Advanced Petrophysics
PGE 381L, Fall 2023
Unique Number: 20215

Homework Assignment No. 7

November 9, 2023
Due on Thursday, November 16, 2023, before 11:00 PM

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UT EID: __ rep2656 ____

Objectives:

- **a)** To understand concepts of wettability and interfacial/surface tension
- **b)** To practice assessment of wettability

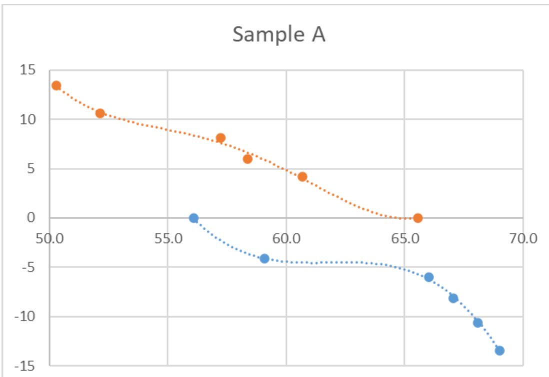
Note: Please scan your homework assignment and upload it as one pdf file on the Canvas website before the deadline. Please name your homework document as follows:

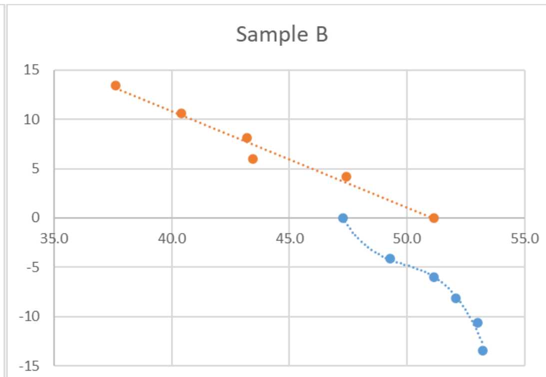
PGE381L_2023_Fall _HW07_lastname_name.pdf

Example: PGE381L_2023_Fall_HW07_Heidari_Zoya.pdf

Question 1: A graduate student in my research team measured the data required for wettability assessment in two rock samples. We labeled them as rock samples A and B. You can find this data in the uploaded Excel file "PGE381L_HW_07_Data". Please note that she tried to organize her measurements in a way to be able to estimate both Amott and USBM wettability indices. She used brine and decane for her measurements. Use the data provided to you and answer the following questions:

a) Create the plots you need for assessment of USBM wettability index. Estimate USBM wettability index for rock samples A and B.





b) Estimate Amott wettability index for rock samples A and B.

$$S_w^a = S_{spw} - S_{wirr} \qquad S_w^b = S_{fw} - S_{sspw} \qquad WI_w = \frac{S_w^a}{S_w^a + S_w^b}$$

$$S_w^c = S_{wor} - S_{spo} \quad , \quad S_w^d = S_{spo} - S_{fo} \quad WI_o = \frac{S_w^c}{S_w^c + S_w^d}$$

Amott-Harvey Index	Va, Vc	Vb, Vd	Sample A			Sample B
Iw	5.1	12.9	0.28158	8.3	6.0	0.58
Io	3.5	15.3	0.18519	2.1	13.5	0.13
AI = Iw-Io			**0.10**			**0.45**

c) Fill out the following two tables and compare the wettability of these two samples.

Amott Index	**Rock Sample A**	**Rock Sample B**
I_w	0.28	0.58
I_o	0.19	0.13
AI = I_w-I_o	0.10	0.45
USBM Index	**Rock Sample A**	**Rock Sample B**
Forced Brine (A_2)	69.5	31.7
Forced Decane (A_1)	99.9	89.2
Wettability Index	0.16	0.45

Both the Ammot and the USBM indices agree in the wettability of the samples. Sample B tends to stronger water-wet, whereas sample A looks more like a mixed-wet to water-wet

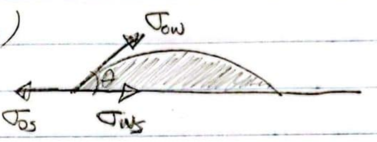
Question 2: The oil/water interfacial tension and contact angle for an oil-water-solid system are 30 dynes/cm and 60˚. How much Super XX water-soluble surfactant would be required to make the system fully water-wet if each part per million (ppm) of Super XX reduces the interfacial tension between water and the solid by 2 dynes/cm, the oil/water interfacial tension by 4 dynes/cm but does not alter the interfacial tension between the oil and the solid?

Q2

$\sigma_{ow} = 30$ dyns/cm^2 $\qquad \theta = 60°$

$\Delta\sigma_{ws} = -2/ppm \qquad \Delta\sigma_{wo} = -4/ppm \qquad \Delta\sigma_{os} = 0.$

Force balance (Young Dupre)



$\sigma_{os} - \sigma_{ws} = \sigma_{ow}\cos\theta$

$\sigma_{os} - \sigma_{ws} = 30 \times \cos 60° = 15$ dyns/cm.

Full water wet : $\cos\theta = 1 \qquad (\theta = 0°)$

$\Rightarrow \sigma_{os} - \sigma_{ws} = \sigma_{ow}$

$(\sigma_{os} + \Delta\sigma_{os} \times ppm) + (\sigma_{ws} - \Delta\sigma_{ws} \times ppm) = 30$

$ppm(-\Delta\sigma_{os} - \Delta\sigma_{ws}) = 30 - 15 = 15$

$ppm\,(6) = 15.$

$XX\,ppm = 15/6 \Rightarrow \boxed{XX_{ppm} = 2.5\ ppm}$

Question 3: We plan to use the setup, illustrated in **Figure 1**, to estimate surface tension of liquid X, σ. In this method, we plan to use the force applied to pull up the triangular plate to estimate surface tension. Assume the system is in equilibrium. Answer the following questions:

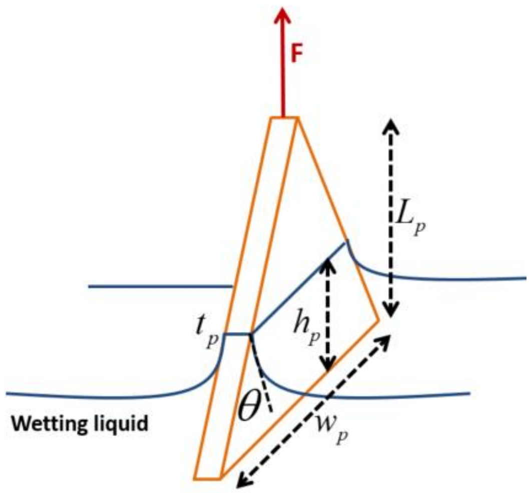


Figure 1: Experimental setup for question 1

a) Estimate surface tension as a function of the following parameters:

Force applied to pull up the plate: F

Thickness of the plate: t_p

Total vertical length of the plate: L_p

Total horizontal length of the plate: w_p

Height of the submerged section of the plate: h_p

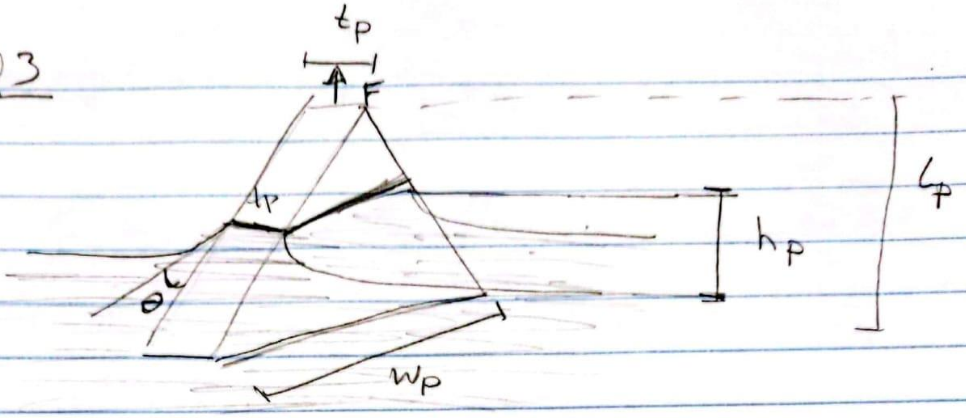
Contact angle between the surface of the plate and the liquid: θ

Density of the plate: ρ_p

Density of the liquid: ρ_L

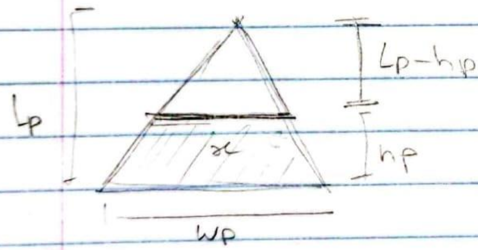
b) Simplify your answer to part (a) assuming that thickness of the plate is negligible compared to its vertical and horizontal lengths.

Q3



$$F = F_{gravity} + F_{capillary} - F_{buoyancy}.$$

$\sigma, S \cos\theta$



$$\frac{x}{w_p} = \frac{L_p - h_p}{L_p}$$

$$x = \frac{w_p}{L_p}(L_p - h_p)$$

$$S = 2x + 2t_p.$$

$$\text{Volume of solid (TOT)} = \frac{w_p \cdot L_p \cdot t_p}{2}$$

$$\text{Volume outside} = \frac{(L_p - h_p) \cdot t_p \cdot x}{2}$$

$$\text{Vol}_{\text{immerse}} = \frac{t_p}{2}\left[w_p \cdot L_p - (L_p - h_p) \cdot \frac{w_p}{L_p}(L_p - h_p)\right]$$

$$= \frac{t_p \cdot w_p}{2}\left[L_p - \frac{(L_p - h_p)^2}{L_p}\right]$$

$F_{Buoyancy} = V_{immersed} \cdot \rho_{liq} \cdot g$

$$F = \rho_p \cdot \frac{W_p L_p t_p}{2} + \sigma \cdot 2 \cdot \left[t_p + \frac{W_p}{L_p}(L_p - h_p)\right]\cos\theta$$

$$- \frac{t_p \cdot W_p}{2}\left[L_p - \frac{(L_p - h_p)^2}{L_p}\right]\rho_L \cdot g$$

Isolate σ:

$$\sigma = \frac{F - \rho_p/2\,(W_p L_p t_p)\,g\ \frac{t_p W_p}{2}\left(L_p - \frac{(L_p - h_p)^2}{L_p}\right)\rho_L g}{2\left(t_p + W_p/L_p\,(L_p - h_p)\right)\cos\theta}$$

(b) $\left.\begin{array}{l} t_p \ll L_p \\ t_p \ll W_p \end{array}\right\} \Rightarrow \boxed{\sigma = \frac{F}{2\,W_p/L_p\,(L_p - h_p)\cos\theta}}$

Optional Question: Figure 2 shows free or spontaneous imbibition of water into two capillary tubes of radii r_1 and r_2 (where $r_1<r_2$) joined through a constriction (pore throat) of radius r_A (where $r_A<r_1$). At this time, the water-gas menisci in tubes 1 and 2 are still moving to the right. The pressure at the inlet and outlet of each tube is constant and equal to the atmospheric pressure. Derive an equation for the distance (x_i) traveled by meniscus i in tube i as a function of time (t) and the other pertinent variables of the problem. Assume x_i = 0 at t= 0. Use the following nomenclature:

ρ_w = water density

σ = water-air interfacial tension

θ = water-air-solid contact angle

r_1 = radius of capillary tube 1

r_2 = radius of capillary tube 2

r_A = radius of constriction at A

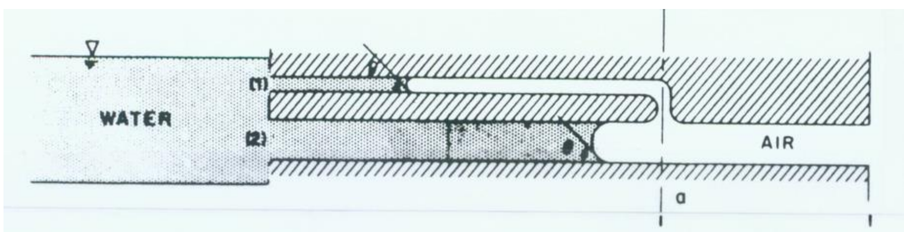
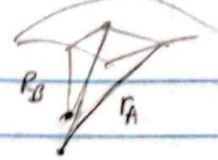


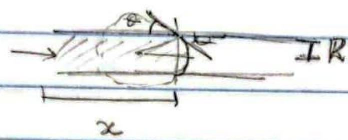
Figure 2

Q4 $P_c = \sigma\left(\frac{1}{r_A} + \frac{1}{r_B}\right)$



In this case: $r_A = r_B \rightarrow P_c = \frac{2\sigma}{r}$

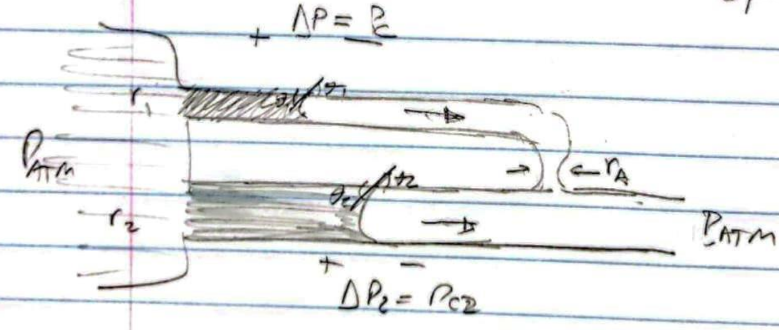
For a tube:



$P_c = \frac{2\sigma \cdot \cos\theta}{R}$

Hagen-Poiseuille's $q = \frac{\pi R^4}{8\mu} \frac{\Delta P}{L} = \frac{dx}{dt} \cancel{\pi} R^2$

$\frac{dx}{dt} = \frac{R^2}{8\mu} \frac{\Delta P}{x}$



IC.

$\boxed{x_1(0) = x_2(0) = 0}$

$\frac{dx_1}{dt} = \frac{r_1^2}{8\mu} \cdot \frac{2\sigma\cos\theta}{\cancel{r_1} \cdot x_1} \rightarrow x_1 dx_1 = \frac{r_1}{8\mu} 2\sigma\cos\theta\, dt$

$\frac{x_1^2}{\cancel{2}} = \frac{r_1}{\cancel{4}\mu_{2}} \cdot \sigma\cos\theta \cdot t + \cancel{c}^{\,0\ (IC)} \rightarrow x_1 = \sqrt{\frac{r_1}{2}\sigma\cos\theta \cdot t}$

The power can be applied to x_2; hence:

$$\boxed{\begin{aligned} x_1 &= \sqrt{\frac{r_1}{2}\,\sigma\cos\theta \cdot t} \\ x_2 &= \sqrt{\frac{r_2}{2}\,\sigma\cos\theta\, t} \end{aligned}}$$

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