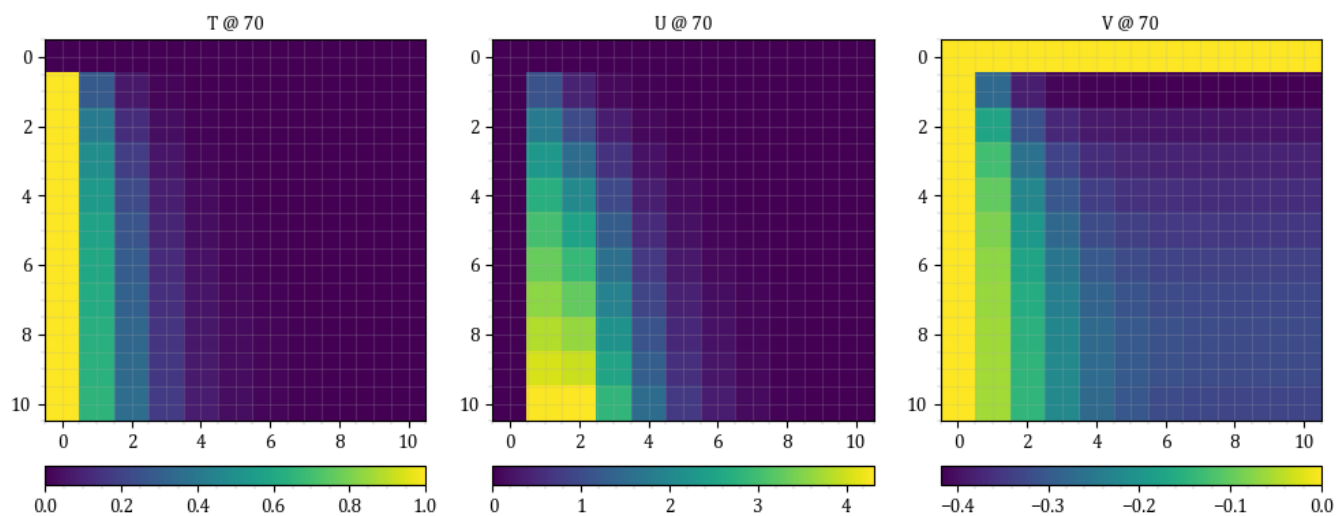
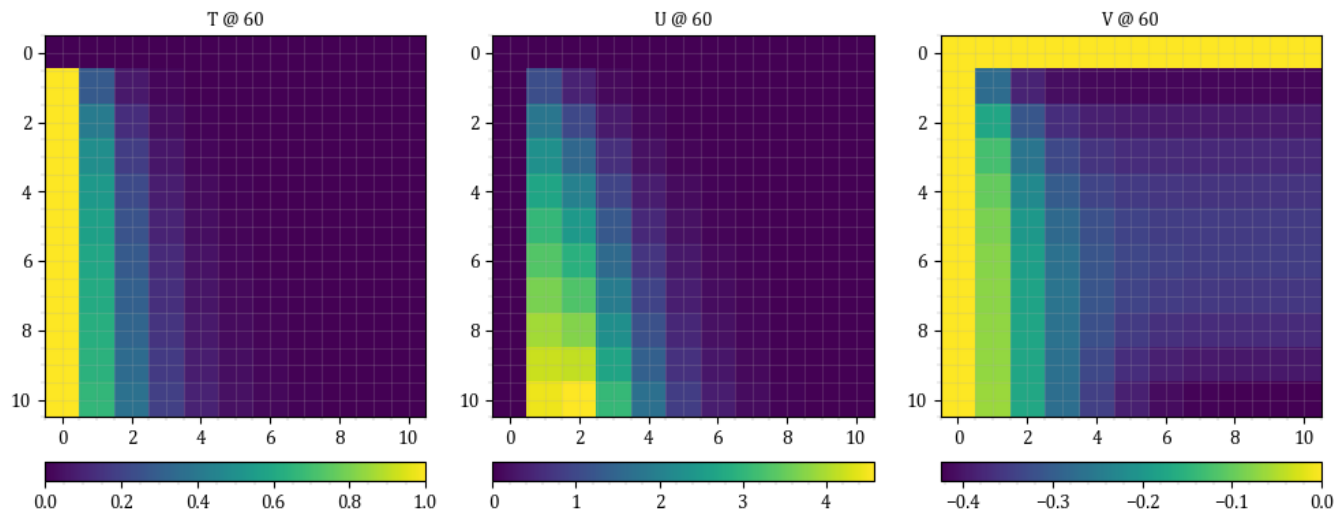
```
ax2.set_title(f"U @ {n}")
ax3.set_title(f"V @ {n}")

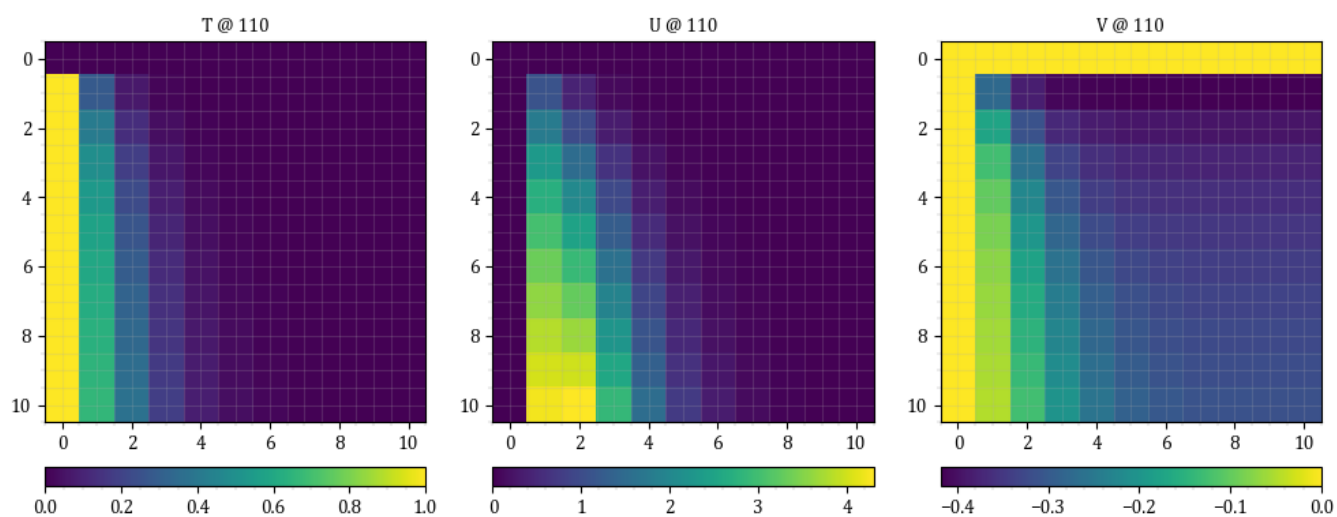
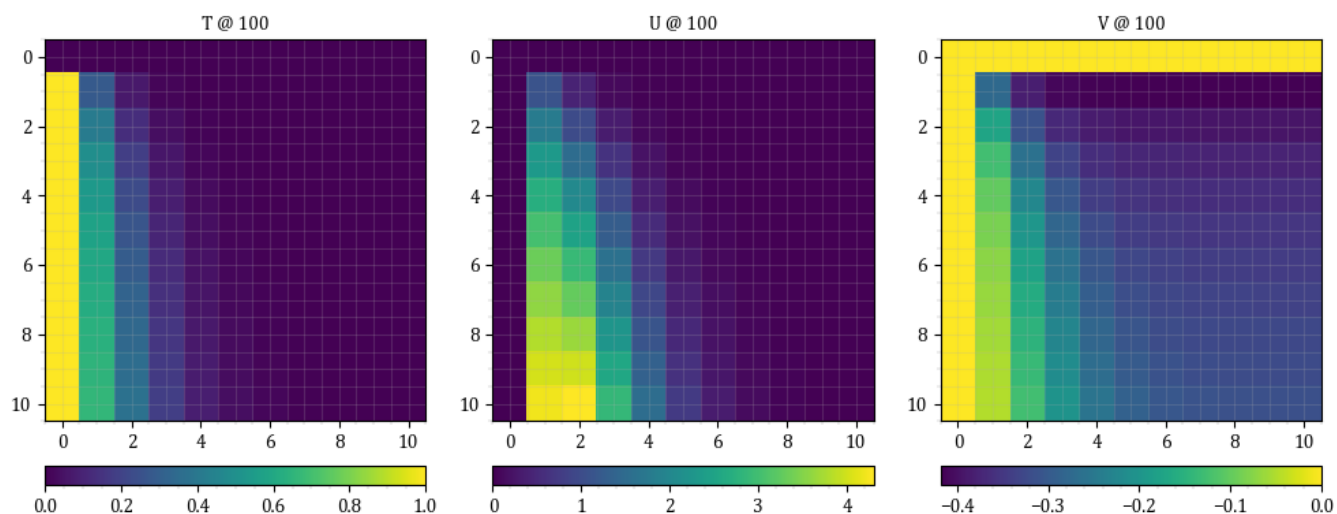
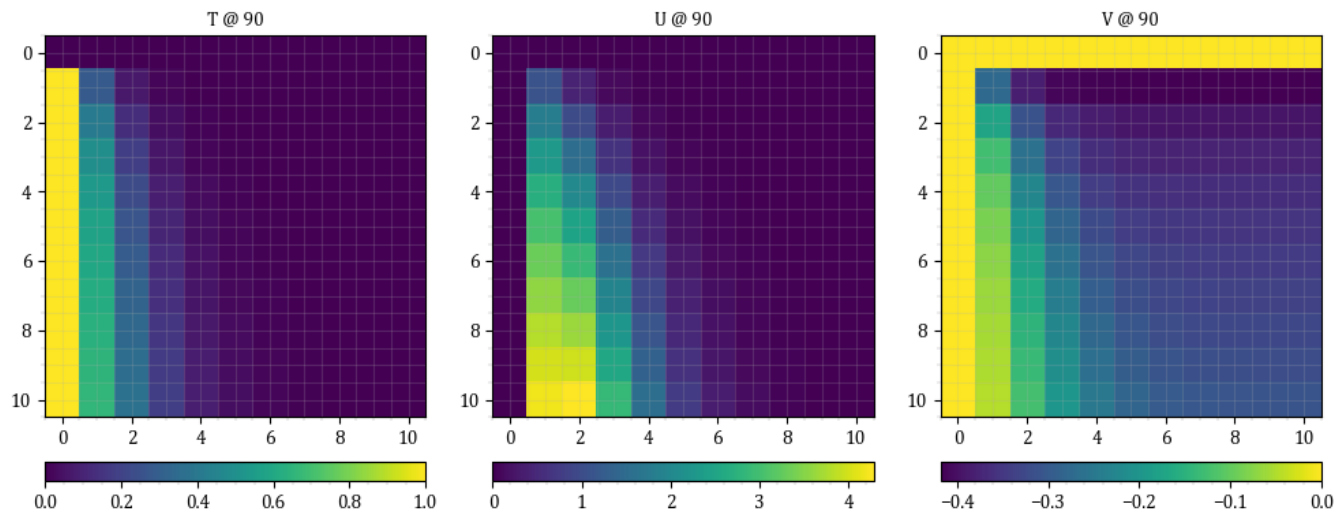
#cb1.mappable.set_clim(1E2,1E9)
fig.tight_layout()
```

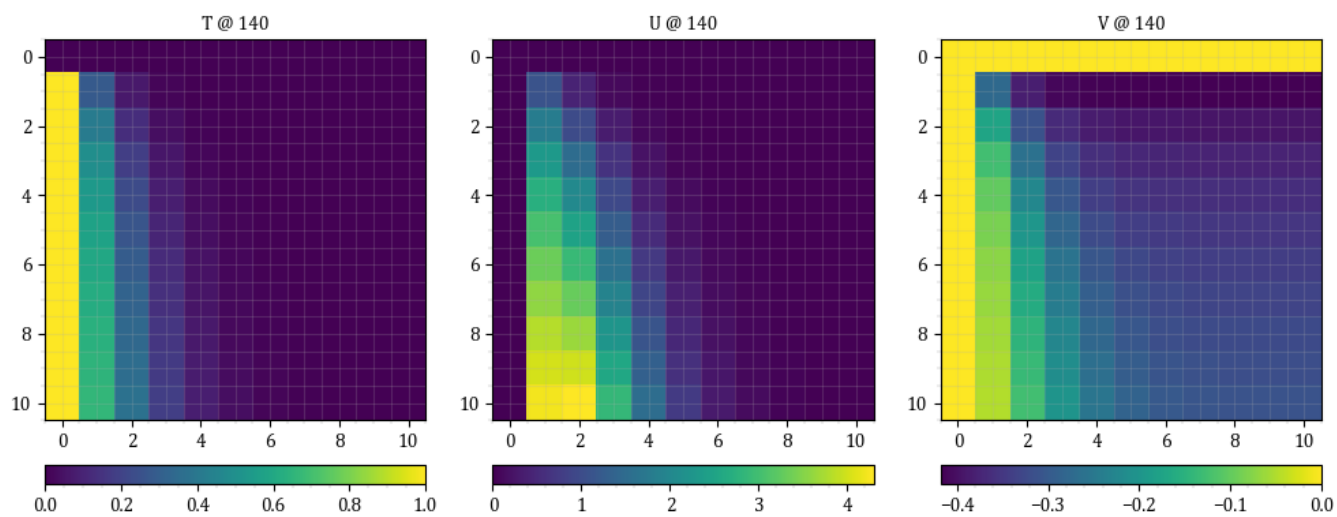
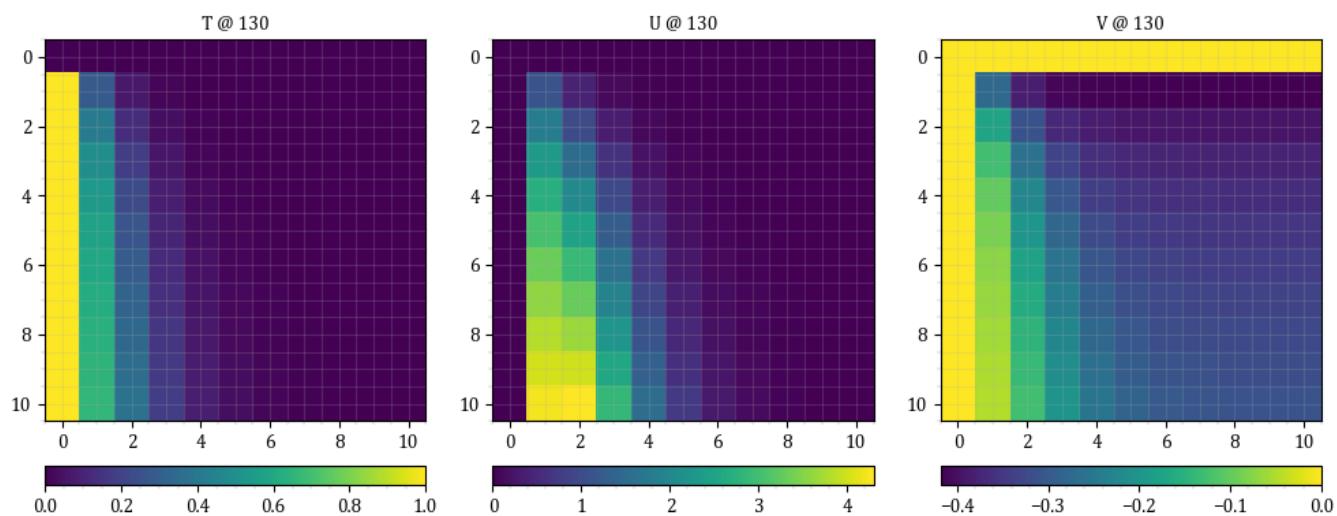
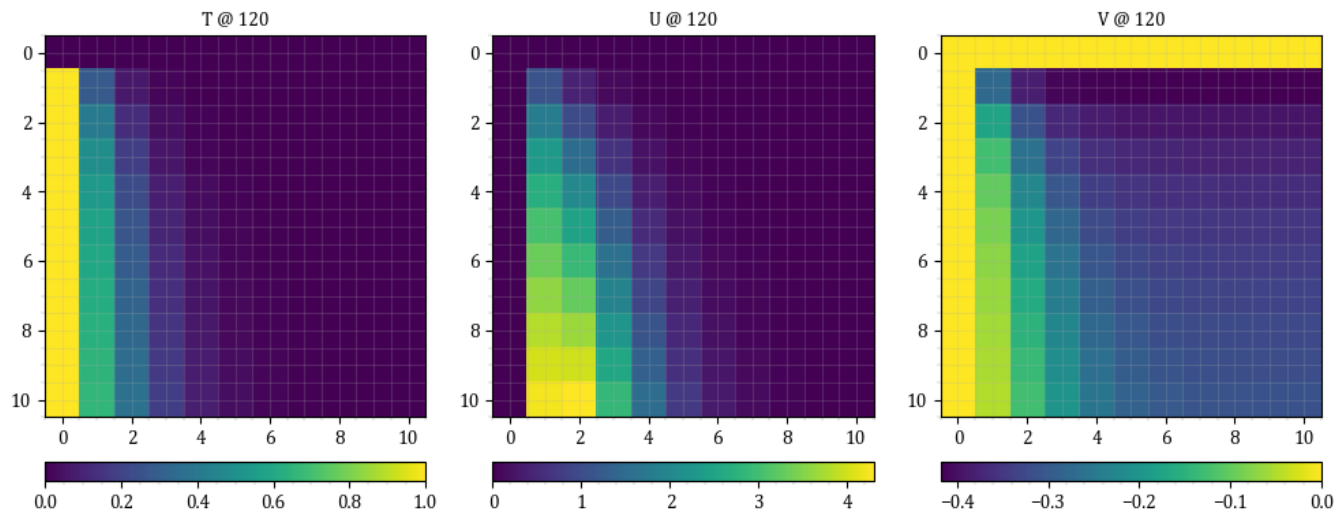


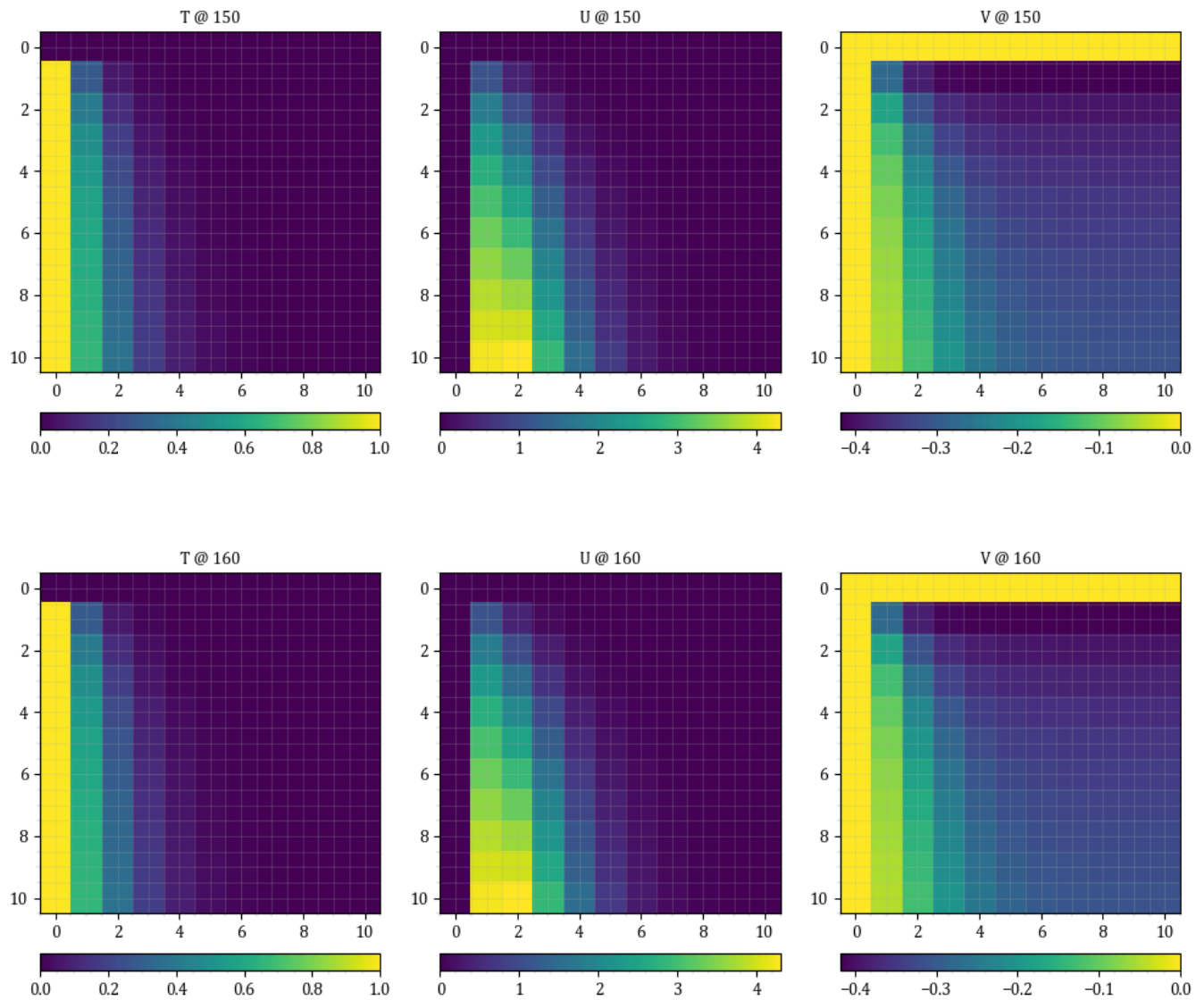












## EXPLICIT SOLUTION

```
[3]: #%%
from math import pi, sin, cos, exp
import numpy as np
from numpy import linspace, zeros, arange
from numpy import ix_ as ix
np.set_printoptions(threshold=10000, linewidth=10000)
from numpy import exp, linspace, vectorize
import matplotlib.pyplot as plt
plt.style.use('paper.mplstyle')

XMAX = 100
YMAX = 25
dx = XMAX/10 ; dy = YMAX/10
Tf = 80 ; Nt=160
dt = Tf/Nt ; Nt = Nt + 1
X = np.arange(0,XMAX+dx,dx) ; Ni = len(X)
Y = np.arange(0,YMAX+dy,dy) ; Nj = len(Y)
Nij = Ni * Nj

beta = 0.5
Pr = 0.733

# Global index
def _(i,j) : return j + Nj*i
```

```

#
# Assign BCs to solution vectors
#
def init_bcs() :
    global Unij, Vnij, Tnij
    Unij = zeros( [Nt,Ni,Nj] )
    Vnij = zeros( [Nt,Ni,Nj] )
    Tnij = zeros( [Nt,Ni,Nj] )

    Unij[0,:,:] = 0 # ic
    Vnij[0,:,:] = 0 # ic
    Tnij[0,:,:] = 0 # ic

    Unij[:,0,:] = 0 # BC , Y=0
    Vnij[:,0,:] = 0 # BC , Y=0
    Tnij[:,0,:] = 1 # BC , Y=0

    Unij[:,Nj-1,:] = 0 # BC , Y=inf
    Tnij[:,Nj-1,:] = 0 # BC , Y=inf

    Unij[:,0,:] = 0 # BC , X=0
    Vnij[:,0,:] = 0 # BC , X=0
    Tnij[:,0,:] = 0 # BC , X=0

#
#
#
# MAIN FLOW
#
#
#
# Global solution vector
init_bcs()

for n in arange(1,Nt) :
    print(f"Solving timestep {n} ...")

    T = Tnij[n-1,:,:]
    U = Unij[n-1,:,:]
    V = Vnij[n-1,:,:]
    # Solve U
    for i in arange(1,Ni) :
        for j in arange(1,Nj-1) :
            Unij[n,i,j] = U[i,j] + dt * (
                T[i,j]
                + (U[i,j-1]-2*U[i,j]+U[i,j+1])/dy/dy
                - U[i,j]*(U[i,j]-U[i-1,j])/dx
                - V[i,j]*(U[i,j]-U[i,j-1])/dy
            )

            # T
            # Uyy
            # - U Ux
            # - U Uy

    # Solve T
    for i in arange(1,Ni) :
        for j in arange(1,Nj-1) :
            Tnij[n,i,j] = T[i,j] + dt*(
                1/Pr*(T[i,j-1]-2*T[i,j]+T[i,j+1])/dy/dy
                - U[i,j]*(T[i,j]-T[i-1,j])/dx
                - V[i,j]*(T[i,j]-T[i,j-1])/dy
            )

            # Tyy/Pr
            # -U Tx
            # -V Ty

    # Solve V
    for i in arange(1,Ni) :
        for j in arange(1,Nj) :
            Vnij[n,i,j] = Vnij[n,i,j-1] - dy/dx*( Unij[n,i,j] - Unij[n,i-1,j])

```

```

Solving timestep 1 ...
Solving timestep 2 ...
Solving timestep 3 ...
Solving timestep 4 ...
Solving timestep 5 ...
Solving timestep 6 ...
Solving timestep 7 ...
Solving timestep 8 ...

```

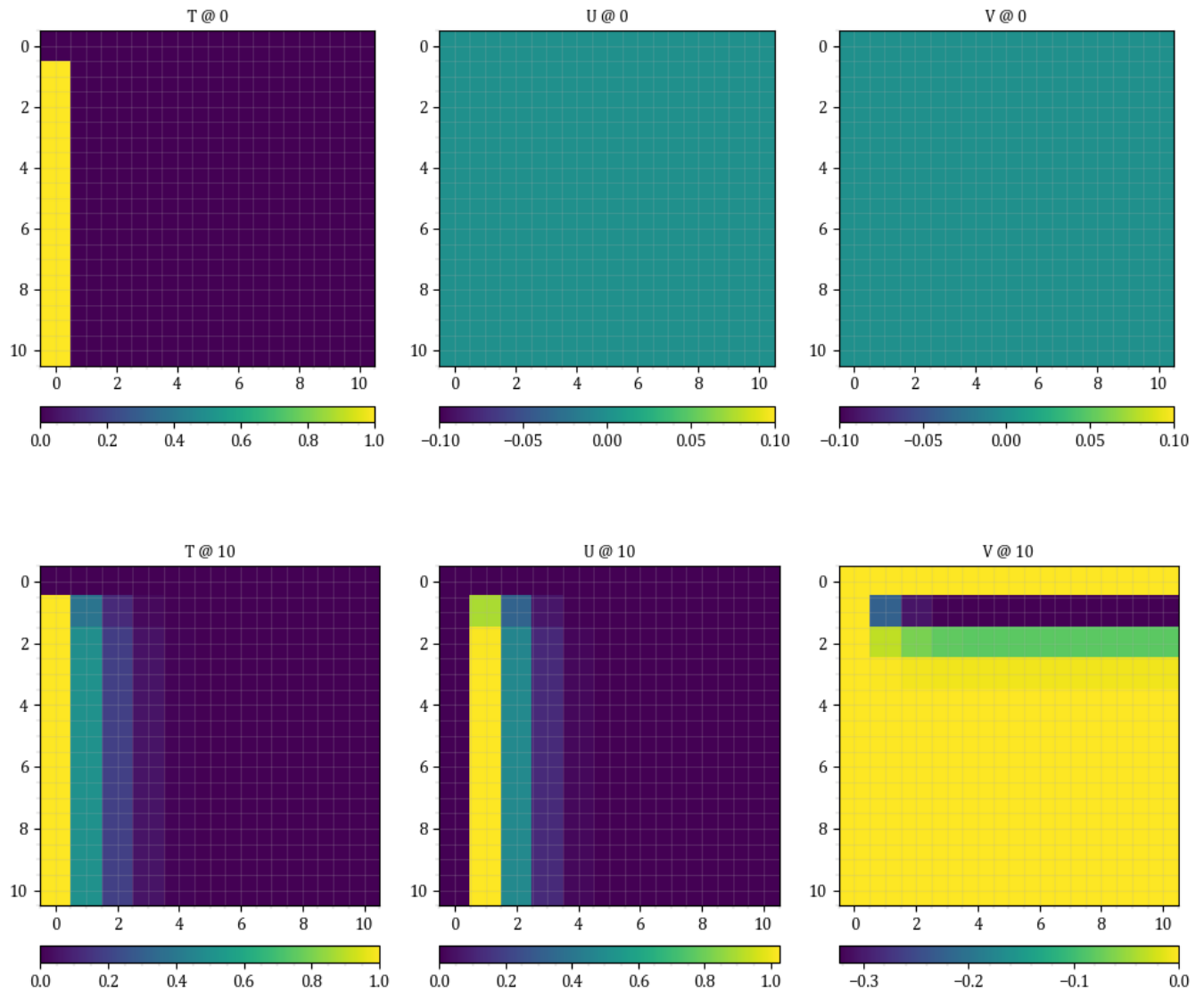
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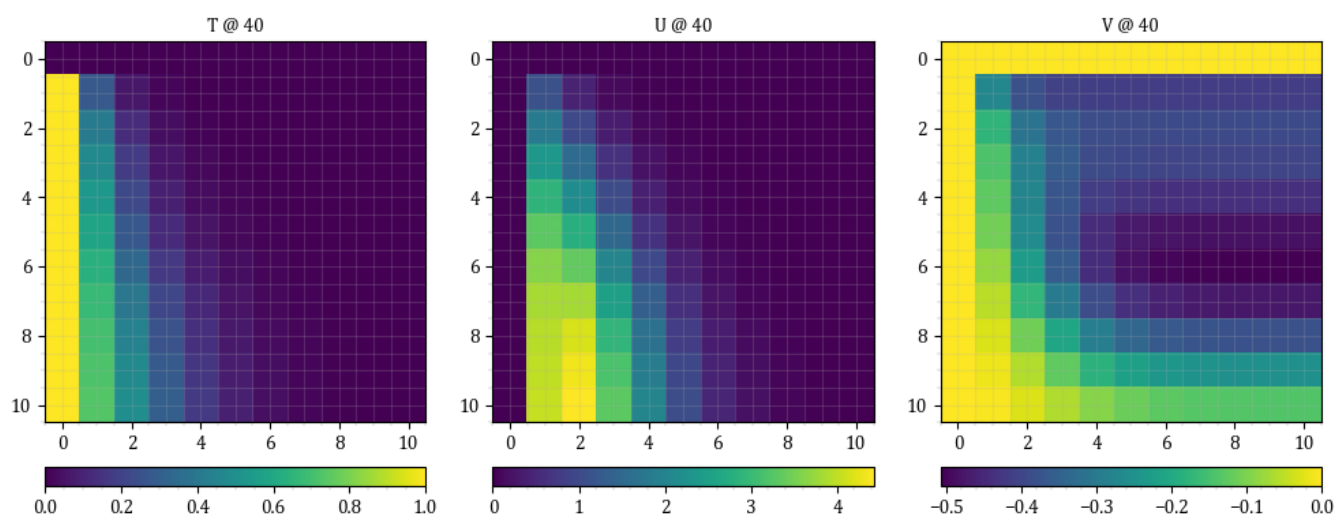
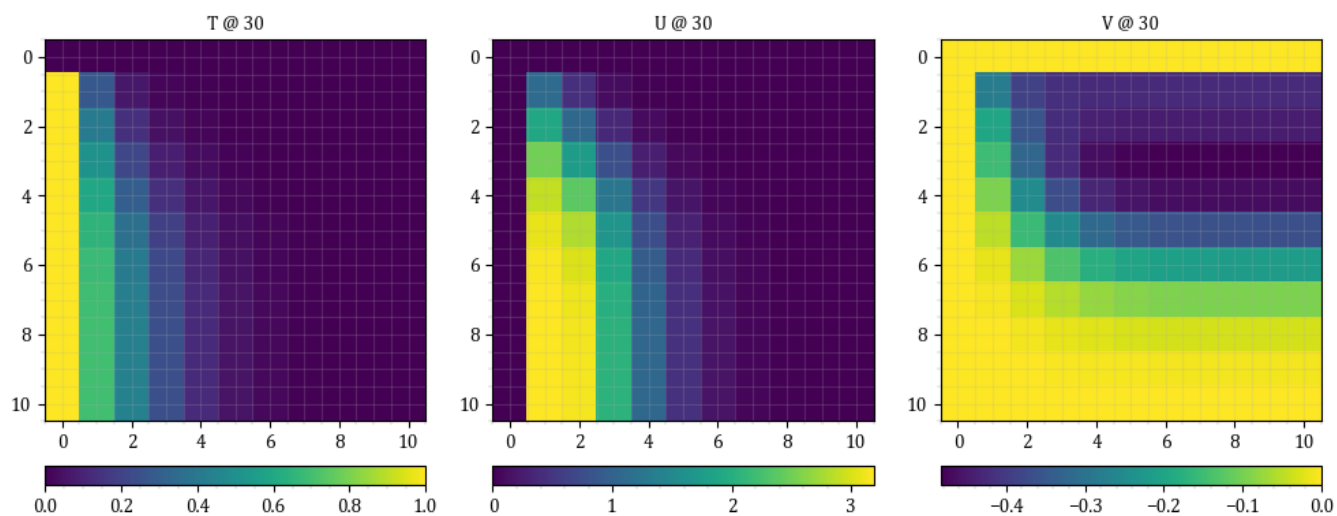
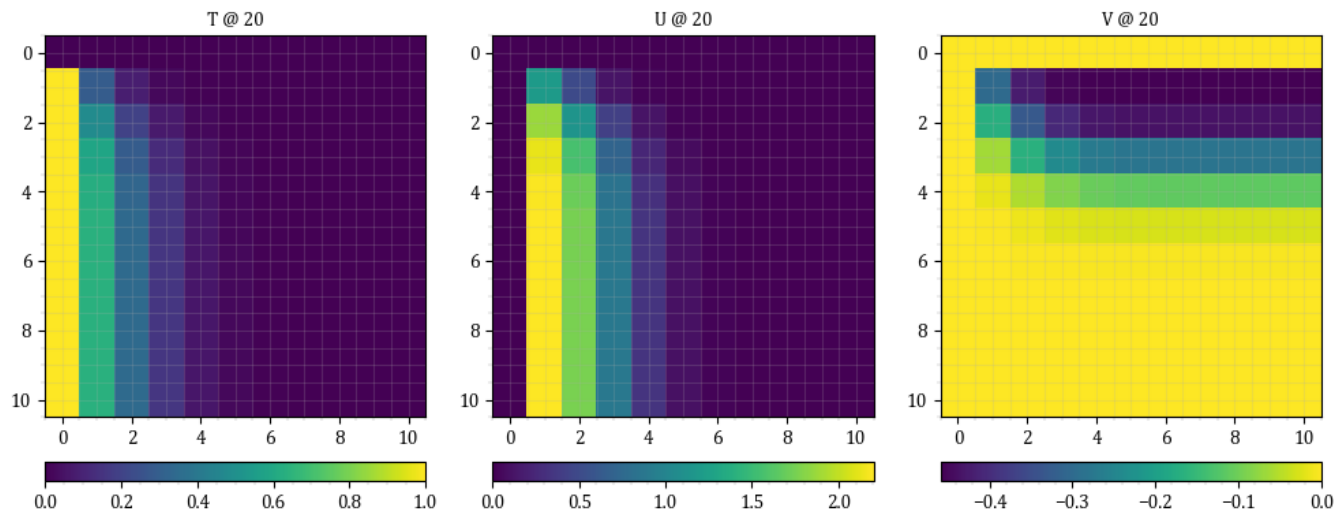
```
[4]: for n in range(0,Nt,10):
fig, [ax1,ax2,ax3] = plt.subplots(1,3, figsize=[10,5])
pcm=ax1.imshow( Tnij[n,:,:] )
cb1 = fig.colorbar(pcm, ax=ax1, location='bottom', pad=.07)
pcm=ax2.imshow( Unij[n,:,:] )
cb2 = fig.colorbar(pcm, ax=ax2, location='bottom', pad=.07)
pcm=ax3.imshow( Vnij[n,:,:] )
cb3 = fig.colorbar(pcm, ax=ax3, location='bottom', pad=.07)

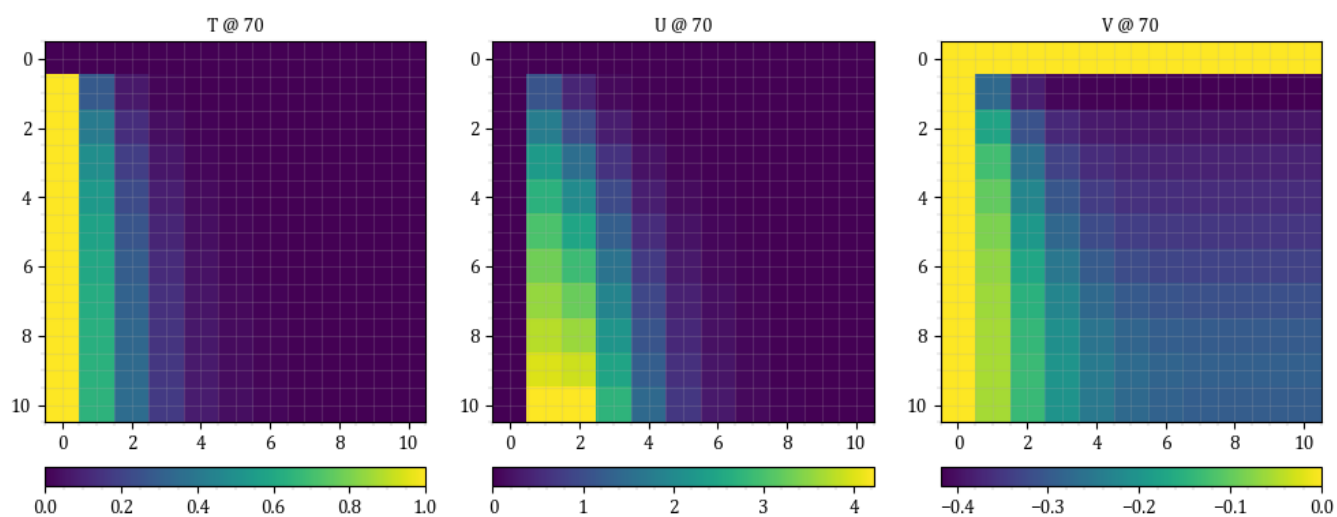
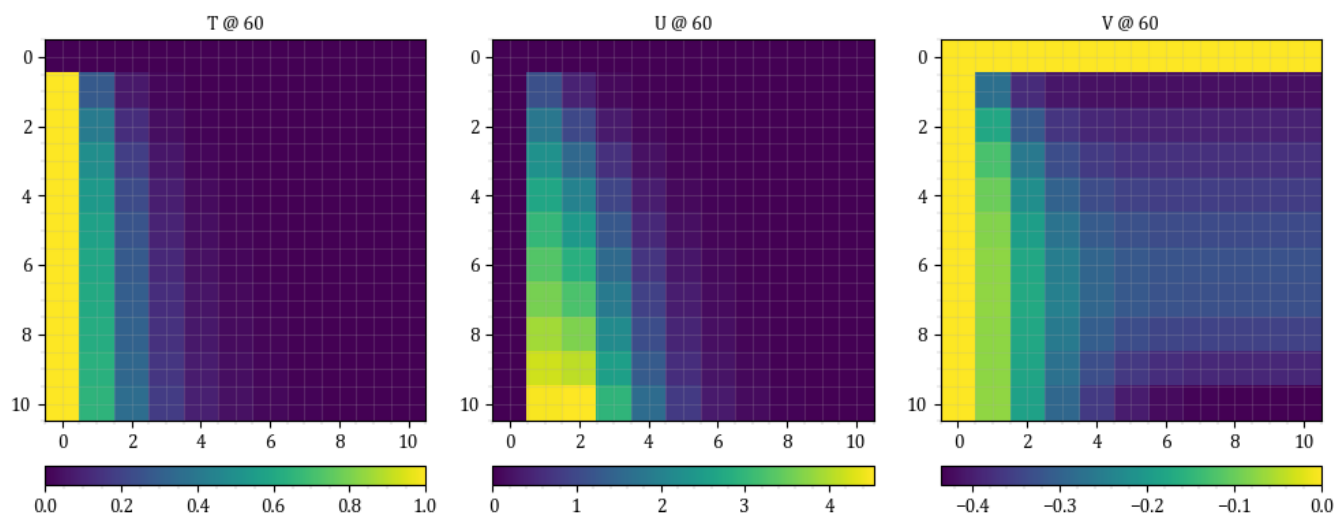
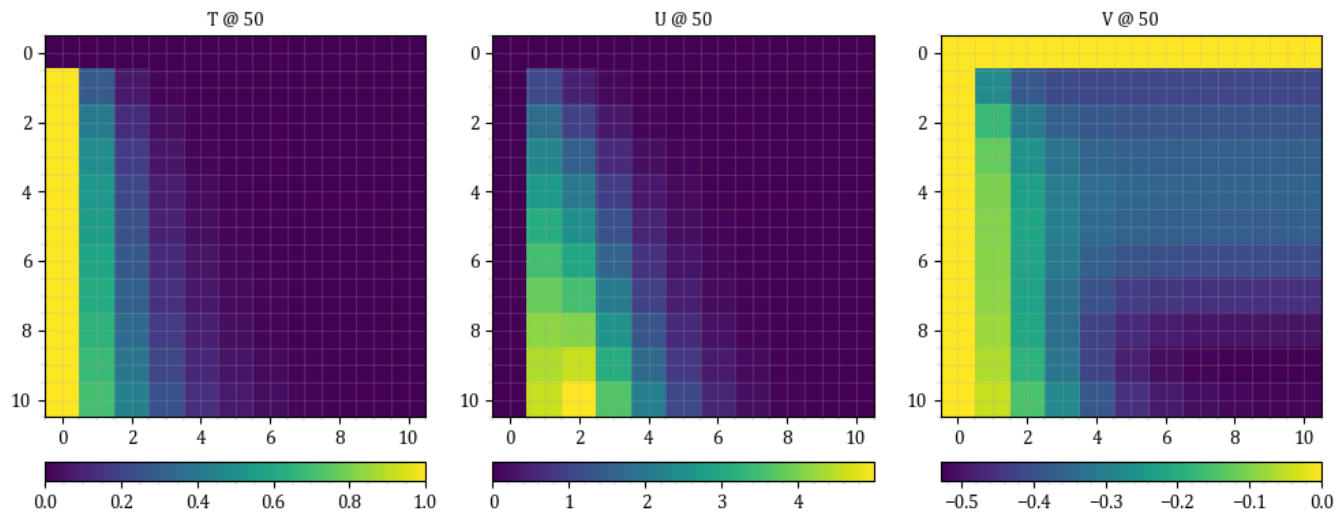
ax1.set_title(f"T @ {n}")
ax2.set_title(f"U @ {n}")
ax3.set_title(f"V @ {n}")

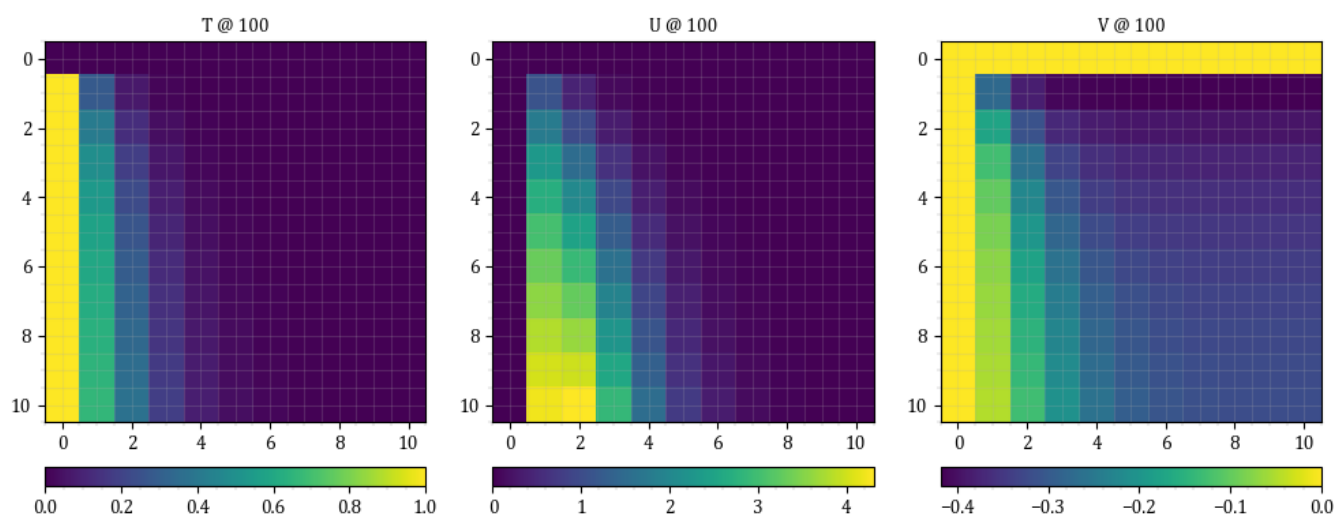
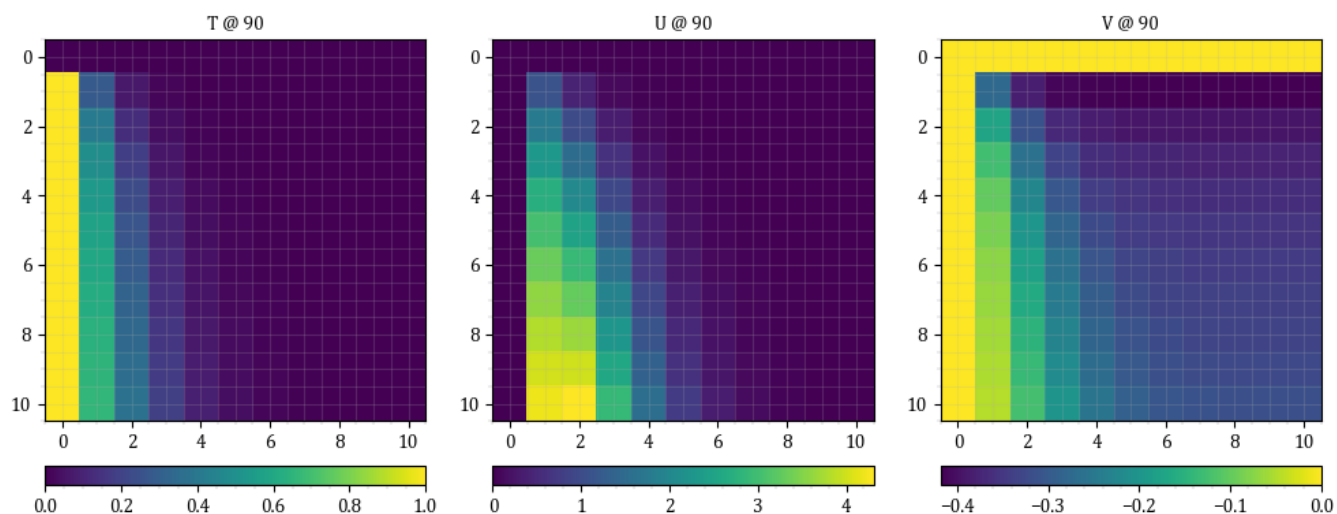
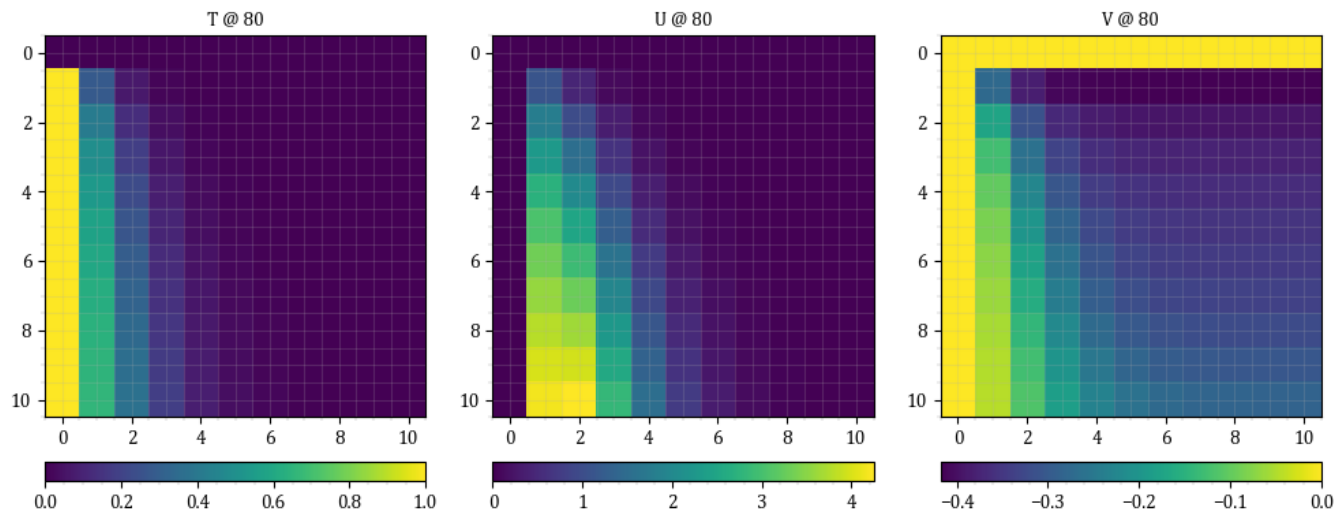
#cb1.mappable.set_clim(1E2,1E9)
fig.tight_layout()
```

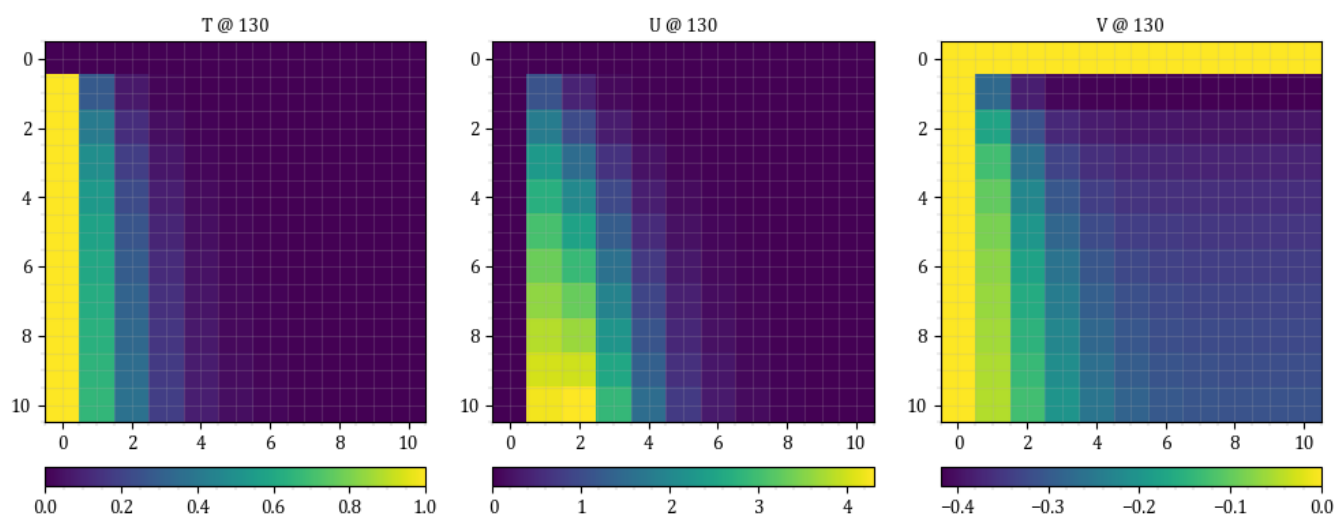
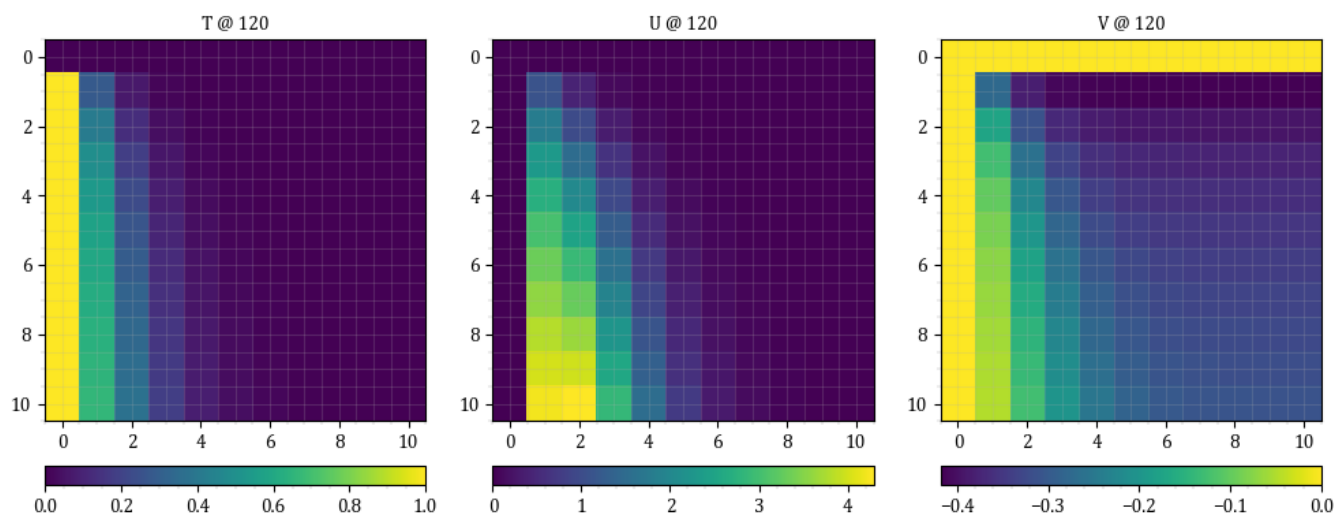
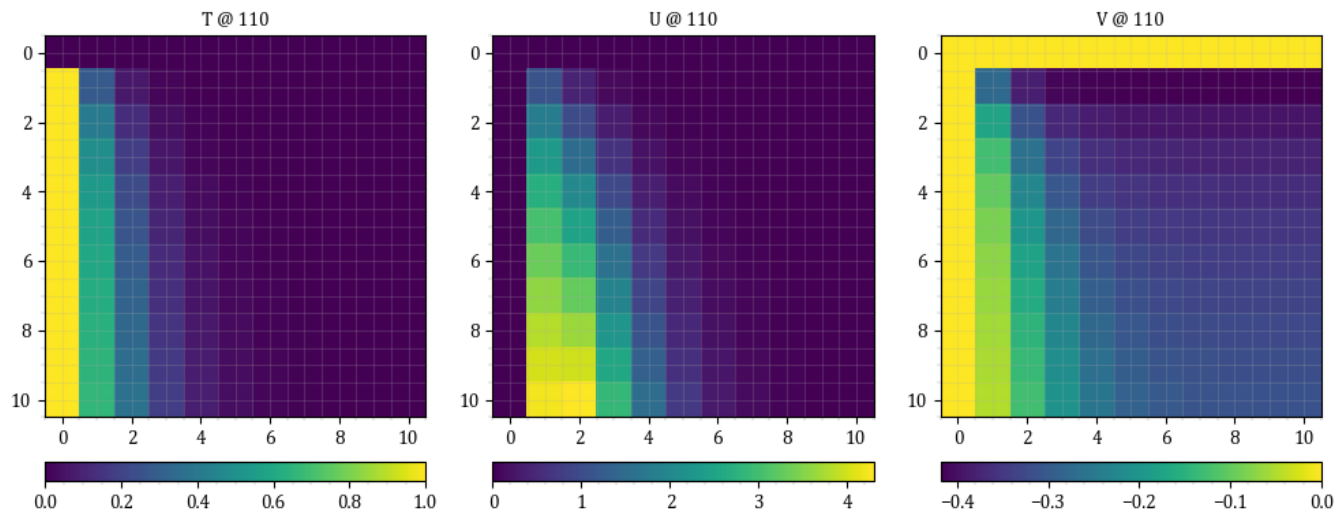


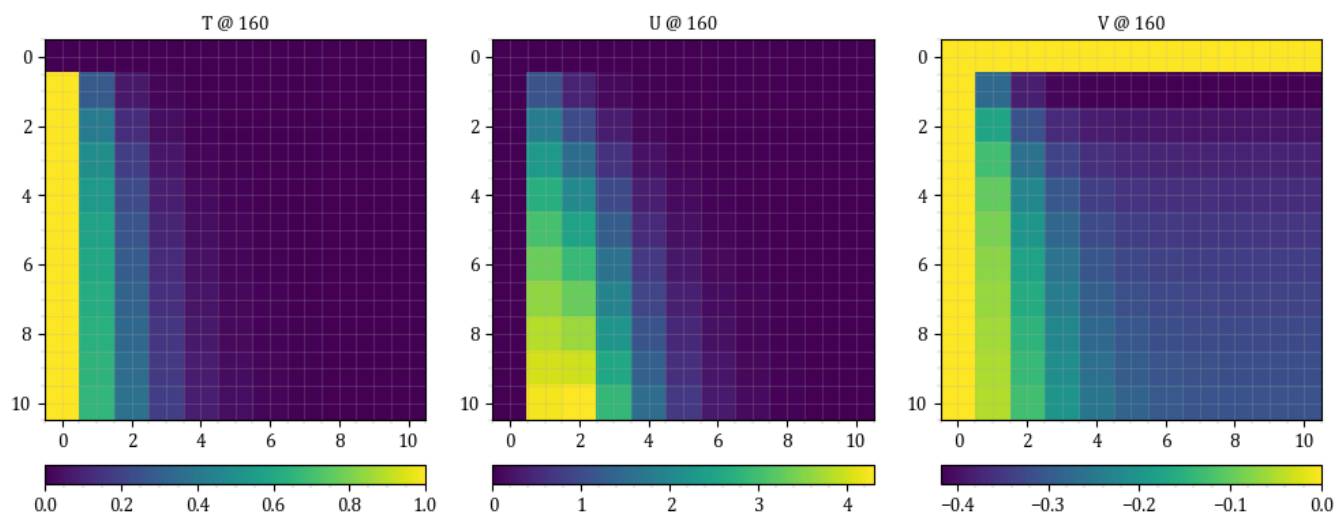
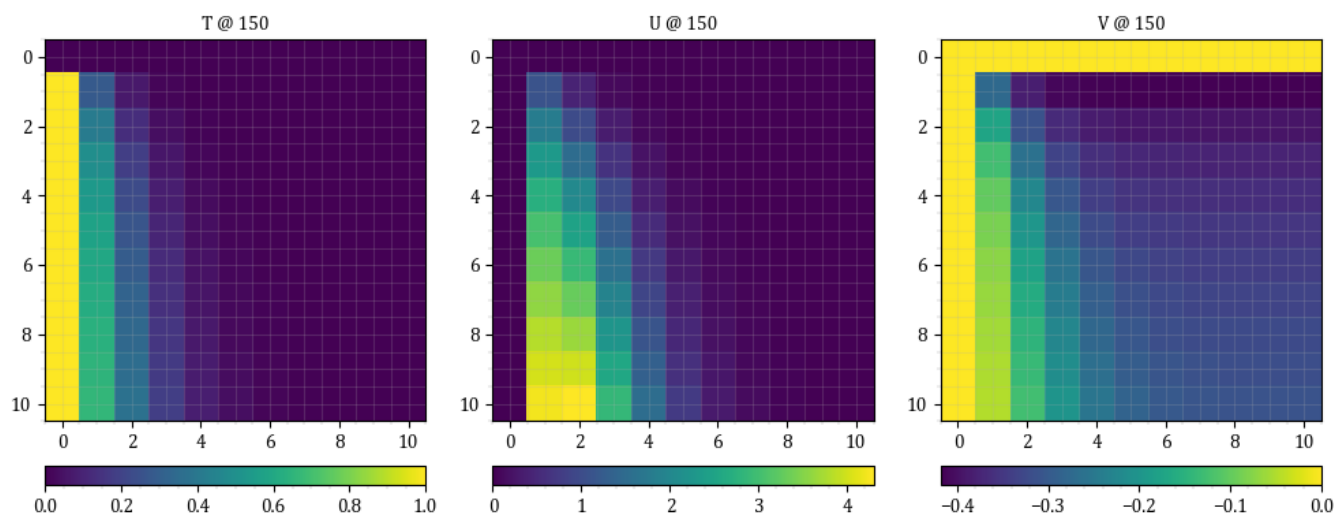
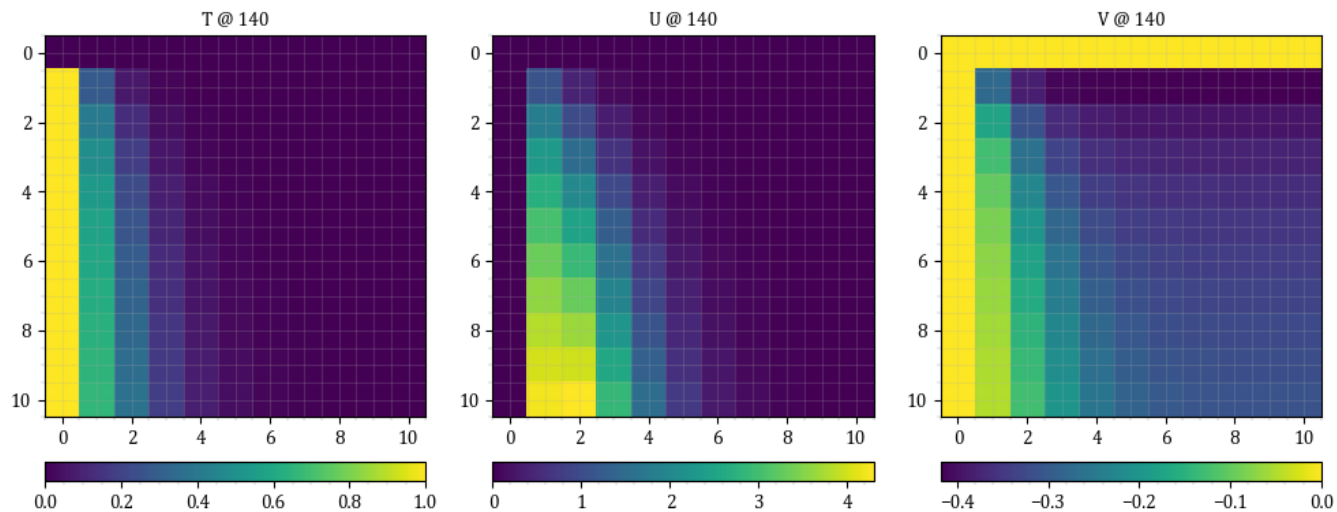












CP#8

Pendo BLU

rep 2656

①

②

③

④

⑤

$$f_i = T + \frac{\partial U}{\partial y^2} - \frac{U}{dx} - U \frac{\partial U}{\partial x} - V \frac{\partial U}{\partial y}$$

$$\textcircled{1} = (\beta) \cdot T + T^n (1-\beta)$$

$$\textcircled{2} = (\beta) \left[ \frac{U_{\ominus j} - 2U + U_{\oplus j}}{h_y^2} \right] + \left[ \frac{U_{\ominus i} - 2U + U_{\oplus i}}{h_x^2} \right]^n (1-\beta)$$

$$\textcircled{3} = \frac{U^n - U}{\Delta t}$$

$\rightarrow -\frac{\beta U^2}{h_x} + \frac{\beta U U_{\oplus i}}{h_x}$

$$\textcircled{4} = -(\beta)U \frac{U - U_{\oplus i}}{h_x} - U^n \frac{U - U_{\oplus i}}{h_x} (1-\beta)$$

$$\textcircled{5} = (\beta)V \frac{U - U_{\oplus j}}{h_y} - V^n \frac{U - U_{\oplus j}}{h_y} (1-\beta)$$

$$U \equiv U_{i,j} \quad U_{i+} = U_{i+1,j} \quad U_{i-} = U_{i-1,j}$$



$$\phi_2 = \frac{1}{Pr} \frac{\partial^2 T}{\partial y^2} - \frac{\partial T}{\partial t} - U \frac{\partial T}{\partial x} - V \frac{\partial T}{\partial y}$$

$$\textcircled{1} = (\beta) \frac{1}{Pr} \frac{T_+ - 2T + T_-}{h_y^2} + \frac{1}{2} \frac{1}{Pr} \left( \frac{T_+^n - 2T + T_-^n}{h_y^2} \right)^n$$

$$\textcircled{2} = \frac{T^n - T}{\Delta t}$$

$$\textcircled{3} = -(\beta) U \frac{T - T_-}{h_x} - \frac{1}{2} U^n \frac{T^n - T_-^n}{h_x (1 - \beta)}$$

$$\textcircled{4} = -(\beta) V \frac{T - T_-}{h_y} - \frac{1}{2} V^n \frac{T^n - T_-^n}{h_y (1 - \beta)}$$

$$\lambda_3 = \frac{\textcircled{1}}{\partial x} + \frac{\textcircled{2}}{\partial y}$$

$$\textcircled{1} = \frac{U - U_{\ominus}}{h_x} (\beta) + (1 - \beta) \frac{U^n - U_{\ominus}^n}{h_x}$$

$$\textcircled{2} = \frac{U - U_{\ominus}}{h_y} (\beta) + (1 - \beta) \frac{U^n - U_{\ominus}^n}{h_x}$$



Jacobian Block:

$$\vec{F} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix}$$

$$\vec{x} = \begin{bmatrix} u_{i,j} \\ u_{i,j-1} \\ u_{i+1,j} \\ u_{i,j-1} \\ u_{i,j+1} \\ v_{i-1,j} \\ v_{i,j} \\ v_{i+1,j} \\ v_{i,j-1} \\ v_{i,j+1} \\ T_{i,j} \\ T_{i,j-1} \\ T_{i+1,j} \\ T_{i,j-1} \\ T_{i,j+1} \end{bmatrix}$$

$$J = \frac{\partial \vec{F}}{\partial \vec{x}}$$

$$q = i + j \times I$$

$$\vec{x} =$$

$$\begin{bmatrix} e_{0j} \\ e_{1j} \\ e_{1j} \\ e_{10} \\ \vdots \end{bmatrix}$$

$$K_1 = \text{MAP AIR} \cdot (J_1)$$

$$K_2 = \text{MAP AIR} \cdot (J_2)$$

$$\vdots$$

$$K = \{ K_1, K_2, K_3 \}^T$$

$$K \delta \vec{x} = -\vec{F}$$

$$J_1 U$$

$$J_1 V$$

$$J_1 T$$

$$J_2 U$$

$$J_2 V$$

$$J_2 T$$

$$\vdots$$

$$\vdots$$

$$\vdots$$

$$\Rightarrow J_1 [e_{0j}, e_{10}] = \partial \phi / \partial e_{0j}$$