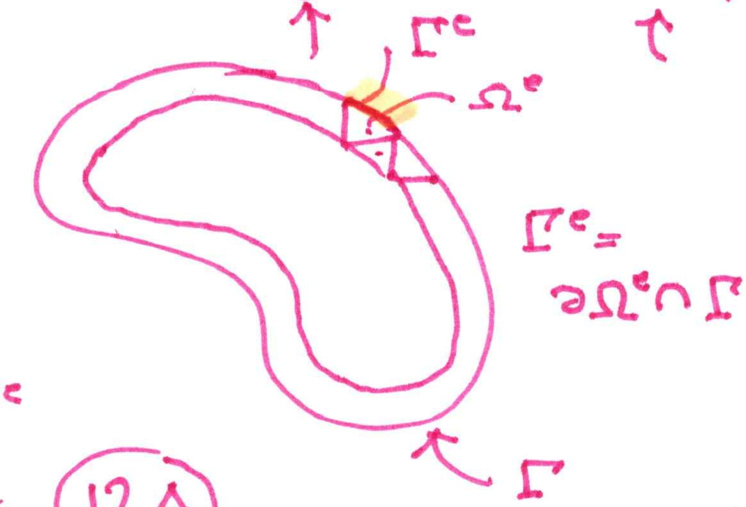
rewrite A in el-by-el fashion

$$A = \sum_e \left(\int_{\Omega^e} \alpha |\nabla w^h|^2 \, d\Omega - \frac{(s+1)}{2} \frac{1}{\varepsilon^e} \| n \cdot \alpha \nabla w^h \|^2_{\Gamma^e} \right)$$



determine $\varepsilon^e \ni$

$$\frac{(s+1)}{2} \frac{1}{\varepsilon^e} \| n \cdot \alpha \nabla w^h \|^2_{\Gamma^e} \le \frac{1}{2} \| \alpha^{1/2} \nabla w^h \|^2_{\Omega^e} \quad ✓ \qquad (12A)$$

$$B = \sum_e \left(\int_{\Gamma^e} \left(\frac{\alpha}{\delta^e} - \frac{s+1}{2} \varepsilon^e \right) d\Gamma \right)$$

select $\delta^e \ni \quad \frac{s+1}{\cancel{2}} \underbrace{\varepsilon^e}_{\text{know from } A} \le \frac{1}{\cancel{2}} \frac{\alpha}{\delta^e}$ ← determine

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$$B_{Nitsche}(w^h, w^h) \ge \frac{1}{2} \sum_e \left(\| \alpha^{1/2} \nabla w^h \|^2_{\Omega^e} + \cancel{\frac{1}{2}} \left\| \left(\frac{\alpha}{\delta^e} \right)^{1/2} w^h \right\|^2_{\Gamma^e} \right)$$

How do you determine ε^e?

1. The fact it is precisely computable emanates from an element eigenprob.

The associated eigenprob.

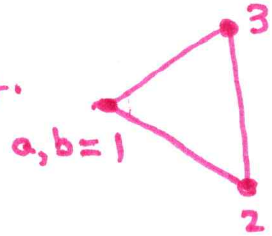
$$0 = \int_{\Gamma^e} (n\cdot \varkappa \nabla w^h)(n\cdot \varkappa \nabla u^h)\, d\Omega$$

$$- \lambda^e \int_{\Omega^e} \varkappa \nabla w^h \cdot \nabla u^h \, d\Omega .$$

eigenvalue

basis fns.

e.g.



$$w^h = \sum_{a=1}^{3} N_a w_a, \quad w_a \in \mathbb{R}$$

$$u^h = \sum_{b=1}^{3} N_b u_b$$

sub.

3 × 3 matrix eigenvalue prob. ✓

$$\| n\cdot \varkappa \nabla w^h \|^2_{\Gamma^e} \le \lambda^e_{max} \underbrace{\| \varkappa^{1/2} \nabla w^h \|^2_{\Omega^e}}$$

$$\max_{w^h} \frac{\| n\cdot \varkappa \nabla w^h \|^2_{\Gamma^e}}{\| \varkappa^{1/2} \nabla w^h \|^2_{\Omega^e}} = \lambda^e_{max}$$

Rayleigh quotient.

(12 A): $\displaystyle \frac{s+1}{2} \frac{1}{\varepsilon^e} \| n\cdot \varkappa \nabla w^h \|^2_{\Gamma^e} \le \underbrace{\frac{s+1}{2} \frac{1}{\varepsilon^e} \lambda^e_{max}}_{\le \frac{1}{2}} \| \varkappa^{1/2} \nabla w^h \|^2_{\Omega^e}$

$$\frac{s+1}{\cancel{2}} \frac{1}{\varepsilon^e} \lambda^e_{max} \le \frac{1}{\cancel{2}}$$

$$\boxed{\frac{1}{\varepsilon^e} \le \frac{1}{(s+1)\lambda^e_{max}}} \quad \text{set } \frac{1}{\varepsilon^e} = \frac{1}{(s+1)\lambda^e_{max}}$$

(B) $\displaystyle \delta^e \le \frac{\varkappa}{(s+1)\varepsilon^e} \le \boxed{\frac{\varkappa}{(s+1)^2} \frac{1}{\lambda^e_{max}} \quad \text{set} = \delta^e}$

2. detour: and calc. the bounds directly for a linear Δ.

an example. write $w^h = \underbrace{c_0 + c_1 x + c_2 y}_{\text{for a lin tri}}$.

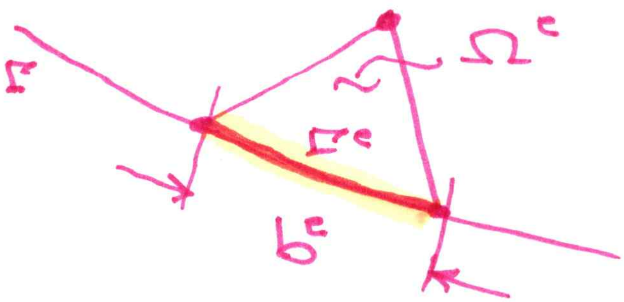
$$\nabla w^h = \begin{Bmatrix} c_1 \\ c_2 \end{Bmatrix} \qquad |\nabla w^h| = |c| = (c_1^2 + c_2^2)^{1/2}$$

$$\| n \cdot \alpha \nabla w^h \|^2_{\Gamma^e}$$

orientation, so we have to assume the worst case, that maximizes, so select $n = \dfrac{\nabla w^h}{|\nabla w^h|} = \dfrac{1}{|c|}\begin{Bmatrix} c_1 \\ c_2 \end{Bmatrix}$

$$n \cdot \nabla w^h = \frac{1}{|c|}(c_1^2 + c_2^2) = \frac{|c|^2}{|c|} = |c|$$

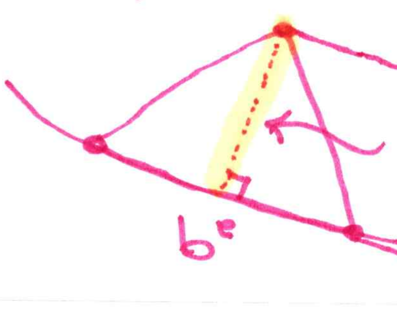
$$\| n \cdot \alpha \nabla w^h \|^2_{\Gamma^e} \leq \int_{\Gamma^e} \underbrace{(\alpha |c|)^2}_{\text{const.}} d\Gamma = \alpha^2 |c|^2 \underbrace{\int_{\Gamma^e} d\Gamma}_{\text{the length of } \Gamma^e}$$



base $= b^e$ $(= h^e_{\parallel})$

$$\boxed{\leq \alpha^2 |c|^2 b^e}$$

$$\| \alpha^{1/2} \nabla w^h \|^2_{\Omega^e} = \int_{\Omega^e} \underbrace{\alpha |c|^2}_{\text{const.}} d\Omega^e = \alpha |c|^2 \underbrace{\int_{\Omega^e} d\Omega^e}_{\text{area of } \Delta}$$



$h^e_{\perp}$ = height of Δ

$$\boxed{= \alpha |c|^2 \frac{1}{2} b^e h^e_{\perp}}$$

127. ~~128.~~

$$① = \int_{\Gamma^e} (n \cdot \alpha \nabla w^s)^2 \, d\Gamma \le |c|^2 \alpha^2 b^e$$

$$② = \int_{\Omega^e} \alpha |\nabla w^s|^2 \, d\Omega = |c|^2 \alpha \, \frac{1}{2} b^e h_\perp^e$$

$$\frac{①}{②} \le \frac{\cancel{|c|^2 \alpha^2 b^e}}{\cancel{|c|^2 \alpha \frac{1}{2} b^e} h_\perp^e} = 2\alpha (h_\perp^e)^{-1}$$

$$\frac{s+1}{2} \frac{1}{\varepsilon^e} \frac{①}{②} \le \cancel{\tfrac{1}{4}} \frac{s+1}{\cancel{2}} \frac{1}{\varepsilon^e} 2\alpha (h_\perp^e)^{-1} \le \frac{1}{\cancel{2}}$$

from 12A

select

12A ✓

$$\frac{1}{\varepsilon^e} \le \left(\frac{\cancel{\alpha\alpha\alpha}(h_\perp^e)^{\cancel{1}}}{(s+1) 2\alpha} \right)$$

set $\frac{1}{\varepsilon^e} =$

$$Ⓑ: \quad \delta^e \le \frac{\alpha}{s+1} \frac{1}{\varepsilon^e} = \frac{\cancel{\alpha}\ \cancel{2}\ \cancel{\alpha(h_\perp^e)^{-1}}\ h_\perp^e}{(s+1)^2\, 2\, \cancel{\alpha}}$$

$$\boxed{\delta^e = \frac{h_\perp^e}{(s+1)^2\, 2}.}$$

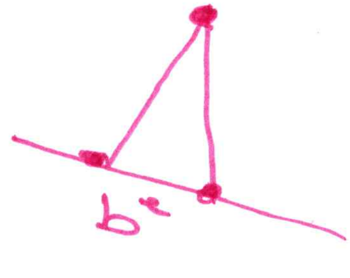
depends exclusively on $h_\perp^e$.

precisely what it is.

$h_\perp^e$ is the same ✓

b^e

very different



b^e

independent of b^e