

Large-Scale Comments

1. Given that many courses using the book might not employ *every* section, it would be extremely helpful to add a glossary of all notation used throughout the book to make it easier to find anything not covered.
2. Throughout the first part of the book, lots of definitions are given with *iffs*. E.g., “Call an object x a _____ iff it satisfies properties _____.” This isn’t as consistently adhered to in the second half.

Chapter 3

1. Notational inconsistency. Section 3.1, page 46: “there is a designated set U_x in \mathbf{X} with $x \in U_x$ such that $f(U_x) \subset V$.” Here, the \mathbf{X} should be in math font, i.e. X .
2. Possible undesirable formatting. Section 3.1, page 48: in the statement of theorem 3.1 (“Let $\{U_i\}_{i=1}^n$ be a finite collection of open sets”), “finite” is not italicized, whereas the surrounding text is. This is likely due to the use of `\emph` inside an italicized environment. **While this is indeed the expected behavior, it might be worth considering using boldface together with italics to emphasize finite in this context — however, it’s certainly a matter of personal preference.**
3. Possibly undesirable notation. Section 3.2, definition of an open ball: “in \mathbb{R}^n , the open ball of radius $\epsilon > 0$ around a point $p \in \mathbb{R}^n$ is the set”

$$B(p, \epsilon) = \{x \in \mathbb{R}^n \mid d(p, x) < \epsilon\}$$

Chapter 4

1. Inconsistent notation. Section 4.4, Theorem 4.30: “Let (X, \mathcal{F}_Y) be a topological space, and (Y, T_Y) be a subspace.” Should be script/cal T_Y .

Chapter 11

1. Possibly ambiguous parse structure. Section 11.4, page 167: “Another category of theorem we will prove is fixed point theorems.” While this sentence is grammatically correct if parsed as “Another (category of (theorem we will prove)) is (fixed point theorems),” **it is easy for a first-time reader to parse the sentence as “Another (category of theorem) we will prove is (fixed point theorems),” which I think has a number agreement error (“category of theorem” is singular, “is” is singular, “fixed point theorems” is plural — easy to not realize “fixed point theorems” is the title of the category).** Not sure if this is actually a problem though.
2. Number agreement error. Section 11.4, page 167: “Another type of theorem that we will prove **are** theorems about geometric separation.” This is sort of the dual of the part above — “type of theorem that we will prove” is singular no matter how you parse it, “are” is plural.

Chapter 12

1. Notational error. Exercise 12.1, page 170: “Show that the torus T^2 is homeomorphic to $\mathbf{s}^1 \times \mathbf{s}^1$.” Torus should be denoted \mathbb{T}^2 .
2. Possible error. Section 12.1, page 172 — “The basis for the topology is the collection of **open cones** with the cone point at the origin.” I believe this should be double cones?

Chapter 15

1. Small typesetting inconsistency. Section 15.1, definition of a standard n -ball:

$$B^n = \{(x_1, \dots, x_n) \in \mathbb{R}^n \mid x_1^2 + \dots + x_n^2 \leq 1\}.$$

In the definition of the n -cube above this, \dots (`\cdots`) is used to indicate continuation of \times . Here, and in the definition of the standard n -sphere below it, \dots (`\ldots`) is used to indicate continuation of $+$. This may be a matter of personal preference, but I've always assumed standard practice is to use \dots (`\ldots`) for the continuation of enumerations (e.g., sets or lists), and \dots (`\cdots`) for the continuation of operations. Either way, it probably ought to be consistent between $+$ and \times .

2. Possible clarification suggestion. Section 15.1, definition of the standard n -sphere. We define

$$S^n = \{(x_0, \dots, x_n) \in \mathbb{R}^{n+1} \mid x_0^2 + \dots + x_n^2 = 1\}.$$

Since we're reindexing from 0 here (instead of adding an x_{n+1} coordinate), it might be worth pointing that out to the reader, especially seeing as both of the previous examples started their indexing at 1. This could easily be incorporated into the note below without adding too much text, however it's non-essential.

3. Possible grammatical error. Section 15.1, definition of a manifold: “An n -dimensional manifold or n -manifold is a separable, metric space, M [...]”. It seems like one or more of these commas should not be there — in this context, isn't “metric” usually used as an adjective? Either way, the comma usage is inconsistent with that in theorem 15.4 (“for a separable, metric space M^n , the following are equivalent”).
4. Notational inconsistency. Section 15.1. “For example, in the closed disk \mathbb{D}^2 , [...]”. We defined \mathbb{D}^2 to refer to the unit square in the section above. Should this be B^2 instead?
5. Possibly incomplete definition. Section 15.2, definition of affinely independent. “recall that a set of points v_0, \dots, v_k in \mathbb{R}^n is *affinely independent* if $\{v_1 - v_0, \dots, v_k - v_0\}$ is a linearly independent set.” As far as I can tell, v_0 is not privileged, hence it should be mentioned that we require this to hold for all v_i ?
6. Possibly incomplete definition. Section 15.2, definition of convex combination. “A *convex combination* of v_0, \dots, v_k is a linear combination of those points whose coefficients sum to 1.” I believe it should also be mentioned that we require said coefficients to be non-negative.
7. Unclear definition. Section 15.2, definition of the underlying space of a simplicial complex. “[...] with a topology of sets whose intersection with each simplex σ in K is open in σ .” From it's unclear from this which topology we're supposed to use to determine if the intersection is open in σ — I'm assuming the standard topology on \mathbb{R}^n ?
8. Simple typo. Section 15.3, definition of PL. “A continuous map $f : |K| \rightarrow |L|$ is called *piecewise linear* if and only **it**” — should be “if.”

Chapter 16

1. Tricky to parse sentence, and also slight inconsistency in phrasing. Section 16.1, intuition: “Although not exactly accurate, a good way to start to understand homology for a space X is to view an n -manifold in X that is not the boundary of an $(n + 1)$ -manifold-with-boundary as capturing some geometry of X while an n -manifold that is the boundary of an $(n + 1)$ -dimensional manifold-with-boundary is not detecting any hole or structure.”

Seems like the two highlighted red regions should be phrased identically, to make the connection between them clear? Also, it might be better to split this up as follows: “Although not exactly

accurate, a good way to start to understand homology for a space X is to consider an n -manifold M . If M is not the boundary of an $(n+1)$ -manifold-with-boundary, then M can capture some geometry of X , while if M is the boundary of some $(n+1)$ -manifold-with-boundary in X , then M doesn't detect a hole or structure. The key here is to notice that boundary relationships between n -manifolds and $(n+1)$ -manifolds in X can carry information about the presence of hollowness and/or holes."

2. Not a big problem but the section does get a bit wordy in the motivating example with Figure 16.1. Maybe it'd be easier to follow with more diagrams? That said, the section as is still does a great job of delivering intuition to the reader.
3. There's a bit of a throwaway comment (in red) and also a typo (in blue) in the same example: "In fact, these two sets of edges differ (**where we take the difference mod 2, meaning** we just look at the set of all the edges that are in one set, but not **it** in the other) [...]" For a reader, it's a bit strange seeing the mention of $\mathbb{Z}/2\mathbb{Z}$ sort of shoehorned in like this. It's great intuition, but it might be easier to digest if given in its own sentence instead of a parenthetical?
4. Suggested additional definition. Section 16.2: before defining an n -chain in K , it might be worth defining what a formal operation is?
5. Suggested clarification: in the definition of an n -chain group, it seems we're actually defining is a module with formal linear combination (even though we still refer to $C_n(K)$ as a group)?
6. Ambiguous diagram. Section 16.2, first example: in figure 16.2, edges and vertices are unlabeled, but we name them in the following example.
7. Suggested additional clarification. In section 16.2, when we define n -cycles and n -boundaries, it might be worth explicitly stating that

$$Z_n(K) = \ker \partial_n \quad \text{and} \quad B_n(K) = \text{im } \partial_{n+1}$$

8. Possible typos. Theorem 16.7: "If K is a one-point space, $H_n(K) \cong 0$ for **$n \geq 0$** and **$H_0(K) \cong \mathbb{Z}$** ." I believe the part in red should be $n > 0$, and the part in blue should be \mathbb{Z}_2 (especially since the next theorem asserts that if K is connected, then $H_0(K) \cong \mathbb{Z}_2$. A one-point simplicial complex should certainly be connected!)
9. Suggested terminology clarification. Theorem 16.8: "If K is **connected** [...]" — it might be worth drawing a distinction between "simplicially connected" and "connected" (in the topological sense).
10. Possible typo. Section 16.2, definition of the simplicial cone operator: "Define the *simplicial cone operator* $\text{Cone}_x : C_n(K) \rightarrow C_{n+1}(w * K)$ by extending the definition of $\text{Cone}_x(\sigma)$ linearly to chains." I believe this should be **$x * K$** ?