The University of Texas at Austin

WHAT STARTS HERE CHANGES THE WORLD

# Advanced Petrophysics: Permeability, Part 1 of 5


Instructor: Zoya Heidari, Ph.D.

Associate Professor
The University of Texas at Austin




## PGE381L Outline

- Introduction to petrophysics, geology, and formation data
- Porosity
- Fluid saturations
- Permeability
- Quantification of heterogeneity, spatial data analysis, and geostatistics
- Interfacial phenomena and wettability
- Capillary pressure
- Relative permeability
- Dispersion in porous media
- Introduction to petrophysics of unconventional reservoirs

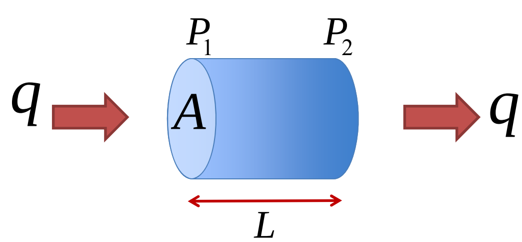
# What Do We Learn in This Lecture?

- What is permeability?
- What rock properties affect absolute permeability?
- Darcy's Law
- Permeability evaluation in tensor form
- Darcy's law for anisotropic porous media
- Non-Darcy flow
- Averaging permeability data
- How to quantify permeability?
  - Analytical models for permeability assessment
  - Permeability assessment in the laboratory?
  - Permeability assessment in-situ condition using pressure transient test
  - Permeability assessment in-situ condition using well logs



# What is Permeability?

Porous medium's ability to transmit fluids!




**Darcy's Law:**

$$q = -\frac{kA}{\mu}\frac{\Delta P}{\Delta x}$$

**Assumptions:**
**Steady-state, linear flow of a single phase liquid, in a horizontal medium**

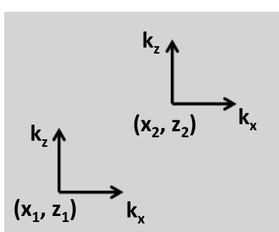
# Horizontal vs. Vertical permeability

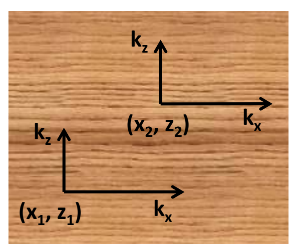
**Permeability is a tensor!**

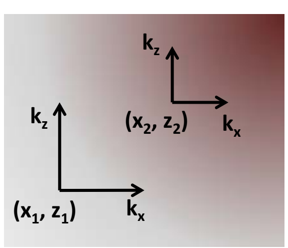
$$\bar{\bar{k}} = \begin{pmatrix} k_{xx} & k_{xy} & k_{xz} \\ k_{yx} & k_{yy} & k_{yz} \\ k_{zx} & k_{zy} & k_{zz} \end{pmatrix}$$

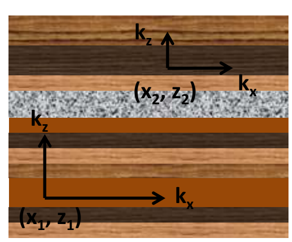


# Heterogeneity and Anisotropy



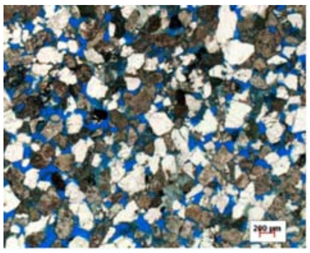



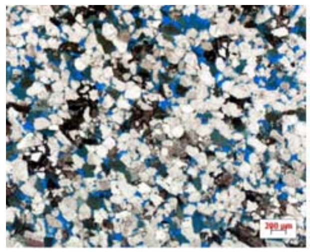



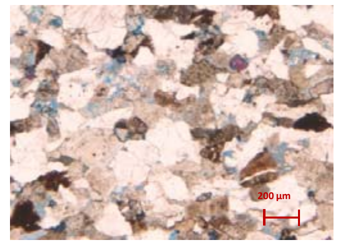



## Parameters Affecting Permeability

- **Which rock has a higher permeability?**





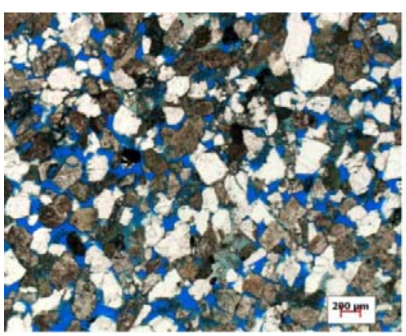



Source: He, W., et al., 2011, SPE 143497.

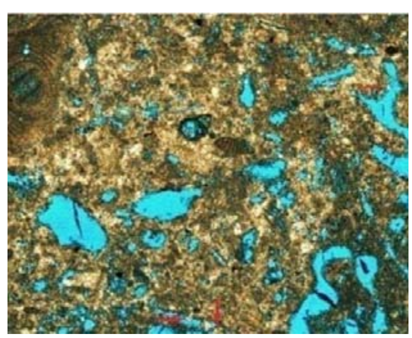


## Parameters Affecting Permeability

- **Which rock has a higher permeability?**



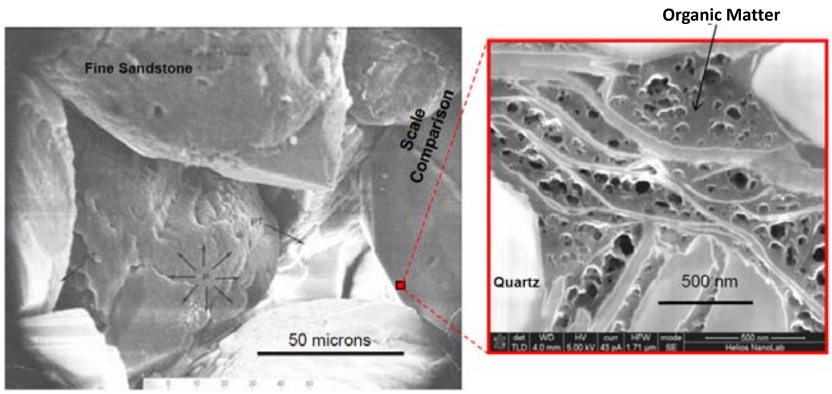
Source: He, W., et al., 2011, SPE 143497.



Source: Rahman et al., 2011, IPTC 14583.

## Parameters Affecting Permeability

- **Which rock has a higher permeability?**




Source: Passey, Q., et al., 2010.



## Parameters Affecting Permeability

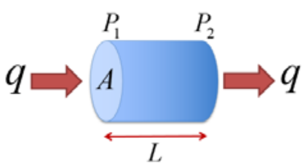
- **How do the following parameters affect permeability?**
  - Compaction
  - Pore size
  - Throat size
  - Sorting
  - Cementation
  - Layering
  - Clay swelling
  - Stress
  - Fractures

# How to Quantify Permeability?

**Darcy's Law:**

$$q = -\frac{kA}{\mu}\frac{\Delta P}{\Delta x}$$

**Assumptions:**
**Steady-state, linear flow of a single phase liquid, in a horizontal medium**




**Absolute Permeability:** **permeability of the medium when it is saturated 100% by a single phase, non-reactive liquid.**

$$k = \frac{q\mu L}{A\Delta P}$$

**How can we quantify it?**

**Question:** **What happens if there is a reaction between rock and liquid?**



# Example

**Results of a laboratory experiment in a sandpack (Peters, 1979) are given as follows. Estimate permeability of this core plug.**

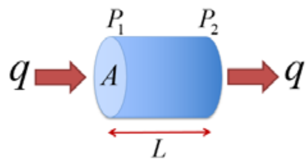
| q ($cm^3/s$) | ΔP (atm) |
|---|---|
| 0 | 0 |
| 0.0014 | 0.0476 |
| 0.0556 | 1.9284 |
| 0.0889 | 3.0573 |
| 0.1333 | 4.5439 |
| 0.2222 | 7.5303 |
| 0.3111 | 10.465 |

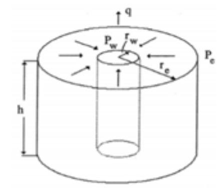
Viscosity = 105.363 cp
Length of the core plug = 115.6 cm
Diameter of the core plug = 4.961 cm
Porosity of the core plug = 37.80%

**Solution:**

**This is an example of a steady-state laboratory measurement technique.**

# More on Darcy's Law



**Darcy's Law in SI units:**

$$q = \frac{kA}{\mu}\frac{\Delta P}{L}$$

$$q = \frac{2\pi kh\left(P_w - P_e\right)}{\mu \ln\left(r_e/r_w\right)}$$

**Let's derive it together!**

**Darcy's Law in oilfield units:**

$$q = \frac{0.001127kA}{\mu B}\frac{\Delta P}{L} = \frac{1}{887.2}\frac{kA}{\mu B}\frac{\Delta P}{L}$$

$$q = \frac{2\pi kh\left(P_w - P_e\right)}{\mu B \ln\left(r_e/r_w\right)} = \frac{1}{141.2}\frac{kh\left(P_w - P_e\right)}{\mu B \ln\left(r_e/r_w\right)}$$

**B = reservoir volume/stock tank volume or reservoir barrels/stock tank barrels**



# Comparison of SI, Darcy, and Oilfield Units

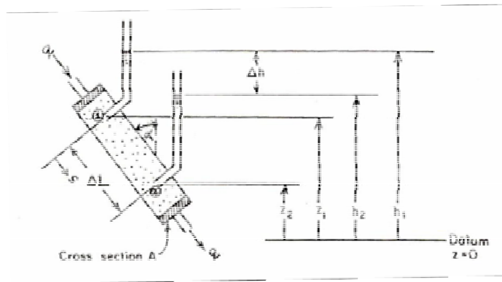
| Quantity | SI Unit | Darcy Unit | Oilfield Unit |
|---|---|---|---|
| Time | s | s | day |
| Length/Thickness/Radius | m | cm | ft |
| Pressure | Pa | atm | psia |
| Flow rate | $m^3/s$ | $cm^3/s$ | STB/day |
| Viscosity | Pa.s | cp | cp |
| Porosity | fraction | fraction | fraction |
| Permeability | $m^2$ | Darcy | Millidarcy |
| Compressibility | $Pa^{-1}$ | $atm^{-1}$ | $psi^{-1}$ |

**1 Darcy = the permeability of a medium that permits a flow rate of 1 $cm^3$/s of a fluid of viscosity 1 cp under a pressure gradient of 1 atm/cm acting on an area of 1 $cm^2$.**

$$1 darcy = 9.869\times10^{-9}\,cm^2 = 9.869\times10^{-13}\,m^2 = 1.062\times10^{-11}\,ft^2$$

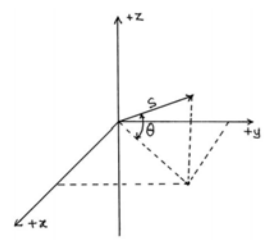
# Darcy's Law for Inclined Flow

$$q = -\frac{kA}{\mu}\frac{d\Phi}{ds}$$




Velocity potential

In Darcy units $$\Phi = P \pm \frac{\rho g z}{1.0133\times 10^{6}}$$



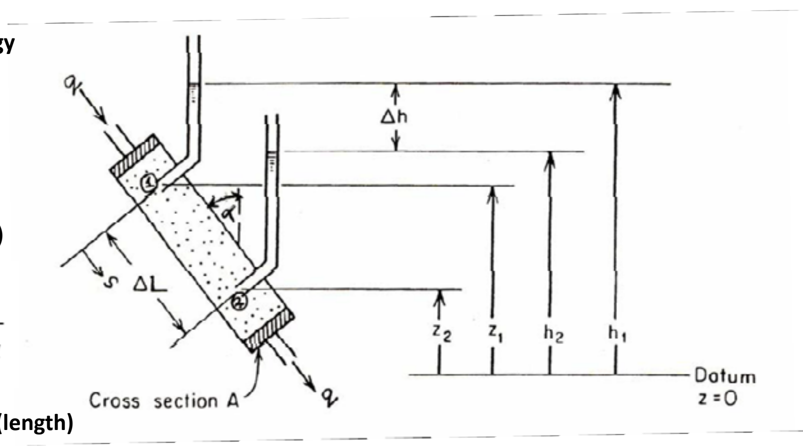

In field units $$\Phi = P \pm 0.433\gamma z$$



# Equivalent Formulation

More common formulation in hydrology

$$q = -KA\frac{\Delta h}{\Delta L}$$




K: hydraulic conductivity (length/time)

$$K = \frac{k\rho g}{1.0133x10^{6}\mu}$$

h: hydraulic head or piezometric head (length)

$$h = \frac{P}{\rho g} \pm z$$

# Example



# Validity of Darcy's Law

**Darcy's law is valid for:**

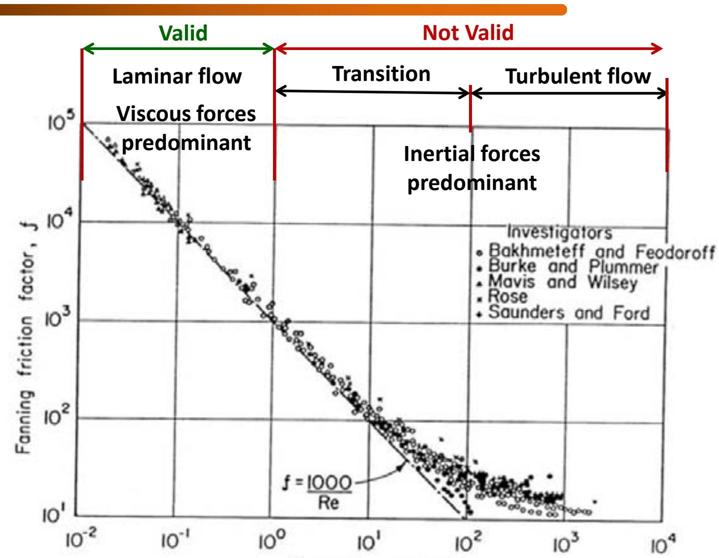
- **Slow, laminar flow in porous media without chemical reaction**

$$\mathrm{Re} = \frac{v \rho D_p}{\mu}$$

Grain Diameter

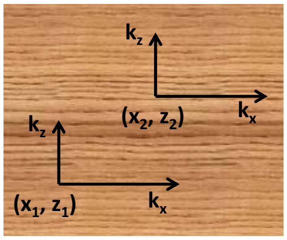
$$f = \frac{\tau}{\rho \frac{v^2}{2}}$$

Shear stress




(Rose, 1945)

# Darcy's Law for Anisotropic Porous Media




$$\vec{v} = -\frac{\overline{k}}{\mu}.\nabla\Phi$$

$$\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = -\frac{1}{\mu}\begin{bmatrix} k_{xx} & k_{xy} & k_{xz} \\ k_{yx} & k_{yy} & k_{yz} \\ k_{zx} & k_{zy} & k_{zz} \end{bmatrix}\begin{bmatrix} \frac{\partial\Phi}{\partial x} \\ \frac{\partial\Phi}{\partial y} \\ \frac{\partial\Phi}{\partial z} \end{bmatrix}$$

**in the coordinates of the principal axes**

$$\begin{bmatrix} v_u \\ v_v \\ v_w \end{bmatrix} = -\frac{1}{\mu}\begin{bmatrix} k_u \frac{\partial\Phi}{\partial u} \\ k_v \frac{\partial\Phi}{\partial v} \\ k_w \frac{\partial\Phi}{\partial w} \end{bmatrix}$$

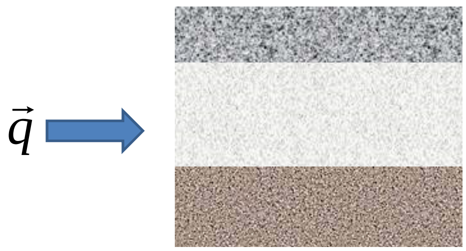
**How to calculate these?**
**Please review vector analysis and coordinate transformation topics in Mathematics.**
**Reading Assignment: Peters Textbook, Chapter 3.14**



# Non-Darcy Flow

**Please take notes!**

## Permeability in Parallel Beds

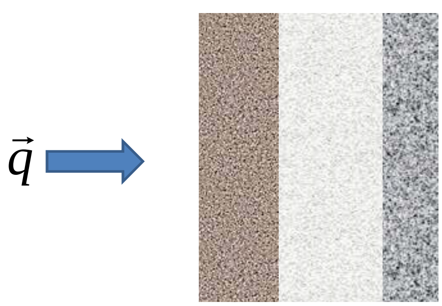


$$k_{avg} = \frac{\sum k_i h_i}{\sum h_i}$$

**Derivation:**



## Permeability in Serial Beds



$$k_{avg} = \frac{\sum h_i}{\sum h_i / k_i}$$

**Derivation:**

## How to Estimate Permeability

**Permeability is not measured!**

**Permeability is estimated!**

- How to Estimate Permeability?
  - Simplified analytical models
  - Laboratory-based permeability assessment
    - Routine core analysis
  - In-situ permeability assessment
    - Pressure Transient Test
    - Well logs





# Advanced Petrophysics: Permeability, Part 2 of 5

Instructor: Zoya Heidari, Ph.D.

Associate Professor
The University of Texas at Austin

## What Do We Learn in This Lecture?

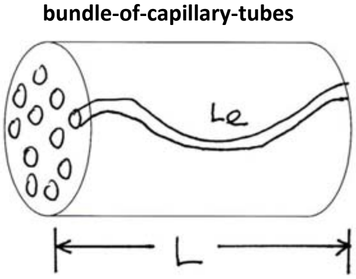
- What is permeability?
- What rock properties affect absolute permeability?
- Darcy's Law
- Permeability evaluation in tensor form
- Darcy's law for anisotropic porous media
- Non-Darcy flow
- Averaging permeability data
- **How to quantify permeability?**
  - **Analytical models for permeability assessment**
  - Permeability assessment in the laboratory?
  - Permeability assessment in-situ condition using pressure transient test
  - Permeability assessment in-situ condition using well logs



## Pore-Geometry-Based Analytical Models for Permeability Assessment

**Hagen-Poiseuille's law for steady flow through a single tortuous, circular capillary tube of radius r**

**bundle-of-capillary-tubes**




**For n tubes**

$$q_i = \frac{\pi r^4}{8\mu}\frac{(p_1 - p_2)}{L_e} = \frac{\pi r^4}{8\mu}\frac{\Delta p}{L_e} \Rightarrow q_T = \frac{n\pi r^4}{8\mu}\frac{(p_1 - p_2)}{L_e} = \frac{n\pi r^4}{8\mu}\frac{\Delta p}{L_e}$$

$$\Rightarrow \quad k = \frac{\phi r^2}{8\tau} \qquad \tau = \left(\frac{L_e}{L}\right)^2$$

**OR**

**Carman-Kozeny Equation**

$$k = \frac{\phi}{2\tau S_p^2} = \frac{\phi}{k_o \tau S_p^2} = \frac{\phi^3}{k_o \tau S^2} = \frac{\phi^3}{k_o \tau S_s^2 (1-\phi)^2}$$

**For non-circular pores** **Kozeny constant**

$S_p$: the wetted surface area of the pores per unit pore volume of the porous medium
$S_s$: the wetted surface area per unit grain volume of the porous medium
S: the wetted surface area per unit bulk volume of the porous medium

## Other Forms of Carman-Kozeny Equation

$$r_H = \frac{\text{Volume of Pore}}{\text{Wetted Surface Area of Pore}} = \frac{1}{S_p}$$

**For cylindrical pores:** $r_H = \frac{n\pi r^2 L_e}{2\pi n r L_e} = \frac{r}{2}$ ⇨ $k = \frac{\phi r_H^2}{2\tau}$

**For granular media:** $S_s = \frac{4\pi\left(D_p/2\right)^2}{\frac{4}{3}\pi\left(D_p/2\right)^3} = \frac{6}{D_p}$ ⇨ $k = \frac{D_p^2\phi^3}{36k_o\tau\left(1-\phi\right)^2}$

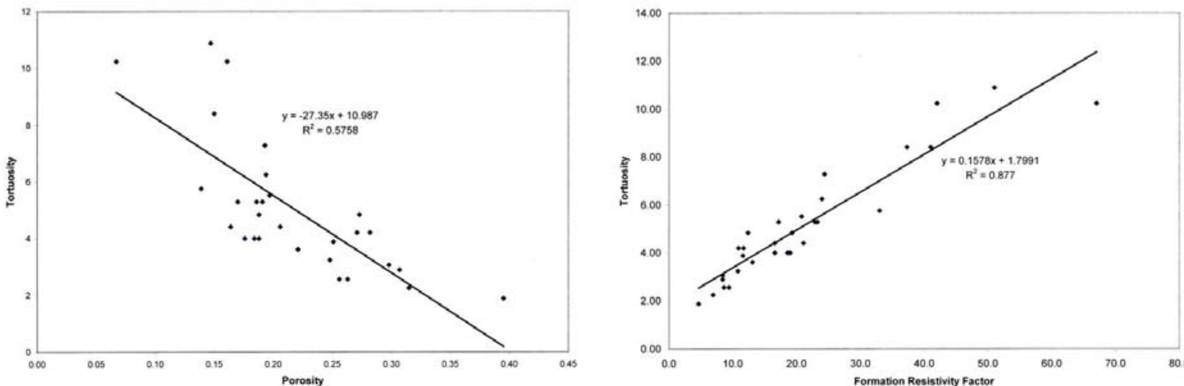
$D_p$: diameter of grains



## Carman-Kozeny Equation

- **Based on Carman-Kozeny equation answer the following questions:**
  - **How does specific surface area of the grains affect permeability?**
  - **Which one has a higher permeability? A medium composed of small grains or one composed of large grains?**
  - **How does tortuosity affect permeability of porous media?**

# How to Estimate Tortuosity?




from Winsauer et al., 1952

- Can we derive this correlation analytically?
- What other methods can be used to estimate tortuosity?

$$\tau_e \approx \phi F$$

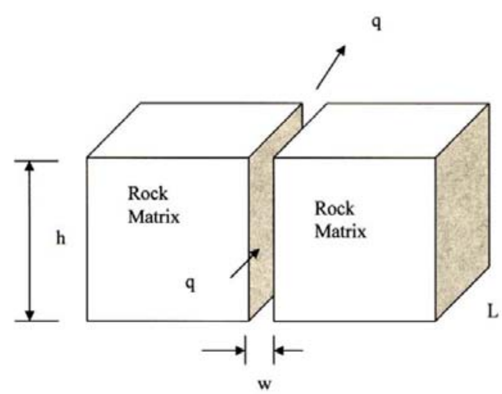
(Wyllie, 1957)



# Flow through Fractures

Hagen-Poiseuille's law for steady state flow through the fracture of width w:

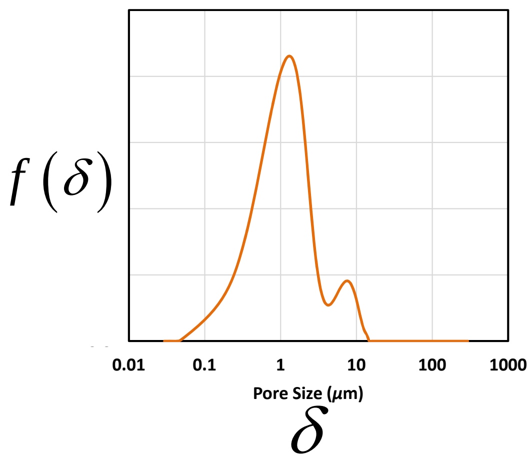
$$\left.\begin{aligned} q &= \frac{w^2 A}{12\mu}\frac{\Delta P}{L} \\ q_T &= \frac{kA_T}{\mu}\frac{\Delta P}{L} \end{aligned}\right\} \Rightarrow k = \frac{w^2}{12}$$




Please derive this equation!

## Impact of Pore-Size Distribution on Permeability

**Probability distribution function for pore-diameter distribution**




$$k = \frac{\phi}{32\tau} \frac{\int_0^\infty f(\delta)\delta^4 d\delta}{\int_0^\infty f(\delta)\delta^2 d\delta}$$

**What are the assumptions?**
**Please derive this equation!**





# Advanced Petrophysics: Permeability, Part 3 of 5


**Instructor: Zoya Heidari, Ph.D.**

Associate Professor
The University of Texas at Austin

# What Do We Learn in This Lecture?

- What is permeability?
- What rock properties affect absolute permeability?
- Darcy’s Law
- Permeability evaluation in tensor form
- Darcy’s law for anisotropic porous media
- Non-Darcy flow
- Averaging permeability data
- **How to quantify permeability?**
  - Analytical models for permeability assessment
  - **Permeability assessment in the laboratory?**
  - Permeability assessment in-situ condition using pressure transient test
  - Permeability assessment in-situ condition using well logs



# Laboratory-Based Permeability Assessment

Transient-state measurements
- Pulse decay
  - Plugs under external stress
- Pressure decay
  - Plugs under external stress
  - Crushed samples No external stress

Steady-state measurements
- Plugs under external stress

## Steady-State Measurements

**Liquid Permeability Measurement**

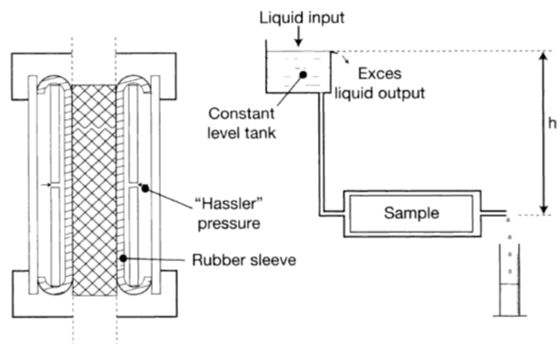



Image source: Zinszner, B. and Pellerin, F. M., 2007, A Geoscientist's Guide to Petrophysics.

**What are the disadvantages of using liquids for permeability measurement?**



## Steady-State Measurements

**Gas Permeability Measurement**

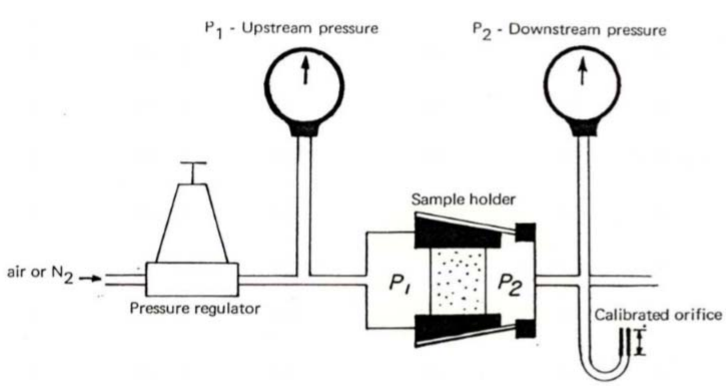



Image source: Peters, E. J., 2012, Advanced Petrophysics. Live Oak Book Company.

**Why do we often use gas and not water?**

# Steady-State Measurements

**Let's write Darcy's equation:**

$$q = -\frac{kA}{\mu}\frac{\Delta P}{\Delta x}$$

Boyle's law:

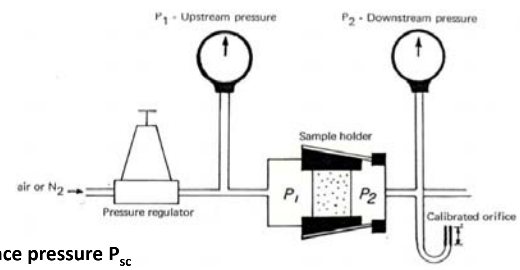
$$qP = q_{sc}P_{sc}$$

Assumptions: ideal gas at a constant temperature

$$\Rightarrow q_{sc} = -\frac{k_g A}{\mu P_{sc}}\left(\frac{1}{2}\frac{dP^2}{dx}\right) \rightarrow q_{sc} = \frac{k_g A}{2\mu P_{sc}}\left(\frac{P_1^2 - P_2^2}{L}\right)$$

$$\rightarrow k_g = \frac{2q_{sc}\mu L P_{sc}}{A\left(P_1^2 - P_2^2\right)}$$

**$P_2$ is the reference pressure, $P_{sc}$**




**$q_{sc}$ is the gas volumetric flow rate at a reference pressure $P_{sc}$**



# Klinkenberg Effect (1941)

- **Pressures applied in the laboratory are lower that those at in-situ condition. How does it impact interaction of gas molecules and rock surface in porous media?**

**The mean free path of gas molecules:**

$$\lambda = \frac{k_B T}{\sqrt{2}\pi d^2 P}$$

Boltzmann's constant

absolute temperature

pressure

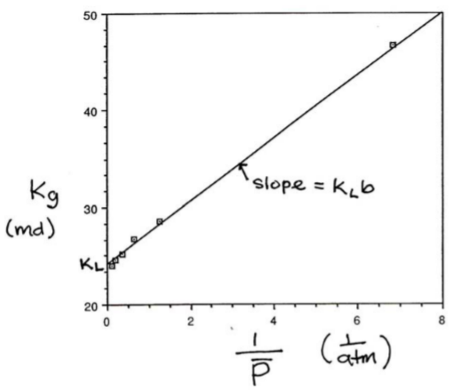
gas molecular diameter

- Put numbers in this equation and calculate mean free path of gas molecules in the laboratory condition.
- Compare this number against pore size

- **How does it affect the estimates of absolute permeability in the laboratory?**
- **How do we take into account this impact on the estimates of permeability?**

## Correction for the Klinkenberg Effect

$$k_g = k_L\left(1+\frac{b}{\bar{P}}\right) \qquad \bar{P} = \frac{P_1+P_2}{2}$$

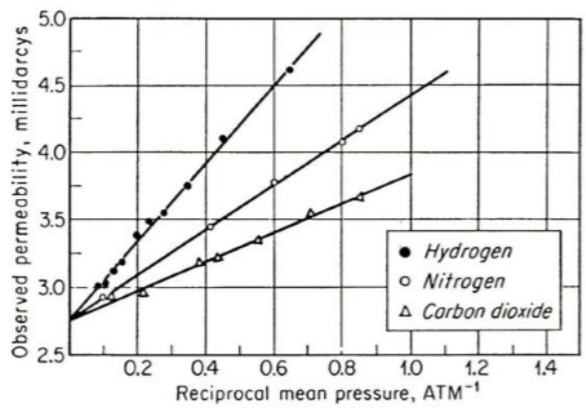



$\Rightarrow$ $K_L$ and $b$

**Does the type of gas affect the slope of this line and the required correction?**



## Impact of Gas Type on Klinkenberg Correction




Absolute permeability of the core to isooctane = 2.55 md

**Klinkenberg (1941)**

## Example



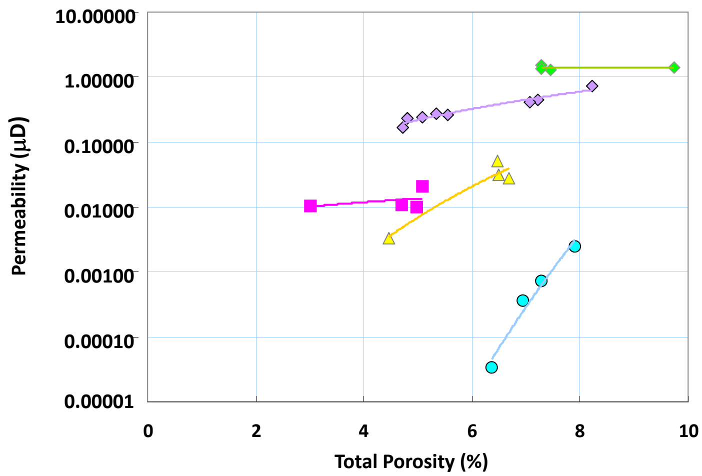
## Permeability Assessment in Tight Formations

OPTIONAL

- Typical techniques applied to tight formations
  - GRI Crushed pressure decay
  - Pressure pulse decay
  - Transient pulse decay
  - Steady-state (?)

## Permeability Assessment: Unconventionals

**OPTIONAL**



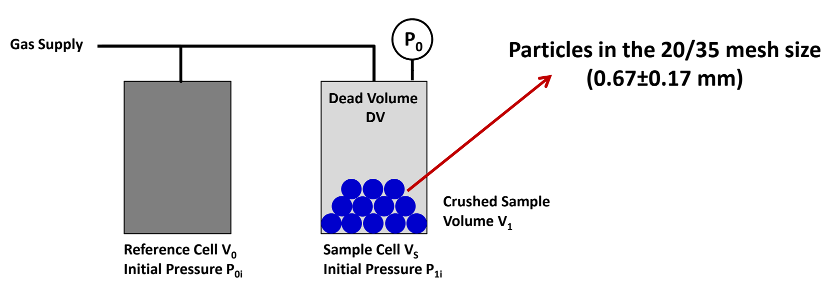

Sources: Passey 2011 (Courtesy Mark Rudnicki; see also Spears et al., 2011)



## GRI Permeability Assessment Method

**OPTIONAL**

**GRI permeability assessment method (Luffel et al., 1993, SPE26633) on crushed rock samples**




**Advantages:**

- **Relatively fast**
- **Not affected by induced fractures during coring**

## GRI method: Limitations

OPTIONAL

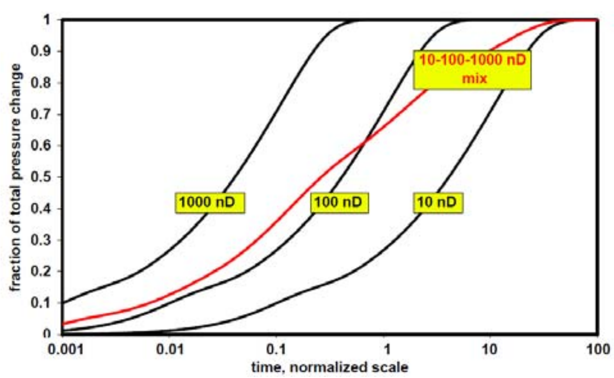
- Absence of overburden stress
- Does not take into account natural fractures
- Darcy's law may not be valid
- Gas slip or Klinkenberg correction is ignored
- Is the original GRI crushed particle size of 20/35 mesh (0.67±0.17 mm) appropriate for all the samples?
- Time scale required for the measurements



## GRI method: Limitations

OPTIONAL

### Effect of rock heterogeneity on the measurements




Sources: Sinha et al., 2012, SPE 152257

# Pressure-Decay vs. Pulse-Decay Measurements

OPTIONAL

## Pulse Decay

- It uses both upstream and downstream reservoirs.
- The sample is filled with gas to a fairly high pressure, 1,000 to 2,000 psig, which reduces gas slippage and compressibility.
- After pressure equilibrium is achieved throughout the system, pressure in the upstream reservoir is increased, typically by 2%-3% of the initial pressure, causing a pressure pulse to flow through the sample.
- Good for low permeability samples, 0.1 md to about 0.01 µd.
- Small differential pressures and low permeabilities virtually eliminate inertial flow resistance.
- "Early time" transients provide information regarding heterogeneity in samples.

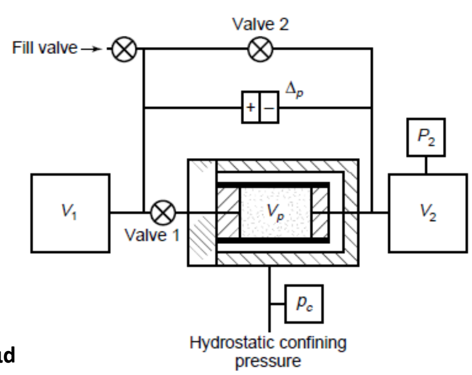
Please see API Recommended Practice 40



# Pulse-Decay Measurements

OPTIONAL

### Pulse-Decay Gas Permeameter




Range of permeability: 0.1 md to 0.01 µd

Please see API Recommended Practice 40

## Advantages and Limitations: Pulse-Decay Gas Permear

OPTIONAL

**Reading Assignment:**
**Please see API Recommended Practice 40**



## Pressure-Decay vs. Pulse-Decay Measuremen

OPTIONAL

**Pressure Decay**

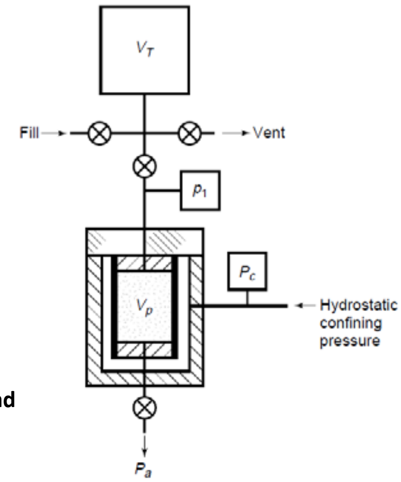
- It uses only upstream reservoir(s). The downstream end of the sample is vented to atmospheric pressure.
- The maximum upstream pressure used is fairly low, 10 to 250 psig (varying inversely with the permeability to be measured).
- Good for permeability range of 0.001 to 30,000 md (through the use of multiple upstream gas reservoirs and pressure transducers) ➔ Complements the pulse-decay method.

Please see API Recommended Practice 40

## Pressure-Decay Measurements

OPTIONAL

**Pressure-Decay Gas Permeameter**




**Range of permeability: 0.001 to 30,000 md**

Please see API Recommended Practice 40



## Advantages and Limitations: Pressure-Decay Measurer

OPTIONAL

**Reading Assignment:**
**Please see API Recommended Practice 40**

## Pressure-Decay Probe Measurements

**OPTIONAL**

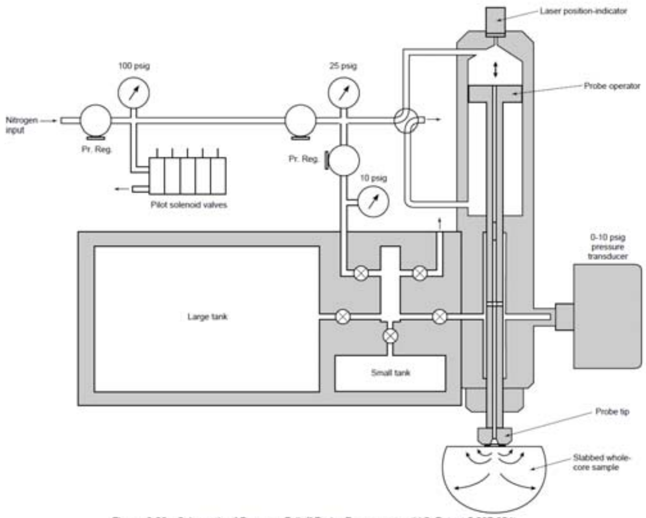



Figure 6-23—Schematic of Pressure-Falloff Probe Permeameter (U.S. Patent 5,237,854)

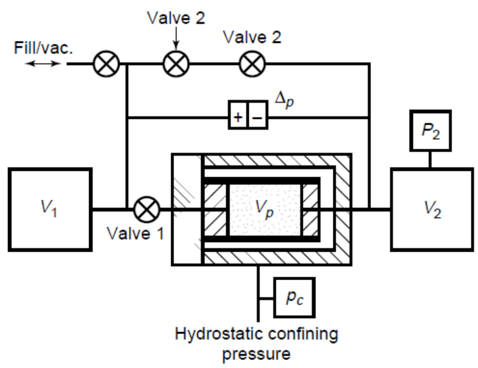
Please see API Recommended Practice 40



## Pulse-Decay Measurements

**OPTIONAL**

**Liquid Pulse-Decay Permeameter**




**Range of permeability: 0.1 md to 0.01 µd**

Please see API Recommended Practice 40

## Advantages and Limitations: Pulse-Decay Liquid Permeameter

**Reading Assignment:**
**Please see API Recommended Practice 40**

**OPTIONAL**





# Advanced Petrophysics: Permeability, Part 4 of 5

**Instructor: Zoya Heidari, Ph.D.**

Associate Professor
The University of Texas at Austin

## What Do We Learn in This Lecture?

- What is permeability?
- What rock properties affect absolute permeability?
- Darcy's Law
- Permeability evaluation in tensor form
- Darcy's law for anisotropic porous media
- Non-Darcy flow
- Averaging permeability data
- **How to quantify permeability?**
  - Analytical models for permeability assessment
  - Permeability assessment in the laboratory?
  - **Permeability assessment in-situ condition using pressure transient test**
  - Permeability assessment in-situ condition using well logs

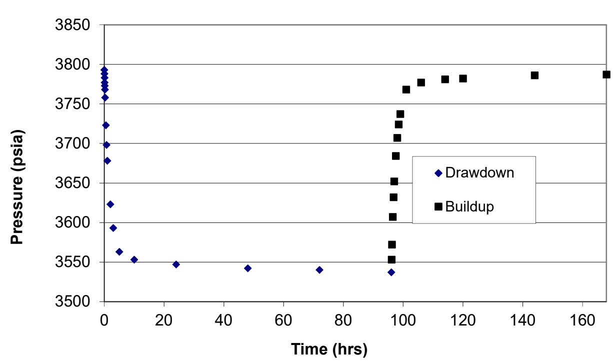


## In-situ Assessment of Permeability: Pressure Transient Test

**Pressure Transient Test:** A pressure transient test consists of changing the flow rate of a well and then recording the bottomhole pressure response as a function of time.

**Data Analysis** → How? → $k$

What laboratory data should I use to verify the reliability of these estimates?



## How to Translate the Collected Pressure Data to Permeability?

Mass conservation equation (continuity equation)

$$\nabla.(\rho\vec{v})+\frac{\partial(\phi\rho)}{\partial t}=0$$

Darcy's law

$$\vec{v}=-\frac{k}{\mu}\nabla P$$

The equation of state for a slightly compressible liquid such as oil or water

$$\rho=\rho_o e^{c(P-P_o)}$$

**Diffusivity Equation for Slightly Compressible Liquid (diffusion equation or the heat conduction equation)**

$$\nabla^2 P=\frac{\phi\mu c_t}{k}\frac{\partial P}{\partial t}=\frac{1}{\alpha}\frac{\partial P}{\partial t}$$

diffusivity constant $\alpha=\frac{k}{\phi\mu c_t}=\frac{\left(\frac{kh}{\mu}\right)}{\phi h c_t}=\frac{T}{S}$

transmissibility

storativity (storage capacity)



## How to Translate the Collected Pressure Data to Permeability?

**For one-dimensional radial flow:**

$$\frac{\partial^2 P}{\partial r^2}+\frac{1}{r}\frac{\partial P}{\partial r}=\frac{\phi\mu c_t}{k}\frac{\partial P}{\partial t}$$

$$P(r,0)=P_i$$

**Assumption: wellbore radius is infinitesimally small.**

$$q_{sf}=\frac{k}{\mu}2\pi rh\frac{\partial P}{\partial r}\rightarrow\lim_{r\to 0}\left(r\frac{\partial P}{\partial r}\right)=\frac{q_{sf}\mu}{2\pi kh}$$

$$\lim_{r\to\infty}P(r,t)=P_i$$

$$t_D=\frac{kt}{\phi\mu c_t r_w^2}$$

$$r_D=\frac{r}{r_w}$$

$$P_D=\frac{P_i-P(r,t)}{\left(\frac{q_{sf}\mu}{2\pi kh}\right)}$$

**In dimensionless forms**

**Why?**

$$\frac{\partial^2 P_D}{\partial r_D^2}+\frac{1}{r_D}\frac{\partial P_D}{\partial r_D}=\frac{\partial P_D}{\partial t_D}$$

$$P_D(r_D,0)=0$$

$$\lim_{r_D\to 0}\left(r_D\frac{\partial P_D}{\partial r_D}\right)=-1$$

$$\lim_{r_D\to\infty}P_D(r_D,t_D)=0$$

# How to Solve this Equation?

$$P_D\left(r_D, t_D\right) = -\frac{1}{2}\mathrm{Ei}\left(-\frac{r_D^2}{4t_D}\right)$$

$$\mathrm{Ei}(x) = -\int_{-x}^{\infty}\frac{e^{-y}}{y}dy$$

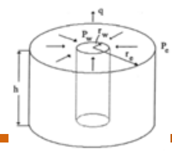
Exponential integral function

$P_D$ and $t_D$ in field units

$$P_D = \frac{P_i - P(r,t)}{141.2\left(\frac{q\mu B}{kh}\right)}$$

$$P(r,t) = P_i - \frac{141.2 q\mu B}{kh}\left[-\frac{1}{2}Ei\left(-\frac{948\phi\mu c_t r^2}{kt}\right)\right]$$

$$t_D = \frac{0.0002637kt}{\phi\mu c_r r_w^2}$$



$$P_{wf}(t) = P\left(r_w, t\right) = P_i - \frac{141.2 q\mu B}{kh}\left[-\frac{1}{2}Ei\left(-\frac{948\phi\mu c_t r_w^2}{kt}\right)\right]$$



# Can we Simplify any Further?

**The Taylor series expansion of Ei:**

$$\mathrm{Ei}(x) = \gamma + \ln|x| + \sum_{k=1}^{\infty}\frac{x^k}{k(k!)} \xrightarrow{x < 0.01} \mathrm{Ei}(x) = \gamma + \ln|x|$$

**Euler-Mascheroni constant (~ 0.5772)**

**At Wellbore ($r_D$ = 1):**

$$P_D\left(1, t_D\right) = \frac{1}{2}\left(\ln t_D + \ln 4 - \gamma\right) = \frac{1}{2}\left(\ln t_D + 0.80907\right) = 1.1513\left[\log t + \log\left(\frac{k}{\phi\mu c_t r_w^2}\right) - 3.23\right]$$

if $t_D \geq 25$

**The flowing bottomhole pressure:**

$$P_{wf}(t) = P\left(r_w, t\right) = P_i - \frac{162.6 q\mu B}{kh}\left(\log t + \log\left(\frac{k}{\phi\mu c_t r_w^2}\right) - 3.23\right) \quad \text{if} \quad t \geq \frac{94805\phi\mu c_t r_w^2}{k}$$

## Can we use Drawdown Test to Estimate Permeability?

$$P_{wf}(t) = P(r_w, t) = P_i - \frac{162.6 q \mu B}{kh}\left(\log t + \log\left(\frac{k}{\phi \mu c_t r_w^2}\right) - 3.23\right)$$

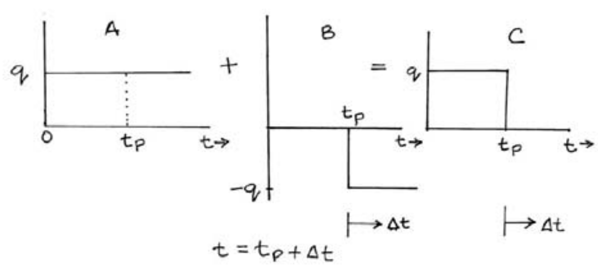
**Slope** ➡ $m = -\dfrac{162.6 q \mu B}{kh}$ ➡ $k = -\dfrac{162.6 q \mu B}{mh}$

at logt = 0 $P_{\text{int}} = P_i + m\left[\log\left(\dfrac{k}{\phi \mu c_t r_w^2}\right) - 3.23\right]$ ➡ $P_i = P_{\text{int}} - m\left[\log\left(\dfrac{k}{\phi \mu c_t r_w^2}\right) - 3.23\right]$

pressure intercept initial pressure



## Pressure Buildup Equation




Drawdown ← $P_{DA}$ + $P_{DB}$ = $P_{DBU}$ → Buildup

$$\frac{1}{2}\left(\ln\left(t_p + \Delta t\right)_D + 0.80907\right) - \frac{1}{2}\left(\ln \Delta t_D + 0.80907\right) = \frac{1}{2}\ln\boxed{\left(\frac{t_p + \Delta t}{\Delta t}\right)}$$

**Horner time**

**The Horner pressure buildup equation for an infinite acting reservoir:** $P_{ws}(\Delta t) = P_i - \dfrac{162.6 q \mu B}{kh}\log\left(\dfrac{t_p + \Delta t}{\Delta t}\right)$

# Can we use Buildup Test to Estimate Permeability?

$$P_{ws}(\Delta t) = P_i - \frac{162.6 q \mu B}{kh} \log\left(\frac{t_p + \Delta t}{\Delta t}\right)$$

**Slope**

$$m = -\frac{162.6 q \mu B}{kh} \quad \Rightarrow \quad k = -\frac{162.6 q \mu B}{mh}$$

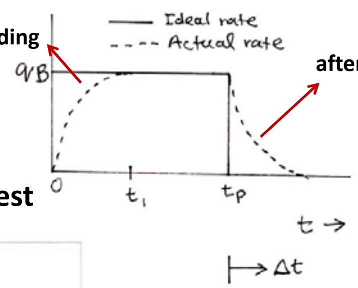
$$at \quad \log\left(\frac{t_p + \Delta t}{\Delta t}\right) = 0 \quad \Rightarrow \quad P_{\text{int}} = P_i$$

pressure intercept    initial pressure



# Wellbore Storage

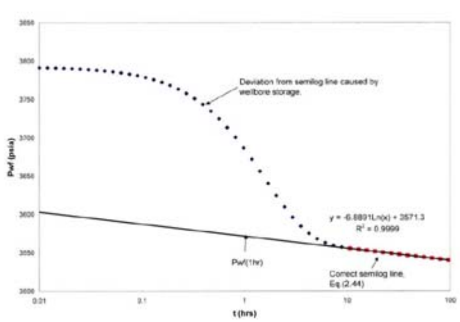
wellbore unloading

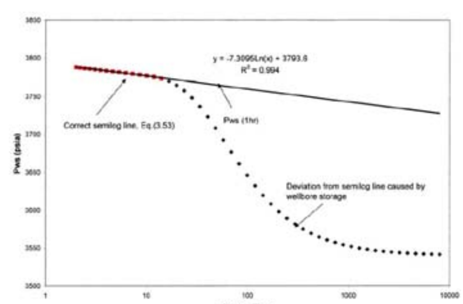



afterflow

**Drawdown (flowing) test**

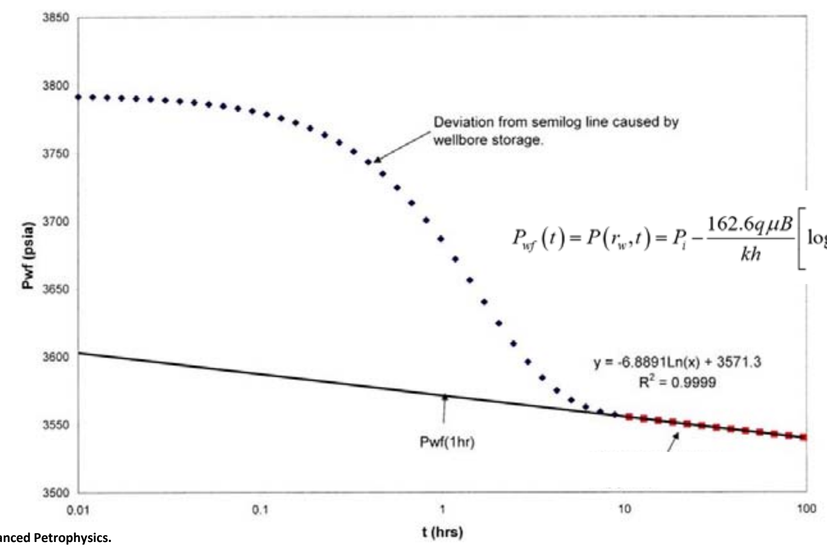
**Buildup (shut in) test**




Source: Peters, E. J., 2012, Advanced Petrophysics.

# Summary: Drawdown Test

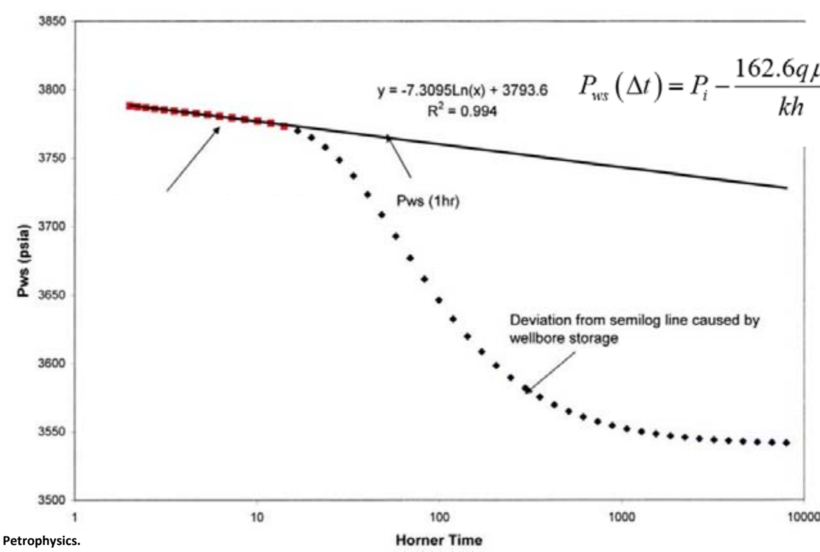



$$P_{wf}(t) = P(r_w,t) = P_i - \frac{162.6 q\mu B}{kh}\left[\log t + \log\left(\frac{k}{\phi\mu c_t r_w^2}\right) - 3.23\right]$$

Source: Peters, E. J., 2012, Advanced Petrophysics.



# Summary: Buildup Test




$$P_{ws}(\Delta t) = P_i - \frac{162.6 q\mu B}{kh}\log\left(\frac{t_p + \Delta t}{\Delta t}\right)$$

Source: Peters, E. J., 2012, Advanced Petrophysics.

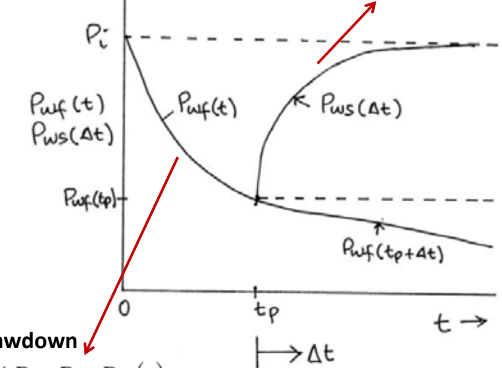
# Diagnostic Plots

## How to make this diagnostic process quantitative?

$$\Delta P' = t_i\left(\frac{d\Delta P}{dt}\right)_{t_i} = t_i\left[\frac{\left(\frac{\Delta P_i-\Delta P_{i-1}}{t_i-t_{i-1}}\right)(t_{i+1}-t_i)+\left(\frac{\Delta P_{i+1}-\Delta P_i}{t_{i+1}-t_i}\right)(t_i-t_{i-1})}{(t_i-t_{i-1})+(t_{i+1}-t_i)}\right]$$

**Buildup**

$$\Delta P = P_{ws}(\Delta t) - P_{wf}(t_p+\Delta t) \simeq P_{ws}(\Delta t) - P_{wf}(t_p)$$



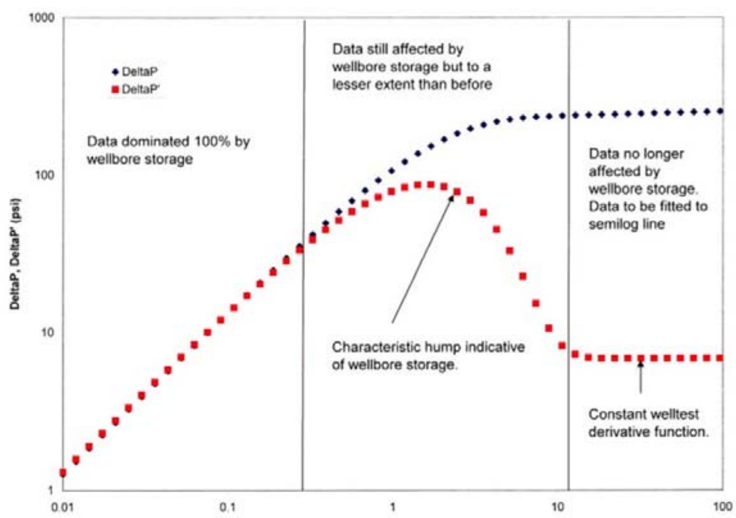

**Drawdown**

$$\Delta P = P_i - P_{wf}(t)$$

Source: Peters, E. J., 2012, Advanced Petrophysics.



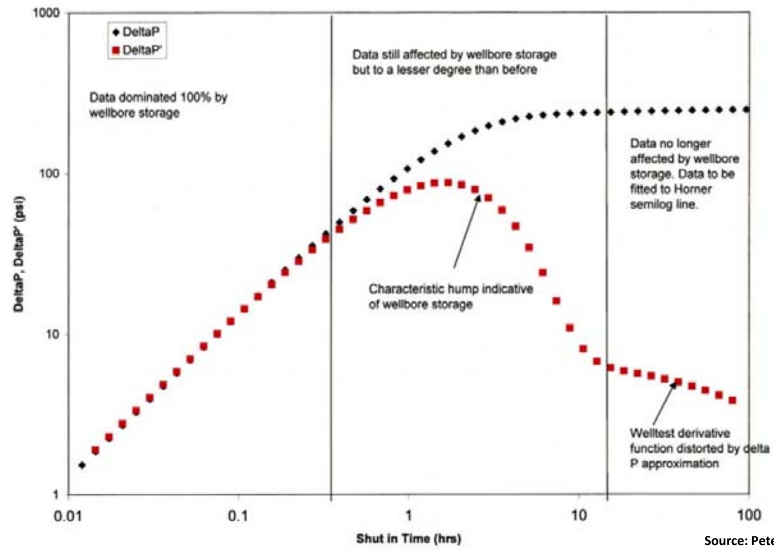
# Diagnostic Plot: Drawdown Test




Source: Peters, E. J., 2012, Advanced Petrophysics.

$$\Delta P' = \frac{d\Delta P}{d\ln t} = t\frac{d\Delta P}{dt} = 70.6\left(\frac{q\mu B}{kh}\right) = \text{ a constant}$$

## Diagnostic Plot: Buildup Test




Source: Peters, E. J., 2012, Advanced Petrophysics.



## Wellbore Storage Coefficient

**Wellbore storage coefficient:**

$$C = \left(\frac{qB}{24}\right)\left(\frac{t}{\Delta P}\right)_{\text{unit slope line}}$$

Unit: RB/psi

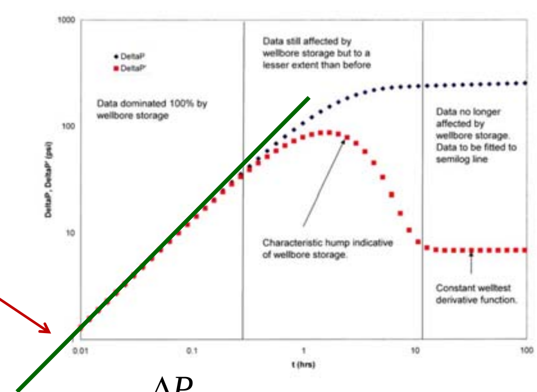
**Dimensionless wellbore storage coefficient:**

$$C_D = \frac{C}{2\pi\phi c_t h r_w^2}$$

**In Darcy units**

$$C_D = \frac{5.615C}{2\pi\phi c_t h r_w^2}$$

**In oil-field units**

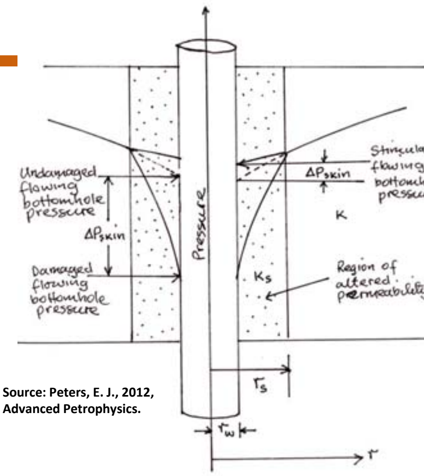



$$slope = \frac{\Delta P}{t}$$

# Skin Factor

- What are the reasons behind damage or stimulation of the near-wellbore region?
- How do they affect permeability of the near-wellbore region?

$$P_{wf} = P_i - \frac{162.6 q \mu B}{kh}\left(\log_{10} t + \log_{10}\left(\frac{k}{\phi \mu c_t r_w^2}\right) - 3.23 + 0.87S\right)$$




Source: Peters, E. J., 2012, Advanced Petrophysics.

**Skin factor from drawdown test**

$$S = 1.1513\left[\frac{P_{wf}(1hr) - P_i}{-\left(\frac{162.6 q \mu B}{kh}\right)} - \log\left(\frac{k}{\phi \mu c_t r_w^2}\right) + 3.23\right]$$

**Skin factor from buildup test**

$$S = 1.1513\left[\frac{P_{wf}(t_p) - P_{ws}(1hr)}{-\left(\frac{162.6 q \mu B}{kh}\right)} - \log\left(\frac{k}{\phi \mu c_t r_w^2}\right) + 3.23\right]$$



# Radius of Investigation of a Welltest

**Approximate radius investigated by a welltest:**

$$r_{inv} = 0.03248\sqrt{\frac{kt}{\phi \mu c_t}}$$

In oil-field units

**Example:** Let's put numbers in this equations!

## Examples

- **Example 1:** Please download the digital data provided to you in Excel format on the Canvas website

- **Example 2:** Please read and solve the field example of a well test analysis from the textbook





# Advanced Petrophysics: Permeability, Part 5 of 5

Instructor: Zoya Heidari, Ph.D.

Associate Professor
The University of Texas at Austin

## What Do We Learn in This Lecture?

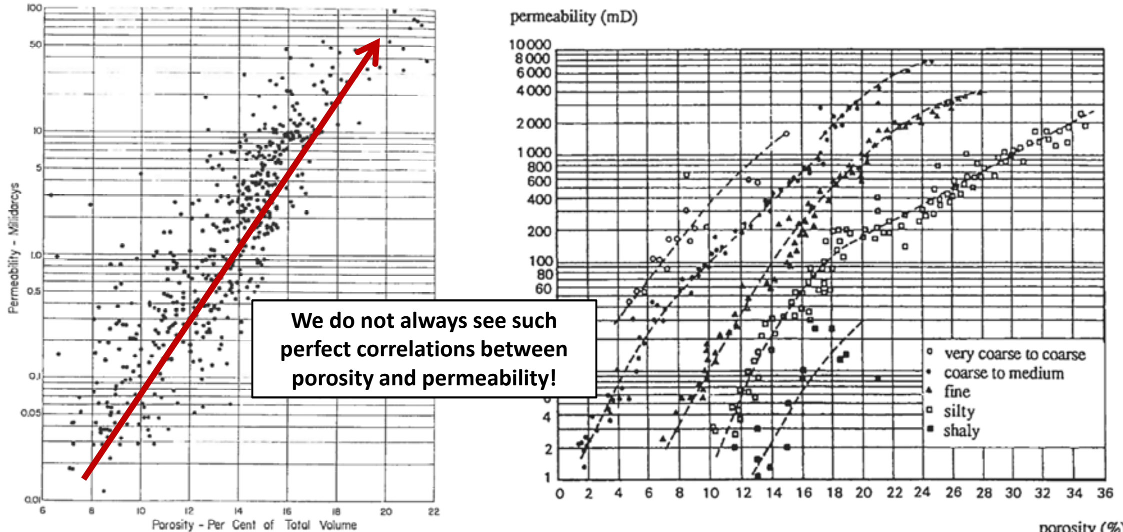
- What is permeability?
- What rock properties affect absolute permeability?
- Darcy’s Law
- Permeability evaluation in tensor form
- Darcy’s law for anisotropic porous media
- Non-Darcy flow
- Averaging permeability data
- **How to quantify permeability?**
  - Analytical models for permeability assessment
  - Permeability assessment in the laboratory?
  - Permeability assessment in-situ condition using pressure transient test
  - **Permeability assessment in-situ condition using well logs**



## Other Methods for in-situ Assessment of Permeability

- **Porosity-permeability correlations**
  - **Combined with rock classification**
- **NMR measurements**
- **Resistivity measurements**
- **Integrated analysis of resistivity and NMR measurements**
- **….**

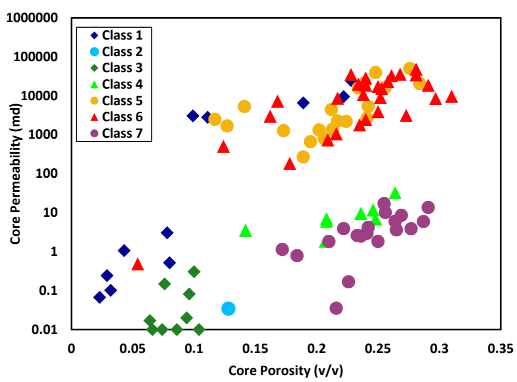
# Is there Any Correlation Between Porosity and Permeability?

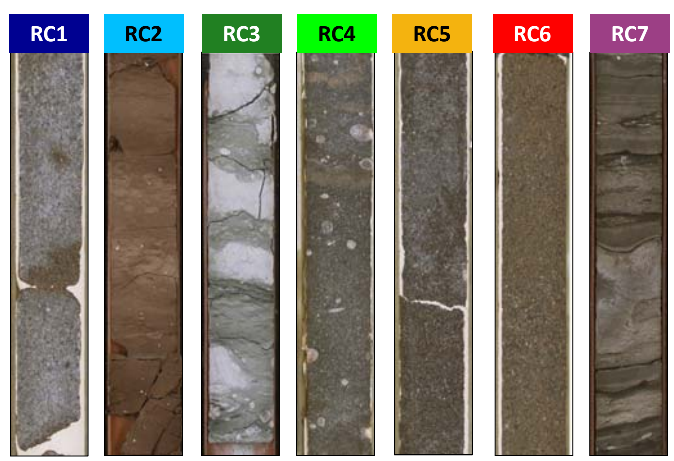



Source: Peters, E. J., 2012, Advanced Petrophysics.



# Is there Any Correlation Between Porosity and Permeability?




Source: Gonzalez et al., 2019, SPWLA (Heidari's research group)

## Permeability Assessment using Well Logs

- **Permeability can be described as a function of porosity and irreducible water saturation in some reservoirs**

- **Wyllie and Rose (1950):**

$$k = a\frac{\phi^b}{S_{wi}^c}$$

  - How to calibrate this model?
  - How to use it to estimate permeability in any depth of interest in the formations?



## Permeability Assessment using Well Logs

- **Empirical formulae for permeability assessment using well logs:**
  - Tixier

Permeability, (D) ← $$k = 62.5\frac{\phi^6}{S_{wirr}^2}$$ → Porosity, ( ); → Irreducible Water Saturation, ( )

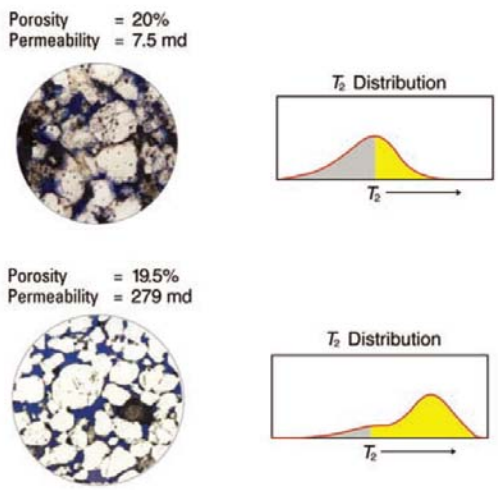
  - Timur

$$k = 8.58\frac{\phi^{4.4}}{S_{wirr}^2}$$

  - Coates

$$k = 4.90\frac{\phi^4\left(1-S_{wirr}\right)^2}{S_{wirr}^4}$$

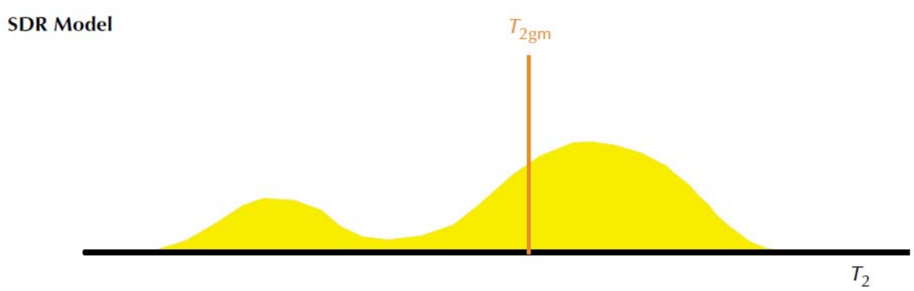
## Effect of Permeability on $T_2$-Distribution




Source: JPT, 2006



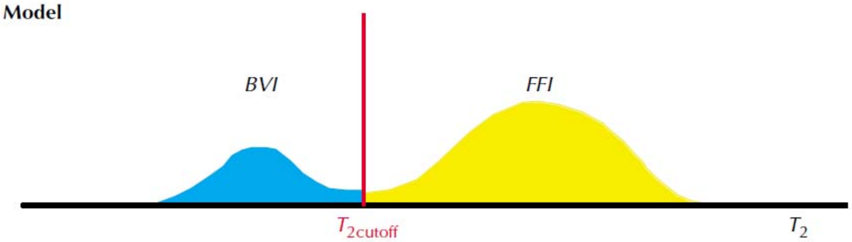
## Permeability Assessment using NMR Measurements




$$k = aT_{2gm}^2 \phi^4$$

# Permeability Assessment using NMR Measurements

Coates Model




% or p.u.

md

$$k = \left[ \left( \frac{\phi}{C} \right)^2 \left( \frac{FFI}{BVI} \right) \right]^2$$

Constant

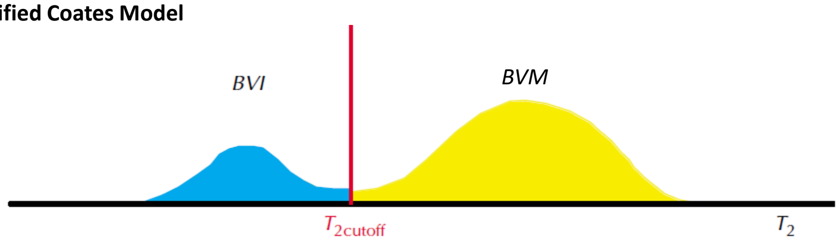
**For younger sediments (Tertiary/Miocene):**
**C = 10**

**For older sediments (Mesozoic):**
**C = 23**



# Permeability Assessment using NMR Measurements

**Modified Coates Model**




$$k_{Coates\_Modified} = \left(\frac{\phi}{C}\right)^4 \left(\frac{\boxed{p} \cdot BVM}{BVI + (1-\boxed{p})BVM}\right)^2$$

**(Chen, et al. 2008)**

## Other Methods/Models

OPTIONAL

**Can we use resistivity measurements to estimate permeability?**
**What other measurements can we possibly use?**
**Can we combine any of these measurements?**

**Please take notes!**



## Complementary References


- **Peters, E. J., 2012, Advanced Petrophysics. Live Oak Book Company. Chapter 3**
- **Zinszner, B. and Pellerin, F. M., 2007, A Geoscientist's Guide to Petrophysics. Editions Technip.**