Lecture #14, March 4, 2024

$$\frac{\alpha}{2}\left(1+\alpha_h\tilde{\xi}\right)\|e_{,x}\| \le c_{int}\left(|a|\left(\frac{h}{L}\right)^{k+1} + (\alpha+T|a|^2)\frac{1}{L}\left(\frac{h}{L}\right)^{k} + T|a|\alpha\frac{1}{L^2}\left(\frac{h}{L}\right)^{k-1}\right)\|u\|_{k+1}$$

recall equiv norm results

$$\frac{1}{L}\frac{1}{2}\|e^h\|_1 \le \frac{|e^h|_1}{L} = \cancel{L}\,\|e^h_{,x}\|$$

$$\frac{1}{2L}\|e^h\|_1 \le \frac{\alpha}{2}\left(1+\alpha_h\tilde{\xi}\right)\cancel{\|e^h_{,x}\|} \le$$

$$\underbrace{\frac{\alpha}{2}\left(1+\alpha_h\tilde{\xi}\right)\|e^h_{,x}\|}_{=\ \text{RHS}}$$

$$\frac{1}{4\cancel{L}}\alpha\left(1+\alpha_h\tilde{\xi}\right)\|e^h\|_1 \le$$

$$\le c_{int}\left(\left(|a|\frac{h}{\cancel{L}} + \frac{1}{\cancel{L}}(\alpha+T|a|^2) + \underbrace{T|a|\alpha\frac{1}{L^2}\left(\frac{h}{L}\right)^{-1}}\right)\left(\frac{h}{L}\right)^{k}\right)\|u\|_{k+1}$$

$$\underbrace{\frac{1}{L^2}\cancel{\cdot}\cancel{\frac{L}{h}}}$$

$$T|a|\alpha\frac{1}{h\cancel{L}}$$

$$T = \frac{h}{2|a|}\tilde{\xi}\kappa$$

Cleaning up

RHS: 2

eventually

$$\cancel{(1+\alpha_h\tilde{\xi})}\|e^h\|_1 \le \text{cint}\Big(\frac{4}{\alpha}h|a| + \frac{4}{\cancel{\alpha}}\big(\cancel{\alpha}^{\,1} + \frac{T|a|^2}{\alpha}\big)\big(\tfrac{h}{L}\big)^k + \frac{4}{\cancel{\alpha}}\big(\frac{T|a|}{\alpha}\frac{1}{h}\alpha\big)\Big)_\wedge\|u\|_{k+1}$$

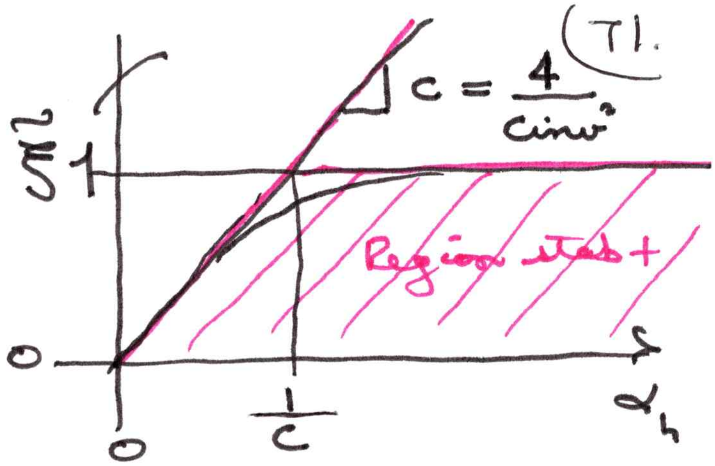
$$\le \text{cint}\Big(8\frac{h|a|}{2\alpha}^{=\alpha_h} + 4\big(1+\cancel{\frac{h}{2|a|}\frac{|a|}{\alpha}}^{\alpha_h}\cancel{\tilde{\xi}}\tilde{\xi}\big) + 4\cancel{\frac{h}{2|a|}\frac{|a|}{\alpha h}}\cancel{\alpha}\tilde{\xi}\Big)_\wedge\big(h/L\big)^k\|u\|_{k+1}$$

$$\le \text{cint}\frac{\big(8\alpha_h + 4(1+\alpha_h\tilde{\xi}) + 4\tilde{\xi}\big)}{(1+\alpha_h\tilde{\xi})}_\wedge\Big(\frac{h}{L}\Big)^k\|u\|_{k+1}$$

$$\le (\text{cint}\,4)\frac{\big(\overset{1}{2\alpha_h} + \overset{2}{(1+\alpha_h\tilde{\xi})} + \overset{3}{\tilde{\xi}}\big)}{(1+\alpha_h\tilde{\xi})}_\wedge\Big(\frac{h}{L}\Big)^k\|u\|_{k+1}$$

T1.

(2) $\dfrac{1+\alpha_h\tilde{\xi}}{1+\alpha_h\tilde{\xi}} \equiv 1.$ ✓



(1) $\dfrac{2\alpha_h}{(1+\alpha_h\tilde{\xi})}$ ✓

(3) $\dfrac{\tilde{\xi}}{1+\alpha_h\tilde{\xi}}$

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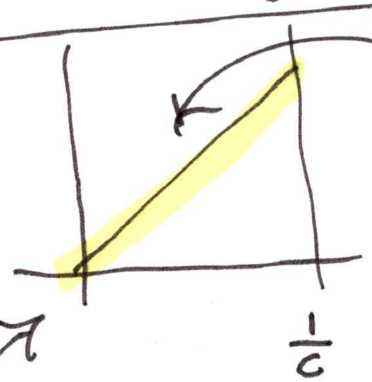
(1)

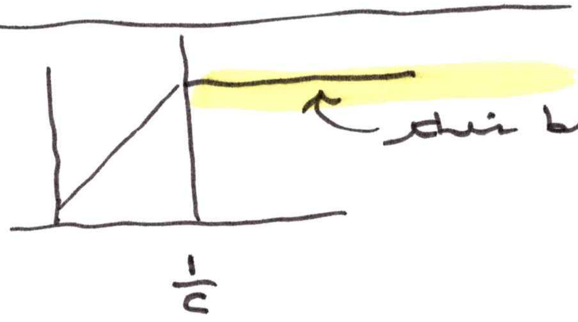


this branch ✓

$$\frac{2\alpha_h}{1+\alpha_h\tilde{\xi}} \le 2\alpha_h \le 2\frac{1}{c}$$

(3) $\dfrac{\tilde{\xi}}{1+\alpha_h\tilde{\xi}} = \dfrac{c\alpha_h}{1+\underbrace{\alpha_h c\alpha_h}} \le c\alpha_h \le 1$

X



this branch

(1) $\dfrac{2\alpha_h}{1+\alpha_h\tilde{\xi}} = \dfrac{2\alpha_h}{1+\alpha_h} \le 2$

(3) $\dfrac{\tilde{\xi}}{1+\alpha_h\tilde{\xi}} = \cancel{\dfrac{c\alpha_h}{1+\alpha_h c\alpha_h} \le \dfrac{1}{\alpha_h} \le c}$ ✓

$$= \frac{1}{1+\alpha_h\cdot 1} \le \frac{1}{1+\frac{1}{c}} \le \frac{c}{c+1} \le 1 \checkmark$$

X

$$\|e^h\|_1 \le C\left(\frac{h}{L}\right)^k \|u\|_{k+1}$$

indep of $\underbrace{\alpha_h, h, a, u, \kappa}$.

SUPG

$\|e\|_1 \le \|\eta\|_1 + \|e^h\|_1$ uniform conv. indep of

X

GLS:

$$B_{GLS}(w^h, u^h) = B(w^h, u^h) + \int_{\Omega'} \tau \mathcal{L} w^h (\mathcal{L} u^h \cancel{- f})\, dx.$$

$$L_{GLS}(w^h, u^h) = (w^h, f) + \int_{\Omega'} \tau \mathcal{L} w^h f\, dx$$

$$\mathcal{L} u^h = a u^h_{,x} - \kappa \underbrace{u^h_{,xx}}.$$

$$B_{GLS}(w^h, w^h) = \kappa \|w^h_{,x}\|^2 \leftarrow \text{Gal.}$$
$$+ \tau \|\mathcal{L} w^h\|^2_{\Omega'} \leftarrow \tau\text{-weighted LS}$$

$$B_{GLS}(e^h, e^h) = \underbrace{\kappa \|e^h_{,x}\|^2 + \tau \|\mathcal{L} e^h\|^2_{\Omega'}}$$

stab is easier!

x