
HOMOLOGY THEORY

NOTES & EXERCISES FROM MY INDEPENDENT STUDY

(OR: *If I could save Klein in a bottle ♪*)

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1. Introduction

This document is a compendium of notes, exercises, and other miscellany from my independent study in Homology Theory.

2. Chapter 12: Classification of 2-Manifolds

2.1 Examples of 2-Manifolds

12.1. Show that the torus \mathbb{T}^2 is homeomorphic to $\mathbb{S}^1 \times \mathbb{S}^1$

Proof. Let \sim be the relation on $[0, 1] \times [0, 1]$ defined as follows: $\forall x \in [0, 1]$,

$$\begin{cases} (0, x) \sim (1, x) \\ (x, 0) \sim (x, 1) \end{cases}$$

and \sim relates no other points. Then $\mathbb{T}^2 \cong ([0, 1] \times [0, 1] / \sim)$ ■