Analysis -1 advanced level, 09.06.2020

Reasoning and justification is needed to each question.

(1) Give an example for a real number set which is bounded below, but it has no minimum

(2) Is it possible that the sum of two divergent number sequences is convergent?

(3) Give an example for a number sequence (a_n), with $a_n > 0$ $(n \in \mathbb{N})$ such that

$$\exists \lim\left(\sqrt[n]{a_n}\right), \text{ but } \nexists \lim\left(\frac{a_{n+1}}{a_n}\right)$$

(4) Give a power series whose convergence set is the interval $[-1, 1)$.

(5) Prove that $\cos(i) \in \mathbb{R}$, $\cos(i) > 1$ (here $i = \sqrt{-1}$)

(6) Give an infinite set in $\mathbb{R}$, which has no interior point.

Theorem to be proved:

The Root Test + some examples for the indeterminate case