AdvGeomec-Poli_Renato_HW5

October 17, 2023

3.6 WP5: Poroelasticity

3.6.1 Exercise 1: Biot coefficient determination The file BiotCoeffExperiment.xlsx has data from a laboratory experiment on a reservoir sandstone that shows axial and radial deformations caused by alternating variations of confining stress P_c and pore pressure P_p.

a. Plot pressure and stresses as a function of time.

b. What are the dPc/dt and dPp/dt loading rates when either confining or pore pressure are increased??

The loading rates are approx. 5 psi/s.

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c. Fit a straight line to the data to obtain a unique relationship between ε_{vol} and σ_{mean} (effective), and calculate the bulk Biot coefficient α. Assuming isotropic elasticity and that the Poisson Ratio is 0.25, what is the Young's modulus?

d. Plot together the volumetric strain with Terzaghi's and Biot's effective stresses.

e. If permeability is k = 100 mD, the fluid is water, porosity is 0.28, and the sample length is 0.05 m with top and bottom drainage, what is the characteristic time for pore pressure diffusion T_{ch}? How does it compare to the pressure/stress loading time? Would it be drained or undrained loading? Note: you need the diffusivity parameter for which you have to look up for properties of water.

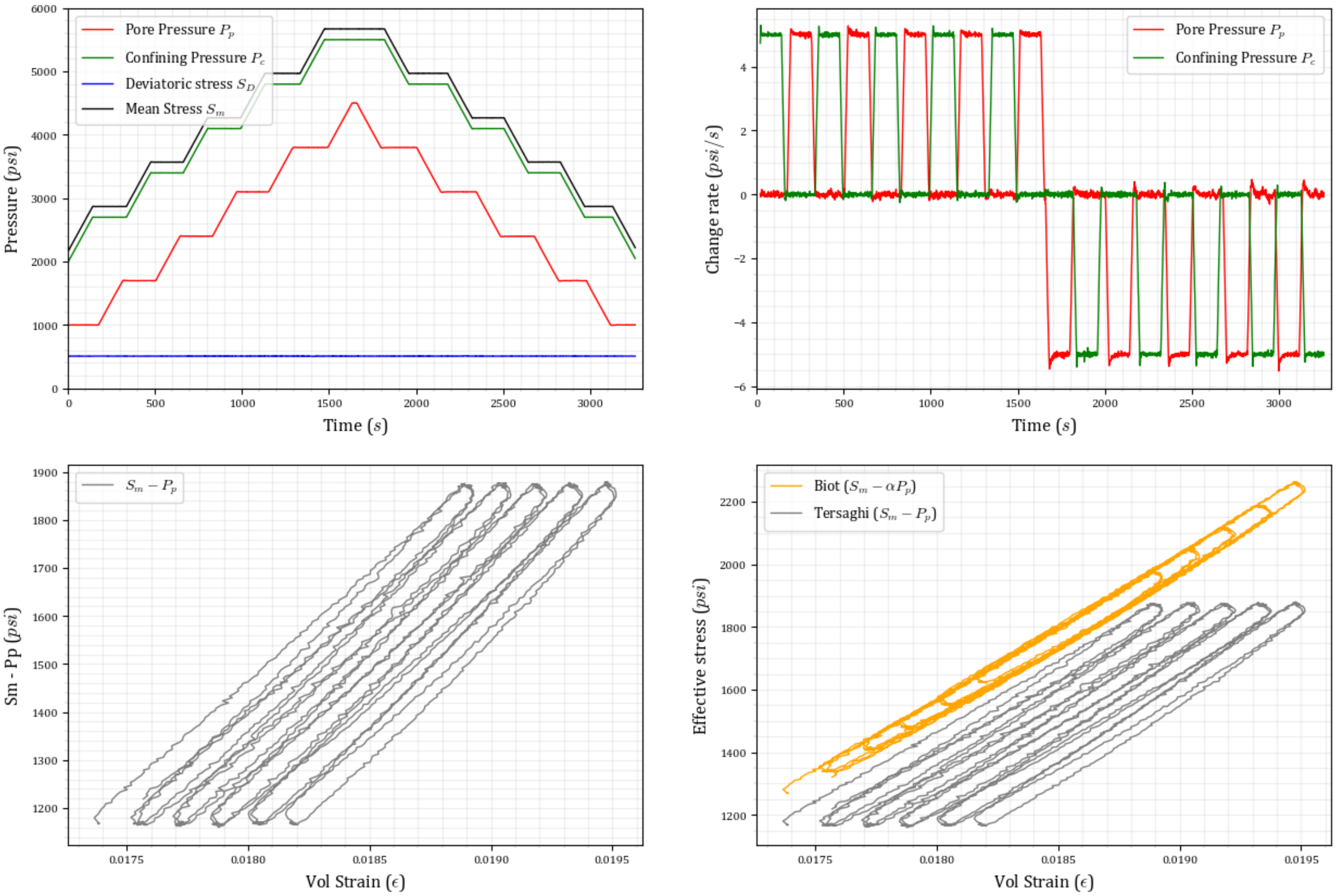
f. EXTRA: Use the theory of transverse isotropic poro-elasticity to figure out the stress paths needed to measure directly α_h and α_v.

2]:
```python
# Support functions
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd

plt.style.use('default')   ## reset!
plt.style.use('paper.mplstyle')

df = pd.read_excel("BiotExp_fmt.xlsx")

df["Pp_psi_s"] = ( df.Pp_psi - df.Pp_psi.shift() ) / ( df.time_s - df.time_s.shift() )
df["Pp_psi_s"] = df.Pp_psi_s.rolling(20).mean()
df["Pc_psi_s"] = ( df.Pc_psi - df.Pc_psi.shift() ) / ( df.time_s - df.time_s.shift() )
df["Pc_psi_s"] = df.Pc_psi_s.rolling(20).mean()

df["strain_vol"] = 2*df.strain_rad + df.strain_ax
df["Sm_psi"] = ( 3*df.Pc_psi + df.Sd_psi ) / 3
df["Sm-Pp"] = df.Sm_psi - df.Pp_psi


# Optimize Biot
from scipy.optimize import minimize
from scipy.stats import linregress
def linreg_biot(alpha) :
    slope, intercept, r_value, p_value, std_err = linregress( df.strain_vol, (df.Sm_psi-alpha* df.Pp_psi) )
    return (1 - r_value**2 )**2
res = minimize( linreg_biot, 0.5 )
alpha = res.x[0]

df["Sm-alpha_Pp"] = df.Sm_psi - alpha* df.Pp_psi
slope, intercept, r_value, p_value, std_err = linregress(df.strain_vol,df["Sm-alpha_Pp"])

from IPython.display import display, Math, Latex
display(Latex(r"\newpage"))

print("### Find Biot coefficient and Young Modulus ###")
print(f"Biot = {alpha:.2f} minimizes gets to the most linear results (R^2 = {r_value**2:.2f}).")

nu = 0.25
K_Pa = slope * 6894.76
E_Pa = K_Pa * 3 * ( 1 - 2 * nu )
print(f"K = slope = {slope:.1f} psi = {K_Pa/1e9:.2f} GPa.")
print(f"E = Young Module = {slope:.1f} psi = {E_Pa/1e9:.2f} GPa.")
```

```python
k_D = 0.1 # Permeability
k_m2 = k_D * 9.869233e-13

phi = 0.28
L_m = 0.05/2
mu_Pa_s = 1e-3
Kf_Pa = 2.1e9 # Pa - water bulk modulus
Ks_Pa = 38e9 # Pa - quartz bulk modulus

M_Pa = 1 / ( ( alpha - phi )/Ks_Pa + phi/Kf_Pa )

Diffusivity = k_m2 * M_Pa / mu_Pa_s
Tch_s = L_m**2 / Diffusivity

print("### Find Relaxation time ###")
print(f"Diffusivity: {Diffusivity:.3f} m2/s")
print(f"Tch: {Tch_s*1e3:.2f} ms")
print("As Tch << T_load, the experiment is DRAINED.")

fig, [[ax1, ax2], [ax3, ax4]] = plt.subplots(2,2)
fig.set_size_inches( 15, 10 )

ax=ax1
ax.plot( df.time_s, df.Pp_psi, c='red', label="Pore Pressure $P_p$" )
ax.plot( df.time_s, df.Pc_psi, c='green',label="Confining Pressure $P_c$" )
ax.plot( df.time_s, df.Sd_psi, c='blue', label="Deviatoric stress $S_D$" )
ax.plot( df.time_s, df.Sm_psi, c='black',label="Mean Stress $S_m$" )


ax.set_ylim( 0,6000 )
ax.set_xlim( 0,3300 )
ax.set_xlabel("Time ($s$)")
ax.set_ylabel("Pressure ($psi$)")
ax.legend()

ax=ax2
ax.plot( df.time_s, df.Pp_psi_s, c='red', label="Pore Pressure $P_p$" )
ax.plot( df.time_s, df.Pc_psi_s, c='green',label="Confining Pressure $P_c$" )

ax.set_xlim( 0,3300 )
ax.set_xlabel("Time ($s$)")
ax.set_ylabel("Change rate ($psi/s$)")
ax.legend()

ax=ax3
ax.plot( df.strain_vol, df["Sm-Pp"], c='gray',label="$S_m-P_p$" )
#ax.set_xlim( 0,3300 )
ax.set_xlabel("Vol Strain ($\epsilon$)")
ax.set_ylabel("Sm - Pp ($psi$)")
ax.legend()

ax=ax4
ax.plot( df.strain_vol, df["Sm-alpha_Pp"], c='orange',label=r"Biot ($S_m-\alpha P_p$)" )
ax.plot( df.strain_vol, df["Sm-Pp"], c='gray',label=r"Tersaghi ($S_m- P_p$)" )
#ax.set_xlim( 0,3300 )
ax.set_xlabel("Vol Strain ($\epsilon$)")
ax.set_ylabel(r"Effective stress ($psi$)")
ax.legend()

''
```

```
### Find Biot coefficient and Young Modulus ###
Biot = 0.90 minimizes gets to the most linear results (R^2 = 0.98).
K = slope = 462381.2 psi = 3.19 GPa.
E = Young Module = 462381.2 psi = 4.78 GPa.
### Find Relaxation time ###
Diffusivity: 0.660 m2/s
Tch: 0.95 ms
As Tch << T_load, the experiment is DRAINED.
```

2]: ''



Advanced Geomechanics – WP5 10/18/2023

3.6.2 Exercise 2: Depletion stress path

For this problem you have to use the geomechanical module of reservoir simulator CMG https://www.cmgl.ca/. The software is available to UT Austin students here: http://pge.utexas.edu/LRC/

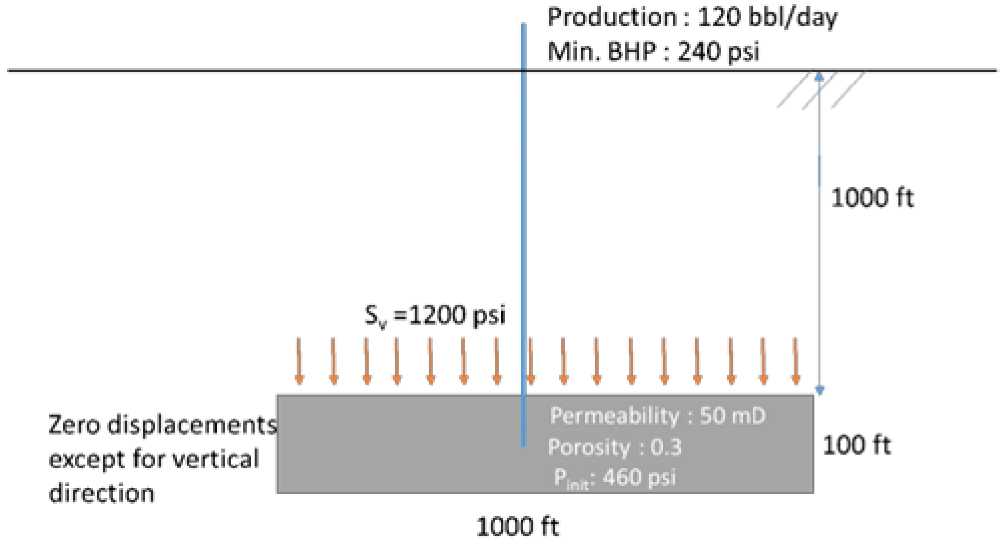
a. Review the files CMG_Geomechanics_Tutorial.pdf and CMG_Running_InputFile.pdf.

OK

b. Change the vertical stress and well schedule as shown in the figure below (example files: Injection1.dat and Production1.dat).

Used Production1.dat to simulate depletion.

Figure 3.28: Schematic cross section of reservoir model for depletion.



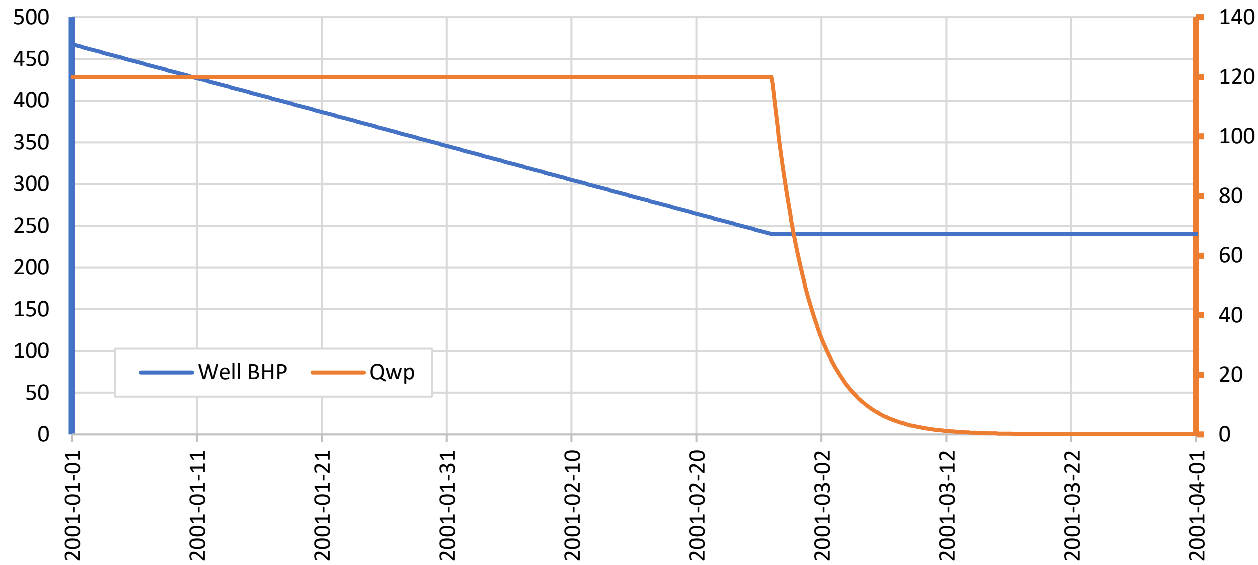
c. What is initial boundary condition in each direction? (i.e. constant stress or zero displacement).

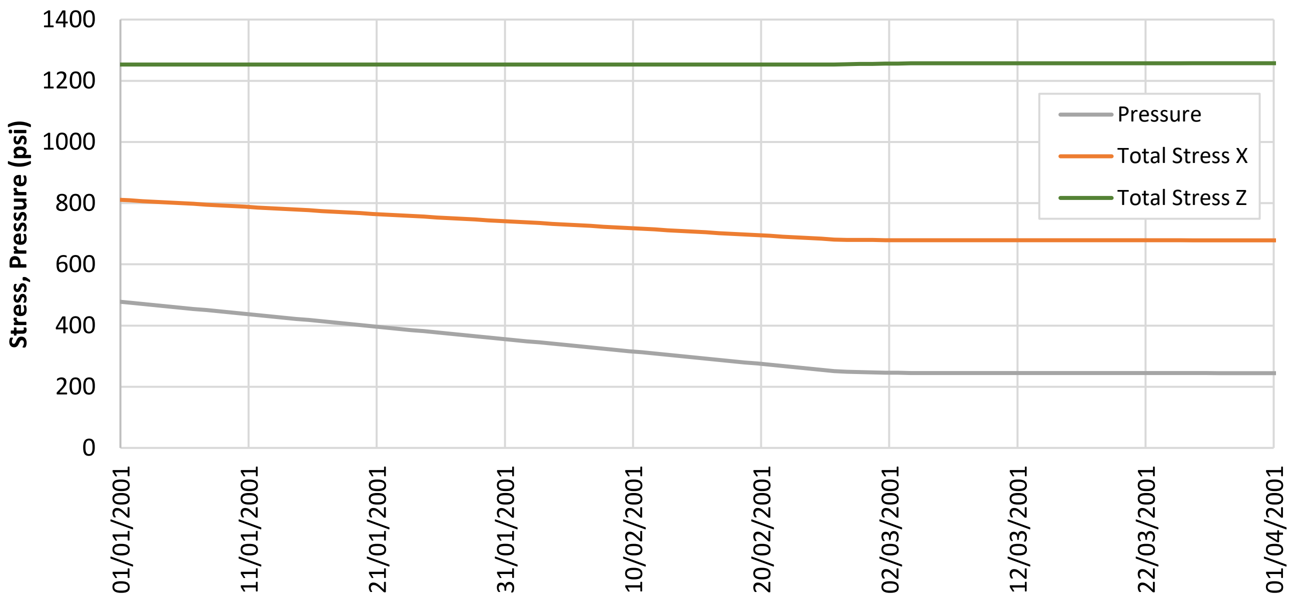
No displacement at front, back, bottom, left and right.
We set constant total stress on top of the reservoir.

```
*RCONBT *ALL ** On the bottom
*RCONLF *ALL ** On the left
*RCONRT *ALL ** On the right
*RCONBK *ALL ** On the back
*RCONFT *ALL ** On the front
```

```
*DLOADBC3D
*IJK 1:21 1:21 1 **top
** node1 node2 node3 node4 load
1      2      3      4      86.455 ** tonf/m2
```

d. Plot 1 - Plot minimum principal total stress (Total stress I), vertical total stress (Total stress K), and pore pressure (Pressure) vs time. (**Note: Please remove initial data (time = 0) when you plot).





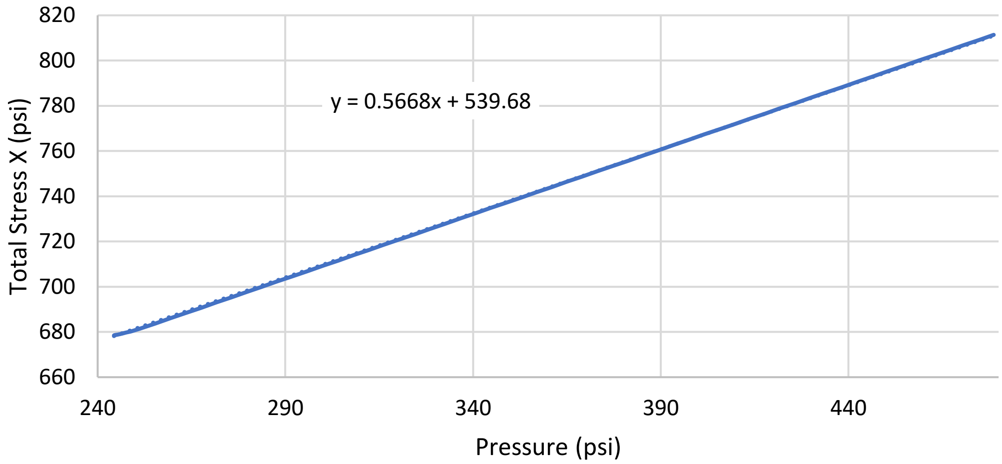
e. Plot 2 - Plot minimum principal stress (y-axis) vs pore pressure (x-axis), and verify the slope of the curve is similar with $\alpha\frac{1-2\nu}{1-\nu}$ (α is the Biot coefficient and ν is Poisson's ratio)

***Note: Please remove initial data (time = 0) when you plot pressure and stresses).*

$$\boldsymbol{\sigma_{min} = 0.5668\,P_p + 539.84 \quad \Longrightarrow \quad \frac{d\sigma_{min}}{dP_p} = 0.5668}$$

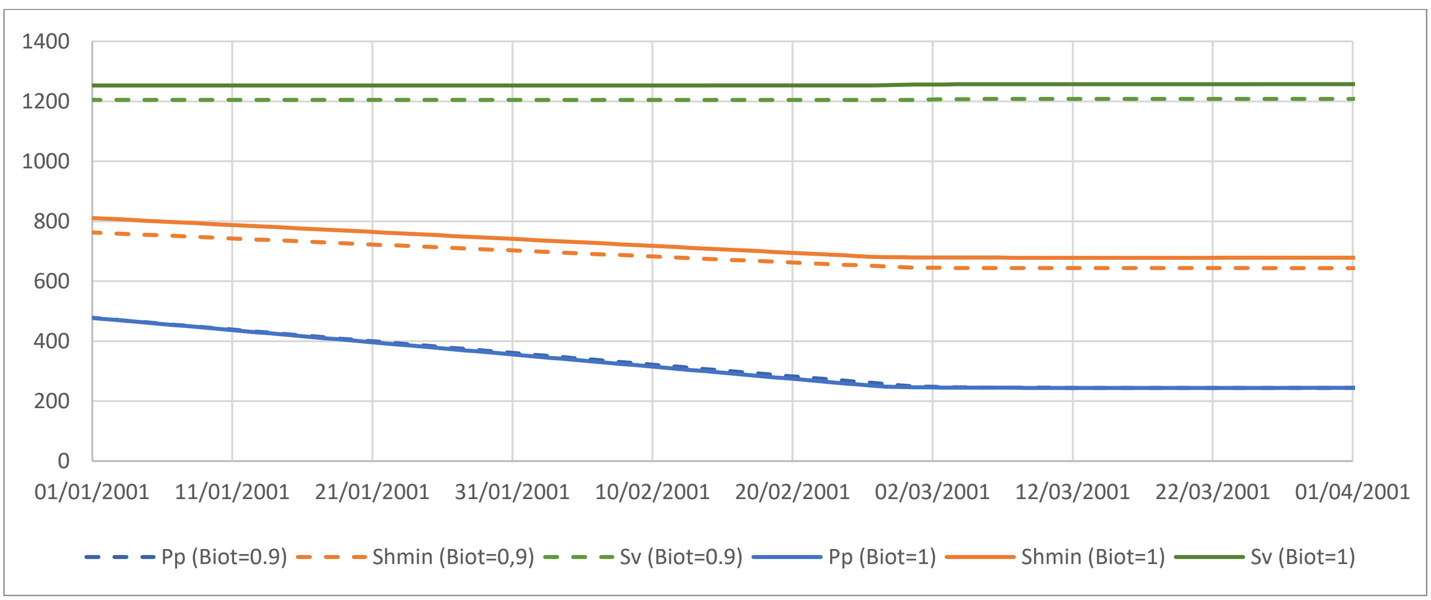
$$\boldsymbol{\alpha = 1 \quad \nu = 0.3 \quad \Longrightarrow \quad \alpha\frac{1-2\nu}{1-\nu} = 0.5714}$$

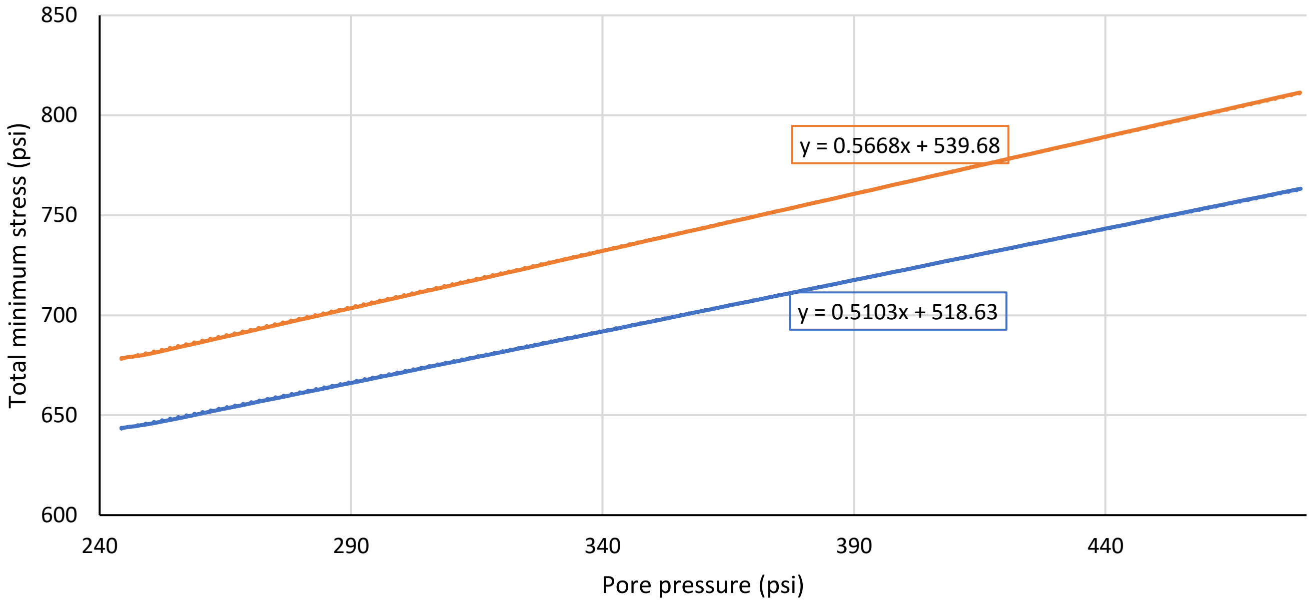
We can see that the slope is similar to $\left[\alpha\frac{1-2\nu}{1-\nu}\right]$.



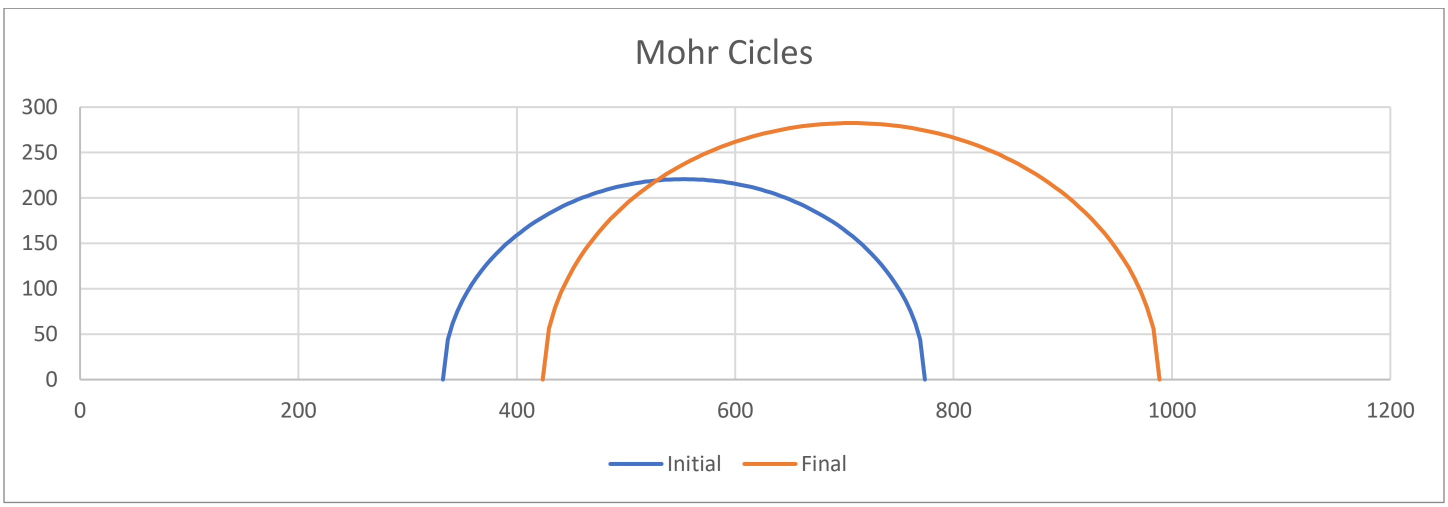
f. Run the simulation again using Biot coefficient from the previous laboratory problem, repeat the question “d” using the new simulation result and plot on the same figure.

Running with $\alpha = 0.9$ ($\alpha \frac{1-2\nu}{1-\nu} = 0.5143$), we get the following plots, and $\frac{d\sigma_{min}}{P_p} = 0.5103$, as expected.



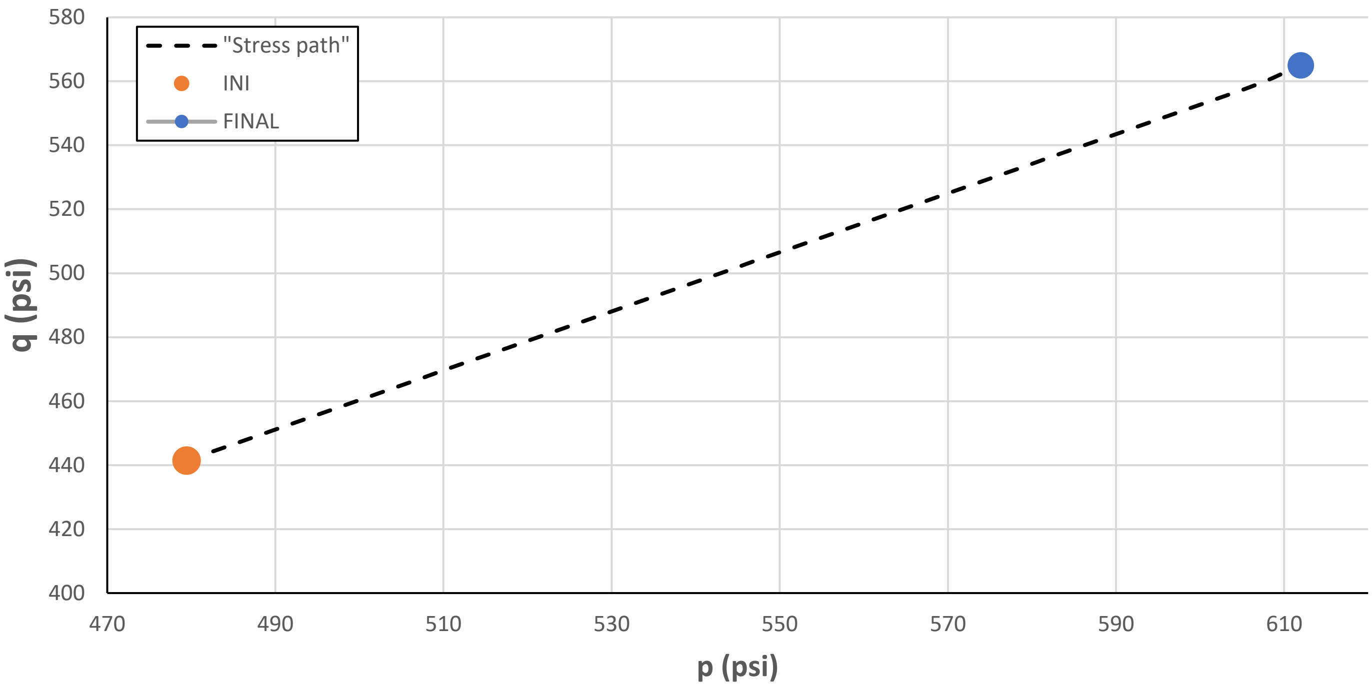


g. Plot the stress path with Mohr circles for the initial (0.1 days) and final time (100 days).



h. Plot the stress path in the (p', q) space for the same period of time.

$$p' = \frac{\sigma_1 + 2\,\sigma_3}{3}$$
$$q = \sigma_1 - \sigma_3$$



i. What is the absolute minimum pressure to create a hydraulic fracture (i.e. minimum principal total stress) at the end of the simulation when bottom-hole pressure is BHP = 240 psi? Compare with the analytical solution.

The total horizontal minimum stress in the end of the simulation is about $\mathbf{643\ psi}$, with $P_p = 244\text{psi}$. The minimum pressure to create a hydraulic fracture would thus be around 643psi.

The analytical estimates would be about $\mathbf{641.5\ psi}$ which is quite similar to the numerical results. The calculations are below.

$$\boldsymbol{S_{22} = \frac{\nu}{1-\nu} S_{33} + \alpha \frac{1-2\nu}{1-\nu} P_p}$$

$$\boldsymbol{S_{22} = \frac{0.3}{1-0.3} 1204 + 0.9 \frac{1-0.6}{1-0.3} 244 = 516 + 125.5 = 641.5\text{psi}}$$