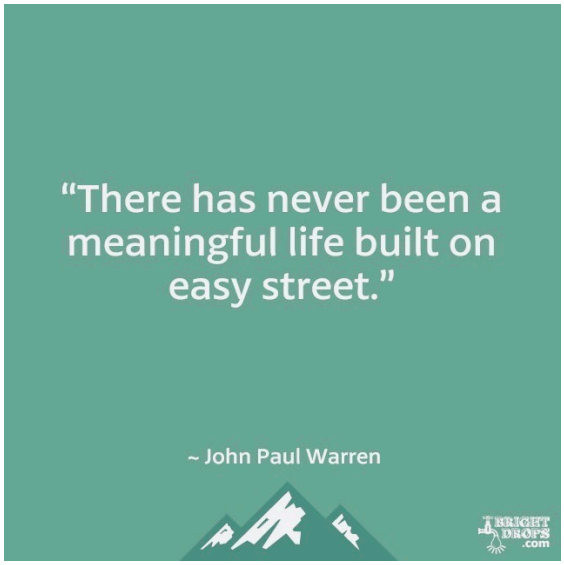
Introduction to Problem Solving



Good Evening

Enjoy the song

Yogesh , NSIT - 2018

Teaching experience → 2+

Work experience in scaler → 1 year

Session will start → 9:05 pm

Today's content →

01. Factors optimization
02. Check if no. is prime
03. Gauss sum
04. logarithmic basics
05. sqrt (n)

FAQ's

(a) Notes will be uploaded right after the session

(b) Assignments will be unlocked once the session ends.

(c) There is no deadline for assignments
↳ would highly appreciate if you guys do it on time. (Try to avoid backlogs)

(d) During doubt session, attendance is not counted
↳ recorded : Yes
↳ Notes : Yes

(e) Language Independent pseudocodes

Count no. of factors

01. Given N, return no. of factors of N?

↳ factor?

Is i a factor of N

↳ if i completely divides N

i is a factor of N if N % i == 0

Is 4 a factor of 24? 24 % 4 == 0

No. of factors

(a) 10 → {1, 2, 5, 10} → 4

(b) 24 → {1, 2, 3, 4, 6, 8, 12, 24} → 8

Idea 1 → Iterate from 1 to N & check every number, if it is completely dividing the N or not.

countfactors(N)

Ass: 10^8 iterations / sec

```
int c=0
for(i=1 : i≤N ; i++){
    // i is a factor of N
    if(N%i==0){c=c+1}
}
return c;
}
```

i : [1 N] ⇒ N iterations

$N = 10^9 \longrightarrow$ # iterations

10^9 iterations

10^8 iteration = 1 sec

1 iteration = $\frac{1}{10^8}$ sec

10^9 iterations = $\frac{1}{10^8} * 10^9$ sec

10^9 iteration = 10 sec

02. 10^{18} iterations

$$1 \text{ iteration} = \frac{1}{10^8} \text{ sec}$$

$$10^{18} \text{ iteration} = \frac{1}{10^8} * 10^{10} \text{ sec}$$

$$= 10^{10} \text{ sec}$$

10^{18} iterations $\approx$ 317 years

You → Child → Granchildren → 3rd gen → 4th gen → 5th gen

Optimization

$a \leq 50$

$\max a = 50$

01. $i * j = N$

(i & j are factors of N)

$j = N/i$

Claim :- If i is a factor of N,

N/i is also a factor of N

N = 24

i	N/i
1	24
2	12
3	8
4	6
6	4
8	3
12	2
24	1

Part A

Part B

Claim → we can get all the factors in Part A

Min	Max
$i = 1$	$i \le \frac{N}{i}$
	$i * i \le N$
	$i^2 \le N$
	$i^2 = N$
	$i = \sqrt{N}$

N = 36

i	N/i
1	36
2	18
3	12
4	9
6	6
9	4
12	3
18	2
36	1

Part A

Part B

```
countFactors(N)
    int c = 0
    for (i = 1 ; i <= √N ; i++) {
        // i is a factor,
        N/i is also a factor
        if (N % i == 0) {
            if (i == N/i) { c = c+1 };
            else { c = c+2 }
        }
    }
    return count;
}
```

$i : [1 \cdot \sqrt{N}]$

$= \sqrt{N}$ iterations

$i \le \sqrt{N}$

Or

$i * i \le N$

$N = 10^{18}$ ⟶ No. of iterations

$$\sqrt{N} = \sqrt{10^{18}} = 10^9$$

for 10^9 iterations ⟶ 10 sec

countfactors(N)
- Idea1 → 317 yrs
- Idea2 → 10 sec

Intermediate

↳ Increase observation skills

↳ Learn new technique

* If a no. is prime or not

Idea → Count the no. of factors

↳ if (c==2) → prime

else → not a prime

* Anirudh → Gauss

$$S = 1 + 2 + 3 + 4 + \dots + 98 + 99 + 100$$

$$S = 100 + 99 + 98 + 97 + \dots + 3 + 2 + 1$$

$$2S = 101 + 101 + 101 + 101 + \dots + 101 + 101$$

$$2S = 101 * 100$$

$$\boxed{S = \frac{101 * 100}{2}}$$

* Sum of first N natural number

$$S = 1 + 2 + 3 + 4 + \dots + (N-2) + (N-1) + N$$

$$S = (N) + (N-1) + (N-2) + \dots + 3 + 2 + 1$$

$$2S = (N+1) + (N+1) + (N+1) + \dots + (N+1) + (N+1)$$

$$2S = N * (N+1)$$

$$\boxed{S = \frac{N * (N+1)}{2}}$$

Perfect squares

01. Given a number N, find sqrt(N)

↓

Perfect square

N = 25 ⟶ 5

N = 36 ⟶ 6

N = 30 ⟶ {Invalid test case}

N = 49 ⟶ 7

```
int sqrt (N)
{
    for (int i=1 ; i ≤ N ; i++) {
        → if (i*i == N) {return i}
    }
}
```

return → breaks the function

Iterations ⟶ (a) ~~N~~ (b) N/2

(c) $\sqrt{N}$ (d) Good question

N = 36 → i: 1 2 3 4 5 6 {return 6} → 6 iteration

N = 25 → i: 1 2 3 4 5 {return 5} → 5 iteration

sqrt(N)

Note :- If N is not a perfect square, then return the floor of sqrt(N)

N = 49 ⟶ 7

N = 60 ⟶ 7.... → floor(7...) ⇓ 7

floor(x) ⟶ just smaller integer value than x

N = 31 ⟶ 5

N = 36 ⟶ 6

```
sqrt(N)
{
    int i=1, ans=0
    while (i*i <= N) {
        ans = i
        i = i+1
    }
    return ans;
}
```

N = 27

i	i*i ≤ 27	ans
1	1*1≤27	1
2	2*2≤27	2
3	3*3≤27	3
4	4*4≤27	4
5	5*5≤27	5
6	6*6≤27	✗

Ans = 5

N = 9

i	i*i ≤ n	ans
1	1*1≤9	1
2	2*2≤9	2
3	3*3≤9	3
4	4*4≤9	✗

10:29 → 10:38 pm

Log Basics

$\log_b^a$ = To what extent, we need to raise b to get a

= To what power, we need to raise b to get a

$$\log_b a = c \leftrightarrow b^c = a$$

01. $\log_2 8 = 2^c = 8$

$$2^c = 2^3$$

$$\boxed{c = 3}$$

02 $\log_5^{25} \Rightarrow 5^c = 25$

$$5^c = 5^2$$

$$\boxed{c = 2}$$

03. $\log_2 32 \longrightarrow 2^c = 32$

$2^c = 2^5$

$c = 5$

04. $\log_2 20$ (a = 20, b = 2)

$2^4 \rightarrow 16$

$2^5 \rightarrow 32$

$\longrightarrow c \simeq 4$

05 $\log_2 2^{10}$ ($\leftarrow a$, $\leftarrow b$) $= 2^c = 2^{10}$

$c = 10$

$\log_a a^n = n$

06. $N = 2^k$, value of k ?

$a = b^c \longrightarrow \log_b a = c$

$\log_2 N = k$

Expectations

01. Attend sessions

02. Revise notes

03. Solve Assignment & Homework

- Mostly discussed in class
- Indirect topics/questions covered in class

04. Doubts

- Content
  - (a) Please ask in live classes
  - (b) Stay back & get it clarified in doubt session
- Assignments
  - → Debug by yourself
  - → Ask Google / ChatGPT
  - → Ask your peers
  - → Raise TA request
  - → Ask me that question in next class DS

05. Join on time (Try to join by 9:05 pm)

06. If you miss
  - → Watch recording
  - → Atleast revise note