11/7/23

$$\sigma^2_{e,min} = 0.977$$

$$z^* \simeq 10\%$$

$$95\%\ CI = 10 \pm z_{95\%}\, \sigma_{e,min}$$

$$= 10 \pm 1.96 \sqrt{0.977}$$

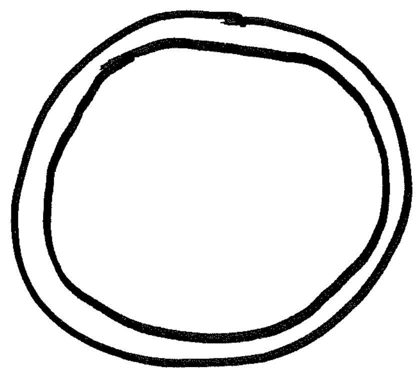
$$\text{dyne}/\text{cm} = \text{mN}/\text{m}$$

$$1\ \text{dyne} = 10^{-5}\ \text{N}$$

$$1\ \text{cm} = 10^{-2}\ \text{m}$$

Example:

$r_1 = 1\ \text{cm} \rightarrow r_2 = 2\ \text{cm}$



Soap bubble

$W = ?$

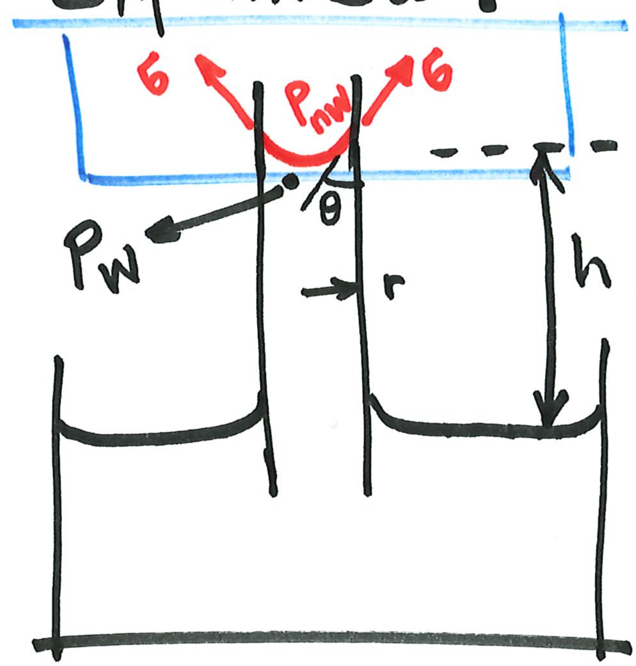
$\sigma = 25\ \text{dynes/cm}$

$$|W| = \sigma\, \Delta A$$

$$|W| = 2\left[4\pi(r_2^2 - r_1^2)\right](25) = 1884.96$$

(dynes.cm)
(ergs)

* Capillary Rise Experiment :



$$\text{Capillary Force} = \sigma (2\pi r) \cos\theta$$

$$\text{Gravitational Force} = \pi r^2 h (\rho_w - \rho_{nw}) g$$

$$\Rightarrow 2\sigma \pi r \cos\theta = \pi r^2 h (\rho_w - \rho_{nw}) g$$

$$\Rightarrow \boxed{\sigma = \frac{r h (\rho_w - \rho_{nw}) g}{2 \cos\theta}}$$

$$\underline{\underline{P_c = P_{nw} - P_w}}$$

Force due to pressure diff =

$$(P_{nw} - P_w)\,\pi r^2 \quad \downarrow$$

$$2\pi r \sigma \cos\theta = \underbrace{(P_{nw} - P_w)}_{P_c}\,\pi r^2$$

$$\Longrightarrow \boxed{P_c = \frac{2\sigma\cos\theta}{r}}$$

if $\rho_{nw} \ll \rho_w$, $\theta \simeq 0 \rightarrow \cos\theta \simeq 1$

$$k^{-1} = \sqrt{\frac{\sigma}{\rho_w g}} = \sqrt{\frac{rh}{2}}$$

Example: water @ 25 °C

$\sigma = 72$ dynes/cm

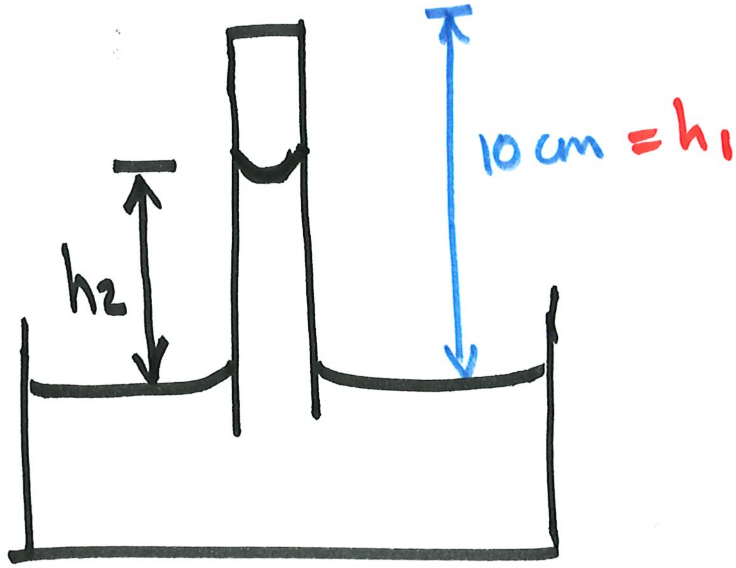
$\rho_w = 1$ g/cm^3

$g = 981$ cm/s^2

$$k^{-1} = \sqrt{\frac{72}{1 \times 981}} = 0.27 \text{ cm} = 2.71 \text{ mm}$$

$\theta \simeq 0$

Example:



$L = 10\,cm$

$d = 0.1\,mm$

$\sigma = 72\,dynes/cm$

$h_2 = ?$

$$P_{g1} V_{g1} = P_{g2} V_{g2}$$

$$P_{g2} = \underset{\downarrow \, P_{atm}}{P_{g1}} \frac{V_{g1} \rightarrow \pi r^2 h_1}{V_{g2} \rightarrow \pi r^2 (h_1 - h_2)}$$

$$\Rightarrow P_{g2} = P_{atm} \frac{\pi r^2 h_1}{\pi r^2 (h_1 - h_2)}$$

$$P_{g2} = P_{atm} \frac{h_1 \swarrow 10}{h_1 - h_2}$$

$$P_c = P_{g2} - P_w$$

Pay attention to this!

$$= P_{g2} - (P_{atm} - \rho_w g h_2)$$

$$\Rightarrow \begin{cases} P_c = P_{atm} \dfrac{10}{10 - h_2} - P_{atm} + \rho_w g h_2 \\ \\ P_c = \dfrac{2\sigma \cos\theta}{r} \end{cases}$$

$$\Rightarrow h_2 = ?$$

$$P_c = \frac{2(72)\cancelto{1}{\cos\theta}}{0.05/10} = 28800 \text{ dynes/cm}^2$$

$$\Rightarrow \boxed{h_2 = 0.28 \text{ cm}}$$

Appendix

$$P_c = P_{atm}\frac{10}{10-h_2} + \rho_w g h_2 - P_{atm}$$

$$10P_c - P_c h_2 = 10\cancel{P_{atm}} + 10\rho_w g h_2$$
$$-\rho_w g h_2^2 - 10\cancel{P_{atm}} + h_2 P_{atm}$$

$$10P_c + h_2(-P_c - 10\rho_w g - P_{atm}) + \rho_w g h_2^2 = 0$$

$$= -28800 - 9810 - 1.0133\times10^6$$

$$= -1051910$$

$$0 = 288000 - 1051910h_2 + 981h_2^2$$

$h_2 = 1072$

$h_2 = 0.27$