CSC 5451, Professor E. Gethner
Assignment 4
21 October 2015
Quiz in class on Thursday 5 November 2015

Please feel free to collaborate with one another on this assignment. Consider writing up the solutions on your own for quiz practice. **Important information regarding the quizzes: Be neat, write complete sentences, and show all of your work. The way you communicate the solution to your answer is as important as the answer itself.**

1. **(Coloring Earth/Moon Graphs)** Let G be a thickness-two graph. Design an algorithm that properly colors the vertices of G with no more than 12 colors. You need not prove that your algorithm is correct, but an incorrect algorithm will receive either partial or no credit. Be sure to write your algorithm in readable and understandable pseudocode.

2. **(Dijsktra Thought)**

   (a) Give an example of a graph with negative edge weights for which Dijstra's algorithm fails. Show a trace of the algorithm on your graph.

   (b) Analyze the worst case runtime of Algorithm Dijkstra.

3. **(Kempe Chain Flaw)** Suppose that the planar graph G in Figure 1 has been run through a four coloring algorithm using the Kempe chain argument given in class (this was Theorem 3, the flawed proof of Kempe that "showed" that all planar graphs can be four colored). All vertices of G except for v have been magically colored. We are faced with coloring the final vertex v. Try to use Kempe chain switches to color v and explain in detail what goes wrong.

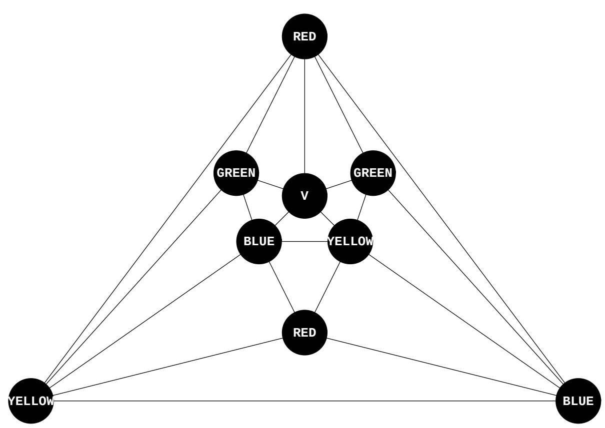

Figure 1: **Color v by using Kempe chain switches??**

4. **(Cycle Search)** Design an algorithm that takes as input a simple, undirected, connected graph G that outputs **TRUE** if G has a cycle with an *odd* number of vertices and **FALSE** otherwise. Prove that your algorithm is correct. Be sure to write your algorithm in readable and understandable pseudocode. What is the runtime of your algorithm? Hint: You may use the fact (without proof) that a graph is 2-colorable if and only if it contains no odd cycle.

5. **(Thickness)** Let G be a simple undirected graph with n vertices labeled 1,2,. . .,n. The graph $G[r_1, r_2, \ldots, r_n]$ is the graph obtained from G by replacing vertex i with K_{r_i} and connecting all possible vertices in neighboring complete graphs. So, for example, $G[1, 1, \ldots, 1] = G$, $K_2[2, 2] = K_4$, $K_2[m, n] = K_{mn}$, and $K_1[n] = K_n$.

   Implementation and Presentations (3 or 4 people in each group). In the problems below, let C_n denote a simple cycle on n vertices. Work the first five problems out below with pencil, paper, and friends and no other resources. Recall that the *thickness* of graph G is the smallest t for which the edges of G can be partitioned into t sets, each of which induces a planar graph; we then write $\theta(G) = t$. **The final three problems are to be implemented.** In particular, do an implementation (your choice of language, method, etc) to work out a partition of the edges into k sets, each of which induces a planar graph (and k is smallest possible) to find the thickness of the graphs in the final three parts of this problem. You should be able to think your way to all of the chromatic numbers in this question.

   Presentations will take place on Thursday, 12 November 2105

   (a) The *independence number* $\alpha(G)$ of a graph G is the size of a largest set of independent (mutually nonadjacent) vertices in G. Prove that $\chi(G) \geq \frac{|V(G)|}{\alpha(G)}$.

   (b) If G is a graph with n vertices, prove that $\alpha(G) = \alpha(G[r_1, r_2, \ldots, r_n])$, where each $r_i \in \mathbb{N}$.

   (c) Find both the chromatic number and thickness of $C_3[2, 2, 2]$ and prove that your answers are correct.

   (d) Find both the chromatic number and thickness of $C_5[2, 2, 2, 2, 2]$ and prove that your answers are correct.

   (e) Find both the chromatic number and thickness of $C_n[2, 2, \ldots, 2]$ and prove that your answers are correct.

   (f) Find both the chromatic number and thickness of $C_5[3, 3, 3, 3, 3]$ and prove that your answers are correct.

   (g) Find both the chromatic number and thickness of $C_5[4, 4, 4, 4, 3]$ and prove that your answers are correct.

   (h) Find both the chromatic number and thickness of $C_7[4, 4, 4, 4, 4, 4, 4]$ and prove that your answers are correct.