Darcy's Law in Anisotropic Porous Media:

$$\vec{v} = -\frac{\bar{\bar{K}}}{\mu} \nabla \vec{\Phi}$$

$$\Rightarrow \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = -\frac{1}{\mu} \begin{bmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{yx} & K_{yy} & K_{yz} \\ K_{zx} & K_{zy} & K_{zz} \end{bmatrix} \begin{bmatrix} \frac{\partial \phi}{\partial x} \\ \frac{\partial \phi}{\partial y} \\ \frac{\partial \phi}{\partial z} \end{bmatrix}$$

Principal Axes of Anisotropy:

$$\bar{\bar{K}}(u, v, w) = \begin{bmatrix} K_u & 0 & 0 \\ 0 & K_v & 0 \\ 0 & 0 & K_w \end{bmatrix}$$

$$\begin{bmatrix} K_u & 0 \\ 0 & K_v \end{bmatrix}$$

Example:

$$K(x,y) = \begin{bmatrix} 200 & 50 \\ 50 & 100 \end{bmatrix} \quad (md)$$

$$\mu = 1.5 \ cp$$

$$\nabla \Phi = -0.004\,\hat{i} + 0.008\,\hat{j} \quad (atm/cm)$$

ⓐ $|\vec{V}_d|$

$$\vec{V}_d = -\frac{\bar{\bar{K}}}{\mu} \vec{\nabla \Phi}$$

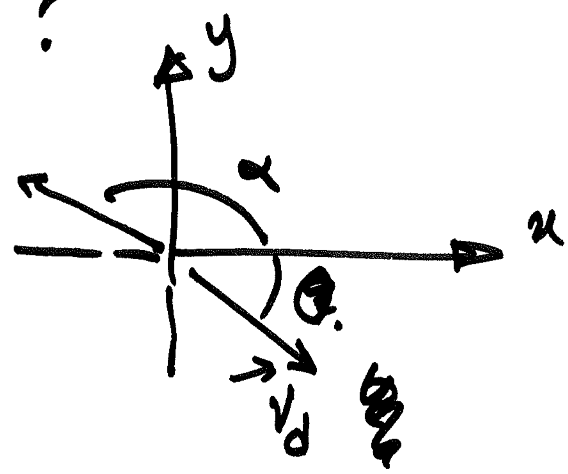
$$= -\frac{1}{1.5} \begin{bmatrix} 0.2 & 0.05 \\ 0.05 & 0.1 \end{bmatrix} \begin{bmatrix} -0.004 \\ 0.008 \end{bmatrix}$$

$$= 2.6 \times 10^{-4}\,\hat{i} - 4 \times 10^{-4}\,\hat{j}$$

$$|\vec{V}_d| = \sqrt{(2.6 \times 10^{-4})^2 + (4 \times 10^{-4})^2}$$

$$= 4.8 \times 10^{-4} \ cm/s$$

(b) direction of flow ?



$$\cos\theta = \frac{\vec{v}_d \cdot \hat{i}}{|\vec{v}_d| \cdot |\hat{i}|}$$

$$= \frac{2.6\times10^{-4}}{4.8\times10^{-4}} = 0.55$$

$$\Longrightarrow \boxed{\theta = 56.3^\circ}$$

(c) $\vec{v}_d$ & $\vec{\nabla\Phi}$? α

$$\cos\alpha = \frac{\vec{v}_d \cdot \vec{\nabla\Phi}}{|\vec{v}_d|\,|\vec{\nabla\Phi}|}$$

$$\cos\alpha = \frac{(2.6\times10^{-4})(-0.004) + (-4\times10^{-4})(0.008)}{(4.8\times10^{-4})\sqrt{(0.004)^2 + (0.008)^2}}$$

$$\Rightarrow \alpha = 172.9^\circ$$

$$\bar{\bar{K}}_{\cancel{xy}} = \begin{bmatrix} 200 & 50 \\ 50 & 100 \end{bmatrix} \longrightarrow \begin{bmatrix} K_{uu} & 0 \\ 0 & K_{vv} \end{bmatrix} = \bar{\bar{K}}_{uv}$$

xy

$$|K_{xy} - \lambda I| = 0$$

$$\begin{vmatrix} 200-\lambda & 50 \\ 50 & 100-\lambda \end{vmatrix} = 0$$

$$(200-\lambda)(100-\lambda) - 2500 = 0$$

$$\rightsquigarrow \lambda_1 = 220.7 \text{ md}$$

$$\lambda_2 = 79.29 \text{ md}$$

$$\bar{\bar{K}}_{uv} = \begin{bmatrix} 220.7 & 0 \\ 0 & 79.29 \end{bmatrix} \text{ md}$$

$\lambda_1 = 220.7$ md

$$\begin{bmatrix} 200-220.7 & 50 \\ 50 & 100-220.7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -20.7 & 50 \\ 50 & -120.7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} -20.7x + 50y = 0 \\ 50x - 120.7y = 0 \end{cases}$$

$$\Rightarrow x = 2.4\,y$$

$$y = 1 \rightarrow x = 2.4 \quad \Rightarrow \vec{u} = \begin{bmatrix} 2.4 \\ 1 \end{bmatrix}$$

$\lambda_2 = 79.29 \text{ md}$

$$\begin{bmatrix} 200-79.29 & 50 \\ 50 & 100-79.29 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

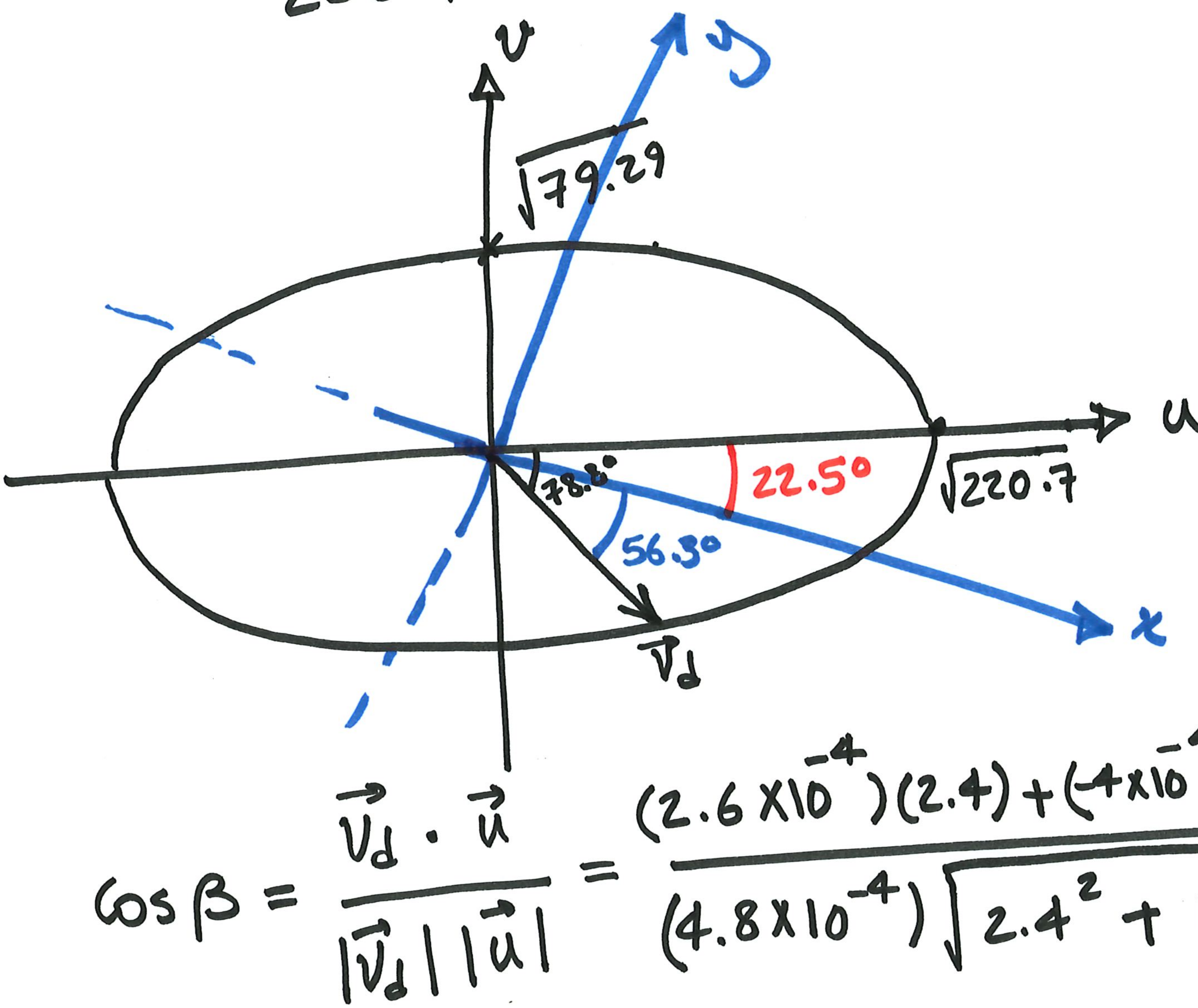
$$\longrightarrow \quad x = -0.4\, y$$

$$\rightsquigarrow \quad \vec{v} = \begin{bmatrix} -0.4 \\ 1 \end{bmatrix}$$

(f) Permeability Ellipse

$$\frac{u^2}{K_u} + \frac{v^2}{K_v} = 1$$

$$\rightarrow \frac{u^2}{220.7} + \frac{v^2}{79.29} = 1$$



$$\cos\beta = \frac{\vec{V}_d \cdot \vec{u}}{|\vec{V}_d|\,|\vec{u}|} = \frac{(2.6\times10^{-4})(2.4) + (-4\times10^{-4})(1)}{(4.8\times10^{-4})\sqrt{2.4^2+1^2}}$$

$$\rightsquigarrow \beta = 78.8^\circ$$

$$\frac{1}{K_s} = \frac{(\cos\beta)^2}{K_u} + \frac{(\sin\beta)^2}{K_V}$$

$$\leadsto \frac{1}{K_s} = \frac{[\cos(78.8)]^2}{220.7} + \frac{(\sin(78.8))^2}{79.29}$$

$$\Longrightarrow K_s = 81.25 \text{ md}$$

Example:

$$\underset{(x,y)}{\overline{\overline{K}}} = \begin{bmatrix} 70 & 20 \\ 20 & 50 \end{bmatrix} (md)$$

$\mu = 1 \ cp$

$$\overrightarrow{\nabla \Phi} = 0.3\,\hat{i} + 0.2\,\hat{j} \quad (atm/cm)$$

82.36   31.7

(a) max & min K & direction

0.0297

(b) $|\vec{V}_d| = ?$ and direction wrt x

212.7°

(c) ($\vec{V}_d$ & $\overrightarrow{\nabla \Phi}$) angle

179°

Example:

$$\bar{\bar{k}}(x,y) = \begin{bmatrix} 100 & 100 \\ 100 & 100 \end{bmatrix} \text{ md}$$

$$\tan 2\theta = \frac{2K_{xy}}{K_{xx} - K_{yy}} = \frac{2(100)}{100-100}$$

$$\theta = 45^\circ$$

$$k_{uu} = \frac{K_{xx}+K_{yy}}{2} + \frac{K_{xx}-K_{yy}}{2}\cos 2\theta + K_{xy}\sin 2\theta$$

$$= \frac{100+100}{2} + \frac{100-100}{2}\cos 90^\circ + 100 \sin 90^\circ$$

$$\Longrightarrow \boxed{K_{uu} = 200 \text{ md}}$$

$$K_{vv} = \frac{K_{xx}+K_{yy}}{2} - \frac{K_{xx}-K_{yy}}{2}\cos 2\theta - K_{xy}\sin 2\theta$$

$$= 0$$

$$k_{uv} = \begin{bmatrix} 200 & 0 \\ 0 & 0 \end{bmatrix} \text{ md}$$