HW7

November 14, 2023

WP7 - Exercise 1

Support functions

[1]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
plt.style.use('default')   ## reset!
plt.style.use('paper.mplstyle')

def shear( sig_n, s1, s2 ) :
    """
    Creates a shear series for given values of principal stresses and sig_n
    """
    center = (s1 + s2)/2
    radius = (s1 - s2)/2
    dx = sig_n - center
    tau_sq = radius * radius - dx * dx
    tau_sq[ tau_sq < 0 ] = None
    return np.sqrt( tau_sq )

def mohr2d(s1, s3, ax, title="") :
    """
    Plot Mohr diagram
    """
    # Setup data
    npts = 1000

    [s1,s3] = sorted([s1,s3], reverse=True)
    step = (s1-s3)/npts
    sig_n = np.sort( np.append( np.arange(s3,s1,step), [s1, s3]) )

    s13_tau = shear( sig_n, s1, s3 )

    # Do the plotting stuff
    ax.plot(sig_n, s13_tau, c='gray')

    ax.set_title(title, fontsize=20)
    ax.set_ylabel("Shear stress ($\\tau$)")
    ax.set_xlabel("Effective normal stress ($\sigma_n$)")

    return ax
```

Determine the best fitting parameters of a Coulomb criterion: $\sigma_1 = UCS + q\sigma_3$.

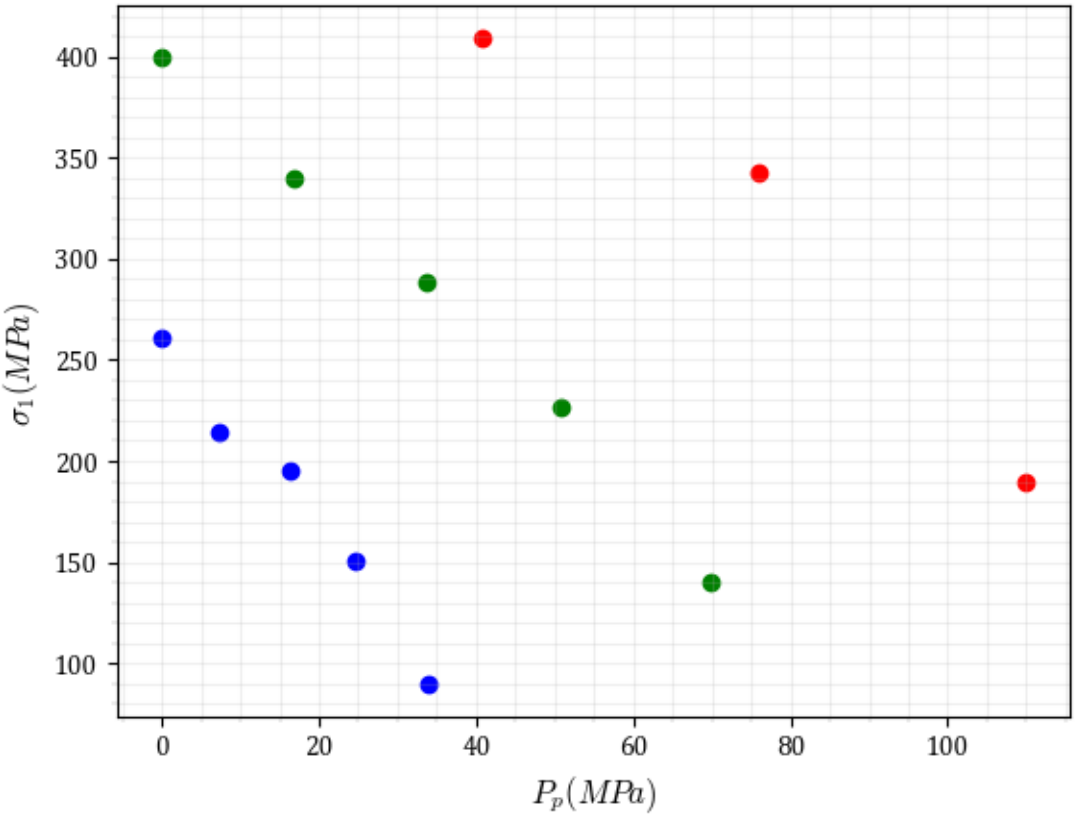
[2]:
```
import pandas as pd
import matplotlib.pyplot as plt
import numpy as np

df = pd.read_excel("DarleyDaleDataset.xlsx")
df["Sig3_MPa"] = df.S3_MPa - df.Pp_MPa
df["Sig1_MPa"] = df.S1_MPa - df.Pp_MPa
# Filter values beyond the linearity
df = df[df.Sig3_MPa < 100]

df["I1_MPa"] = df.Sig1_MPa + 2*df.Sig3_MPa
s1=df.Sig1_MPa
s3=df.Sig3_MPa
df["J2_MPa2"] = 1/6*( 2 * (s1-s3)**2 )
df["sqrt_J2_MPa"] = np.sqrt(df.J2_MPa2)

fig, ax = plt.subplots()
```

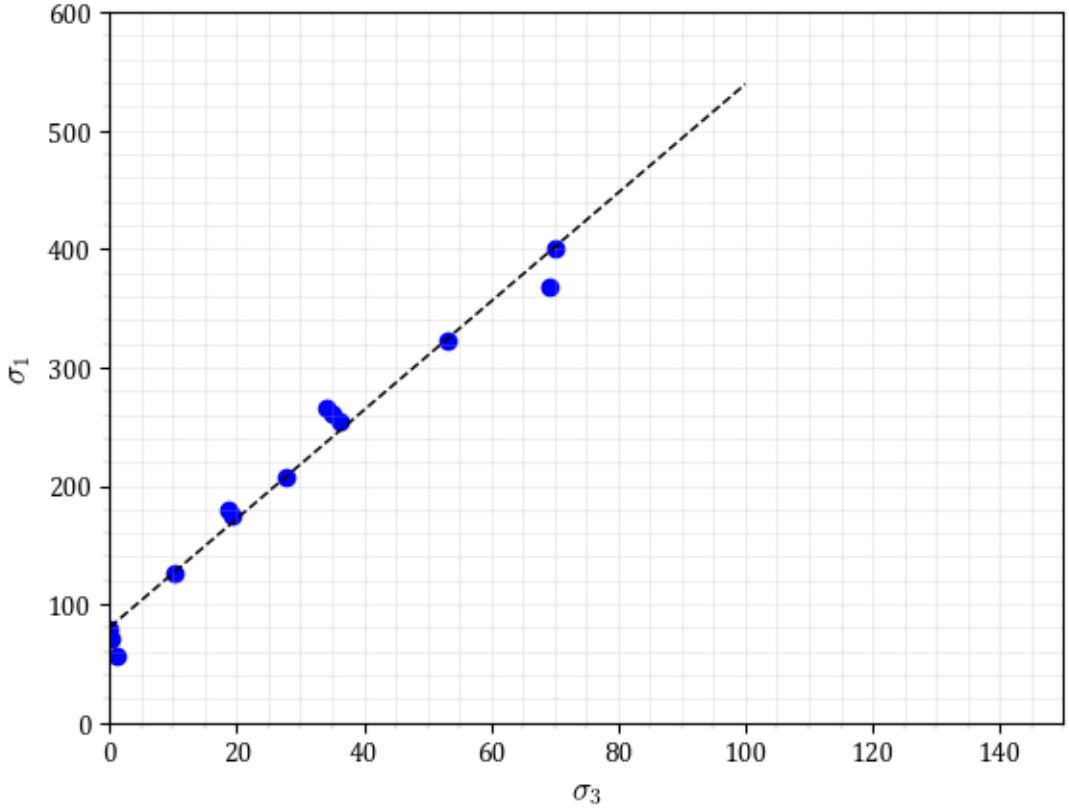
```
for name, g in df.groupby('S3_MPa'):
    ax.scatter(g.Pp_MPa, g.S1_MPa)
ax.set_xlabel("$P_p (MPa)$")
ax.set_ylabel("$\sigma_1 (MPa)$")

from scipy.stats import linregress
Q, UCS, r_value, p_value, std_err = linregress( df.Sig3_MPa, df.Sig1_MPa )
Sig3_fit = np.linspace(0,100,10)
Sig1_fit = Sig3_fit * Q + UCS

fig, ax = plt.subplots()
ax.scatter(df.Sig3_MPa, df.Sig1_MPa)
ax.plot(Sig3_fit, Sig1_fit, ls='--', color='k')
ax.set_xlabel("$\sigma_3$")
ax.set_ylabel("$\sigma_1$")
ax.set_xlim(0,150)
ax.set_ylim(0,600)

print(f"UCS={UCS:.2f} MPa q={Q:.4f}")
df
```

```
UCS=80.44 MPa q=4.5881
```

2]:

	S3_MPa	Pp_MPa	S1_MPa	Sig3_MPa	Sig1_MPa	I1_MPa	J2_MPa2	sqrt_J2_MPa
1	110	40.730779	409.023317	69.269221	368.292538	506.830979	29804.981366	172.641193
2	110	76.019958	342.219591	33.980042	266.199633	334.159717	17975.312781	134.072043
3	110	110.014132	189.773495	-0.014132	79.759362	79.731098	2121.270155	46.057249
4	70	-0.055399	399.750377	70.055399	399.805776	539.916574	36245.103735	190.381469
5	70	16.932524	339.904257	53.067476	322.971732	429.106683	24282.769258	155.829295
6	70	33.748003	288.242137	36.251997	254.494134	326.998129	15876.543480	126.002157
7	70	50.737176	226.162799	19.262824	175.425623	213.951271	8128.939967	90.160634
8	70	69.921900	140.320493	0.078100	70.398593	70.554794	1648.323920	40.599556
9	35	-0.145370	260.542030	35.145370	260.687400	330.978141	16956.402398	130.216752
10	35	7.266436	214.585236	27.733564	207.318800	262.785928	10750.285693	103.683584
11	35	16.341879	195.471680	18.658121	179.129801	216.446042	8583.720043	92.648368
12	35	24.760448	150.286070	10.239552	125.525621	146.004725	4430.292616	66.560443
13	35	33.859217	89.485789	1.140783	55.626571	57.908136	989.567059	31.457385

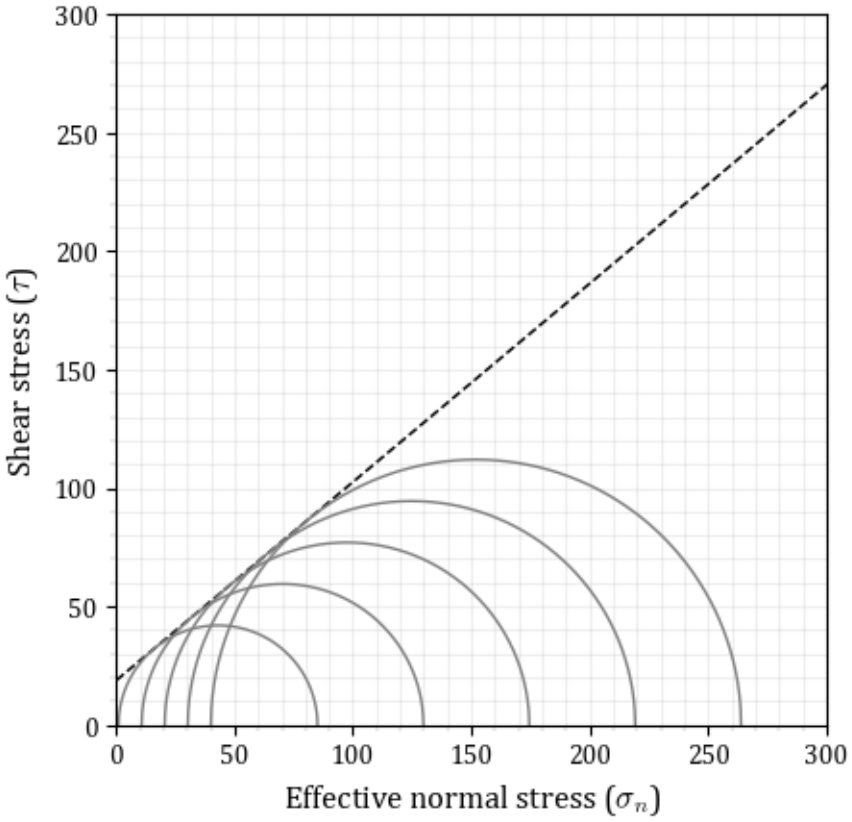

400
350
300
250
200
150
100
$\sigma_1(MPa)$
0
20
40
60
80
100
$P_p(MPa)$


600
500
400
300
200
100
0
σ_1
0
20
40
60
80
100
120
140
σ_3

Determine the corresponding values of cohesive strength S_0 and friction angle φ from point (1) and draw the shear failure line together with Mohr circles at failure.

3]:
```
S0 = UCS / 2 / np.sqrt(Q)
MU = (Q-1) / 2 / np.sqrt(Q)

PHI = np.arctan(MU)

print(f"S0={S0:.2f} MU={MU:.4f}  PHI={PHI*180/np.pi:.2f} deg")

sign=np.linspace(0,400,10)
tau = S0 + MU * sign

fig, ax = plt.subplots()
ax.plot(sign, tau, ls='--', color='k')
ax.set_xlim(0,300)
ax.set_ylim(0,300)

sig1_sig3 = Q
for sig3 in np.linspace(1,40,5) :
#    sig1 = sig1_sig3 * ( sig3 + S0 )
    sig1 = sig1_sig3 * ( sig3 + S0/MU ) - S0/MU
    mohr2d(sig1, sig3, ax)

ax.set_aspect('equal', 'box')
```

```
S0=18.78 MU=0.8376  PHI=39.95 deg
```

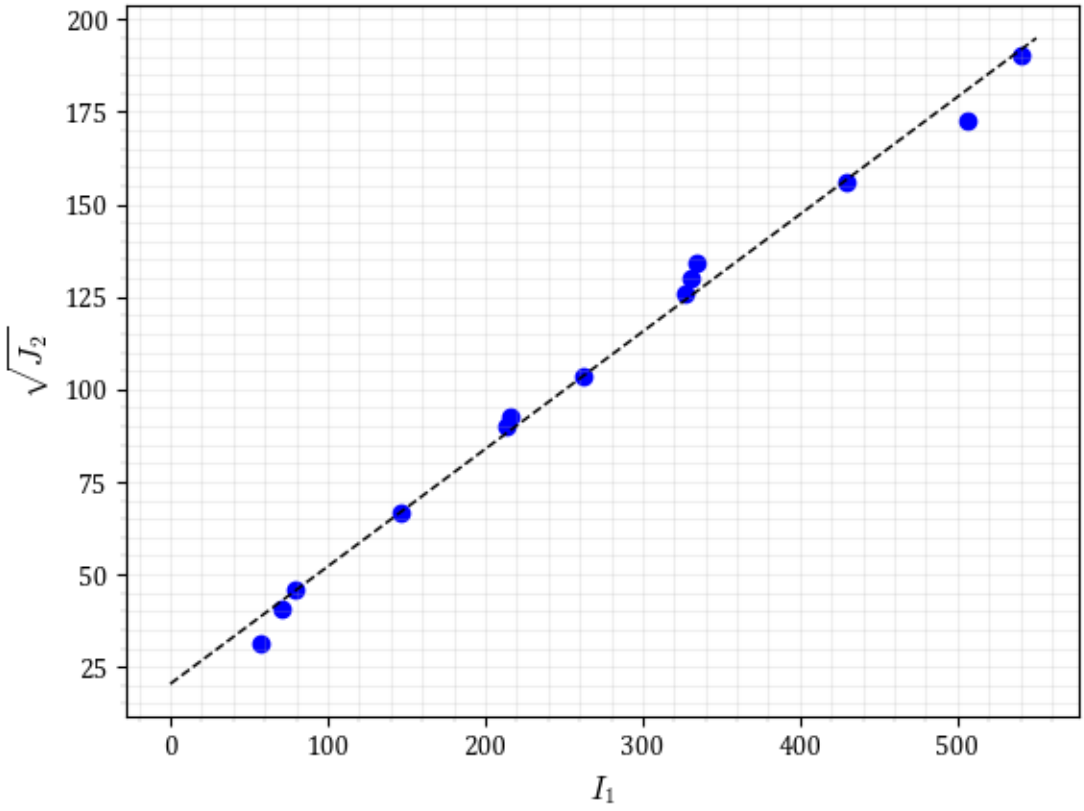


Determine the corresponding fitting parameters for a Drucker-Prager criterion and plot experimental data in a I_1 v.s. $(J_2)^{1/2}$ space.

4]:
```
fig, ax = plt.subplots()
ax.scatter( df.I1_MPa, df.sqrt_J2_MPa )

from scipy.stats import linregress
C4, C3, r_value, p_value, std_err = linregress( df.I1_MPa, df.sqrt_J2_MPa  )
i1 = np.linspace(0,550,10)
sqrt_j2 = C3 + C4 * i1
ax.plot( i1, sqrt_j2, c='k', ls='--' )
```

```
ax.set_xlabel("$I_1$")
ax.set_ylabel("$\sqrt{J_2}$")

print("Drucker-Prager:")
print(f"   C3={C3:.2f} C4={C4:.4f}")
```

```
Drucker-Prager:
   C3=20.35 C4=0.3174
```



Compute the corresponding tensile strength assuming a reasonable ratio of unconfined compression strength to tensile strength (browse Jaeger's book).

As of Section 4.5 and 4.6 of Jaeger's book, we can understand that a reasonable ration for $C_0/T_0 = [\sqrt{\mu^2+1}+\mu]^2$, or a reference value $C_0/T_0 = 15$.

5]:
```
c0_t0 = ( np.sqrt( MU**2 + 1 ) + MU ) ** 2

C0 = UCS
T0 = C0 / c0_t0

print(f"\nC_0/T_0 = {c0_t0:.2f}")
print(f"UCS = {UCS:.2f}   T_0 = {T0:.2f} MPa")

# Using the ratio from a table in the book
c0_t0 = 15
C0 = UCS
T0 = C0 / c0_t0

print(f"\n\nC_0/T_0 = {c0_t0:.2f}")
print(f"UCS = {UCS:.2f}   T_0 = {T0:.2f} MPa")
```

```
C_0/T_0 = 4.59
UCS = 80.44   T_0 = 17.53 MPa


C_0/T_0 = 15.00
UCS = 80.44   T_0 = 5.36 MPa
```

WP7 - Exercise 2

For this assignment you have to be able to calculate principal stresses on the wall of a wellbore at an arbitrary deviation and azimuth (See Chapter 6 of my notes "Wellbore stability" (https://dnicolasespinoza.github.io/) and Zoback's book Ch. 8).

Develop a script to compute and illustrate graphically (lower hemisphere projection):

1. The likelihood of tensile fractures considering wellbore mud pressure is equal to pore pressure in the formation $P_W = P_p$ and a simple tensile strength criterion (independent of intermediate and maximum principal stresses), i.e., required T_s.
2. The likelihood of breakouts considering wellbore mud pressure is equal to pore pressure in the formation $P_W = P_p$ and a simple unconfined compression strength criterion, i.e., required UCS.
3. The wellbore breakout angle w_{BO} considering a Mohr-Coulomb shear failure criterion and the properties of the Darley Dale Sandstone from Exercise 1.
4. The wellbore breakout angle w_{BO} considering a linear I_1 v.s. $(J_2)^{1/2}$ shear failure criterion and the properties of the Darley Dale Sandstone from Exercise 1.
5. The wellbore breakout angle w_{BO} considering a Modified Lade shear failure criterion (Zoback's book, p. 100) and the properties of the Darley Dale Sandstone from point 1.

Limit your wellbore breakout angle plots to $0° < w_{BO} < 60°$ because predictions with linear elasticity over $w_{BO} > 60°$ are likely highly inaccurate. Test your script with the three examples given in Zobacks's book in Figures 8.2 and 10.4 (normal faulting, strike-slip faulting, and reverse faulting). When computing breakouts assume $P_W = 32$ MPa (NF), $P_W = 40$ MPa (SSF), and $P_W = 52$ MPa (RF). Use a Poisson ratio of 0.25 for stress calculations.

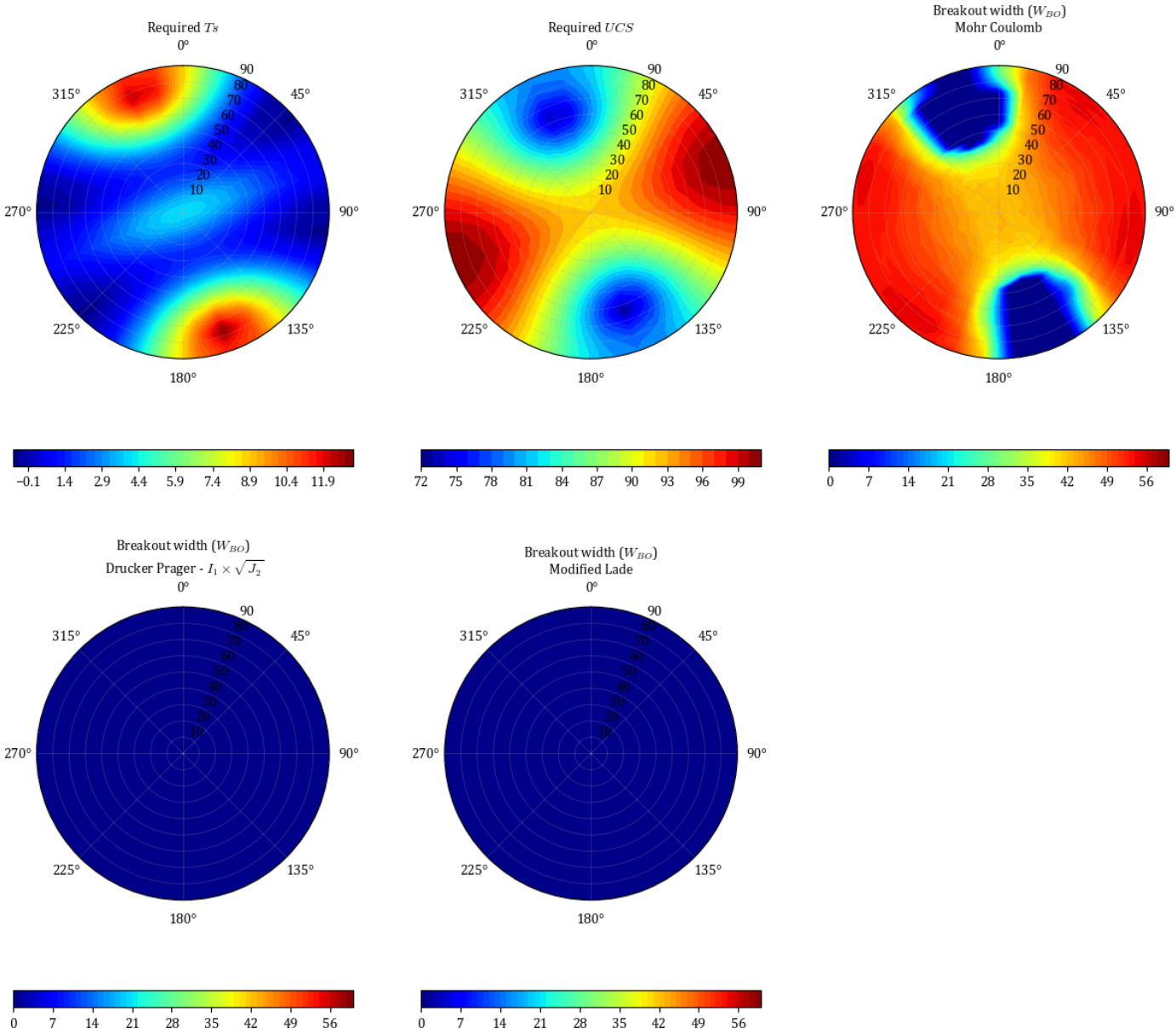
6]:
```
#
#
#
def Sp_to_Sg(Sp, alpha, beta, gamma) :
  # ROTATION MATRIX
  CA, SA = np.cos(alpha), np.sin(alpha)
  CB, SB = np.cos(beta),  np.sin(beta)
  CG, SG = np.cos(gamma), np.sin(gamma)
  RPG = np.array(
        [ [ CA*CB,                SA*CB            ,   -SB  ],
          [ CA*SB*SG - SA*CG,     SA*SB*SG + CA*CG , CB*SG  ],
          [ CA*SB*CG + SA*SG,     SA*SB*CG - CA*SG , CB*CG  ]]
  )

  # MATRIX MULTIPLICATION - STRESS IN GEOGRAPHIC COORDINATES
  Sg = RPG.T @ Sp @ RPG
  return Sg

#
#
#
def Sg_to_Sw(Sg, delta, phi) :
  # ROTATION MATRIX
  CD = np.cos(delta)
  SD = np.sin(delta)
  CP = np.cos(phi)
  SP = np.sin(phi)

  Rb = np.array(
        [ [ -CD*CP,   -SD*CP ,  SP  ],
          [ SD,       -CD    ,   0  ],
          [ CD*SP,     SD*SP ,  CP  ]]
  )

  Sw = Rb @ Sg @ Rb.T
  return Sw

def range_levels(min, max, n_steps) :
    n_dec = 0
    _n = n_steps
    while ( True ) :
        step = ( max - min ) / n_steps
        step = round( step, n_dec )
        if ( not step ) :
            n_dec += 1
            continue
        _n = ( max - min ) / step
        levels = np.arange(np.floor(min*10**n_dec)/10**n_dec, max+step, step)

        if ( np.abs( _n - n_steps ) < n_steps*.5 ) : break    # Accept 20% deviation
        n_dec += 1
    return levels
```

7]:
```
import numpy as np
import pandas as pd
from numpy import sin, cos, pi
from numpy.linalg import eigvals
import matplotlib.pyplot as plt

def build_polar_plots() :

    global NU
    global S1, S2, S3, AZIMUTH, REGIME
    global PW, PP

    # Mohr coulomb failure criteria: Sig1 = UCS + Q * Sig3
    global MC_UCS, MC_Q
    # I1 x J2 Drucker Parger failure criteria: SQRT_J2 = C3 + V4 * I1
    global DP_C3, DP_C4, DP_I1, DP_SJ2

    DP_I1 = []
    DP_SJ2 = []

    MC_S0 = MC_UCS / 2 / np.sqrt(MC_Q)
    MC_MU = (MC_Q-1) / 2 / np.sqrt(MC_Q)
    MC_PHI = np.arctan(MC_MU)

    # Modified Lade
    ML_S = MC_S0 / MC_MU
    ML_ETA = 4 * MC_MU**2 * ( 9 - 7 * np.sin(MC_PHI) ) / ( 1 - np.sin(MC_PHI) )
    # MPa

    Delta_P = PW - PP

    # Sp
    Sp = [[ S1, 0, 0 ],[ 0, S2, 0],[ 0, 0, S3]]

    # Normal        : az, pi/2, 0
    # Strike slip   : az, 0,    pi/2
    # Reverse       : az, 0,    0
    az = AZIMUTH / 180 * np.pi
    if (REGIME == "NORMAL")     : Sg = Sp_to_Sg( Sp, az, pi/2  , 0 )
    if (REGIME == "STRIKESLIP") : Sg = Sp_to_Sg( Sp, az, 0     , pi/2 )
    if (REGIME == "REVERSE")    : Sg = Sp_to_Sg( Sp, az, 0     , 0 )

    phimesh, deltamesh = np.meshgrid( np.linspace(0,pi/2,30), np.linspace(0, 2*np.pi, 30) )
    Ts = np.zeros_like( deltamesh )
    UCS = np.zeros_like( deltamesh )
    MC_WBO = np.zeros_like( deltamesh )
    DP_WBO = np.zeros_like( deltamesh )
    ML_WBO = np.zeros_like( deltamesh )

    print("Calculating ... ")

    for i, j in np.ndindex(deltamesh.shape) :
        delta = deltamesh[i,j] ; phi = phimesh[i,j]
        Sw = Sg_to_Sw(Sg, delta, phi)
        [ [ sig11, sig12, sig13 ], [ sig21, sig22, sig23 ], [ sig31, sig32, sig33] ] = Sw - PP * np.identity(3)
        for theta in np.linspace(0,2*np.pi,180):  # Angle around the borehole
            srr = Delta_P
            stt = sig11 + sig22 - 2*(sig11-sig22)*cos(2*theta) - 4*sig12*sin(2*theta) - Delta_P
            szt = 2 * ( sig23*cos(theta) - sig13*sin(theta) )
            szz = sig33 - 2*NU*(sig11-sig22)*cos(2*theta) - 4*NU*sig12*sin(2*theta)

            ST = [ [ stt, szt ], [ szt, szz] ]
            EV = sorted(eigvals(ST), reverse=True)

            if theta==0 : Ts[i,j] = EV[1] ; UCS[i,j] = EV[0] # Initialize
            Ts[i,j] = np.minimum( Ts[i,j], EV[1] )
            UCS[i,j] = np.maximum( UCS[i,j], EV[0] )

            [ sig1, sig2, sig3 ] = sorted( [ EV[0], EV[1], srr ], reverse=True )
            #print(f"{sig1} - {sig2} - {sig3}")
            # Mohr coulomb criteria
            mc_sig1_max = MC_UCS + MC_Q * sig3
            if sig1 > mc_sig1_max : MC_WBO[i,j] += 1

            # Drucker Prager
```

```
            i1 = sig1 + sig3 + sig2
            sqrt_j2 = np.sqrt ( 1/6*( (sig1-sig2)**2 + (sig1-sig3)**2 + (sig2-sig3)**2 ) )
            sqrtj2_max = DP_C3 + DP_C4 * i1
            #print(f"i1:{i1:.2f} sqrt_j2:{sqrt_j2:.2f}  max:{sqrtj2_max:.2f}")
            if sqrt_j2 > sqrtj2_max : DP_WBO[i,j] += 1
            DP_I1.append(i1)
            DP_SJ2.append(sqrt_j2)

            # Modified Lade
            [ sig1s, sig2s, sig3s ] = np.array([ sig1, sig2, sig3 ]) + ML_S
            i1s = sig1s + sig2s + sig3s
            i3s = sig1s * sig2s * sig3s
            i1s_max_3 = (27 + ML_ETA) * i3s
            i1s_max = np.sign(i1s_max_3) * np.abs(i1s_max_3)**(1/3)
            if i1s > i1s_max : ML_WBO[i,j] += 1

    MC_WBO[ MC_WBO > 60 ] = 60
    DP_WBO[ DP_WBO > 60 ] = 60
    ML_WBO[ ML_WBO > 60 ] = 60

    print("Data build ... [ok]")

    fig = plt.figure(figsize=(15, 20))

    ##
    def polar_plot( pos, m1, m2, Y, title, range=None, n_levels=50 ) :
        ax = plt.subplot(pos, projection='polar')
        if not range :
            range = [ np.min(Y), np.max(Y)]
        levels = range_levels( range[0], range[1], n_levels )

        CS = ax.contourf( m1, m2, Y, levels, cmap='jet' ) # contourf or pcolormesh
        cbar = fig.colorbar(CS, orientation="horizontal", pad=0.2)

        ax.set_title( title )
        ax.set_theta_zero_location("N")
        ax.set_theta_direction(-1)
    ##

    polar_plot(331, deltamesh, phimesh*180/pi, Ts, "Required $Ts$", None, 50)
    polar_plot(332, deltamesh, phimesh*180/pi, UCS, "Required $UCS$", None, 50)
    polar_plot(333, deltamesh, phimesh*180/pi, MC_WBO, "Breakout width ($W_{BO}$)\nMohr Coulomb", [0,60])
    polar_plot(334, deltamesh, phimesh*180/pi, DP_WBO, "Breakout width ($W_{BO}$)\n Drucker Prager - $I_1 \\times \sqrt{J_2}$", [0,60])
    polar_plot(335, deltamesh, phimesh*180/pi, ML_WBO, "Breakout width ($W_{BO}$)\nModified Lade", [0,60] )

    print("Done")
```

8]:
```
# Mohr coulomb failure criteria: Sig1 = UCS + Q * Sig3
MC_UCS=80.44
MC_Q=4.5881

# I1 x J2 Drucker Parger failure criteria: SQRT_J2 = C3 + V4 * I1
DP_C3=20.35
DP_C4=0.3174

# Parameters to the Modified Lade
S1 = 70 # Sv
S2 = 67 # Shmax
S3 = 45 # Shmin
AZIMUTH = 160
REGIME = 'NORMAL'

PP = 32
PW = 32
NU = 0.25

print ("ZOBACK - EXAMPLE 1 (NORMAL)")
build_polar_plots()
```

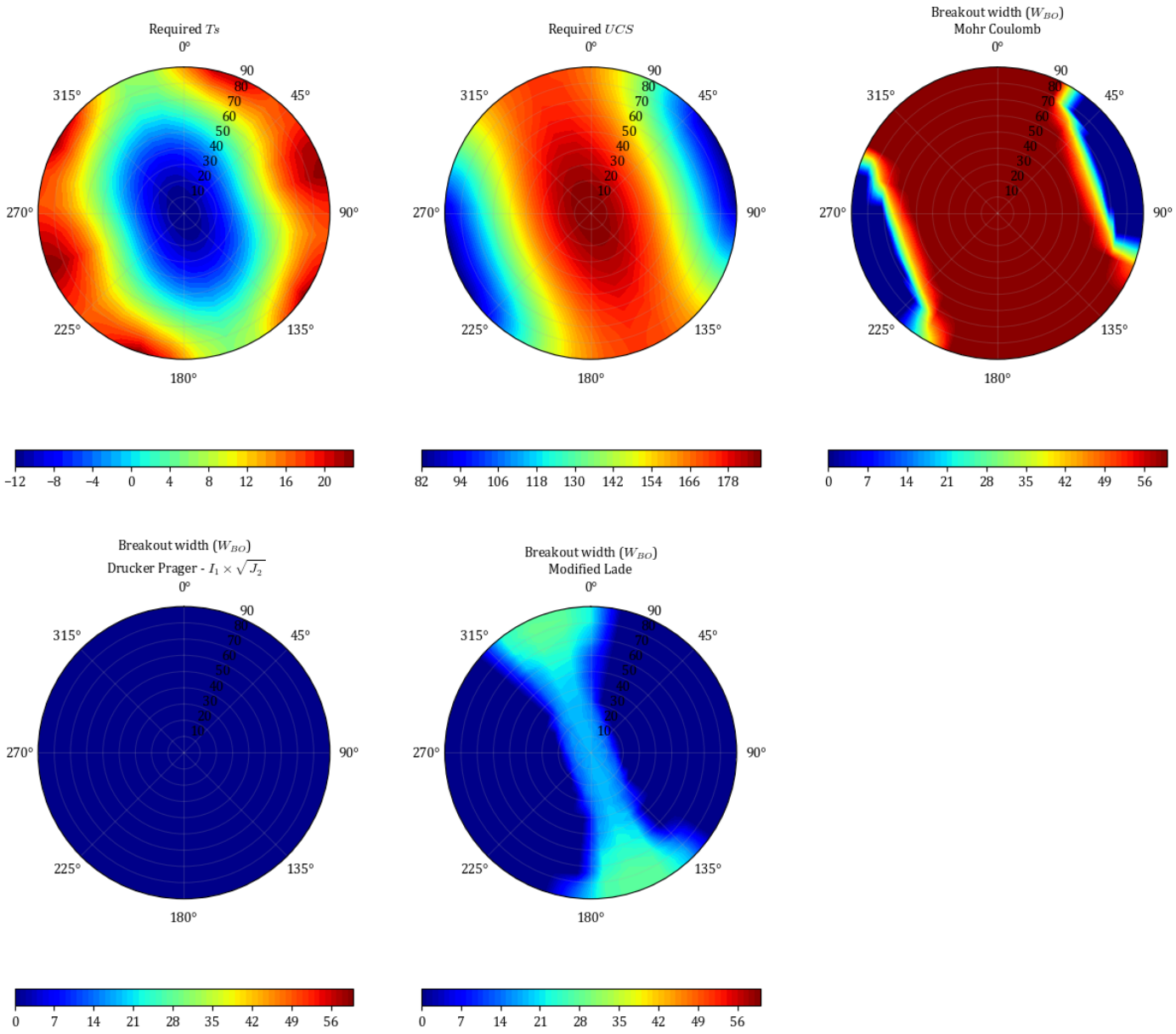
```
ZOBACK - EXAMPLE 1 (NORMAL)
Calculating ...
Data build ... [ok]
Done
```



9]:
```
# Mohr coulomb failure criteria: Sig1 = UCS + Q * Sig3
MC_UCS=80.44
MC_Q=4.5881

# I1 x J2 Drucker Parger failure criteria: SQRT_J2 = C3 + V4 * I1
DP_C3=20.35
DP_C4=0.3174

# Parameters to the Modified Lade
S1 = 105    # Shmax
S2 = 70     # Sv
S3 = 55     # Shmin
AZIMUTH = 70
REGIME = 'STRIKESLIP'

PP = 32
PW = 40
NU = 0.25

print ("ZOBACK - EXAMPLE 2 (STRIKE SLIP)")
build_polar_plots()
```

```
ZOBACK - EXAMPLE 2 (STRIKE SLIP)
Calculating ...
```

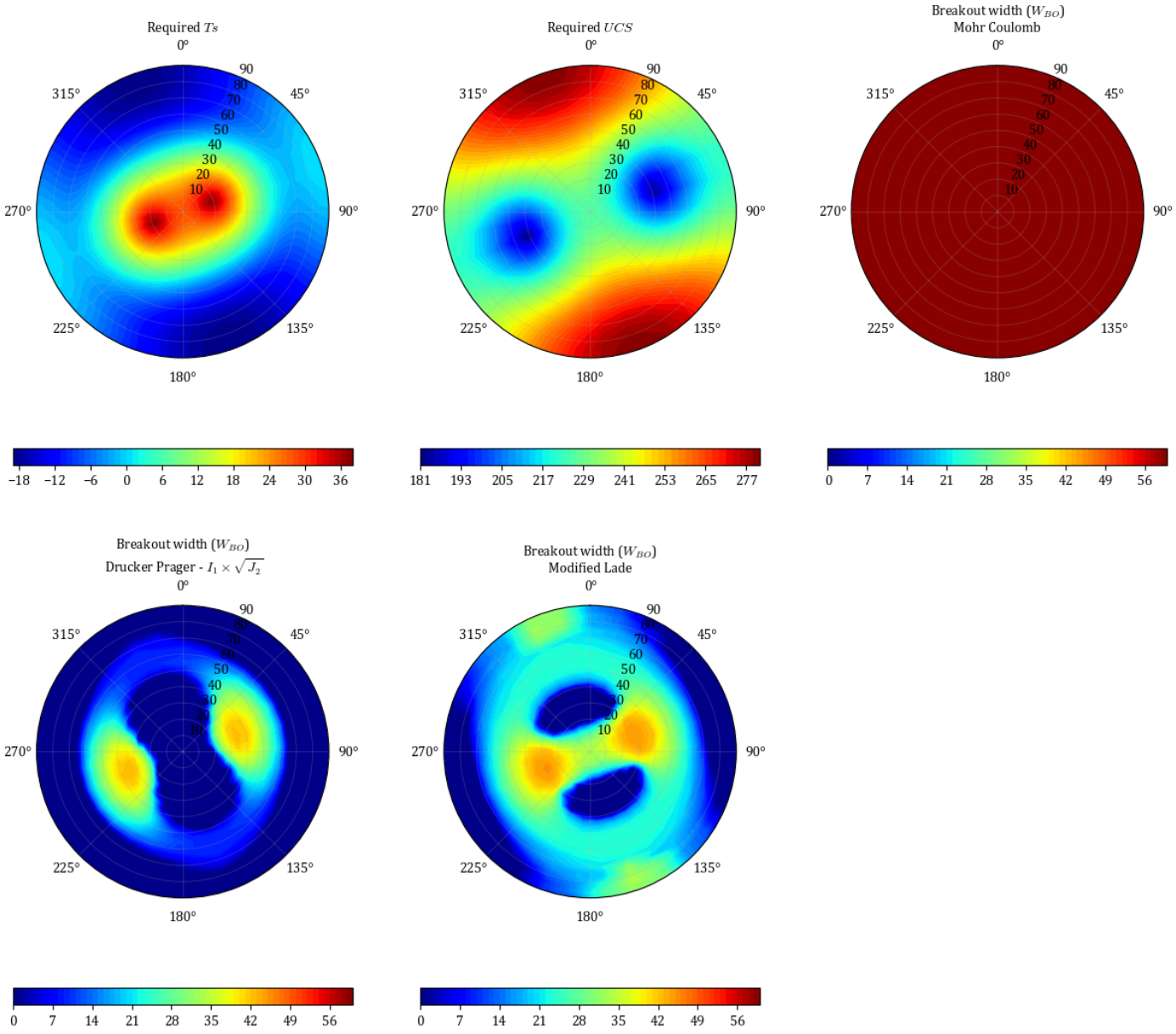
```
Data build ... [ok]
Done
```



0]:
```
# Mohr coulomb failure criteria: Sig1 = UCS + Q * Sig3
MC_UCS=80.44
MC_Q=4.5881

# I1 x J2 Drucker Parger failure criteria: SQRT_J2 = C3 + V4 * I1
DP_C3=20.35
DP_C4=0.3174

# Parameters to the Modified Lade
S1 = 145    # Shmax
S2 = 125    # Shmin
S3 = 70     # Sv
AZIMUTH = 70
REGIME = 'REVERSE'

PP = 32
PW = 52
NU = 0.25

print ("ZOBACK - EXAMPLE 2 (REVERSE)")
```

```
build_polar_plots()
```

```
ZOBACK - EXAMPLE 2 (REVERSE)
Calculating ...
Data build ... [ok]
Done
```



1]:
```
# Mohr coulomb failure criteria: Sig1 = UCS + Q * Sig3
MC_UCS=80.44
MC_Q=4.5881

# I1 x J2 Drucker Parger failure criteria: SQRT_J2 = C3 + V4 * I1
DP_C3=20.35
DP_C4=0.3174

# Parameters to the Modified Lade
S1 = 70   # Shmax
S2 = 67   # Shmin
S3 = 45    # Sv
AZIMUTH = 70
REGIME = 'NORMAL'

PP = 32
```

```
PW = 60
NU = 0.25

print ("BREAKOUT ANALYSIS")
build_polar_plots()
```

```
BREAKOUT ANALYSIS
Calculating ...
Data build ... [ok]
Done
```