

## Large-Scale Comments

1. Given that many courses using the book might not employ *every* chapter, it might be helpful to add a glossary of all notation defined throughout the book to make it easier to find anything not covered.
2. Throughout the first part of the book, lots of definitions are given with *iffs*. E.g., “Call an object  $x$  a \_\_\_\_\_ iff it satisfies properties \_\_\_\_\_.” This isn’t as consistently adhered to in the second half.

## Chapter 3

1. Notational inconsistency. Section 3.1, page 46: “there is a designated set  $U_x$  in  $\mathbf{X}$  with  $x \in U_x$  such that  $f(U_x) \subset V$ .” Here, the  $\mathbf{X}$  should be in math font, i.e.  $X$ .
2. Possibly undesirable notation. Section 3.2, definition of an open ball: “in  $\mathbb{R}^n$ , the open ball of radius  $\epsilon > 0$  around a point  $p \in \mathbb{R}^n$  is the set”

$$B(p, \epsilon) = \{x \in \mathbb{R}^n \mid d(p, x) < \epsilon\}$$

## Chapter 4

1. Inconsistent notation. Section 4.4, Theorem 4.30: “Let  $(X, \mathcal{F}_X)$  be a topological space, and  $(Y, T_Y)$  be a subspace.” Should be script/cal  $T_Y$ .

## Chapter 7

1. Incomplete definition. Section 8.5, definition of a cone. “Given a topological space  $X$ , consider the quotient space  $X \times [0, 1]$  such that all points  $(x, 0)$  are identified to a single point  $p$ .” The word “cone” is never mentioned explicitly here!
2. Undefined notation. Section 8.5, right after the definition of a cone:  $\mathbf{s}^1$  is used before it has been defined.
3. Possibly ungrammatical sentence: “It is called a lemma, because it first appeared in a paper in which Urysohn used it to prove a theorem about the existence of metrics on certain kinds of spaces.” Seems the comma shouldn’t be there.

## Chapter 11

1. Possibly ambiguous parse structure? Section 11.4, page 167: “Another category of theorem we will prove is fixed point theorems.” While this sentence is grammatically correct if parsed as “Another (category of (theorem we will prove)) is (fixed point theorems),” it is easy for a first-time reader to parse the sentence as “Another (category of theorem) we will prove is (fixed point theorems),” which I think has a number agreement error (“category of theorem” is singular, “is” is singular, “fixed point theorems” is plural — easy to not realize “fixed point theorems” is the title of the category). Not sure if this is actually a problem though.
2. Number agreement error. Section 11.4, page 167: “Another type of theorem that we will prove are theorems about geometric separation.” This is sort of the dual of the part above — “type of theorem that we will prove” is singular no matter how you parse it, “are” is plural.

## Chapter 12

1. Notational error. Exercise 12.1, page 170: “Show that the torus  $T^2$  is homeomorphic to  $\mathbb{S}^1 \times \mathbb{S}^1$ .” Torus should be denoted  $\mathbb{T}^2$ .
2. Possible error. Section 12.1, page 172 — “The basis for the topology is the collection of open cones with the cone point at the origin.” I believe this should be double cones?

## Chapter 15

1. Small typesetting inconsistency. Section 15.1, definition of a standard  $n$ -ball:

$$B^n = \{(x_1, \dots, x_n) \in \mathbb{R}^n \mid x_1^2 + \dots + x_n^2 \leq 1\}.$$

In the definition of the  $n$ -cube above this,  $\dots$  (`\cdots`) is used to indicate continuation of  $\times$ . Here, and in the definition of the standard  $n$ -sphere below it,  $\dots$  (`\ldots`) is used to indicate continuation of  $+$ . This may be a matter of personal preference, but I’ve always assumed standard practice is to use  $\dots$  (`\ldots`) for the continuation of enumerations (e.g., sets or lists), and  $\cdots$  (`\cdots`) for the continuation of operations. Either way, it probably ought to be consistent between  $+$  and  $\times$ .

2. Possible clarification suggestion. Section 15.1, definition of the standard  $n$ -sphere. We define

$$\mathbb{S}^n = \{(x_0, \dots, x_n) \in \mathbb{R}^{n+1} \mid x_0^2 + \dots + x_n^2 = 1\}.$$

Since we’re reindexing from 0 here (instead of adding an  $x_{n+1}$  coordinate), it might be worth pointing that out to the reader, especially seeing as both of the previous examples started their indexing at 1. This could easily be incorporated into the note below without adding too much text, however it’s non-essential.

3. Possible grammatical error. Section 15.1, definition of a manifold: “An  $n$ -dimensional manifold or  $n$ -manifold is a separable, metric space,  $M$  [...]”. It seems like one or more of these commas should not be there — in this context, isn’t “metric” usually used as an adjective? Either way, the comma usage is inconsistent with that in theorem 15.4 (“for a separable, metric space  $M^n$ , the following are equivalent”).
4. Notational inconsistency. Section 15.1. “For example, in the closed disk  $\mathbb{D}^2$ , [...]”. We defined  $\mathbb{D}^2$  to refer to the unit square in the section above. Should this be  $B^2$  instead?
5. Possibly incomplete definition. Section 15.2, definition of affinely independent. “recall that a set of points  $v_0, \dots, v_k$  in  $\mathbb{R}^n$  is *affinely independent* if  $\{v_1 - v_0, \dots, v_k - v_0\}$  is a linearly independent set.” As far as I can tell,  $v_0$  is not privileged, hence it should be mentioned that we require this to hold for all  $v_i$ ?
6. Possibly incomplete definition. Section 15.2, definition of convex combination. “A *convex combination* of  $v_0, \dots, v_k$  is a linear combination of those points whose coefficients sum to 1.” I believe it should also be mentioned that we require said coefficients to be non-negative.
7. Simple typo. Section 15.3, definition of PL. “A continuous map  $f : |K| \rightarrow |L|$  is called *piecewise linear* if and only **it**” — should be “if.”
8. Simple typo. Section 15.4, exercise 15.23: “Let  $K$  is a complex [...]” — should be “Let  $K$  be a complex.”
9. Inconsistent use of notation. Section 15.4, exercise 15.28: “If the simplex  $\sigma = \{v_0, \dots, v_n\}$  is the minimal face of [...]” — should be  $\{v_0 \cdots v_k\}$  (no commas)

## Chapter 16

1. Tricky to parse sentence, and also slight inconsistency in phrasing. Section 16.1, intuition: “Although not exactly accurate, a good way to start to understand homology for a space  $X$  is to view an  $n$ -manifold in  $X$  that is not the boundary of an  $(n+1)$ -manifold-with-boundary as capturing some geometry of  $X$  while an  $n$ -manifold that is the boundary of an  $(n+1)$ -dimensional manifold-with-boundary is not detecting any hole or structure.”

Seems like the two highlighted red regions should be phrased identically, to make the connection between them clear? Also, it might be better to split this up as follows: “Although not exactly accurate, a good way to start to understand homology for a space  $X$  is to consider an  $n$ -manifold  $M$ . If  $M$  is not the boundary of an  $(n+1)$ -manifold-with-boundary, then  $M$  can capture some geometry of  $X$ , while if  $M$  is the boundary of some  $(n+1)$ -manifold-with-boundary in  $X$ , then  $M$  doesn’t detect a hole or structure. The key here is to notice that boundary relationships between  $n$ -manifolds and  $(n+1)$ -manifolds in  $X$  can carry information about the presence of hollowness and/or holes.”

2. There’s a bit of a throwaway comment (in red) and also a typo (in blue) in the motivating example: “In fact, these two sets of edges differ (where we take the difference mod 2, meaning we just look at the set of all the edges that are in one set, but not it in the other) [...]” For a reader, it’s a bit strange seeing the mention of  $\mathbb{Z}/2\mathbb{Z}$  sort of shoehorned in like this. It’s great intuition, but it might be easier to digest if given in its own sentence instead of a parenthetical?
3. Suggested additional definition. Section 16.2: before defining an  $n$ -chain in  $K$ , it might be worth defining what a formal operation is?
4. Ambiguous diagram. Section 16.2, first example: in figure 16.2, edges and vertices are unlabeled, but we name them in the following example.
5. Typographical error: Section 16.2, definition of the boundary operator:

$$\partial(\sigma) = \sum_{i=0}^n \{v_0 \cdots \widehat{v_i} \cdots v_n\}$$

I believe that it should look like

$$\partial(\sigma) = \sum_{i=0}^n \{v_0 \cdots \widehat{v_i} \cdots v_n\}$$

which can be generated with the code `\cdots \widehat{v_i} \cdots`

6. Suggested additional clarification. In section 16.2, when we define  $n$ -cycles and  $n$ -boundaries, it might be worth explicitly stating that

$$Z_n(K) = \ker \partial_n \quad \text{and} \quad B_n(K) = \text{im } \partial_{n+1}$$

7. Possible typos. Theorem 16.7: “If  $K$  is a one-point space,  $H_n(K) \cong 0$  for  $n \geq 0$  and  $H_0(K) \cong \mathbb{Z}$ .” I believe the part in red should be  $n > 0$ , and the part in blue should be  $\mathbb{Z}_2$  (especially since the next theorem asserts that if  $K$  is connected, then  $H_0(K) \cong \mathbb{Z}_2$ ).
8. Suggested terminology clarification. Theorem 16.8: “If  $K$  is connected [...]” — it might be worth drawing a distinction between “simplicially connected” and “connected” (in the topological sense).
9. Possible typo. Section 16.2, definition of the simplicial cone operator: “Define the *simplicial cone operator*  $\text{Cone}_x : C_n(K) \rightarrow C_{n+1}(w * K)$  by extending the definition of  $\text{Cone}_x(\sigma)$  linearly to chains.” I believe this should be  $x * K$ ?
10. Possible typo. Section 16.2, exercise 16.19: “Suggest a homomorphism from  $C_n(K) \rightarrow C_n(\text{sd } K)$  that commutes with  $\partial$ . Could its induced homomorphism on homology be an inverse for  $\lambda_*$ ?” — seems like this should be “on the homology group?” Not sure though.

11. Inconsistent notation. Section 16.2, exercise 16.21: “Show that  $\lambda_{\#} \circ \text{SD} = \text{id}$ , the identity map on  $C_n(K)$ , and therefore  $\lambda_* \circ \text{SD}_* = \text{id}_*$ , the identity map on  $H_n(K)$ ” — these both should be in sans-serif font:  $C_n(K)$  and  $H_n(K)$ , respectively.

12. Undefined term. Section 16.4, Exercise 16.32: “Can a trivial cycle in  $A \cap B$  be non-trivial in  $A$ ?” This is the first time this term is introduced.

Also, as a general note, the wording in this question is a bit confusing. Are we looking at  $n$ -cycles? Or  $n$ -chains? Or are we treating elements of  $H_n(A \cap B)$ ? Furthermore: there are two somewhat-reasonable ways to parse the wording of this question. For the following, let  $\iota : A \cap B \hookrightarrow A$ , and let  $\pi : A \twoheadrightarrow A \cap A$ . In the question, we define some  $\sigma \in A \cap B$ , but it’s not completely clear based on the context whether we’re being asked to consider

- (1) Whether  $\iota(\sigma)$  can be non-trivial in  $A$ , or
  - (2) Whether there exists non-trivial  $\tau \in A$  such that  $\pi(\sigma) = \iota$ .
13. Simple typo. Section 16.3, Corollary 16.31: “If  $K$  is a strong deformation retract of  $L$ . Then  $K$  and  $L$  have isomorphic  $\mathbb{Z}/2\mathbb{Z}$  homologies.” Either the period should be a comma, or the “If” should be a “Let” or something.
  14. Ungrammatical sentence. Section 16.4, “A second observation starts with a cycle in  $A$ , and asks if, in  $K$ , that cycle in  $A$  is homologous to a cycle in  $B$ , and, if so, is there a cycle in their intersection that is homologous to both?”