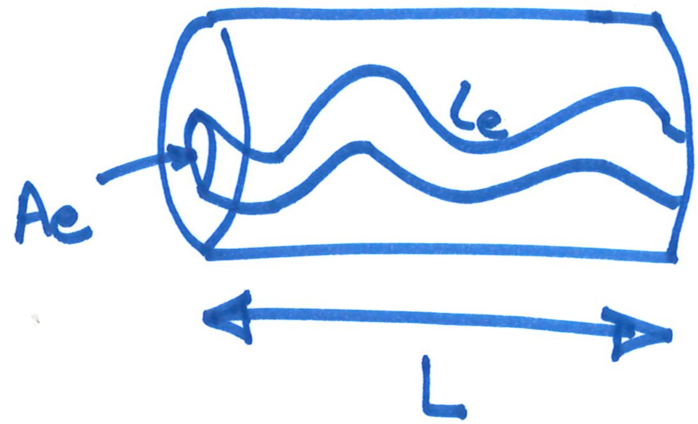
10/5/23



$$\frac{1}{r_0} = \frac{1}{r_w} + \cancel{\frac{1}{r_m}}^{\,0}$$

$$\Rightarrow r_0 = r_w$$

$$r_0 = R_0 \frac{L}{A} \quad , \quad r_w = R_w \frac{L_e}{A_e}$$

$$\left.\begin{aligned} \Rightarrow R_0 \frac{L}{A} &= R_w \frac{L_e}{A_e} \\ \underline{A_e L_e} &= \phi \underline{AL} \end{aligned}\right\} \rightarrow R_0 \frac{L}{A} = R_w \frac{L_e}{\frac{\phi AL}{L_e}}$$

$$\Rightarrow \underbrace{\frac{R_o}{R_w}}_{F} = \underbrace{\left(\frac{L_e}{L}\right)^2}_{\tau} \left(\frac{1}{\phi}\right)$$

$$\Rightarrow \boxed{\tau = F\phi}$$

Example: proppant pack

$\phi = 24\%$

Grain $D = 180\ \mu m$

$F = 32$

$k = ?$

$$k = \frac{D^2 \phi^3}{72\, \tau\, (1-\phi)^2} \quad , \quad \tau = F\phi$$

$$k = \frac{(180 \times 10^{-6})^2 (0.24)^3}{72\,(32)(0.24)\,(1-0.24)^2}$$

$\rightsquigarrow k = 1.4 \times 10^{-12}\ m^2$

$$\boxed{k = 1.4\ D}$$

Example: Unconsolidated Sand
Uniform grain size
closest packing possible

$L = 2$ miles $= 10560$ ft

$W = 1$ mile $= 5280$ ft

$t = 200$ ft

$D_g = 1/8$ mm $\rightsquigarrow$ $r = 2.05 \times 10^{-4}$ ft

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$\phi = ?$

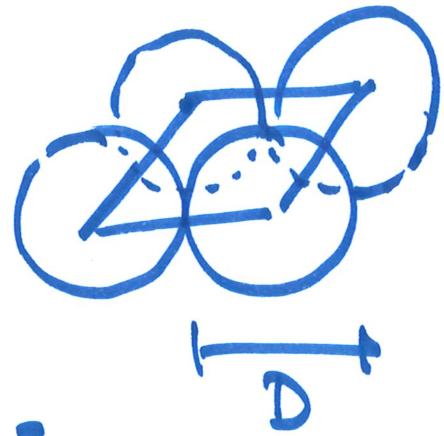
Wetted surface area $= ?$

$K = ?$

$$V_b = D^2 \cdot D/\sqrt{2} = \frac{D^3}{\sqrt{2}}$$

$\swarrow \sin\theta$

$$V_g = \frac{\pi D^3}{6}$$



$$\phi = \frac{V_b - V_g}{V_b} = 1 - \frac{\pi D^3/6}{D^3/\sqrt{2}}$$

$$\rightarrow \boxed{\phi = 25.9\%}$$

$$S = \frac{\text{Wetted surf. area}}{\text{Bulk vol.}}$$

$$= \frac{4\pi r^2}{\frac{4}{3}\pi r^3/(1-\phi)} = 3(1-\phi)/r$$

$$\text{Wetted Surf. area} = S \cdot V_b$$

$$= 3(1-\phi)/r \cdot V_b$$

$$= 3(1-0.259)/2.05\times10^{-4}$$

$$\times(10560 \times 5280 \times 200)$$

$$\Rightarrow \underbrace{WSA = 1.209 \times 10^{14}\ ft^2}$$

$K = ?$

$$K = \frac{\phi}{K_o \tau S_p^2}$$

$$\Rightarrow K = \frac{\phi^3}{\underbrace{K_o \tau}_{5} S^2}$$

$$S = \frac{WSA}{Vol.} = \frac{1.209\times10^{14}}{(10560 \times 5280 \times 200)}$$

$$= 10814.2\ ft^2/ft^3$$

$$K = \frac{(0.259)^3}{5(10814.2)^2}$$

$$\Rightarrow K = 2.95 \times 10^{-11} \ ft^2$$

$$= 2.78 \ D$$

Example:

$$f(\delta) = 3\left(\frac{\delta}{10}\right)^2\left(1-\frac{\delta}{10}\right)^2$$

$$0 \leq d \leq 10\,\mu m$$

$$\phi = 25\%$$

$$k = ?$$

$$q_i = \frac{\pi r_i^4 \Delta P}{8\mu L} = \frac{\pi \delta^4 \Delta P}{128\mu L}$$

$$Q = \sum_{i=1}^{N} q_i = \sum \frac{\pi \delta^4 \Delta P}{128\mu L}$$

$$= \frac{\pi \Delta P}{128\mu L}\sum_{i=1}^{N} \delta^4$$

$$\Rightarrow Q = \frac{\pi \Delta P}{128 \mu L} \int_{\delta_{min}}^{\delta_{max}} f(\delta)\, \delta^4 \, d\delta$$

$$\int_0^{10} 3\left(\frac{\delta}{10}\right)^2 \left(1-\frac{\delta}{10}\right)^2 \delta^4 \, d\delta = \frac{10^5}{84}$$

Darcy's Law:

$$Q = \frac{K' A_t \, \Delta P}{\mu L}$$

$$A_t = \frac{\sum A_i}{\phi}$$

$$A_i = \pi r_i^2$$

$$A_t = \frac{1}{\phi} \int_{\delta_{min}}^{\delta_{max}} \pi \frac{\delta^2}{4} f(\delta)\, d\delta$$

$$A_t = \frac{1}{\Phi}\,\frac{\pi}{4}\int_0^{10} \delta^2 \left[3\left(\frac{\delta}{10}\right)^2\left(1-\frac{\delta}{10}\right)^2\right] d\delta$$

$$\leadsto \quad A_t = \frac{1}{\Phi}\,\frac{\pi\,(10)^3}{140}$$

$$\frac{\cancel{\pi}\,\cancel{\Delta P}}{128\,\cancel{\mu L}}\,\frac{10^5}{84} = \frac{K}{\Phi}\,\frac{\cancel{\pi}(10^3)}{140}\,\frac{\cancel{\Delta P}}{\cancel{\mu L}}$$

$\Phi \nearrow 0.25$

$$\Longrightarrow \quad \underline{\underline{K = 0.325\,(\mu m)^2}}$$