

SUBARRAY

"Success is the sum of small efforts, repeated day in and day out."

~ Robert Collier



A very good
Evening to
Everyone ☺

Today's Content

01. Subarray basics
02. Printing all subarrays
03. Printing all subarrays sums
04. Return total sum of all subarrays sums

↳ Google
=====

Subarray

→ Continuous part of an array is called a subarray

01. Single element is also a subarray

02. Whole array is also a subarray

03. Subarray is considered from left to right

$\text{arr}[9] = \begin{bmatrix} 4 & 1 & 2 & 3 & -1 & 6 & 9 & 8 & 12 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{bmatrix}$

Check if below subarrays are correct or not

Subarrays Indices
 $\{2, 3, -1, 6\}$ → Yes $[2 \ 5]$

$\{2 \ 9\}$ → No

$\{10\}$ → No

$\{9\}$ → Yes

$\{8, 12\}$ → Yes

Properties of subarray

01 Given st index & end index of subarr

$$\text{len} = \text{end} - \text{st} + 1$$

02. $\text{arr} = \{ 4, 2, 10, 3, 12, -2, 5 \}$
 0 1 2 3 4 5 6

subarrays starting with index 1

$$(1, 1) = \{ 2 \}$$

$$(1, 5) = \{ 2, 10, 3, 12, -2 \}$$

$$(1, 2) = \{ 2, 10 \}$$

$$(1, 6) = \{ 2, 10, 3, 12, -2, 5 \}$$

$$(1, 3) = \{ 2, 10, 3 \}$$

Ans # 6

$$(1, 4) = \{ 2, 10, 3, 12 \}$$

03. $\text{arr} = \{ 4, 2, 10, 3, 12, -2, 5 \}$
 0 1 2 3 4 5 6

subarr starting from index 3

$$(3, 3) = \{ 3 \}$$

$$(3, 6) = \{ 3, 12, -2, 5 \}$$

$$(3, 4) = \{ 3, 12 \}$$

Ans # 4

$$(3, 5) = \{ 3, 12, -2 \}$$

04

$$\text{arr} = \{ 4, 2, 10, 3 \}$$

0 1 2 3

$st=0$	$st=1$	$st=2$	$st=3$
$(0,0)$	$(\cancel{1},0)$	$(2,2)$	$(3,3)$
$(0,1)$	$(1,1)$	$\underbrace{(2,3)}_1$	
$(0,2)$	$(1,2)$	$\underbrace{2}$	
$(0,3)$	$(1,3)$		
$\underbrace{4}$	$\underbrace{3}$		

$$\begin{aligned} \text{Total no. of subarrays} &= 4 + 3 + 2 + 1 \\ &= 10 \end{aligned}$$

Total no. of subarrays of (array of size n)

$$= \frac{n * (n+1)}{2}$$

Q1. Given an $\text{arr}[n]$, s, e , print subarr from $[s \ e]$
 $s \rightarrow$ start point
 $e \rightarrow$ end point

$\text{arr} = \{ 4, 2, 10, 3, 12, -2, 5 \}$ $s=3$
 0 1 2 3 4 5 6 $e=6$

$\text{arr} = \{ 4, 2, 10, 3, 12, -2, 5 \}$ $s=2$
 0 1 2 3 4 5 6 $e=5$

Idea \rightarrow Iterate from s to e & print each & every element

void print (arr[], s, e)

 for (int i=s ; i<=e ; i++)

 TC = O(n)

 print (arr[i]);

SC = O(1)

}

}

Q2. Given $ar[N]$, print all subarrays

Note:- Print each subarr in new line

Constraints :- $1 \leq N \leq 10^2$

$1 \leq A[i] \leq 10^5$

Eg:- $ar[4] : \{ 2 \underset{0}{\textcircled{}} \underset{1}{\textcircled{}} \underset{2}{\textcircled{}} \underset{3}{\textcircled{}} 8 \underset{-1}{\textcircled{}} 4 \}$

$$st=0 \rightarrow (0,0) = \{2\}$$

$$(0,1) = \{2,8\}$$

$$(0,2) = \{2,8,-1\}$$

$$(0,3) = \{2,8,-1,4\}$$

$$\underline{st=1} \quad (1,1) = \{8\}$$

$$(1,2) = \{8,-1\}$$

$$(1,3) = \{8,-1,4\}$$

$$st=2 \quad (2,2) = \{-1\}$$

$$(2,3) = \{-1,4\}$$

$$(3,3) = \{4\}$$

Idea → To print a subarr, we need the st & end point

$\overset{\circ}{i}$ $\overset{\circ}{j}$	$\overset{\circ}{i}$ $\overset{\circ}{j}$	$\overset{\circ}{i}$ $\overset{\circ}{j}$	$\overset{\circ}{i}$ $\overset{\circ}{j}$
(0, 0)	(1, 1)	(2, 2)	(3, 3)
(0, 1)	(1, 2)	(2, 3)	
(0, 2)	(1, 3)		
(0, 3)			

$i \rightarrow 0 \text{ to } n-1$

$j \rightarrow i \text{ to } n-1$

void printallsubarr (arr[])

```
for ( i=0 ; i<n ; i++ ) { // i → st pt  
    for ( j=i ; j<n ; j++ ) { // j → end pt
```

st = i , end = j

for (k=st ; k≤end ; k++) {

print(arr[k]);

}

println();

}

}

$$TC = O(n^3)$$

$$SC = O(1)$$

Best TC

Total no. of subarrays = $\frac{n(n+1)}{2}$

TC to print 1 subarray = $O(n)$

Total TC = $\frac{n(n+1)}{2} * n$

$$\leq n^2 * n$$

$$\leq O(n^3)$$

Q2. Given N arr elements, print each subarr sum

Note :- Print each subarr sum in new line

Constraints :- $1 \leq N \leq 10^3$

$1 \leq arr[i] \leq 10^5$

arr [4] =

6	8	-1	7
0	1	2	3

subarr sum

02. Use prefix sum (Since calculating the sum for a given range)

`int [] psum = new int [n]; → TODO`

void prefsumsub (int [] arr)

O1. Construct psum array → {TODO}

for (i=0 ; i<n ; i++) { // i → st pt

 for (j=i ; j<n ; j++) { // j → end pt

 if (i==0) pointen (psum[j]);

 else pointen (psum[j] - psum[i-1]);

 }

}

TC : O(n²)

SC : O(n)

SC : O(1)

O3. Use carry forward approach

Q → Given an arr [N], print all subarrays

sums starting from index 3

arr [10] =

3	8	4	7	9	4	3	2	10	6
---	---	---	---	---	---	---	---	----	---

sum = 0

subarr

sum

(3, 3)	7	sum = 0 + 7
(3, 4)	16	sum = 7 + 9
(3, 5)	20	sum = 16 + 4
(3, 6)	23	sum = 20 + 3
(3, 7)	25	sum = 23 + 2
(3, 8)	35	sum = 25 + 10
(3, 9)	41	sum = 35 + 6

index = 3
 ↗

```
void printsum ( int arr, int k )
```

```

    int sum = 0
    for ( j = k ; j < n ; j++ )
        sum = sum + arr[j]
    print ( sum )
}
  
```

TC = O(n)
 SC = O(1)

$K=3 \rightarrow$ print all subarr sum starting from 3

$K=4 \rightarrow$ print all subarr sum starting from 4

$K=5 \rightarrow$ print all subarr sum starting from 5

$K=i \rightarrow$ print all subarr sum starting from i

* Back to original question

```
for (int i=0; i<n; i++) {
```

```
    int sum = 0
```

```
    for (j=2; j<n; j++)
```

```
        sum = sum + arr[j]
```

```
        print(sum);
```

3

$TC = O(n^2)$

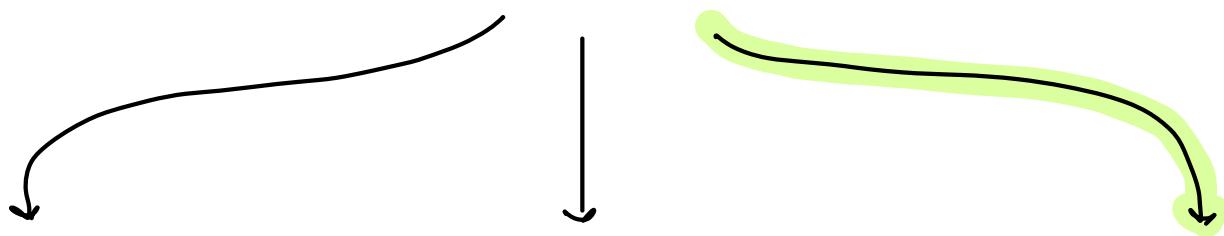
$SC = O(1)$

Q4 Given an arr[n] , return sum of all subarr sum .

$$\text{arr}[4] = \{ 6, 8, -1, 7 \} \longrightarrow 94$$

subarr	sum	
[0 0]	6	
[0 1]	14	Return sum of all
[0 2]	13	subarrays sums
[0 3]	20	
[1 1]	8	
[1 2]	7	
[1 3]	14	
[2 2]	-1	
[2 3]	6	
[3 3]	7	

Brute force \rightarrow For every subarray . get the sum & add it to the total sum



Approach 1

3 nested loops

$\rightarrow \text{TC} : O(n^3)$

$\rightarrow \text{SC} : O(1)$

Approach 2

Prefix sum

$\text{TC} : O(n^2)$

$\text{SC} : O(n)$

Approach 3

Carry forward

$\text{TC} : O(n^2)$

$\text{SC} : O(1)$

$$\text{totsum} = 0$$

```
for (int i=0; i<n; i++) {  
    int sum = 0  
    for (j=i; j<n; j++)  
        sum = sum + ar[j]  
    totsum = totsum + sum;  
}
```

$\text{TC} : O(n^2)$

$\text{SC} : O(1)$

TODO → { Don't take total sum, instead use
the sum outside for the loop }

2 | 3 | -1 → Wrong ans

Google → Constraint → TC: O(n)
SC: O(1)

Final Idea → { Contribution technique }

arr = { 3 -2 4 -1 2 6 }
 0 1 2 3 4 5

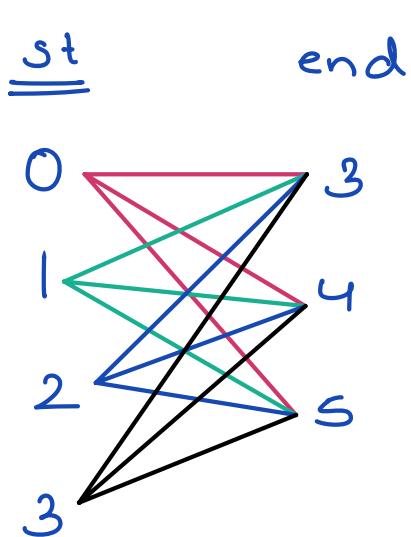
In how many subarrays, index 1 will
be present?

<u>st</u>	<u>end</u>
0	1
1	2
2	3
3	4
4	5

$$\text{Total no. of subarr} = 2 * 5 = 10$$

$$\text{arr} = \{ 3, -2, 4, \boxed{-1}, 2, 6 \}$$

In how many subarrays, index 3 will be present?



(0,3)	(1,3)	(2,3)	(3,3)
(0,4)	(1,4)	(2,4)	(3,4)
(0,5)	(1,5)	(2,5)	(3,5)

Total no. of subarrays

$$4 * 3 = 12$$

* Given $\text{arr}[n]$, how many subarr will contain the index i ?

$$\text{arr} = \{ a_0, a_1, a_2, \dots, a_{i-1}, \boxed{a_i}, a_{i+1}, \dots, a_{n-2}, a_{n-1} \}$$

st $[0-i]$ end $[i-n-1]$
 $\downarrow (i-0+1)$ $\downarrow (n-1-i+1)$
 $\# (i+1)$ $(n-i)$

Total no. of subarray containing index i = $(i+1) * (n-i)$

$$\text{arr}[4] = \{6, 8, -1, 7\}$$

0 1 2 3

$$(i+1) = 1 2 3 4$$

$$(n-i) = \cancel{4} \quad \cancel{3} \quad \cancel{2} \quad \cancel{1}$$

Total no. of subarr = 4 6 6 4 → In 4 subarr,
of subarr 7 will be present

4 is the no. of times 6
is going to contribute

6 is the no. of times 8
is going to contribute

In 6 subarr -1, will be present

$\text{arr}[4] = \{ 6, 8, -1, 7 \}$

subarr

sum

$$[0 \ 0] \quad 6 \longrightarrow \{ 6 \} = 6$$

$$[0 \ 1] \quad 14 \longrightarrow \{ 6, 8 \} = 6 + 8$$

$$[0 \ 2] \quad 13 \longrightarrow \{ 6, 8, -1 \} = 6 + 8 + (-1)$$

$$[0 \ 3] \quad 20 \longrightarrow \{ 6, 8, -1, 7 \} = 6 + 8 + (-1) + 7$$

$$[1 \ 1] \quad 8 \longrightarrow \{ 8 \}$$

$$[1 \ 2] \quad 7 \longrightarrow \{ 8, -1 \}$$

$$[1 \ 3] \quad 14 \longrightarrow \{ 8, -1, 7 \}$$

$$[2 \ 2] \quad -1 \longrightarrow \{ -1 \}$$

$$[2 \ 3] \quad 6 \longrightarrow \{ -1, 7 \}$$

$$[3 \ 3] \quad 7 \longrightarrow \{ 7 \}$$

Total contribution of
each & every ele

no. of times an ele is contributed

$$\begin{aligned}
 &= 6 * 4 \\
 &+ 8 * 6 \\
 &+ -1 * 6 \\
 &+ 7 * 4
 \end{aligned}$$

94

```

int total sum = 0

for ( i=0 ; i<n ; i++ ) {
    int nt = ( i+1 ) * ( n-i );
    totsum += nt * ar[i];
}

```

$$TC = O(n)$$

$$SC = O(1)$$

Contribution technique \rightarrow Participation of each & every element

```
for ( i=0; i<n; i++ ) {  
    → int sum = i;  
    {≡ }  
}
```

3

$i = 0$
 $i = 1$

