The University of Texas at Austin — WHAT STARTS HERE CHANGES THE WORLD

Advanced Petrophysics: Dispersion in Porous Media

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PGE381L Outline

- Introduction to petrophysics, geology, and formation data
- Porosity
- Fluid saturations
- Permeability
- Quantification of heterogeneity, spatial data analysis, and geostatistics
- Interfacial phenomena and wettability
- Capillary pressure
- Relative permeability
- Dispersion in porous media
- Introduction to petrophysics of unconventional reservoirs

What do we learn in this lecture?

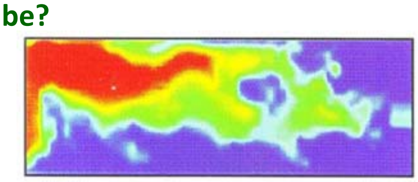
- What is dispersion?
- Factors affecting efficiency of a miscible displacement
- Origins of dispersion in porous media
- Advection-Dispersion equation
- Solution to advection-dispersion equation
- Dispersion coefficient and dispersivity
- Examples

What is Dispersion?

Dispersion: Mixing and spreading of the injected fluid, when a miscible fluid displaces another in a porous medium.

Examples:

- **Contaminant transport in aquifers**
- **Improved oil recovery from petroleum reservoirs**
  - **What is the capillary force to trap the displaced oil in a miscible displacement process?**
  - **What would you expect the recoverable oil to be?**
  - **Then what is the problem?**
- **Mud-filtrate invasion**
- **Other examples?**



0.25 pore volume injected

Factors Affecting Efficiency of a Miscible Displacement

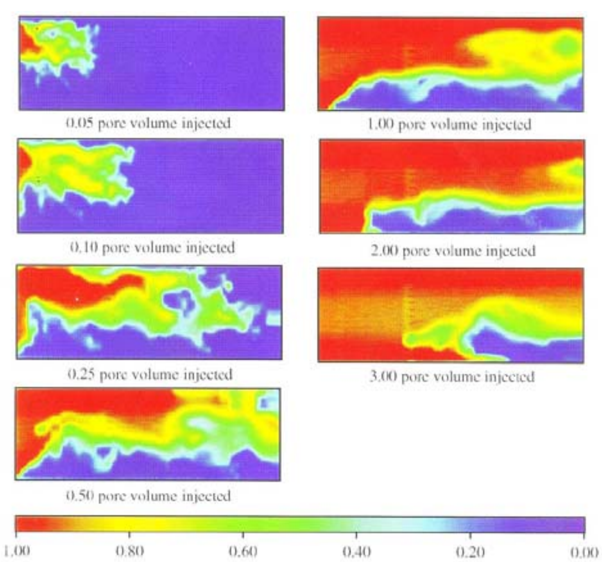
- **Effect of gravity**
- **Dispersion of the porous medium**
- **Heterogeneity of the porous medium**
- **Mobility ratio**

Mobility of a fluid phase $$\lambda_{phase} = \frac{k_{phase}}{\mu_{phase}}$$

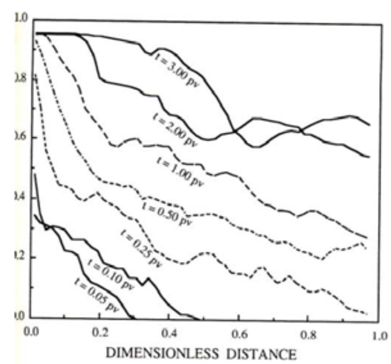
Mobility ratio $$M = \frac{\lambda_{displacing\ fluid}}{\lambda_{displaced\ fluid}} = \frac{\mu_{displaced\ fluid}}{\mu_{displacing\ fluid}}$$

Mobility ratio $\downarrow$ ➡ **Displacement efficiency** $\uparrow$

Examples: Impact of Gravity

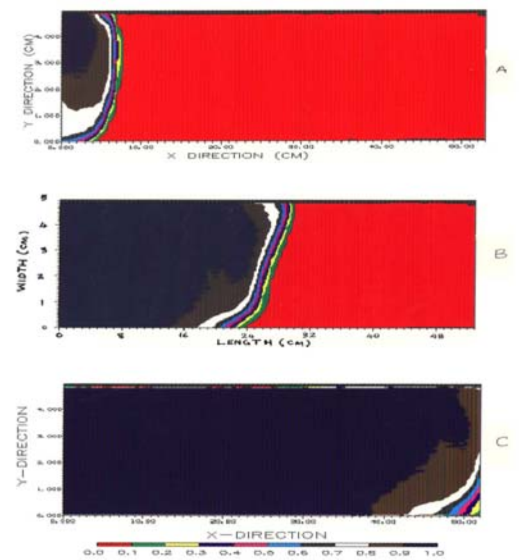


First-contact miscible displacement in a sandpack
M = 54

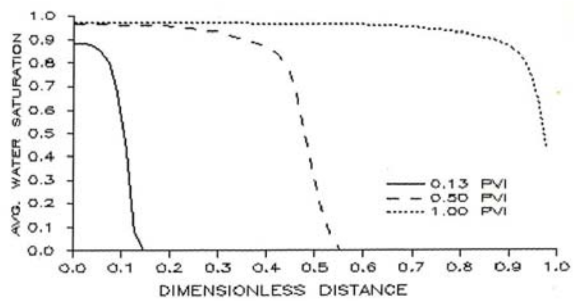


Source: Peters and Afzal, 1992; Peters, E. J., 2012, Advanced Petrophysics

Examples: Impact of Mobility Ratio

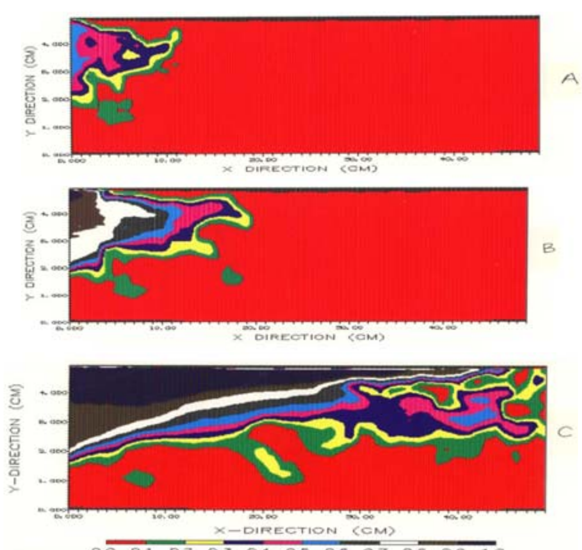


First-contact miscible displacement in a sandpack
M = 0.82

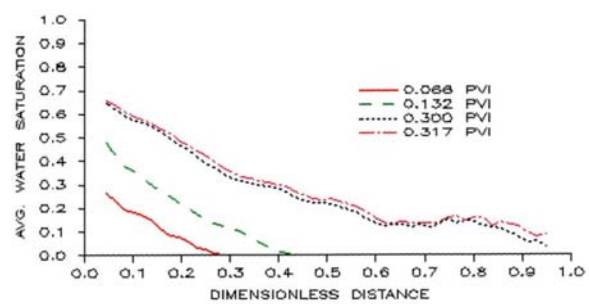


Source: Peters and Hardham, 1990; Peters, E. J., 2012, Advanced Petrophysics

Examples: Impact of Mobility Ratio



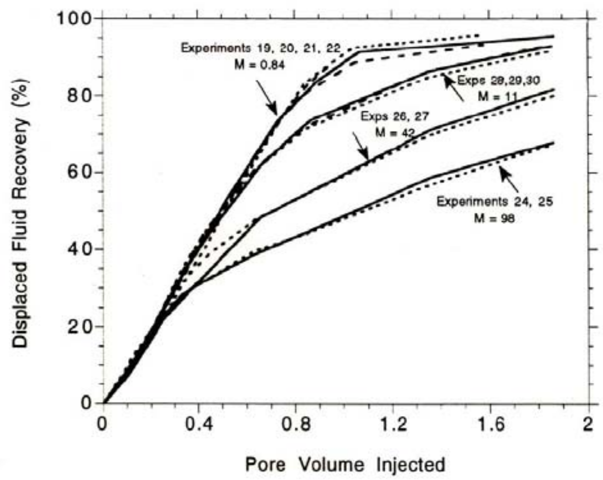
First-contact miscible displacement in a sandpack
M = 74



Source: Peters and Hardham, 1990; Peters, E. J., 2012, Advanced Petrophysics

Examples: Impact of Mobility Ratio

First-contact miscible displacement in a layered sandstone core



Source: Peters and Li, 1996; Peters, E. J., 2012, Advanced Petrophysics

Dispersion in Porous Media

- **What are the origins of dispersion in porous media?**
  - **Molecular diffusion**
  - **Local velocity gradients within given pores**
  - **Locally heterogeneous streamline lengths and velocities**
  - **Mechanical mixing in pore bodies**

Mechanical dispersion

Molecular Diffusion

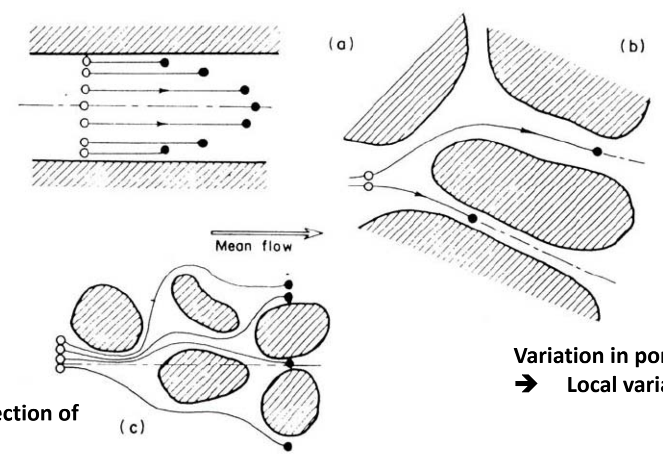
Molecular diffusion: A physiochemical dispersion caused by chemical potential gradient, which is correlated to the chemical concentration of the solute being transported

Concentration C_1 > Concentration C_2

What is the direction of net movement of molecules?

Mechanical Dispersion

velocity gradient



Variation in pore dimensions
➔ Local variation in velocity

Tortuosity affects direction of particle flow

Advection-Dispersion Equation

Transport mechanisms for chemical species in a flowing system:

Advective mass flux: $\vec{J}_a = \phi \vec{u} C$

Dispersive mass flux: $\vec{J}_d = -\phi \bar{D} \nabla C$ $\qquad \bar{D} = D_d + \bar{D}_m$

Continuity equation:

$$\frac{\partial(\phi C)}{\partial t} + \nabla . \vec{J} = 0 \quad \Rightarrow \quad \frac{\partial C}{\partial t} + \nabla . (\vec{u} C) - \nabla . (\bar{D} \nabla C) = 0$$

advection-dispersion equation

How to Solve Advection-Dispersion Equation?

1D transport in the x direction:

$$\frac{\partial C}{\partial t} + u_x \frac{\partial C}{\partial x} - D_L \frac{\partial^2 C}{\partial x^2} = 0 \quad \text{OR} \quad \frac{\partial C}{\partial t} + \frac{v_x}{\phi} \frac{\partial C}{\partial x} - D_L \frac{\partial^2 C}{\partial x^2} = 0$$

D_L: longitudinal dispersion

Boundary conditions:

$$C(x,0) = C_i \qquad C(0,t) = C_j \qquad C(\infty,t) = C_i$$

1D Advection-Dispersion Equation in Dimensionless Form

$$\frac{\partial C_D}{\partial t_D} + \frac{\partial C_D}{\partial x_D} - \frac{1}{N_{Pe}} \frac{\partial^2 C_D}{\partial x_D^2} = 0$$

Boundary conditions:

$$C(x_D, 0) = 0 \qquad C_D(0, t_D) = 1 \qquad C(\infty, t_D) = 0$$

Dimensionless parameters:

Peclet Number

$$C_D = \frac{C(x,t) - C_i}{C_j - C_i} \qquad t_D = \frac{u_x t}{L} = \frac{qt}{A\phi L} \qquad x_D = \frac{x}{L} \qquad N_{Pe} = \frac{u_x L}{D_L} = \frac{qL}{A\phi D_L}$$

Peclet Number

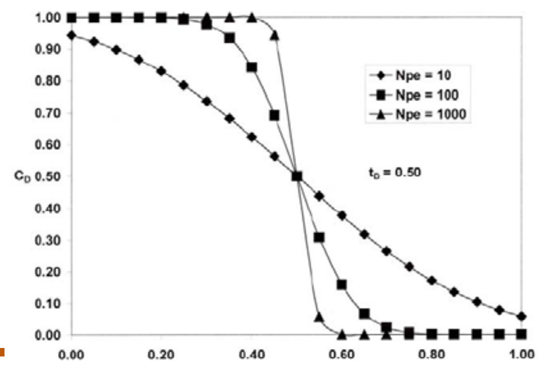
$$N_{Pe} = \frac{u_x L}{D_L} = \frac{qL}{A\phi D_L}$$

General Form

$$N_{Pe} = \frac{u_x D_p}{D_L}$$

Characteristic dimension of the porous medium

What would you consider to be used as the characteristic dimension of the porous medium?



Solution to Advection-Dispersion Equation

$$C(x,t)=\frac{C_j}{2}\left[erfc\left(\frac{x-u_x t}{2\sqrt{D_L t}}\right)+e^{\frac{u_x x}{D_L}}erfc\left(\frac{x+u_x t}{2\sqrt{D_L t}}\right)\right]$$

$$erfc(u)=\frac{2}{\sqrt{\pi}}\int_u^{\infty}e^{-u^2}du$$

complementary error function

Negligible

Negligible

Dimensionless form:

$$C_D(x_D,t_D)=\frac{1}{2}\left[erfc\left(\frac{x_D-t_D}{2\sqrt{\frac{t_D}{N_{Pe}}}}\right)+e^{x_D N_{Pe}}erfc\left(\frac{x_D+t_D}{2\sqrt{\frac{t_D}{N_{Pe}}}}\right)\right]$$

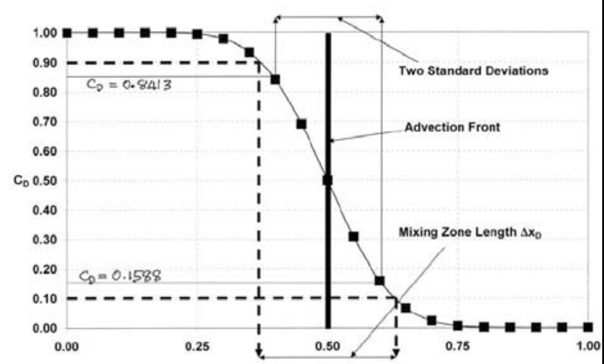
Solution to Advection-Dispersion Equation

$$C_D(x_D,t_D)=\frac{1}{2}erfc\left(\frac{x_D-t_D}{2\sqrt{t_D/N_{Pe}}}\right)$$

equivalent to the CDF of a normal distribution

$$\frac{2\sqrt{2}}{\sqrt{N_{Pe}}}=\frac{2\sqrt{2D_L}}{\sqrt{u_x L}}=\left(\frac{1-t_D}{\sqrt{t_D}}\right)_{C_D=0.84}-\left(\frac{1-t_D}{\sqrt{t_D}}\right)_{C_D=0.16}$$

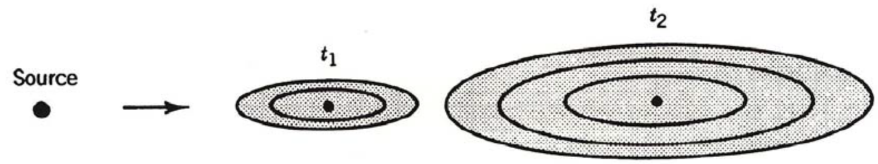
$$D_L=\frac{u_x L}{8}\left[\left(\frac{1-t_D}{\sqrt{t_D}}\right)_{C_D=0.84}-\left(\frac{1-t_D}{\sqrt{t_D}}\right)_{C_D=0.16}\right]^2$$



Dispersion Coefficient and Dispersivity

Dispersion coefficient tensor

$$\bar{D}(x,y,z)=\begin{bmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{yx} & D_{yy} & D_{yz} \\ D_{zx} & D_{zy} & D_{zz} \end{bmatrix} \qquad \bar{D}(u,v,w)=\begin{bmatrix} D_L & 0 & 0 \\ 0 & D_T & 0 \\ 0 & 0 & D_W \end{bmatrix}$$



Constant velocity flow system

Dispersion Coefficient and Dispersivity

$$D_L = D_{Ld} + D_{Lm}$$

$$D_T = D_{Td} + D_{Tm}$$

Experimental Observations

$$\frac{D_L}{D_0} = C_1 + C_2\left(\frac{uD_p}{D_0}\right)^{\beta}$$

$$\frac{D_T}{D_0} = C_3 + C_4\left(\frac{uD_p}{D_0}\right)^{\beta}$$

D_o: Effective binary molecular diffusion coefficient between the miscible displacing fluid and the displaced fluid

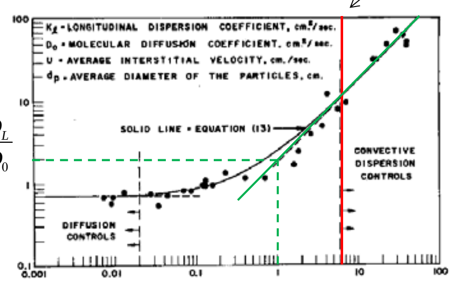
D_p: Mean particle diameter of the porous medium

Experimental Observations: $C_1 = \frac{1}{\phi F_L} = \frac{1}{\tau_L}$ $\quad C_3 = \frac{1}{\phi F_T} = \frac{1}{\tau_T}$

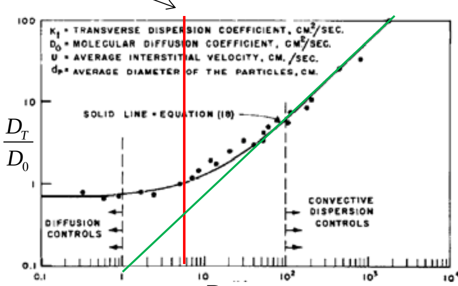
Dispersion Coefficient and Dispersivity

Peters' rule of thumb: $N_{Pe} > 6$ in most reservoir applications

$\frac{D_L}{D_0}$



$\frac{D_T}{D_0}$

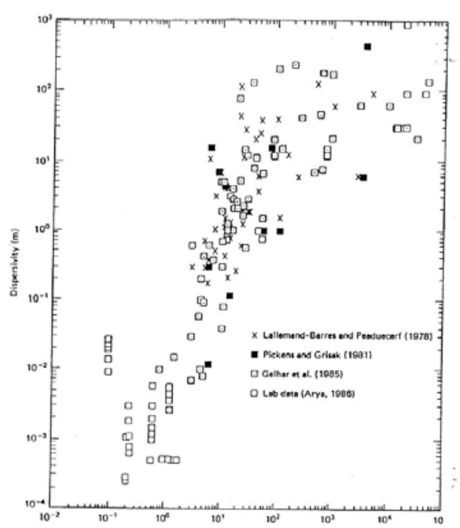


$\frac{uD_p}{D_0} = N_{Pe}$ **Source: Perkins and Johnston, SPE J., 1963** $\frac{uD_p}{D_0} = N_{Pe}$

$$D_L = C_2 \left(\frac{uD_p}{D_0} \right)^{\beta} D_0 \cong \alpha_L u \qquad D_T = C_4 \left(\frac{uD_p}{D_0} \right)^{\beta} D_0 \cong \alpha_T u \qquad N_{Pe} = \frac{L}{\alpha_L}$$

Longitudinal dispersivity **Transverse dispersivity**

Dispersivity is Scale Dependent!



Example 1

The following data were obtained in a 1D core tracer test. Determine the Peclet number for this tracer test.

t_D	C_D
0.600	0.010
0.650	0.015
0.700	0.037
0.800	0.066
0.900	0.300
1.000	0.502
1.100	0.685
1.200	0.820
1.300	0.906
1.400	0.988
1.500	0.997

Example 2

A tracer test is conducted in a relatively homogeneous cylindrical sandpack using continuous injection of a nonreactive tracer. The injected tracer and the displaced liquid have the same density and viscosity. The relative concentration C/C_0 = 0.75 was observed to arrive at the outlet end of the core after 0.9 hours from the start of injection. Other data about the test:

Sandpack length = 30 cm
Sandpack diameter = 10 cm
Volumetric injection rate = 1000 cm^3/hour
Porosity = 35%
Hydraulic gradient = -0.1 cm/cm
Fluid viscosity = 1.0 cp
Fluid density = 1.0 g/cm^3

a. **What are the dispersion coefficient and dispersivity?**
b. **What is the permeability of the sandpack?**

Example 3

Fresh water at relative concentration $C/C_0 = 0$ is injected into a sandpack saturated with salt water at relative concentration $C/C_0 = 1$. As the saltwater is displaced the concentration measurements below were made at a specific time *t*. The average interstitial velocity of flow was 1.6 cm/min.

a. How long after the initiation of flow were these readings taken?
b. Determine the dispersion coefficient and the dispersivity of the sandpack.

Distance from the core (cm)	C/Co (%)
48.2	0.8
49.7	2.2
51.2	3.5
53.6	9.5
55.4	21.7
57.2	50.3
59.3	78
61.3	94.5
63.2	94.5
65.4	98.7
68.2	98.7
73.4	100

Complementary References

- Peters, E. J., 2012, Advanced Petrophysics. Live Oak Book Company. Chapter 5