PGE381L – CHEAT SHEET MIDTERM FALL 2023

Units

	SI	Darcy	oilfield
Time	s	s	day
Length	m	cm	ft
Pressure	Pa	atm	psia
Flow rate	m³/s	cm³/s	stb/day
Viscosity	Pa.s	cp	cp
Permeability	m²	Darcy (d)	Millidarcy (md)
Compressibility	Pa^{-1}	atm^{-1}	psi^{-1}
$q = -\frac{k\ A}{\mu}\frac{d\Phi}{ds}$	$\Phi = P \pm \rho g z$	$\Phi = P \pm \frac{\rho g z}{1.0133\times 10^6}$	$\Phi = P \pm 0.433\gamma z$
Gravity $[L/T^2]$	9.807	980.9	-

* 1 darcy = 9.869E-9 cm² = 9.869E-13 m²=1.062E-11 ft²

Porosity

Total, Effective *(contribution to flow)*, Interconnected.

Cubic packing: maximum porosity. Best sorting, higher ϕ

$\phi = \frac{Vp}{Vb} = \frac{Vp}{Vp+Vg} = \frac{V_b - V_g}{V_b}$ $\quad V_g = \frac{W_{dry}}{\rho_g}$ $\quad V_p = V_b - V_g$

$V_b = \frac{W_s - W_i}{\rho_f}$ $\quad W_s$: weight of saturated , W_i: weight immersed

Gas expansion - calibration $\quad V_s = V_1 - V_2\left(\frac{P_2}{P_1 - P_2}\right)$ $\quad V_s$: volume of solid

In situ:

$\rho_b = \phi\rho_{fluid} + (1-\phi)\rho_{matrix} \quad \Longrightarrow \quad \phi = \frac{\rho_b - \rho_m}{\rho_f - \rho_m}$ (1) $\quad rhob \leftrightarrow \phi_D$

$\rho_m = \sum C_i \rho_i$, C_i: % of the mineral i

$\rho_f = S_w\rho_w + (1-S_w)\rho_o$ (2)

Density and Neutron log – calibrated to $S_w = 100\%$

$\phi_N > \phi_D$ → wrong lithology

$\phi_N < \phi_D$ → $\phi = \sqrt{\frac{\phi_N^2 + \phi_D^2}{2}}$

$\phi_N = \phi_D$ → direct reading, $Sw = 100\%$

Resistivity log – ILD(deep) > ILM > SFLU(shallow) ;;; AT90 > AT30 > AT10

Workflow: ϕ_D and ϕ_N → $\sqrt{\dots}$ → S_W with Archie → ρ_f with (1) → ρ_{HC} with (2)

Low GR: clay free ; resistivities overlap: no invasion

Shaly sands

$C_{sh} = \frac{GR - GR_{clean\ sand}}{GR_{shale} - GR_{clean\ sand}}$ Volumetric concentration of shale

$\rho_B = \rho_{sh}C_{sh} + \rho_{sand}(1 - C_{sh})$

$\rho_{sand} = \rho_m(1 - \phi_{sand}) + \phi_{sand}\rho_f$

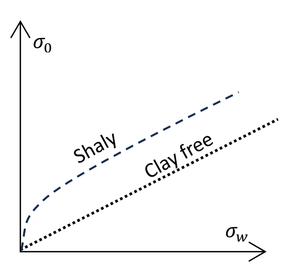
$\phi_D^{sh} = \frac{\phi_D - C_{sh}(\phi_D)_{sh}}{1 - C_{sh}}$ $\quad \phi^{sh}$: corrected porosity;

$\phi_N^{sh} = \frac{\phi_N - C_{sh}(\phi_N)_{sh}}{1 - C_{sh}}$ $\quad (\phi)_{sh}$: measurement in shale

Conductivity $\sigma = \frac{1}{R}$

$\sigma_0 = \frac{\sigma_w}{F} + C$

$C = 0$ for clay free rocks



Impact of stress, compressibility

$C_f = \frac{1}{V_p}\frac{\partial V_p}{\partial P} = \frac{1}{\phi}\frac{\partial \phi}{\partial P}$

Saturation

Clay free! Log always read R_t!

Formation Factor $F = \frac{R_0}{R_w} = \frac{a}{\phi^m}$, R_0: Sw=100% $\quad R_w$: resistivity of the water

Resistivity Index $I_R = \frac{R_t}{R_0} = \frac{1}{S_w^n}$

Archie: $R_t = R_w \frac{a}{\phi^m S_w^n}$

Picker Plot: $\log R_t$ vs. $\log \phi$

$r = R\frac{L}{A}$ $\quad r$ is the resistance of the resistor ; R is the resistivity

Shallow resistivity: $R_{shallow} = R_{mud}\frac{a}{\phi^m S_{xo}^n}$

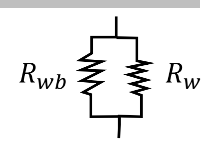
Deep resistivity: $R_{deep} = R_w \frac{a}{\phi^m S_w^n}$

$S_{movable} = S_{xo} - S_w$

Electrical double layers around clay crystals

$\frac{1}{R_t} = \frac{\phi_T^m S_{wt}^n}{a\ R_w}\left(1 + BQ_V \frac{R_w}{S_{wt}}\right)$ $\quad \boldsymbol{S_{wt}}$ is the unknown

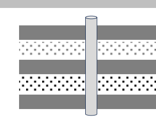
$\frac{1}{R_{weq}} = \frac{1}{R_{wb}} + \frac{1}{R_w}$

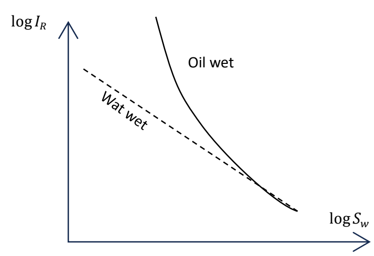


Lamination

$R_V = C_{sh}R_{sh} + (1 - C_{sh})R_{sand}$

$\frac{1}{R_H} = \frac{C_{sh}}{R_{sh}} + \frac{1 - C_{sh}}{R_{san}}$





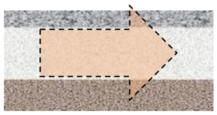
Darcy *(§107)*

$q = -\frac{kA}{\mu}\frac{\Delta P}{\Delta x}$ Linear

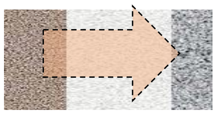
$q \ln\frac{r_e}{r_w} = -\frac{k}{\mu} 2\pi\, h(P_e - P_w)$ Cylindrical

$\mathrm{v_d} = \frac{q}{A} = -\frac{\bar{\bar{k}}}{\mu}\,\overline{\nabla}\Phi$ Inclined, anisotropic

Permeability average



$$k_{avg} = \frac{\sum k_i h_i}{\sum h_i}$$



$$k_{avg} = \frac{\sum h_i}{\sum h_i/k_i}$$

Carman-Kozeny §104

$k = \frac{\phi r^2}{8\tau} = \frac{\phi}{k_0 \tau S_P^2} = \frac{\phi^3}{k_0 \tau\, S_S^2 (1-\phi)^2}$ $\tau = \left(\frac{L_e}{L}\right)^2$ Tortuosity

r: radius of the capillar

S_P: wetted surface area of the pores per unit pore volume

S_S: wetted surface area per unit grain volume

S: wetted surface área per unit bulk volume ($S = S_P\ \phi$)

Granular media: $k = \frac{D^2\phi^3}{36\,k_0 \tau (1-\phi)^2} \implies D$: diameter of grains

$\tau = \phi\, F$ *(§107)*

Pore circular in cross section → $k_0 = 2$

Cylindrical pores: $k = \frac{\phi r_H^2}{2\tau} \implies r_H = \frac{1}{S_P}$

Flow through a cilinder(Hagen-Poiseuille): $q = \frac{\pi R^2}{8\mu}\frac{\Delta P}{L}$

Flow through fractures (Hagen-Poiseuille): $q = \frac{w^2 A}{12\mu}\frac{\Delta P}{L} \implies k = \frac{w^2}{12}$

Permeability given pore distribution: $k = \frac{\phi}{32\tau}\frac{\int_0^\infty f(\delta)\delta^4 d\delta}{\int_0^\infty f(\delta)\delta^2 d\delta}$, $\int_0^\infty f(\delta) d\delta = 1$

Laboratory assessment (steady state)

Darcy and Boile: $q = -\frac{k_L A}{\mu}\frac{\Delta P}{\Delta x}$ (líq) $q = \frac{k_G A}{2\mu}\frac{P_{in}^2 - P_{out}^2}{L}$ (gás)

Klinkenberg: $k_G = k_L\left(1 + \frac{b}{\bar{P}}\right)$, $\bar{P} = \frac{P_{in}+P_{out}}{2}$

Workflow: measure P_1, P_2, q ; Calc k_G ; Extrapolate k_L

Anisotropy

$\bar{\bar{k}}(u,v,w) = \begin{bmatrix} k_u & 0 & 0 \\ 0 & k_v & 0 \\ 0 & 0 & k_w \end{bmatrix}$ Principal axes of anisotropy

$\cos\theta_{12} = \frac{\overline{\mathrm{v}}_1 \cdot \overline{\mathrm{v}}_2}{|\overline{\mathrm{v}}_1| \cdot |\overline{\mathrm{v}}_2|}$

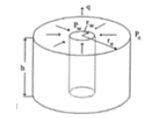
$\frac{1}{k_s} = \frac{\cos^2\beta}{k_u} + \frac{\sin^2\beta}{k_v}$ Permeability ellipse

Eigen values analysis to find the principal axes: $\det(k(x,y,z) - \lambda I) = 0$

Pressure transient analysis (PTA) → *Field units*

Drawdown:

$P(r,t) = P_i - \frac{141.2\,q\,\mu\,B}{k\,h}\left(-\frac{1}{2}E_i\left(-\frac{948\,\phi\,\mu\,c_t\,r^2}{kt}\right)\right)$



$P_{wf}(t) = P(r_w, t)$

$E_i(x) = 0.5722 + \ln\,|x|$, $x < 0.01$

$P_{wf}(t) = P_i - \frac{162.6\,q\,\mu\,B}{k\,h}\left(\log t + \log\frac{k}{\phi\,\mu\,c_t\,r_w^2} - 3.23\right)$

Slope: $m = -\frac{162.6\,q\,\mu\,B}{k\,h}$

$S = 1.1513\left(\frac{P_{wf}(1\text{hr}) - P_i}{-\left(\frac{162.6\,q\,\mu\,B}{k\,h}\right)} - \log\frac{k}{\phi\,\mu\,c_t\,r_w^2} + 3.23\right)$

At $\log t = 0$, $(t = 1)$, $P_i = P_1 - m\left(\log\frac{k}{\phi\,\mu\,c_t\,r_w^2} - 3.23\right)$ (initial pressure)

Buildup

$t^* = \frac{t_P + \Delta t}{t_P}$ (Horner time)

$P_{ws}(\Delta t) = P_i - \frac{162.6\,q\,\mu\,B}{k\,h}\log t^*$ (infinite res)

$S = 1.1513\left(\frac{P_{wf}(t_P) - P_{ws}(1\text{hr})}{-\left(\frac{162.6\,q\,\mu\,B}{k\,h}\right)} - \log\frac{k}{\phi\,\mu\,c_t\,r_w^2} + 3.23\right)$

Radius of investigation

$r_{inv} = 0.03248\sqrt{\frac{kt}{\phi\,\mu\,c_t}}$