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GPU-accelerated

Fractional Differencing for Time Series Stationarity

Ritchie Ng, Jie Fu, Tat-Seng Chua

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Links

Links

Github Repository:

https://github.com/ritchieng/fractional_differencing_gpu

Presentation:
https://www.researchgate.net/publication/335159299_GFD_GPU_Fractional_Differencing_for_Rapid_Large-scale_Stationarizing_of_Time_Series_Data_while_Minimizing_Memory_Loss

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INTRO

Author, Collaborators and Supporters

Main Author

Ritchie Ng

- Chief AI Officer and Portfolio Manager, ensemblecap.ai
- Deep Learning Research Scholar in NExT++, NUS School of Computing
- NVIDIA Deep Learning Institute Instructor

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Co-author

Jie Fu, Postdoc Quebec Artificial Intelligence Institute (Mila)

Tat-Seng Chua, KITHCT Chair Professor at the School of Computing, NUS

Teaching Assistants

Timothy, Quantitative Engineer Goldman Sachs

Si An, AI Pod, Temasek Holdings

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STATIONARITY

Common Approaches and Pitfalls

Achieving Stationarity

Common Approaches

Why

Typically we attempt to achieve some form of stationarity via a transformation on our time series.

How

Common methods include integer differencing. For example to attempt to make S&P 500 time series stationary, we may take the one day difference yielding daily returns.

Problem

However, integer differencing often removes too much memory in the time series. Often, we can achieve stationarity without losing too much memory via fractional differencing.

Achieving Stationarity

Common Approaches

S&P 500 Absolute Levels (Zero Differencing)

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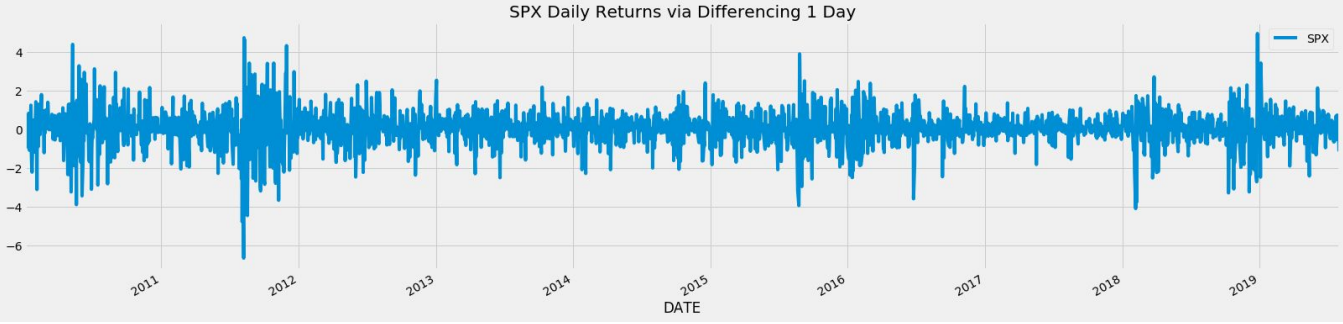
Achieving Stationarity

Common Approaches

S&P 500 Daily Returns (Integer Differencing, d=1)



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“Integer differencing unnecessarily removes too much memory while trying to make a time-series stationary. An alternative would be fractional differencing.

- Ritchie Ng

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STATIONARITY

Fractional Differencing to Achieve Maximum Memory with Stationarity

Achieving Stationarity

Fractional Differencing

Why

Fractional differencing allows us to achieve stationarity while maintaining the maximum amount of memory compared to integer differencing.

Where

This was originally introduced in 1981 in his paper "Fractional Differencing" by J. R. M. Hosking[1] and subsequent work by others concentrated on fast and efficient implementations for fractional differentiation for continuous stochastic processes.

Recently, fractional differencing was introduced for financial time series through the fixed window fractional differencing instead of the expanding window method by Marcos Lopez de Prado[2].

Expanding Window

Fractional Differencing

How?

Step 1: Calculating Weights Array

Essentially, independent of any time series, we can calculate the weights array via this iterative equation.

$$w_k = -w_{k-1}\frac{d-k+1}{k}$$

w: weight at lag k

k: lag

d: fractional differencing value where 0 implies no differencing and above 1 implies integer differencing

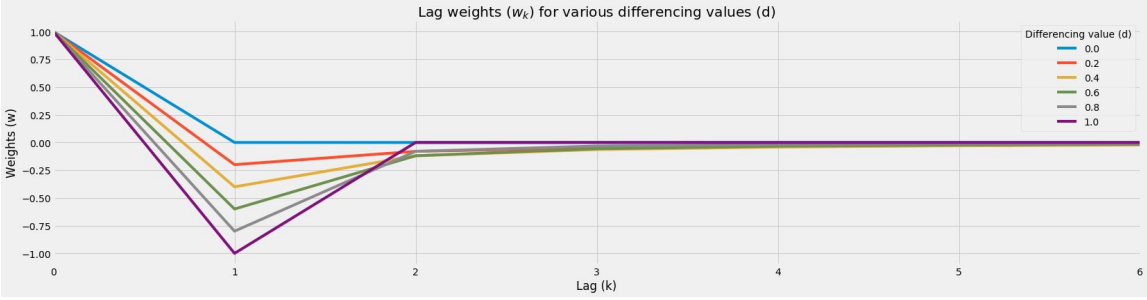
Expanding Window

Fractional Differencing

How?

Step 1: Calculating Weights Array

Essentially, independent of any time series, we can calculate the weights array via this iterative equation.



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Expanding Window

Fractional Differencing

How?

Step 1: Calculating Weights Array

Essentially, independent of any time series, we can calculate the weights array via this iterative equation.

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Expanding Window

Fractional Differencing

How?

Step 2: Rolling Dot Product of Weights Array and Time Series Array

When we take the dot product of the weights array and the time series array, we get a single value at lag k = 0. We do this for all lags k > 0, until we reach the beginning of the time series.

Problem?

Notice how this is very computationally expensive as we even take parts of the weights array for our dot product where the values are extremely small? And this window keeps expanding as we move further down the time series timeline. The alternative to this is the fixed-window fractional differencing method.

Fixed Window

Fractional Differencing

How?

Step 1: Calculating Weights Array with Threshold

Essentially, independent of any time series, we can calculate the weights array via this iterative equation and put a floor to stop calculating when the weights are too small.

$$w_k = -w_{k-1}\frac{d-k+1}{k}$$

$$w_k > \tau$$

w: weight at lag k

k: lag

d: fractional differencing value where 0 implies no differencing and above 1 implies integer differencing

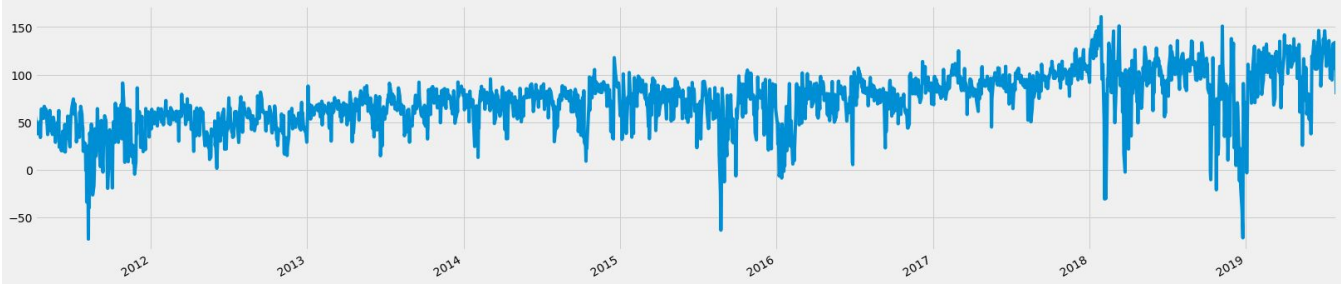
τ: the threshold to stop calculating

Fixed Window

Fractional Differencing

Example

Applying a fixed window fractional differencing on S&P 500, we get the following.



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Fixed Window

Fractional Differencing

ADF Tests: S&P 500 (2012-2019)

Comparing the three ADF test with constant order only included in the regression :

no differencing, integer differencing (d=1) and fractional differencing (d=0.5, τ = 5e-5).

	No Differencing	Integer Differencing	Fractional Differencing
t Statistic	-0.11	-11.12	-3.86
Critical Values	1%: -3.43 5%: -2.86 10%: -2.57		

Important: there are other ways to check for stationarity like Kwiatkowski–Phillips–Schmidt–Shin (KPSS) tests for trend stationarity, Phillips-Perron test for higher order autocorrelation and Augmented Dickey–Fuller test (ADF) with linear/quadratic trend order to include in the regression. But they are not covered as it is not the main point. The point is to show how we can minimize memory loss while reaching stationarity with fractional differencing.

Achieving Stationarity

Fractional Differencing

Derivation of Fractional Differencing Weights Formula

(refer to Hosking[1] paper)

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“Existing CPU-based implementations are inefficient for running fractional differencing on many large-scale time-series. GPU-based implementations provide an avenue to adapt to this century’s big data requirements.

- Ritchie Ng

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PERFORMANCE

GPU vs CPU Implementation

Fixed Window Fractional Differencing

Improvements

Improvement Indicators

- 6x-7x speed-up on a dataset of S&P 500 (2000-2019)

Fixed Window Fractional Differencing

Improvements

	Mean (seconds)	Standard Deviation (seconds)
Google Colab: 2x vCPUs	2.9105	0.0265
Google Colab: 1x T4 GPU	0.3704	0.0448
GCP 4x vCPUs, 15 GB RAM	2.0105	0.0551
GCP 8x vCPUs, 15 GB RAM	2.0774	0.0419
GCP 1x Tesla V100 GPU	0.3111	0.0080

Fixed Window Fractional Differencing

Improvements

	Speed-up
Speed-up Colab 1x T4 vs Colab 2vCPUs	7.8581x
Speed-up GCP 1x V100 vs GCP 4vCPUs	6.4636x
Speed-up GCP 1x V100 vs GCP 8vCPUs	6.6786x

References

References

Hosking, J. R. M. Fractional Differencing. Biometrika 68, no. 1 (1981): 165-76.

Marcos Lopez de Prado. 2018. Advances in Financial Machine Learning (1st ed.). Wiley Publishing.

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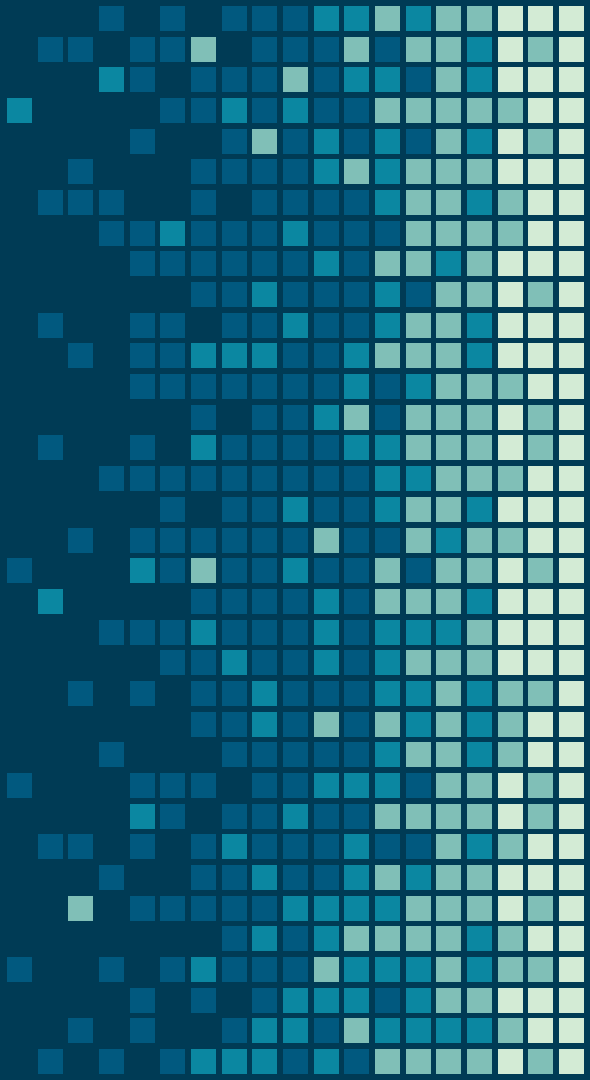
Corporate: ritchie@ensemblecap.ai

Academic: ritchieng@u.nus.edu

CONTACT



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