QP6

CASE 1 :- Backdate non linearities
- 2 step solution (eq1 → eq2)

EQN 1

u_t : LHS: $u^{n+1}\left(\frac{1}{\Delta t}\right)$ RHS: $u^n\left(\frac{1}{\Delta t}\right)$

$v^2 u_{xx}$: LHS : $-\frac{v^2}{\Delta x^2}\left(u_{i-1} - 2u_i + u_{i+1}\right)$

$2 v v_x u_x$: LHS : $-2v\left(\frac{v_i - v_{i-1}}{\Delta x}\right)\left[\frac{u - u_{i-1}}{\Delta x}\right]$

$-u_v$: LHS : $v(u_i)$

u^2 : LHS : $-u_i^n\left(u^{n+1}\right)$

10 : RHS : 10 .

EQN 2

V_t : LHS : $V^{n+1}\left(\frac{1}{\Delta t}\right)$ RHS : $V^{n}\left(\frac{1}{\Delta t}\right)$

$u^2 V_{xx}$: LHS $\ominus \frac{u^2}{\Delta x^2}\left(V_{i-1} - 2V_i + V_{i+1}\right)$

$2uu_x V_x$: LHS : $\ominus 2u\left(\frac{u - u_{i-1}}{\Delta x}\right)\left(\frac{V - V_{i-1}}{\Delta x}\right)$

u_{xx} : RHS : $\frac{u_{i-1} - 2u_i + u_{i+1}}{\Delta x^2}$

uv : LHS : $-u\, V^{n+1}$

$-V^2$: LHS : $V^n\, V^{n+1}$

BCs:

$$u_x + \sin(uv) = 1/2$$

$$\frac{u_{i+1} - u_i}{\Delta x} + \sin(u_i v_i) = 1/2 \quad @\ x = 1$$

— x —

i:	-1	0	1	2		$i-1$	i	$i+1$			$n-1$	n	$n+1$
ξ	0	1	2	3		$\xi-1$	ξ	$\xi+1$	$\dots$				

$$\xi = i + 1 \quad ; \quad [0, nx+2]$$

— x —

At $i = N$

$$\frac{u_{N+1} - u_N}{\Delta x} + \sin(u_N v_N) = 1/2$$

$$\boxed{u_{N+1} - u_N = \Delta x/2 - \Delta x \sin(u_N v_N)} \leftarrow \boxed{\text{LHS}}$$

— x —

$$\boxed{V_{N+1} - V_N = \Delta x \cos(U_N V_N) + \Delta x} \leftarrow \text{LHS}.$$

Case 2 $\boxed{f\ \text{eqn}\ 1}$

$$f_1 = -u_t + v^2 u_{xx} + 2v v_x u_x - u\,v + u^2 + 10$$

$$f_{1i} = -\frac{u - u^n}{\Delta t} + v^2\left(\frac{u_{i+1} - 2u_i + u_{i-1}}{h^2}\right) + 2v\left(\frac{v - v_{i-1}}{h}\right)\left(\frac{u - u_{i-1}}{h}\right)$$

$$-u.v + u^2 + 10$$

— x —

$$\frac{\partial f_{1i}}{\partial u_{i-1}} = \frac{v^2}{h^2} + 2v\left(\frac{v - v_{i-1}}{h}\right)\left(-\frac{1}{h}\right)$$

$$\frac{\partial f_{1i}}{\partial u} = -\frac{1}{\Delta t} - \frac{2v^2}{h^2} + \frac{2v\,dv}{h^2} - v + 2u$$

$$\frac{\partial f_{1i}}{\partial u_{i+1}} = \frac{v^2}{h^2}$$

$$\frac{\partial f_{1i}}{\partial v_{i-1}} = -\frac{2v}{h^2}\,d\,u$$

$$\frac{\partial f_{1i}}{\partial v_i} = \left(\frac{u_{i+1} - 2u + u_{i-1}}{h^2}\right) 2v + \left(\frac{u - u_{i-1}}{h}\right)\left(\frac{4v}{h}\right) - \frac{2v_{i-1}}{h^2}(u_i - u_{i-1})$$

$$-u$$

Case 2 - eqn 2

$$f_2 = -v_t + u^2 v_{xx} + 2u u_x v_x + u_{xx} + uv - v^2$$

$$f_2 = -\frac{v - v^n}{\Delta t} + u^2\left(\frac{v_{i-1} - 2v + v_{i+1}}{h^2}\right) + 2u\left(\frac{u - u_{i-1}}{h}\right)\left(\frac{v - v_{i-1}}{h}\right) +$$

$$+ \frac{u_{i-1} - 2u + u_{i+1}}{h^2} + uv - v^2$$

$$\frac{\partial f_2}{\partial u_{i-1}} = -\left(\frac{1}{h}\right) 2u\left(\frac{v - v_{i-1}}{h}\right) + \frac{1}{h^2}$$

$$\boxed{-2u_{i-1}\, dv/h^2}$$

$$\frac{\partial f_2}{\partial u_i} = 2u\left(\frac{v_{i-1} - 2v + v_{i+1}}{h^2}\right) + \frac{4u}{h}\left(\frac{v - v_{i-1}}{h}\right) - \frac{2}{h^2} + v$$

$$\frac{\partial f}{\partial v_{i-1}} = \frac{u^2}{h^2} + \left(-\frac{1}{h}\right)(2u)\left(\frac{u - u_{i-1}}{h}\right)$$

$$\frac{\partial f}{\partial v} = \left(-\frac{1}{\Delta t}\right) + (-2)\left(\frac{u^2}{h^2}\right) + \left(\frac{1}{h}\right)(2u)\left(\frac{u - u_{i-1}}{h}\right) + u - 2v$$

$$\frac{\partial f}{\partial v_{i+1}} = \left(\frac{u^2}{h^2}\right)$$

$$\frac{\partial f}{\partial u_{i+1}} = \left(\frac{1}{h^2}\right)$$

Boundary conditions

@ $x=1$

$$f_{B1} = u_x + \sin(uv) - 1/2$$

$$f_{B1} = \frac{u_N - u_{N-1}}{h} + \sin(u_N v_N) - 1/2$$

$$\frac{\partial f_{B1}}{\partial u_N} = \frac{1}{h} + v_N \cos(u_N v_N)$$

$$\frac{\partial f_{B1}}{\partial u_{N-1}} = -\frac{1}{h}$$

$$\frac{\partial f_{B1}}{\partial v_N} = u_N \cos(u_N v_N)$$

Boundary Condition 2 :

$$f_{B2} = V_x - \cos(UV) - 1$$

$$= \frac{V_N - V_{N-1}}{h} - \cos(U_N V_N) - 1$$

$$\frac{\partial f_{B2}}{\partial V_N} = \frac{1}{h} + U_N \sin(U_N V_N)$$

$$\frac{\partial f_B}{\partial V_{N-1}} = -\frac{1}{h}$$

$$\frac{\partial f_B}{\partial U_N} = V_N \sin(U_N V_N)$$

Newton Solver

$\underset{\sim}{R}(\underset{\sim}{r}^{k}) \to 0$

$\underline{K}(\underset{\sim}{r}^{k})\, \delta^{k+1} = \underset{\sim}{R}(\underset{\sim}{r}^{k})$

$\underset{\sim}{r}^{k+1} = r^{k} + \delta^{k+1}$

$\underset{\sim}{r} = \begin{bmatrix} \underset{\sim}{r}_F \\ \underset{\sim}{r}_P \end{bmatrix}$ $\begin{matrix} \to \text{free} \\ \to \text{prescribed.} \end{matrix}$ $\underset{\sim}{r}_P = \bar{\underset{\sim}{r}}_P$

$\underline{K} = \begin{bmatrix} \underline{K}_{FF} & \underline{K}_{FP} \\ \underline{K}_{PF} & K_{PP} \end{bmatrix}$

$\begin{bmatrix} \underline{K}_{FF} & \underline{K}_{FP} \\ \underline{K}_{PF} & \underline{K}_{PP} \end{bmatrix} \begin{bmatrix} \underset{\sim}{\delta}_F \\ \underset{\sim}{\Delta}_P \end{bmatrix} = \begin{bmatrix} \underset{\sim}{R}_F \\ \underset{\sim}{R}_P \end{bmatrix}$

Solve first line:

$$\boxed{K_{FF}\, \underset{\sim}{\delta}_F = \underset{\sim}{R}_F - \underline{K}_{FP}\, \underset{\sim}{\delta}_P .}$$

BC: become homogeneous (solving for $\underset{\sim}{\delta}$)