Lecture #12, Feb. 26, 2024

$$\ldots \leq \varkappa \|\cancel{w} e^h_{,x}\|^2 \leq \left| B(e^h, \eta) \right|$$

$$= \left| \int_0^L \left(-a \cancel{w} e^h_{,x} \eta + e^h_{,x} \varkappa \eta_{,x} \right) dx \right|$$

a, $\varkappa$ const's >0

$$\leq |a| \left| (e^h_{,x}, \eta) \right| + \varkappa \left| (e^h_{,x}, \eta_{,x}) \right|$$

$$\leq |a| \|e^h_{,x}\| \|\eta\| + \varkappa \|e^h_{,x}\| \|\eta_{,x}\| \quad \text{by C-S}$$

$$\varkappa \|e^h_{,x}\| \leq \cancel{\|e^h_{,x}\|} \left(|a| \|\eta\|_0 + \varkappa \|\eta_{,x}\|_0 \right)$$

$$\frac{\varkappa}{2L} \|e^h\|_1 \leq \quad |a| \|\eta\|_0 + \varkappa \frac{1}{L} \|L \eta_{,x}\|_0 + \frac{\varkappa}{L} |\eta|_1$$

↖ equiv norms

↙ equiv norms.

$$\leq \|\eta\|_1$$

$$\leq |a| \left(c_{int} \left(\frac{h}{L}\right)^{k+1-0} \|u\|_{k+1} \right) + \frac{\varkappa}{L} \left(c_{int} \left(\frac{h}{L}\right)^{k+\cancel{1}-\cancel{1}} \|u\|_{k+1} \right)$$

$$= C_{int} \left(|a| \left(\frac{h}{L}\right)^{k+1} + \left(\frac{h}{L}\right)^k \right) \|u\|_{k+1}$$

$$\frac{\cancel{\varkappa}}{2\cancel{L}} \|e^h\|_1 \leq C_{int} \left(\frac{|a|}{\varkappa} \frac{h}{\cancel{L}} + \frac{\cancel{\varkappa}\,1}{\cancel{L}} \right) \left(\frac{h}{L}\right)^k \|u\|_{k+1}$$

$$\boxed{\|e^h\|_1 \leq 2 C_{int} \left(1 + 2\alpha_h\right) \left(\frac{h}{L}\right)^k \|u\|_{k+1}}$$

$$\alpha_h = \frac{|a| h}{2\varkappa}$$

$$\|e\|_1 = \|e^h + \eta\|_1$$

$$\le \|e^h\|_1 + \|\eta\|_1$$

$$\|e\|_1 \le c_{int}\left(2 + 4\alpha_h\right)\left(\frac{h}{L}\right)^k \|u\|_{k+1} + c_{int}\left(\frac{h}{L}\right)^k \|u\|_{k+1}$$

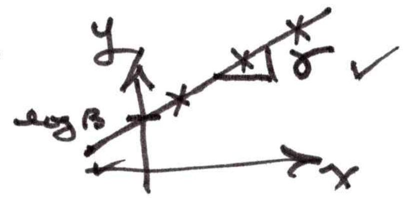
$$\|e\|_1 \le C_{int}\left(3 + 4\alpha_h\right)\left(\frac{h}{L}\right)^k \|u\|_{k+1}$$

uh oh! α_h can be large $\frac{|a|h}{2\alpha} \gg 1$.

Goal. $$\boxed{\|e\|_1 \le C\left(1 + \alpha_h\right)\left(\frac{h}{L}\right)^k \|u\|_{k+1}}$$

γ ← conv. rate

$$\underbrace{|||e|||}_{\text{meas.}} \sim \beta \underbrace{\left(\frac{h}{L}\right)}_{\text{meas.}}^{\gamma}$$



$$\log |||e||| \sim \log\beta + \gamma \log\left(\frac{h}{L}\right)$$

$$y \sim \underset{\text{y-intercept}}{b} + \underset{\text{slope}}{m}\, x$$

$y_1, y_2, \ldots$ $x_1, x_2, \ldots$

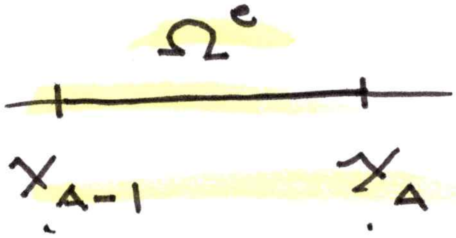
Toward anal. of SUPG, we need an inverse est.

$$|w^h|_{H^s(\Omega^e)} \le C_{inv}\left(\frac{h}{L}\right)^{r-s} |w^h|_{H^r(\Omega^e)}$$

$\forall\, w^h \in \mathcal{V}^h, \mathcal{S}^h$

C_{inv}: non-dim, indep of h, but is a ftn. of k, r, s.

element result.



$$\left(|w^h|_{H^s(\Omega^e)}\right)^2 = \int_{\Omega^e} L^{2s} (w^h_{,x\ldots x})^2\, dx$$

$$= L^{2s} \|w^h_{,x\ldots x}\|^2_{\Omega^e}$$

same but w. r

$$\cancel{L^{2s}} \|\underbrace{w^h_{,x\ldots x}}_{s\,x}\|^2_{\Omega^e} \le C_{inv}^2 \left(\frac{h^e}{\cancel{L}}\right)^{2(r-s)} \cancel{L^{2(r-s)}} \|\underbrace{w^h_{,x\ldots x}}_{r\,x}\|^2_{\Omega^e}$$

$$\|\underbrace{w^h_{,x\ldots x}}_{s\,x}\|_{\Omega^e} \le \boxed{C_{inv}\left(\frac{h^e}{L}\right)^{r-s}} \|\underbrace{w^h_{,x\ldots x}}_{r\,x}\|_{\Omega^e}$$

computable

$$\boxed{\|w^h_{,xx}\|_{\Omega^e} \le C_{inv}\left(\frac{h}{L}\right)^{-1} \|w^h_{,x}\|_{\Omega^e}}$$

important in Nietsche, DG.