

Darcy's Law for Inclined flow

$$\text{In Darcy units : } q = -\frac{kA}{\mu} \left(\frac{dP}{ds} - \frac{\rho g}{1.0133 \times 10^6} \frac{dz}{ds} \right)$$

$$\text{In oilfield units : } q = -0.001127 \frac{kA}{\mu B} \left(\frac{dP}{ds} - 0.433 \gamma \frac{dz}{ds} \right)$$

$$k [D] = k [md] \times \left[\frac{10}{1000 md} \right] = 10^{-3} k [md]$$

$$A [cm^2] = A [ft^2] \times \left[\frac{(30.48)^2 cm^2}{1 ft^2} \right] = 929.03 A [ft^2]$$

$$\begin{aligned} \frac{dP}{ds} \left[\frac{atm}{cm} \right] &= \frac{dP}{ds} \left[\frac{Psi}{ft} \right] \times \left[\frac{1 ft}{(30.48) cm} \right] \times \left[\frac{1 atm}{14.7 psi} \right] \\ &= 2.23 \times 10^{-3} \frac{dP}{ds} \left[\frac{Psi}{ft} \right] \end{aligned}$$

$$\begin{aligned} \frac{\rho g}{1.0133 \times 10^6} \frac{dz}{ds} \left[\frac{atm}{cm} \right] &= \left(\frac{32.185 \times 62.428}{32.185 \times 12^2} \gamma \right) \frac{dz}{ds} \left[\frac{Psi}{ft} \right] \\ &\times \left[\frac{1 atm}{14.7 psi} \right] \times \left[\frac{1 ft}{30.48 cm} \right] \\ &= (2.23 \times 10^{-3}) (0.433 \gamma) \frac{dz}{ds} \left[\frac{Psi}{ft} \right] \end{aligned}$$

$$\begin{aligned}
 q \left[\frac{\text{cm}^3}{\text{s}} \right] &= q \left[\frac{\text{STB}}{\text{day}} \right] B \left[\frac{\text{RB}}{\text{STB}} \right] \times \left[\frac{5.615 \text{ ft}^3}{1 \text{ RB}} \right] \\
 &\times \left[\frac{(30.48)^3 \text{ cm}^3}{1 \text{ ft}^3} \right] \times \left[\frac{1 \text{ day}}{24(3600) \text{ s}} \right] \\
 &= 1.84 q B \left[\frac{\text{RB}}{\text{day}} \right]
 \end{aligned}$$

$$q = - \frac{kA}{\mu} \left[\frac{dP}{ds} - \frac{\rho g}{1.0133 \times 10^6} \frac{dz}{ds} \right]$$

$$1.84 q B \left[\frac{\text{STB}}{\text{day}} \right] = - \frac{10^{-3} k [\text{md}] \times 929.03 A [\text{ft}^2]}{\mu} \times$$

$$\left[2.23 \times 10^{-3} \frac{dP}{ds} \left[\frac{\text{Psi}}{\text{ft}} \right] - (2.23 \times 10^{-3})(0.4338) \times \frac{dz}{ds} \left[\frac{\text{Psi}}{\text{ft}} \right] \right]$$

$$\Rightarrow q = - 0.001127 \frac{kA}{\mu B} \left[\frac{dP}{ds} - 0.4338 \frac{dz}{ds} \right]$$