**CSC 5451, Professor E. Gethner**
**Assignment 0: All About Induction**
**18 August 2015**
**Quiz related to this homework in class on Thursday, 3 September 2015**

Please feel free to collaborate with one another on this assignment. Consider writing up the solutions on your own for quiz practice. **Important information regarding the quizzes: Be neat, write complete sentences, and show all of your work. The way you communicate the solution to your answer is as important as the answer itself.** Good luck!

1. Find a closed form formula for $\sum_{i=1}^{n} i^2$ and prove by induction that your formula is correct for all $n \in \mathbb{Z}^+$.

2. Prove by induction that $5^n + 3$ is divisible by 4 for all $n \in \mathbb{Z}^+$.

3. Derive a formula for the number of subsets of an $n$-element set and prove that your formula is correct (for all $n \in \mathbb{Z}^+$) by induction. For example, $\{1, a, \text{chocolate}, \pi\}$ is a 4 element set. Note also that the empty set (denoted by $\{\}$ or $\varnothing$) is a valid subset of any set.

4. **Second Principle of Mathematical Induction:**

   - Suppose $P(n_0)$ is true.

   - If for any integer $k \geq n_0$, $P(n_0)$, $P(n_0 + 1)$, $P(n_0 + 2), \ldots, P(k)$ are true imply that $P(k + 1)$ is true, then $P(n)$ is true for all $n \geq n_0$.

   Prove by the Second Principle of Mathematical Induction that any postage charges of greater than or equal to 5 cents can be made using only 2- and 5-cent stamps.

5. If $a$ and $r$ are real numbers (that is, $a, r \in \mathbb{R}$) such that $r \neq 1$, then prove by induction on $n$ that
$$a + ar + ar^2 + \cdots + ar^n = \frac{ar^{n+1} - a}{r - 1}.$$

   Note that the formula above is the summation formula for a finite geometric series with first term $a$, ratio $r$, and with $n + 1$ terms. We'll need this formula later in the semester.