



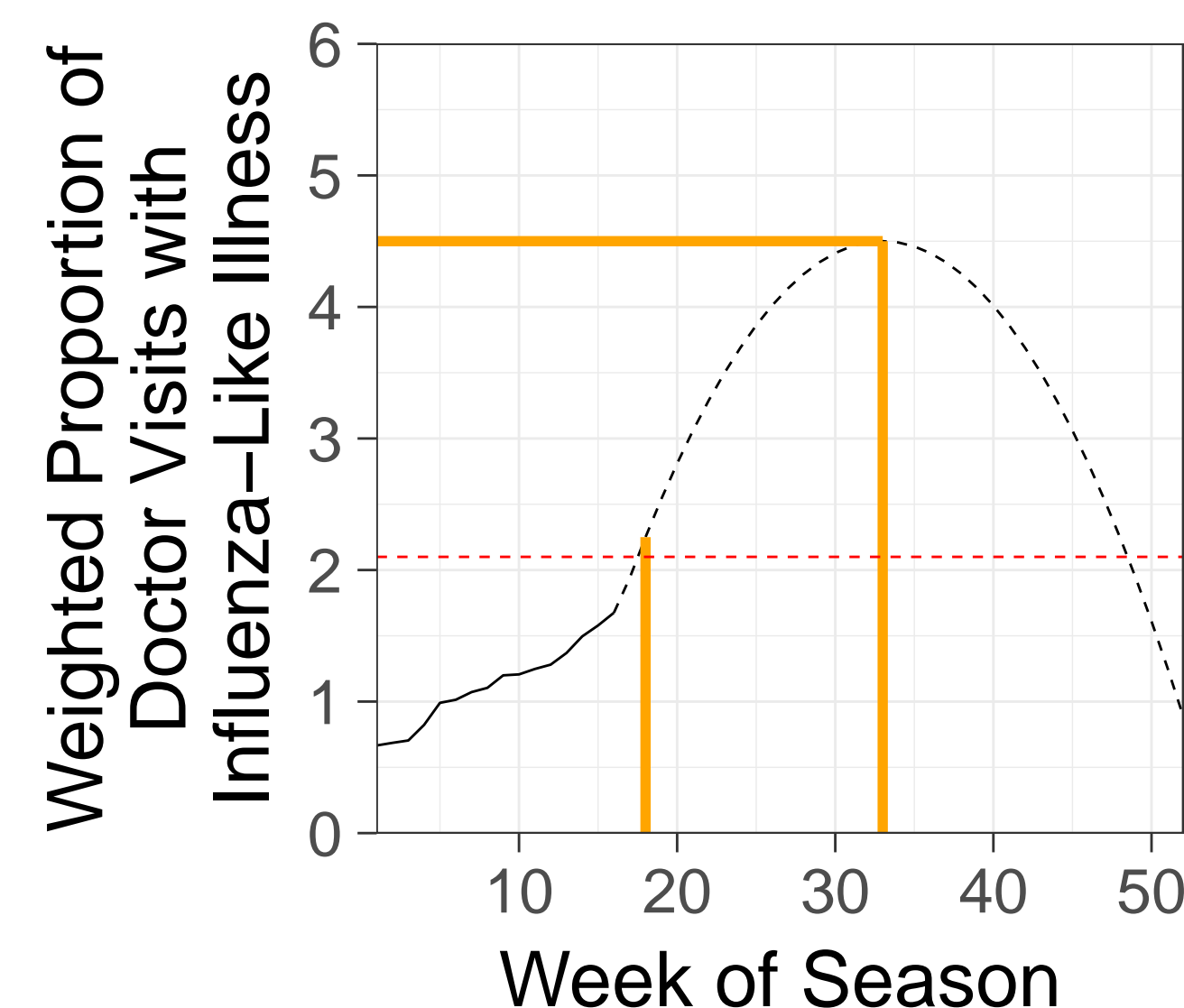
# Model Averaging for Probabilistic Time Series Forecasts

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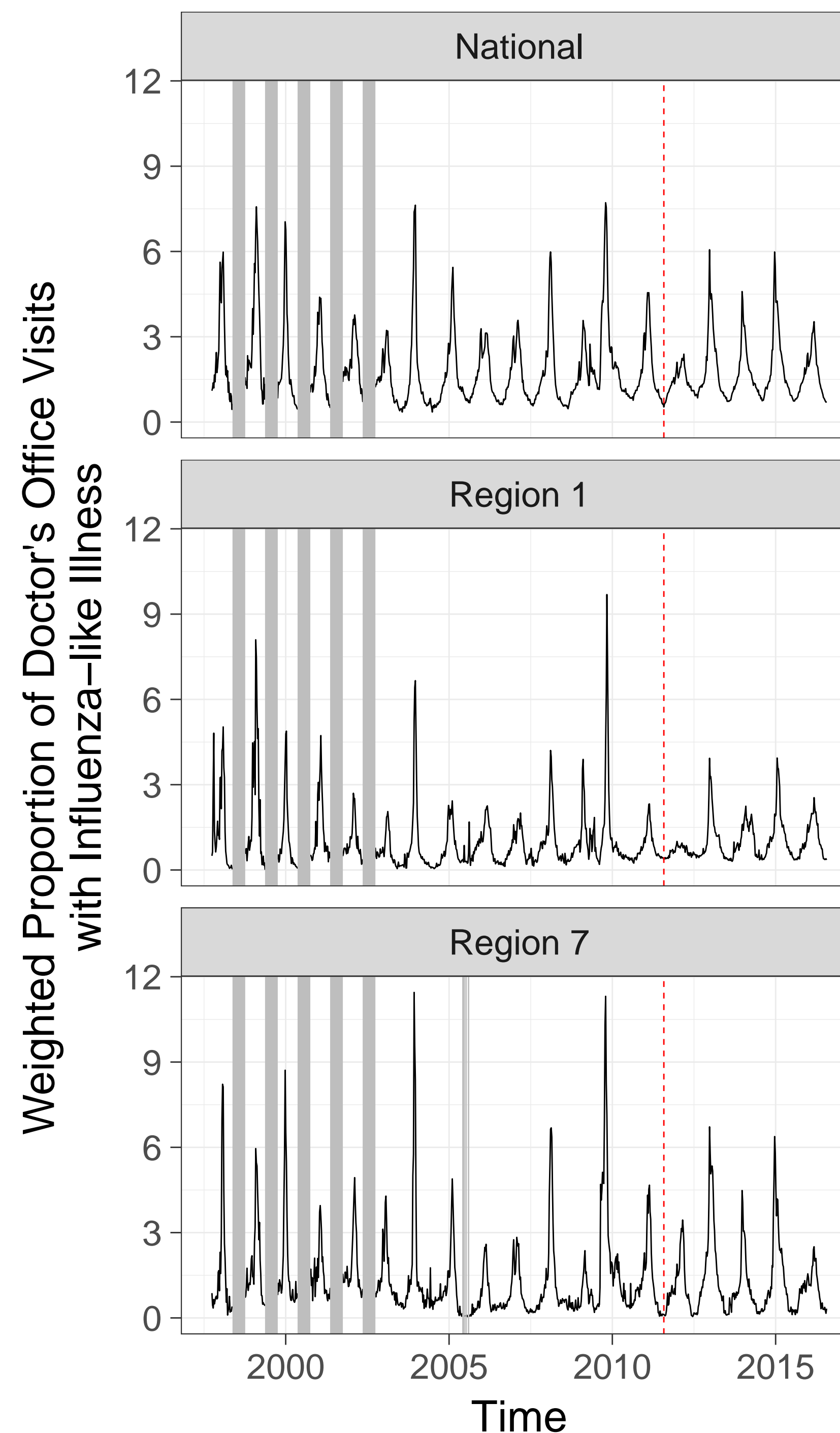
## Introduction

- Predictions of infectious disease are helpful to public health decision makers.
- We focus on three quantities:
  - 1 timing of season onset
  - 2 timing of season peak week
  - 3 incidence in season peak week



- Predictions updated weekly.
- Relative performance of different models varies.
- Can we combine predictions from these models to improve performance?

## Data



## Component Models

- Our ensembles combine predictions from three component models:
  - 1 Kernel Density Estimation (KDE)
    - separate distribution estimates for each target based on observed values in training-phase seasons
    - predictions do not change over the season
  - 2 Kernel Conditional Density Estimation (KCDE) with Copulas
    - KCDE: separate predictive distributions for flu incidence in each future week of the season given recent observations of wILI and the current week of the season.
    - Copula: model dependence among incidence in different weeks
    - Get predictive distributions for onset timing, peak timing, and peak incidence as integrals of joint distribution for incidence in all remaining weeks
  - 3 Seasonal Auto-Regressive Integrated Moving Average (SARIMA)
    - Log-transform wILI, first-order seasonal differencing
    - Integrate to obtain predictive distributions for onset timing, peak timing, and peak incidence as with KCDE

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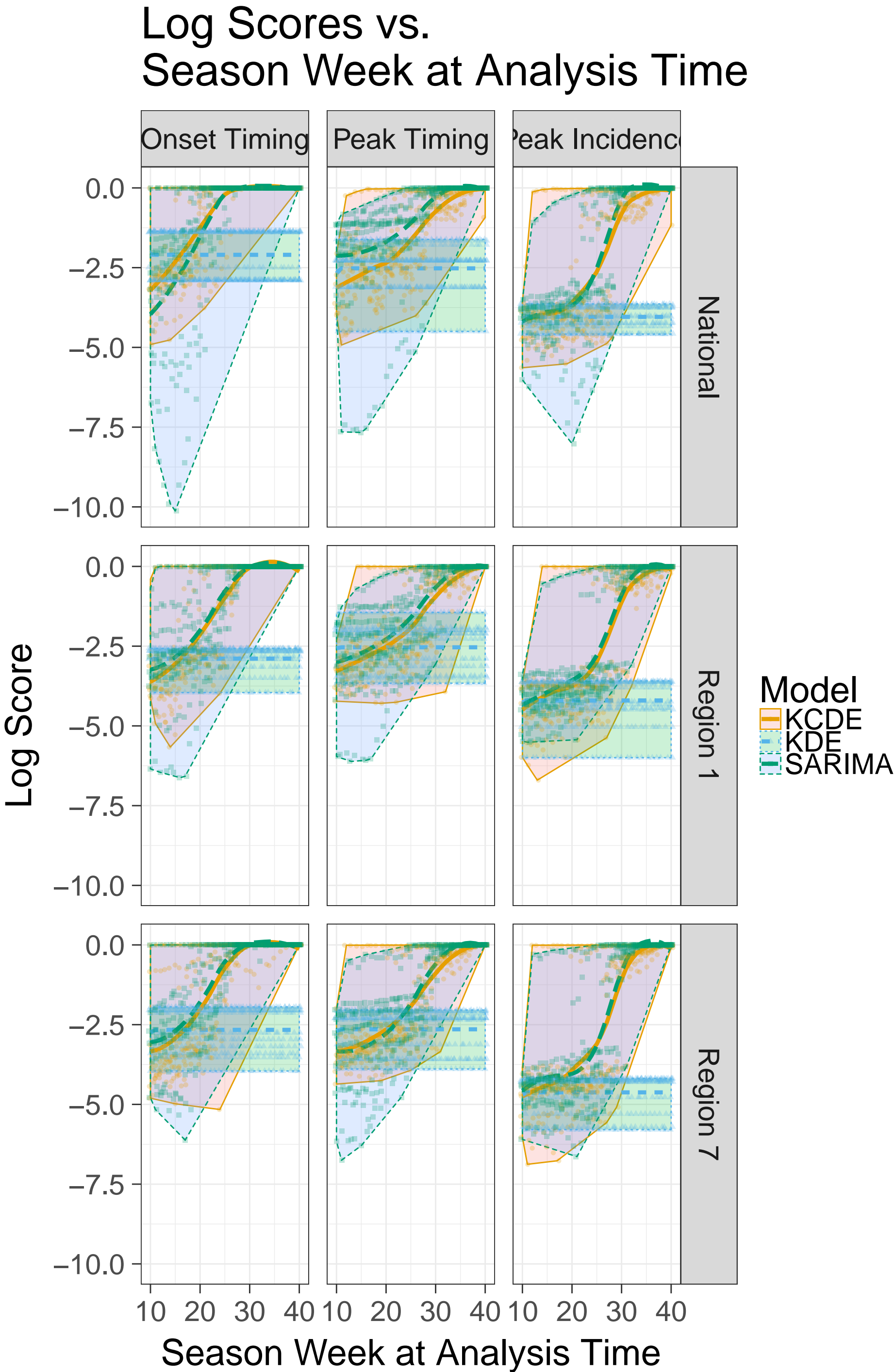




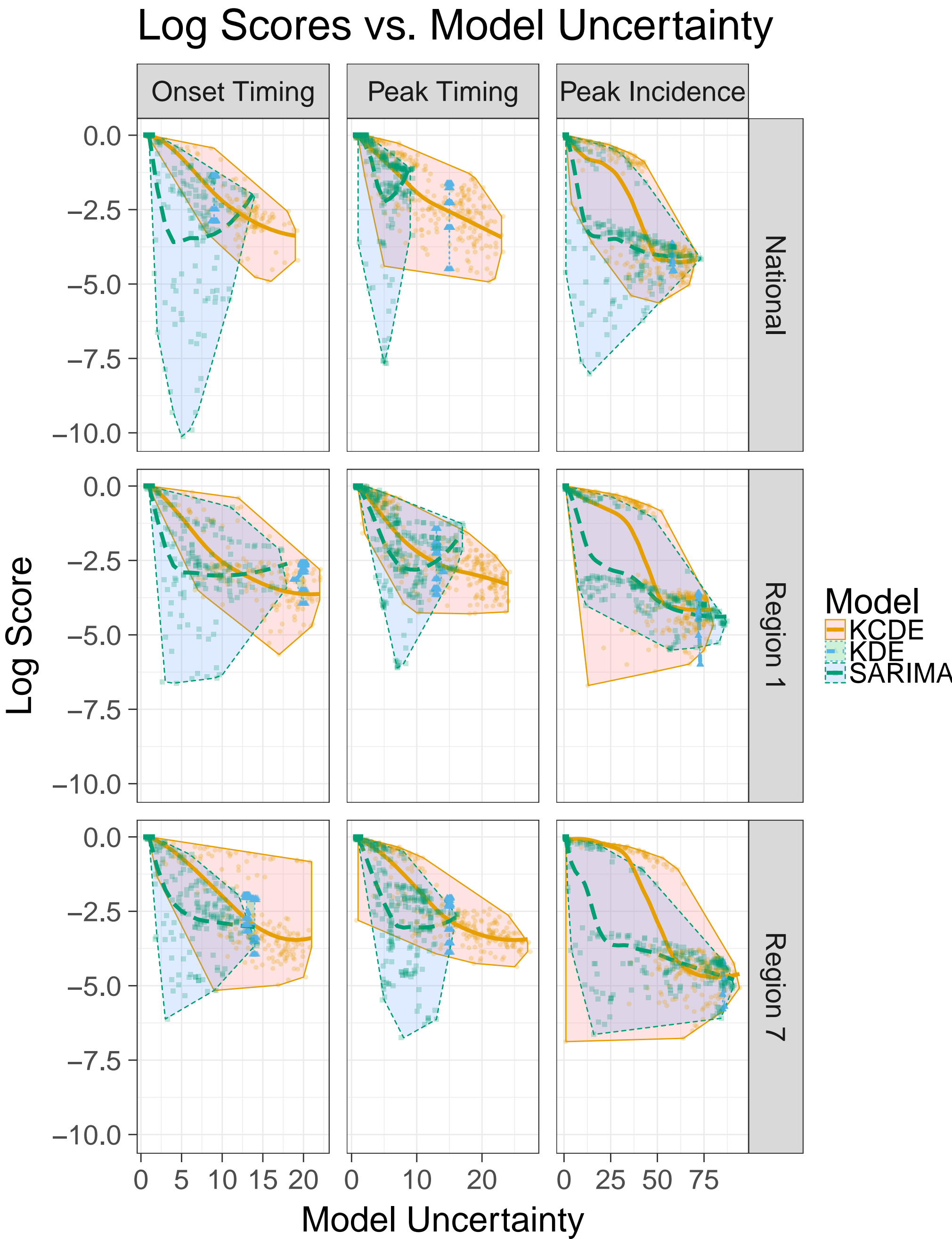
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## Performance Varies with Week



## Performance Varies with Uncertainty



## Ensemble Models

- The ensemble models we consider are *weighted averages* of the component models:

$$f(y|\mathbf{x}) = \sum_{m=1}^M \pi_m(\mathbf{x}) f_m(y|\mathbf{x}), \text{ where}$$

- $y$  is a possible value for one of the prediction targets
- $\mathbf{x}$  is a vector of covariates such as recent incidence, time of year at which we are making the predictions, ...
- $f_m(\cdot)$  are predictive distributions from  $M = 3$  component models
- $\pi_m(\mathbf{x})$  are model weights with  $\sum_{m=1}^M \pi_m(\mathbf{x}) = 1 \forall \mathbf{x}$
- We consider six variations:

Component Model Weights Vary with...						
Model	Region	Prediction Week of SARIMA    KDE    Current				
		Target	Season	Uncertainty	Uncertainty	wILI
EW						
CW	X	X				
FW	X	X	X	X	X	
FW-reg-w	X	X	X			
FW-reg-wu	X	X	X	X	X	
FW-reg-wui	X	X	X	X	X	X

- Weighting functions estimated via gradient tree boosting



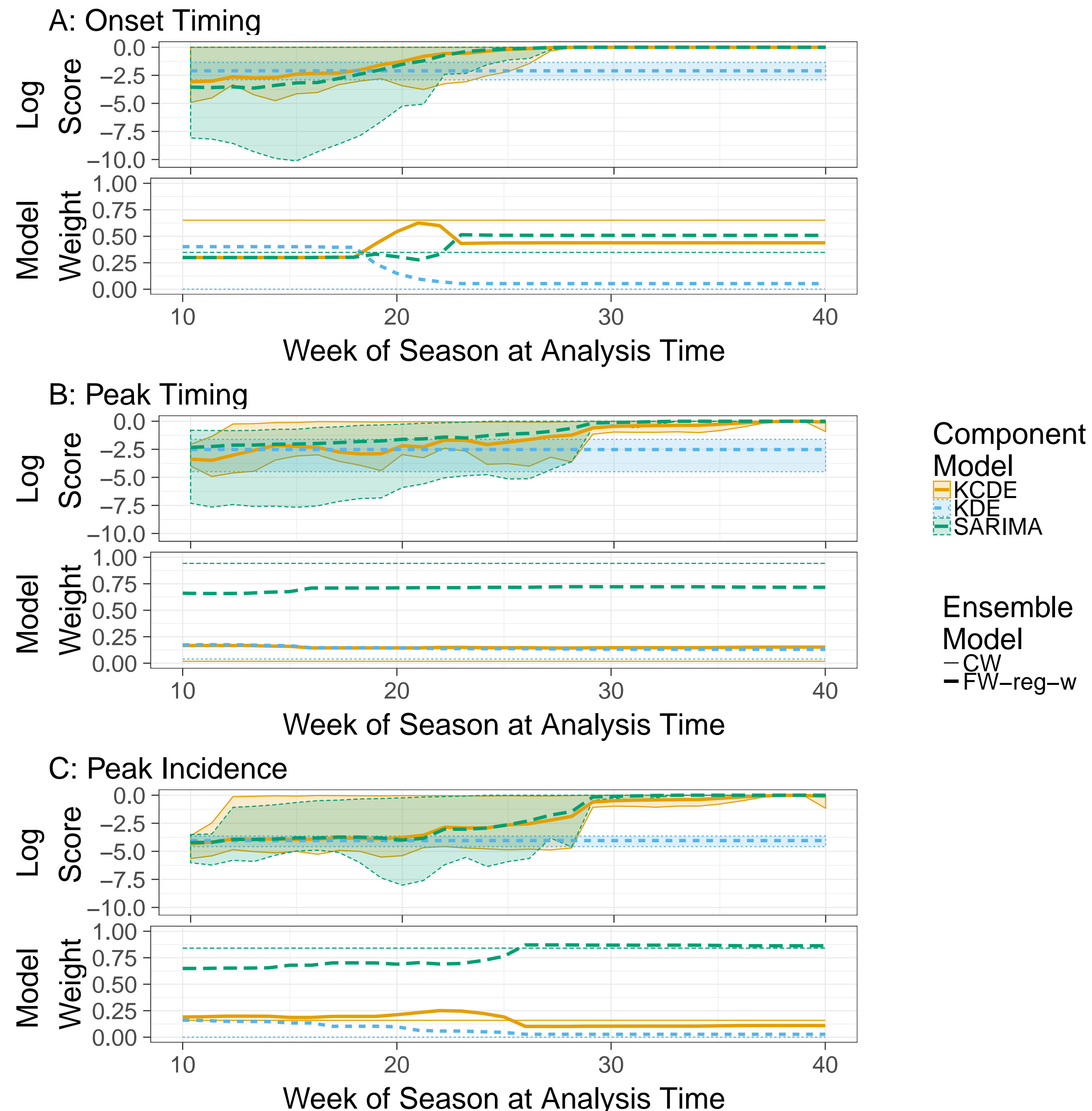


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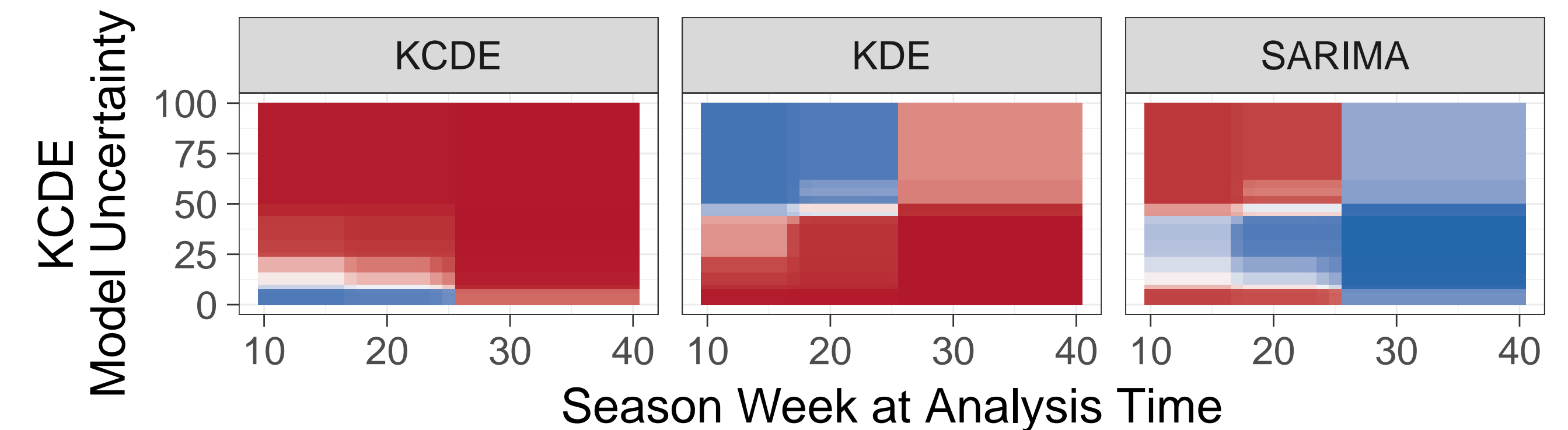
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## Weights as a Function of Week

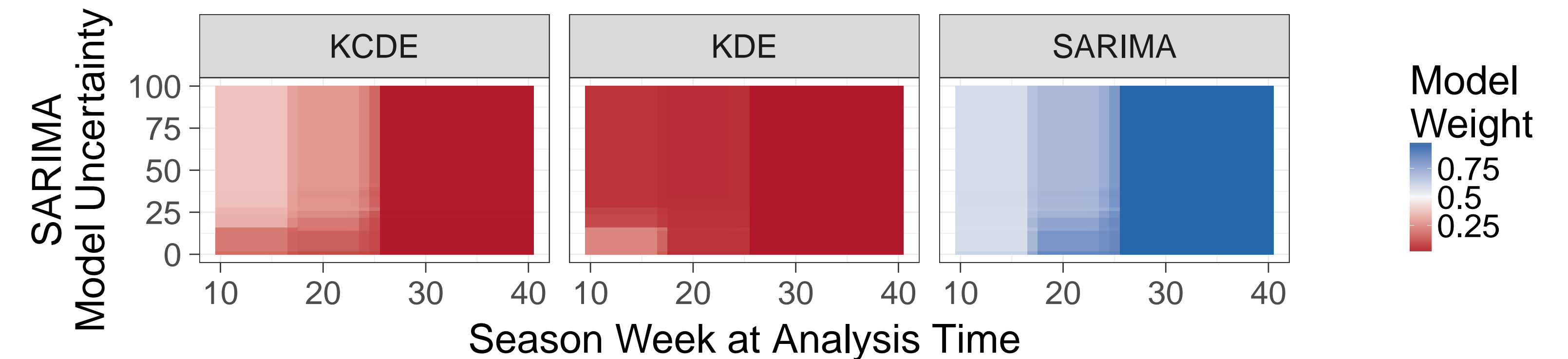


## Weights as a Function of Week & Uncertainty

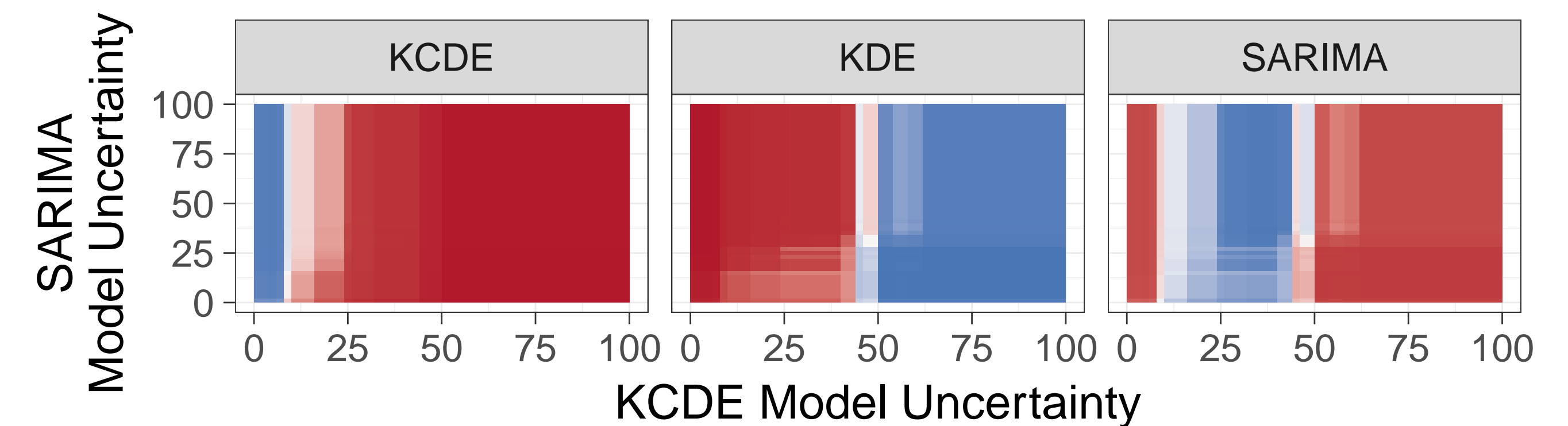
A: SARIMA Model Uncertainty Fixed at 20



B: KCDE Model Uncertainty Fixed at 20



C: Season Week Fixed at 17

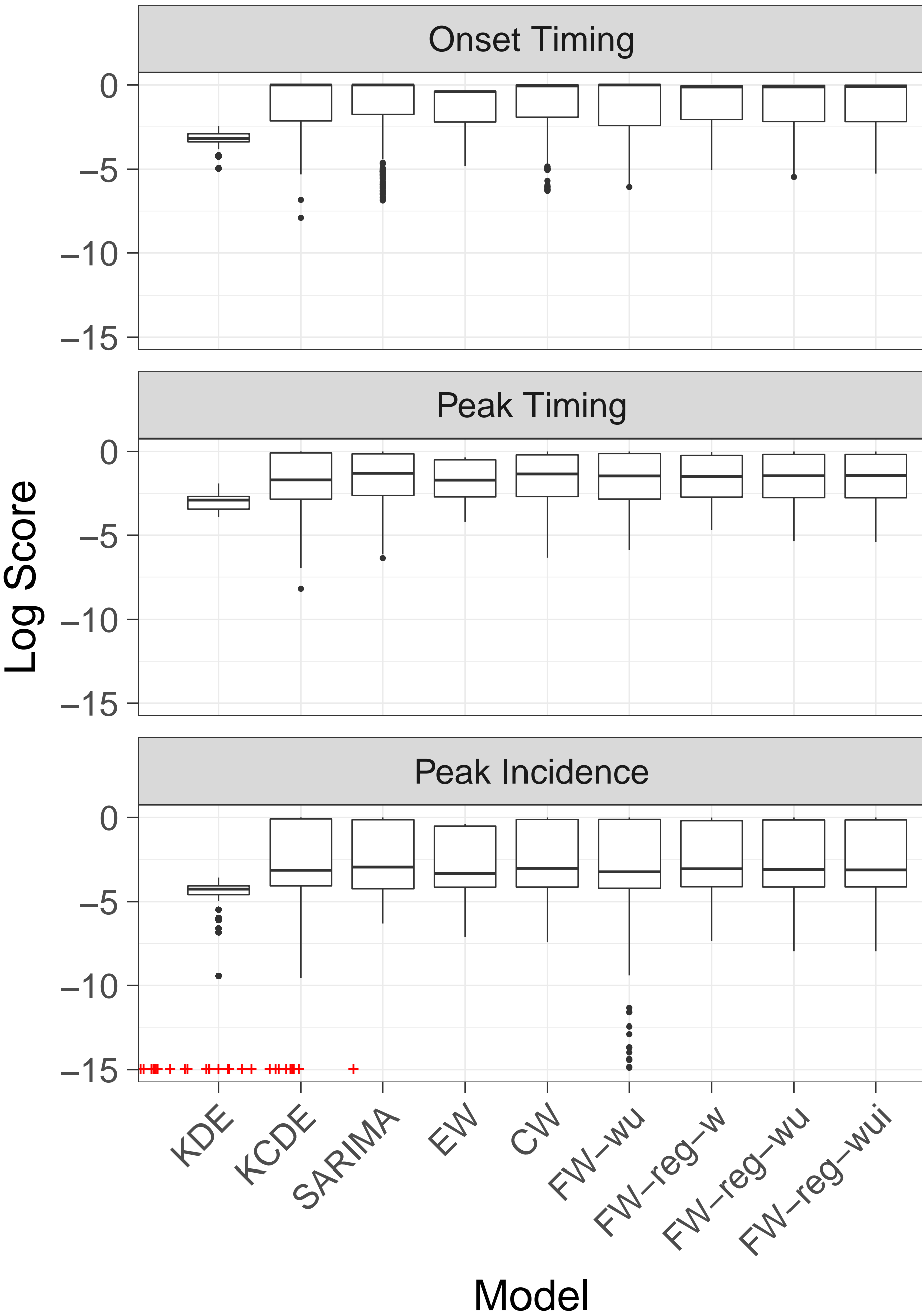




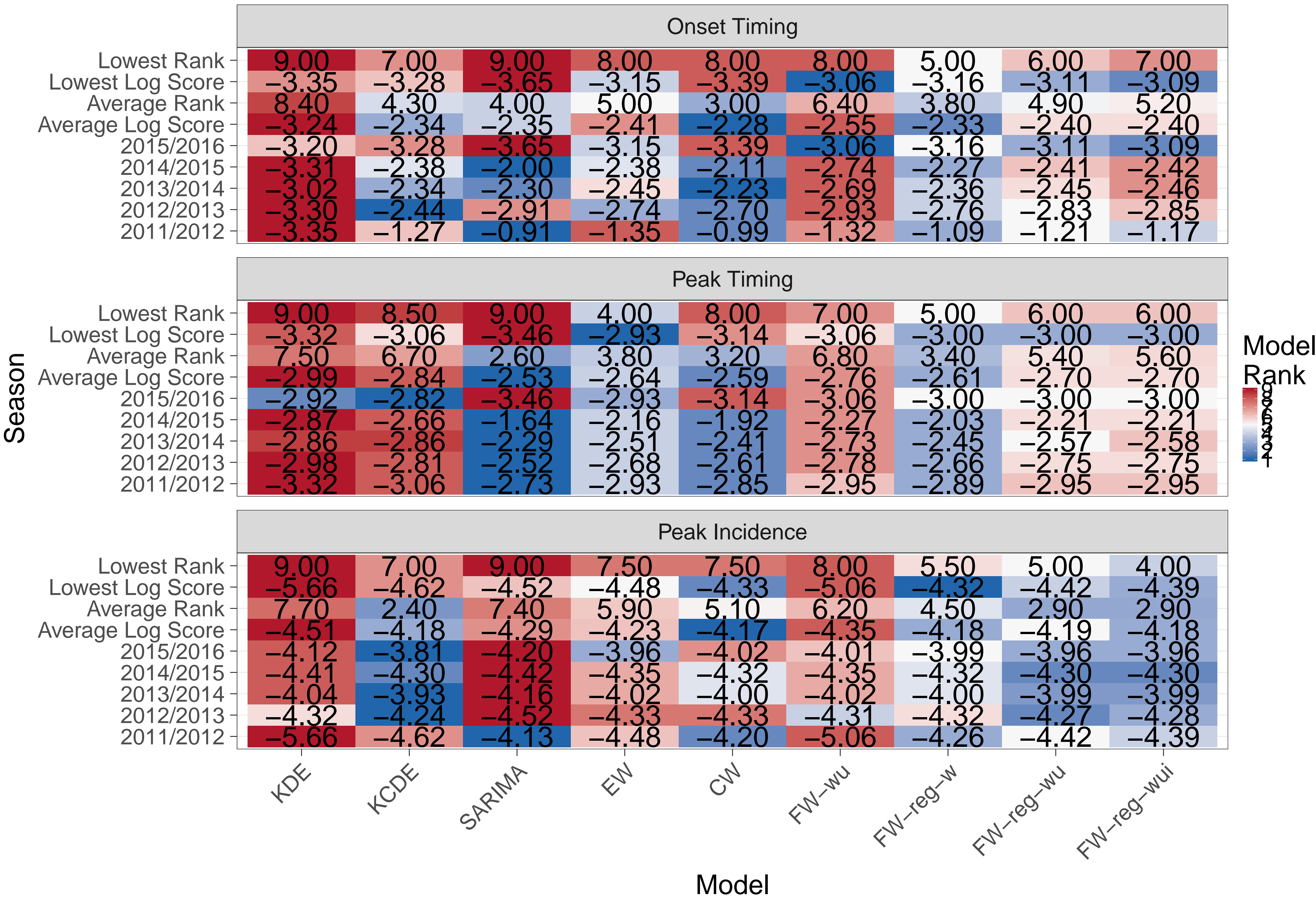
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## Aggregated Results – All Regions and Test Phase Seasons



## Results By Test Phase Season





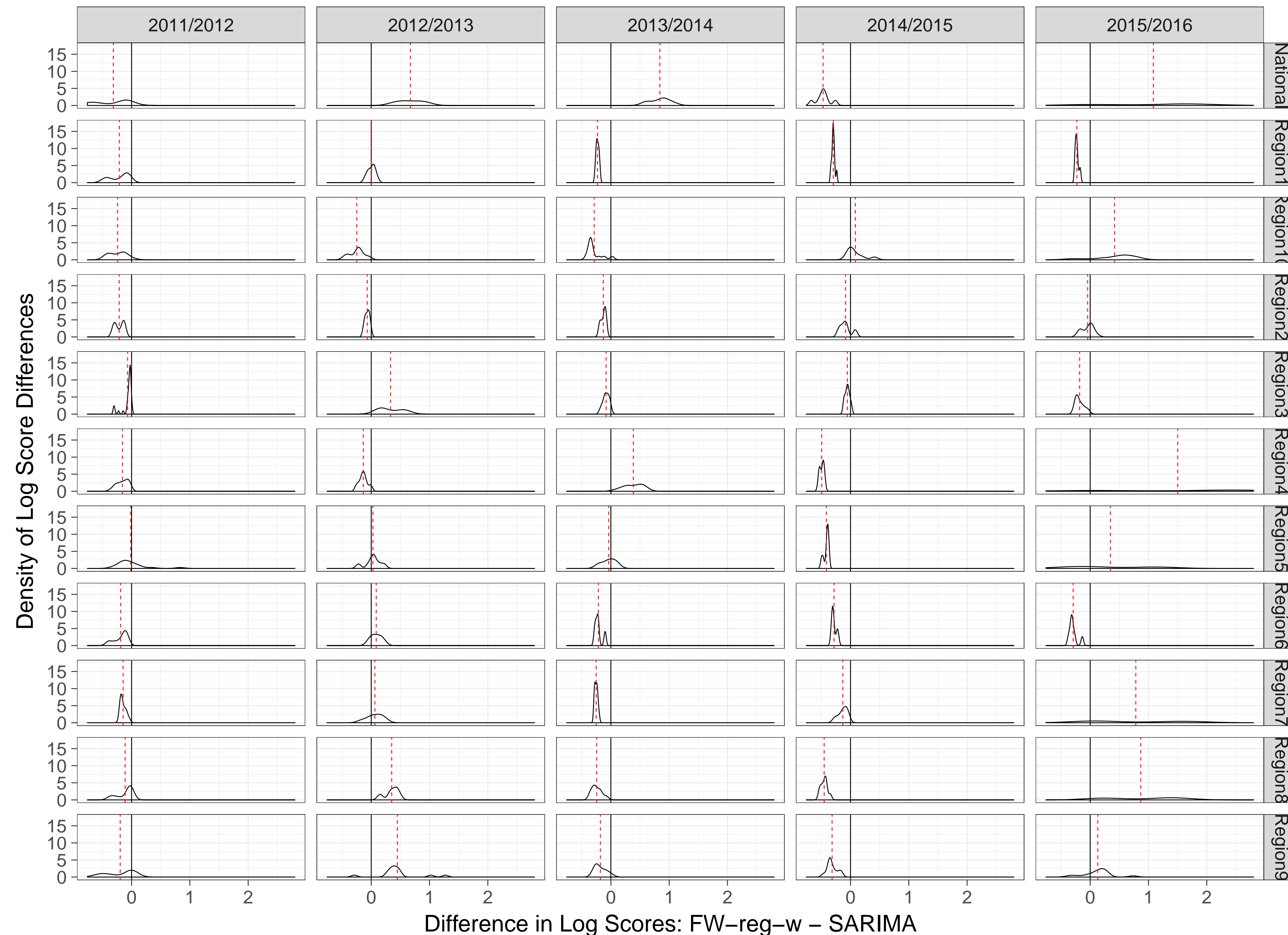


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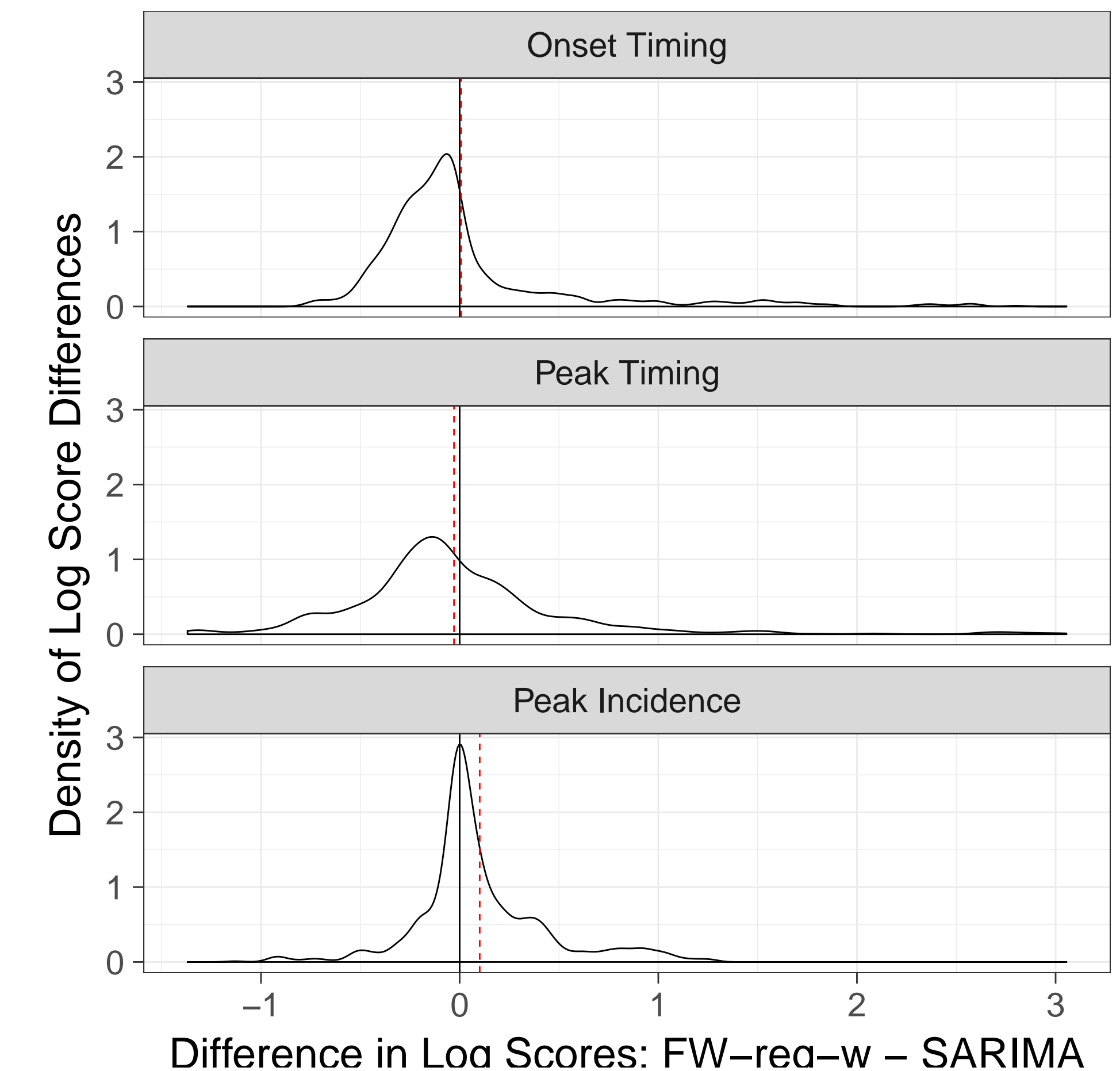
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## Log Score Differences for Onset Timing via SARIMA and FW-reg-w



## Log Score Differences for SARIMA and FW-reg-w



## Conclusions

- Ensemble methods had similar performance as the best of the component models in aggregate
- Ensemble methods had more stable performance across region-seasons than the component models
- In future work, would benefit from using a more diverse set of component models.