Advanced Petrophysics
PGE 381L, Fall 2023
Unique Number: 20215

Homework Assignment No. 5

October 10, 2023
Due on Thursday, October 19, 2023, before 11:00 PM

Name: ________________**SOLUTION**______________________________

UT EID: ____

Objectives:

a) To practice application of Darcy's law
b) To practice assessment of permeability using analytical models
c) To practice analysis of experimental data for permeability assessment
d) To practice analysis of pressure transient data for assessment of permeability

Note: Please scan your homework assignment and upload it as one pdf file on the Canvas website before the deadline. Please name your homework document as follows:

PGE381L_2023_Fall _HW05_lastname_name.pdf

Example: PGE381L_2023_Fall_HW05_Heidari_Zoya.pdf

Question 1: Resistivity of a formation at depth X, which is above the capillary transition zone, is equal to 117 ohm-m. You can assume the clay-free rocks at depth X are at irreducible water saturation which is estimated to be 20%. Some other formation properties are given as follows:

Grain diameter = 120 μm
Formation water resistivity = 0.03 ohm-m
Archie's model parameters: a=1, m=2, n=2

Estimate absolute permeability (in Darcy) of the formation at depth X. You can assume that the pore network is circular in cross section.

$$\tau = F\phi$$

$$K = \frac{D^2 \phi^3}{72\tau(1-\phi)^2}.$$

$$R_t = \frac{a R_w}{\phi^m S_w^n}, \quad S_w = S_{irr} = 0.2.$$

$$\Rightarrow 117 = \frac{0.03}{\phi^2 (0.2)^2} \Rightarrow \phi = 0.08.$$

$$F = R_o/R_w = \frac{(R_w \cdot a/\phi^m)}{R_w}$$

$$= \frac{1}{(0.08)^2} = 156.25$$

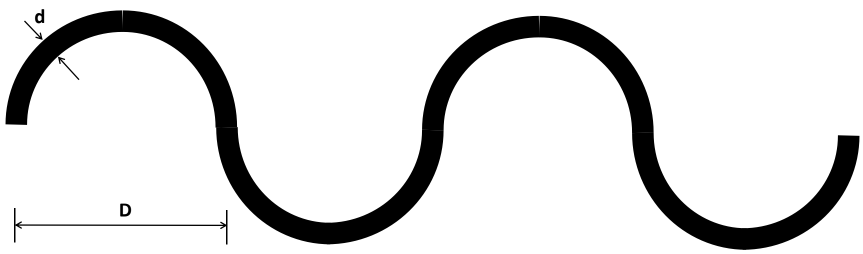
$$\tau = F\phi = 12.5$$

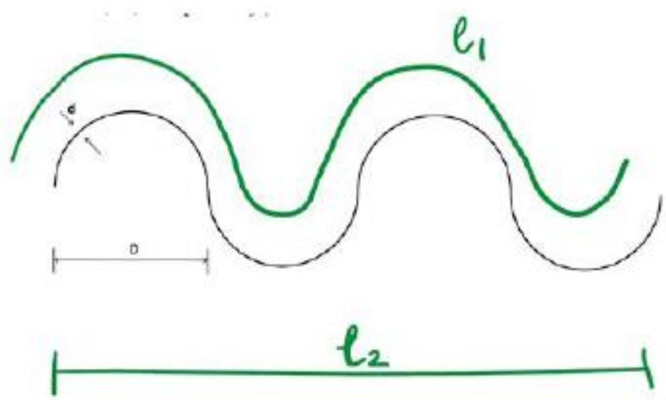
$$K = \frac{(120 \times 10^{-6})^2 (0.08)^3}{(72)(12.5)(1-0.08)^2}$$

$$= 9.68 \times 10^{-15}\ m^2 = 0.0098\ D$$

$$= 9.8\ mD$$

Question 2: A porous medium is composed of *n* parallel connected tubes with the following shape. Assume the tubes are circular in cross section. Derive an expression for the permeability of this porous medium in terms of *d*, *D*, and porosity ϕ.





Circular cross section $\longrightarrow$ $k_0 \simeq 2$

$$\left.\begin{aligned} l_1 &= 2\pi D \\ l_2 &= 4D \end{aligned}\right\} \Rightarrow \tau = \left(\frac{l_1}{l_2}\right)^2 = \left(\frac{\pi}{2}\right)^2$$

$$\frac{V}{S} = \frac{\pi r^2}{2\pi r} = \frac{r}{2} = \frac{d}{4} \quad \text{OR} \quad S_p = \frac{2\pi n r l_e}{\pi r^2 n l_e} = \frac{2}{r} = \frac{4}{d}$$

$$k = \frac{\phi}{2\tau S_p^2} = \frac{\phi}{2\left(\frac{\pi}{2}\right)^2}\left(\frac{d}{4}\right)^2 = \frac{\phi d^2}{8\pi^2}$$

$$\Rightarrow \boxed{k = \phi d^2 / 8\pi^2}$$

Question 3: Figure 1 shows the probability distribution function for pore diameter in a given rock sample. You can assume that this porous media consists of a bundle of straight capillary tubes in a solid impermeable material. On average the cross-sectional areas of the tubes occupy 23% of the bulk cross-sectional area. Estimate the following properties of this rock sample:

- **a)** Porosity
- **b)** Permeability

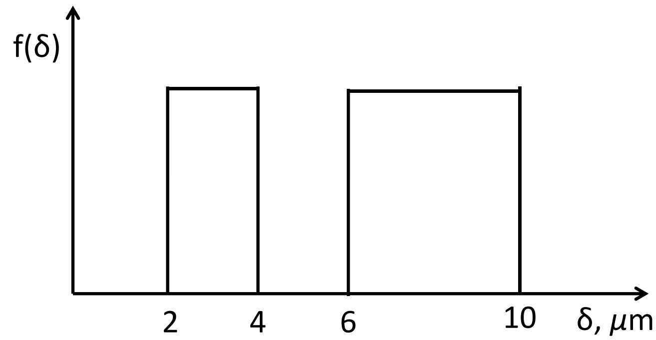


Figure 1: The probability distribution function for pore diameter in a rock sample.

$$\int_0^\infty f(\delta)\, d\delta = 1$$

$$\rightarrow 2C + 4C = 1$$

$$\rightarrow C = 1/6$$

(a) $\phi = 23\%$

(b) $$K = \frac{\phi}{32\tau}\left[\frac{\int_2^4 f_1(\delta)\, \delta^4 d\delta + \int_6^{10} f_2(\delta)\, \delta^4 d\delta}{\int_2^4 f_1(\delta)\, \delta^2 d\delta + \int_6^{10} f_2(\delta)\, \delta^2 d\delta}\right], \quad f_1 = C, \quad f_2 = C$$

$\tau = 1$

$$\Rightarrow K = \frac{0.23}{32\,(1)}\left[\frac{C\, \delta^5/5\,\big|_2^4 + C\, \delta^5/5\,\big|_6^{10}}{C\, \delta^3/3\,\big|_2^4 + C\, \delta^3/3\,\big|_6^{10}}\right]$$

$$\Rightarrow K = \frac{0.23}{32(1)}\left[\frac{\frac{1}{5}(4^5 - 2^5) + \frac{1}{5}(10^5 - 6^5)}{\frac{1}{3}(4^3 - 2^3) + \frac{1}{3}(10^3 - 6^3)}\right]$$

$$\Rightarrow K = 4.78 \times 10^{-13}\ m^2 = 0.485\ D = \underline{\underline{485\ mD}}$$

Question 4: The following data were obtained in a gas permeameter experiment for the determination of the permeability of a clean core plug.

Core diameter	=	2.54 cm
Core length	=	2.54 cm
Gas viscosity	=	0.018 cp
Atmospheric pressure	=	760 mm Hg (mercury)

Upstream Pressure (mmHg)	Downstream Pressure	Flow Rate (Atmospheric Pressure, cc/min)
101	Atmospheric	6.4
507	Atmospheric	35.6
1520	Atmospheric	132.8

Determine the absolute permeability of the core plug in millidarcy. Did you make the Klinkenberg correction?

$$q_{sc} = \frac{k_g A}{2 P_{sc} \mu L} \left(P_1^2 - P_2^2\right)$$

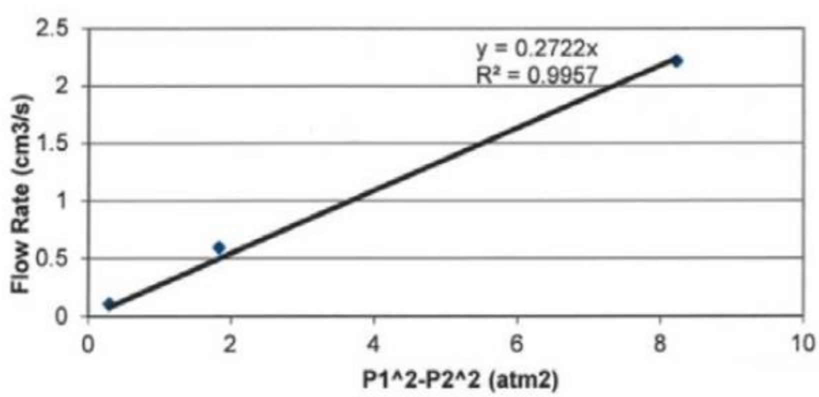
$P_{sc} = P_2 = 760$ mmHg $= 1.01325$ atm

$P_1 =$ Upstream pressure + Atmospheric pressure

$P_2 = 760$ mmHg $= 1.01325$ atm

$\mu = 0.018$ cp

$L = 2.54$ cm $\longrightarrow$ $A = \pi r^2 = \pi \left(\frac{2.54}{2}\right)^2 = 5.067$ cm^2



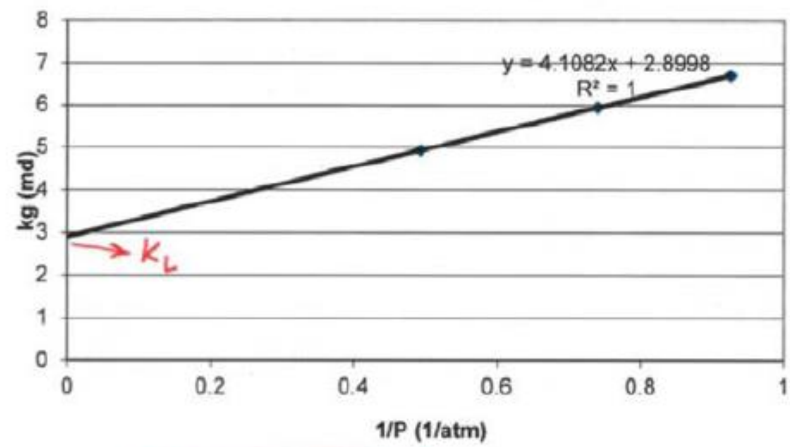
$$\frac{k_g A}{2 P_{sc} \mu L} = 0.2722$$

$$\rightarrow \quad k_g = \frac{2(1.01325)(0.018)(2.54)(0.2722)}{5.067}$$

$$\rightarrow \quad \boxed{k_g = 4.977 \text{ mD}}$$

We have to make Klinkenberg correction to estimate k_L:

$$k_g = k_L \left(1 + \frac{b}{P}\right)$$



$$\Rightarrow \quad \boxed{k_L = 2.9 \text{ mD}}$$

Bonus Questions:

You do not need to submit solutions to the following questions, unless you are interested in receiving bonus credit. These questions are aimed to provide an opportunity of additional practice.

Question 5: The permeability tensor for a 2D reservoir in the *x*-*y* Cartesian coordinate system is given by

$$\bar{\bar{k}}(x,y)=\begin{bmatrix}70 & 20\\ 20 & 50\end{bmatrix}md$$

Viscosity of the reservoir fluid can be assumed to be 1 cp. The potential gradient is given by

$$\nabla\vec{\Phi}=0.3i+0.2j(atm/cm)$$

Answer the following questions:

a) The magnitude and direction of Darcy velocity with respect to *x* coordinate.

b) What angle does the flow direction make with respect to the potential gradient?

c) Estimate the maximum directional permeability that can be observed in this reservoir and its direction with respect to *x* coordinate.

$$\begin{vmatrix}70-\lambda & 20\\ 20 & 50-\lambda\end{vmatrix}=0 \rightarrow \lambda^2+3500-120\lambda-400=0$$

$$\rightarrow \lambda^2-120\lambda+3100=0$$

2

$$\rightarrow \boxed{\lambda_1=82.36},\ \lambda_2=37.64\ md$$

$$\begin{bmatrix}70-82.36 & 20\\ 20 & 50-82.36\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$-12.36x+20y=0 \rightarrow \vec{u}=\begin{bmatrix}20/12.36\\ 1\end{bmatrix}$$

$$\tan 2\theta=\frac{2(20)}{70-50}=2 \Rightarrow 2\theta=63.4^\circ$$

$$\Rightarrow \boxed{\theta=31.7}$$

2

$$\cos\beta = \frac{\nabla\Phi \cdot V_d}{|\vec{\nabla\Phi}||\vec{V_d}|} = \frac{(-0.025)(0.3) + (-0.016)(0.2)}{(0.0297)\sqrt{0.3^2 + 0.2^2}}$$

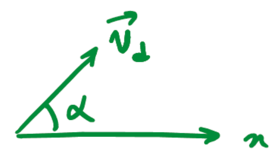
$$\Rightarrow \boxed{\beta \simeq 179^0} \swarrow 2$$

$$\vec{V_d} = -\frac{k}{\mu}\vec{\nabla\Phi} = \frac{-1}{1}\begin{bmatrix} 0.07 & 0.02 \\ 0.02 & 0.05 \end{bmatrix}\begin{bmatrix} 0.3 \\ 0.2 \end{bmatrix}$$

$$\Rightarrow \vec{v_d} = \begin{bmatrix} -0.025 \\ -0.016 \end{bmatrix} (cm/s)$$

$$|\vec{v_d}| = \sqrt{0.025^2 + 0.016^2} = \underline{\underline{0.0297}}\ cm/s \swarrow 2$$

$$\cos\alpha = \frac{\vec{v_d} \cdot \vec{u}}{|\vec{v_d}|} = \frac{-0.025}{0.0297}$$



$$\Rightarrow \boxed{\alpha = 212.7^0} \leftarrow 2 \quad \text{OR} \quad 147.3^0$$

Question 6: Consider the pressure transient data provided to you for practice purpose during the lecture time. Answer the following questions:

a) Analyze the drawdown and buildup tests and determine the formation permeability, the total skin factor and the average reservoir pressure.

b) Is this well damaged or stimulated? Justify your answer.

c) Perform the log-log diagnostic plots for the drawdown and buildup tests, separately. Compute the wellbore storage coefficients and the dimensionless wellbore storage coefficients for the tests.

Note: We will solve this question in a team effort during our lecture time on October 19th. The purpose for having this question is to give you a chance to review that process individually.

Question 7: Figure 2 shows the cross section of a cylindrical porous medium which consists of five capillary tubes (labeled 1, 2, 3, 4 and 5) encased in a solid impermeable matrix. The dimensions of the capillary tubes and the porous medium are given in Table 1. (Question 3.30 in your textbook)

Table 1

Item	Dimension in millimeters
Diameter of porous medium	50
Length of porous medium	200
Diameter of capillary tube 1	6
Diameter of capillary tubes 2,3,4 and 5	3

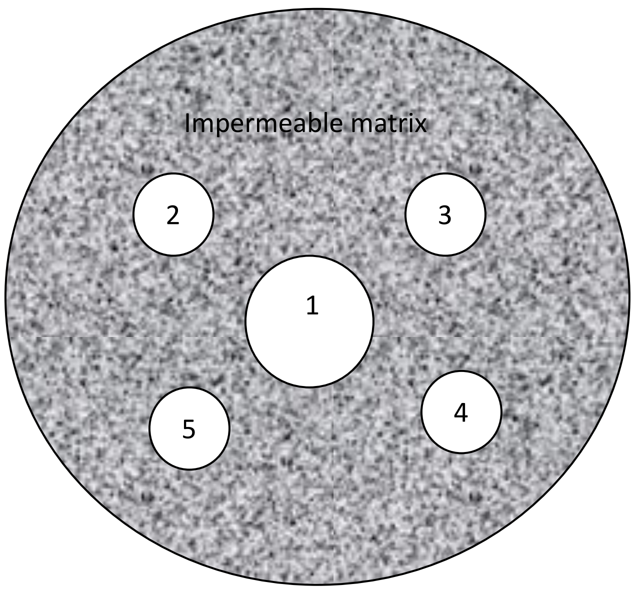


Figure 2: Porous medium for Question 7

Answer the following questions:

a) Calculate porosity of this porous medium.

b) Calculate Permeability of this porous medium in darcies.

c) Calculate specific surface area per unit bulk volume of this porous medium. Please state your units for specific surface area.

a) $$\phi = \frac{4\pi(3/2)^2 + \pi(6/2)^2}{\pi(50/2)^2} = 2.88\%$$

b) $$\left.\begin{aligned} q &= \frac{\pi r^4}{8\mu}\frac{\Delta P}{L} \\ q &= \frac{kA}{\mu}\frac{\Delta P}{L} \end{aligned}\right\} \rightarrow k = \frac{\pi}{8A}\sum_{i=1}^{5} r_i^4$$

$$k = \frac{\pi}{8\pi\left(\frac{50/2}{10}\right)^2}\left[4\left(\frac{3}{2\times10}\right)^2 + \left(\frac{6}{2\times10}\right)^2\right]$$

mm to cm

$$\Rightarrow k = 2.0255\times10^{-4}\ cm^2 = 20,524\ D$$

c) $$S_p = \frac{4(2\pi(0.15)L) + (2\pi(0.3)L)}{4(\pi(0.15)^2L) + (\pi(0.3)^2L)} = \frac{8(0.15)+2(0.3)}{4(0.15)^2+(0.3)^2} = 10\ cm^{-1}$$

cm

$$S = \phi S_p = 10\phi = 10(0.0288) \longrightarrow \boxed{S = 0.288\ cm^{-1}}$$

$$S_s = \frac{A\phi}{V_p(1-\phi)} = \frac{S_p\phi}{(1-\phi)} = \frac{S}{(1-\phi)} = \frac{0.288}{(1-0.0288)} = 0.3\ cm^{-1}$$

Per grain volume