SE499 Report — Path Following Controllers

Rollen S. D'Souza
Software Engineering Undergraduate
Department of Electrical & Computer Engineering
`rs2dsouz@edu.uwaterloo.ca`

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Rollen S. D'Souza
Software Engineering Undergraduate
University of Waterloo
`rs2dsouz@edu.uwaterloo.ca`

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Nomenclature

$\mathcal{T}$	Mobile robot task space.
$\mathcal{T}_{(w,h)}$	Rectangular mobile robot task space with width w and height h.
$\mathbf{M}$	A matrix. Can be assumed real unless stated otherwise.
$\mathbf{M}_{(i,j)}$	The value at the i-th row and j-th column of matrix $\mathbf{M}$. The variables may have a : substituted to indicate selection of all values in that dimension, similar to that found in the Matlab grammar.
$\mathbf{x}$	A vector in $\mathbb{R}$.
$\mathbf{x}_i$	The i-th component of vector $\mathbf{x}$

Chapter 1

Introduction

1.1 Background

A common control objective for mobile robots involves tracking a trajectory in the space the robot operates in. This space is defined as the task space of the robot, denoted as $\mathcal{T}$. Often the structure of this space is unknown — along with the goal — and the robot must first explore the world using an exploration and mapping algorithm before deciding on a goal and path. This paper is instead concerned with a restricted subset of this problem.

Consider a differential drive robot starting at location $(0, 0)$, placed in a world with an unknown number of obstacles, that is given the objective to reach position (x, y) in task space without colliding with obstacles. A two-pronged approach can be taken to safe-guard from collisions:

1. Plan a path that does not intersect with any obstacles for all future time.

2. Design a controller that ensures sufficiently fast convergence to the path and provides a guarantee, under reasonable assumptions, that the robot will not leave the path.

Section 2.2 covers the techniques used in planning a safe path for all future time. In implementation, planning generally precedes control however in this report the preceding section, Section 2.1, concerns itself with controller design. This out-of-order content placement is intended to give the reader a better intuition for the decisions made at planning stage. With this framework in mind, the problem may be stated formally.

1.2 Notation

1.3 Problem Statement

Let the task space, without loss of generality, be the rectangular space $\mathcal{T}_{(w,h)} = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq w \wedge 0 \leq y \leq h\}$. For simplicity, consider the kinematic model of a differential drive robot with the combined position and orientation state vector $\mathbf{x} = (x, y, \theta)^\intercal$. The robot has a forward velocity $v \in \mathbb{R}$, $v > 0$ and a controllable turning rate $u : (\mathbf{x}, t) \to \mathbb{R}$. The kinematic dynamics follow as,

$$\dot{\mathbf{x}} = \begin{bmatrix} v\ cos(\mathbf{x}_3) \\ v\ sin(\mathbf{x}_3) \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \tag{1.1}$$

Chapter 2

Approach Description

2.1 Control

The most basic of controllers is the

2.2 Planning

Chapter 3