CSCI 5451 Fall 2015

Week 4 Notes

Professor Ellen Gethner

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Dynamic Programming: Second Big Example

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- **Set-up.** Let $A = a_1 a_2 a_3 \ldots a_n$ and let $B = b_1 b_2 b_3 \ldots b_m$.
- Assume the universe of characters is finite.
- We want to change A, character by character, to B.

Operations on Strings

- We'll allow the following *edit steps* (and one more later).

1. **Insert.** Insert one character in the string.
2. **Delete.** Delete one character from the string.
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- **Example.** Transform *abbc* into *babb*.

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- The cost of the above set of edit steps is 3.
- But were we efficient?
- No: *abbc* $\rightarrow$ *abb* $\rightarrow$ *babb* has a cost of 2, and is best possible. Why?

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- such as the *diff* file, which is based on today's (upcoming) algorithm.

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- $B(i) = b_1 b_2 \ldots b_i$ (i.e. $B(i)$ is the first i characters of string B).
- **Restatement of the Problem.** Change $A(n)$ to $B(m)$ with the minimum number of edit steps.

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- Then with one deletion, namely a_n from $A(n)$, we know how to convert $A(n)$ to $B(m)$.
- But the above idea may not be the best way! What else can happen?

Optimization ideas, continued

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- Let $C(i,j)$ be the minimum cost of changing $A(i)$ to $B(j)$.
- **Goal.** Find a relation between $C(n,m)$ and some of the $C(i,j)$'s for some smaller values of i and j.

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- **Case 1: Delete.** If a_n is deleted in a minimum change from A to B then $C(n, m) = C(n-1, m) + 1$.

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- **Case 4: Match.** If $a_n = b_m$ then $C(n,m) = C(n-1,m-1)$.

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- $C(0,j) = j$ for $j = 0, 1, \ldots, m$.
- Have we captured all possibilities for the edit steps?

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- If a_n is mapped to an earlier character, then this is Case 2.

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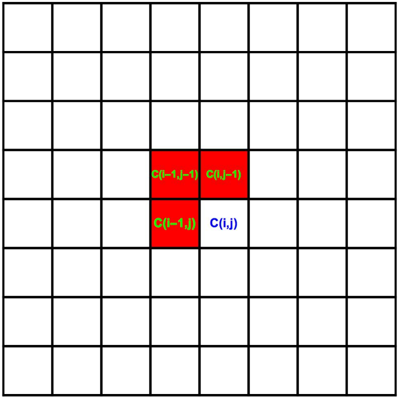
- we have reduced one problem of size n to three problems of similar size,
- so the faux algorithm is exponential.
- **Reality.** There are only $(n+1) \times (m+1)$ subproblems so we should keep track of information as we acquire it.
- To do so, we'll use an $n+1$ by $m+1$ array such that the (i, j)th entry contains information about $C(i, j)$.

Artist's interpretation of the array



$C(i,j)$ is computed from information in the red area and in an implementation you maintain an array C such that $C[i,j]$ contains information about the problem $C(i,j)$.

Algorithm MinEdDis(A, n, B, m)

- **Input** A (string of length n) and B (string of length m)
- **Output** C, the $n+1 \times m+1$ cost matrix.
- **begin**
-     **for** $i = 0$ **to** n **do** $C[i, 0] = i$
-     **for** $j = 0$ **to** m **do** $C[0, j] = j$
-     **for** $i = 1$ **to** n **do**
-         **for** $j = 1$ **to** m **do**
-             $x = C[i-1, j] + 1$
-             $y = C[i, j-1] + 1$
-             **if** $a_i = b_i$ **then** $z = C[i-1, j-1]$
-                 **else** $z = C[i-1, j-1] + 1$
-         $C[i, j] = min(x, y, z)$
- **end**

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- **Exercise.** Think about how you would keep track of each edit step.

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- there are several possible legal solutions,
- there is a value (usually numerical) associated with each solution, and
- we want to find a legal solution with the minimum or maximum possible value, depending on the context of the problem.

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- The ith job has start time s_i and finish time f_i.
- **Goal.** Find a maximum set of nonoverlapping jobs,
- where job i and job j are considered to be nonoverlapping iff $[s_i, f_i) \cap [s_j, f_j) = \varnothing$.
- Alternatively, jobs i and j are nonoverlapping as long as either $s_i \geq f_j$ or $s_j \geq f_i$.

Artist's Interpretation of the Scheduling Problem

- Here are three jobs with start and finish times, respectively, s_1, f_1, s_2, f_2, and s_3, f_3.

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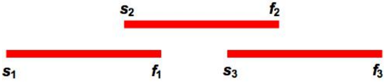
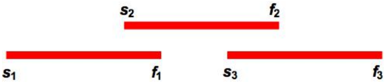
- Here's a slight tweak so you can see all three jobs.

- 

Scheduling, continued

- 

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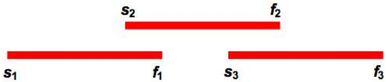
- 

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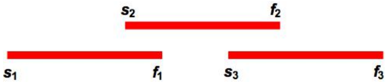
- 

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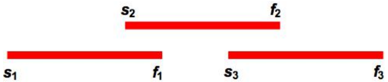
- 

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- Next, an algorithm to maximize the number of non-overlapping jobs...

Algorithm GreedySched(S)

- **Input:** $S = \{(s_1, f_1), (s_2, f_2), \ldots, (s_n, f_n)\}$.
- **Output:** A largest set of non-overlapping jobs.
- Sort jobs by finish time: $f_1 \leq f_2 \leq \cdots \leq f_n$
- $A = \varnothing$
- **for** $i = 1$ **to** n **do**
- **if** job i does not overlap any job in A
- **then** $A = A \cup \{i\}$
- **return** A

Proof that Algorithm GreedySched is correct

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- Theorem

  The algorithm GreedySched produces an optimal set of nonoverlapping jobs. In particular, $|G(S)| = |OPT(S)|$ where $OPT(S)$ is some largest set of nonoverlapping jobs in S.

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- So it remains to show that $|OPT(S)| \leq |G(S)|$.
- To that end, suppose BWOC that $|OPT(S)| > |G(S)|$.

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- **Set-up.** Choose $OPT(S)$ to be an optimal set of jobs that agrees with $G(S)$ on the largest possible number of jobs.

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- **Set-up.** Choose $OPT(S)$ to be an optimal set of jobs that agrees with $G(S)$ on the largest possible number of jobs.
- Since $G(S)$ and $OPT(S)$ are different sets, there exists a job i_0 that is not in both of the sets $G(S)$ and $OPT(S)$.

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- Suppose BWOC that $|OPT(S)| > |G(S)|$.

- **Set-up.** Choose $OPT(S)$ to be an optimal set of jobs that agrees with $G(S)$ on the largest possible number of jobs.

- Since $G(S)$ and $OPT(S)$ are different sets, there exists a job i_0 that is not in both of the sets $G(S)$ and $OPT(S)$.

- Assume i_0 is the first job on which $G(S)$ and $OPT(S)$ disagree.

Proof of correctness, continued

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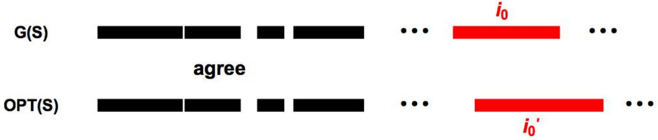

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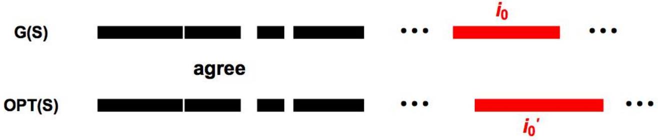
- 

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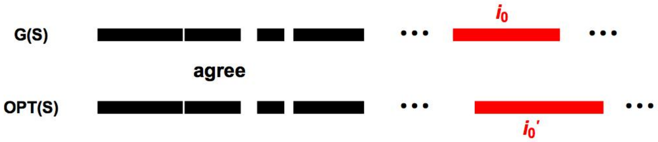

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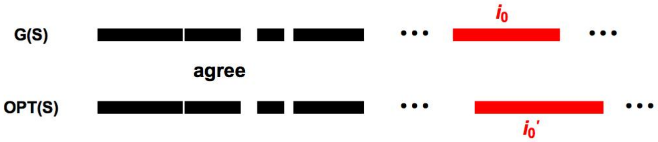
- 

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- In that case, we simply replace i_0' with i_0 in $OPT(S)$ thus producing a new $OPT(S)$ that agrees with one more job in $G(S)$, a contradiction.

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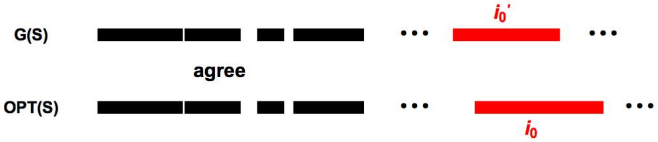

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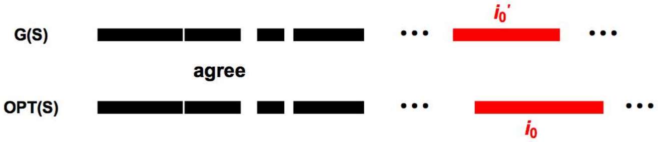
  - 

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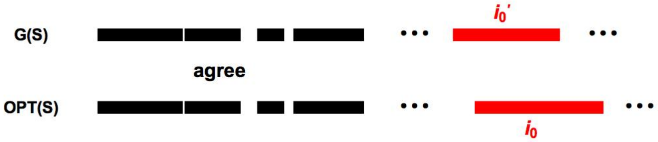

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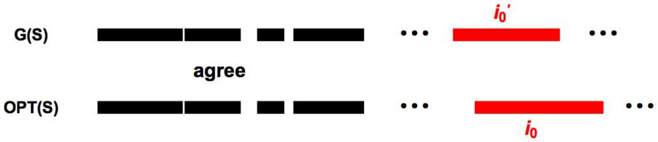

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Proof of Correctness of GreedySched concluded

- We have shown that $|G(S)| = |OPT(S)|$ and thus the algorithm GreedySched(S) produces a set of nonoverlapping jobs that is equal in cardinality to a largest set of nonoverlapping jobs.

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- **Exercise:** what if you want to maximize the sum of the lengths of the nonoverlapping jobs? Can you think of a dynamic programming solution?

Next

Data Compression