$$(W) \quad B(w, \underset{\cap}{u}) = L(w) \qquad \forall w \in \mathcal{V}$$

$$\mathcal{S} = \mathcal{V} \oplus \{g^{ext}\} \in \mathcal{S} \qquad g^{ext} = g \text{ on } \Gamma_g, \ = 0 \text{ on } \Gamma_h$$

$$\underset{\Psi}{u} = \underset{\Psi}{v}$$

homogeneous Dirichlet BCs on Γ_g

Gal. $B(w^h, \underline{u^h}) = L(w^h) \quad \forall w^h \in \mathcal{V}^h \subset \mathcal{V}.$

(G) ✓

$$\mathcal{S}^h = \mathcal{V}^h \oplus \{g^{ext}\}$$

same guy as above.

$$\underset{\Psi}{u^h} = \underset{\Psi}{v^h}$$

$(W)^h$: $B(w^h, \underline{u}) = L(w^h) \quad \forall w^h \in \mathcal{V}^h \subset \mathcal{V}.$

$$B(w^h, u^h) - B(w^h, u) = 0$$

$$B(w^h, u^h - u) = 0 \qquad \underline{\text{Gal. orthog.}}$$

$$e = e^h + \eta$$

consist. of FEM.

We need more stab.

$$(G) + (\tau \mathbb{L} w^h, \mathcal{L} u^h - f)$$

$\Omega' =$ el. interiors

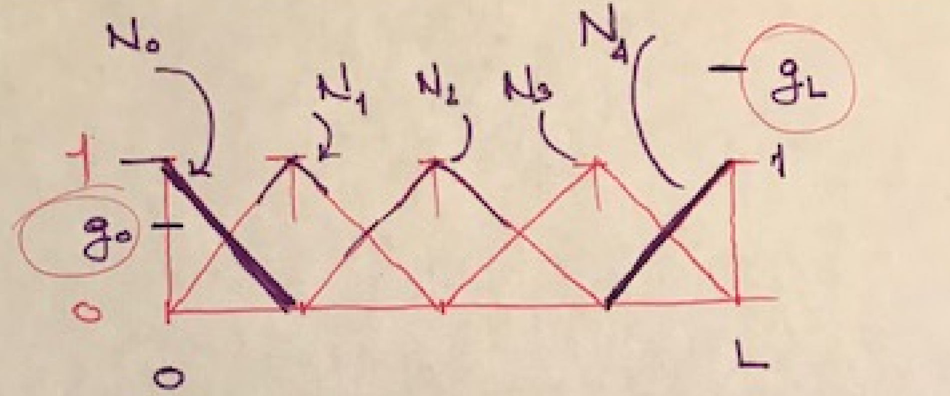
$\mathbb{L} = \mathcal{L}$ GLS

$= \mathcal{L}_{adv}$ $\boxed{a \cdot \nabla w^h}$ SUPG

$= -\mathcal{L}^* -$ MS.

$$\mathcal{L} w^h = a \cdot \nabla w^h - \kappa \Delta w^h$$

$$-\mathcal{L}^* w^h = a \cdot \nabla w^h \underline{\underline{+}} \kappa \Delta w^h$$



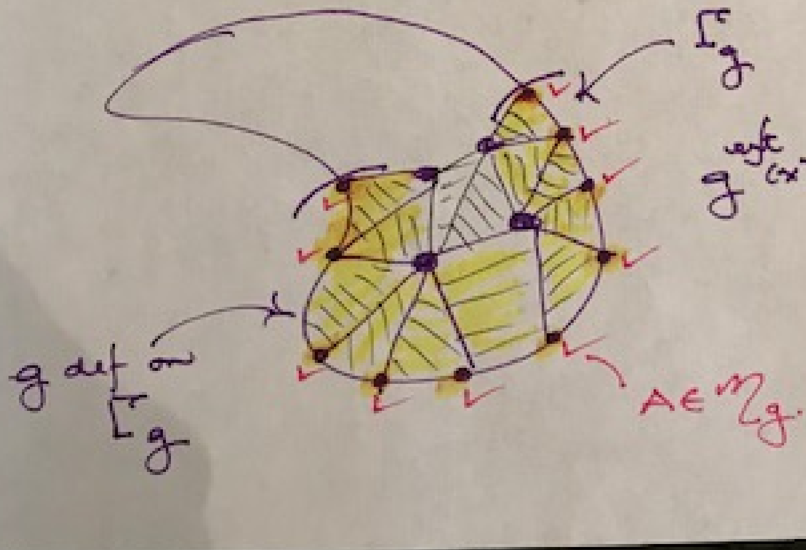
$A = 0 \quad 1 \quad 2 \quad 3 \quad 4 = N$

$$g^{ext} = g_0 N_0 + g_L N_4$$

$$g^{ext}(x) = g_0 N_0(x) + g_L \underbrace{N_4(x)}_{} \qquad x \in \Gamma_g$$

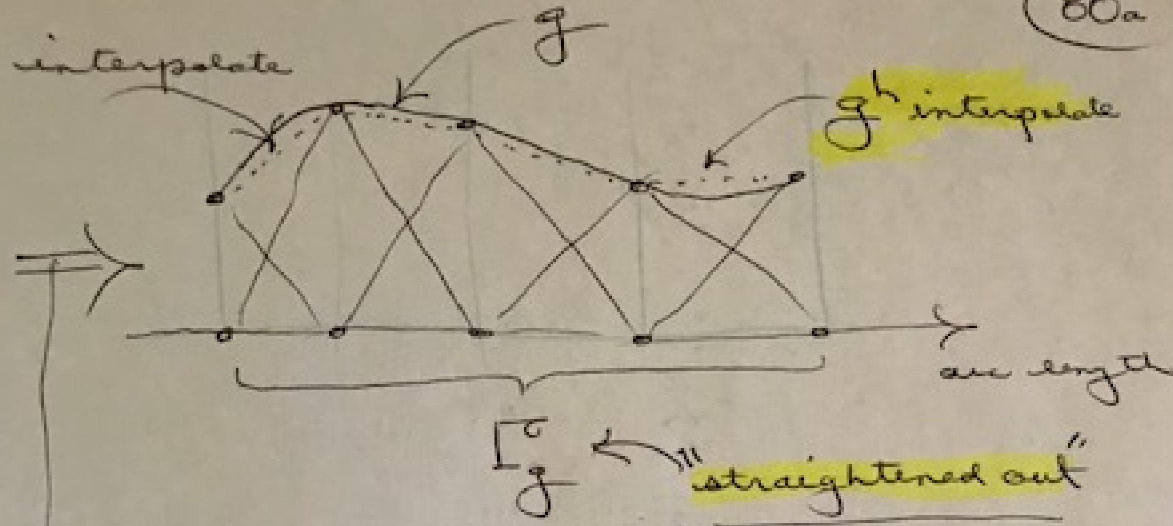
$$g^{ext}(0) = g_0 1 + 0$$
$$= g_0 \quad ✓$$

$$g^{ext}(L) = 0 + g_L 1 \; ✓$$



$$g^{ext}(x) = \sum_{A \in \eta_g} g_A N_A$$

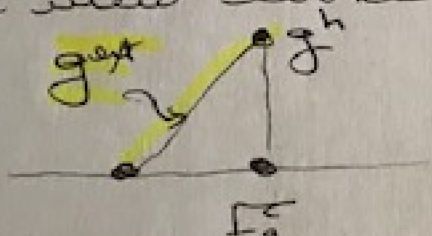
set of nodes on the Γ_g part of bndy.



$\Longrightarrow$

g is only defined on Γ_g^c
but g^{ext} extends it into the
domain. (Natural to use
basis functions of nodes on
Γ_g^c, $N_A(x)$, $x_A \in \eta_g$.

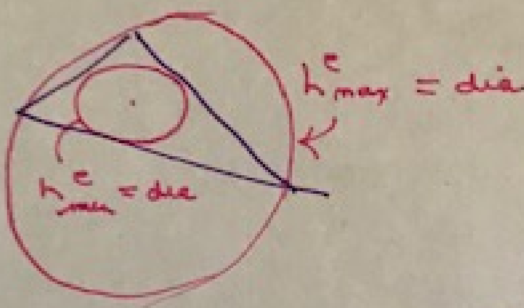
$\Longrightarrow$ side view looks like



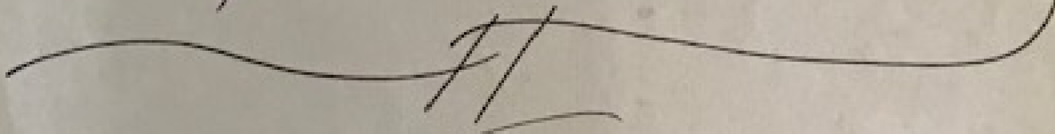
recall in 1D

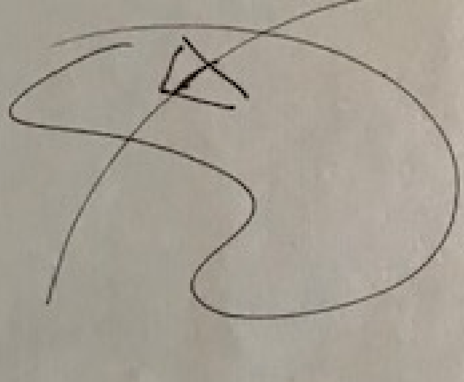
$$\tau = \underbrace{\frac{h}{2|a|}}\ \underbrace{\tilde{\xi}}_{\coth \alpha_h - \frac{1}{\alpha_h}}(\alpha_h)$$

$$\alpha_h = \frac{h|a|}{2\alpha}.$$



$$C_1 < \max_e \left(\frac{h^e_{max}}{h^e_{min}}\right) < C_2$$



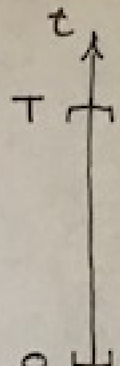


Model prob. pure adv. (82)

(1) $\overset{>0}{a}\, u_{,x} = f$, $x \in [0,L]$

$u(0) = g_0$

Another model prob.

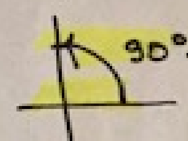


(2) $u_{,t} = f = f(t)$, $t \in [0,T]$

$u(0) = u_0$

What's the diff between (1) & (2)?

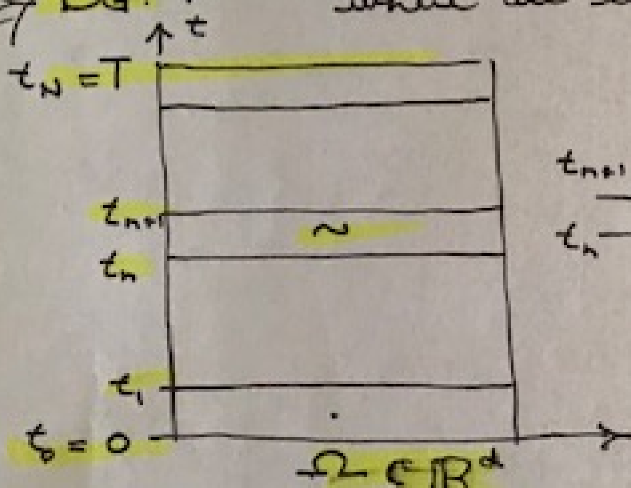
Absolutely nothing.

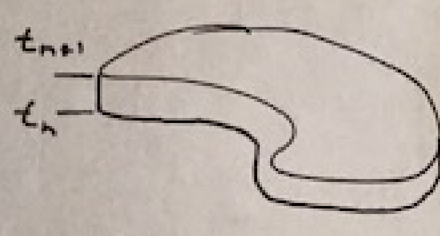


$a \leftarrow 1$
$x \leftarrow t$
$L \leftarrow T$.

DG. where are we going

$n = 0, 1, 2, \dots, N.$

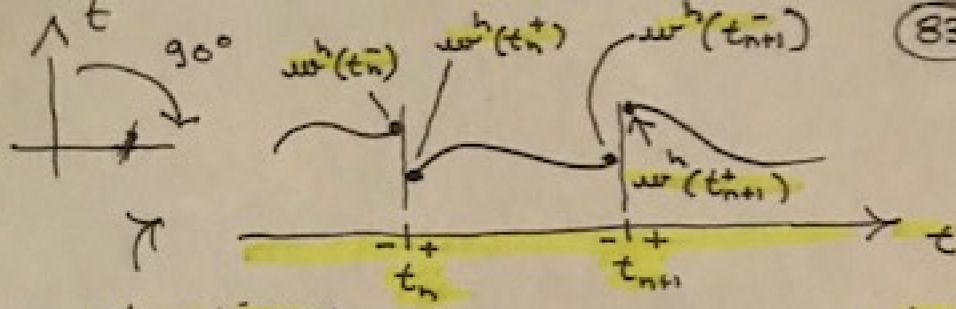




$\mathbb{R}^d$ d = # of space dim.

$\Omega \subset \mathbb{R}^d$

sp. time = $\Omega \times [0,T] =: Q$



discontinuous

10. $$w^h, u^h \in \mathcal{V}^h = \mathcal{S}^h = \bigoplus_{n=0}^{N-1} \mathcal{P}^k(t_n, t_{n+1})$$

degree

poly's

no continuity.

our exact prob is similar

$$w, u \in \mathcal{V} = \mathcal{S} = \bigoplus_{n=0}^{N-1} H^1(t_n, t_{n+1})$$

also, no continuity