Advanced Petrophysics:
Capillary Pressure

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PGE381L Outline

- Introduction to petrophysics, geology, and formation data
- Porosity
- Fluid saturations
- Permeability
- Quantification of heterogeneity, spatial data analysis, and geostatistics
- Interfacial phenomena and wettability
- Capillary pressure
- Relative permeability
- Dispersion in porous media
- Introduction to petrophysics of unconventional reservoirs

What do we learn in this lecture?

- What is capillary pressure?
- Parameters affecting capillary pressure
- Leverett J-Function
- How to quantify capillary pressure
  - Laboratory measurements
  - Well-log-based saturation-height analysis
- Capillary trapping
- Empirical capillary pressure models
- Assessment of rock properties from capillary pressure
  - Permeability, pore-throat-size distribution, and relative permeability

Capillary Pressure

- **Capillary Pressure:** The pressure difference between two sides of the interface separating two immiscible fluids

Young-Laplace equation:

$$P_c = P_2 - P_1 = \sigma\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

curvature of the interface

r_1 and r_2 :the principal radii of curvature of the interface

Let's derive this equation!

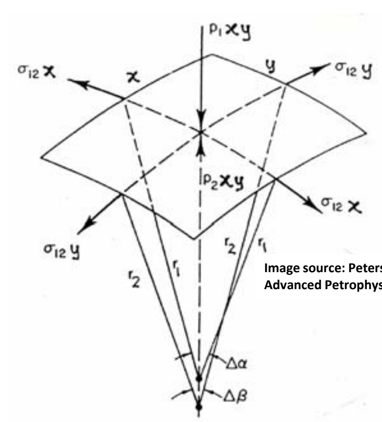


Image source: Peters, E. J., 2012, Advanced Petrophysics

Which one has a larger value? P1 or P2?

Laplace’s Equation in Special Cases

Spherical Liquid Drop:

$$r_1 = r_2 = r \longrightarrow P_c = \frac{2\sigma}{r}$$

Spherical Soup Bubble:

$$r_1 = r_2 = r \longrightarrow P_c = 2\left(\frac{2\sigma}{r}\right) = \frac{4\sigma}{r}$$

two gas-liquid interfaces

Flat Surface:

$$r_1 = r_2 = \infty \longrightarrow P_c = 0$$

Laplace’s Equation in Special Cases

Capillary Rise Experiment:

$$r_1 = r_2 = \frac{r}{\cos\theta}$$

$$\longrightarrow P_c = \frac{2\sigma\cos\theta}{r}$$

Assumption: The interface lies on a sphere

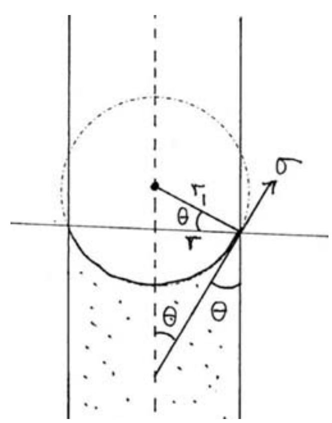
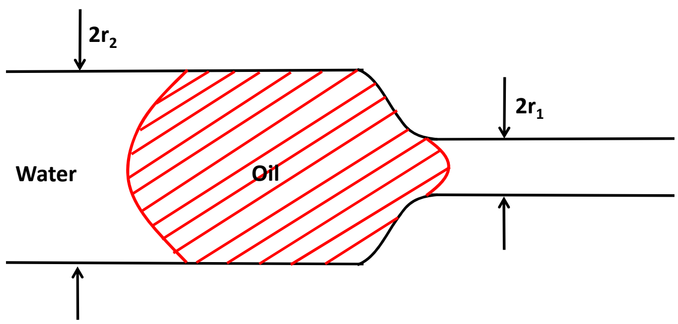


Image source: Peters, E. J., 2012, Advanced Petrophysics

Example: Mobilization of Residual Non-Wetting Phase

Example: **Calculate the pressure gradient required to mobilize a trapped oil blob in the following waterflood process.**

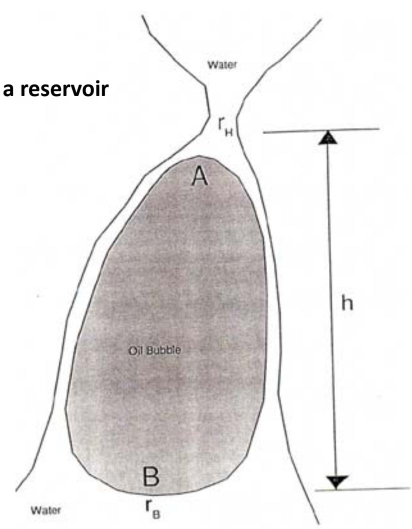
Is the pressure gradient generated by the waterflood sufficient to mobilize the oil droplet.



Example: Oil Migration

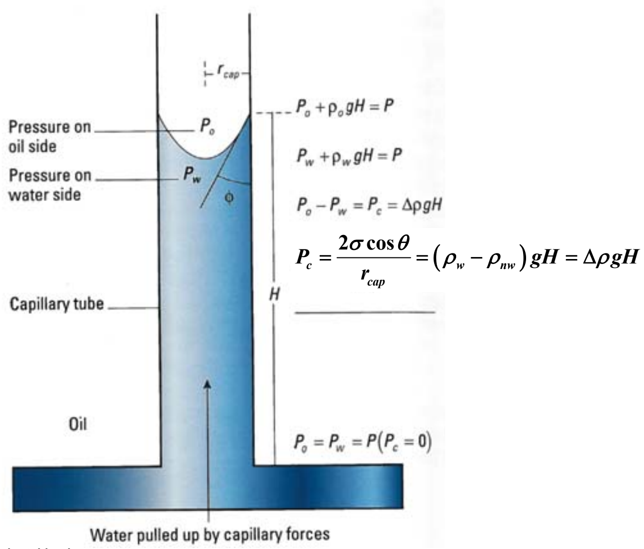
upward migrating oil bubble from a source rock into a reservoir initially fully saturated with water

Example: **Calculate the length of the oil blob required for the blob to pass through the pore throat of radius r_H and continue its upward migration.**



Source: Peters, E. J., 2012, Advanced Petrophysics

Capillary Pressure

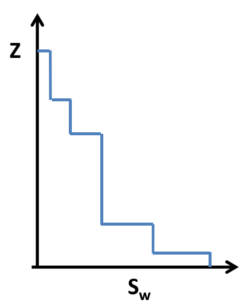


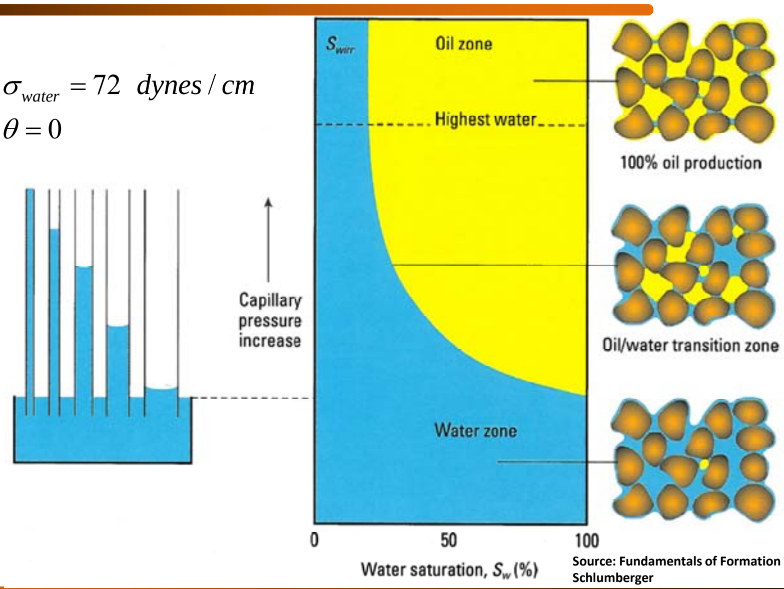
Source: Fundamentals of Formation Testing by Schlumberger

Saturation-Dependent Capillary Pressure

$$z = \frac{2\sigma\cos\theta}{r\rho_w g} = \frac{0.1468}{r} cm$$

$$\sigma_{water} = 72 \ \ dynes/cm$$
$$\theta = 0$$

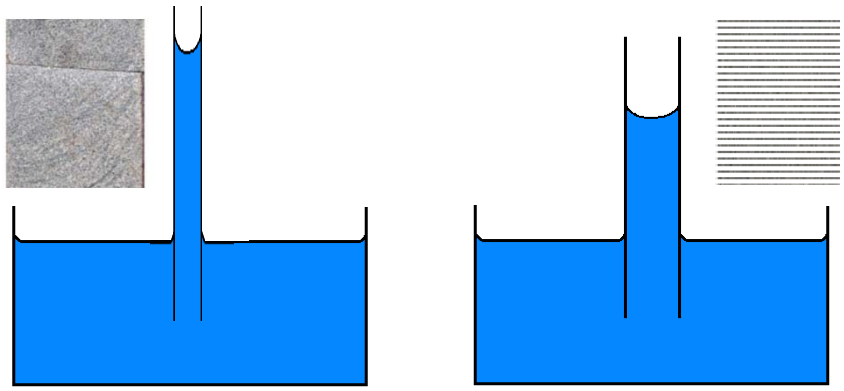




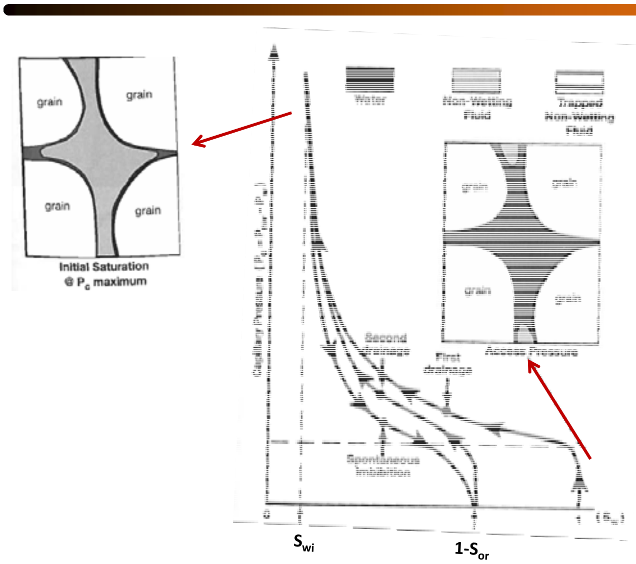
$$P_c = \frac{2\sigma\cos\theta}{r} = \frac{144}{r} \ dynes/cm^2$$

Source: Fundamentals of Formation Testing by Schlumberger

Capillary Pressure in Porous Media with Different Grain Sizes



Drainage Capillary Pressure



Drainage: A process in which the wetting phase saturation decreases

Imbibition: A process in which the wetting phase saturation increases

Source: Zinszner, B. and Pellerin, F. M., 2007, A geoscientist's guide to petrophysics: IFP Publications.

Capillary Pressure and Height above FWL

How to estimate saturation-dependent capillary pressure from well logs?

Please take notes!

Capillary Pressure and Height above FWL

$$P_c = (\rho_w - \rho_{nw})\, gz$$

Capillary Pressure, (Pa)

Density, (kg/m3)

Height above FWL, (m)

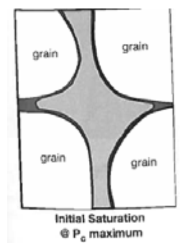
$$P_c = \frac{(\rho_w - \rho_{nw})\, z}{144}$$

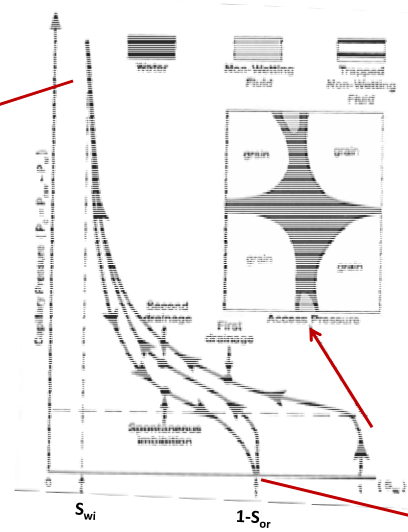
Capillary Pressure, (psi)

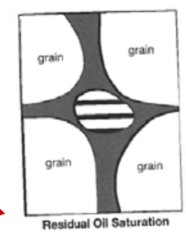
Density, (lb/ft³)

Height above FWL, (ft)

Drainage and Imbibition Capillary Pressure







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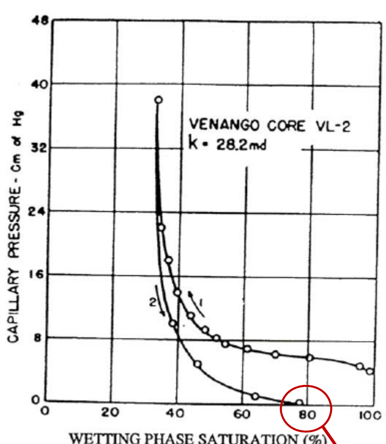
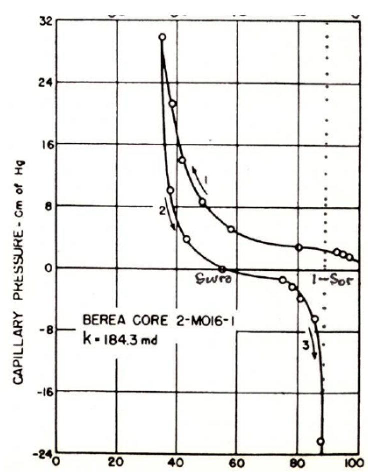
Drainage and Imbibition Capillary Pressure



Forced Imbibition

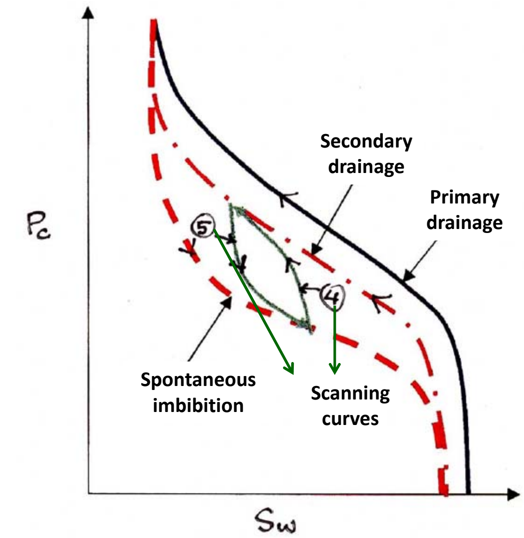
Is this value equal to residual non-wetting phase?



WETTING PHASE SATURATION (%)

Source: Peters, E. J., 2012, Advanced Petrophysics

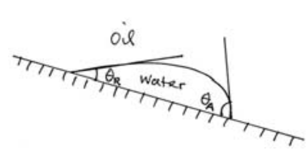
Cycles of Capillary Pressure Measurements



Source: Peters, E. J., 2012, Advanced Petrophysics

Capillary Pressure Hysteresis

- **Which one requires more work to be done?**
  - **Non-wetting phase to displace a wetting phase OR a wetting phase to displace a non-wetting phase?**
  - **Drainage OR imbibition capillary pressure measurement?**
- **Which contact angle do we experience during drainage and imbibition capillary pressure measurements**

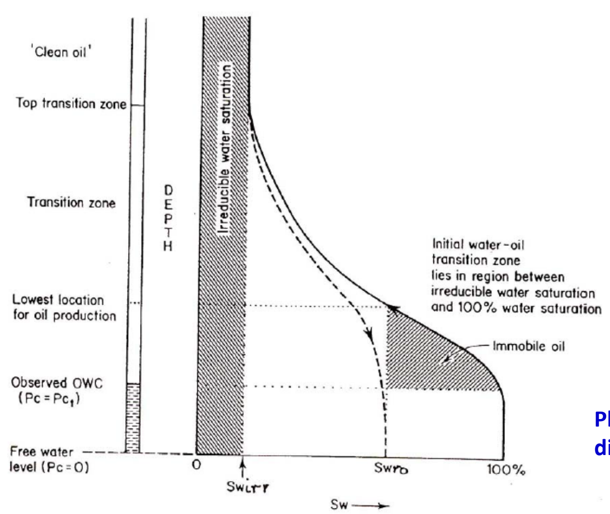


Parameters Affecting Capillary Pressure Hysteresis

- **Fluids**
- **Contact angle/Wettability**
- **Nature of immiscible displacement and trapping of fluids in the pores**
- **Pore structure**

Question: What are the applications of drainage and imbibition capillary pressure measurements?

What Do We Produce at Different Depths?



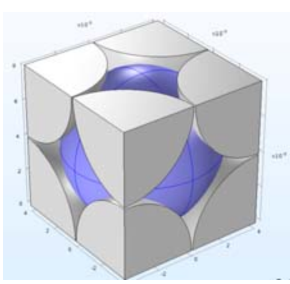
Please see the well-log example distributed in the class!

Source: Peters, E. J., 2012, Advanced Petrophysics

Parameters Affecting Capillary Pressure

- Pore size/structure
- Pore-size distribution
- Tortuosity
- Cementation
- Dead-end pores
- Fluid saturation
- Wettability of the rock-fluid system
- Interfacial tension of the fluids
- Porosity, permeability, fluid types
- …

Pore Throat vs. Pore Body Size



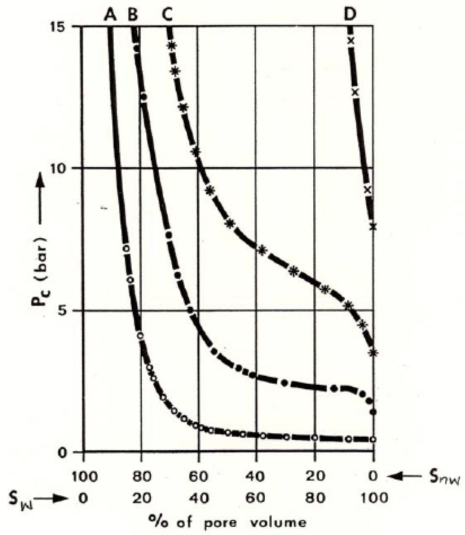
Source of image: Bellini et al., 2018

Calculate pore-body diameter:

Calculate pore-throat diameter:

- Compare these two values?
- How do each one contribute to capillary pressure?

Impact of Pore Structure on Capillary Pressure



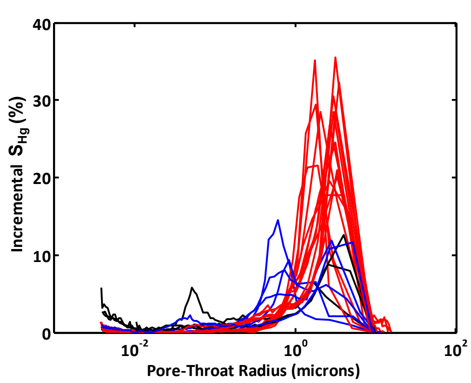
Source: Peters, E. J., 2012, Advanced Petrophysics

Which rock type has the

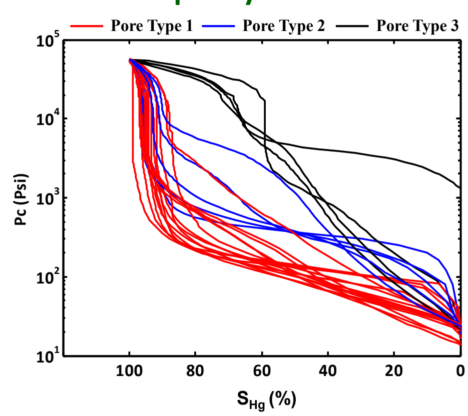
- largest pore throats?
- Smallest pore throats?
- Narrowest pore-size distribution?
- Broadest pore-size distribution?
- Most uniform pore-size distribution?
- Least uniform pore-size distribution?
- Highest irreducible wetting phase saturation?
- Lowest irreducible wetting phase saturation?
- Best sorting?
- Worst sorting?

Capillary Pressure and Pore Structure

Pore-Throat Radius

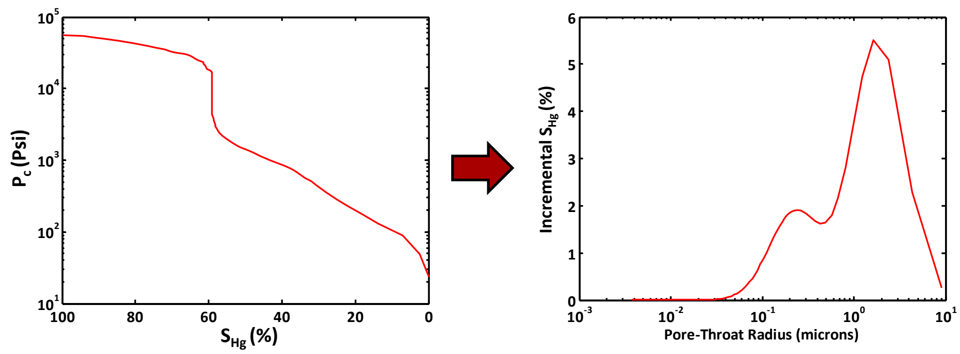


Capillary Pressure



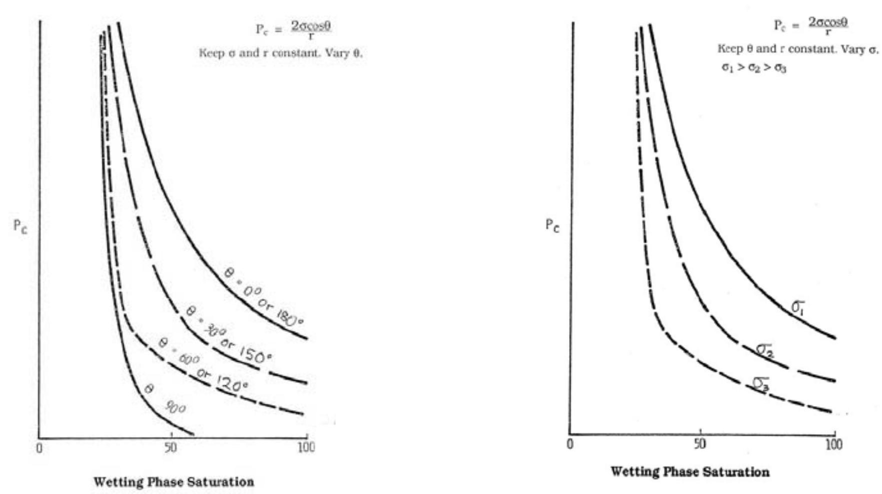
Source: Saneifar and Heidari, 2015

Capillary Pressure and Pore Structure



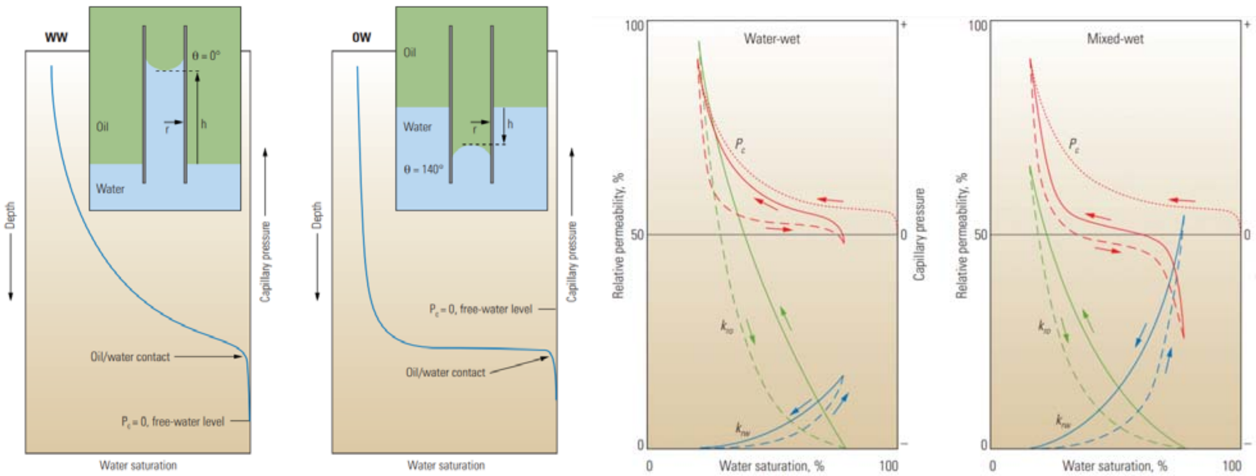
Source: Saneifar and Heidari, 2015

Impact of Interfacial Tension and Wettability on Capillary Pressure



Source: Peters, E. J., 2012, Advanced Petrophysics

Impact of Wettability on Capillary Pressure and Kr



Source: Schlumberger Oilfield Review, Fundamentals of Wettability.

Leverett J-Function (Leverett, 1941)

Capillary Pressure, (psi)

Leverett J-Function, (dimensionless)

$$J(S_w) = 0.21655 \frac{P_c}{\sigma \cos\theta} \sqrt{\frac{k}{\phi}}$$

Permeability, (md)

Porosity, ( )

Interfacial Tension, (dynes/cm)

Contact Angle, (degrees)

- In a given rock type from a given reservoir, porosity and permeability might change, and consequently, capillary pressure will change.
- J-function is a dimensionless saturation-dependent capillary pressure for a given rock type from a given reservoir.

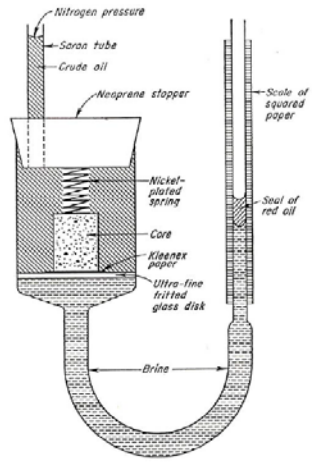
How to Estimate Capillary Pressure?

- How to Estimate Capillary Pressure?
  - Laboratory-based capillary pressure assessment
    - Restored state method (porous plate method)
    - Mercury injection method
    - Centrifuge method
  - In-situ capillary pressure assessment
    - Interpretation of well logs and saturation-height analysis

Laboratory-based Capillary Pressure Assessment

- Laboratory-based capillary pressure assessment
  - Restored state method (porous plate method)
  - Mercury injection method
  - Centrifuge method

Restored State Method (Porous Plate Method)



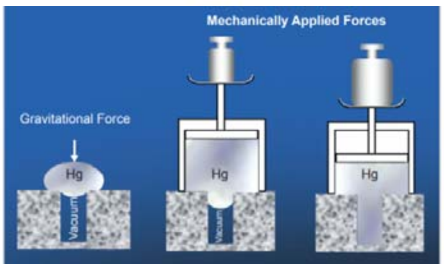
Source: Peters, E. J., 2012, Advanced Petrophysics; Raza et al., 1968

Restored State Method (Porous Plate Method)

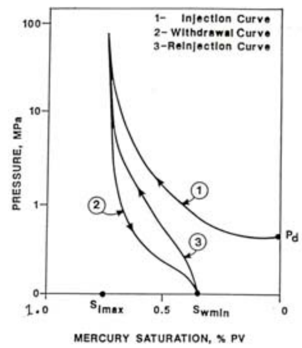
- Advantages
  - Both Drainage and Imbibition P_c curves
  - Very accurate
  - Reliable estimates of $S_{w,irr}$
  - Can use reservoir fluids

- Disadvantages
  - Very slow (days, weeks, months)
  - The maximum capillary pressure that can be measured is limited by displacement pressure of porous disk
  - Usually only goes up to Pc of about 200 psi

Mercury Injection Method



Source: www.Micromeritics.com

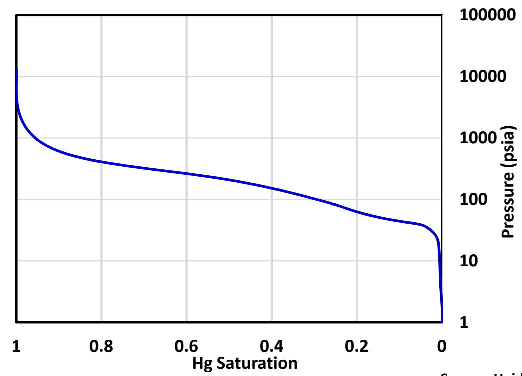


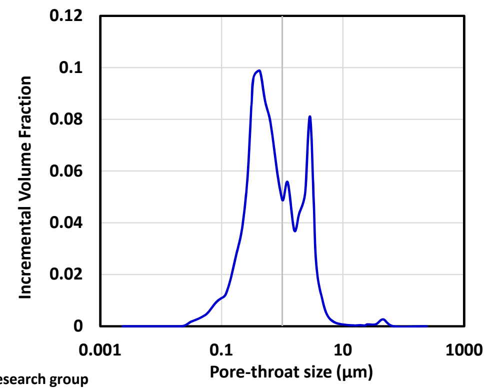
- Why mercury?
- What are the disadvantages?

Mercury Injection Method

- Advantages
  - A fast method (minutes, hours)
  - Method is reasonably accurate
  - Very high range of capillary pressures
  - Goes up to 55,000 or 60,000 psi
  - Can perform multiple intrusion-extrusion cycles
- Disadvantages
  - After this test, core cannot be used for other tests
  - Hazardous testing material (mercury)
  - Conversion required between mercury/air capillary data to reservoir fluid systems
  - It cannot be used for assessment of $S_{w,irr}$
  - High pressures can destroy small pores

Example: Mercury Injection Method



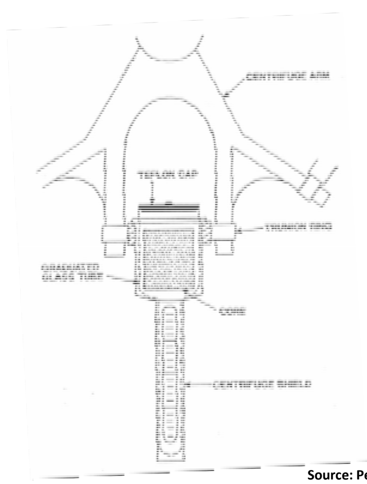


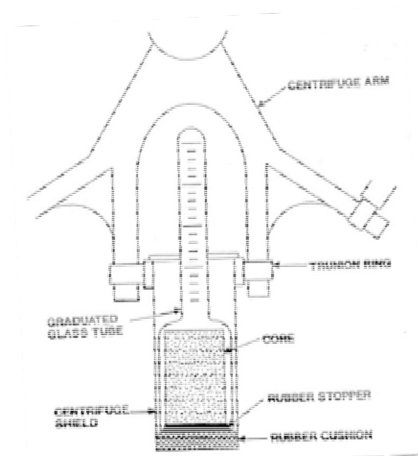
Source: Heidari's research group

- How can we convert capillary pressure measurements to pore-throat-size distribution?
- What are the assumptions?
- What are the limitations?

Centrifuge Method

Oil displacing water **Water displacing oil**





Source: Peters, E. J., 2012, Advanced Petrophysics

How to Analyze the Data Collected from Centrifuge Measurements?

$$\left.\begin{aligned}\frac{dP_w}{dr} &= -\rho_w \omega^2 r \\ \frac{dP_o}{dr} &= -\rho_o \omega^2 r\end{aligned}\right\} \Rightarrow \frac{dP_c}{dr} = -(\rho_w - \rho_o)\omega^2 r = -\Delta\rho\omega^2 r \Rightarrow P_c = -\frac{\Delta\rho\omega^2 r}{2} + C$$

Hassler-Brunner boundary condition (1945)

At the outlet face of the core: $r = r_{outlet} \longrightarrow P_c = 0 \Rightarrow P_c = \frac{\Delta\rho\omega^2}{2}\left(r_{outlet}{}^2 - r^2\right)$

At the inlet face of the core: $P_{c,inlet} = \frac{\Delta\rho\omega^2}{2}\left(r_{outlet}{}^2 - r_{inlet}{}^2\right) = P_{c,\max}$

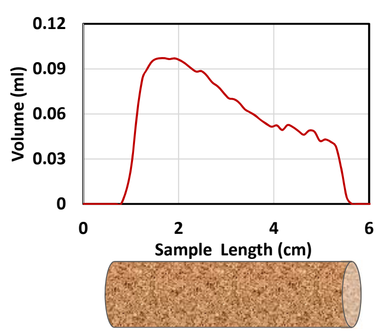
What is water saturation at the core inlet?

How to Analyze the Data Collected from Centrifuge Measurements?

$$S_{w,avg} = \frac{\int_{r_1}^{r_2} S_w dr}{r_2 - r_1}$$

$$\Rightarrow \Delta\rho\omega^2 L r_{inlet} S_{w,avg} = \int_{r_1}^{r_2} S_w \Delta\rho\omega^2 r_{inlet} dr \qquad L = r_2 - r_1$$

$\Delta\rho\omega^2 L r_{inlet} \rightarrow P_{c,inlet}$ $\qquad \Delta\rho\omega^2 r_{inlet} dr \rightarrow dP_{c,inlet}$



Source: Heidari's research group

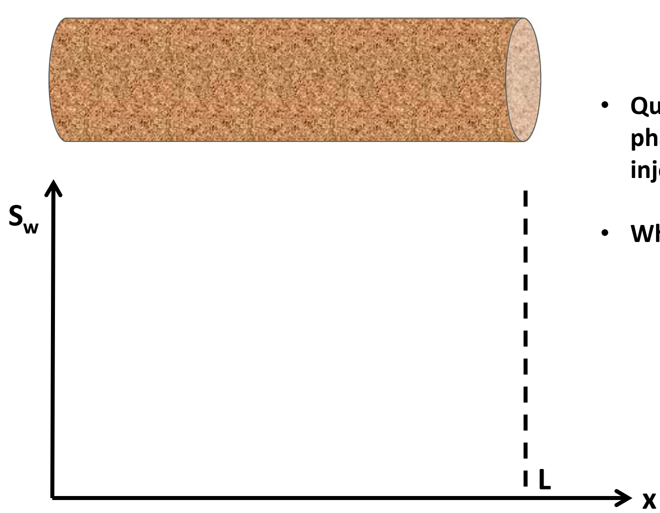
Assumption: *L* is small relative to *r₁* and *r₂*

$$\Rightarrow P_{c,inlet} S_{w,avg} = \int_0^{P_{c,inlet}} S_w P_c dP_c \xrightarrow{\text{Differentiate}} S_{w,inlet} = \frac{d\left(P_{c,inlet} S_{w,avg}\right)}{d\left(P_{c,inlet}\right)} = S_{w,avg} + P_{c,inlet}\frac{d\left(S_{w,avg}\right)}{d\left(P_{c,inlet}\right)}$$

Centrifuge Method

- Advantages
  - The method is fast (hours, days, weeks)
  - Reasonably accurate
  - Can use reservoir fluids
  - It can measure large P_c
  - Good for assessment of $S_{w,irr}$
- Disadvantages
  - Not useful for unconsolidated rocks
  - Inability to measure displacement pressure
  - The calculated water saturation at the core inlet is an approximation
  - The Hassler-Brunner boundary condition at the core outlet may be violated at high centrifuge speeds
  - Inability to obtain spontaneous imbibition capillary pressure curve

Capillary End Effect



- Qualitatively plot saturation of wetting phase as a function of length at different injection times.
- What challenges can it cause?

Capillary End Effect

- **What challenges can capillary end effect cause?**
  - **The wetting phase saturation will be higher towards the core outlet than in the rest of the core.**
  - **The observed breakthrough recovery of the non-wetting phase will be falsely high.**
  - **The breakthrough recovery will be too large and will give a false sense of the displacement efficiency.**
  - **In the unsteady state method for relative permeability assessment, the calculated relative permeabilities will be wrong.**

Quantitative Analysis of Capillary End Effect

OPTIONAL

PDE for the wetting phase saturation for two phase immiscible displacement:

$$\frac{\partial S_w}{\partial t_D} + \left(\frac{dF_w}{dS_w}\right)\frac{\partial S_w}{\partial x_D} + N_{cap}\frac{\partial}{\partial x_D}\left(F_w k_{rnw}\frac{dJ}{dS_w}\frac{\partial S_w}{\partial x_D}\right) = 0$$

Please see the derivation after we cover relative permeability topic.

dimensionless time $t_D = \frac{qt}{A\phi L}$

true fractional flow of the wetting phase $f_w = \frac{q_w}{q}$

dimensionless distance $x_D = \frac{x}{L}$

approximate fractional flow of the wetting phase $F_w = \frac{1}{1+\frac{k_{rnw}\mu_w}{k_{rw}\mu_{nw}}} = \frac{1}{1+\frac{1}{\underset{\text{Mobility}}{M}}}$

$$N_{cap} = \frac{A\sigma\cos\theta\sqrt{k\phi}}{q\mu_{nw}L}$$

Quantitative Analysis of Capillary End Effect

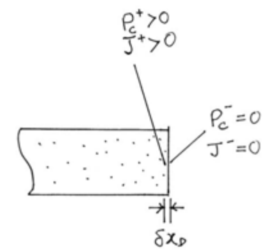
OPTIONAL

$$f_w = F_w\left[1 + N_{cap}k_{rnw}\frac{\partial J}{\partial x_D}\right] \Rightarrow f_w = F_w\left[1 + N_{cap}k_{rnw}\left(\frac{J^- - J^+}{\delta x_D}\right)\right] \Rightarrow f_w = F_w\left[1 - N_{cap}k_{rnw}\frac{J^+}{\delta x_D}\right]$$

$$1 - N_{cap}k_{rnw}\frac{J^+}{\delta x_D} \geq 0 \Rightarrow$$ **The wetting phase will flow out.**

$$1 - N_{cap}k_{rnw}\frac{J^+}{\delta x_D} = 0 \Rightarrow$$ **The wetting phase cannot flow out. It accumulates there raising the wetting phase saturation to an abnormal level.**

$$1 - N_{cap}k_{rnw}\frac{J^+}{\delta x_D} < 0 \Rightarrow$$ **The wetting phase cannot flow out. It accumulates there raising the wetting phase saturation to an abnormal level.**



How to Eliminate Capillary End Effect?

OPTIONAL

$$1 - N_{cap}k_{rnw}\frac{J^+}{\delta x_D} > 0 \Rightarrow \boxed{N_{cap} < \frac{\delta x_D}{k_{rnw}J^+}}$$

$$N_{cap} = \frac{A\sigma\cos\theta\sqrt{k\phi}}{q\mu_{nw}L} \Rightarrow \boxed{q > \frac{Ak_{rnw}J^+\sigma\cos\theta\sqrt{k\phi}}{\mu_{nw}L\delta x_D}}$$

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Capillary Pressure and Height above FWL

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Capillary Pressure and Height above FWL

Capillary Pressure, (Pa)

$$P_c = (\rho_w - \rho_{nw})\,gz$$

Density, (g/cc)

Height above FWL, (m)

Density, (lb/ft^3)

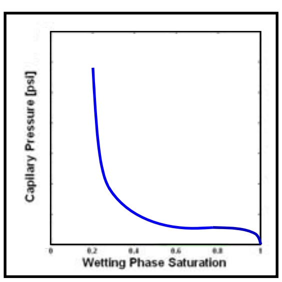
Capillary Pressure, (psi)

$$P_c = \frac{(\rho_w - \rho_{nw})\,z}{144}$$

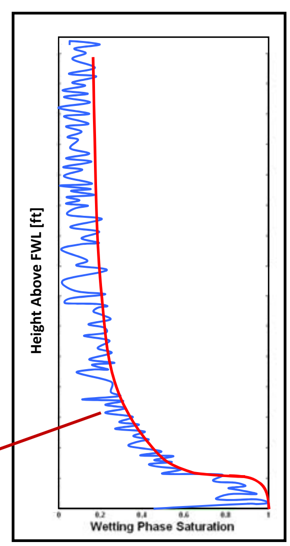
Height above FWL, (ft)

Saturation-Dependent Capillary Pressure

Saturation-dependent capillary pressure at reservoir condition



$$H = 144P_c / (\Delta\rho g)$$



Vertical distribution of water saturation can be estimated using well logs

Example

Laboratory P_c vs. Reservoir P_c

Converting laboratory capillary pressure data to reservoir conditions:

$$\left(P_c\right)_{lab} = \frac{2\left(\sigma\cos\theta\right)_{lab}}{r_m}$$

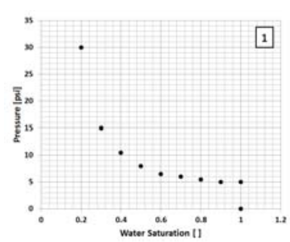
$$\left(P_c\right)_{reservoir} = \frac{2\left(\sigma\cos\theta\right)_{reservoir}}{r_m}$$

$$\Rightarrow \quad \left(P_c\right)_{reservoir} = \left(P_c\right)_{lab}\frac{\left(\sigma\cos\theta\right)_{reservoir}}{\left(\sigma\cos\theta\right)_{lab}}$$

Example

Example: Figure 1 shows drainage capillary pressure at reservoir condition. Average porosity and permeability of this reservoir is 0.25 and 300 md, respectively. Assume that FWL is located at the depth 9500 ft. Densities of oil and water are 45 and 64 lb/ft^3, respectively. Interfacial tension and contact angle at the reservoir condition are 30 dynes/cm and 0°, respectively.

- Calculate depth of WOC.
- Estimate water saturation at 20 ft above FWL.
- Estimate water saturation at 40 ft above WOC.
- Estimate water saturation at 30 ft above WOC.
- Estimate water saturation at 300 ft above WOC.

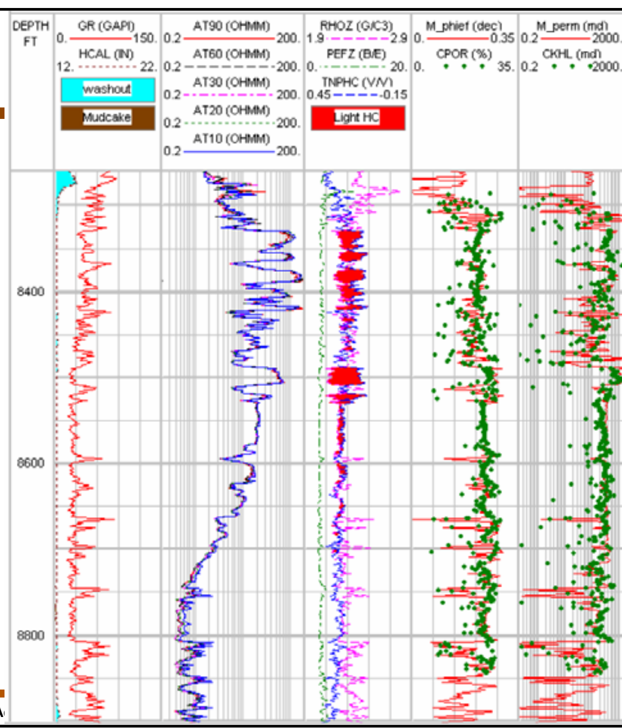


Example

Example: **Now, we take a core plug from this formation to the laboratory. Porosity and permeability of this core plug is 0.15 and 100 md, respectively. Interfacial tension and contact angle at the laboratory are 72 dynes/cm and 0°, respectively.**

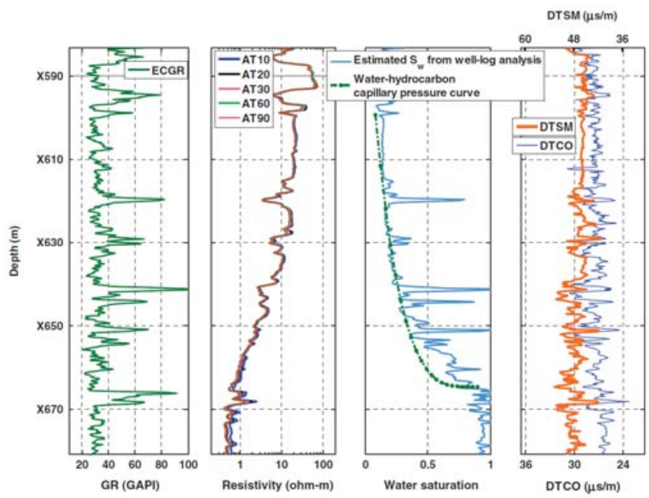
- **Calculate and plot the saturation-dependent Leverett J-function.**
- **What do you expect to measure for saturation-dependent capillary pressure in the laboratory?**

Field Example



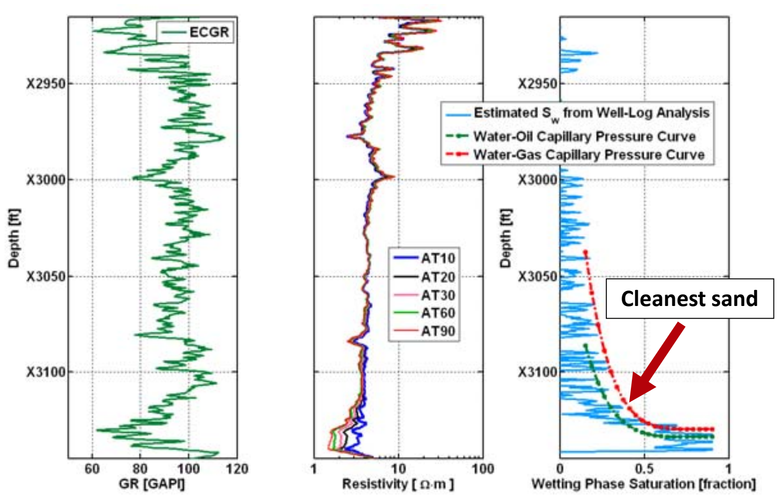
Source: Salazar J. M., Torres-Verdín C., and Wang G. L., 2011, Effects of Surfactant-Emulsified Oil-Based Mud on Borehole Resistivity Measurements, SPE 109946.

Field Example



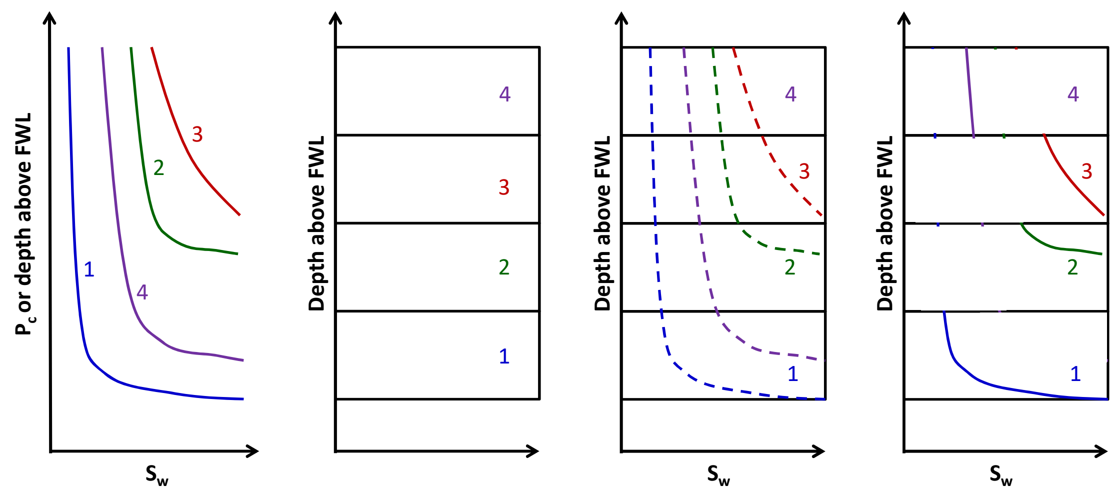
Source: Heidari Z. and Torres-Verdín C., 2012, Estimation of dynamic petrophysical properties of water-bearing sands invaded with oil-base mud from multi-physics borehole geophysical measurements: *Geophysics*, vol. 77, no. 6.

Field Example



Source: Heidari Z. and Torres-Verdín C., 2012, Estimation of dynamic petrophysical properties of water-bearing sands invaded with oil-base mud from multi-physics borehole geophysical measurements: *Geophysics*, vol. 77, no. 6.

Saturation-Height Analysis in Layered Reservoirs



Saturation-Height Analysis in Layered Reservoirs

1. Using the displacement pressure of the bottom layer, calculate the free water level using

$$d_o = \frac{P_d}{\Delta \rho g}$$

2. Take a small value of z measured from the free water level.
3. Calculate the capillary pressure at that level using $P_c(z) = (\rho_w - \rho_o) gz = \Delta \rho gz$
4. Determine the layer in which z occurs.
5. Using the capillary pressure curve for the layer in which z occurs, read or calculate the water saturation for the value of capillary pressure from step 3.
6. If z is at the boundary of two layers, there will be a saturation discontinuity at that value of z. Two saturation values should be calculated one from each of the capillary pressure curves of the two layers involved.
7. Increase the value of z and repeat steps 3 through 6 until z reaches the top of the reservoir.

Example From Your Textbook (Example 7.1): Tasks

- Calculate and plot the graph of the Leverett J-function for the reservoir.
- Calculate and plot the capillary pressure curves for Layers 2, 3 and 4, together with that of Layer 1.
- Calculate the depth of the free water level for the reservoir.
- Calculate and plot graphs of the initial water and oil saturations in the reservoir from 8000 ft to the free water level assuming the reservoir is in capillary equilibrium.
- Calculate and plot graphs of the water and oil pressures at the initial reservoir conditions.
- A well drilled into the reservoir has been perforated from 8090 to 8110 ft. Determine the type of reservoir fluid that will be produced initially.

Let's Practice!

Please take notes!

Other Methods for in-situ Assessment of Capillary Pressure

- **Integration of NMR and electrical measurements**
- **Low-frequency dielectric measurements**
- **Analyze the impact of mud-filtrate invasion on well logs using numerical modeling**

Please see the additional references uploaded on the Canvas website.

OPTIONAL

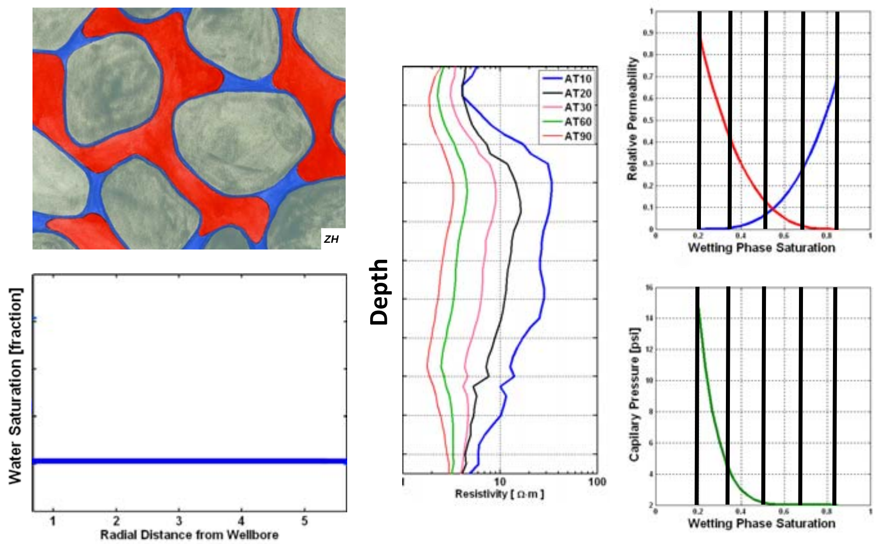
Analyze the Impact of Mud-filtrate Invasion on Well Logs using Numerical Modeling

References:

- Salazar, J. M., Torres-Verdín, C., Alpak, F. O., Habashy, T. M., and Klein, J. D., 2006, Estimation of permeability from array induction measurements: applications to the petrophysical assessment of tight-gas sands: *Petrophysics*, vol. 47, no. 6, pp. 527–544.
- Heidari Z., Torres-Verdín C., Mendoza A., and Wang G. L., 2011, Assessment of residual hydrocarbon saturation with the combined quantitative interpretation of resistivity and nuclear logs: *Petrophysics*, vol. 52, no. 3, pp. 1-35.
- Heidari Z. and Torres-Verdín C., 2012, Estimation of dynamic petrophysical properties of water-bearing sands invaded with oil-base mud from multi-physics borehole geophysical measurements: *Geophysics*, vol. 77, no. 6.

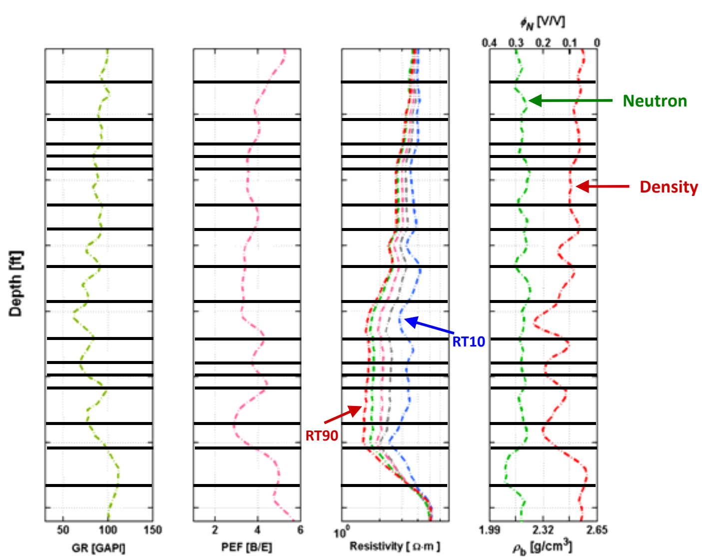
Impact of Capillary Pressure on Well Logs

OPTIONAL



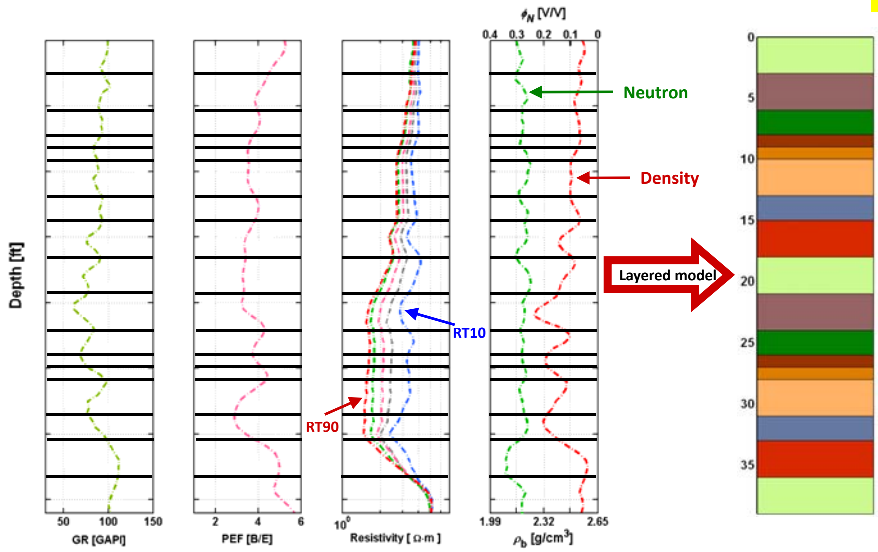
Method

OPTIONAL



Method

OPTIONAL



Method

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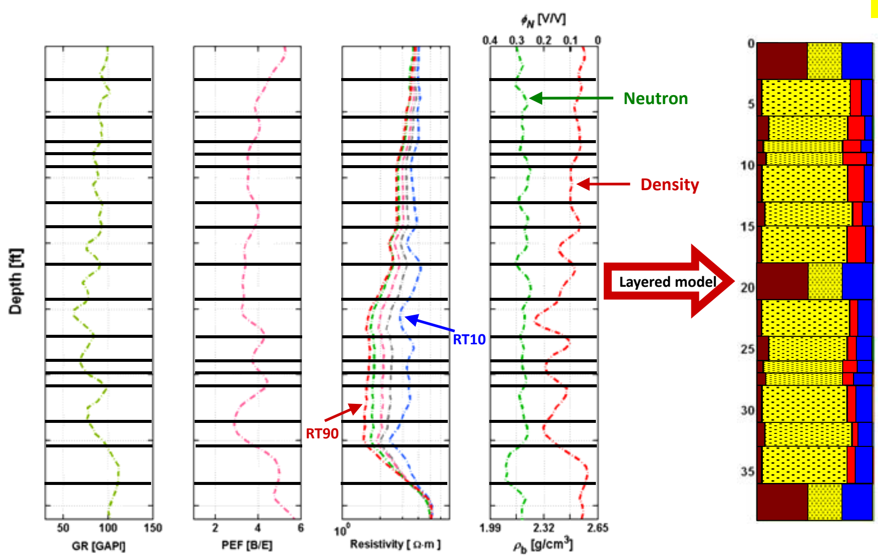

Method

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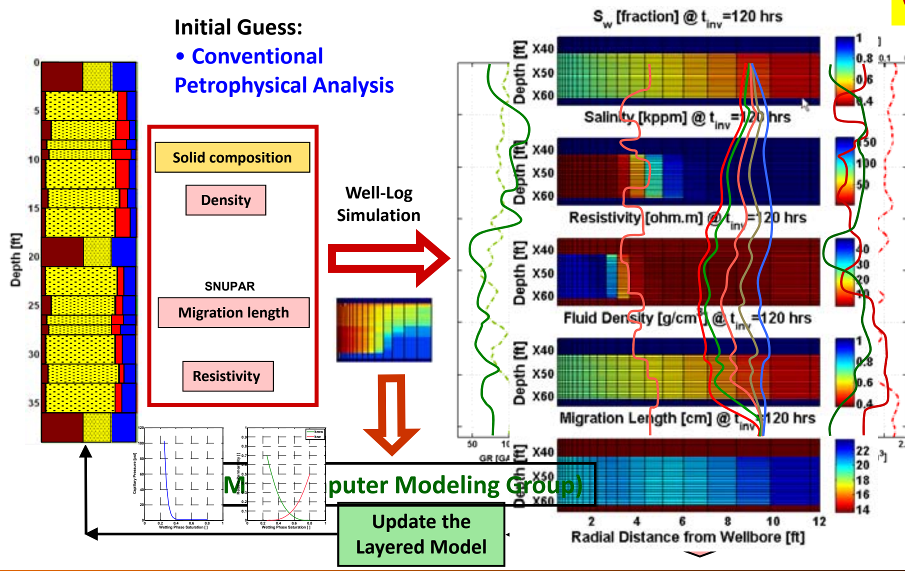

Method

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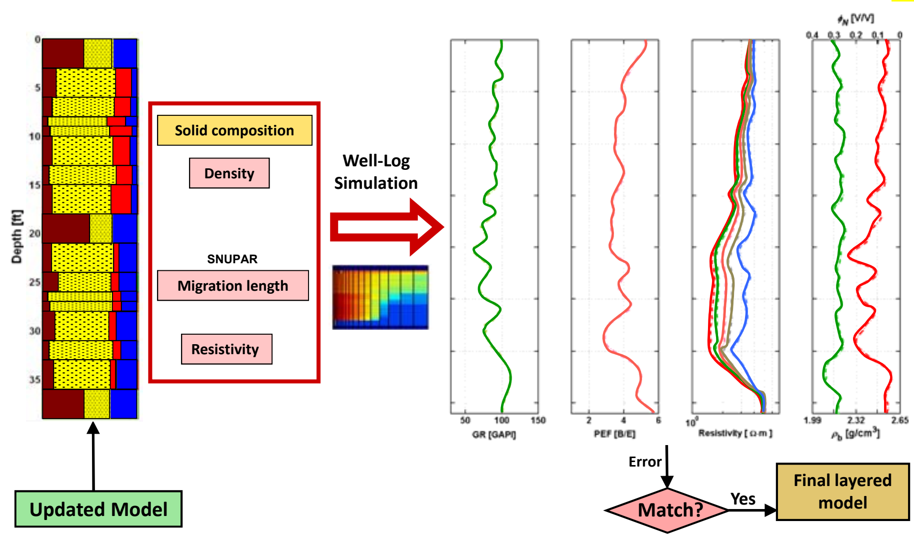
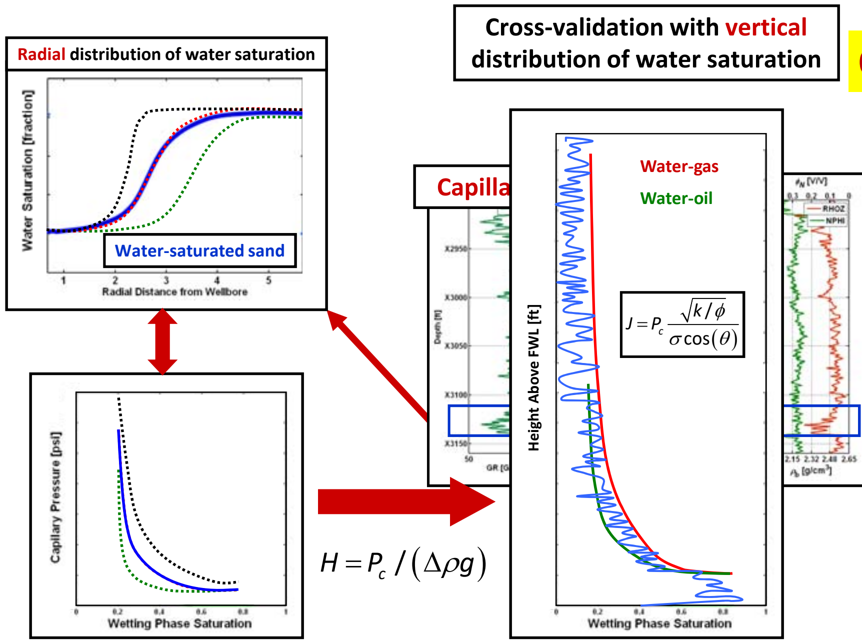


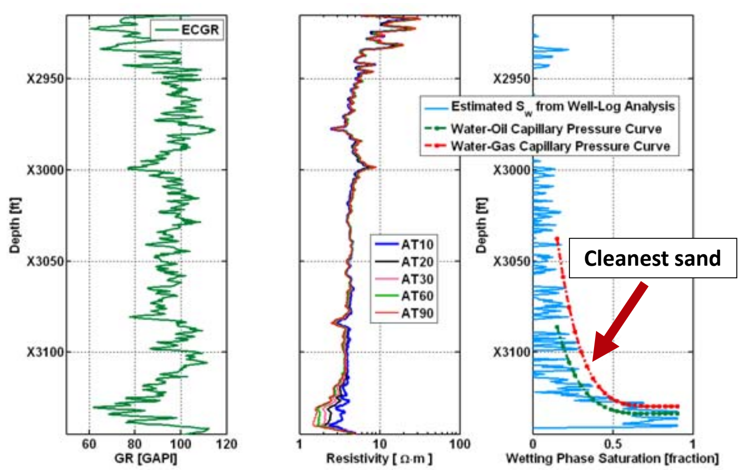


OPTIONAL

Field Application

OPTIONAL

Field Example: Cross-Validation



Source: Heidari Z. and Torres-Verdín C., 2012, Estimation of dynamic petrophysical properties of water-bearing sands invaded with oil-base mud from multi-physics borehole geophysical measurements: *Geophysics*, vol. 77, no. 6.

OPTIONAL

Integration of NMR and Electrical Measurements

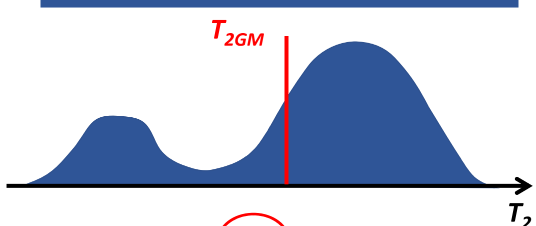
Reference:

Garcia, A. P., Han, Y., and Heidari, Z. 2018. An Integrated Workflow to Estimate Permeability through Quantification of Rock Fabric Using Joint Interpretation of Nuclear Magnetic Resonance and Electric Measurements. *Petrophysics* **59** (5): 672-693. DOI:10.30632/PJV59N5-2018a7.

Interpretation of Nuclear Magnetic Resonance Measurements

OPTIONAL

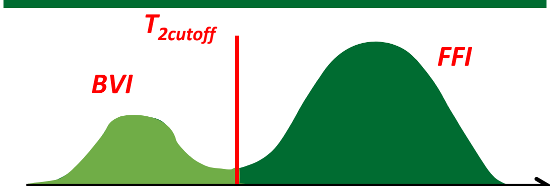
SDR Model



$$k = aT_{2GM}^2\phi^4$$

Keyson et al. (1988)

Coates Model



$$k = \left[\left(\frac{\phi}{C}\right)^2\left(\frac{FFI}{BVI}\right)\right]^2$$

Coates et al. (1997)

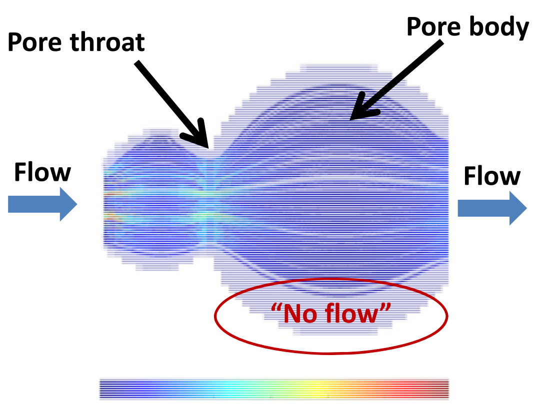
These models rely on calibration efforts such as detection of cutoff values and assessment of constant model parameters.

NMR measurements are sensitive to pore-size distribution, causing uncertainties in permeability assessment in carbonates!

Fluid Flow versus Electric Current Flux

OPTIONAL

Fluid Flow

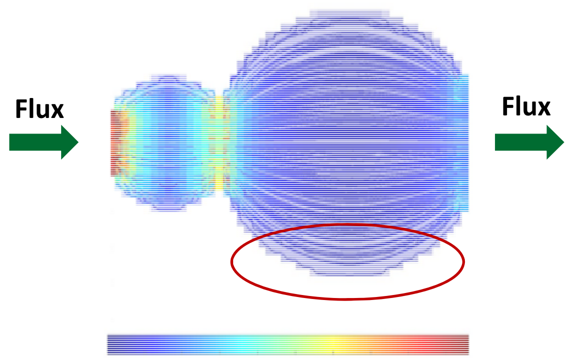


Velocity

No slip boundary condition

Electric Current Flux

Analogous to fluid flow

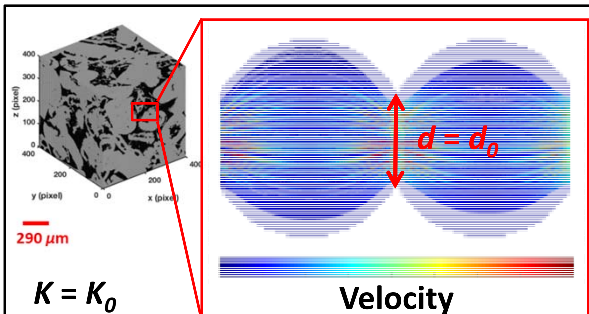


Current

No flux perpendicular to the grain

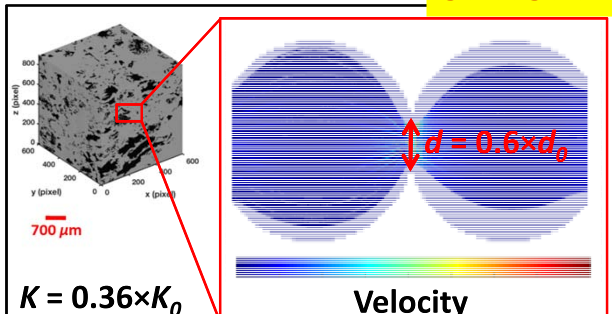
Impact of Pore-Throat Size on Rock Properties

OPTIONAL



$K = K_0$

$\sigma = \sigma_0$



$K = 0.36 \times K_0$

$\sigma = 0.36 \times \sigma_0$

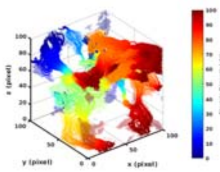
K – permeability, σ – electrical conductivity

Pore-throat assessment is important for reliable petrophysical evaluation of formations with complex pore structure!

Are There Non-Intrusive Methods?

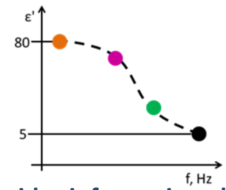
OPTIONAL

Electrical Resistivity



Provides information about the throat-to-pore ratio, but not the throat-size distribution!

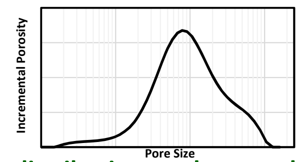
Dielectric Permittivity



Provides information about the throat-size distribution, but still requires calibration!

NMR

NMR: Nuclear Magnetic Resonance



T_2 distribution can be translated into pore-size distribution, but not throat-size distribution!

Let's combine them to estimate pore-throat-size distribution and saturation-dependent capillary pressure!

Workflow for Estimating Throat-Size Distribution

OPTIONAL

NMR Measurements

Calculate porosity and pore-size distribution:

$$r_P = \frac{3\rho T_{2b} T_2}{T_{2b} - T_2}$$

Dielectric Permittivity

Calculate tortuosity (τ_{eW}):

$$F_C = \frac{\varepsilon_f}{\varepsilon_R} = \frac{\tau_{eW}^2}{\phi_T}$$

Bitterlich and Wobking (1970)

Electrical Resistivity

Calculate constriction factor (C_E):

$$\frac{\sigma_R}{\sigma_W} = \frac{\phi_c}{C_E \tau_E^2}$$

ϕ_c - connected porosity

Berg (2012)

Calculate throat-size distribution:

$$r_T = \frac{1}{\sqrt{C_H}} \frac{3\rho T_{2b} T_2}{T_{2b} - T_2}$$

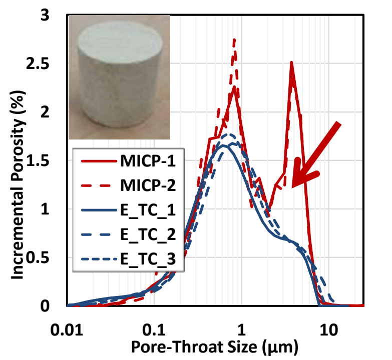
Main Simplifying Assumptions

1) All the pores are represented by the same shape (e.g., sphere) for approximating pore-body-size distribution from NMR measurements.
2) Constriction factor is assumed constant for a particular sample.
3) Hydraulic and electrical parameters (e.g., tortuosity, constriction factor) are assumed to be approximately the same.

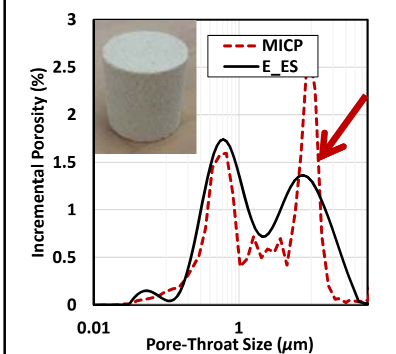
Core-Scale Model Verification: Pore-Throat-Size Distribution

OPTIONAL

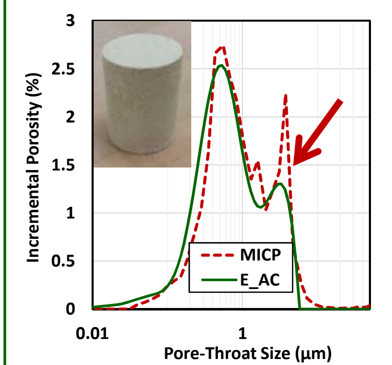
Texas Cream Limestone



Estaillades Limestone



Austin Chalk



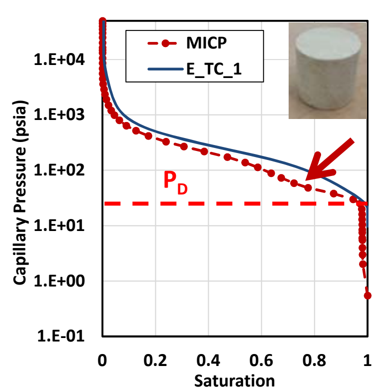
Pore-throat-size distribution can be translated to capillary pressure through the Washburn Equation:

$$P_C = \frac{2\gamma}{r}\cos\phi$$

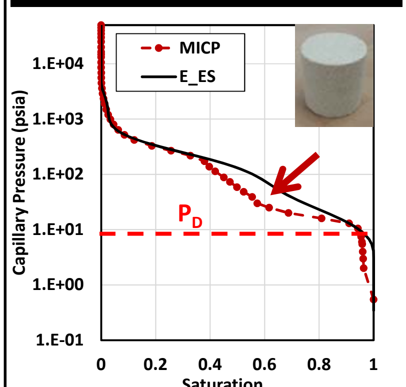
Model Verification: Capillary Pressure Curve

OPTIONAL

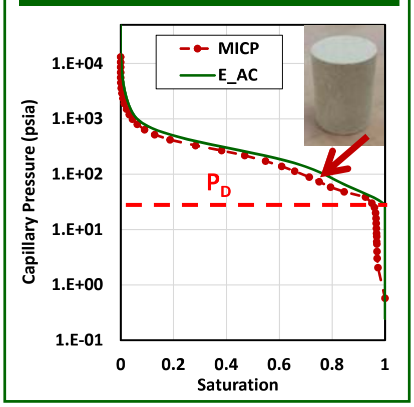
Texas Cream Limestone



Estaillades Limestone



Austin Chalk

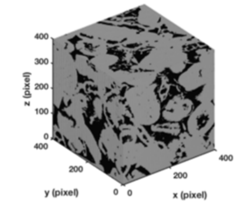


Saturation-dependent capillary pressure and displacement pressure are in good agreement with mercury injection capillary pressure (MICP)!

Pore-Scale Model Verification: Pore-Throat-Size Distribution

OPTIONAL

Carbonate C1



290 μm Dong (2007)

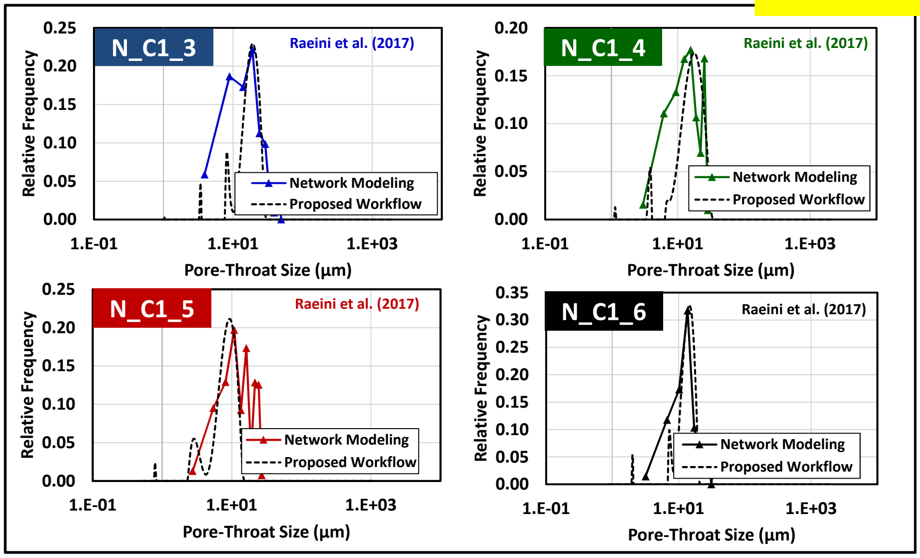
Simulations

NMR

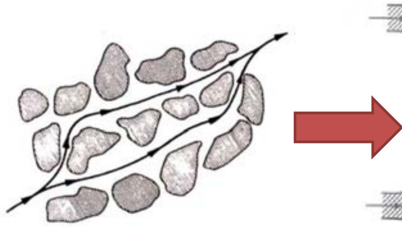
Dielectric Permittivity

Electrical Resistivity

Network Modeling

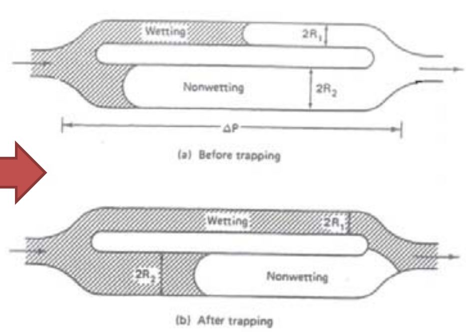


Capillary Trapping in Porous Media



Source: Peters, E. J., 2012, Advanced Petrophysics

Pore Doublet Model



Source: Lake L. W. , 1989, Enhanced Oil Recovery

$$N_{vcap} = \frac{q\mu L}{\pi r_1^3 \sigma \cos\theta}$$

$$N_{vcapcritical} = \frac{\beta\left(\beta^2+1\right)}{4\left(\beta+1\right)}$$

$$\beta = \left(\frac{r_2}{r_1}\right)$$

Assumption: wetting and non-wetting phases have the same viscosity

$N_{vcap} < N_{vcap,critical}$ **non-wetting phase traps in the larger pore**

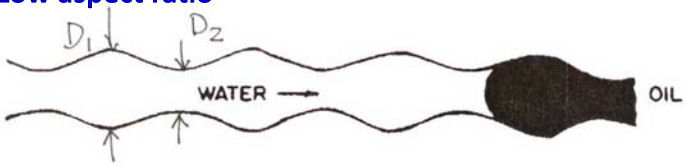
$N_{vcap} > N_{vcap,critical}$ **non-wetting phase traps in the smaller pore**

- Where does the non-wetting phase get trapped?
- Let's calculate velocities and see which flow pathway wins the competition?
- How does the flow rate affect the trapping mechanism?

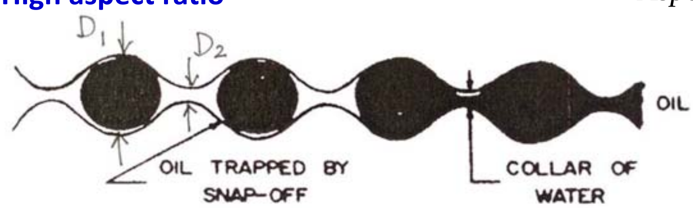
Capillary Trapping in Porous Media

Snap-off Model

Low aspect ratio



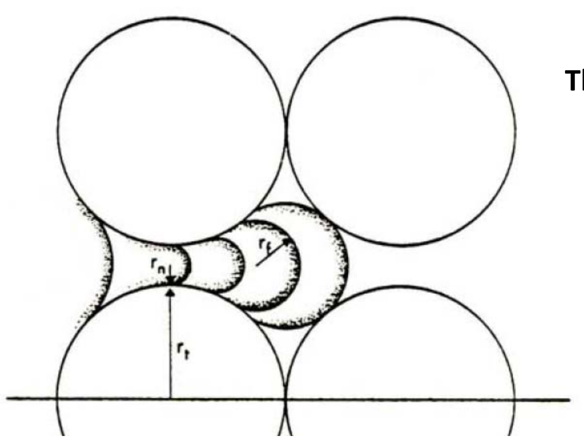
High aspect ratio



$$Aspect\ ratio = \frac{Pore\ Body\ Diameter}{Pore\ Throat\ Diameter} = \frac{D_1}{D_2}$$

Source: Peters, E. J., 2012, Advanced Petrophysics; Chatzis et al., 1983

Example: Snap-off Model



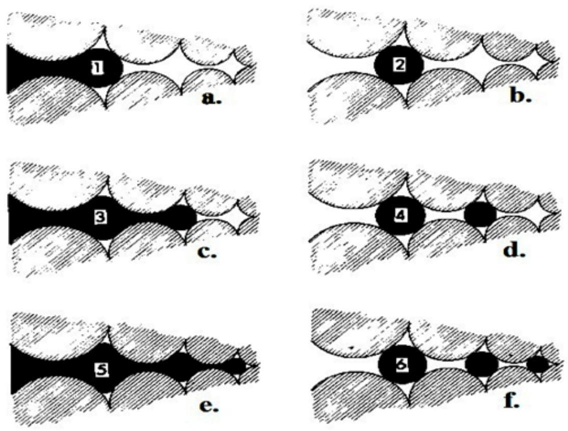
The condition for snap-off:

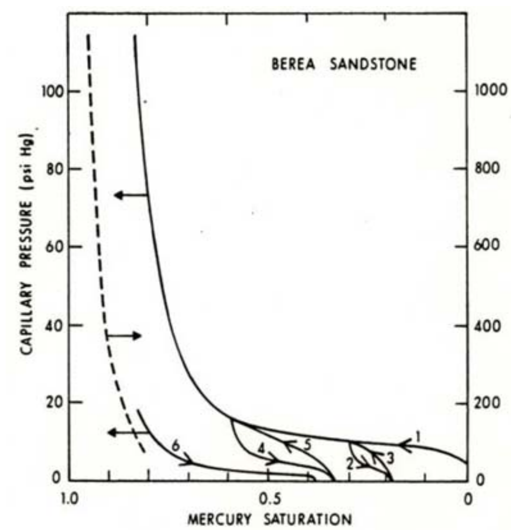
$$\underbrace{P_{cn} = \sigma\left(\frac{1}{r_n} - \frac{1}{r_t}\right)} > \underbrace{\frac{2\sigma}{r_f}}$$

Capillary pressure at the pore neck

Capillary pressure at the leading edge of the drop

Example: Snap-off Model





Empirical Capillary Pressure Models

Brooks and Corey (1966) Model:

Drainage capillary pressure model $P_c = P_e\left(S_w^*\right)^{-\frac{1}{\lambda}}$ → Pore-size distribution index

Imbibition capillary pressure model $P_c = P_e\left[\left(S_e\right)^{-\frac{1}{\lambda}} - 1\right]$

Constant

$$S_e = \frac{S_w - S_{wirr}}{1 - S_{wirr} - S_{nwr}}$$

Effective wetting phase saturation

residual non-wetting phase saturation

$$S_w^* = \frac{S_w - S_{wirr}}{1 - S_{wirr}}$$

Reduced wetting phase saturation

How would you calibrate these models?

Empirical Capillary Pressure Models

van Genuchten (1980) Model:

$$S_w^* = \left[\frac{1}{1+(\alpha P_c)^n}\right]^m$$

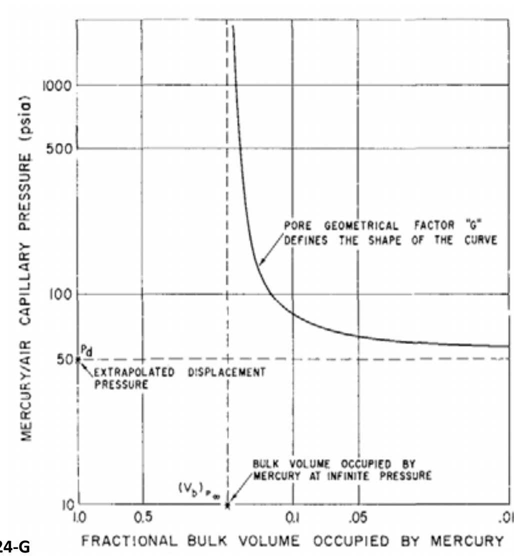
Fitting parameters

Fitting parameters

Empirical Capillary Pressure Models

Thomeer (1960) Model:

$$\frac{(V_b)_{P_c}}{(V_b)_{P_\infty}} = e^{-G/\mathrm{Log}(P_c/P_d)}$$



Source: Thomeer, 1960, SPE-1324-G

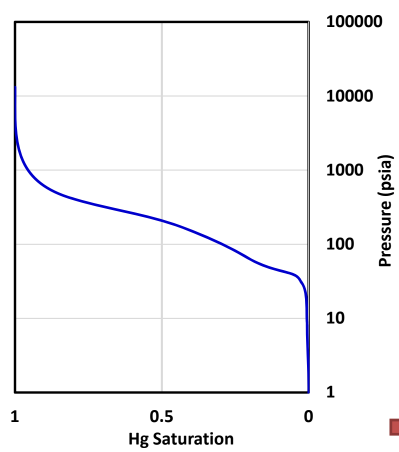
Other Applications of Capillary Pressure

- **What other rock properties can be estimated from capillary pressure data**
  - **Pore-size (pore-throat-size) distribution**
    - ➔ **Be careful about this! Check the assumptions!**
  - **Permeability**
  - **Relative Permeability**

Assessment of Pore-size Distribution from Capillary Pressure

OPTIONAL

Method 1:



$$P_c = \frac{2\sigma|\cos\theta|}{R} \Rightarrow R = \frac{2\sigma|\cos\theta|}{P_c}$$

Pore-throat radius

$$\Rightarrow P_c R = 2\sigma\cos\theta = const. \Rightarrow P_c dR + R dP_c = 0$$

$$S_w = \int_{R_{final}}^{R} f(R)\, dR$$

probability density function for the pore volume distribution

$$\Rightarrow f(R) = \frac{dS_w}{dR} = -\frac{dS_{nw}}{dR}$$

Assessment of Pore-size Distribution from Capillary Pressure

OPTIONAL

$$f(R)=\frac{dS_w}{dR}=-\frac{dS_{nw}}{dR}$$

$$P_c=\frac{2\sigma|\cos\theta|}{R}$$

$$P_c dR + R dP_c = 0$$

$$\Rightarrow$$

$$f(R)=\frac{dS_w}{dR}=-\frac{P_c}{R}\frac{dS_w}{dP_c}=-\frac{2\sigma|\cos\theta|}{R^2}\frac{dS_w}{dP_c}$$

$$f(R)=-\frac{dS_{nw}}{dR}=\frac{P_c}{R}\frac{dS_w}{dP_c}=\frac{2\sigma|\cos\theta|}{R^2}\frac{dS_{nw}}{dP_c}$$

Assessment of Pore-size Distribution from Capillary Pressure

OPTIONAL

Method 2: Bundle of Capillary Tubes Model

- **Step 1:** Pick a high $P_c(S_w)$ value corresponding to a low wetting phase saturation, S_w, and a small pore size, R.
- **Step 2:** Calculate the pore radius, R, using $$P_c(S_w)=\frac{2\sigma|\cos\theta|}{R}$$
- **Step 3:** Calculate the derivative of the capillary pressure curve with respect to the wetting phase saturation at the value of the $P_c(S_w)$ in step 1.
- **Step 4:** Calculate $\delta(R)/\bar{R}^2$ using $$\frac{\delta(R)}{\bar{R}^2}=-\frac{P_c(S_w)}{R^3\left[\frac{dP_c(S_w)}{dS_w}\right]}$$ $$\bar{R}^2=\int_0^\infty R^2\delta(R)dR=\text{a constant}$$

Probability density function for the pore radius distribution

Please review the derivation!

Assessment of Pore-size Distribution from Capillary Pressure

OPTIONAL

Method 2: Bundle of Capillary Tubes Model

- **Step 5:** Pick lower values of $P_c(S_w)$ and repeat steps 2 through 4 until the entire capillary pressure curve has been used in the pore size distribution calculation.
- **Step 6:** Plot the graph of $\delta(R)/\bar{R}^2$ versus R. Calculate the area under the graph, A_g. Using A_g, calculate the constant $\bar{R}^2$ so as to satisfy equation

$$\int_0^\infty \delta(R)dR = 1$$

Pore-throat radius

Probability density function for the pore size distribution

- **Step 7:** Using the value of $\bar{R}^2$, calculate and plot the graph of $\delta(R)$ versus R, which is the required probability density function for the pore size distribution.

Assessment of Permeability from Capillary Pressure

OPTIONAL

The effective permeability to the wetting phase:

$$k_w = \frac{\left(2\sigma\left|\cos\theta\right|\right)^{2+\alpha}}{8a}\phi\int_0^{S_w}\frac{dS_w}{P_c^{2+\alpha}}$$

Tortuosity

$$\tau(R) = \frac{a}{R^\alpha}$$

Constants

The absolute permeability of the porous medium (for the medium fully saturated by the wetting phase)

$$k = \frac{\left(2\sigma\left|\cos\theta\right|\right)^{2+\alpha}}{8a}\phi\int_0^1\frac{dS_w}{P_c^{2+\alpha}} \xrightarrow[1/a\ =\ F_1]{\alpha\ =\ 0} k = \frac{\left(2\sigma\left|\cos\theta\right|\right)^2}{8}F_1\phi\int_0^1\frac{dS_w}{P_c^2}$$

In field units:

$$k = 10.6566\left(\sigma\left|\cos\theta\right|\right)^2 F_1\phi\int_0^1\frac{dS_w}{P_c^2}$$

In the case of mercury injection: Purcell's equation (1949)

$$k = 1.441x10^6 F_1\phi\int_0^1\frac{dS_w}{P_c^2}$$

Assessment of Relative Permeability from Capillary Pressure

OPTIONAL

$$k_{rw}(S_w) = \frac{k_w}{k} = \frac{\int_0^{S_w} \frac{dS_w}{P_c^{2+\alpha}}}{\int_0^{1} \frac{dS_w}{P_c^{2+\alpha}}}$$

$$k_{rnw}(S_w) = \frac{k_{nw}}{k} = \frac{\int_{S_w}^{1} \frac{dS_w}{P_c^{2+\alpha}}}{\int_0^{1} \frac{dS_w}{P_c^{2+\alpha}}}$$

- **What are the limitations of these models?**
- **Can they be addressed?**

Complementary References

- **Peters, E. J., 2012, Advanced Petrophysics. Live Oak Book Company. Chapter 7**
- **Zinszner, B. and Pellerin, F. M., 2007, A Geoscientist's Guide to Petrophysics. Editions Technip.**