CP8

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PGE 382 - Numerical Methods in Petroleum and Geosystems Engineering

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[1]:
```
from math import pi, sin, cos, exp
import numpy as np
from numpy import linspace, zeros, arange
from numpy import ix_ as ix
np.set_printoptions(threshold=10000, linewidth=10000)
from numpy import exp, linspace, vectorize
import matplotlib.pyplot as plt
plt.style.use('paper.mplstyle')

XMAX = 100
YMAX = 25
dx = XMAX/10 ; dy = YMAX/10
Tf = 80 ; Nt=160
dt = Tf/Nt ; Nt = Nt + 1
X = np.arange(0,XMAX+dx,dx) ; Ni = len(X)
Y = np.arange(0,YMAX+dy,dy) ; Nj = len(Y)
Nij = Ni * Nj

beta = .5
Pr = 0.733

# Global index
def _(i,j) : return j + Nj*i

# Calculate F1
def cF1(i, j, U, V, T, Un, Vn, Tn) :
    _ij = _(i,j)
    _0j = _(i-1,j)
    _i0 = _(i,j-1)
    _i1 = _(i,j+1)

    ret = 0
    #1 = T
    ret += beta * T[_ij]
    ret += (1-beta) * Tn[_ij]
    #2 = d2U/dy2
    ret += beta * ( U[_i0] - 2*U[_ij] + U[_i1] ) / dy / dy
    ret += (1-beta) * ( Un[_i0] - 2*Un[_ij] + Un[_i1] ) / dy / dy
    #3 = -dU/dt
    ret += Un[_ij]/dt - U[_ij]/dt
    #4 = -U dU/dx
    ret += -beta     * U[_ij]  * ( U[_ij]  - U[_0j] )/dx
    ret += -(1-beta) * Un[_ij] * ( Un[_ij] - Un[_0j] )/dx
    #5 = -V dU/dy
    ret += -beta     * V[_ij]  * ( U[_ij]  - U[_i0])/dy
    ret += -(1-beta) * Vn[_ij] * ( Un[_ij] - Un[_i0])/dy
    return ret

# Calculate F2
def cF2(i, j, U, V, T, Un, Vn, Tn) :
    _ij = _(i,j)
    _0j = _(i-1,j)
    _1j = _(i+1,j)
    _i0 = _(i,j-1)
    _i1 = _(i,j+1)

    ret = 0
    #1 = 1/Pr d2T/dy2
    ret += beta * (1/Pr) * ( T[_i0] - 2*T[_ij] + T[_i1] ) / dy / dy
    ret += (1-beta) * (1/Pr) * ( Tn[_i0] - 2*Tn[_ij] + Tn[_i1] ) / dy / dy
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    #2 = -dT/dt
    ret += Tn[_ij]/dt - T[_ij]/dt
    #3 = -U dT/dx
    ret += -beta     * U[_ij]  * ( T[_ij]  - T[_0j] )/dx
    ret += -(1-beta) * Un[_ij] * ( Tn[_ij] - Tn[_0j] )/dx
    #4 = -V dT/dy
    ret += -beta     * V[_ij]  * ( T[_ij]  - T[_i0] )/dy
    ret += -(1-beta) * Vn[_ij] * ( Tn[_ij] - Tn[_i0] )/dy
    return ret

# Calculate F3
def cF3(i, j, U, V, T, Un, Vn, Tn) :
    _ij = _(i,j)
    _0j = _(i-1,j)
    _i0 = _(i,j-1)

    ret = 0
    #1 = dU/dx
    ret += beta * ( U[_ij] - U[_0j] ) / dx
    ret += (1-beta) * ( Un[_ij] - Un[_0j] ) / dx
    #2 = dV/dy
    ret += beta * ( V[_ij] - V[_i0] ) / dy
    ret += (1-beta) * ( Vn[_ij] - Vn[_i0] ) / dy
    return ret

# The list of free and prescribed dofs
def build_Df() :
    global U_doff, V_doff, T_doff, Ni, Nj

    U_doff=[]
    T_doff=[]
    for i in arange(1,Ni) :
        for j in arange(1,Nj-1) :
            U_doff.append( _(i,j) )
            T_doff.append( _(i,j) )

    V_doff=[]
    for i in arange(1,Ni) :
        for j in arange(1,Nj) :
            V_doff.append( _(i,j) )


#
# Assign BCs to solution vectors
#
def init_bcs() :
    global Unij, Vnij, Tnij
    Unij = zeros( [Nt,Ni,Nj] )
    Vnij = zeros( [Nt,Ni,Nj] )
    Tnij = zeros( [Nt,Ni,Nj] )

    Unij[0,:,:] = 0 # ic
    Vnij[0,:,:] = 0 # ic
    Tnij[0,:,:] = 0 # ic

    Unij[:,:,0] = 0 # BC , Y=0
    Vnij[:,:,0] = 0 # BC , Y=0
    Tnij[:,:,0] = 1 # BC , Y=0

    Unij[:,:,-1] = 0 # BC , Y=inf
    Tnij[:,:,-1] = 0 # BC , Y=inf

    Unij[:,0,:] = 0 # BC , X=0
    Vnij[:,0,:] = 0 # BC , X=0
    Tnij[:,0,:] = 0 # BC , X=0

#
#
#
#
def build_force() :
    global F1, F2, F3, Ni, Nj, Nij
    # Fx
    F1 = zeros( Nij )
    F2 = zeros( Nij )
```

```
    F3 = zeros( Nij )
    for i in arange(1,Ni) :
        for j in arange(1,Nj-1) :
            _ij = _(i,j)
            F1[_ij] += cF1(i,j,Uk,Vk,Tk,Un,Vn,Tn)
            F2[_ij] += cF2(i,j,Uk,Vk,Tk,Un,Vn,Tn)
    for i in arange(1,Ni) :
        for j in arange(1,Nj) :
            F3[_ij] += cF3(i,j,Uk,Vk,Tk,Un,Vn,Tn)

#
#
#
#
def build_jacobian() :
    global J1U, J1V, J1T, J2U, J2V, J2T, J3U, J3V, J3T

    # DFx/DU
    J1U = zeros([ Nij, Nij ] )
    J2U = zeros([ Nij, Nij ] )
    J3U = zeros([ Nij, Nij ] )
    # DFx/DV
    J1V = zeros([ Nij, Nij ] )
    J2V = zeros([ Nij, Nij ] )
    J3V = zeros([ Nij, Nij ] )
    # DFx/DT
    J1T = zeros([ Nij, Nij ] )
    J2T = zeros([ Nij, Nij ] )
    J3T = zeros([ Nij, Nij ] )

    for i in arange(1,Ni) :
        for j in arange(1,Nj-1) :
            _ij = _(i,j)
            _0j = _(i-1,j)
            _1j = _(i+1,j)
            _i0 = _(i,j-1)
            _i1 = _(i,j+1)

            # F1
            #
            #1 = T
            J1T[_ij,_ij] += beta
            #2 = d2U/dy2
            J1U[_ij,_i0] += beta/dy/dy
            J1U[_ij,_ij] += beta*(-2/dy/dy)
            J1U[_ij,_i1] += beta/dy/dy
            #3 = -dU//dt
            J1U[_ij,_ij] += -1/dt
            #4 = -U dU/dx --
            #             ./dUij = beta*(-2U/dx + U0j/dx)
            #             ./dU0j = beta*U/dx
            J1U[_ij,_ij] += beta * (-2*Uk[_ij] + Uk[_0j])/dx
            J1U[_ij,_0j] += beta * Uk[_ij]/dx
            #5 = -V dU/dy
            #             ./dUij = -beta*V/dy
            #             ./dUj0 = beta*V/dy
            #             ./dVij = -beta*(U-Ui0)/dy
            J1U[_ij,_ij] += -beta * Vk[_ij]/dy
            J1U[_ij,_i0] += beta * Vk[_ij]/dy
            J1V[_ij,_ij] += beta * (-Uk[_ij]+Uk[_i0])/dy

            # F2
            #
            #1 = 1/Pr d2T/dy2
            J2T[_ij,_ij] += beta * (-2) / Pr / dy / dy
            J2T[_ij,_i0] += beta * 1 / Pr / dy / dy
            J2T[_ij,_i1] += beta * 1 /Pr / dy / dy
            #2 = -dT/dt
            J2T[_ij,_ij] += -1/dt
            #3 = -U dT/dx
            #             (3)/dUij = -beta*(T-T0j)/dx
            #             (3)/dTij = -beta*Uij/dx
            #             (3)/dT0j = beta*Uij/dx
            J2U[_ij,_ij] += -beta * ( Tk[_ij]  - Tk[_0j] )/dx
            J2T[_ij,_ij] += -beta * Uk[_ij] /dx
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            J2T[_ij,_0j] += beta * Uk[_ij] /dx
            #4 = -V dT/dy
            #             (4)/dVij = -beta*(T-Ti0)/dy
            #             (4)/dTij = -beta*Uij/dy
            #             (4)/dTi0 = beta*Uij/dy
            J2V[_ij,_ij] += -beta * ( Tk[_ij]  - Tk[_i0] )/dy
            J2T[_ij,_ij] += -beta * Vk[_ij] /dy
            J2T[_ij,_i0] += beta * Vk[_ij] /dy

    for i in arange(1,Ni) :
        for j in arange(1,Nj) :
            _ij = _(i,j)
            _0j = _(i-1,j)
            _1j = _(i+1,j)
            _i0 = _(i,j-1)
            _i1 = _(i,j+1)
            # F3
            #
            #1 = dU/dx
            J3U[_ij,_ij] += beta/dx
            J3U[_ij,_0j] += -beta/dx
            #2 = dU/dy
            J3V[_ij,_ij] += beta / dy
            J3V[_ij,_i0] += -beta / dy



#
#
#
#
def linear_solve( ) :
    global Nij
    global J1U, J1V, J1T, J2U, J2V, J2T, J3U, J3V, J3T
    global F1, F2, F3
    global JAC, FORCE
    global U_doff, V_doff, T_doff

    JAC = np.block([[ J1U[ix(U_doff,U_doff)], J1V[ix(U_doff,V_doff)], J1T[ix(U_doff,T_doff)]],
                    [ J2U[ix(T_doff,U_doff)], J2V[ix(T_doff,V_doff)], J2T[ix(T_doff,T_doff)]],
                    [ J3U[ix(V_doff,U_doff)], J3V[ix(V_doff,V_doff)], J3T[ix(V_doff,T_doff)]] ])
    FORCE = np.block([ F1[ix(U_doff)], F2[ix(T_doff)], F3[ix(V_doff)] ])

    dX = np.linalg.solve( JAC, -FORCE )

    # Extract free dofs
    dUk = np.zeros(Nij)
    dVk = np.zeros(Nij)
    dTk = np.zeros(Nij)
    bl1 = len(U_doff) # block length
    bl2 = len(V_doff) # block length
    bl3 = len(T_doff) # block length
    dUk[ix(U_doff)] = dX[:bl1]
    dVk[ix(V_doff)] = dX[bl1:(bl1+bl2)]
    dTk[ix(T_doff)] = dX[(bl1+bl2):]

    err = np.linalg.norm(dX)

    return dUk, dVk, dTk, err



#
#
#
#  MAIN FLOW
#
#
#

# Global solution vector
init_bcs()
build_Df()

for n in arange(1,Nt) :
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```
    print(f"Solving timestep {n} ...")

    # Solution from the previous TS
    Un = Unij[n-1,:,:].flatten()
    Vn = Vnij[n-1,:,:].flatten()
    Tn = Tnij[n-1,:,:].flatten()
    # Initial guess for newton-raphson
    Uk = Un.copy()
    Vk = Vn.copy()
    Tk = Tn.copy()

    nk = 0 #newton loop index
    while(1) :
        build_jacobian()
        build_force()
        dUk, dVk, dTk, err = linear_solve()

        # Update results
        Uk += dUk
        Vk += dVk
        Tk += dTk

        # Check for convergence
        nk += 1
        print(f"   Newton iteration #{nk} ... (err={err:.3e})")
        if err < 1e-13 : break
        if nk > 50 : break

    Unij[n,:,:] = Uk.reshape(Ni,Nj)
    Vnij[n,:,:] = Vk.reshape(Ni,Nj)
    Tnij[n,:,:] = Tk.reshape(Ni,Nj)
```

```
Solving timestep 1 ...
   Newton iteration #1 ... (err=3.211e-01)
   Newton iteration #2 ... (err=6.590e-05)
   Newton iteration #3 ... (err=4.760e-11)
   Newton iteration #4 ... (err=1.870e-17)
Solving timestep 2 ...
   Newton iteration #1 ... (err=3.266e-01)
   Newton iteration #2 ... (err=2.126e-04)
   Newton iteration #3 ... (err=6.699e-10)
   Newton iteration #4 ... (err=5.655e-17)
Solving timestep 3 ...
   Newton iteration #1 ... (err=3.633e-01)
   Newton iteration #2 ... (err=3.574e-04)
   Newton iteration #3 ... (err=1.959e-09)
   Newton iteration #4 ... (err=6.383e-17)
Solving timestep 4 ...
   Newton iteration #1 ... (err=4.026e-01)
   Newton iteration #2 ... (err=4.686e-04)
   Newton iteration #3 ... (err=3.389e-09)
   Newton iteration #4 ... (err=1.044e-16)
Solving timestep 5 ...
   Newton iteration #1 ... (err=4.366e-01)
   Newton iteration #2 ... (err=5.306e-04)
   Newton iteration #3 ... (err=4.419e-09)
   Newton iteration #4 ... (err=1.086e-16)
Solving timestep 6 ...
   Newton iteration #1 ... (err=4.641e-01)
   Newton iteration #2 ... (err=5.444e-04)
   Newton iteration #3 ... (err=4.754e-09)
   Newton iteration #4 ... (err=1.719e-16)
Solving timestep 7 ...
   Newton iteration #1 ... (err=4.861e-01)
   Newton iteration #2 ... (err=5.235e-04)
   Newton iteration #3 ... (err=4.365e-09)
   Newton iteration #4 ... (err=1.787e-16)
Solving timestep 8 ...
   Newton iteration #1 ... (err=5.037e-01)
   Newton iteration #2 ... (err=4.887e-04)
   Newton iteration #3 ... (err=3.421e-09)
   Newton iteration #4 ... (err=2.168e-16)
Solving timestep 9 ...
   Newton iteration #1 ... (err=5.180e-01)
   Newton iteration #2 ... (err=4.607e-04)
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   Newton iteration #3 ... (err=2.282e-09)
   Newton iteration #4 ... (err=1.762e-16)
Solving timestep 10 ...
   Newton iteration #1 ... (err=5.298e-01)
   Newton iteration #2 ... (err=4.494e-04)
   Newton iteration #3 ... (err=1.616e-09)
   Newton iteration #4 ... (err=3.584e-16)
Solving timestep 11 ...
   Newton iteration #1 ... (err=5.396e-01)
   Newton iteration #2 ... (err=4.505e-04)
   Newton iteration #3 ... (err=1.889e-09)
   Newton iteration #4 ... (err=2.641e-16)
Solving timestep 12 ...
   Newton iteration #1 ... (err=5.478e-01)
   Newton iteration #2 ... (err=4.535e-04)
   Newton iteration #3 ... (err=2.307e-09)
   Newton iteration #4 ... (err=2.495e-16)
Solving timestep 13 ...
   Newton iteration #1 ... (err=5.544e-01)
   Newton iteration #2 ... (err=4.520e-04)
   Newton iteration #3 ... (err=2.347e-09)
   Newton iteration #4 ... (err=3.206e-16)
Solving timestep 14 ...
   Newton iteration #1 ... (err=5.597e-01)
   Newton iteration #2 ... (err=4.458e-04)
   Newton iteration #3 ... (err=2.012e-09)
   Newton iteration #4 ... (err=2.933e-16)
Solving timestep 15 ...
   Newton iteration #1 ... (err=5.638e-01)
   Newton iteration #2 ... (err=4.381e-04)
   Newton iteration #3 ... (err=1.574e-09)
   Newton iteration #4 ... (err=4.601e-16)
Solving timestep 16 ...
   Newton iteration #1 ... (err=5.666e-01)
   Newton iteration #2 ... (err=4.314e-04)
   Newton iteration #3 ... (err=1.374e-09)
   Newton iteration #4 ... (err=4.946e-16)
Solving timestep 17 ...
   Newton iteration #1 ... (err=5.682e-01)
   Newton iteration #2 ... (err=4.261e-04)
   Newton iteration #3 ... (err=1.414e-09)
   Newton iteration #4 ... (err=3.881e-16)
Solving timestep 18 ...
   Newton iteration #1 ... (err=5.688e-01)
   Newton iteration #2 ... (err=4.215e-04)
   Newton iteration #3 ... (err=1.410e-09)
   Newton iteration #4 ... (err=4.399e-16)
Solving timestep 19 ...
   Newton iteration #1 ... (err=5.682e-01)
   Newton iteration #2 ... (err=4.170e-04)
   Newton iteration #3 ... (err=1.274e-09)
   Newton iteration #4 ... (err=6.491e-16)
Solving timestep 20 ...
   Newton iteration #1 ... (err=5.666e-01)
   Newton iteration #2 ... (err=4.125e-04)
   Newton iteration #3 ... (err=1.116e-09)
   Newton iteration #4 ... (err=6.151e-16)
Solving timestep 21 ...
   Newton iteration #1 ... (err=5.639e-01)
   Newton iteration #2 ... (err=4.081e-04)
   Newton iteration #3 ... (err=1.042e-09)
   Newton iteration #4 ... (err=7.127e-16)
Solving timestep 22 ...
   Newton iteration #1 ... (err=5.602e-01)
   Newton iteration #2 ... (err=4.040e-04)
   Newton iteration #3 ... (err=1.008e-09)
   Newton iteration #4 ... (err=7.795e-16)
Solving timestep 23 ...
   Newton iteration #1 ... (err=5.554e-01)
   Newton iteration #2 ... (err=4.001e-04)
   Newton iteration #3 ... (err=9.438e-10)
   Newton iteration #4 ... (err=9.421e-16)
Solving timestep 24 ...
   Newton iteration #1 ... (err=5.495e-01)
   Newton iteration #2 ... (err=3.964e-04)
   Newton iteration #3 ... (err=8.659e-10)
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   Newton iteration #4 ... (err=8.389e-16)
Solving timestep 25 ...
   Newton iteration #1 ... (err=5.426e-01)
   Newton iteration #2 ... (err=3.928e-04)
   Newton iteration #3 ... (err=8.115e-10)
   Newton iteration #4 ... (err=6.900e-16)
Solving timestep 26 ...
   Newton iteration #1 ... (err=5.345e-01)
   Newton iteration #2 ... (err=3.894e-04)
   Newton iteration #3 ... (err=7.711e-10)
   Newton iteration #4 ... (err=8.993e-16)
Solving timestep 27 ...
   Newton iteration #1 ... (err=5.253e-01)
   Newton iteration #2 ... (err=3.862e-04)
   Newton iteration #3 ... (err=7.247e-10)
   Newton iteration #4 ... (err=7.703e-16)
Solving timestep 28 ...
   Newton iteration #1 ... (err=5.148e-01)
   Newton iteration #2 ... (err=3.830e-04)
   Newton iteration #3 ... (err=6.791e-10)
   Newton iteration #4 ... (err=6.570e-16)
Solving timestep 29 ...
   Newton iteration #1 ... (err=5.032e-01)
   Newton iteration #2 ... (err=3.798e-04)
   Newton iteration #3 ... (err=6.427e-10)
   Newton iteration #4 ... (err=8.664e-16)
Solving timestep 30 ...
   Newton iteration #1 ... (err=4.902e-01)
   Newton iteration #2 ... (err=3.765e-04)
   Newton iteration #3 ... (err=6.088e-10)
   Newton iteration #4 ... (err=9.217e-16)
Solving timestep 31 ...
   Newton iteration #1 ... (err=4.758e-01)
   Newton iteration #2 ... (err=3.729e-04)
   Newton iteration #3 ... (err=5.746e-10)
   Newton iteration #4 ... (err=7.662e-16)
Solving timestep 32 ...
   Newton iteration #1 ... (err=4.601e-01)
   Newton iteration #2 ... (err=3.686e-04)
   Newton iteration #3 ... (err=5.432e-10)
   Newton iteration #4 ... (err=7.853e-16)
Solving timestep 33 ...
   Newton iteration #1 ... (err=4.430e-01)
   Newton iteration #2 ... (err=3.632e-04)
   Newton iteration #3 ... (err=5.141e-10)
   Newton iteration #4 ... (err=8.414e-16)
Solving timestep 34 ...
   Newton iteration #1 ... (err=4.244e-01)
   Newton iteration #2 ... (err=3.565e-04)
   Newton iteration #3 ... (err=4.859e-10)
   Newton iteration #4 ... (err=8.231e-16)
Solving timestep 35 ...
   Newton iteration #1 ... (err=4.044e-01)
   Newton iteration #2 ... (err=3.481e-04)
   Newton iteration #3 ... (err=4.599e-10)
   Newton iteration #4 ... (err=9.425e-16)
Solving timestep 36 ...
   Newton iteration #1 ... (err=3.831e-01)
   Newton iteration #2 ... (err=3.378e-04)
   Newton iteration #3 ... (err=4.351e-10)
   Newton iteration #4 ... (err=7.489e-16)
Solving timestep 37 ...
   Newton iteration #1 ... (err=3.605e-01)
   Newton iteration #2 ... (err=3.254e-04)
   Newton iteration #3 ... (err=4.066e-10)
   Newton iteration #4 ... (err=9.176e-16)
Solving timestep 38 ...
   Newton iteration #1 ... (err=3.368e-01)
   Newton iteration #2 ... (err=3.108e-04)
   Newton iteration #3 ... (err=3.707e-10)
   Newton iteration #4 ... (err=8.838e-16)
Solving timestep 39 ...
   Newton iteration #1 ... (err=3.124e-01)
   Newton iteration #2 ... (err=2.937e-04)
   Newton iteration #3 ... (err=3.303e-10)
   Newton iteration #4 ... (err=7.794e-16)
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Solving timestep 40 ...
   Newton iteration #1 ... (err=2.874e-01)
   Newton iteration #2 ... (err=2.736e-04)
   Newton iteration #3 ... (err=2.950e-10)
   Newton iteration #4 ... (err=1.035e-15)
Solving timestep 41 ...
   Newton iteration #1 ... (err=2.625e-01)
   Newton iteration #2 ... (err=2.502e-04)
   Newton iteration #3 ... (err=2.704e-10)
   Newton iteration #4 ... (err=9.743e-16)
Solving timestep 42 ...
   Newton iteration #1 ... (err=2.380e-01)
   Newton iteration #2 ... (err=2.236e-04)
   Newton iteration #3 ... (err=2.481e-10)
   Newton iteration #4 ... (err=1.614e-15)
Solving timestep 43 ...
   Newton iteration #1 ... (err=2.148e-01)
   Newton iteration #2 ... (err=1.944e-04)
   Newton iteration #3 ... (err=2.166e-10)
   Newton iteration #4 ... (err=1.167e-15)
Solving timestep 44 ...
   Newton iteration #1 ... (err=1.936e-01)
   Newton iteration #2 ... (err=1.642e-04)
   Newton iteration #3 ... (err=1.748e-10)
   Newton iteration #4 ... (err=1.389e-15)
Solving timestep 45 ...
   Newton iteration #1 ... (err=1.751e-01)
   Newton iteration #2 ... (err=1.348e-04)
   Newton iteration #3 ... (err=1.309e-10)
   Newton iteration #4 ... (err=1.068e-15)
Solving timestep 46 ...
   Newton iteration #1 ... (err=1.601e-01)
   Newton iteration #2 ... (err=1.085e-04)
   Newton iteration #3 ... (err=9.304e-11)
   Newton iteration #4 ... (err=8.483e-16)
Solving timestep 47 ...
   Newton iteration #1 ... (err=1.490e-01)
   Newton iteration #2 ... (err=8.672e-05)
   Newton iteration #3 ... (err=6.415e-11)
   Newton iteration #4 ... (err=1.324e-15)
Solving timestep 48 ...
   Newton iteration #1 ... (err=1.416e-01)
   Newton iteration #2 ... (err=7.038e-05)
   Newton iteration #3 ... (err=4.308e-11)
   Newton iteration #4 ... (err=1.007e-15)
Solving timestep 49 ...
   Newton iteration #1 ... (err=1.376e-01)
   Newton iteration #2 ... (err=5.903e-05)
   Newton iteration #3 ... (err=2.832e-11)
   Newton iteration #4 ... (err=1.035e-15)
Solving timestep 50 ...
   Newton iteration #1 ... (err=1.360e-01)
   Newton iteration #2 ... (err=5.144e-05)
   Newton iteration #3 ... (err=1.876e-11)
   Newton iteration #4 ... (err=1.616e-15)
Solving timestep 51 ...
   Newton iteration #1 ... (err=1.357e-01)
   Newton iteration #2 ... (err=4.630e-05)
   Newton iteration #3 ... (err=1.272e-11)
   Newton iteration #4 ... (err=1.050e-15)
Solving timestep 52 ...
   Newton iteration #1 ... (err=1.359e-01)
   Newton iteration #2 ... (err=4.286e-05)
   Newton iteration #3 ... (err=8.529e-12)
   Newton iteration #4 ... (err=1.391e-15)
Solving timestep 53 ...
   Newton iteration #1 ... (err=1.358e-01)
   Newton iteration #2 ... (err=4.085e-05)
   Newton iteration #3 ... (err=6.013e-12)
   Newton iteration #4 ... (err=1.259e-15)
Solving timestep 54 ...
   Newton iteration #1 ... (err=1.350e-01)
   Newton iteration #2 ... (err=4.008e-05)
   Newton iteration #3 ... (err=5.640e-12)
   Newton iteration #4 ... (err=1.061e-15)
Solving timestep 55 ...
```

```
   Newton iteration #1 ... (err=1.332e-01)
   Newton iteration #2 ... (err=4.020e-05)
   Newton iteration #3 ... (err=6.156e-12)
   Newton iteration #4 ... (err=1.004e-15)
Solving timestep 56 ...
   Newton iteration #1 ... (err=1.303e-01)
   Newton iteration #2 ... (err=4.068e-05)
   Newton iteration #3 ... (err=6.296e-12)
   Newton iteration #4 ... (err=8.489e-16)
Solving timestep 57 ...
   Newton iteration #1 ... (err=1.263e-01)
   Newton iteration #2 ... (err=4.100e-05)
   Newton iteration #3 ... (err=5.966e-12)
   Newton iteration #4 ... (err=8.502e-16)
Solving timestep 58 ...
   Newton iteration #1 ... (err=1.214e-01)
   Newton iteration #2 ... (err=4.078e-05)
   Newton iteration #3 ... (err=5.563e-12)
   Newton iteration #4 ... (err=1.118e-15)
Solving timestep 59 ...
   Newton iteration #1 ... (err=1.156e-01)
   Newton iteration #2 ... (err=3.985e-05)
   Newton iteration #3 ... (err=5.395e-12)
   Newton iteration #4 ... (err=1.396e-15)
Solving timestep 60 ...
   Newton iteration #1 ... (err=1.092e-01)
   Newton iteration #2 ... (err=3.817e-05)
   Newton iteration #3 ... (err=5.411e-12)
   Newton iteration #4 ... (err=8.696e-16)
Solving timestep 61 ...
   Newton iteration #1 ... (err=1.022e-01)
   Newton iteration #2 ... (err=3.584e-05)
   Newton iteration #3 ... (err=5.375e-12)
   Newton iteration #4 ... (err=1.101e-15)
Solving timestep 62 ...
   Newton iteration #1 ... (err=9.483e-02)
   Newton iteration #2 ... (err=3.297e-05)
   Newton iteration #3 ... (err=5.124e-12)
   Newton iteration #4 ... (err=1.057e-15)
Solving timestep 63 ...
   Newton iteration #1 ... (err=8.726e-02)
   Newton iteration #2 ... (err=2.976e-05)
   Newton iteration #3 ... (err=4.635e-12)
   Newton iteration #4 ... (err=1.245e-15)
Solving timestep 64 ...
   Newton iteration #1 ... (err=7.962e-02)
   Newton iteration #2 ... (err=2.635e-05)
   Newton iteration #3 ... (err=3.973e-12)
   Newton iteration #4 ... (err=7.691e-16)
Solving timestep 65 ...
   Newton iteration #1 ... (err=7.204e-02)
   Newton iteration #2 ... (err=2.291e-05)
   Newton iteration #3 ... (err=3.233e-12)
   Newton iteration #4 ... (err=1.075e-15)
Solving timestep 66 ...
   Newton iteration #1 ... (err=6.461e-02)
   Newton iteration #2 ... (err=1.956e-05)
   Newton iteration #3 ... (err=2.506e-12)
   Newton iteration #4 ... (err=8.903e-16)
Solving timestep 67 ...
   Newton iteration #1 ... (err=5.744e-02)
   Newton iteration #2 ... (err=1.640e-05)
   Newton iteration #3 ... (err=1.854e-12)
   Newton iteration #4 ... (err=8.256e-16)
Solving timestep 68 ...
   Newton iteration #1 ... (err=5.061e-02)
   Newton iteration #2 ... (err=1.351e-05)
   Newton iteration #3 ... (err=1.315e-12)
   Newton iteration #4 ... (err=1.087e-15)
Solving timestep 69 ...
   Newton iteration #1 ... (err=4.419e-02)
   Newton iteration #2 ... (err=1.093e-05)
   Newton iteration #3 ... (err=8.977e-13)
   Newton iteration #4 ... (err=9.297e-16)
Solving timestep 70 ...
   Newton iteration #1 ... (err=3.823e-02)
```

```
   Newton iteration #2 ... (err=8.690e-06)
   Newton iteration #3 ... (err=5.941e-13)
   Newton iteration #4 ... (err=1.020e-15)
Solving timestep 71 ...
   Newton iteration #1 ... (err=3.279e-02)
   Newton iteration #2 ... (err=6.790e-06)
   Newton iteration #3 ... (err=3.845e-13)
   Newton iteration #4 ... (err=1.079e-15)
Solving timestep 72 ...
   Newton iteration #1 ... (err=2.791e-02)
   Newton iteration #2 ... (err=5.219e-06)
   Newton iteration #3 ... (err=2.466e-13)
   Newton iteration #4 ... (err=8.786e-16)
Solving timestep 73 ...
   Newton iteration #1 ... (err=2.362e-02)
   Newton iteration #2 ... (err=3.954e-06)
   Newton iteration #3 ... (err=1.566e-13)
   Newton iteration #4 ... (err=9.749e-16)
Solving timestep 74 ...
   Newton iteration #1 ... (err=1.997e-02)
   Newton iteration #2 ... (err=2.959e-06)
   Newton iteration #3 ... (err=9.915e-14)
Solving timestep 75 ...
   Newton iteration #1 ... (err=1.697e-02)
   Newton iteration #2 ... (err=2.197e-06)
   Newton iteration #3 ... (err=6.158e-14)
Solving timestep 76 ...
   Newton iteration #1 ... (err=1.464e-02)
   Newton iteration #2 ... (err=1.624e-06)
   Newton iteration #3 ... (err=3.710e-14)
Solving timestep 77 ...
   Newton iteration #1 ... (err=1.294e-02)
   Newton iteration #2 ... (err=1.200e-06)
   Newton iteration #3 ... (err=2.164e-14)
Solving timestep 78 ...
   Newton iteration #1 ... (err=1.181e-02)
   Newton iteration #2 ... (err=8.906e-07)
   Newton iteration #3 ... (err=1.207e-14)
Solving timestep 79 ...
   Newton iteration #1 ... (err=1.112e-02)
   Newton iteration #2 ... (err=6.707e-07)
   Newton iteration #3 ... (err=6.231e-15)
Solving timestep 80 ...
   Newton iteration #1 ... (err=1.073e-02)
   Newton iteration #2 ... (err=5.250e-07)
   Newton iteration #3 ... (err=2.555e-15)
Solving timestep 81 ...
   Newton iteration #1 ... (err=1.048e-02)
   Newton iteration #2 ... (err=4.436e-07)
   Newton iteration #3 ... (err=1.116e-15)
Solving timestep 82 ...
   Newton iteration #1 ... (err=1.028e-02)
   Newton iteration #2 ... (err=4.126e-07)
   Newton iteration #3 ... (err=1.094e-15)
Solving timestep 83 ...
   Newton iteration #1 ... (err=1.006e-02)
   Newton iteration #2 ... (err=4.103e-07)
   Newton iteration #3 ... (err=1.181e-15)
Solving timestep 84 ...
   Newton iteration #1 ... (err=9.765e-03)
   Newton iteration #2 ... (err=4.161e-07)
   Newton iteration #3 ... (err=1.392e-15)
Solving timestep 85 ...
   Newton iteration #1 ... (err=9.392e-03)
   Newton iteration #2 ... (err=4.172e-07)
   Newton iteration #3 ... (err=1.263e-15)
Solving timestep 86 ...
   Newton iteration #1 ... (err=8.939e-03)
   Newton iteration #2 ... (err=4.078e-07)
   Newton iteration #3 ... (err=1.495e-15)
Solving timestep 87 ...
   Newton iteration #1 ... (err=8.414e-03)
   Newton iteration #2 ... (err=3.871e-07)
   Newton iteration #3 ... (err=1.653e-15)
Solving timestep 88 ...
   Newton iteration #1 ... (err=7.833e-03)
```

```
   Newton iteration #2 ... (err=3.565e-07)
   Newton iteration #3 ... (err=1.625e-15)
Solving timestep 89 ...
   Newton iteration #1 ... (err=7.217e-03)
   Newton iteration #2 ... (err=3.189e-07)
   Newton iteration #3 ... (err=1.354e-15)
Solving timestep 90 ...
   Newton iteration #1 ... (err=6.584e-03)
   Newton iteration #2 ... (err=2.775e-07)
   Newton iteration #3 ... (err=1.439e-15)
Solving timestep 91 ...
   Newton iteration #1 ... (err=5.956e-03)
   Newton iteration #2 ... (err=2.353e-07)
   Newton iteration #3 ... (err=1.353e-15)
Solving timestep 92 ...
   Newton iteration #1 ... (err=5.353e-03)
   Newton iteration #2 ... (err=1.949e-07)
   Newton iteration #3 ... (err=1.026e-15)
Solving timestep 93 ...
   Newton iteration #1 ... (err=4.791e-03)
   Newton iteration #2 ... (err=1.581e-07)
   Newton iteration #3 ... (err=8.992e-16)
Solving timestep 94 ...
   Newton iteration #1 ... (err=4.288e-03)
   Newton iteration #2 ... (err=1.260e-07)
   Newton iteration #3 ... (err=9.977e-16)
Solving timestep 95 ...
   Newton iteration #1 ... (err=3.856e-03)
   Newton iteration #2 ... (err=9.918e-08)
   Newton iteration #3 ... (err=1.110e-15)
Solving timestep 96 ...
   Newton iteration #1 ... (err=3.501e-03)
   Newton iteration #2 ... (err=7.744e-08)
   Newton iteration #3 ... (err=1.192e-15)
Solving timestep 97 ...
   Newton iteration #1 ... (err=3.227e-03)
   Newton iteration #2 ... (err=6.027e-08)
   Newton iteration #3 ... (err=1.120e-15)
Solving timestep 98 ...
   Newton iteration #1 ... (err=3.027e-03)
   Newton iteration #2 ... (err=4.697e-08)
   Newton iteration #3 ... (err=8.645e-16)
Solving timestep 99 ...
   Newton iteration #1 ... (err=2.889e-03)
   Newton iteration #2 ... (err=3.683e-08)
   Newton iteration #3 ... (err=8.474e-16)
Solving timestep 100 ...
   Newton iteration #1 ... (err=2.798e-03)
   Newton iteration #2 ... (err=2.931e-08)
   Newton iteration #3 ... (err=1.360e-15)
Solving timestep 101 ...
   Newton iteration #1 ... (err=2.736e-03)
   Newton iteration #2 ... (err=2.406e-08)
   Newton iteration #3 ... (err=9.369e-16)
Solving timestep 102 ...
   Newton iteration #1 ... (err=2.688e-03)
   Newton iteration #2 ... (err=2.085e-08)
   Newton iteration #3 ... (err=6.995e-16)
Solving timestep 103 ...
   Newton iteration #1 ... (err=2.642e-03)
   Newton iteration #2 ... (err=1.932e-08)
   Newton iteration #3 ... (err=8.737e-16)
Solving timestep 104 ...
   Newton iteration #1 ... (err=2.590e-03)
   Newton iteration #2 ... (err=1.889e-08)
   Newton iteration #3 ... (err=1.093e-15)
Solving timestep 105 ...
   Newton iteration #1 ... (err=2.528e-03)
   Newton iteration #2 ... (err=1.893e-08)
   Newton iteration #3 ... (err=1.242e-15)
Solving timestep 106 ...
   Newton iteration #1 ... (err=2.452e-03)
   Newton iteration #2 ... (err=1.897e-08)
   Newton iteration #3 ... (err=1.042e-15)
Solving timestep 107 ...
   Newton iteration #1 ... (err=2.363e-03)
```

```
   Newton iteration #2 ... (err=1.875e-08)
   Newton iteration #3 ... (err=8.506e-16)
Solving timestep 108 ...
   Newton iteration #1 ... (err=2.261e-03)
   Newton iteration #2 ... (err=1.815e-08)
   Newton iteration #3 ... (err=1.006e-15)
Solving timestep 109 ...
   Newton iteration #1 ... (err=2.149e-03)
   Newton iteration #2 ... (err=1.719e-08)
   Newton iteration #3 ... (err=7.962e-16)
Solving timestep 110 ...
   Newton iteration #1 ... (err=2.027e-03)
   Newton iteration #2 ... (err=1.593e-08)
   Newton iteration #3 ... (err=7.786e-16)
Solving timestep 111 ...
   Newton iteration #1 ... (err=1.900e-03)
   Newton iteration #2 ... (err=1.444e-08)
   Newton iteration #3 ... (err=9.482e-16)
Solving timestep 112 ...
   Newton iteration #1 ... (err=1.769e-03)
   Newton iteration #2 ... (err=1.283e-08)
   Newton iteration #3 ... (err=1.061e-15)
Solving timestep 113 ...
   Newton iteration #1 ... (err=1.637e-03)
   Newton iteration #2 ... (err=1.119e-08)
   Newton iteration #3 ... (err=7.996e-16)
Solving timestep 114 ...
   Newton iteration #1 ... (err=1.507e-03)
   Newton iteration #2 ... (err=9.583e-09)
   Newton iteration #3 ... (err=1.016e-15)
Solving timestep 115 ...
   Newton iteration #1 ... (err=1.380e-03)
   Newton iteration #2 ... (err=8.077e-09)
   Newton iteration #3 ... (err=1.053e-15)
Solving timestep 116 ...
   Newton iteration #1 ... (err=1.258e-03)
   Newton iteration #2 ... (err=6.709e-09)
   Newton iteration #3 ... (err=1.198e-15)
Solving timestep 117 ...
   Newton iteration #1 ... (err=1.143e-03)
   Newton iteration #2 ... (err=5.503e-09)
   Newton iteration #3 ... (err=8.845e-16)
Solving timestep 118 ...
   Newton iteration #1 ... (err=1.036e-03)
   Newton iteration #2 ... (err=4.465e-09)
   Newton iteration #3 ... (err=9.094e-16)
Solving timestep 119 ...
   Newton iteration #1 ... (err=9.368e-04)
   Newton iteration #2 ... (err=3.593e-09)
   Newton iteration #3 ... (err=8.410e-16)
Solving timestep 120 ...
   Newton iteration #1 ... (err=8.472e-04)
   Newton iteration #2 ... (err=2.876e-09)
   Newton iteration #3 ... (err=9.005e-16)
Solving timestep 121 ...
   Newton iteration #1 ... (err=7.672e-04)
   Newton iteration #2 ... (err=2.297e-09)
   Newton iteration #3 ... (err=1.106e-15)
Solving timestep 122 ...
   Newton iteration #1 ... (err=6.966e-04)
   Newton iteration #2 ... (err=1.835e-09)
   Newton iteration #3 ... (err=1.209e-15)
Solving timestep 123 ...
   Newton iteration #1 ... (err=6.354e-04)
   Newton iteration #2 ... (err=1.471e-09)
   Newton iteration #3 ... (err=1.291e-15)
Solving timestep 124 ...
   Newton iteration #1 ... (err=5.829e-04)
   Newton iteration #2 ... (err=1.186e-09)
   Newton iteration #3 ... (err=6.786e-16)
Solving timestep 125 ...
   Newton iteration #1 ... (err=5.385e-04)
   Newton iteration #2 ... (err=9.634e-10)
   Newton iteration #3 ... (err=1.118e-15)
Solving timestep 126 ...
   Newton iteration #1 ... (err=5.014e-04)
```

```
   Newton iteration #2 ... (err=7.893e-10)
   Newton iteration #3 ... (err=8.579e-16)
Solving timestep 127 ...
   Newton iteration #1 ... (err=4.706e-04)
   Newton iteration #2 ... (err=6.533e-10)
   Newton iteration #3 ... (err=9.321e-16)
Solving timestep 128 ...
   Newton iteration #1 ... (err=4.452e-04)
   Newton iteration #2 ... (err=5.474e-10)
   Newton iteration #3 ... (err=1.011e-15)
Solving timestep 129 ...
   Newton iteration #1 ... (err=4.241e-04)
   Newton iteration #2 ... (err=4.654e-10)
   Newton iteration #3 ... (err=1.155e-15)
Solving timestep 130 ...
   Newton iteration #1 ... (err=4.064e-04)
   Newton iteration #2 ... (err=4.025e-10)
   Newton iteration #3 ... (err=1.026e-15)
Solving timestep 131 ...
   Newton iteration #1 ... (err=3.915e-04)
   Newton iteration #2 ... (err=3.548e-10)
   Newton iteration #3 ... (err=9.402e-16)
Solving timestep 132 ...
   Newton iteration #1 ... (err=3.786e-04)
   Newton iteration #2 ... (err=3.189e-10)
   Newton iteration #3 ... (err=1.394e-15)
Solving timestep 133 ...
   Newton iteration #1 ... (err=3.672e-04)
   Newton iteration #2 ... (err=2.918e-10)
   Newton iteration #3 ... (err=9.712e-16)
Solving timestep 134 ...
   Newton iteration #1 ... (err=3.569e-04)
   Newton iteration #2 ... (err=2.712e-10)
   Newton iteration #3 ... (err=1.046e-15)
Solving timestep 135 ...
   Newton iteration #1 ... (err=3.474e-04)
   Newton iteration #2 ... (err=2.552e-10)
   Newton iteration #3 ... (err=1.091e-15)
Solving timestep 136 ...
   Newton iteration #1 ... (err=3.384e-04)
   Newton iteration #2 ... (err=2.422e-10)
   Newton iteration #3 ... (err=1.096e-15)
Solving timestep 137 ...
   Newton iteration #1 ... (err=3.298e-04)
   Newton iteration #2 ... (err=2.314e-10)
   Newton iteration #3 ... (err=1.172e-15)
Solving timestep 138 ...
   Newton iteration #1 ... (err=3.214e-04)
   Newton iteration #2 ... (err=2.219e-10)
   Newton iteration #3 ... (err=9.943e-16)
Solving timestep 139 ...
   Newton iteration #1 ... (err=3.132e-04)
   Newton iteration #2 ... (err=2.133e-10)
   Newton iteration #3 ... (err=7.761e-16)
Solving timestep 140 ...
   Newton iteration #1 ... (err=3.050e-04)
   Newton iteration #2 ... (err=2.053e-10)
   Newton iteration #3 ... (err=8.653e-16)
Solving timestep 141 ...
   Newton iteration #1 ... (err=2.969e-04)
   Newton iteration #2 ... (err=1.976e-10)
   Newton iteration #3 ... (err=9.726e-16)
Solving timestep 142 ...
   Newton iteration #1 ... (err=2.887e-04)
   Newton iteration #2 ... (err=1.901e-10)
   Newton iteration #3 ... (err=1.202e-15)
Solving timestep 143 ...
   Newton iteration #1 ... (err=2.806e-04)
   Newton iteration #2 ... (err=1.826e-10)
   Newton iteration #3 ... (err=9.846e-16)
Solving timestep 144 ...
   Newton iteration #1 ... (err=2.724e-04)
   Newton iteration #2 ... (err=1.752e-10)
   Newton iteration #3 ... (err=7.946e-16)
Solving timestep 145 ...
   Newton iteration #1 ... (err=2.642e-04)
```

```
   Newton iteration #2 ... (err=1.676e-10)
   Newton iteration #3 ... (err=8.778e-16)
Solving timestep 146 ...
   Newton iteration #1 ... (err=2.559e-04)
   Newton iteration #2 ... (err=1.600e-10)
   Newton iteration #3 ... (err=7.902e-16)
Solving timestep 147 ...
   Newton iteration #1 ... (err=2.477e-04)
   Newton iteration #2 ... (err=1.523e-10)
   Newton iteration #3 ... (err=9.553e-16)
Solving timestep 148 ...
   Newton iteration #1 ... (err=2.395e-04)
   Newton iteration #2 ... (err=1.445e-10)
   Newton iteration #3 ... (err=9.574e-16)
Solving timestep 149 ...
   Newton iteration #1 ... (err=2.313e-04)
   Newton iteration #2 ... (err=1.367e-10)
   Newton iteration #3 ... (err=9.319e-16)
Solving timestep 150 ...
   Newton iteration #1 ... (err=2.231e-04)
   Newton iteration #2 ... (err=1.289e-10)
   Newton iteration #3 ... (err=1.262e-15)
Solving timestep 151 ...
   Newton iteration #1 ... (err=2.151e-04)
   Newton iteration #2 ... (err=1.212e-10)
   Newton iteration #3 ... (err=1.055e-15)
Solving timestep 152 ...
   Newton iteration #1 ... (err=2.071e-04)
   Newton iteration #2 ... (err=1.136e-10)
   Newton iteration #3 ... (err=1.095e-15)
Solving timestep 153 ...
   Newton iteration #1 ... (err=1.993e-04)
   Newton iteration #2 ... (err=1.061e-10)
   Newton iteration #3 ... (err=1.337e-15)
Solving timestep 154 ...
   Newton iteration #1 ... (err=1.915e-04)
   Newton iteration #2 ... (err=9.888e-11)
   Newton iteration #3 ... (err=1.117e-15)
Solving timestep 155 ...
   Newton iteration #1 ... (err=1.840e-04)
   Newton iteration #2 ... (err=9.190e-11)
   Newton iteration #3 ... (err=1.280e-15)
Solving timestep 156 ...
   Newton iteration #1 ... (err=1.766e-04)
   Newton iteration #2 ... (err=8.520e-11)
   Newton iteration #3 ... (err=9.097e-16)
Solving timestep 157 ...
   Newton iteration #1 ... (err=1.695e-04)
   Newton iteration #2 ... (err=7.882e-11)
   Newton iteration #3 ... (err=1.209e-15)
Solving timestep 158 ...
   Newton iteration #1 ... (err=1.625e-04)
   Newton iteration #2 ... (err=7.277e-11)
   Newton iteration #3 ... (err=1.037e-15)
Solving timestep 159 ...
   Newton iteration #1 ... (err=1.558e-04)
   Newton iteration #2 ... (err=6.707e-11)
   Newton iteration #3 ... (err=9.900e-16)
Solving timestep 160 ...
   Newton iteration #1 ... (err=1.492e-04)
   Newton iteration #2 ... (err=6.171e-11)
   Newton iteration #3 ... (err=1.035e-15)
```

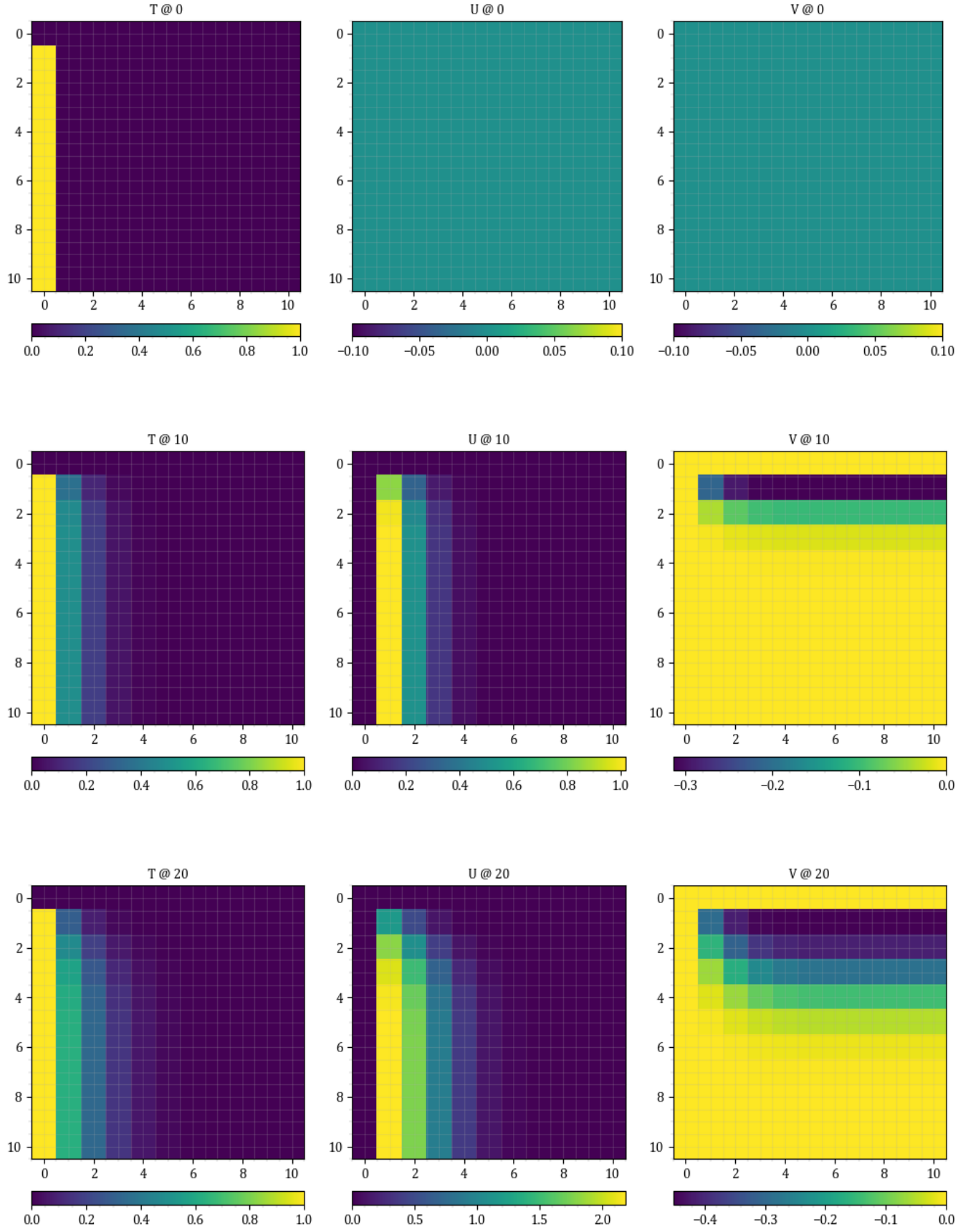
[2]:

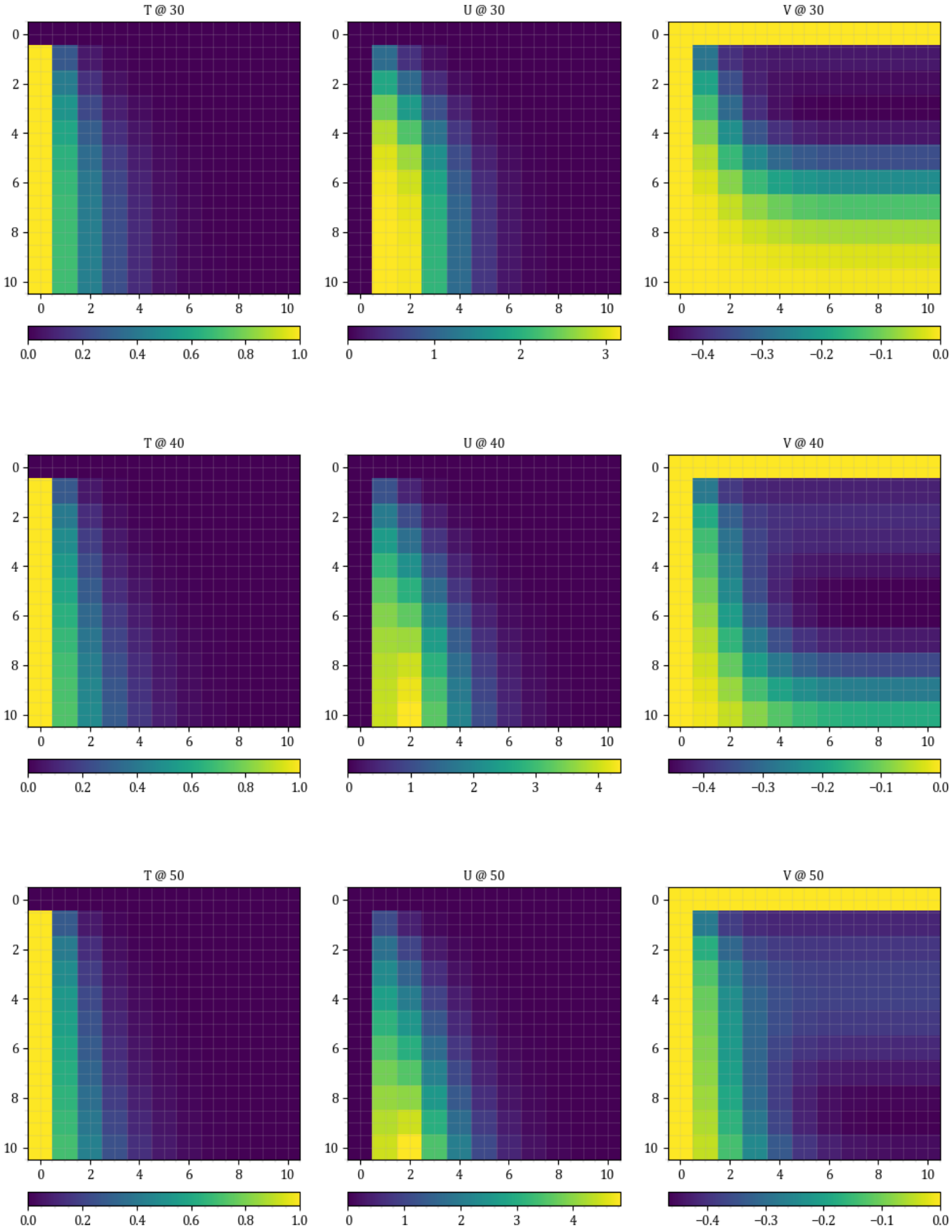
```
#%%
for n in arange(0,Nt,10) :
    fig, [ax1,ax2,ax3] = plt.subplots(1,3, figsize=[10,5])
    pcm=ax1.imshow( Tnij[n,:,:] )
    cb1 = fig.colorbar(pcm, ax=ax1, location='bottom', pad=.07)
    pcm=ax2.imshow( Unij[n,:,:] )
    cb2 = fig.colorbar(pcm, ax=ax2, location='bottom', pad=.07)
    pcm=ax3.imshow( Vnij[n,:,:] )
    cb3 = fig.colorbar(pcm, ax=ax3, location='bottom', pad=.07)

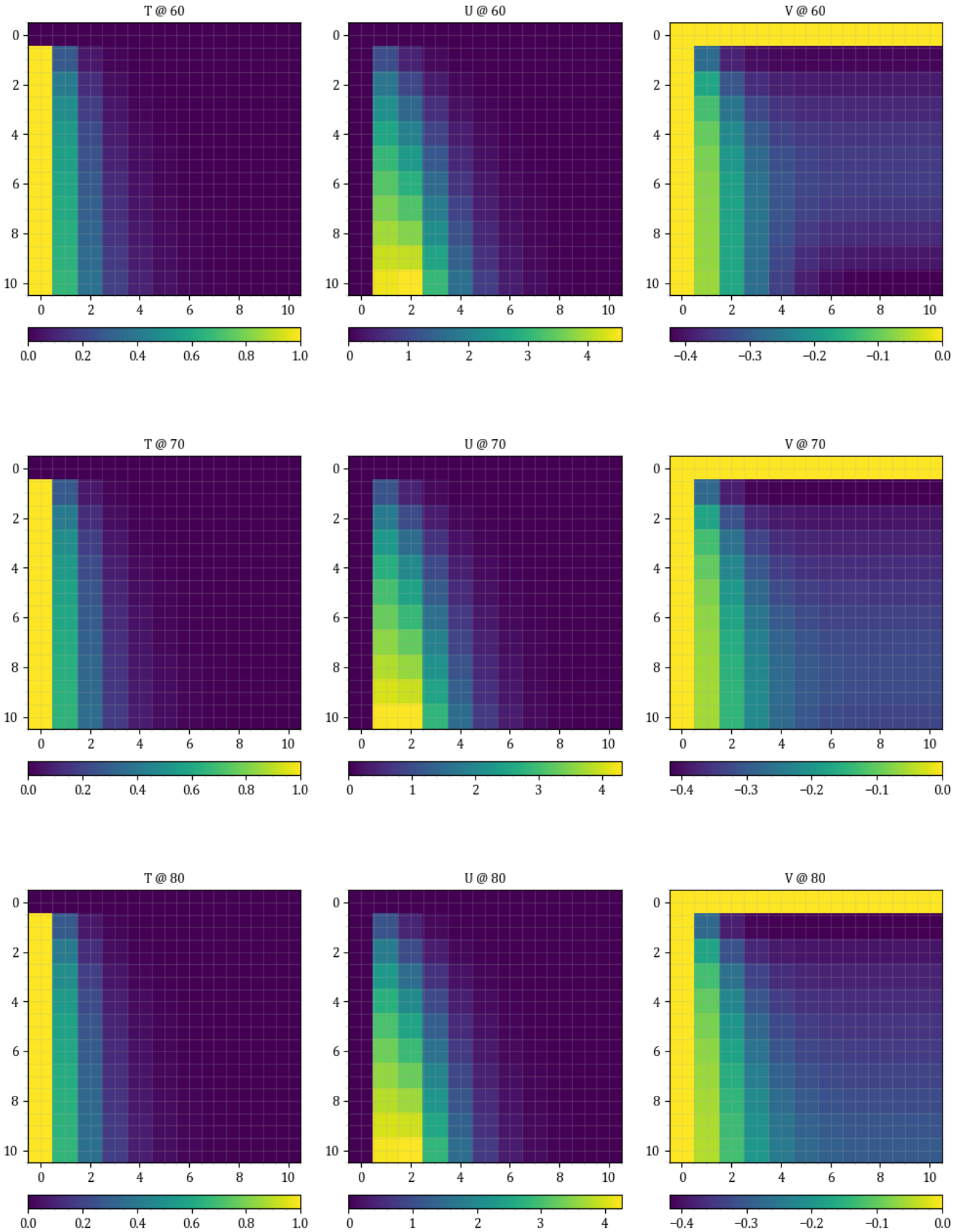
    ax1.set_title(f"T @ {n}")
```

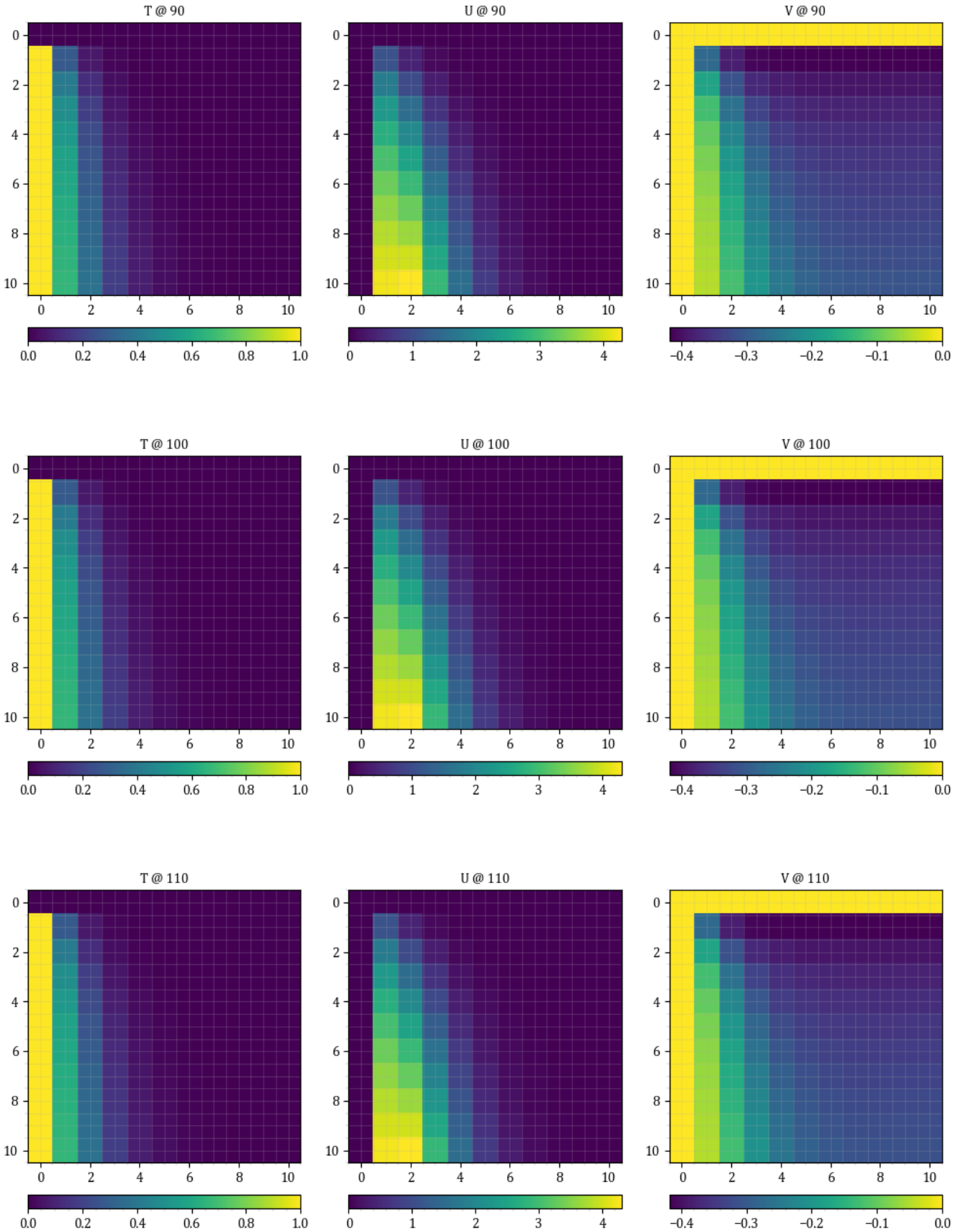
```
    ax2.set_title(f"U @ {n}")
    ax3.set_title(f"V @ {n}")

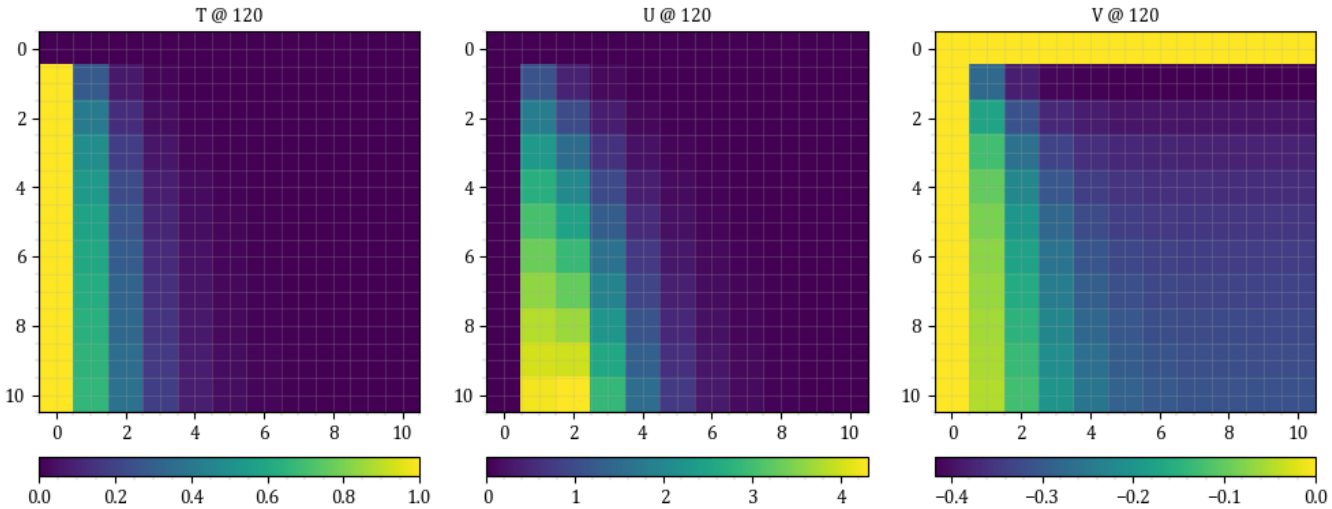
    #cb1.mappable.set_clim(1E2,1E9)
    fig.tight_layout()
```

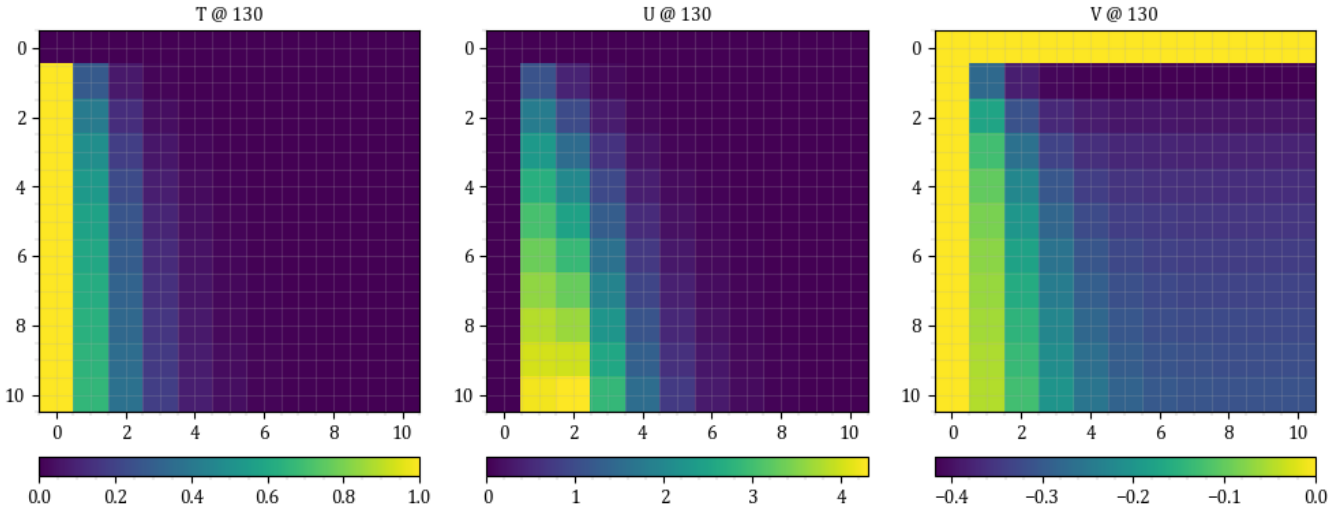


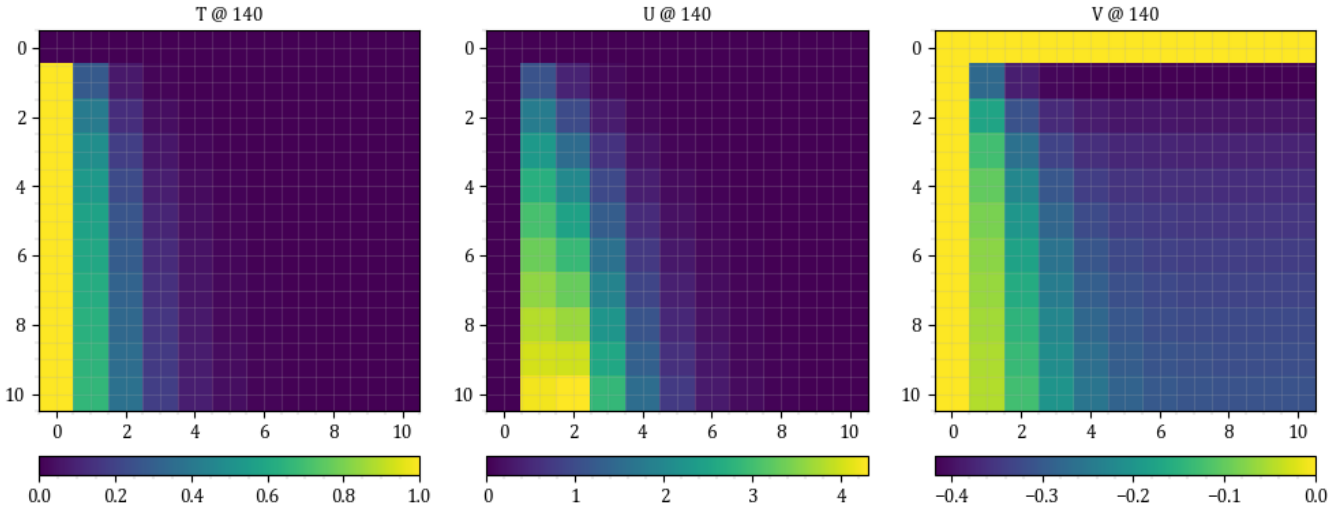

T @ 30
U @ 30
V @ 30
T @ 40
U @ 40
V @ 40
T @ 50
U @ 50
V @ 50

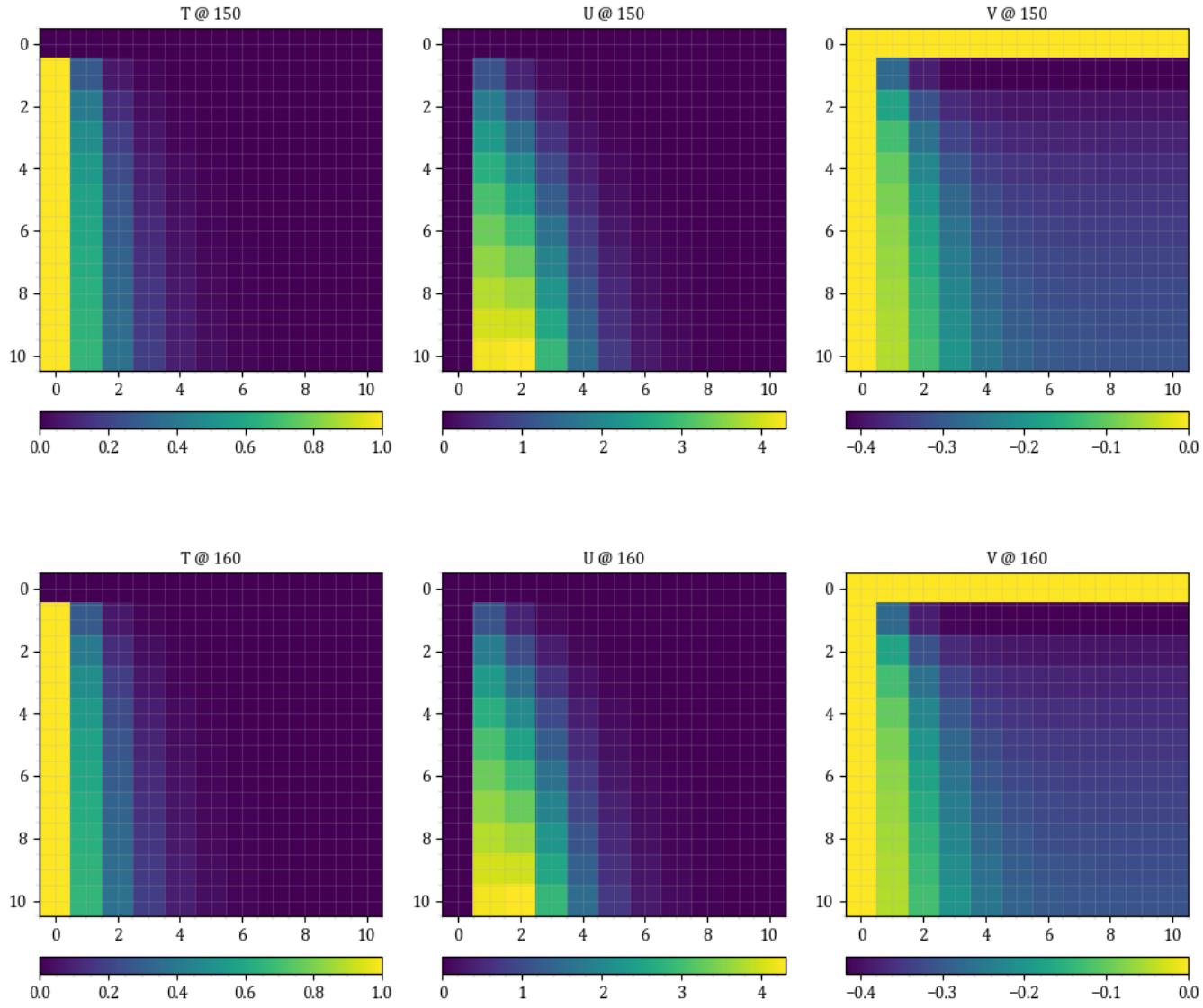

T @ 60
U @ 60
V @ 60
T @ 70
U @ 70
V @ 70
T @ 80
U @ 80
V @ 80


T @ 90
U @ 90
V @ 90
T @ 100
U @ 100
V @ 100
T @ 110
U @ 110
V @ 110


T @ 120
U @ 120
V @ 120


T @ 130
U @ 130
V @ 130


T @ 140
U @ 140
V @ 140



EXPLICIT SOLUTION

[3]:
```
#%%
from math import pi, sin, cos, exp
import numpy as np
from numpy import linspace, zeros, arange
from numpy import ix_ as ix
np.set_printoptions(threshold=10000, linewidth=10000)
from numpy import exp, linspace, vectorize
import matplotlib.pyplot as plt
plt.style.use('paper.mplstyle')

XMAX = 100
YMAX = 25
dx = XMAX/10 ; dy = YMAX/10
Tf = 80 ; Nt=160
dt = Tf/Nt ; Nt = Nt + 1
X = np.arange(0,XMAX+dx,dx) ; Ni = len(X)
Y = np.arange(0,YMAX+dy,dy) ; Nj = len(Y)
Nij = Ni * Nj

beta = 0.5
Pr = 0.733

# Global index
def _(i,j) : return j + Nj*i
```

```
#
# Assign BCs to solution vectors
#
def init_bcs() :
    global Unij, Vnij, Tnij
    Unij = zeros( [Nt,Ni,Nj] )
    Vnij = zeros( [Nt,Ni,Nj] )
    Tnij = zeros( [Nt,Ni,Nj] )

    Unij[0,:,:] = 0 # ic
    Vnij[0,:,:] = 0 # ic
    Tnij[0,:,:] = 0 # ic

    Unij[:,:,0] = 0 # BC , Y=0
    Vnij[:,:,0] = 0 # BC , Y=0
    Tnij[:,:,0] = 1 # BC , Y=0

    Unij[:,:,-1] = 0 # BC , Y=inf
    Tnij[:,:,-1] = 0 # BC , Y=inf

    Unij[:,0,:] = 0 # BC , X=0
    Vnij[:,0,:] = 0 # BC , X=0
    Tnij[:,0,:] = 0 # BC , X=0


#
#
#
#  MAIN FLOW
#
#
#

# Global solution vector
init_bcs()

for n in arange(1,Nt) :
    print(f"Solving timestep {n} ...")

    T = Tnij[n-1,:,:]
    U = Unij[n-1,:,:]
    V = Vnij[n-1,:,:]
    # Solve U
    for i in arange(1,Ni) :
        for j in arange(1,Nj-1) :
            Unij[n,i,j] = U[i,j] + dt * (
                            + T[i,j]                                    # T
                            + (U[i,j-1]-2*U[i,j]+U[i,j+1])/dy/dy        # Uyy
                            - U[i,j]*(U[i,j]-U[i-1,j])/dx               # - U Ux
                            - V[i,j]*(U[i,j]-U[i,j-1])/dy               # - U Uy
                        )

    # Solve T
    for i in arange(1,Ni) :
        for j in arange(1,Nj-1) :
            Tnij[n,i,j] = T[i,j] + dt*(
                + 1/Pr*(T[i,j-1]-2*T[i,j]+T[i,j+1])/dy/dy               # Tyy/Pr
                - U[i,j]*(T[i,j]-T[i-1,j])/dx                           # -U Tx
                - V[i,j]*(T[i,j]-T[i,j-1])/dy                           # -V Ty
            )

    # Solve V
    for i in arange(1,Ni) :
        for j in arange(1,Nj) :
            Vnij[n,i,j] = Vnij[n,i,j-1] - dy/dx*( Unij[n,i,j] - Unij[n,i-1,j])
```

```
Solving timestep 1 ...
Solving timestep 2 ...
Solving timestep 3 ...
Solving timestep 4 ...
Solving timestep 5 ...
Solving timestep 6 ...
Solving timestep 7 ...
Solving timestep 8 ...
```

```
Solving timestep 9 ...
Solving timestep 10 ...
Solving timestep 11 ...
Solving timestep 12 ...
Solving timestep 13 ...
Solving timestep 14 ...
Solving timestep 15 ...
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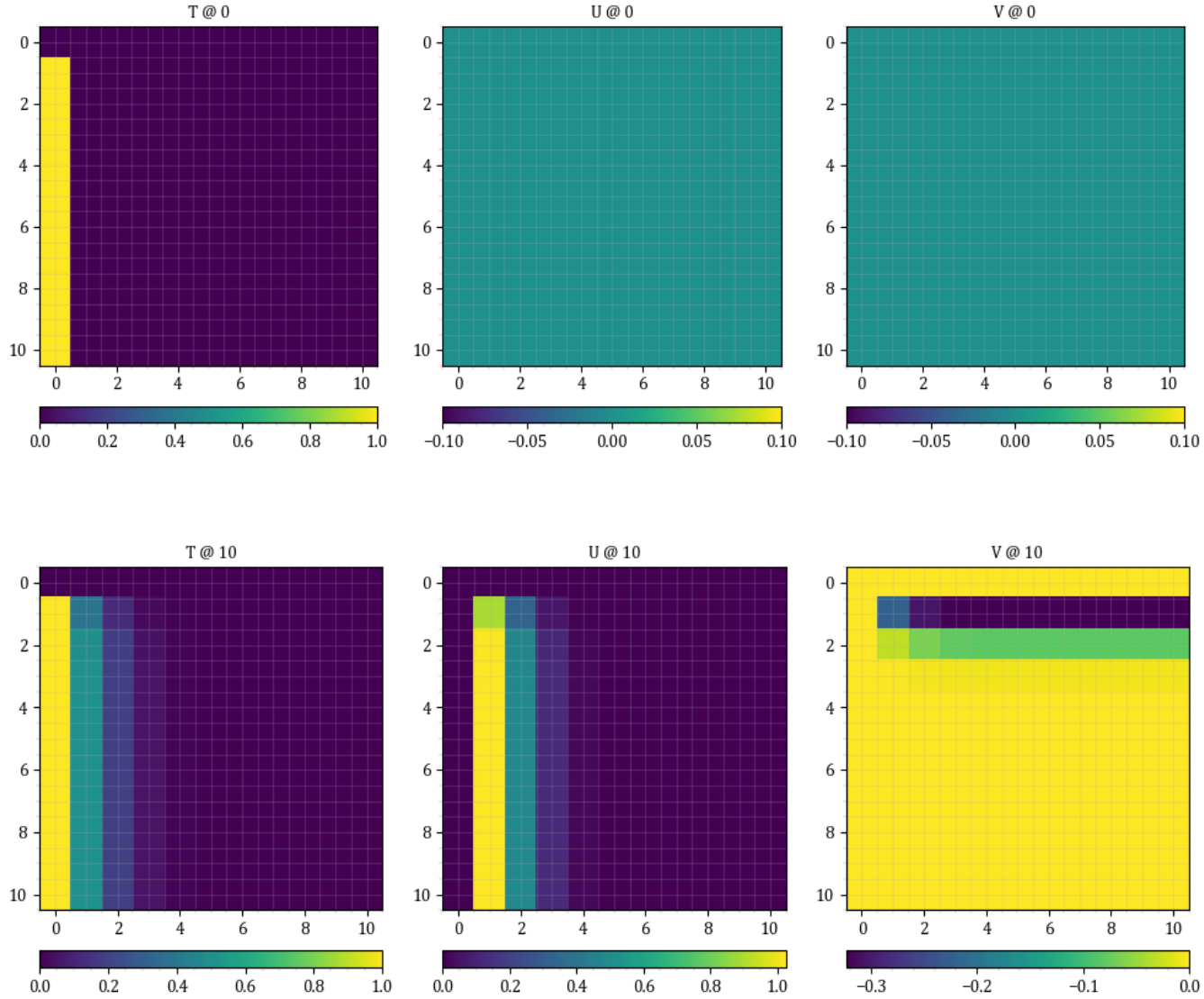
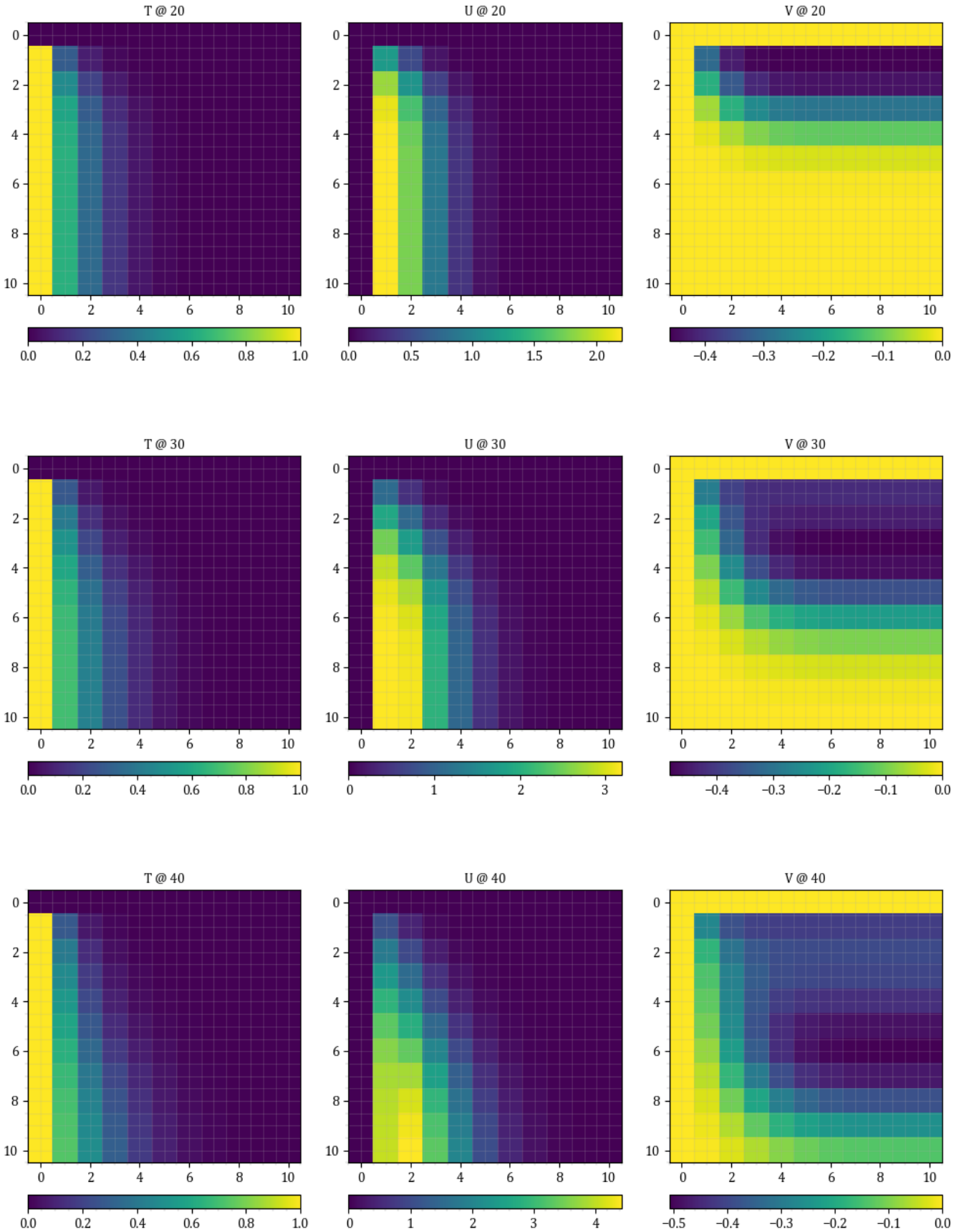
Solving timestep 157 ...
Solving timestep 158 ...
Solving timestep 159 ...
Solving timestep 160 ...
```

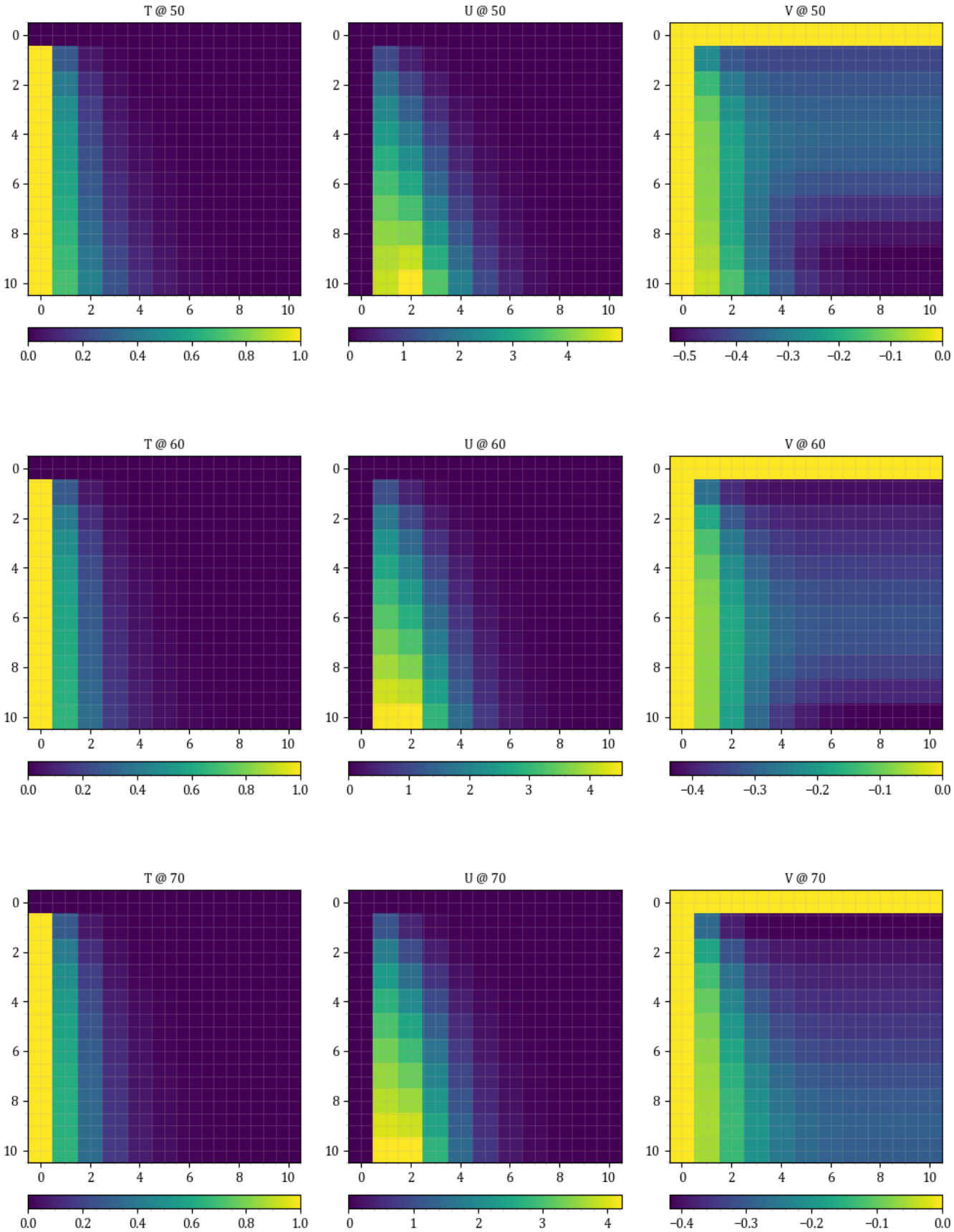
[4]:
```
for n in arange(0,Nt,10) :
    fig, [ax1,ax2,ax3] = plt.subplots(1,3, figsize=[10,5])
    pcm=ax1.imshow( Tnij[n,:,:] )
    cb1 = fig.colorbar(pcm, ax=ax1, location='bottom', pad=.07)
    pcm=ax2.imshow( Unij[n,:,:] )
    cb2 = fig.colorbar(pcm, ax=ax2, location='bottom', pad=.07)
    pcm=ax3.imshow( Vnij[n,:,:] )
    cb3 = fig.colorbar(pcm, ax=ax3, location='bottom', pad=.07)

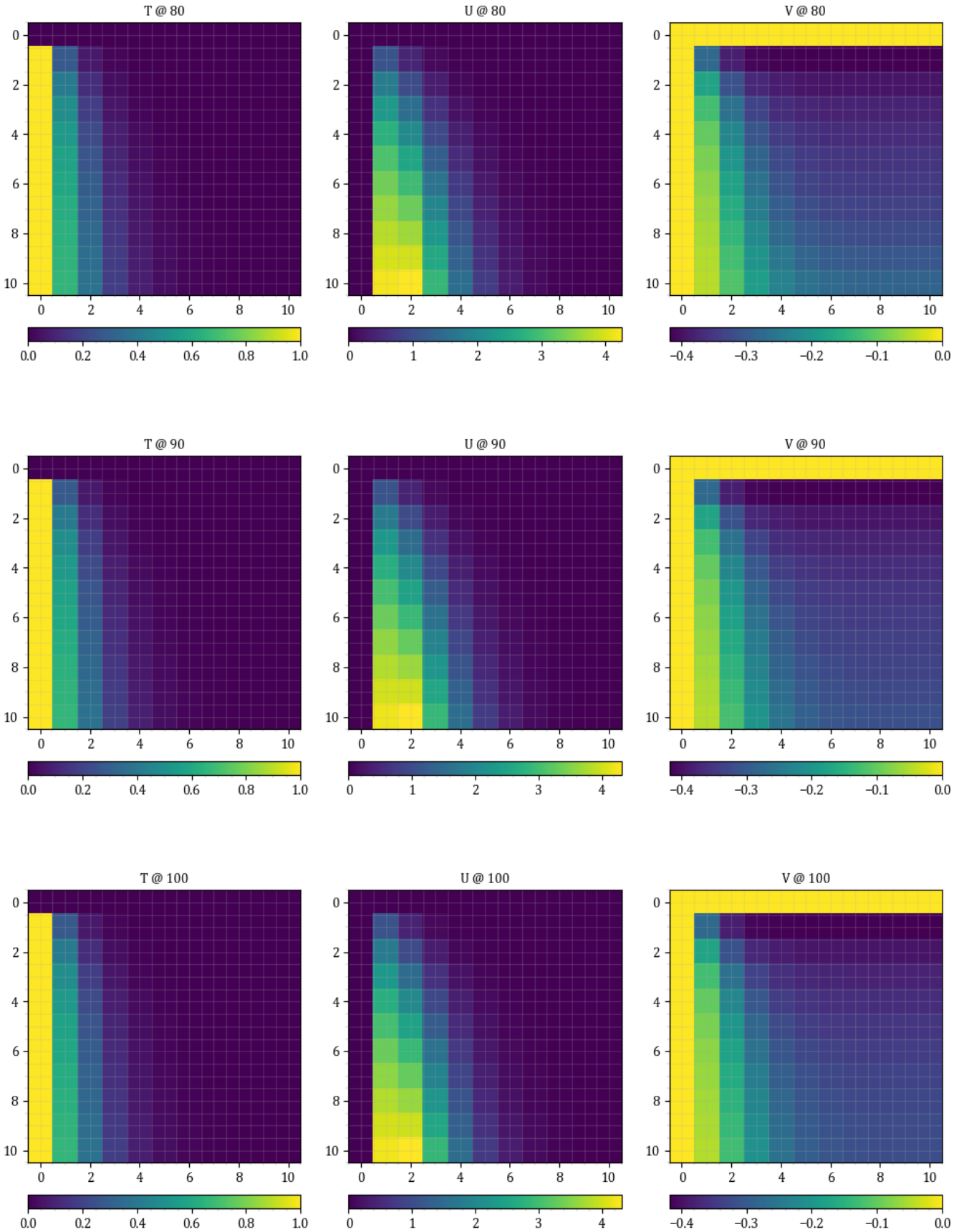
    ax1.set_title(f"T @ {n}")
    ax2.set_title(f"U @ {n}")
    ax3.set_title(f"V @ {n}")

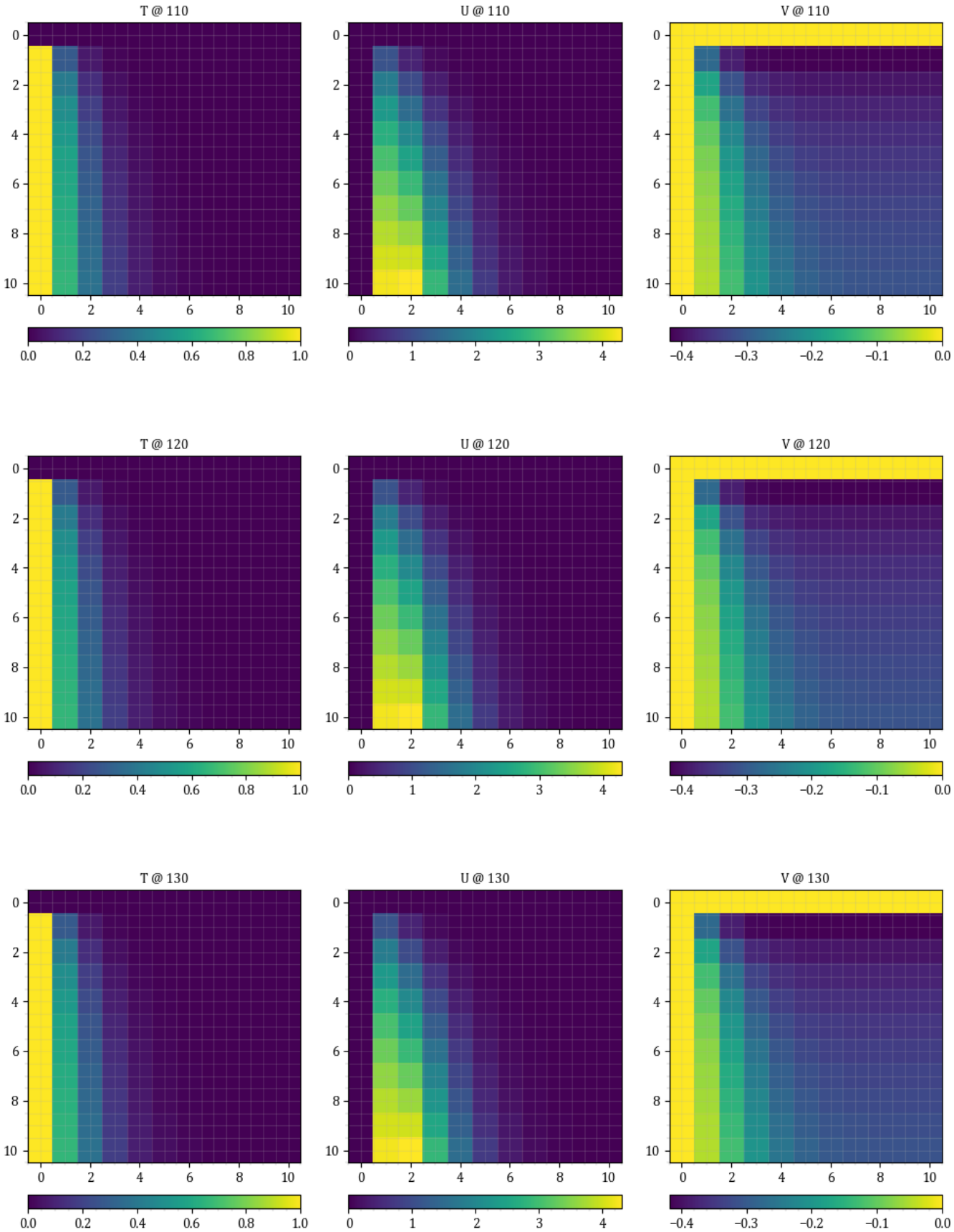
    #cb1.mappable.set_clim(1E2,1E9)
    fig.tight_layout()
```

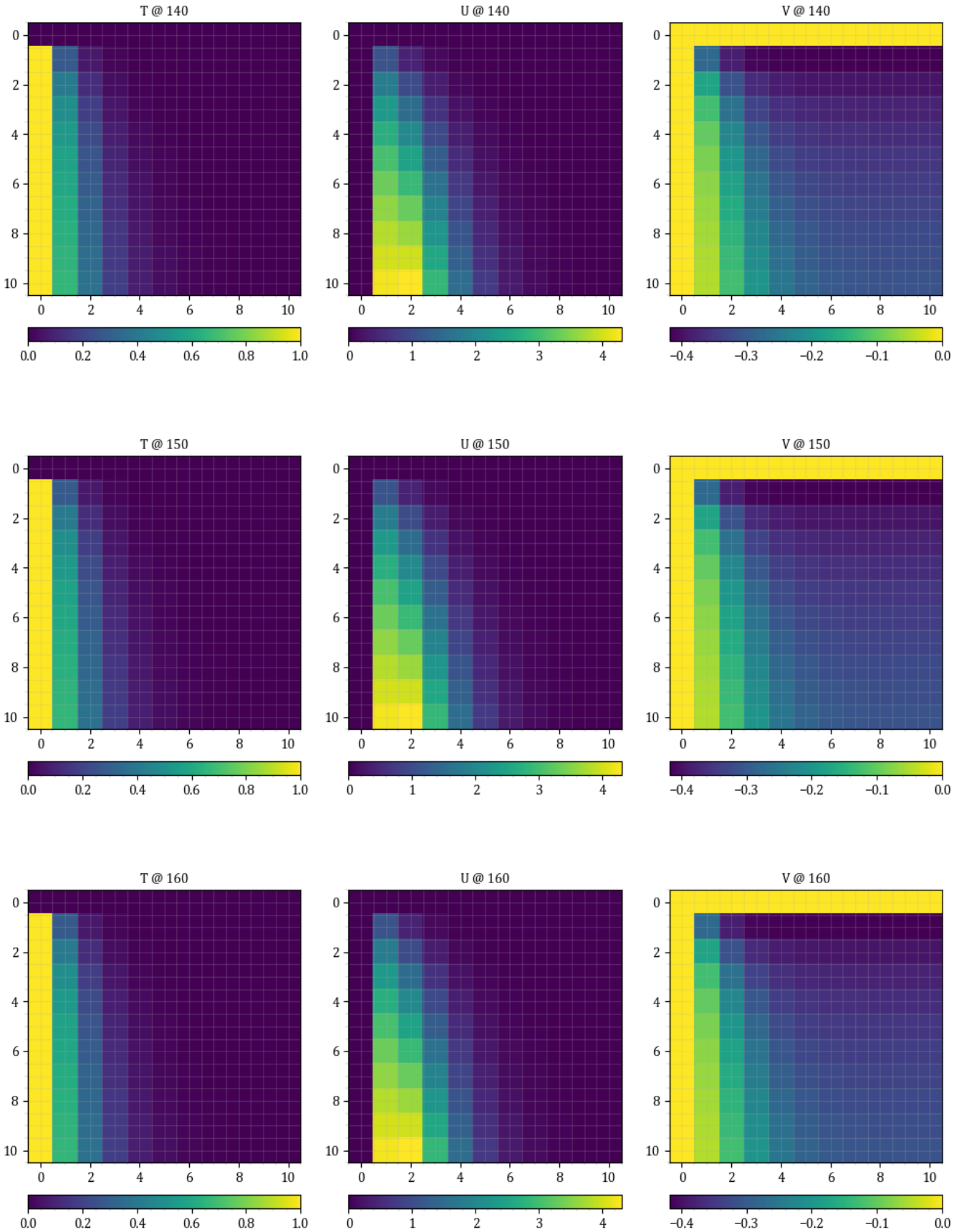



T @ 20
U @ 20
V @ 20
T @ 30
U @ 30
V @ 30
T @ 40
U @ 40
V @ 40


T @ 50
U @ 50
V @ 50
T @ 60
U @ 60
V @ 60
T @ 70
U @ 70
V @ 70


T @ 80
U @ 80
V @ 80
T @ 90
U @ 90
V @ 90
T @ 100
U @ 100
V @ 100


T @ 110
U @ 110
V @ 110
T @ 120
U @ 120
V @ 120
T @ 130
U @ 130
V @ 130


T @ 140
U @ 140
V @ 140
T @ 150
U @ 150
V @ 150
T @ 160
U @ 160
V @ 160