11/2/23

ϕ_1 . . . . ϕ_{10}

$|x_2 - x_1|$

$|x_3 - x_1|$

$\vdots$

$|x_{10} - x_1|$

$\dfrac{(\phi_2 - \phi_1)^2}{2}$

$\vdots$

$\dfrac{(\phi_{10} - \phi_1)^2}{2}$

→ Array B

Array H

ϕ_2 . . . . ϕ_{10}

$|x_3 - x_2|$

$|x_4 - x_2|$

$\vdots$

$|x_{10} - x_2|$

$(\phi_3 - \phi_2)^2/2$

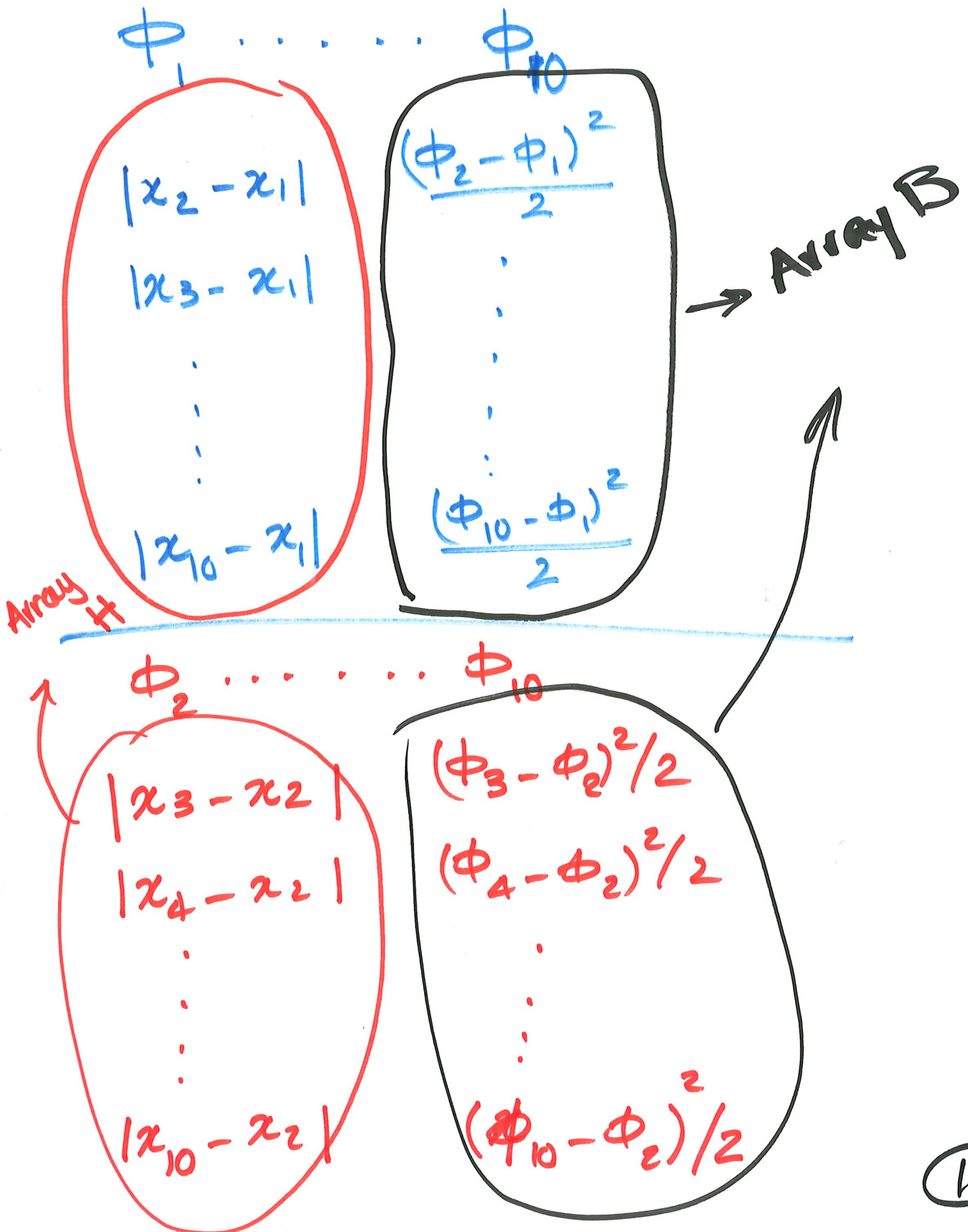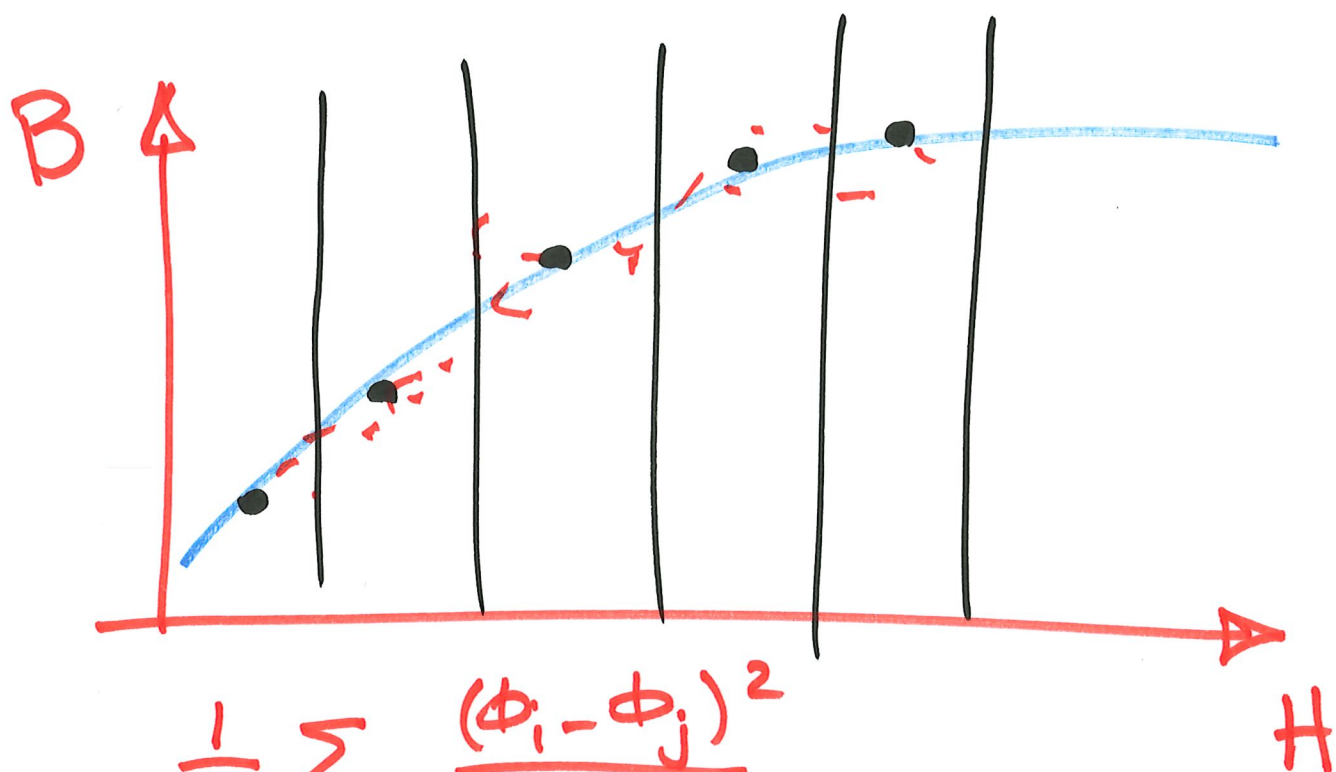$(\phi_4 - \phi_2)^2/2$

$\vdots$

$(\phi_{10} - \phi_2)^2/2$

①

$$\vdots$$

$$|x_{10} - x_9| \rightarrow (\phi_{10} - \phi_9)^2/2$$

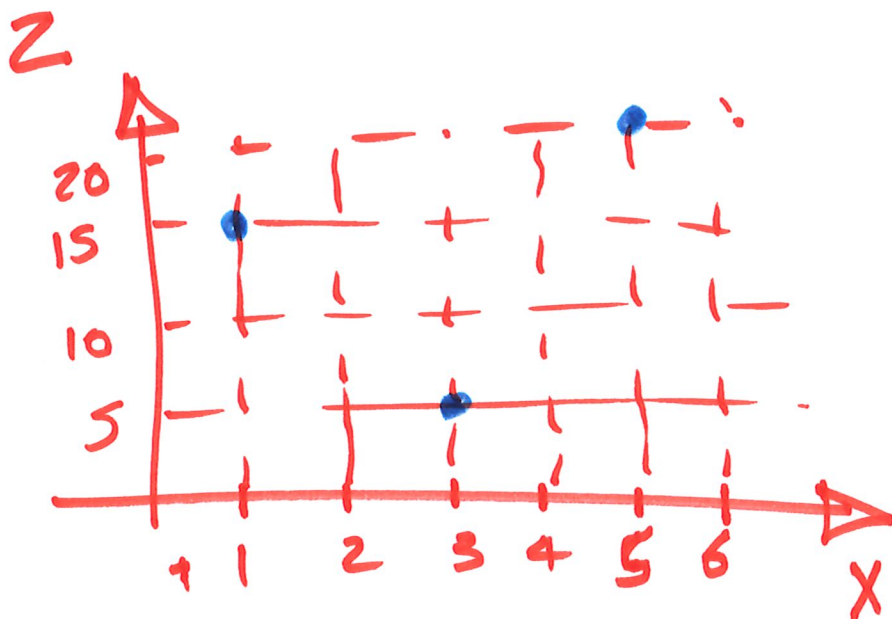

$$\frac{1}{n} \sum_n \frac{(\phi_i - \phi_j)^2}{2}$$

$$Y$$

$$X$$

$$\frac{1}{n} \sum h_{ij}$$

$$|x_i - x_j|$$

②

$$Z(x=2) = ?$$

$$Z(x=4) = ?$$

$$Z(x=6) = ?$$

$$Z_2^* = Z_1 + \left(\frac{x_2 - x_1}{x_3 - x_1} \right)(Z_3 - Z_1)$$

$$\underbrace{\frac{h_{21}}{h_{31}}}_{} = \lambda_3$$

$$Z_2^* = Z_1 \underbrace{(1 - \lambda_3)}_{\lambda_1} + \lambda_3 Z_3$$

$$= Z_1 \lambda_1 + Z_3 \lambda_3$$

$$\lambda_1 + \lambda_3 = 1$$

$$\text{if} \quad \lambda_1 = \tfrac{1}{2}, \quad \lambda_3 = \tfrac{1}{2} \implies Z_2^* = \tfrac{1}{2}(5 + 15) = 10$$
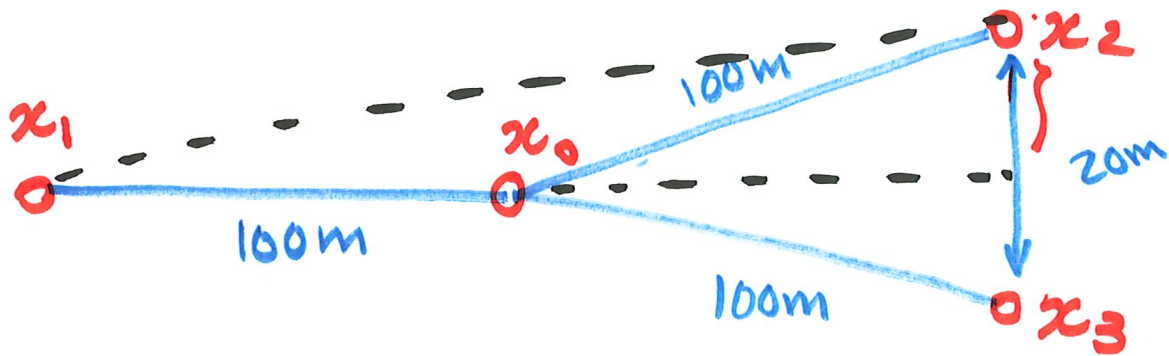
(3)

$$z^*(x_0) = \sum_{i=1}^{N} \lambda_i \, z(x_i)$$

$$, \quad \sum_{i=1}^{N} \lambda_i = 1$$

Find λ_i

Example:

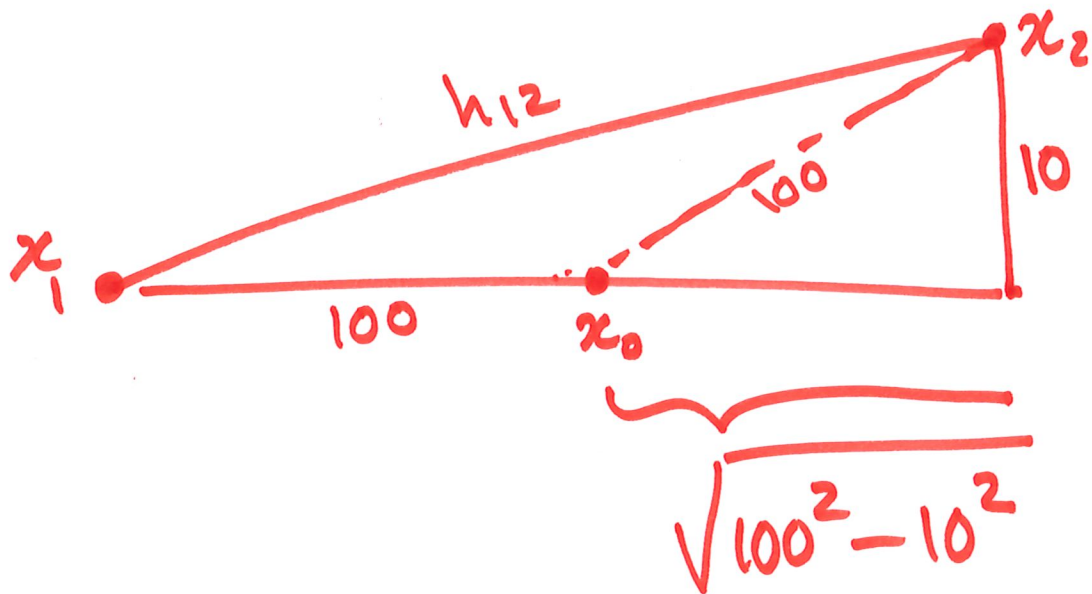$V(x_1) = 10\%$ $V(x_2) = 5\%$

$V(x_3) = 15\%$

~~$h_{10} = 100$~~   ~~$h_{20} = 100$~~   ~~$h_{30} = 100$~~

$$\gamma(h) = \begin{cases} 0.01\,h & h \leqslant 400 \\ 4 & h > 400 \end{cases}$$

$h_{10} = h_{20} = h_{30} = 100$

$h_{23} = 20$

$h_{12} = h_{13} = \sqrt{10^2 + \left(100 + \sqrt{100^2 - 10^2}\right)^2}$

$= 199.75 \text{ m}$

⑤

$$h_{12} = \sqrt{10^2 + \left[100 + \sqrt{100^2 - 10^2}\right]^2}$$

$$= 199.75 \text{ m}$$

$$\begin{bmatrix} h_{11} & h_{12} & h_{13} & h_{10} \\ h_{21} & h_{22} & h_{23} & h_{20} \\ h_{31} & h_{32} & h_{33} & h_{30} \end{bmatrix} = $$

$$\begin{bmatrix} 0 & 199.75 & 199.75 & 100 \\ 199.75 & 0 & 20 & 100 \\ 199.75 & 20 & 0 & 100 \end{bmatrix}$$

⑥

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} & | & C_{10} \\ C_{21} & C_{22} & C_{23} & | & C_{20} \\ C_{31} & C_{32} & C_{33} & | & C_{30} \end{bmatrix}$$

$$Cov(h) = Cov(0) - \gamma(h)$$

$$Cov(h) = 4 - \gamma(h)$$

$$\begin{bmatrix} 4 & 2.01 & 2.01 & 3 \\ 2.01 & 4 & 3.8 & 3 \\ 2.01 & 3.8 & 4 & 3 \end{bmatrix}$$

$$Cov(199.75) = 4 - 0.01(199.75)$$
$$= 2.01$$

$$Cov(20) = 4 - 0.01(20) = 3.8$$

⑦

$$\begin{bmatrix} 4 & 2.01 & 2.01 & 1 \\ 2.01 & 4 & 3.8 & 1 \\ 2.01 & 3.8 & 4 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \beta \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 1 \end{bmatrix}$$

$$\Rightarrow \quad \lambda_1 = 0.487$$

$$\lambda_2 = 0.256$$

$$\lambda_3 = 0.256$$

$$\beta = 0.026$$

$$z^*(x_0) = 0.487(10) + 0.256(5)$$
$$+ 0.256(15)$$

$$\Rightarrow \quad z^*(x_0) \simeq 10\%$$

(8)

$$\sigma^2_{e,min} = \sigma^2 - \beta - \sum_{i=1}^{N} \lambda_i C(h_{io})$$

$$= 4 - 0.026 -$$

$$[0.487(3) + 0.256(3) + 0.256(3)]$$

$$= 0.977$$

$$95\% \ CI = 10 \pm 1.96 \sqrt{0.977}$$

$$\implies 10 \pm 1.94$$