Theory & Steps

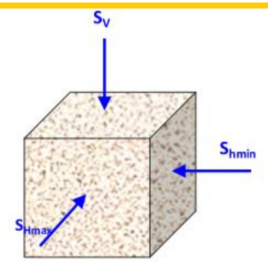
Agustin Garbino

1. Principal Stresses

$$S_v(z) = \int_0^z \rho_{bulk}(z) g \, dz$$

Assuming elasticity and isotropy of mechanical properties:

$$\begin{cases} \sigma_{Hmax} = \dfrac{\nu}{1-\nu}\sigma_v + E'\varepsilon_{Hmax} + \nu E'\varepsilon_{hmin} \\ \sigma_{hmin} = \dfrac{\nu}{1-\nu}\sigma_v + \nu E'\varepsilon_{Hmax} + E'\varepsilon_{hmin} \end{cases}$$



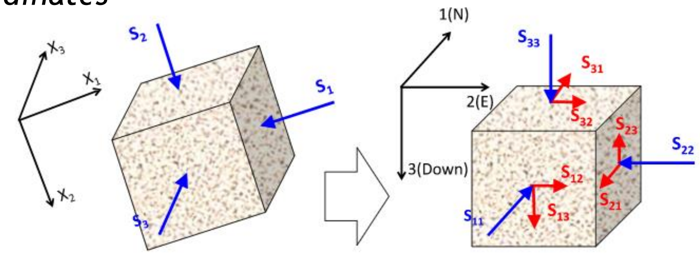
Rock mechanical properties:

$$E_{dyn} = \rho \, V_s^2 \frac{3V_p^2 - 4V_s^2}{V_p^2 - V_s^2}$$

$$\nu_{dyn} = \frac{V_p^2 - 2V_s^2}{2V_p^2 - 2V_s^2}$$

2. Rotation of Principal Stresses into Geographical Coordinates

$$\mathrm{R}_{PG} = \begin{bmatrix} \cos\alpha\cos\beta & \sin\alpha\cos\beta & -\sin\beta \\ \cos\alpha\sin\beta\sin\gamma - \sin\alpha\cos\gamma & \sin\alpha\sin\beta\sin\gamma + \cos\alpha\cos\gamma & \cos\beta\sin\gamma \\ \cos\alpha\sin\beta\cos\gamma + \sin\alpha\sin\gamma & \sin\alpha\sin\beta\cos\gamma - \cos\alpha\sin\gamma & \cos\beta\cos\gamma \end{bmatrix}$$

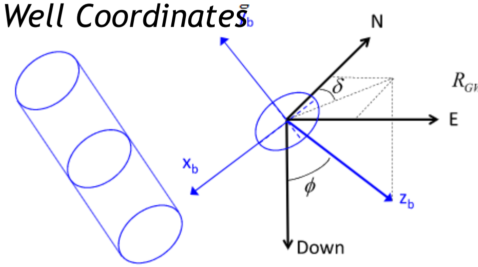


$$\underline{\underline{S}}_G = R_{PG}^T \underline{\underline{S}}_P R_{PG}$$

$$\underline{\underline{S}}_P = \begin{bmatrix} S_1 & 0 & 0 \\ 0 & S_2 & 0 \\ 0 & 0 & S_3 \end{bmatrix} \qquad \underline{\underline{S}}_G = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

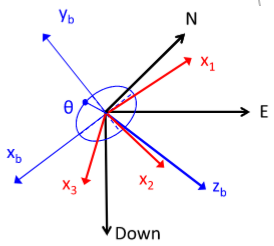
3. Rotation of Principal Stresses from Geo to Well Coordinates

$$\underline{\underline{S}}_W = R_{GW} \underline{\underline{S}}_G R_{GW}^T$$



$$R_{GW} = \begin{bmatrix} -\cos\delta\cos\phi & -\sin\delta\cos\phi & \sin\phi \\ \sin\delta & -\cos\delta & 0 \\ \cos\delta\sin\phi & \sin\delta\sin\phi & \cos\phi \end{bmatrix}$$

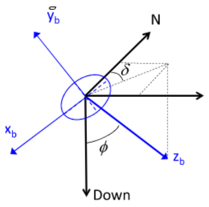
4. Kirsch Equations - Well Principal Stresses



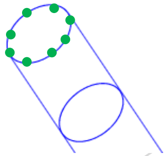
Stresses at the wellbore wall (Kirsch[PP]+Kirsch[S])

$$\begin{cases} \sigma_{rr} = \Delta P \\ \sigma_{\theta\theta} = \sigma_{11} + \sigma_{22} - 2(\sigma_{11} - \sigma_{22})\cos 2\theta - 4\sigma_{12}\sin 2\theta - \Delta P \\ \tau_{\theta z} = 2(\sigma_{23}\cos\theta - \sigma_{13}\sin\theta) \\ \sigma_{zz} = \sigma_{33} - 2\nu(\sigma_{11} - \sigma_{22})\cos 2\theta - 4\nu\sigma_{12}\sin 2\theta \end{cases}$$

5. Principal Stresses around the deviated wellbore perimeter



Wellbore principal stresses calculated for all possible values of azimuth and inclination



$$\begin{bmatrix} \sigma_{rr} & 0 & 0 \\ 0 & \sigma_{\theta\theta} & \sigma_{\theta z} \\ 0 & \sigma_{z\theta} & \sigma_{zz} \end{bmatrix} \xrightarrow{\text{Eigen values}} \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$$

$$\boxed{UCS = \max(\max\_shear\ stress)}$$

$$\boxed{TS = \min(\min\_shear\ stress)}$$

Source: *https://dnicolasespinoza.github.io/AdvancedGeomech/*