Advanced Petrophysics:
Quantification of Heterogeneity, Spatial Data Analysis, and Geostatistics
Part 1: Measures of Heterogeneity

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PGE381L Outline

- Introduction to petrophysics, geology, and formation data
- Porosity
- Fluid saturations
- Permeability
- Quantification of heterogeneity, spatial data analysis, and geostatistics
- Interfacial phenomena and wettability
- Capillary pressure
- Relative permeability
- Dispersion in porous media
- Introduction to petrophysics of unconventional reservoirs

What do we Learn in this Lecture?

- What is Heterogeneity?
- Measures of Heterogeneity
  - Variance
  - The Coefficient of Variation
  - Dykstra-Parsons Coefficient of Variation
  - Lorenz Coefficient
- Are these measures sufficient to quantify spatial heterogeneity? What is the next step?

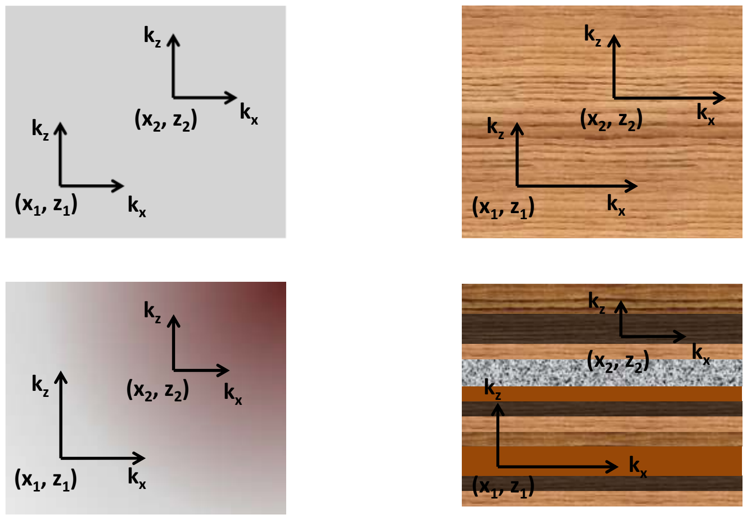
Measures of Heterogeneity

- Measures of Heterogeneity, Variability, or Spread
  - Static Measures
    - Variance
    - The Coefficient of Variation
    - Dykstra-Parsons Coefficient of Variation
    - Lorenz Coefficient
    - Gelhar-Axness Coefficient

    based on variability in properties of permeable media (e.g., permeability)
  - Dynamic Measures
    - Koval Factor
    - Dispersion

    based on variability of flood front

Heterogeneity and Anisotropy



Variance

$$s^2 = \sum_{i=1}^{N} \frac{\left(x_i - \overline{x}\right)^2}{N-1}$$

$|s| \longrightarrow$ **Standard Deviation**

The Coefficient of Variation

$$C_v = \frac{\sigma}{\mu}$$

σ → Standard deviation

C_v → Coefficient of variation

μ → Arithmetic mean

General Form:

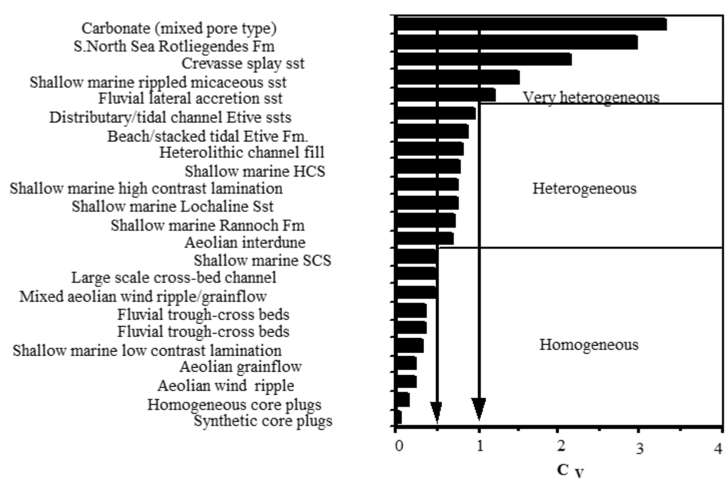
$$C_v = \frac{\sqrt{\mathrm{var}(k)}}{E(k)}$$

Examples:

$$C_v = \frac{\sigma(k)}{\bar{k}} = \frac{\sigma(k)}{\mu(k)} \quad \textbf{OR} \quad C_v = \frac{\sigma(k/\phi)}{\mu(k/\phi)}$$

$$C_v = \frac{\sigma(k_1, k_2, \ldots, k_N)}{\mu(k_1, k_2, \ldots, k_N)} \quad \textbf{OR} \quad C_v = \frac{\sigma(k_1/\phi_1, k_2/\phi_2, \ldots, k_N/\phi_N)}{\mu(k_1/\phi_1, k_2/\phi_2, \ldots, k_N/\phi_N)}$$

Coefficient of Variation: Typical Values



Source: Jensen, J. R., Lake, L. W., Corbett P. M. W., and Goggin, D. J., 2000, Statistics for Petroleum Engineers and Geoscientists, Elsevier.

Example No. 1

- Calculate coefficient of variation for the two sample sets uploaded on the Canvas website. These sample sets include core porosity and permeability measurements in a carbonate and a sandstone formation. Which formation is more heterogeneous?

  – Carbonate Formation: (Carbonate 1)

  coefficient of variation =

  – Sandstone Formation:

  coefficient of variation =

Dykstra-Parsons Coefficient of Variation

- Dykstra-Parsons Coefficient of Variation is a popular method for assessment of permeability variation

$$V = \frac{k_{50} - k_{84.1}}{k_{50}}$$

Median of permeability

Median of permeability + standard deviation of the data

84.1% of the data is above $k_{84.1}$

$$0 \le V \le 1$$

Homogeneous reservoir

Extremely heterogeneous reservoir

Example No. 2

Example: Quantify the permeability heterogeneity in the following permeability data (Carbonate 2)

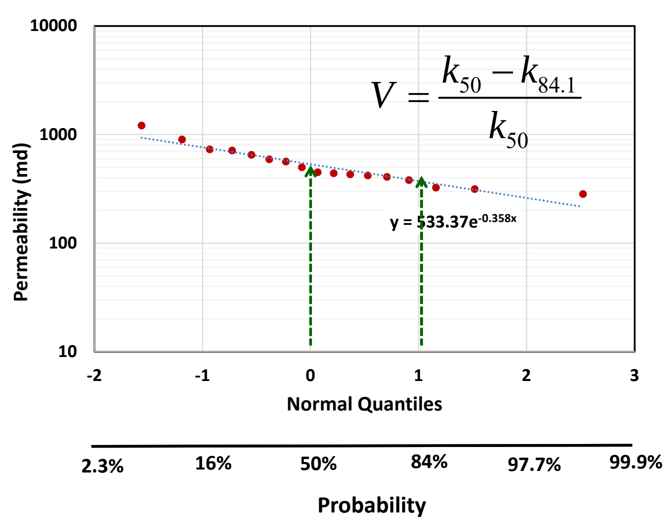
Method 1:

- Arrange the permeability data in descending order.
- Compute the percent of total number of k-values equal or exceeding each permeability.

Solution:

k (md)	k_sorted	N(>=k)	%>=k
650	1212	1	0.06
430	900	2	0.12
500	730	3	0.18
1212	714	4	0.23
283	650	5	0.29
407	591	6	0.35
381	565	7	0.41
315	500	8	0.47
440	450	9	0.53
730	440	10	0.58
714	430	11	0.64
565	420	12	0.70
324	407	13	0.76
591	381	14	0.82
900	324	15	0.88
450	315	16	0.94
420	283	17	0.99

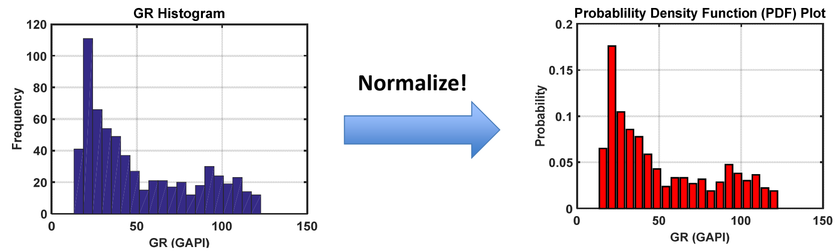
Dykstra-Parsons Coefficient of Variation



$$V = \frac{k_{50} - k_{84.1}}{k_{50}}$$

Reminder: Probability Density Function (PDF)

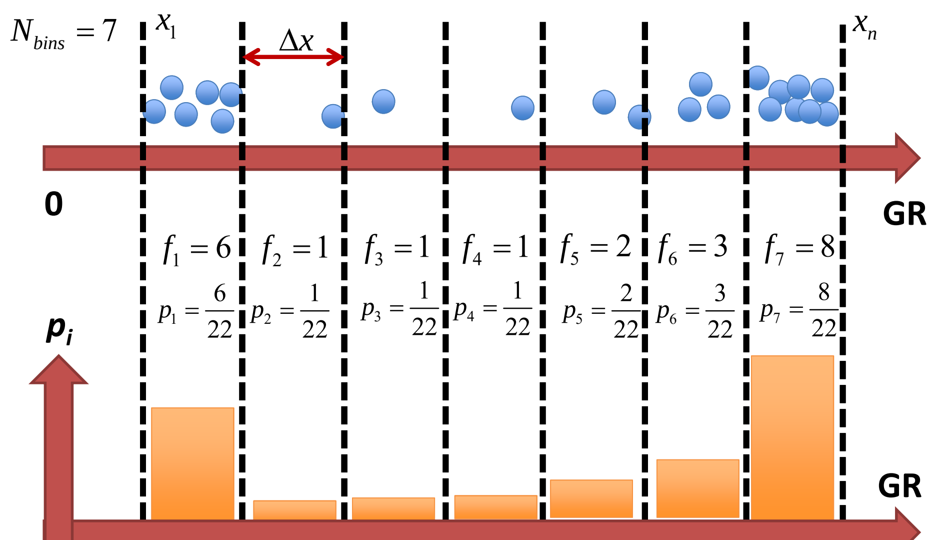
- **The probability density function f(x) is the probability that the variate has the value equal to x.**



Normalized histogram of a discrete set of data is equivalent to its PDF function

- **For discrete variables** $\sum_i p_i = 1$
- **For continuous variables** $\int_{-\infty}^{+\infty} f(x)\,dx = 1$

Reminder: How to Generate a PDF?



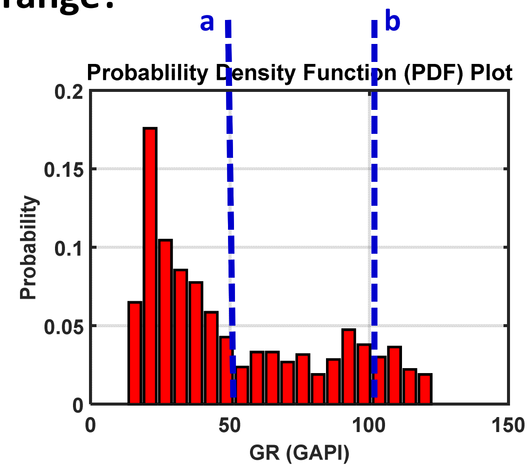
Reminder: Application of PDF

- **What is the probability that a randomly selected sample falls between a specific range?**

$$P(a < X \le b) = ?$$

$$P(a < X \le b) = \int_a^b f(x)\,dx$$

$$P(a < X \le b) = \sum_{i, a < X \le b} p_i$$



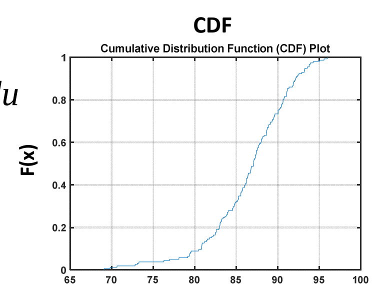
Reminder: Cumulative Distribution Function (CDF)

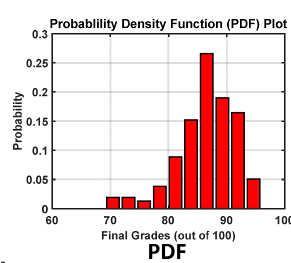
Cumulative Distribution Function:

The cumulative distribution function (CDF) is the sum of a discrete PDF or the integral of a continuous PDF. The cumulative distribution function $F_X(x)$ is the probability that the variable takes a value less than or equal to x.

$$F_X(x) = \int_{-\infty}^{x} f(u)\,du$$

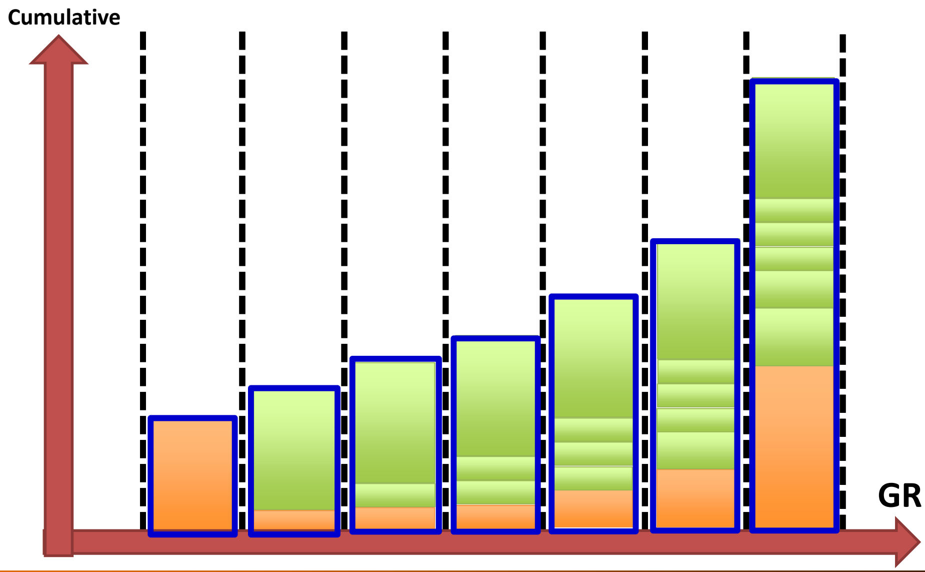
$$\int_{-\infty}^{+\infty} f(x)\,dx = 1$$





➔ **The shape of CDF is uniform and not affected by bin size.**

Reminder: How to Generate a Cumulative Function?



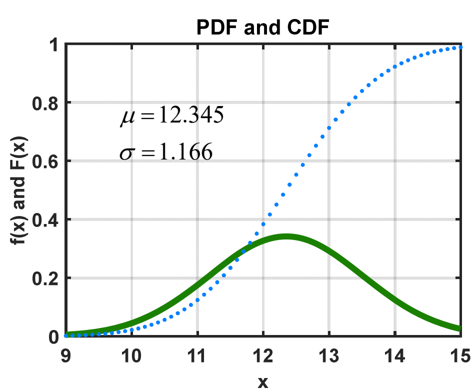
Reminder: Normal or Gaussian Distribution

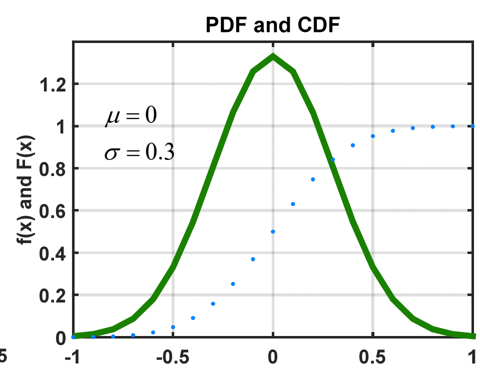
Mean

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right] \qquad F(x)=\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{x}\exp\left[-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2\right]dy$$

Standard Deviation

$$-\infty < x < +\infty$$





Reminder: Log-Normal Distribution

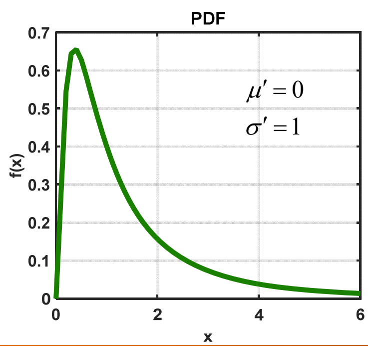
Mean of ln(x)

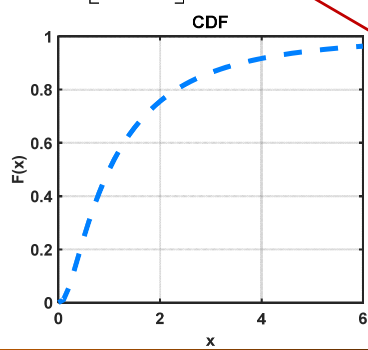
$$f(x)=\frac{1}{\sigma'\sqrt{2\pi}\,x}\exp\left[-\frac{1}{2}\left(\frac{\ln x-\mu'}{\sigma'}\right)^2\right] \qquad F(x)=\frac{1}{\sigma'\sqrt{2\pi}}\int_{-\infty}^{x}\frac{1}{y}\exp\left[-\frac{1}{2}\left(\frac{\ln y-\mu'}{\sigma'}\right)^2\right]dy$$

Standard deviation of ln(x) $\quad x>0$

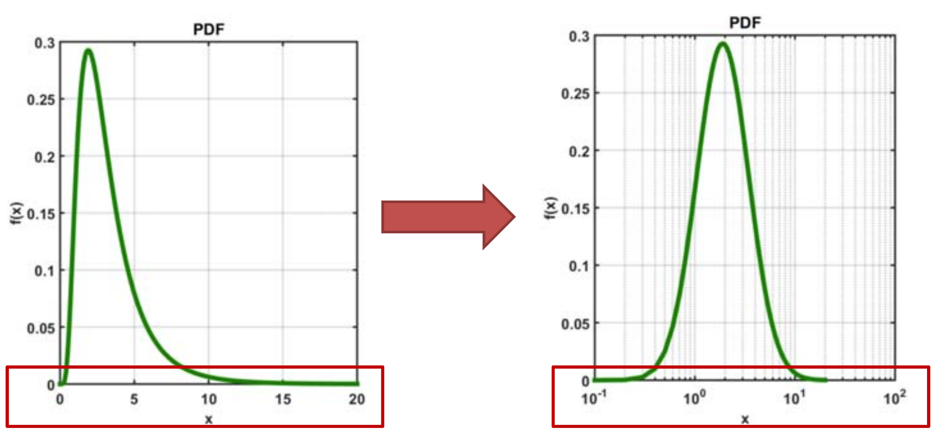
Standard deviation of x $\leftarrow \sigma^2=\mu^2\left[e^{\sigma'^2}-1\right]$ $\qquad \mu=\exp\left[\mu'+\frac{\sigma'^2}{2}\right]$

Mean of x

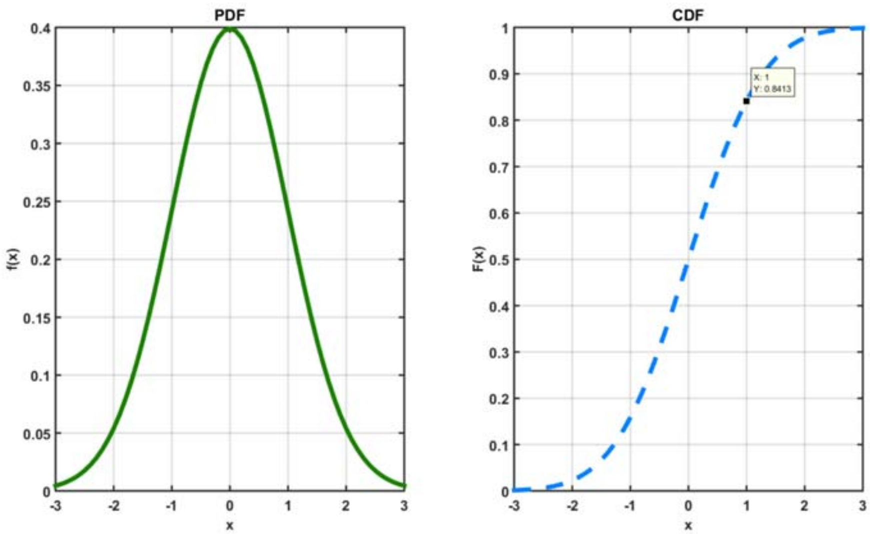




Reminder: Log-Normal Distribution



Inverse of the Standard Normal CDF

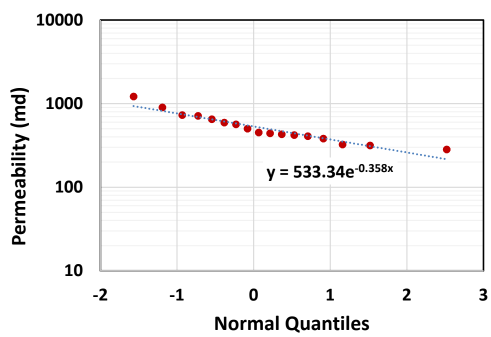


Example No. 2

Method 2:

- Arrange the permeability data in descending order.
- Compute the percent of total number of k-values equal or exceeding each permeability.
- Calculate the equivalent standard deviation scale.
- Find the equation of the line passing through the data points.

Solution:



Example No. 3

Example: **Quantify the permeability heterogeneity in the Carbonate formation of Example 1 (Carbonate 1), which is shared with you on the Canvas website.**

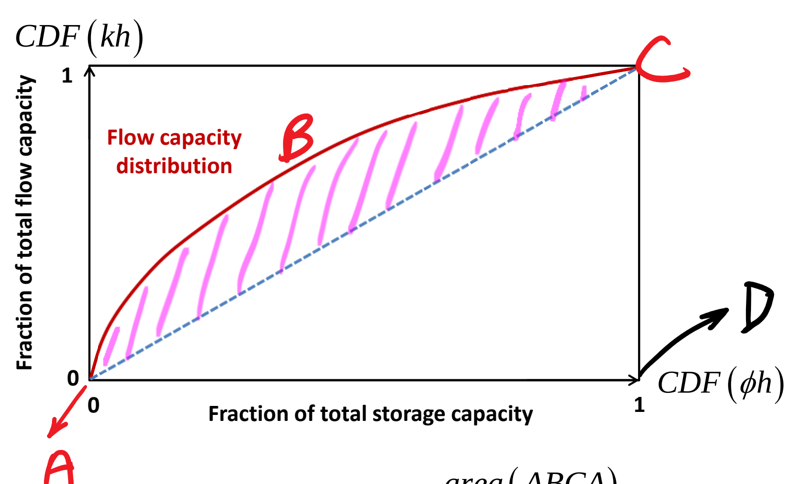
Compare the heterogeneity of this formation to that of formation from Example No. 2

Solution:

- **You can use any one of the previously discussed approaches to calculate the Dykstra-Parsons Coefficient of Variation.**

$$V = \frac{k_{50} - k_{84.1}}{k_{50}}$$

Lorenz Coefficient



$$0 < Lorenz\ Coefficient = \frac{area(ABCA)}{area(ADCA)} < 1$$

Lorenz Coefficient

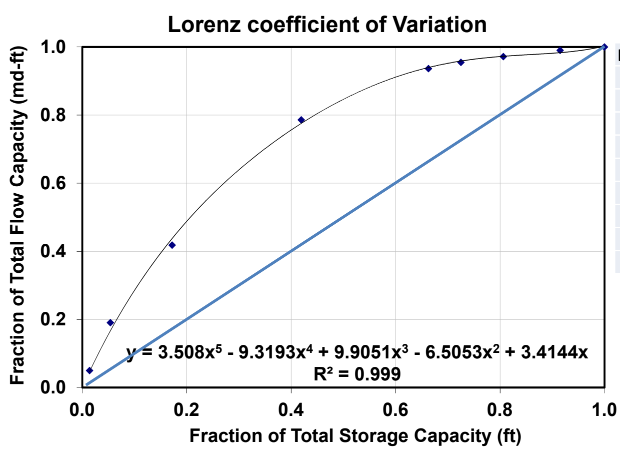
First sort the data in decreasing order of k/ϕ

n	h_n	ϕ_n	k_n	$k_n h_n$	$\sum_{i=1}^{n} k_i h_i$	$\phi_n h_n$	$\sum_{i=1}^{n} \phi_i h_i$	$\frac{\sum_{i=1}^{n} \phi_i h_i}{\sum_{i=1}^{N} \phi_i h_i}$	$\frac{\sum_{i=1}^{n} k_i h_i}{\sum_{i=1}^{N} k_i h_i}$
1	h_1	ϕ_1	k_1	$k_1 h_1$	$\sum_{i=1}^{1} k_i h_i$	$\phi_1 h_1$	$\sum_{i=1}^{1} \phi_i h_i$	$\frac{\sum_{i=1}^{1} \phi_i h_i}{\sum_{i=1}^{N} \phi_i h_i}$	$\frac{\sum_{i=1}^{1} k_i h_i}{\sum_{i=1}^{N} k_i h_i}$
2	h_2	ϕ_2	k_2	$k_2 h_2$	$\sum_{i=1}^{2} k_i h_i$	$\phi_2 h_2$	$\sum_{i=1}^{2} \phi_i h_i$	$\frac{\sum_{i=1}^{2} \phi_i h_i}{\sum_{i=1}^{N} \phi_i h_i}$	$\frac{\sum_{i=1}^{2} k_i h_i}{\sum_{i=1}^{N} k_i h_i}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$		
N	h_N	ϕ_N	k_N	$k_N h_N$	$\sum_{i=1}^{N} k_i h_i$	$\phi_N h_N$	$\sum_{i=1}^{N} \phi_i h_i$	$\frac{\sum_{i=1}^{N} \phi_i h_i}{\sum_{i=1}^{N} \phi_i h_i}$	$\frac{\sum_{i=1}^{N} k_i h_i}{\sum_{i=1}^{N} k_i h_i}$

Plot these values on the x axis

Example No. 4

- Calculate the Lorenz coefficient for the data uploaded on the Canvas website.

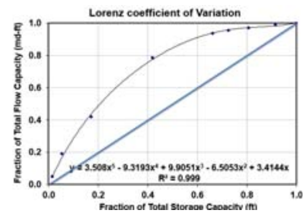


Layer	h (m)	k (md)	phi	k/phi
5	0.28	750	0.26	2884.62
4	0.74	800	0.28	2857.14
9	2.28	420	0.27	1555.56
7	5.15	300	0.25	1200.00
1	5.29	120	0.24	500.00
3	1.41	54	0.23	234.78
6	1.69	42	0.25	168.00
8	2.57	30	0.22	136.36
2	2.11	20	0.21	95.24

We need to sort the data based on k/phi

Example No. 4 (Continued)

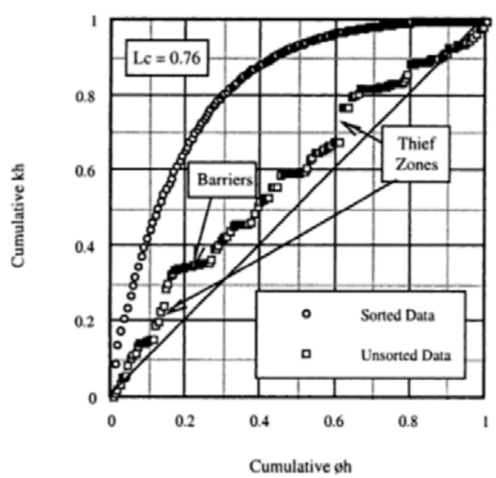
Layer	h (m)	k (md)	phi	k/phi	kh (md-m)	phi*h	cum(kh)/sum	cum(phi*h)/sum
5	0.28	750	0.26	2884.62	210	0.07	0.050	0.014
4	0.74	800	0.28	2857.14	592	0.21	0.191	0.054
9	2.28	420	0.27	1555.56	958	0.62	0.418	0.172
7	5.15	300	0.25	1200.00	1545	1.29	0.786	0.419
1	5.29	120	0.24	500.00	635	1.27	0.937	0.663
3	1.41	54	0.23	234.78	76	0.32	0.955	0.725
6	1.69	42	0.25	168.00	71	0.42	0.972	0.806
8	2.57	30	0.22	136.36	77	0.57	0.990	0.915
2	2.11	20	0.21	95.24	42	0.44	1.000	1.000



$$y = 3.508x^5 - 9.3193x^4 + 9.9051x^3 - 6.5053x^2 + 3.4144x$$

Solution:

Other Applications of Lorenz Plot



Source: Jensen, J. R., Lake, L. W., Corbett P. M. W., and Goggin, D. J., 2000, Statistics for Petroleum Engineers and Geoscientists, Elsevier.

Lorenz Coefficient

- Lorenz Coefficient is not a unique measure of reservoir heterogeneity.
  - Different permeability distributions can give the same Lorenz Coefficient.
- In the case of log-normal permeability distribution:
  - Lorenz Coefficient is very similar to Dykstra-Parsons Coefficient.
- Sample size affects Lorenz coefficient.

Neither Lorenz Coefficient nor Dykstra-Parsons Coefficient provides spatial correlation between permeability data.

Let's talk about methods for quantifying spatial correlation between formation data.

Advantages of Lorenz Coefficient

- It can be calculated reliably for any distribution.
- It does not rely on best fit procedures and it typically contains less calculation error compared to Dykstra-Parsons Coefficient.
- It takes into account porosity heterogeneity and thickness of different layers.

Complementary References

- Peters, E. J., 2012, Advanced Petrophysics. Live Oak Book Company. Chapter 4
- Jensen, J. R., Lake, L. W., Corbett P. M. W., and Goggin, D. J., 2000, Statistics for Petroleum Engineers and Geoscientists, Elsevier. Chapter 6